A BAYESIAN STATISTICAL MODEL FOR RECONSTRUCTING AND ANALYSING FORMER SEA LEVELS

33rd annual meeting of the Irish Mathematical Society

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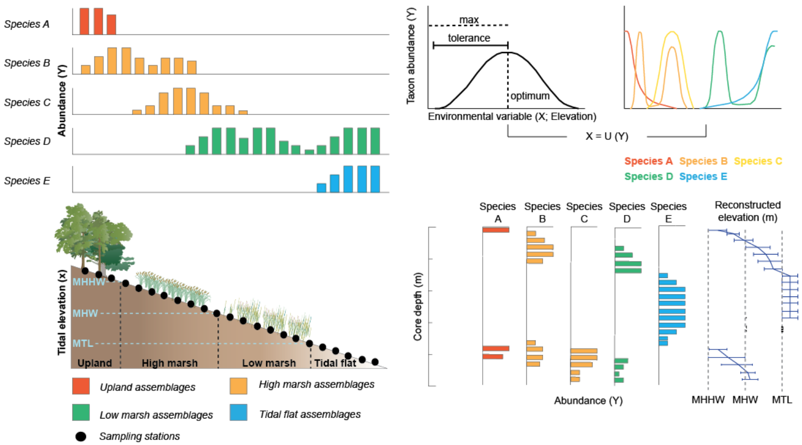
January 14th 2021

Reconstructing Relative Sea Level



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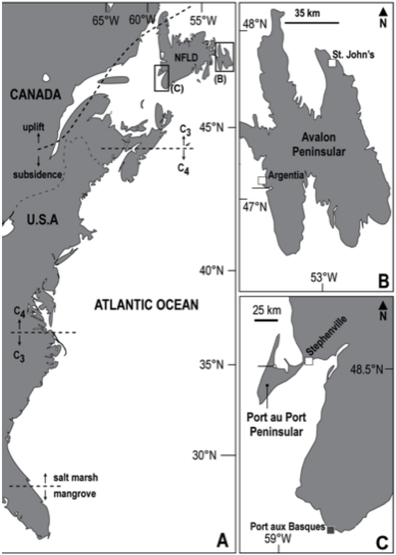
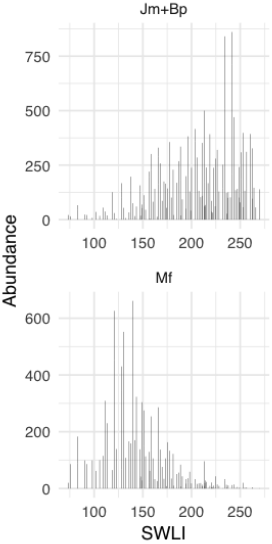
[1] Figure credit: Dr Isabel Hong

