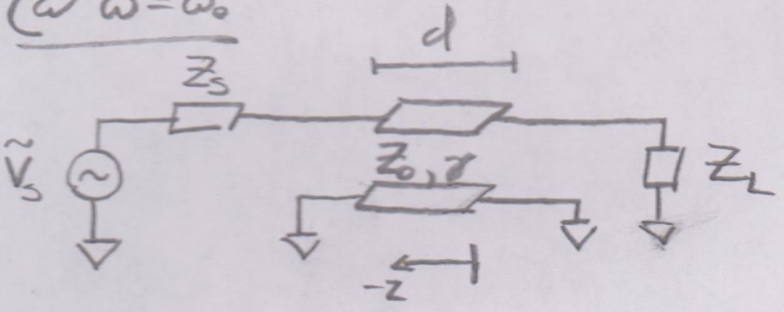
Assignment 1

Nicol Cardamone
3000 60019

$\underline{@ \omega = \omega_0}$

1)



Is there always a non zero $q(z)$ on the line? (for all z)

yes / \ no

from continuity equ'n

$$\frac{\partial q(z,t)}{\partial t} = -\frac{\partial i(z,t)}{\partial z}$$

$\Downarrow$ FT in time

$$j\omega q(z,\omega) = -\frac{\partial \tilde{I}(z,\omega)}{\partial z}$$

$$q(z,\omega) = -\frac{1}{j\omega}\frac{\partial \tilde{I}(z,\omega)}{\partial z}$$

if $\tilde{V}_s = 0$ and noise doesn't matter ($T \approx 0$)

$q(z) = 0$

Conditions for $q(z) = 0$

1) $q(z) \approx 0$ if $\omega >>>> 1$

2) $\frac{\partial \tilde{I}}{\partial z} = 0$

Conditions for $\frac{\partial \tilde{I}}{\partial z} = 0$

1) At DC / Low frequency

However, $\omega = 0$

$$g(t) = \frac{-1}{j0} \cdot 0$$

= undefined / too lazy to do l'hospital's

2) If $\tilde{V}_s = 0$

$$\tilde{I}(z,\omega) = \text{some function}(\tilde{V}_s, Z_L, Z_0, \dots)$$

that I'm too lazy to look up

when $\tilde{V}_s = 0$ there is no input voltage, so no current

$$\frac{\partial 0}{\partial z} = 0 \Rightarrow g(z) = 0$$

Since $\tilde{V}_s$ is not 0, $g(z) \neq 0$

Since $q(z) = \frac{-1}{j\omega} \frac{\partial \tilde{I}}{\partial z}$

If $\tilde{I}$ is a standing wave
it will look something
like this

$$\tilde{I}(z) = \tilde{I}_0 \cos(\beta z + \theta) \leftarrow \text{standing wave}$$

$$\frac{\partial \tilde{I}(z)}{\partial z} = \text{some other const} \cos(\beta z + \text{some other phase})$$

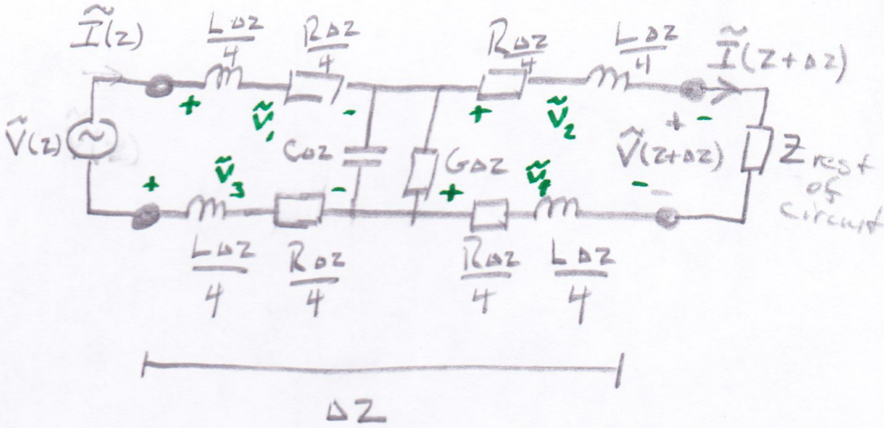
still standing wave

$\Rightarrow$ if $\tilde{I}$ is a standing wave, q will
also be a standing wave

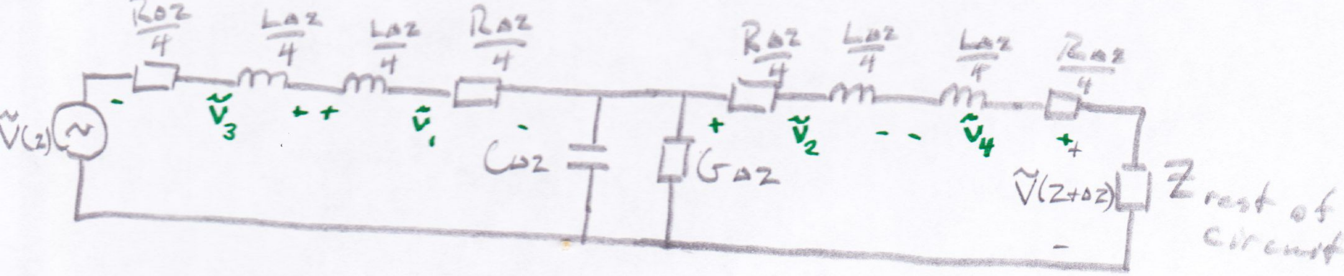
$\tilde{I}$ is a ~~of~~ standing wave when
$\tilde{V}$ is ~~$(\tilde{I} =$~~

$\tilde{V}$ is a standing wave
when $j\,\mathrm{Im}(Z_L) = Z_L$ $(|\Gamma_L| = 1)$

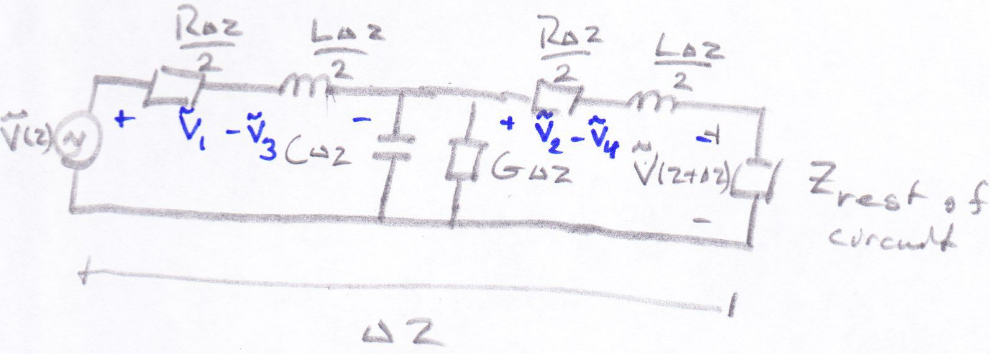
(2)



