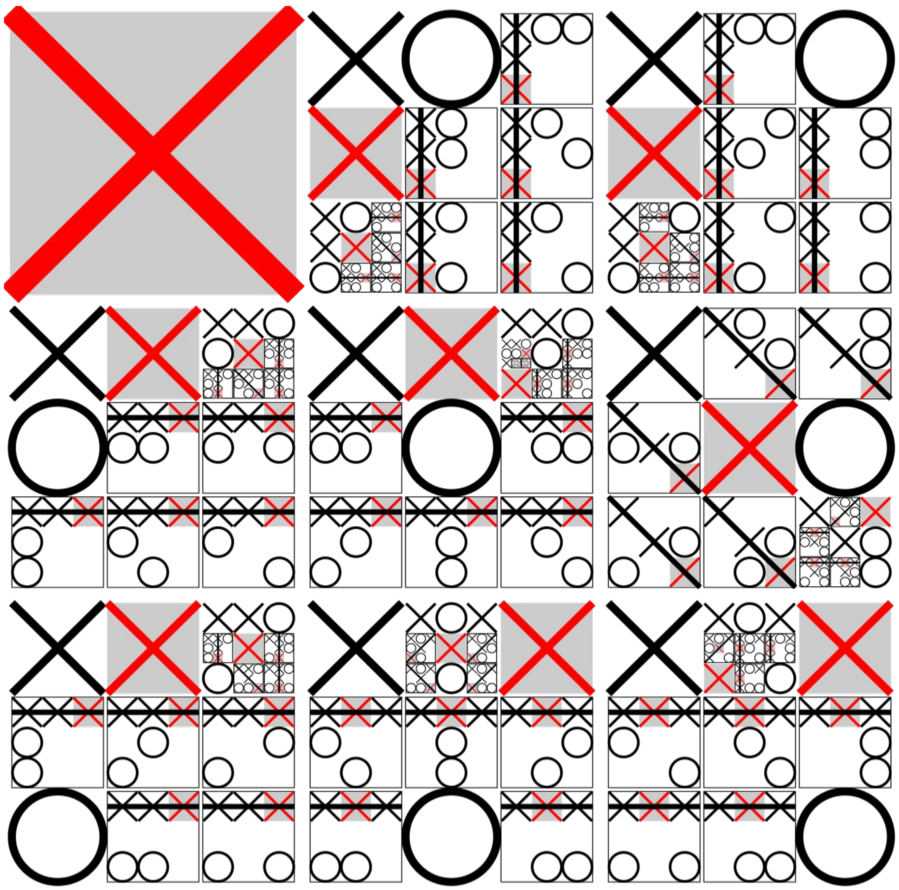
Tic-Tac-Toe Tactics



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3x3 Tic-Tac-Toe

- Rules
  - Players take turns placing an X or O in an empty grid space
  - The player who places three marks in a horizonal, vertical, or diagonal row wins
- Pedagogical value
  - Artificial intelligence
  - Game theory
  - And, according to Wikipedia, good sportsmanship
- Trivially solvable

Some Terms

- Solved game
  - A game whose outcome (win, lose, or draw) can be correctly predicted from any position when each side plays optimally.
  - Ultra-weak, weak, and strong
- Perfect/optimal play
  - The behavior or strategy of a player which leads to the best possible outcome for that player regardless of the response by the opponent.
- Perfect information
  - Player has available the same information to determine all the possible games as would be available at the end of a game (examples: chess, go, tic-tac-toe; counterexample: most card games).

3x3x3 Tic-Tac-Toe

- Also trivially solvable...
  1. First player picks center
  2. Second player picks anywhere
  3. First player takes corner for which block will not give opponent two in a row
  4. First player creates fork, wins
- “Checked” with computer - under “non-stupid” play, every initial configuration (1x26x25=650) results in a win for player 1.

Qubic (4x4x4 Tic-Tac-Toe)

- Much more interesting!

Tic-Tac-Toe Strategy

- According to Wikipedia:

1. Win: If the player has two in a row, play the third to get three in a row.
2. Block: If the [opponent] has two in a row, play the third to block them.
3. Fork: Create an opportunity where you can win in two ways.
4. Block opponent's Fork:
   - Option 1: Create two in a row to force the opponent into defending, as long as it doesn't result in them creating a fork or winning. For example, if "X" has a corner, "O" has the center, and "X" has the opposite corner as well, "O" must not play a corner in order to win. (Playing a corner in this scenario creates a fork for "X" to win.)
   - Option 2: If there is a configuration where the opponent can fork, block that fork.
5. Center: Play the center. (If it is the first move of the game, playing on a corner gives "O" more opportunities to make a mistake and may therefore be the better choice; however, it makes no difference between perfect players.)
6. Opposite corner: If the opponent is in the corner, play the opposite corner.
7. Empty corner: Play in a corner square.
8. Empty side: Play in a middle square on any of the 4 sides.

