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ELG 6367 – Fundamentals of Antenna Engineering
Assignment#1 Winter 2024

1.1 A transmission line of length *d*, characteristic impedance Z_o, propagation factor γ, termination Z_L, and source with Thevenin parameters V_g and Z_g, operates at a frequency ω. Is there always a non-zero charge distribution $q(z)$ on the TL? If so, can we say that the charge distribution forms a standing wave. Explain using mathematical expressions in support of your explanation. **[10]**

1.2 Derive the transmission line equations (2.2-1) and (2.2-2) of Appendix 2 using the equivalent circuit shown in Fig.2 as the starting point, applying the KVL and KCL, and then letting $\Delta z \to 0$. **[10]**

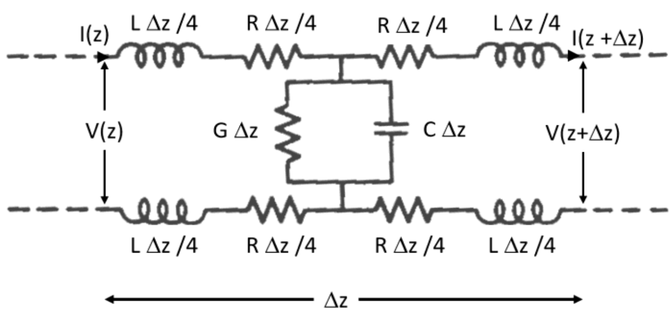

Fig.2 : Lumped equivalent circuit of an infinitesimal length of transmission line that has per unit length parameters R, L, C and G.

1.3 We are given the following fields of a circularly-polarised plane wave in free space :

$$\overline{E}(z) = E_o e^{-j\beta z}(\hat{x} - j\hat{y})$$

$$\overline{H}(z) = \eta^{-1} E_o e^{-j\beta z}(j\hat{x} + \hat{y})$$

with E_o a known real constant. The wave is clearly travelling in positive z-direction. Determine the wave impedance "looking" in the direction of propagation. **[5]**

1.4 Use Maxwell's curl equation $\nabla \times \bar{H}(\bar{r}) = j\omega\varepsilon_0 \bar{E}(\bar{r})$ to show that if, in free space, the electric and magnetic fields only have components transverse to $\hat{r}$, they are related as $\bar{H} = \eta^{-1}\hat{r} \times \bar{E}$. Furthermore, show that the wave impedance in the direction $\hat{r}$ will always be η. **[15]**

1.5 Consider a z-directed filament in Fig.3 whose current distribution is given by

$$I(z) = \begin{cases} \dfrac{I_o \sin k(z_3 - z)}{\sin kh_2} & z_2 \le z \le z_3 \\[2ex] \dfrac{I_o \sin k(z - z_1)}{\sin kh_1} & z_1 \le z \le z_2 \end{cases}$$

where I_o is a constant that in this case can be considered to be real, and is the value of the current at $z = z_2$. The "arm" lengths are $h_1 = z_2 - z_1$ and $h_2 = z_3 - z_2$. The current is seen to be zero at $z = z_1$ and $z = z_3$, which are the locations of the ends of the filament. The current is sinusoidal on each of the arms, and continuous at $z = z_2$, but has a derivative discontinuity at $z = z_2$. This is often used to **approximate** the current distribution on a straight wire dipole, with the terminals at $z = z_2$.

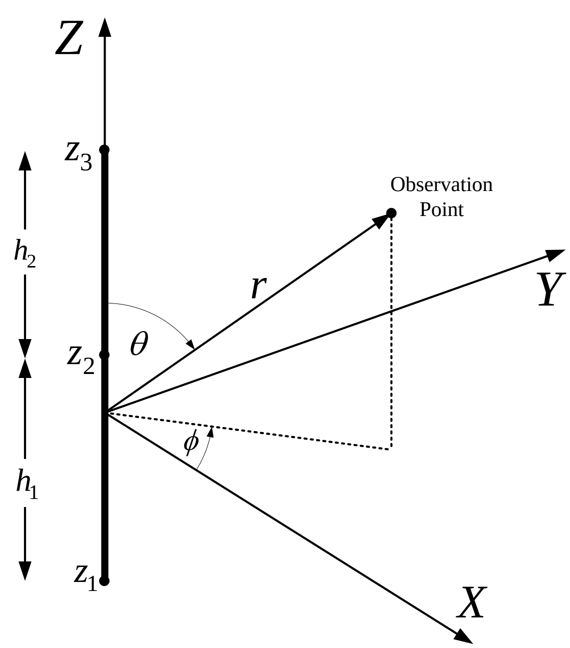

Fig.3 : Filamentary current distribution located on, and parallel to, the z-axis.