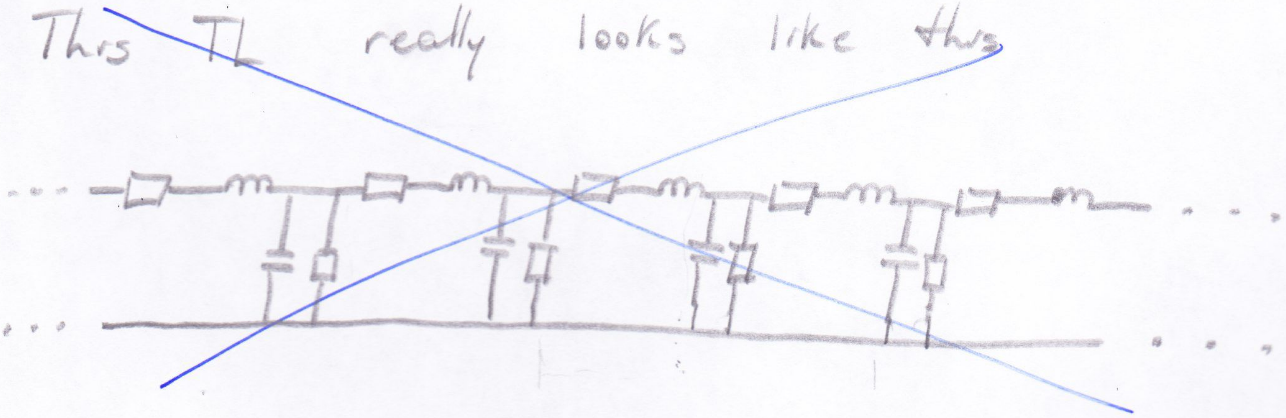
Since this is a Linear circuit, component's in Series can be moved like this



Now, let's combind elements in series

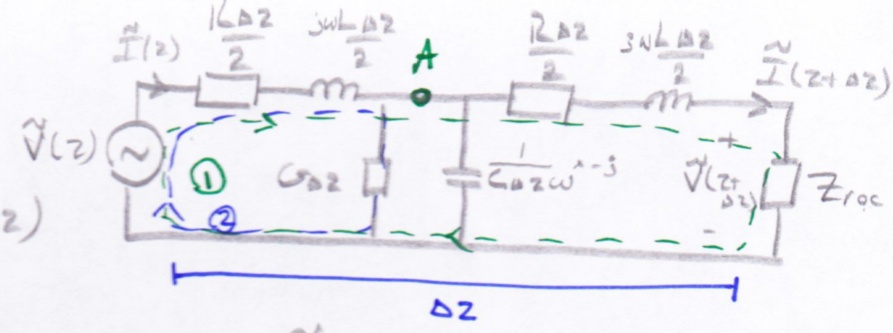


~~This TL really looks like this~~



NVM

KVL @ D



$$\tilde{V}(z) = \frac{\tilde{I}(z)}{2}(R\Delta z + j\omega L\Delta z) + \frac{\tilde{I}(z+\Delta z)}{2}(R\Delta z + j\omega L\Delta z) + \tilde{V}(z+\Delta z)$$

$$\tilde{V}(z) - \tilde{V}(z+\Delta z) = (\tilde{I}(z) + \tilde{I}(z+\Delta z))(R\Delta z + j\omega L\Delta z) \times \frac{1}{2}$$

difference quotient for $\frac{\partial \tilde{V}}{\partial z}$

$$\frac{\tilde{V}(z+\Delta z) - \tilde{V}(z)}{\Delta z} = -\frac{1}{2}(R + j\omega L)(I(z) + I(z+\Delta z)) \quad ①$$

KCL @ A

$$\tilde{I}(z) - \tilde{I}(z+\Delta z) = \frac{\tilde{V}_A}{\frac{1}{G\Delta z + j\omega C\Delta z}} = \Delta z(G + j\omega C)\tilde{V}_A$$

KVL @ ②

$$\tilde{V}_A = \tilde{V}(z) - \frac{I(z)}{2}(R\Delta z + j\omega L\Delta z)$$

difference quotient for $\frac{\partial \tilde{I}}{\partial z}$

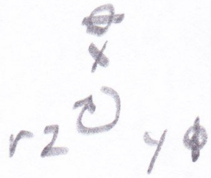
$$\frac{\tilde{I}(z) - \tilde{I}(z+\Delta z)}{\Delta z} = (G + j\omega C)\left(\tilde{V}(z) - \frac{I(z)\Delta z}{2}(R + j\omega L)\right) \quad ②$$

Now take $\lim_{z \to 0}$

① $\frac{\partial \tilde{V}}{\partial z} = -\frac{1}{2}(R + j\omega L) \times 2\tilde{I} \implies \boxed{\frac{\partial \tilde{V}}{\partial z} = -(R + j\omega L)\tilde{I}}$

② $\frac{\partial \tilde{I}}{\partial z} = (G + j\omega C)(\tilde{V} - 0) \implies \boxed{\frac{\partial \tilde{I}}{\partial z} = (G + j\omega C)\tilde{V}}$

