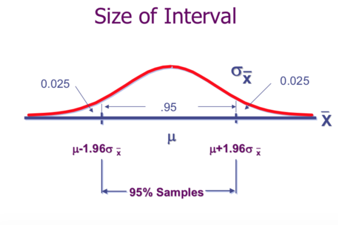
Overview

Outline:

- Two-sided Confidence Interval
- One-sided Confidence Interval

Two-sided Confidence Interval

- Based on Z-statistic: $(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}})$

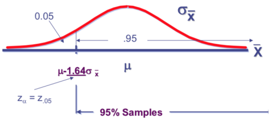


- Based on T-statistic: $(\bar{X} - t_{n-1,\alpha/2}\frac{\hat{\sigma}}{\sqrt{n}},\ \bar{X} + t_{n-1,\alpha/2}\frac{\hat{\sigma}}{\sqrt{n}})$

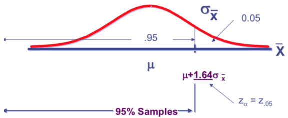
One-sided Confidence Interval

- Based on Z-statistic
  - Lower interval: $(-\infty,\ \underbrace{\bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}}}_{\text{Upper bound}})$
  - Upper interval: $(\underbrace{\bar{X} - z_\alpha \frac{\sigma}{\sqrt{n}}}_{\text{Lower bound}},\ \infty)$

Upper Interval



Lower Interval



- Based on T-statistic:
  - Lower interval: $(-\infty,\ \underbrace{\bar{X} + t_{n-1,\alpha} \frac{\hat{\sigma}}{\sqrt{n}}}_{\text{Upper bound}})$
  - Upper interval: $(\underbrace{\bar{X} - t_{n-1,\alpha} \frac{\hat{\sigma}}{\sqrt{n}}}_{\text{Lower bound}},\ \infty)$

Example

Problem: Suppose the mean of an i.i.d. sample of $n = 100$ is $\bar{x} = 50$ with sample standard deviation 10. Set up an upper 95%-CI estimate for the population mean μ.

Answer: Assume the observation $X_i \sim_{\text{i.i.d.}} N(\mu, \sigma^2)$ for all $i = 1, \ldots, 100$. Since σ is unknown, we consider the T-statistic. Note that $t_{99,0.05} = 1.66$ and $\hat{\sigma} = 10$. So the 95%-CI for μ is

$$(\bar{x} - t_{99,0.05} * \frac{\hat{\sigma}}{\sqrt{n}}, \infty) = (50 - 1.66 * \frac{10}{\sqrt{100}},\ \infty) = (48.34,\ \infty).$$