Bayesian Data Analysis

Dr Niamh Cahill (she/her)

Model Checking

Posterior Predictive Checking

The idea behind posterior predictive checking is simple: if a model is a good fit then we should be able to use it to generate data that looks a lot like the data we observed.

To generate the data used for posterior predictive checks (PPCs) we simulate from the posterior predictive distribution.

For each draw (simulation) $s = 1, \ldots, S$ of the parameters from the posterior distribution, we draw an entire vector of N outcomes $\tilde{y}^{(s)}$ from the posterior predictive distribution by simulation from the data model, conditional on parameters.

The result is an $S \times N$ matrix of draws $\tilde{y}$

When simulating from the posterior predictive distribution using the same values of the predictors X that we used when fitting the model we denote the simulations y^{rep}.

When predicting new or future observations we denote the simulations $\tilde{y}$.

We will use the `bayesplot` package to create various graphical displays for posterior predictive checks (PPCs).

Posterior Predictive Checking

Let's consider again the model for Kid IQ

$y_i \sim N(\mu_i, \sigma)$

$$\mu_i = \underbrace{\alpha_{j[i]} + \beta_{j[i]}}_{\text{j[i] = HS for obs i}} (x_i - mean(x_i))$$

$\alpha_j \sim N(100, 30^2)$, for $j = 1, 2$

$\beta_j \sim N(0, 2^2)$, for $j = 1, 2$

We can generate "replicates" of this dataset (y^{rep}) from the specified model

- draw $\mu_i^{(s)}$ and $\sigma^{(s)}$ from the posterior $p(\mu, \sigma | y)$
- draw $y_i^{rep(s)}$ from $N(\mu_i^{(s)}, \sigma^{(s)})$

JAGS model to include y^{rep}

```
lrmodel2 ="
model{
    for (i in 1:n){
        y.i[i] ~ dnorm(mu.i[i], sigma^-2)
        mu.i[i] <- alpha.j[hs_index[i]] + beta.j[hs_index[i]]*(x.i[i] - mean(x.i))
    }

#Priors
for(j in 1:m)
{
alpha.j[j] ~ dnorm(80, 30^-2)
beta.j[j] ~ dnorm(0, 2^-2)
}
sigma ~ dt(30,10^-2,1)T(0,) #truncated t-distribution

## predictive distribution
for (i in 1:n) {yrep[i] ~ dnorm(mu.i[i], sigma^-2)}
}
"
```

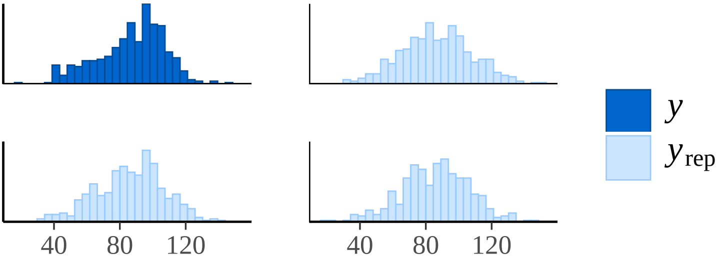
Posterior predictive checking

To use the PPC function from the `bayesplot` package we need the outcome values `y` and a matrix of replicates `yrep`

```
y <- kidiq$kid_score
yrep <- mod$BUGSoutput$sims.list$yrep
```

Now let's look at histograms of some of the `yrep` datasets and see how they compare to `y`

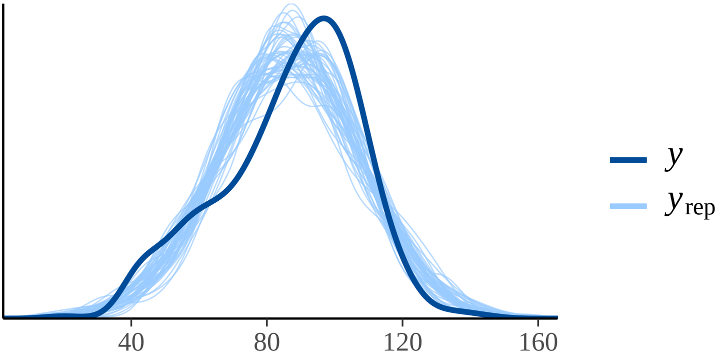
```
library(bayesplot)
color_scheme_set("brightblue")
ppc_hist(y, yrep[1:3, ])
```