③ $$\bar{E}(z) = \tilde{E}_0 e^{-j\beta z}(\hat{x} - j\hat{y})$$



$$\bar{H}(z) = \frac{\tilde{E}_0 e^{-j\beta z}(j\hat{x} + \hat{y})}{\eta}$$

Method A : $$Z_{wave} = \frac{|\bar{E}|}{|\bar{H}|} = \sqrt{\frac{\tilde{E}_0 e^{-j\beta z}(\hat{x} - j\hat{y}) \cdot \tilde{E}_0^* e^{j\beta z}(\hat{x}+j\hat{y})}{\frac{\tilde{E}_0 e^{-j\beta z}(j\hat{x}+\hat{y}) \cdot \tilde{E}_0^* e^{j\beta z}(-j\hat{x}+\hat{y})}{\eta^2}}}$$

$$= \frac{2|\tilde{E}_0|}{\frac{2|\tilde{E}_0|}{\eta^{2/2}}} = \eta$$

Method B : $$Z_{wave} = \frac{\hat{z}\cdot(\bar{E}\times\bar{H}^*)}{|\hat{z}\times\bar{H}|^2}$$

$$\bar{E}(z) = \begin{bmatrix} \tilde{E}_0 e^{-j\beta z} \\ -j\tilde{E}_0 e^{-j\beta z} \\ 0 \end{bmatrix} = \tilde{E}_0 e^{-j\beta z}\begin{bmatrix} 1 \\ -j \\ 0 \end{bmatrix}$$

$$\bar{H}^*(z) = \frac{\tilde{E}_0^* e^{+j\beta z}}{\eta}\begin{bmatrix} -j \\ 1 \\ 0 \end{bmatrix}$$

$$\bar{E}\times\bar{H}^* = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & -j & 0 \\ -j & 1 & 0 \end{vmatrix} \times \frac{|\tilde{E}_0|^2}{\eta} \times e^0$$

$$= \left(\overset{1}{\cancel{1\times 1}} - (\overset{-1}{\cancel{+j\times j}})\right)\hat{z}\left(\frac{|E_0|^2}{\eta}\right)$$

$$= 2\frac{|\tilde{E}_0|^2}{\eta}\hat{z}$$

$$\hat{z} \times \bar{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ j & 1 & 0 \end{vmatrix} \frac{\tilde{E}_0 e^{-j\beta z}}{\eta}$$

$$= \begin{bmatrix} -1 \\ j \\ 0 \end{bmatrix}$$

$$|\hat{z} \times \bar{H}|^2 = \frac{\tilde{E}_0 e^{-j\beta z}}{\eta} \begin{bmatrix} -1 \\ j \\ 0 \end{bmatrix} \cdot \frac{\tilde{E}_0^* e^{j\beta z}}{\eta} \begin{bmatrix} -1 \\ -j \\ 0 \end{bmatrix}$$

$$= \frac{|\tilde{E}_0|^2}{\eta^2} (+1 + 1)$$

$$= \frac{2|\tilde{E}_0|^2}{\eta^2}$$

$$Z_{wave} = \frac{\cancel{\hat{z}} \cdot \frac{\cancel{2|\tilde{E}_0|^2} \cancel{\hat{z}}}{\cancel{\eta}}}{\frac{\cancel{2|\tilde{E}_0|^2}}{\cancel{\eta^2}}}$$

$$= \eta$$

④ $\nabla \times \bar{H}(\vec{r}) = j\omega\varepsilon_0 E(\vec{r})$, $\bar{H} = \dfrac{\hat{r} \times \bar{E}}{\eta}$

$\hat{\theta}$ x

r^2 $y\phi$

$$\bar{E} = \left(\tilde{E}_\phi(\theta,\phi)\hat{\phi} + \tilde{E}_\theta(\theta,\phi)\hat{\theta}\right)\frac{e^{-jkr}}{r}$$

$$\bar{H} = \left(\tilde{H}_\phi(\theta,\phi)\hat{\phi} + \tilde{H}_\theta(\theta,\phi)\hat{\theta}\right)\frac{e^{-jkr}}{r}$$

(farfield)

$E_r = H_r = 0$

Steps

1) $\nabla \times \bar{H}$

2) $\dfrac{\hat{r} \times \bar{E}}{\eta}$

3) $\nabla \times \left(\dfrac{\hat{r} \times \bar{E}}{\eta}\right)$

4) compare

from wikipedia

↓

$$1)\ \nabla \times \bar{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\hat{r} + \frac{1}{r}\left(0 - \frac{\partial}{\partial r}(rA_\phi)\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) + 0\right)\hat{\phi}$$

$$\nabla \times \bar{H} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\tilde{H}_\phi \frac{e^{-jkr}}{r}\sin\theta\right) - \frac{\partial}{\partial\phi}\left(\tilde{H}_\theta\frac{e^{-jkr}}{r}\right)\right)\hat{r} + \frac{1}{r}\left(-\frac{\partial}{\partial r}\left(\cancel{r}\tilde{H}_\phi \frac{e^{-jkr}}{\cancel{r}}\right)\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(\cancel{r}\tilde{H}_\theta\frac{e^{-jkr}}{\cancel{r}}\right)\right)\hat{\theta}$$

$$\nabla \times \bar{H} = \frac{e^{-jkr}}{r^2 \sin\theta}\left(\frac{\partial}{\partial\theta}\left(\tilde{H}_\phi \sin\theta\right) - \frac{\partial}{\partial\phi}\tilde{H}_\phi\right)\hat{r}$$

$$- \frac{1}{r}\left(-jk\tilde{H}_\phi e^{-jkr}\right)\hat{\theta}$$

$$+ \frac{1}{r}\left(-jk e^{-jkr}\tilde{H}_\theta\right)\hat{\phi}$$

$$= \frac{e^{-jkr}}{r}\left(\frac{\frac{\partial}{\partial\theta}\left(H_\phi \sin\theta\right) - \frac{\partial}{\partial\phi}\tilde{H}_\theta}{r\sin\theta}\hat{r} + jk\tilde{H}_\phi\hat{\theta} - jk\tilde{H}_\theta\hat{\phi}\right)$$

2) $$\frac{\hat{r}\times\bar{E}}{\eta} = \left(\frac{\tilde{E}_\phi}{\eta}\hat{r}\times\hat{\phi} + \frac{\tilde{E}_\theta}{\eta}\hat{r}\times\hat{\theta}\right)\frac{e^{-jkr}}{r}$$

$$= \frac{e^{-jkr}}{r}\left(\underbrace{-\frac{\tilde{E}_\phi}{\eta}}_{X_\theta}\hat{\theta} + \underbrace{\frac{\tilde{E}_\theta}{\eta}}_{X_\phi}\hat{\phi}\right)$$

3) $$\nabla\times\left(\frac{\hat{r}\times\bar{E}}{\eta}\right) = \nabla\times\bar{X}$$

$$= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(X_\phi \sin\theta\right) - \frac{\partial}{\partial\phi}X_\theta\right)\hat{r}$$

$$+ \frac{1}{r}\left(0 - \frac{\partial}{\partial r}\left(rX_\phi\right)\right)\hat{\theta}$$