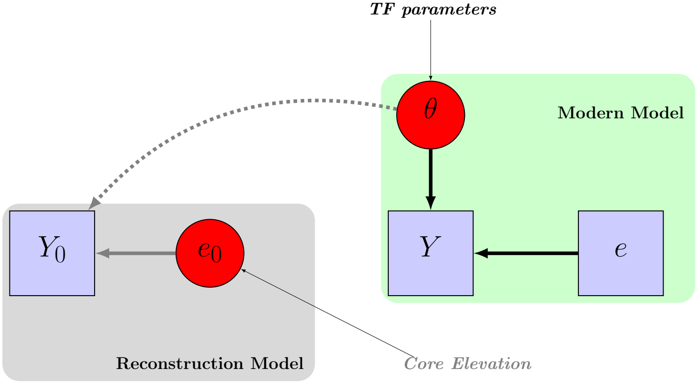
Case Study: Newfoundland



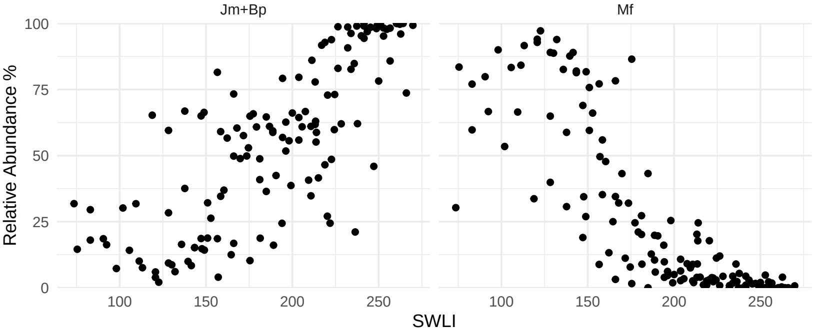


A Bayesian Transfer Function (BTF)

$$\underbrace{P(parameters|Data)}_{\text{Posterior}} \propto \underbrace{P(Data|parameters)}_{\text{Likelihood}} \underbrace{P(parameters)}_{\text{Prior}}$$



Species Response Curves



The Process Model

- For modelling puposes we consider a latent species response variable
- Each latent species variable has a functional relationship with elevation:

$$\mathbf{s}_j = f_{\theta_j}(\text{elevation}) + \epsilon_j$$

- f is a penalised-spline function and θ_j is a vector of parameters controlling the shape of the spline for species j.
- $\epsilon_{ij} \sim N(0, \sigma_j^2)$ and is added to account for overdispersion (i.e, the data here are likely to exhibit more variation than may be capturped with the data model).

The Data Model

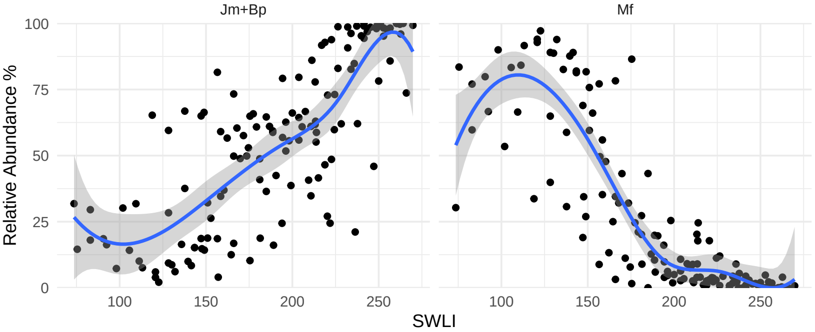
- The observed species abundances are multinomial

$$(y_{i1}, ..., y_{iM}) \sim Multi(p_{i1}, ..., p_{iM}, N_i)$$

- The probabilites of the multinomial distribution are estimated as a function of the latent species variable via a softmax transformation where