The electric field due to this filamentary current has been found to be given (in cylindrical coordinates) by the field components $E_z(\rho,z)$ and $E_\rho(\rho,z)$ provided in the frame below[1].

Case#1 : Use $z_2 = 0$, $z_1 = -h$, $z_3 = h$, and $h = \lambda/4$.

(a). Plot $I(z)$ versus z for $-h \le z \le h$. **[5]**

(b). Plot (in rectangular format) the fields $|E_\theta(r,0,0)|$ and $|E_r(r,0,0)|$, in dB, versus r for $r > 0^+$. **[15]**

(c). Plot (in polar format) the normalized fields $20\log\dfrac{|E_\theta(r,\theta,0)|}{|E_\theta(r,0,0)|}$ versus θ over the angular range for $-\pi \le \theta \le \pi$, for several increasing values of r (starting with $r = h^+$). **[15]**

Case#2 : Use $z_2 = 0$, $z_1 = -h$, $z_3 = h$, and $h = 0.625\lambda$.

(d). Repeat (a) for the length $h = 0.625\lambda$. **[2]**

(e). Repeat (c) for the length $h = 0.625\lambda$. **[5]**

Very Important : Comment on your results. Also, you must write out any mathematical results you use (in your Matlab code) in the usual mathematical symbols. Do not say "see my Matlab code for the analysis"!

$$E_z(\rho,z) = j30I_o\left\{\left(\cot kh_1 + \cot kh_2\right)\frac{e^{-jkR_2}}{R_2} - \frac{e^{-jkR_1}}{R_1\sin kh_1} - \frac{e^{-jkR_3}}{R_3\sin kh_2}\right\}$$

$$E_\rho(\rho,z) = \frac{j30I_o}{\rho}\left\{\frac{(z-z_1)e^{-jkR_1}}{R_1\sin kh_1} - (z-z_2)\left(\cot kh_1 + \cot kh_2\right)\frac{e^{-jkR_2}}{R_2} + \frac{(z-z_3)e^{-jkR_3}}{R_3\sin kh_2}\right\}$$

$$E_\phi(\rho,z) = 0$$

where

$$\rho = \sqrt{x^2+y^2} \qquad R_1 = \sqrt{x^2+y^2+(z_1-z)^2}$$

$$R_2 = \sqrt{x^2+y^2+(z_2-z)^2} \qquad R_3 = \sqrt{x^2+y^2+(z_3-z)^2}$$

[1] These apply at any distance from the filament, and not only in the far-zone.

If we have a general vector function expressed in terms of its cylindrical coordinate components by $\bar{A} = A_\rho \hat{\rho} + A_\phi \hat{\phi} + A_z \hat{z}$, then it can be expressed in terms of its spherical coordinate components as $\bar{A} = A_r \hat{r} + A_\theta \hat{\theta} + A_\theta \hat{\phi}$, with

$$A_r = A_\rho \sin\theta + A_z \cos\theta$$
$$A_\theta = A_\rho \cos\theta - A_z \sin\theta$$
$$A_\phi = A_\phi$$

and the spatial coordinates themselves are related as

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

1.6 Measurements on a transmission line of characteristic impedance $Z_0 = 75\Omega$ connected to the input port of a particular antenna reveal that the input reflection coefficient of the antenna is $0.12e^{j35^\circ}$ at a frequency of 900MHz.

(a). What is the value of the input reactance X_{in} of the antenna at the frequency in question?

(b). What percentage of the power <u>incident</u> on the antenna's input port is reflected at the antenna port?

(c). If the incident power in (b) is 12mW, how much power actually enters the antenna terminals (i.e. the input/output port)? Express your answer in dBm.

[8]