$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rX_\theta\right)\right)\hat{\phi}$$

$$\nabla \times \left(\frac{\hat{r} \times \bar{E}}{\eta}\right) = \hat{r}\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\frac{e^{-jkr}}{r}\frac{\tilde{E}_\theta}{\eta}\right) - \frac{\partial}{\partial\phi}\left(\frac{e^{-jkr}}{r}\frac{-\tilde{E}_\phi}{\eta}\right)\right)$$

$$- \frac{1}{r}\left(\frac{\partial}{\partial r}\left(\cancel{r}\frac{e^{-jkr}}{\cancel{r}}\frac{\tilde{E}_\theta}{\eta}\right)\right)\hat{\theta}$$

$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{e^{-jkr}}{r}\cdot\frac{-\tilde{E}_\phi}{\eta}\right)\right)\hat{\phi}$$

$$= \frac{e^{-jkr}}{r}\left(\frac{\frac{\partial}{\partial\theta}\left(\frac{\tilde{E}_\theta}{\eta}\right) + \frac{\partial}{\partial\phi}\left(\frac{\tilde{E}_\phi}{\eta}\right)}{r\sin\theta}\hat{r} \;?\right.$$

$$\left. + jk\frac{\tilde{E}_\theta}{\eta}\hat{\theta} + jk\frac{\tilde{E}_\phi}{\eta}\hat{\phi}\right)$$

$$\nabla \times \bar{H} = \frac{e^{-jkr}}{r}\left(\frac{\frac{\partial}{\partial\theta}\left(\tilde{H}_\phi\sin\theta\right) - \frac{\partial}{\partial\phi}\tilde{H}_\theta}{r\sin\theta}\hat{r} \;?\right.$$

$$\left. + jk\tilde{H}_\phi\hat{\theta} - jk\tilde{H}_\theta\hat{\phi}\right)$$

$$Z_{wave} = \frac{\hat{k} \cdot (\bar{E} \times \bar{H}^*)}{|\hat{k} \times \bar{H}|^2}$$

θ

$r \quad \phi$

when $\hat{k} = \hat{r}$

$$Z_{wave} = \frac{\hat{r} \cdot (\bar{E} \times \bar{H}^*)}{(\hat{r} \times \bar{H}) \cdot (\hat{r} \times \bar{H}^*)}$$

$$= \frac{\hat{r} \cdot \left(\bar{E}(\theta,\phi) \cancel{\frac{e^{-jkr}}{r}} \times \bar{H}^*(\theta,\phi) \cancel{\frac{e^{jkr}}{r}} \right)}{\left(\hat{r} \times \bar{H}(\theta,\phi) \cancel{\frac{e^{-jkr}}{r}}\right) \cdot \left(\hat{r} \times \bar{H}^*(\theta,\phi) \cancel{\frac{e^{jkr}}{r}}\right)}$$

$$= \frac{\hat{r} \cdot \left((\tilde{E}_\theta(\theta,\phi)\hat{\theta} + \tilde{E}_\phi(\theta,\phi)\hat{\phi}) \times (\tilde{H}^*_\theta(\theta,\phi)\hat{\theta} + \tilde{H}^*_\phi(\theta,\phi)\hat{\phi}) \right)}{\left(\hat{r} \times (\tilde{H}_\theta(\theta,\phi)\hat{\theta} + \tilde{H}_\phi(\theta,\phi)\hat{\phi})\right) \cdot \left(\hat{r} \times (\tilde{H}^*_\theta(\theta,\phi)\hat{\theta} + \tilde{H}^*_\phi(\theta,\phi)\hat{\phi})\right)}$$

$$= \frac{\hat{r} \cdot \left(\tilde{E}_\theta(\theta,\phi)\tilde{H}^*_\phi(\theta,\phi)\hat{r} - \tilde{E}_\phi(\theta,\phi)\tilde{H}^*_\theta(\theta,\phi)\hat{r} \right)}{\left(\tilde{H}_\theta(\theta,\phi)\hat{\phi} - \tilde{H}_\phi(\theta,\phi)\hat{\theta}\right) \cdot \left(\tilde{H}^*_\theta(\theta,\phi)\hat{\phi} - \tilde{H}^*_\phi(\theta,\phi)\hat{\theta}\right)}$$

$$= \frac{\tilde{E}_\theta(\theta,\phi)\tilde{H}^*_\phi(\theta,\phi) - \tilde{E}_\phi(\theta,\phi)\tilde{H}^*_\theta(\theta,\phi)}{\tilde{H}_\theta(\theta,\phi)\tilde{H}^*_\theta(\theta,\phi) + \tilde{H}_\phi(\theta,\phi)\tilde{H}^*_\phi(\theta,\phi)}$$

Now let's try using eta η

$$\tilde{E}_\phi(\theta,\phi) = -\eta \tilde{H}_\theta(\theta,\phi)$$

$$\tilde{E}_\theta(\theta,\phi) = \eta \tilde{H}_\phi(\theta,\phi)$$

$$Z_{wave} = \frac{\eta \tilde{H}_\phi(\theta,\phi)\tilde{H}_\phi^*(\theta,\phi) - -\eta \tilde{H}_\theta(\theta,\phi)\tilde{H}_\theta^*(\theta,\phi)}{|\tilde{H}_\theta(\theta,\phi)|^2 + |\tilde{H}_\phi(\theta,\phi)|^2}$$

$$= \eta \cancel{\frac{|\tilde{H}_\phi|^2 + |\tilde{H}_\theta|^2}{|\tilde{H}_\phi|^2 + |\tilde{H}_\theta|^2}} \to 1$$

$$= \eta$$

I feel like we have different definations of what "rewarding" is . . .

6

$$Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}}$$

$$= 90.4 + j12.6\,\Omega$$

$Z_0 = 75\Omega$

$\Gamma_{in} = 0.12\,e^{j35°}$

$f = 900$ MHz

a) $\boxed{X_{in} = 12.6\,\Omega}$

b) $P_{refl} = |\Gamma_{in}|^2 P_{inc} \Rightarrow$ $\boxed{\text{1.44\% of incident power will reflect}}$