$$p_{ij} = \frac{e^{s_{ij}}}{\sum_{j=1}^{M} e^{s_{ij}}}$$

Species Response Curves



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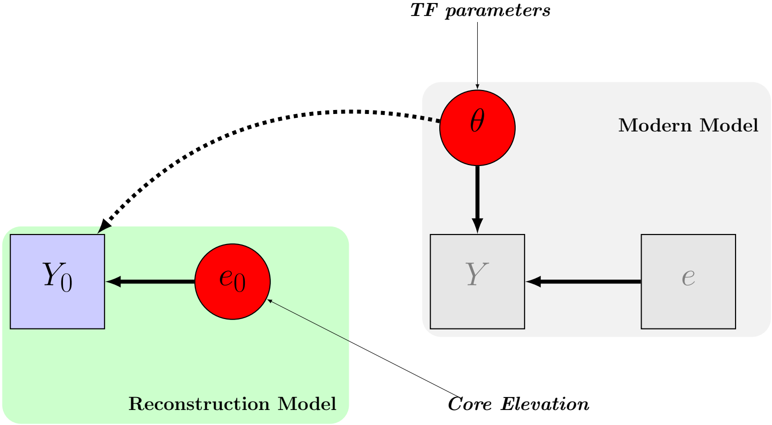
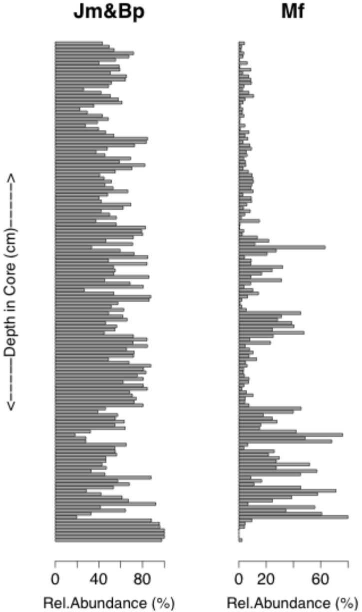
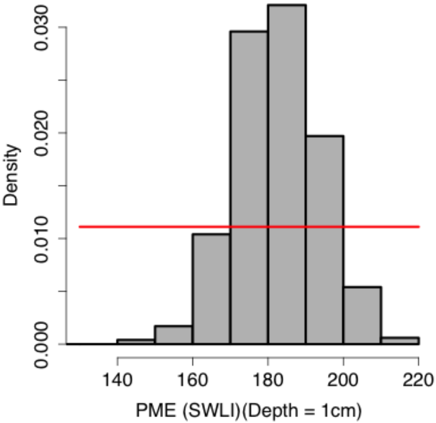


Core Results for Newfoundland





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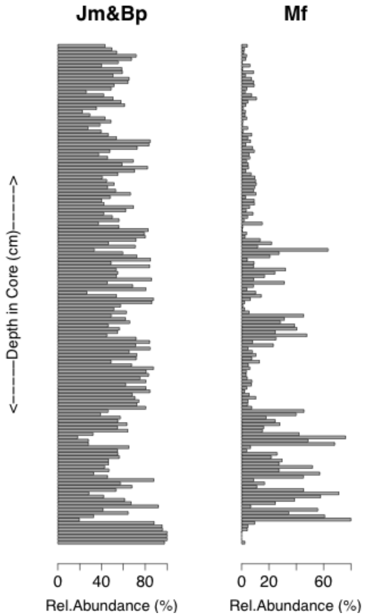
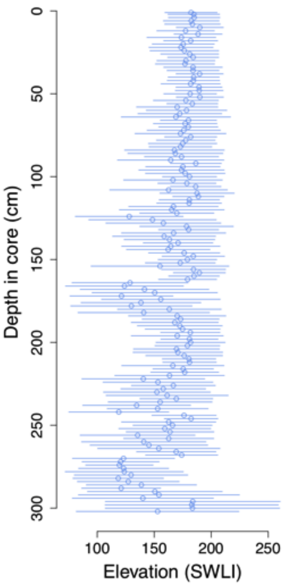
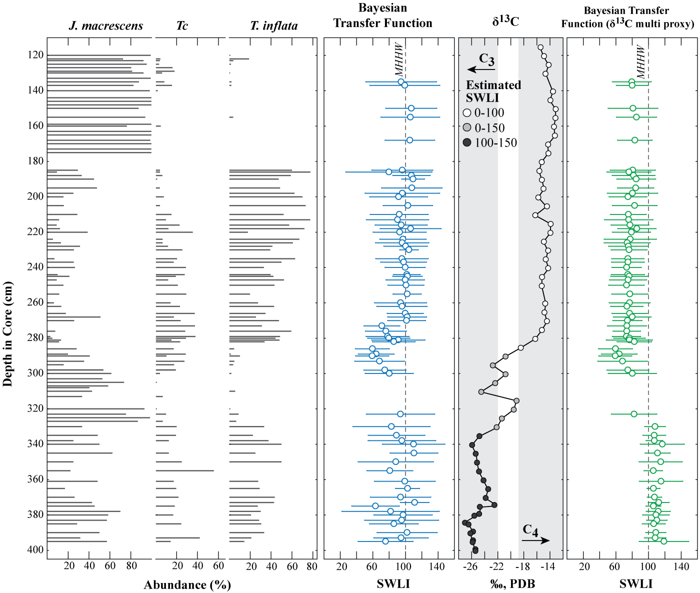