Qubic Strategy

1. First four rules the same
2. Create two in a row using one of 8 center or 8 corner positions (strongest points – contained in most winning lines)
3. Create two in a row anywhere else
4. Block opponent's two in a row in any corner/center position
5. Block opponent's two in a row anywhere else
6. Play a random corner/center position
7. Play any random position

Player vs. Computer

- Me vs. Computer
  - 0 – too many to count – 0
- Anthony vs. Computer
  - 0 – 5(ish?) – 0
- Luke vs. Computer
  - 0 – 1 – 0
- Students in my classes vs. Computer
  - 0 – 15 – 0
- In need of more volunteers.

Computer vs. Computer

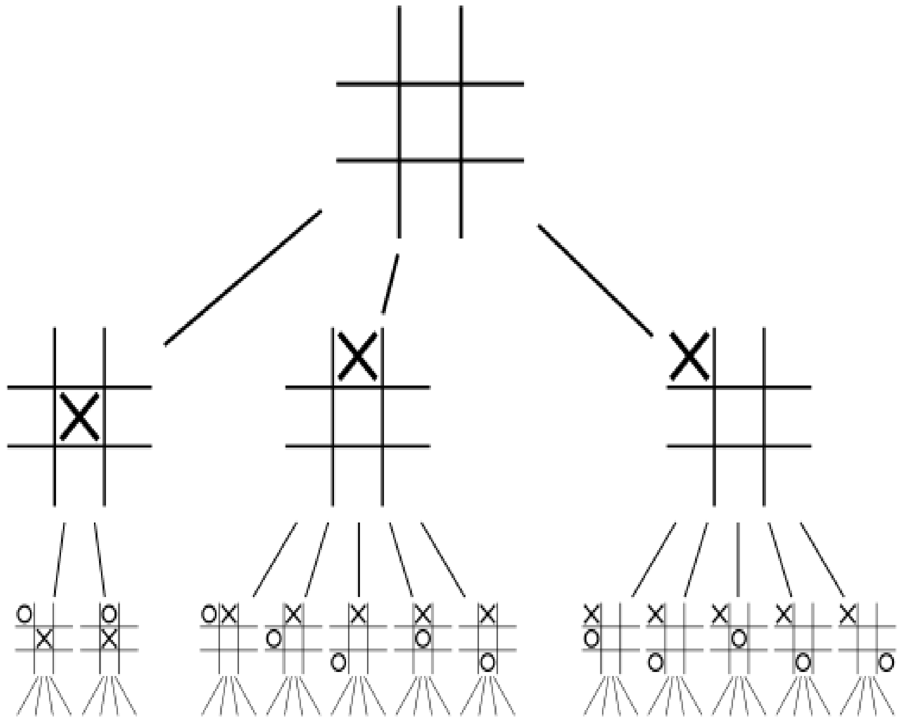
- Defense vs. Defense
  - 213 – 56 – 731
- Offense vs. Offense
  - 588 – 192 – 220
- Defense vs. Offense
  - 131 – 666 – 203
- Offense vs. Defense
  - 895 – 42 – 63
- Well-rounded vs. Offense
  - 612 – 180 – 208
- Offense vs. Well-rounded
  - 625 – 161 – 214
- Easy vs. Defense
  - 10 – 909 – 81
- Easy vs. Offense
  - 77 – 914 – 9
- Easy vs. Well-rounded
  - 61 – 924 – 15

Solving Qubic

- “Hardest” k^n game I could find that's been solved
- Problem consists of determining which “class” Qubic falls under
  - Class 1 - No draw games exist. Player 1 always wins.
  - Class 2 - Draw positions exist, but player 1 can still always win with optimal play.
  - Class 3 -Player 2 can always force a draw.
  - Player 2 can never force a win.

Brute Force?

- Game Tree
  - Unreasonable
  - Even after just 7 moves, 3x10^12 positions in tree (naïve upper limit results in 64! ~ 10^89)
  - However, many board positions are inaccessible
  - Also, symmetries (192 automorphisms, most unintuitive – proved by mathematicians)
  - Still unreasonable



Solved!

(but not by me)

- Weakly solved by Oren Patashnik [1] in 1980
  - 1500 hours of 1980 computing time
  - Relied on both human and computer input
  - Only tried to find a single distinct-position tree whose terminal positions were first-player wins
  - Employed a search based on “forced sequences”

[1] Oren Patashnik, Qubic: 4 x 4 x 4 Tic-Tac-Toe, Mathematical Magazine 53 (1980) 202216

Questions