c) $P_{inc} = 12$ mW

$$P_{in} = (1-|\Gamma|^2)P_{inc}$$

$$\boxed{= 10.7\text{ dBm}}$$

Question 5

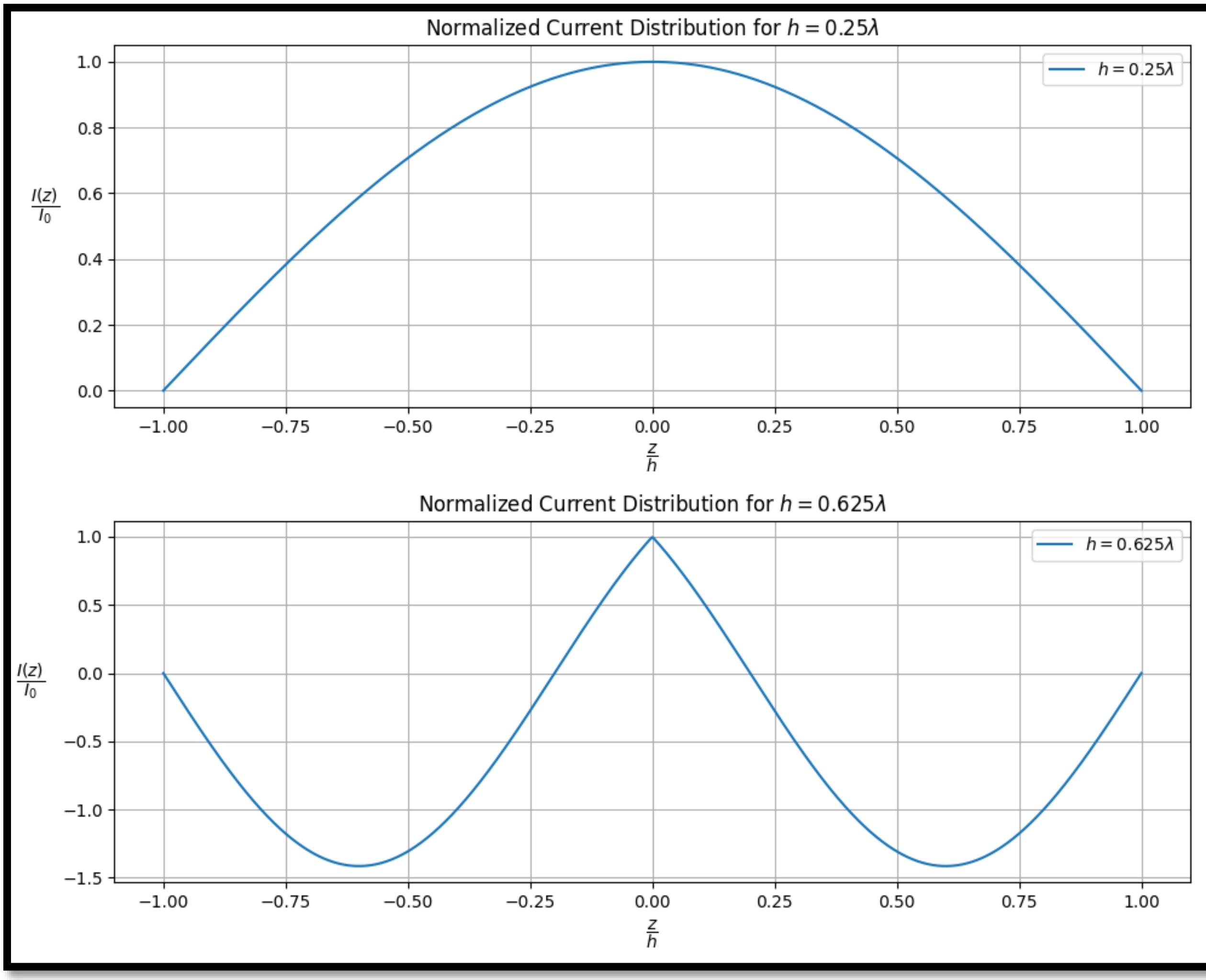


Figure 1

I'm skeptical of the current distribution for the 1.25λ case since I would expect it to be somewhere in between the λ and 1.5 λ cases (see figure 1). Maybe I've just inverted the plot somewhere?

