Chapter 3

Linear and nonlinear microwave circuit design

LINEAR MICROWAVE CIRCUIT DESIGN

After introducing the device models, the next step would be to include them in linear and nonlinear circuits for small-signal/large-signal simulation, optimization, and statistical design.

LINEAR MICROWAVE CIRCUIT DESIGN

Linear circuit design has been widely investigated in the last 50 years and many methods have been proposed to designers. Three of them are now mainly used to design linear microwave circuits and systems.

They are namely the **connection-scattering matrix** approach, the **multiport connection** method and the **sub-network** method.

Connection-scattering matrix approach

This approach is applicable when the network contains arbitrarily interconnected multi-ports and independent generators. Consider a network with M multiport components. For the k^{th} component having n_k ports, the incoming and outgoing wave vectors (**[a$_k$]** and **[b$_k$]** respectively) at its ports are related by

$$[\mathbf{b}_k] = [\mathbf{S}_k][\mathbf{a}_k]$$

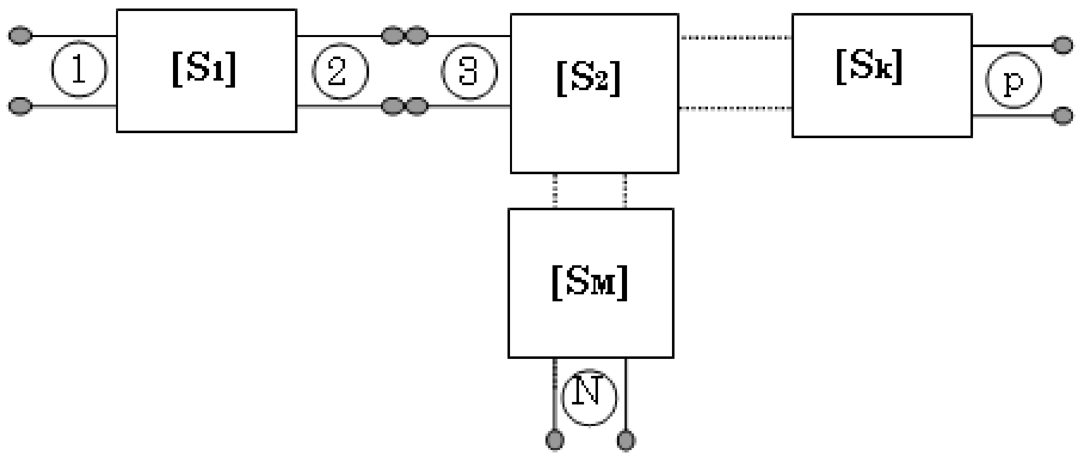
This is valid for all the components except the internal independent generators where the above relation is replaced by

$$[\mathbf{b}_k] = [\mathbf{S}_k][\mathbf{a}_k] + [\mathbf{c}_k]$$

where **[c$_k$]** is the wave vector impressed by the generators. It should be pointed out that no unconnected (i.e., external) ports are allowed in this network.

$$[\mathbf{b}] = [\mathbf{S}]\,[\mathbf{a}] + [\mathbf{c}] \quad \Leftrightarrow \quad \begin{bmatrix} [\mathbf{b}_1] \\ \vdots \\ [\mathbf{b}_M] \end{bmatrix} = \begin{bmatrix} [\mathbf{S}_1] & [\mathbf{0}] & \cdots & [\mathbf{0}] \\ [\mathbf{0}] & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & [\mathbf{0}] \\ [\mathbf{0}] & \cdots & [\mathbf{0}] & [\mathbf{S}_M] \end{bmatrix} \begin{bmatrix} [\mathbf{a}_1] \\ \vdots \\ [\mathbf{a}_M] \end{bmatrix} + \begin{bmatrix} [\mathbf{c}_1] \\ \vdots \\ [\mathbf{c}_M] \end{bmatrix}$$

This equation refers to all individual components but does not take into account the **interconnections**.