Posterior predictive checking

We can also look at is a comparison of the distribution of `y` and the distributions of some of the simulated datasets in the `yrep` matrix via density plots.

```
ppc_dens_overlay(y, yrep[1:50, ])
```



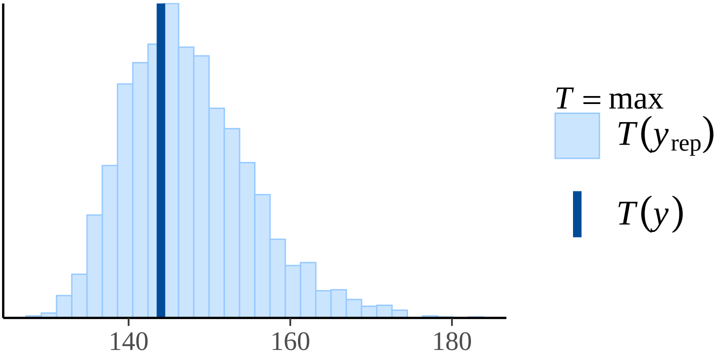
Posterior predictive checking with test statistic

Decide on a test quantity (min, max) $T(y, \theta)$. A good statistic would ideally be independent of the parameters of the model.

- obtain the summary quantity for the observed data $T(y)$
- obtain the summary quantity for the replicated data $T(y, \theta)$

Let's consider the maximum Kid IQ score as the test quantity

```
max_y <- max(kidiq$kid_score)
ppc_stat(y,yrep, stat = "max")
```

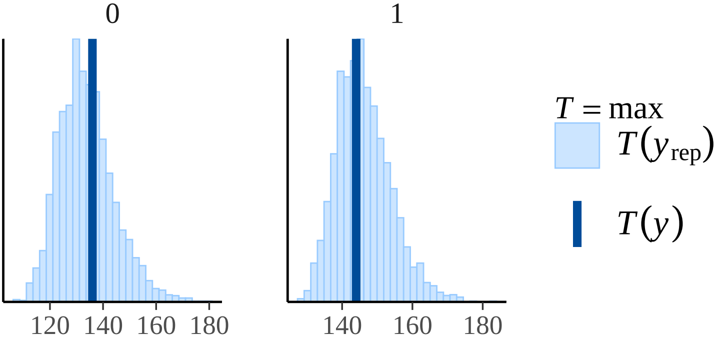


Posterior predictive checking with test statistic

Many of the available PPCs can also be carried out within levels of a grouping variable.

For example we can compute the test statistic within levels of the highschool grouping variable and a separate plot is made for each level.

```
max_y <- max(kidiq$kid_score)
ppc_stat_grouped(y,yrep,group = kidiq$mom_hs,  stat = "max")
```



Sensitivity Analysis

How much do different choices in model structure and priors affect the results?

- test different models and priors
- compare the sensitivity of essential inference quantities
- extremes are more sensitive than measures of central tendency (means, medians)
- extrapolation (beyond the range of the observed predictors) is more sensitive than interpolation (within the range of the observed predictors)

Prior predictive checks

Prior predictive checks allow us to look at the situation prior to any data being observed in order to see what data distribution is being implied by the choice of priors and likelihood. Ideally this distribution would have at least some mass around all plausible data sets.

JAGS has a simulation functionality that allows us to easily generate prior predictive distributions based on our specified model. The likelihood and priors can be specified in a "data" block so that we can simulate what data generated from the likelihood would look like, based solely on our prior assumptions.

```
lrmodel_prior ="
data{
    for (i in 1:n){
      yrep_prior[i] ~ dnorm(mu.i[i], sigma^-2)
        mu.i[i] <- alpha.j[hs_index[i]] + beta.j[hs_index[i]]*(x.i[i] - mean(x.i))
    }
#Priors
for(j in 1:m)
{
alpha.j[j] ~ dnorm(80, 30^-2)
beta.j[j] ~ dnorm(0, 2^-2)
}
sigma ~ dt(30,10^-2,1)T(0,) #truncated t-distribution
} # end data block
model{
fake <- 0
}
"
```

Prior predictive checks

We can use the `run.jags()` function from the `runjags` package to generate the simulations.