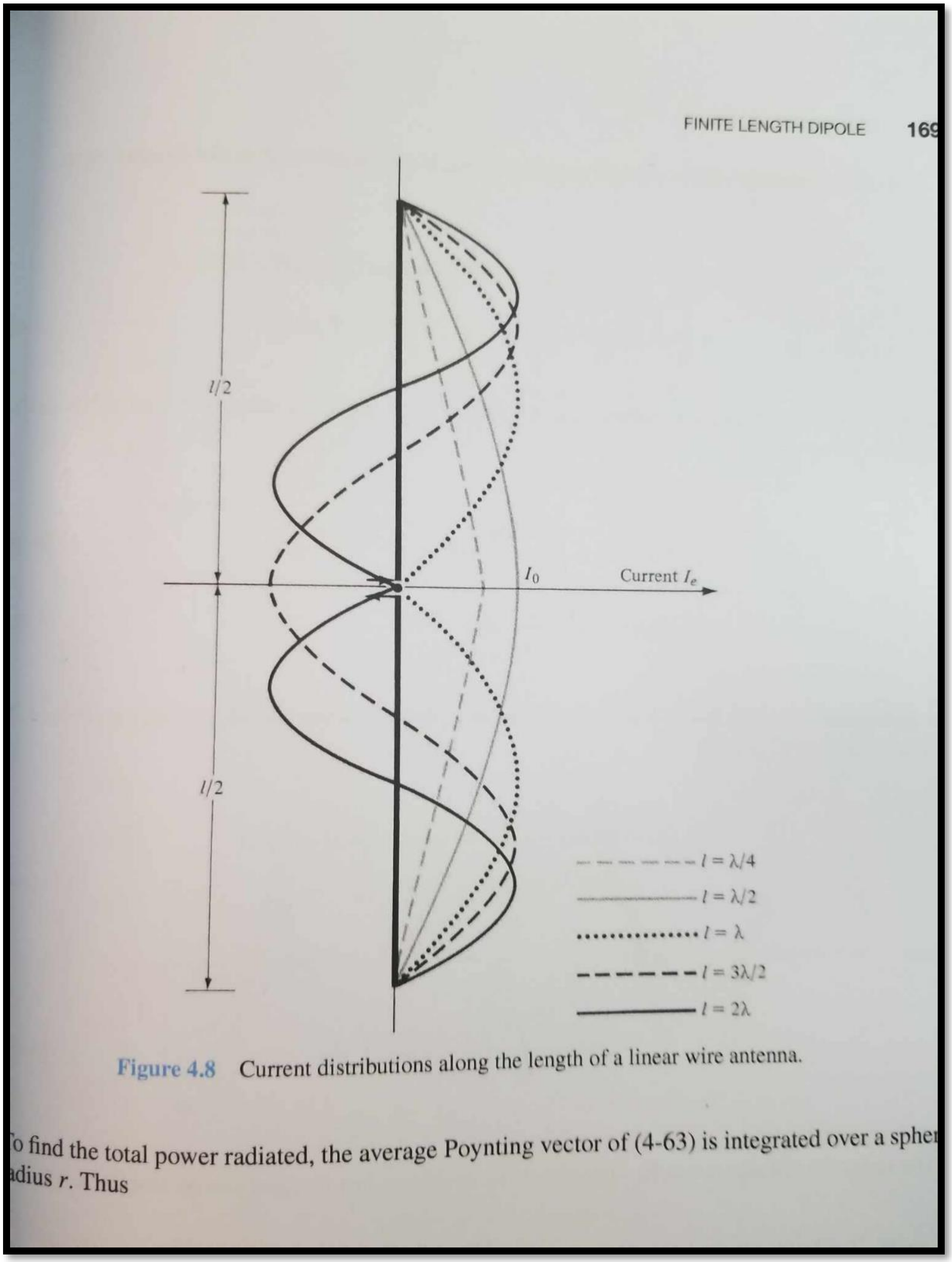


Figure 4.8 Current distributions along the length of a linear wire antenna.

o find the total power radiated, the average Poynting vector of (4-63) is integrated over a sphe
adius r. Thus

Figure 2

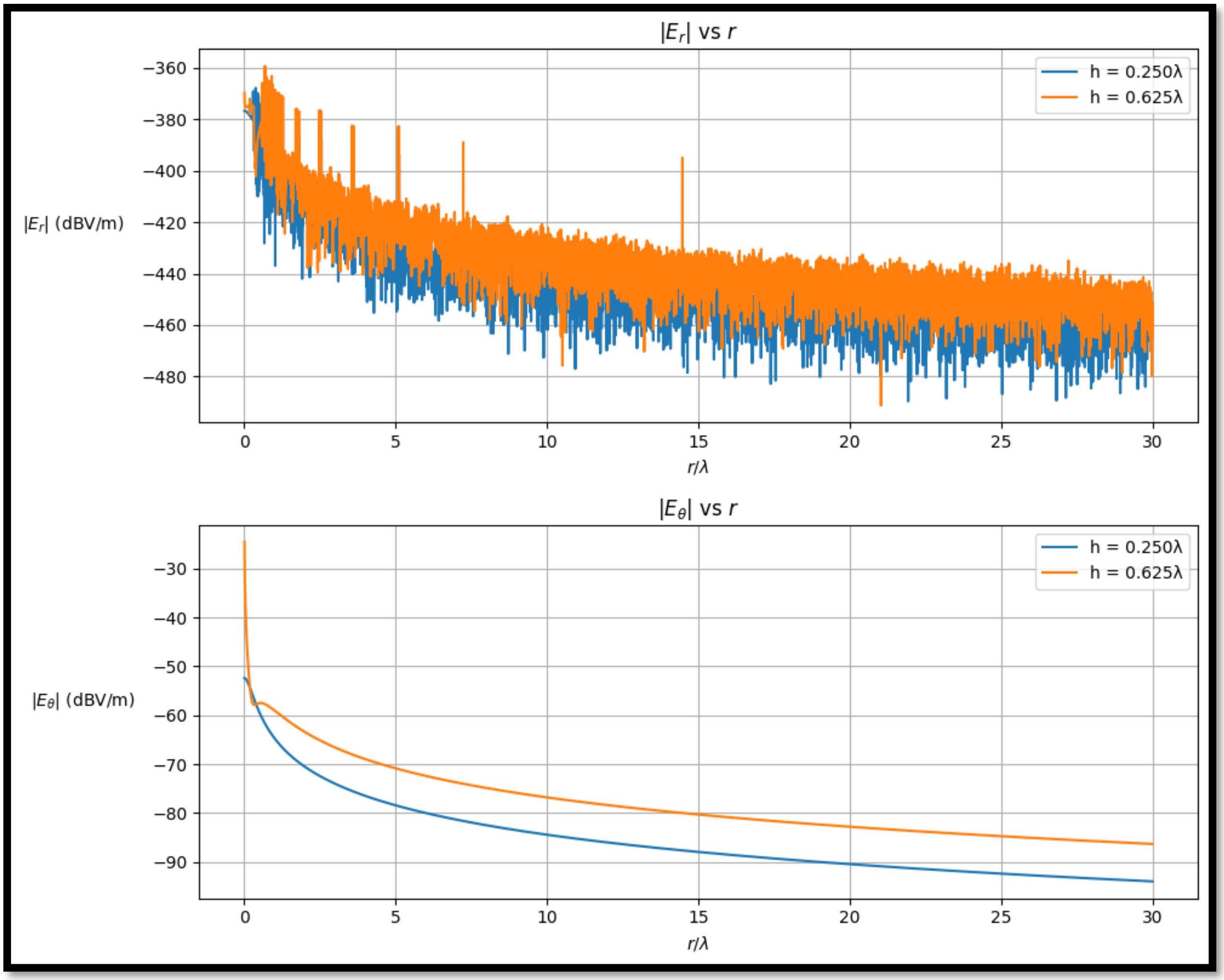


Figure 3

This does make sense. Radial component of E field should be 0 far from the antenna. Not sure what that spiking on the top plot of figure 3 is though

