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Faculty of Engineering

School of Electrical Engineering
and Computer Science

ELG 6367 – Fundamentals of Antenna Engineering

Assignment#2 Winter 2024

Hand-Out Date : 12th February 2024
Due Date : 28th February 2024

Your assignment report must be uploaded on Brightspace as a single PDF document before or on 28th February 2024.

If you wish to meet with me to discuss this assignment you should make an appointment with me, but only after you have attempted it yourself. Bring preliminary results with you to such a meeting.

Please attach a copy of this page to your solution, with your details filled in, and the declaration signed and dated.

Lastname: ……………………………………..

First Name : …………………………………...

Student Number : ……………………………..

*I hereby confirm that this is entirely **my own work**. I did not discuss my solution with anyone else other than the ELG 6367 instructor, and I did not copy it from someone else's solution.*

Signature : ………………………………….

Date : ……………………………………….

Problem 2.1

The far-zone radiation patterns of any antenna can be expressed in the form (2.4-5) and (2.4-6) of Chapter 2. A number of classes of conical horn antenna can be modelled as having

$$F_\theta(\theta,\phi) = C_e(\theta)\cos\phi \tag{1}$$

and

$$F_\phi(\theta,\phi) = C_h(\theta)\sin\phi \tag{2}$$

Note that (1) and (2) have the form of the product of a function dependent on θ only, and one that depends on ϕ only. We cannot assume this to be so for antennas in general.

Fig.2.1-1 : Smooth-walled conical horn antennas. Photo courtesy of MDA-Space, Canada.

(a). A widely-used model (the so-called "raised-cosine model") for conical horns uses $C_e(\theta) = (\cos\theta)^{q_e}$ and $C_h(\theta) = (\cos\theta)^{q_h}$, over the angular section $0 \le \theta \le \pi/2$, and zero over $\pi/2 < \theta \le \pi$ (that is, elsewhere). The quantities q_e and q_h need not be integers, and can be adjusted to alter the beamwidths of the antenna being modelled[1]. Derive an expression for the directivity $D(\theta,\phi)$ for the conical horn antenna when this raised-cosine model applies. In what direction is $D(\theta,\phi)$ a maximum? Give an expression for this maximum value in terms of q_e and q_h. In light of what we have said in Chapter 2 about the relation between beamwidth and directivity, does your result make sense?

[1] They do not signify electric charge!☺

(b). Suppose we wish to express $\bar{F}(\theta,\phi)$ as in (2.5-18) of Chapter 2 with the particular basis vectors[2]

$$\hat{u}_{co}(\theta,\phi) = \sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi} \tag{3}$$

and

$$\hat{u}_{cr}(\theta,\phi) = \cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi} \tag{4}$$

Show that these definitions satisfy the orthogonality requirements of polarization basis vectors.

(c). Using the basis vectors (3) and (4), write down expressions for $F_{co}(\theta,\phi)$ and $F_{cr}(\theta,\phi)$ when $F_{\theta}(\theta,\phi)$ and $F_{\phi}(\theta,\phi)$ are[3] as in (1) and (2). Simplify your expressions.

(d). Examine the expressions for $F_{co}(\theta,\phi)$ and $F_{cr}(\theta,\phi)$ form (c) when $F_{\theta}(\theta,\phi)$ and $F_{\phi}(\theta,\phi)$ are indeed the raised-cosine forms, and plot (in dB) the normalized forms[4] $F_{co}(\theta,\phi)/|F_{co}(0,\phi)|$ and $F_{cr}(\theta,\phi)/|F_{co}(0,\phi)|$. Plot in the planes $\phi = 0^{\circ}$ and $\phi = 90^{\circ}$ (the two principal or cardinal planes) and the $\phi = 45^{\circ}$ plane (a diagonal plane, one of the so-called inter-cardinal planes). Do this for the case $q_e = 3.2$ and $q_h = 2.16$.

(e). If the walls of the horn are corrugated[5], it is possible to obtain $C_e(\theta) = C_h(\theta)$, so that both can be denoted by $C(\theta)$. What do your expressions for $F_{co}(\theta,\phi)$ and $F_{cr}(\theta,\phi)$ from (c) tell you about the polarization of a well-designed corrugated horn antenna.

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[2] The so-called "Ludwig III" choice.
[3] Do not assume the raised-cosine forms for $C_e(\theta)$ and $C_h(\theta)$.
[4] Observe that $|F_{co}(0,\phi)|$ is used to normalize both plots.
[5] P.J.B.Clarricoats & A.D.Olver, *Corrugated Horns for Microwave Antennas* (IEE Press, 1984).