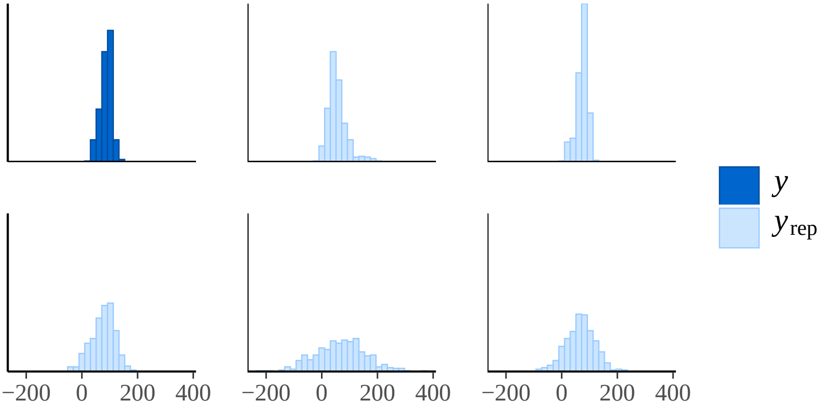
```
jags.data <- list(x.i = kidiq$mom_iq,
                  hs_index = as.numeric(kidiq$mom_hs + 1),
                  n = nrow(kidiq),
                  m = 2)
yrep_prior <- matrix(NA, nrow = 50, ncol = nrow(kidiq))
for(i in 1:50){
out <- runjags::run.jags(lrmodel_prior,
                         data = jags.data,monitor=c("yrep_prior"),
                         sample=1,
                         n.chains=1,
                         summarise=FALSE)

yrep_prior[i,] <- coda::as.mcmc(out)}
```

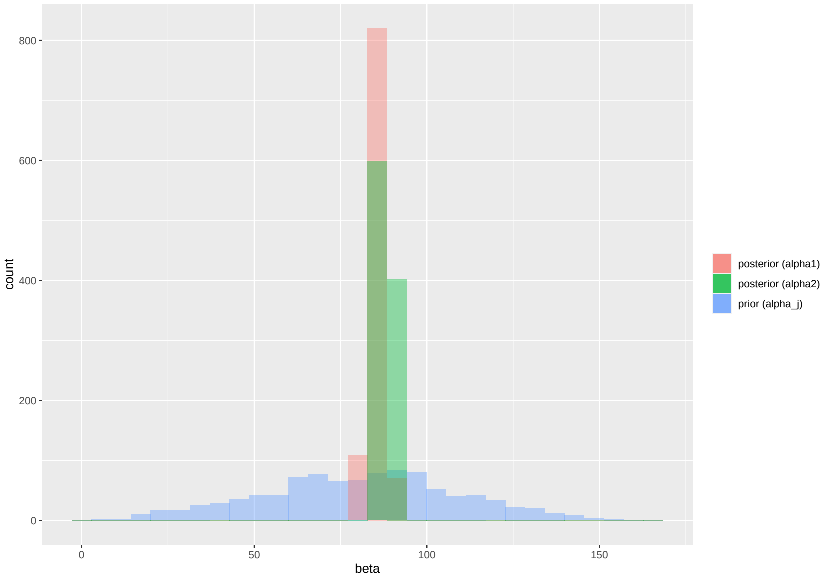
Prior predictive checks

We can then use the PPC functions from the `bayesplot` package by providing replicates `yrep_prior` instead of `y_rep`

```
ppc_hist(y, yrep_prior[1:5, ])
```



Prior vs Posterior for parameters (α_j)



Prior vs Posterior for parameters (β_j)