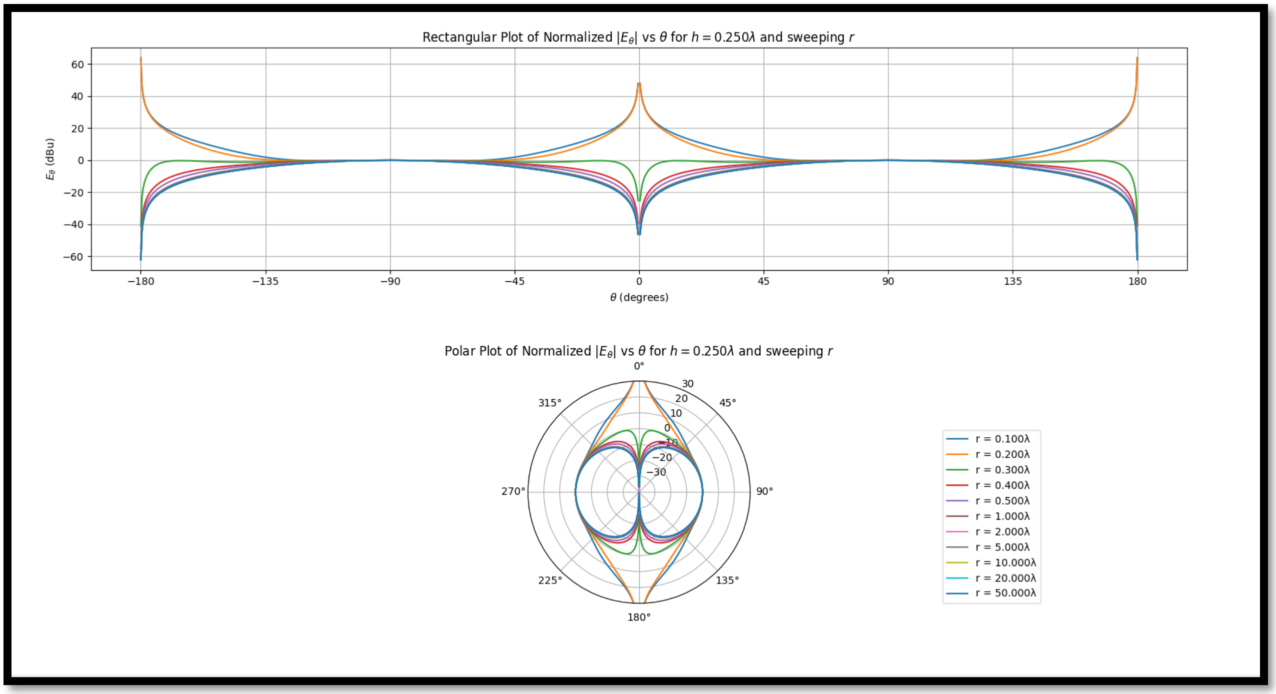


Figure 4

This makes sense. Radiation pattern should be a torus, but only close to the antenna. When looking at figure 6, the most E field is in the theta hat direction along the z azis

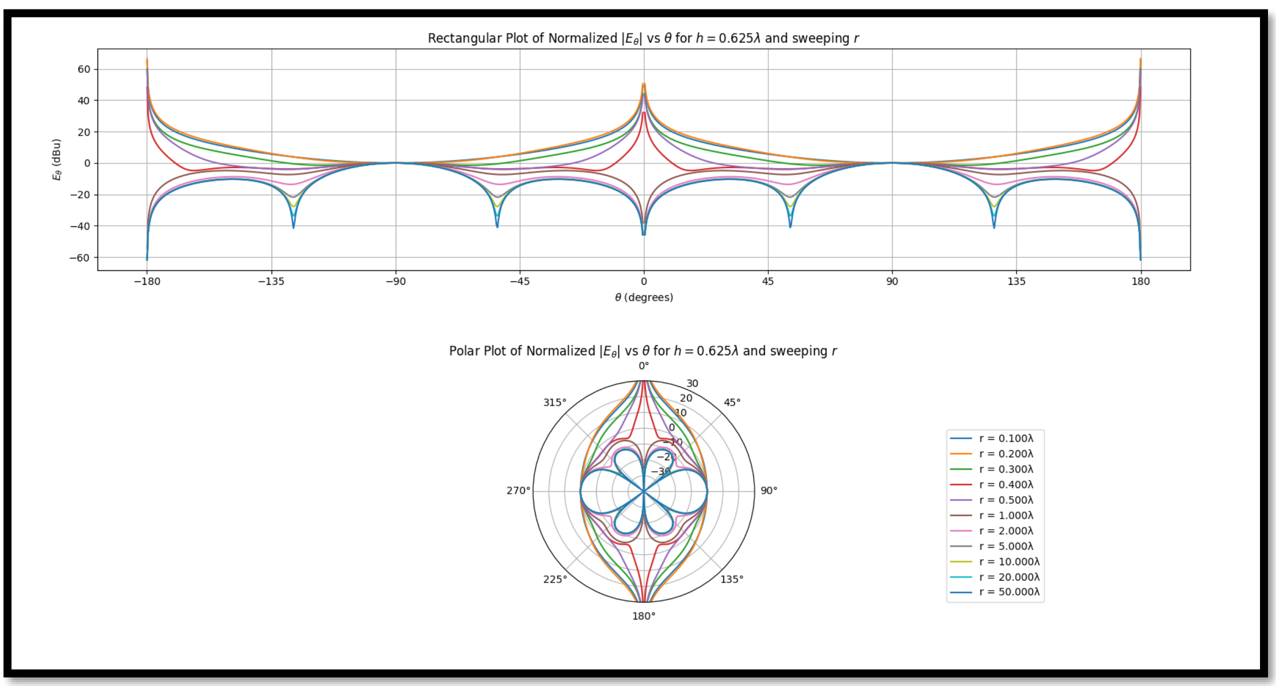


Figure 5

Makes sense again. Looks like textbook (figure 7)