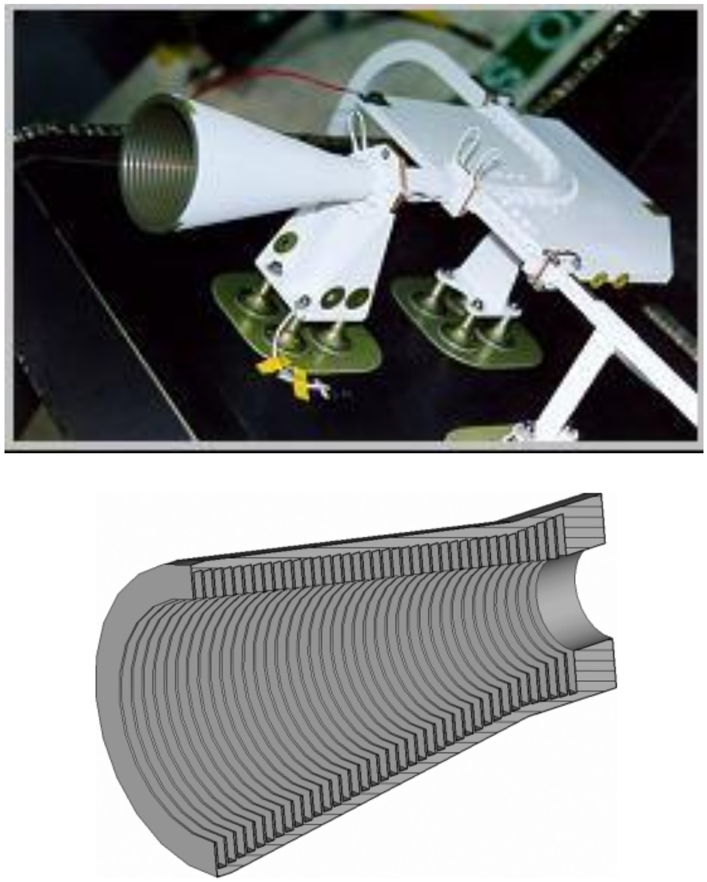

Fig. 2.1-2 : Corrugated conical corrugated horn (top) courtesy of MDA-Space, Canada. Drawing (bottom) from source whose details have unfortunately been lost.

Problem 2.2

A theoretical construct known as a Huygens source consists of spatially orthogonal infinitesimal electric and magnetic dipole sources, as depicted in Fig.2.2-1, that have their dipole moments adjusted so that $C_e / C_m = \eta$. Huygens sources are idealized sources, but have certain properties that are favourable in many real-world antenna applications. In such applications engineers therefore develop elements that attempt to approximate Huygens sources. These are of course of finite size and not infinitesimal, but are usually electrically very small[6]. It is for this reason that you might often see such elements referred to as being “sub-wavelength”; others even refer to them as “meta-atoms” (especially when several such sources

[6] Dimensions of a quarter- or half-wavelength would not be considered electrically small. If you determine the radius (say *a*) of the smallest sphere that just encloses the element, then the element would be considered electrically small if $ka < 1$ (although some insist it must be $ka < 0.5$). The *IEEE Standard for Definitions of Terms for Antennas* simply defines an electrically small antenna as one whose dimensions are such that it can be contained within a sphere whose diameter is small compared to a wavelength at the frequency of operation.

are used together) but this is an unnecessary proliferation of terminology. Approximately realized Huygens sources have been used individually[7] and as elements in an array of Huygens sources[8].

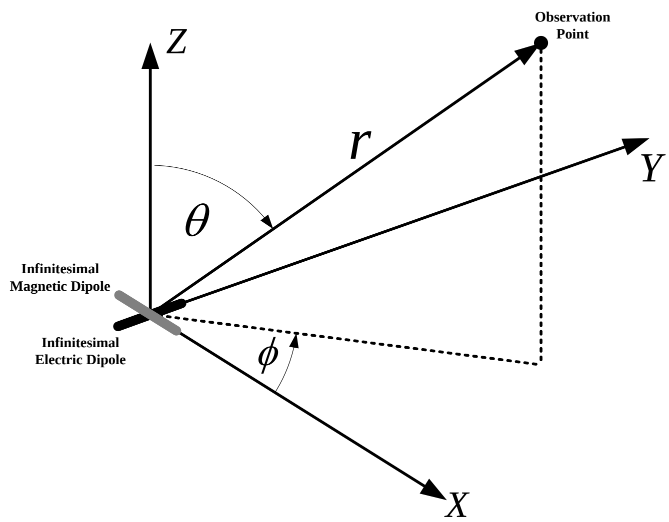

Fig.2.2-1 : Discrete infinitesimal Huygens source.

Derive an expression for $\bar{F}(\theta,\phi)$, and hence $\bar{E}(r,\theta,\phi)$, for a Huygens source with $C_m = 1$, and provide a set of pattern plots in any form you wish, but such that "the reader" is able to fully appreciate how such a source radiates. Is there a backlobe?

Hints : Express the current densities for each of the infinitesimal sources in a manner similar to that in Section 4.13.4 of Appendix 4. Remember that we are only interested in the far-zone fields, and so the far-zone forms of the source-field integrals (the "radiation integrals") in Section 4.22 of Appendix 4. If you wish you can find the field of the infinitesimal electric dipole first, and then use the duality principle to find that for the infinitesimal magnetic dipole, but this it up to you. Importantly, remember that the dipoles are not parallel to the z-axis, and so the expressions for their fields that we derived elsewhere do not apply.

Some Possibly Useful Mathematical Handbook Type Information

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \sin^3 ax\,dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a}$$

$$\int \cos^2 ax \sin^2 ax\; dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \qquad \int \cos^n ax \sin ax\; dx = -\frac{\cos^{n+1} ax}{(n+1)a} \qquad n \neq -1$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \qquad \int \sin ax\,dx = -\frac{\cos ax}{a} \qquad \int_{-b}^{b} e^{ja\xi} d\xi = 2b\ \mathrm{sinc}(ab)$$

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[7] P.Jin and R.W.Ziolkowski, "Metamaterial-inspired, electrically small Huygens sources", IEEE Antennas and Wireless Propagation Letters, Vol. 9, pp.501-505, 2010.

[8] C.Pfeiffer, and A.Grbic, "Metamaterial Huygens' surfaces: Tailoring wave fronts with reflectionless sheets," Phys. Rev. Lett., vol. 110, 197401, May 2013.