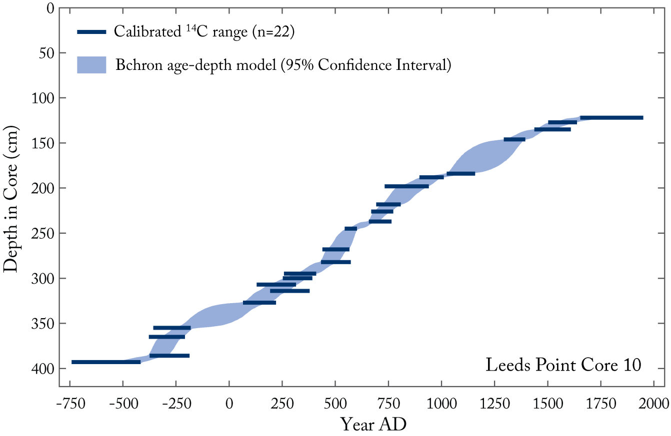
Using Secondary Proxies

A Case Study from New Jersey, USA

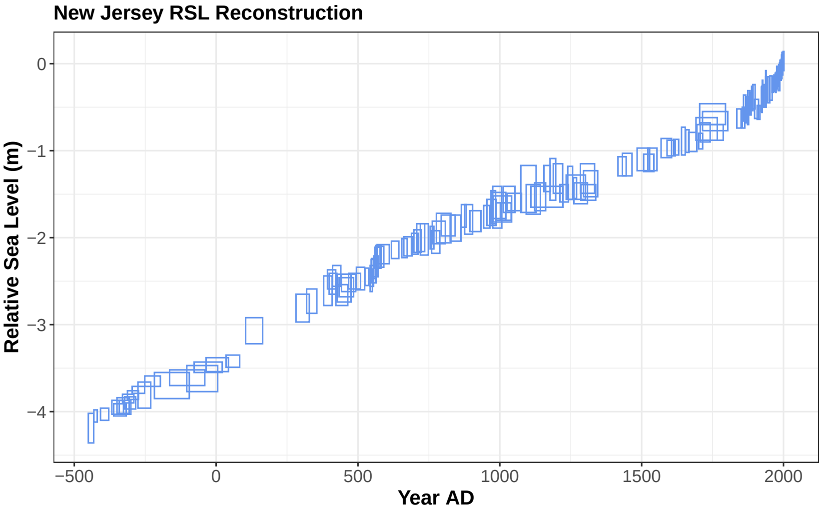
Core Results for New Jersey



Developing a Chronology for New Jersey



A RSL Reconstruction for New Jersey



Gaussian processes

Consider the multivariate normal distribution for some k-dimension random vector y

$$y \sim MVN(0, \Sigma)$$

The matrix Sigma currently looks like this:

$$\Sigma = \begin{bmatrix} \sigma^2 & 0 & \dots & 0 \\ 0 & \sigma^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma^2 \end{bmatrix}$$

- The *key idea* in Gaussian Processes is to change the off-diagonals of Σ so that the you get higher correlations between y_i and y_j when t_i and t_j are close.

Gaussian Processes cont.