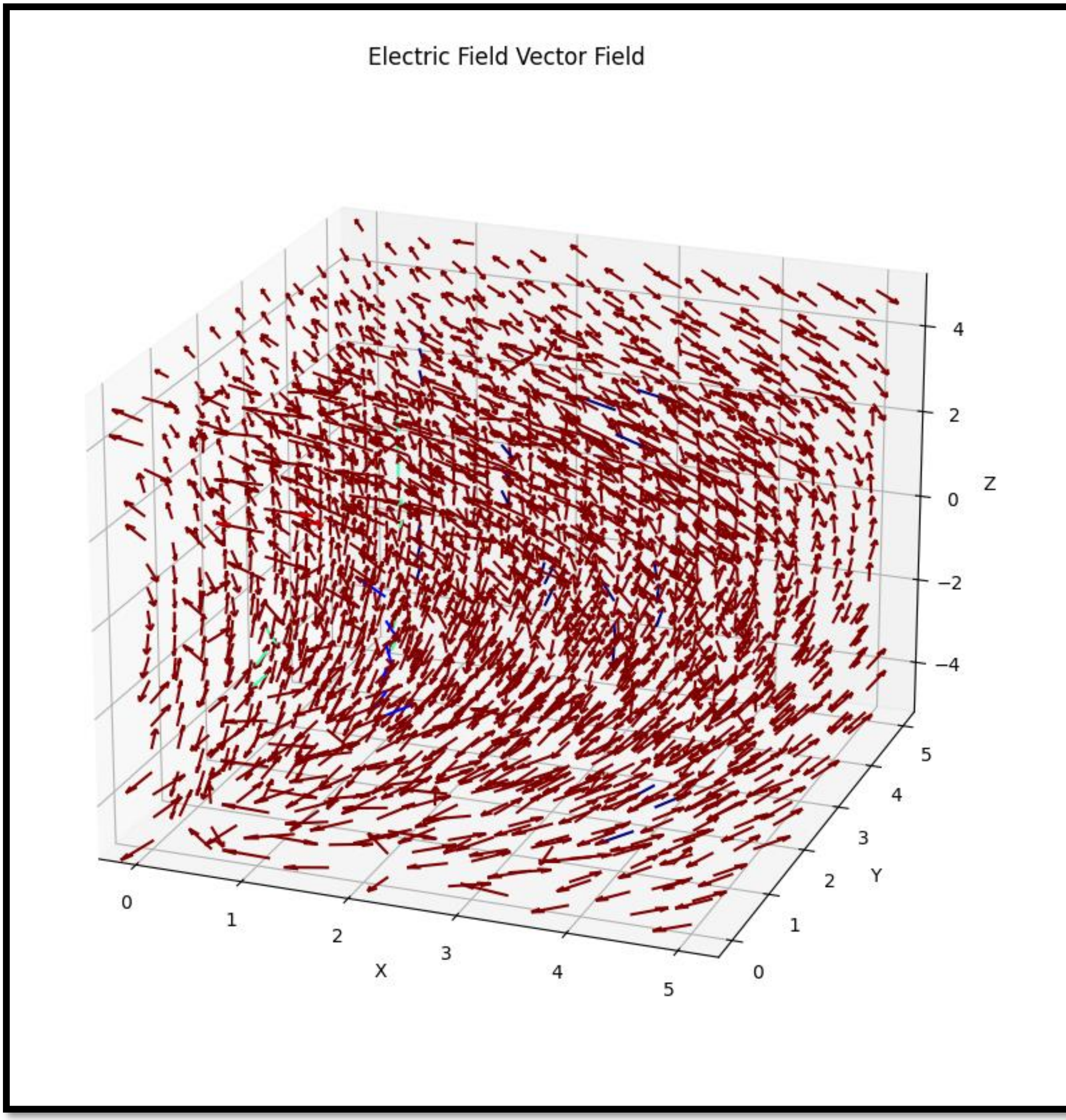


Figure 6

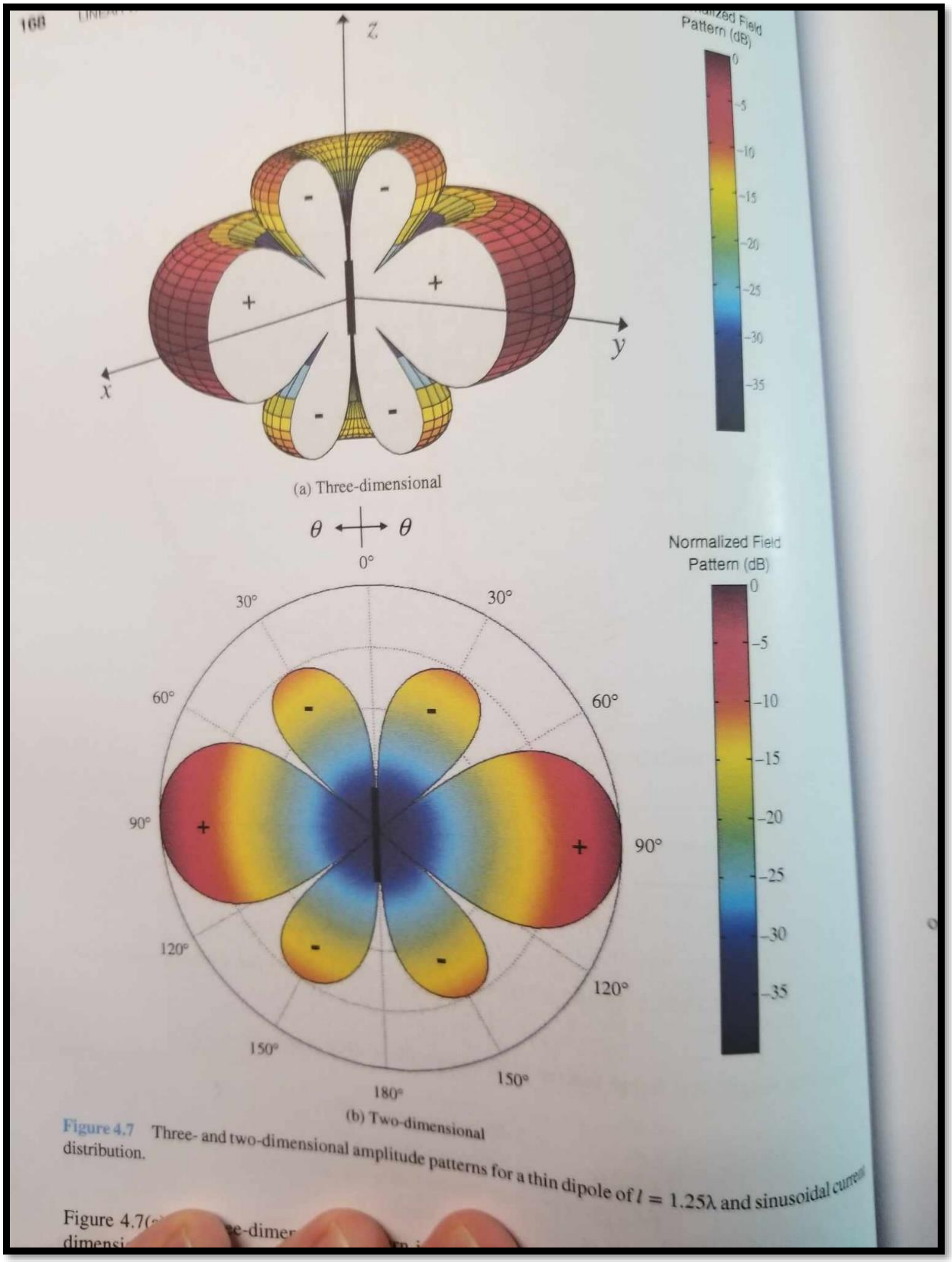

168
z
y
x
+
-
Pattern (dB)
0
-5
-10
-15
-20
-25
-30
-35
(a) Three-dimensional
θ
θ
0°
30°
30°
60°
60°
90°
90°
120°
120°
150°
150°
180°
Normalized Field
Pattern (dB)
(b) Two-dimensional
Figure 4.7 Three- and two-dimensional amplitude patterns for a thin dipole of l = 1.25λ and sinusoidal current distribution.

Figure 7