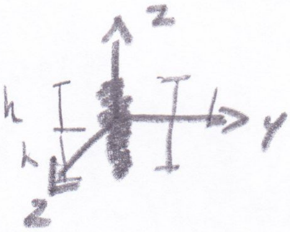
② $\tilde{I}(z) = \tilde{I}_0 e^{-j\beta z}$ for $z \in [-h, h]$



$$u_{pp} = \frac{K}{\beta} c_0$$

$$u_p = \frac{\omega}{\beta}$$

a) Show $\tilde{E}_{F\theta} = \dfrac{j\eta \tilde{I}_0 KL}{4\pi} \sin\theta \dfrac{\sin(Kh\alpha)}{Kh\alpha}$

where $\alpha = \cos\theta - \beta/K$

$$\tilde{E}_{F\phi} = 0$$

$$\bar{J} = \frac{\tilde{I}(z)}{L}\hat{z} = \frac{\tilde{I}(z)}{2h}\hat{z} = \frac{\tilde{I}_0}{2h} e^{-j\beta z}\hat{z}$$

$$f_x = f_y = 0$$

$$f_z = \int_{-h}^{h} J_z(z') e^{jK\cos\theta z'} dz'$$

$$= \int_{-h}^{h} \frac{\tilde{I}_0}{2h} e^{-j\beta z'} \cdot e^{jK\cos\theta z'} dz'$$

$$= \frac{\tilde{I}_0}{2h} \cdot \frac{j e^{-jz(\beta - K\cos\theta)}}{\beta - K\cos\theta}\Bigg|_{-h}^{h}$$

$\beta - K\cos\theta = K(\beta/K - \cos\theta)$
$= -\alpha K$

$$= \frac{-j\tilde{I}_0}{2Kh\alpha} \cdot e^{+jzK\alpha}\Big|_{-h}^{h}$$

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$$= \frac{-j\tilde{I}_0}{2Kh\alpha}\left(e^{jKh\alpha} - e^{-jKh\alpha}\right)$$

$$= \frac{\tilde{I}_0}{Kh\alpha}\left(\frac{-je^{jKh\alpha}}{2} + \frac{je^{-jKh\alpha}}{2}\right)$$

$$= \frac{\tilde{I}_0}{Kh\alpha}\sin(Kh\alpha) = f_z$$

$$f\lambda = c \qquad K = \frac{2\pi}{\lambda}$$

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\omega = 2\pi f = \frac{2\pi c}{\lambda} = Kc = \frac{K}{\sqrt{\mu\varepsilon}}$$

$$\mu\omega = K\frac{\mu}{\sqrt{\mu\varepsilon}} = K\sqrt{\mu/\varepsilon} = K\eta$$

$$\tilde{E}_{F\theta} = C_0\left(-0 - 0 + \sin\theta f_z\right)$$

$$C_0 = \frac{j\omega\mu_0}{4\pi} = \frac{j\eta K}{4\pi}$$

$$= \frac{jK\eta\tilde{I}_0}{4\pi}\sin\theta\cdot\frac{\sin(Kh\alpha)}{Kh\alpha}$$

↑

there is a missing $L = 2h$ here and I don't know why...

But I'll just add it in any ~~way~~ way

$$\tilde{E}_{F\phi} = f_x\cdot\text{Junk} + f_y\cdot\text{Junk} = 0$$

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$$\tilde{E}_{F\theta} = \frac{j\eta k L \tilde{I}_0}{4\pi} \sin\theta \frac{\sin(kh\alpha)}{kh\alpha} \qquad \alpha = \cos\theta - \beta/k$$

$$= j\frac{2hk\eta\tilde{I}_0}{4\pi} \sin\theta \frac{\sin(kh\alpha)}{\cancel{kh}\alpha} \qquad \tilde{E}_{F\phi} = 0$$

$$\boxed{= j\frac{\eta\tilde{I}_0}{2\pi} \sin\theta \frac{\sin(kh\alpha)}{\alpha}}$$

$$U = \frac{|\tilde{E}_F|^2}{2\eta} = \frac{\left| j\frac{\eta\tilde{I}_0}{2\pi} \sin\theta \frac{\sin(kh\alpha)}{\alpha}\right|^2}{2\eta}$$

$$= \frac{\eta^2|\tilde{I}_0|^2}{4\pi^2 \cdot 2\eta} \sin^2\theta \cdot \frac{\sin^2(kh\alpha)}{\alpha^2}$$

$$\boxed{= \frac{\eta|\tilde{I}_0|^2}{8\pi^2} \sin^2\theta \frac{\sin^2(kh\alpha)}{\alpha^2}}$$