- A zero-mean Gaussian process is defined as follows:

$$y \sim MVN(0, K)$$

- one commonly used option is the squared exponential covariance function:

$$K_{i,j} = \sigma_g^2 exp\Big(- \rho^2 (t_i - t_j)^2 \Big)$$

- σ_g^2 should be high for functions that cover a broad range on the y-axis
- if t_i is distant from t_j then $K_{i,j} \approx 0$
- the effect of $t_i - t_j$ on $K_{i,j}$ will depend on ρ

The Process model

- We assume a GP model for the rate of sea-level change

$$\omega(t) \sim MVN(0, K)$$

Where $K_{i,j} = \sigma_g^2 exp\Big(- \rho^2 (t_i - t_j)^2 \Big)$

- Then the sea-level process is obtained by integrating the rate process

$$s(t) = \int_0^t \omega(u) du$$

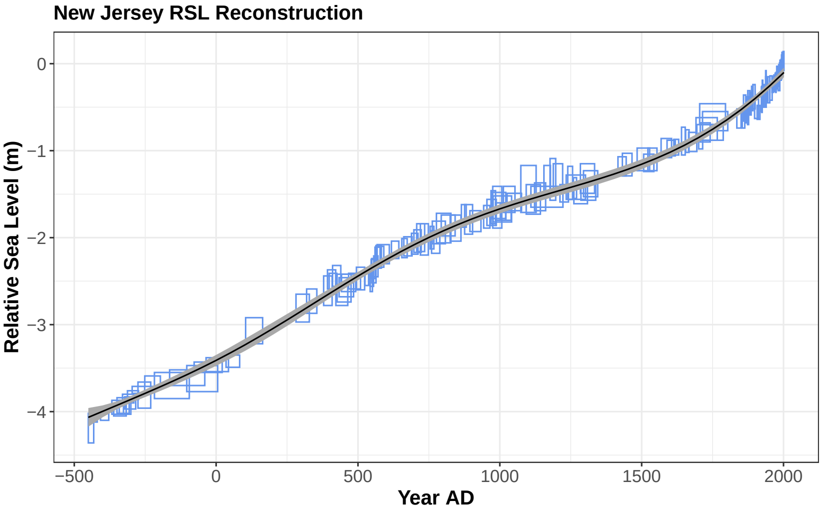
The Data Model

- The observed sea-level values are assumed to be normally distributed

$$y_i \sim N(s(t_i), \sigma_i^2)$$

where σ_i^2 will capture the variation of the observed sea-level values around the mean.

Model Results for New Jersey



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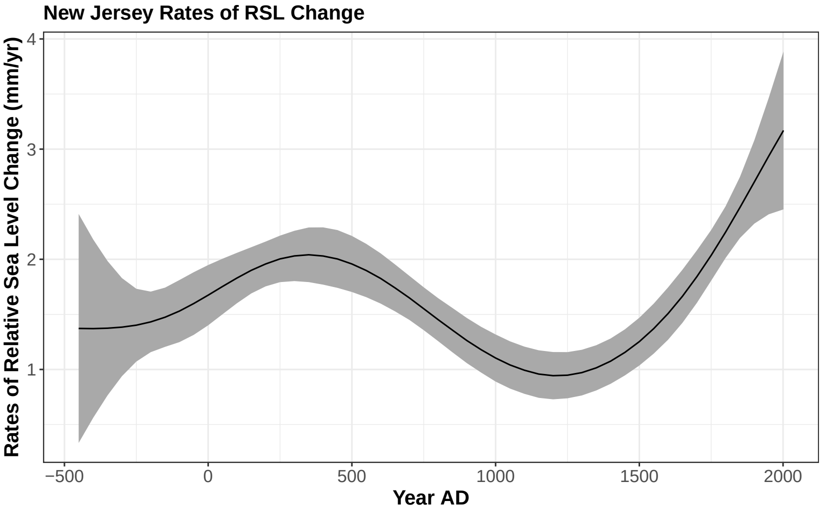

References

N Cahill, A C. Kemp, B P. Horton and A C. Parnell. A Bayesian Hierarchical Model for Reconstructing Relative Sea Level: From Raw Data to Rates of Change.Climate of the Past} , 12(2):525-542, 2016.

N Cahill, A C. Kemp, B P. Horton and A C. Parnell. Modeling sea-level change using errors-in-variables integrated Gaussian processes. Annals of Applied Statistics, 9(2): 547-571, 2015.