For a pair of connected ports, the outgoing wave variable at one port must equal the incoming wave variable at the other. For example, if port j of one component is connected to port k of another component, we should have

$$\begin{bmatrix} b_j \\ b_k \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_j \\ a_k \end{bmatrix}$$

The elements of this matrix are all “0’s" or “1’s".

For the overall circuit, we can then introduce a connection matrix **[Γ]**, which describes the topology

$$\mathbf{[b] = [\Gamma]\ [a]} \qquad \begin{bmatrix} b_j \\ b_k \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_j \\ a_k \end{bmatrix}$$

In each row of **[Γ]** all elements are "0" except the element $\Gamma_{jk} = 1$ in the column indicating the interconnection between ports j and k : b_j (going out from port j) = a_k (entering port k)

Substituting for **[Γ]**, we obtain

$$\mathbf{\{\,[\Gamma] - [S]\,\}\ [a] = [W]\ [a] = [c]} \quad \Longrightarrow \quad \mathbf{[a] = [W]^{-1}\ [c]}$$

[W] is called the connection scattering matrix. The main diagonal elements in **[W]** are the negative of the reflection coefficients at the various component ports.

It allows to obtain the vector **[a]** and then, to deduce the vector **[b]**. Therefore, the overall scattering matrix $\mathbf{[S_g]}$ of the circuit is totally defined by the following relation

$$\mathbf{[b] = [S_g][a]}$$

Multiport connection approach

In this method also, the global scattering matrix is determined by the individual S-matrices of each component. This method is applicable when the network contains arbitrarily interconnected multiport components **without independent generators**. In this case, incoming and outgoing waves can be separated into two groups; the first is corresponding to the *p* external ports, and the second to *c* internally connected ports:

$$\begin{bmatrix} [\mathbf{b_p}] \\ [\mathbf{b_c}] \end{bmatrix} = \begin{bmatrix} [\mathbf{S_{pp}}] & [\mathbf{S_{pc}}] \\ [\mathbf{S_{cp}}] & [\mathbf{S_{cc}}] \end{bmatrix} \begin{bmatrix} [\mathbf{a_p}] \\ [\mathbf{a_c}] \end{bmatrix}$$

where **[S_{pp}]**, **[S_{pc}]**, **[S_{cp}]** and **[S_{cc}]** are the S-sub-matrices.

A connection matrix **[Γ]** allows taking into account the interconnections between the connections

$$[\mathbf{b_c}] = [\mathbf{\Gamma}]\,[\mathbf{a_c}] = [\mathbf{S_{cp}}][\mathbf{a_p}] + [\mathbf{S_{cc}}]\,[\mathbf{a_c}] \quad \Longrightarrow \quad [\mathbf{a_c}] = \{\,[\mathbf{\Gamma}] - [\mathbf{S_{cc}}]\,\}^{-1}\,[\mathbf{S_{cp}}][\mathbf{a_p}]$$

$$\Longrightarrow \quad [\mathbf{b_p}] = [\mathbf{S_{pp}}][\mathbf{a_p}] + [\mathbf{S_{pc}}][\mathbf{a_c}] = \left\{[\mathbf{S_{pp}}] + [\mathbf{S_{pc}}]\left([\mathbf{\Gamma}] - [\mathbf{S_{cc}}]\right)^{-1}[\mathbf{S_{cp}}]\right\}[\mathbf{a_p}]$$