$$D = \frac{4\pi U}{P_{rad}} \qquad P_{rad} = \frac{\eta|\tilde{I}_0|^2}{4\pi}\left(1.415 + \ln\left(\frac{2kh}{\pi}\right) - C_i(4kh) + \frac{\sin(4kh)}{4kh}\right)$$

$$D = \frac{4\pi \cdot \frac{\cancel{\eta|\tilde{I}_0|^2}}{8\cancel{\pi^2}} \cdot \sin^2\theta \cdot \frac{\sin^2(kh\alpha)}{\alpha^2}}{\frac{\cancel{\eta|\tilde{I}_0|^2}}{\cancel{4\pi}} \cdot \left(1.415 + \ln\left(\frac{2kh}{\pi}\right) - C_i(4kh) + \frac{\sin(4kh)}{4kh}\right)}$$

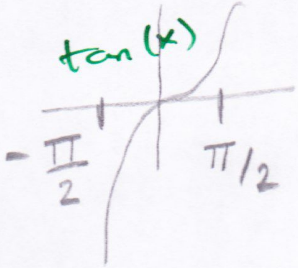
$$\boxed{= \frac{2\sin^2\theta \cdot \frac{\sin^2(kh\alpha)}{\alpha^2}}{1.415 + \ln\left(\frac{2kh}{\pi}\right) - C_i(4kh) + \frac{\sin(4kh)}{4kh}}}$$

$$\int_{-\infty}^{t} f(x)\,dx = \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

$$\int_{a}^{b} f(x)\,dx = F(b) - F(a)$$

$$\int_{0}^{\pi} u_0\,d\theta = \underbrace{\int_{-\infty}^{\pi} u_0\,d\theta}_{} - \underbrace{\int_{-\infty}^{0} u_0\,d\theta}_{}$$

$$\int_{-\infty}^{\infty} \sin^2\theta\,\frac{\sin^2(kh\alpha)}{\alpha^2}\,d\theta$$ ← try to use Parseval's thm for this



$\theta = 0 \rightarrow x \rightarrow -\infty$

$\theta = \pi \rightarrow x \rightarrow \infty$

$$x = \tan\left(\theta - \frac{\pi}{2}\right) \qquad dx = \csc^2\theta\,d\theta$$

$$\int_{0}^{\pi} \sin^2\theta\,\frac{\sin^2\left(kh\left(\cos\theta - \beta/k\right)\right)}{\left(\cos\theta - \beta/k\right)^2}\,d\theta$$

$$\Theta = \tan^{-1}(x) + \frac{\pi}{2}$$

$$d\Theta = \sin^2\Theta \, dx = \sin^2\left(\tan^{-1}(x) + \frac{\pi}{2}\right) dx$$

$$= \frac{1}{x^2+1} dx$$

$$\int_0^{\pi} \sin^2\Theta \frac{\sin^2(kh(\cos\Theta - \beta/k))}{(\cos\Theta - \beta/k)^2} d\Theta = \int_{-\infty}^{\infty} \sin^2\left(\tan^{-1}(x) + \frac{\pi}{2}\right) \times \frac{\sin^2\left(kh\left(\cos\left(\tan^{-1}(x) + \frac{\pi}{2}\right) - \beta/k\right)\right)}{\left(\cos\left(\tan^{-1}(x) + \pi/2\right) - \beta/k\right)^2} \times \frac{1}{x^2+1} dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} \times \frac{\sin^2\left(\frac{-khx - \beta h}{\sqrt{x^2+1}}\right)}{\left(\frac{-x}{\sqrt{x^2+1}} - \beta/k\right)^2} dx$$

$$\cos\left(\tan^{-1}(x) + \frac{\pi}{2}\right) = \frac{-x}{\sqrt{x^2+1}}$$

$$= \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{-khx - \beta h}{\sqrt{x^2+1}}\right)}{(x^2+1)^2\left(\frac{x}{\sqrt{x^2+1}} + \beta/k\right)^2} dx$$

$$u^2 = x^2 + 1$$

$$2u\,du = 2x\,dx$$

$$\frac{u\,du}{x} = dx = \frac{u}{\sqrt{u^2-1}} du$$

$$= \int_{-\infty}^{\infty} \frac{\sin^2\left(\frac{-khx - \beta h}{\sqrt{x^2+1}}\right)}{(x^2+1)\left(\beta/k\sqrt{x^2+1} + x\right)^2} dx$$

$$= \int_{-\infty}^{\infty} \left(\underbrace{\frac{\sin\left(\frac{khx + \beta h}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\left(\beta/k\sqrt{x^2+1} + x\right)}}_{f(x)}\right)^2 dx = \int_{-\infty}^{\infty} |F(f)|^2 df$$