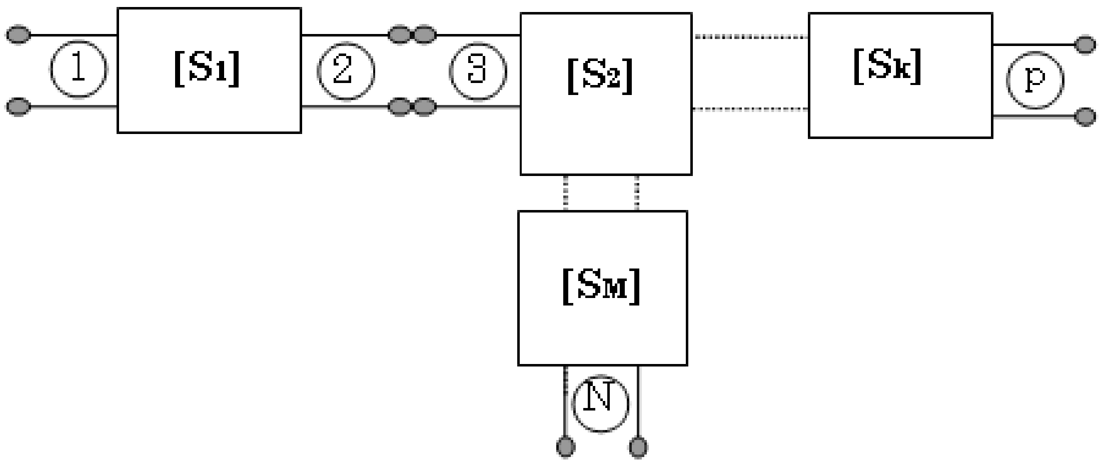
$$\Longrightarrow \quad [\mathbf{S_g}] = [\mathbf{S_p}] = [\mathbf{S_{pp}}] + [\mathbf{S_{pc}}]\left([\mathbf{\Gamma}] - [\mathbf{S_{cc}}]\right)^{-1}[\mathbf{S_{cp}}]$$

Sub-network growth method

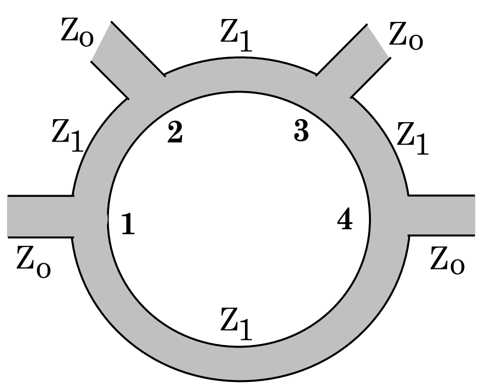
In the two first methods, to obtain the S-matrix of a network requires to invert a matrix of order equal to the number of interconnected ports. When the network contains many interconnected ports, the order of the matrix to be inverted will become quite large. The computational effort of inverting the matrix can be reduced considerably if the entire network is not taken at once, but it portioned into a number of sub-networks. The S-matrices of the sub-networks are obtained separately and are then combined to obtain the overall S-matrix.

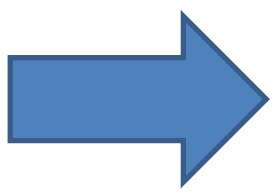
EXAMPLE: RAT-RACE COUPLER

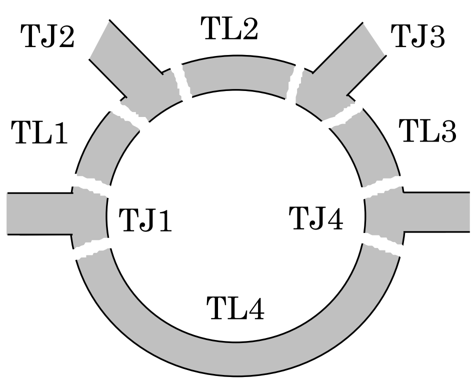
Using the Connection-scattering matrix method



- Four Te junctions (TJ1 to TJ4) with characteristic impedances Z_o (external line) and Z_1 (internal connected lines).
- Four transmission lines (TL1 to TL4) with characteristic impedances Z_1. The three first have a λ/4 length and the last 3λ/4 length







NONLINEAR MICROWAVE CIRCUIT DESIGN

NONLINEAR APPROACHES TO CIRCUIT ANALYSIS

Linear :

S-parameters, Y-parameters, Z-parameters, H-parameters, voltages, currents.

So we could use:

In Simulations:

voltages, currents, power

reflection/transmission coefficients

In measurements:

reflection/transmission coefficients

power

Tools:

Smith Chart

Nonlinear :

S-parameters, Y-parameters, Z-parameters, H-parameters, voltages, currents, **Harmonics**

So we could use:

In Simulations:

voltages, currents, power, **Harmonic responses**

reflection/transmission coefficients

In measurements:

reflection/transmission coefficients

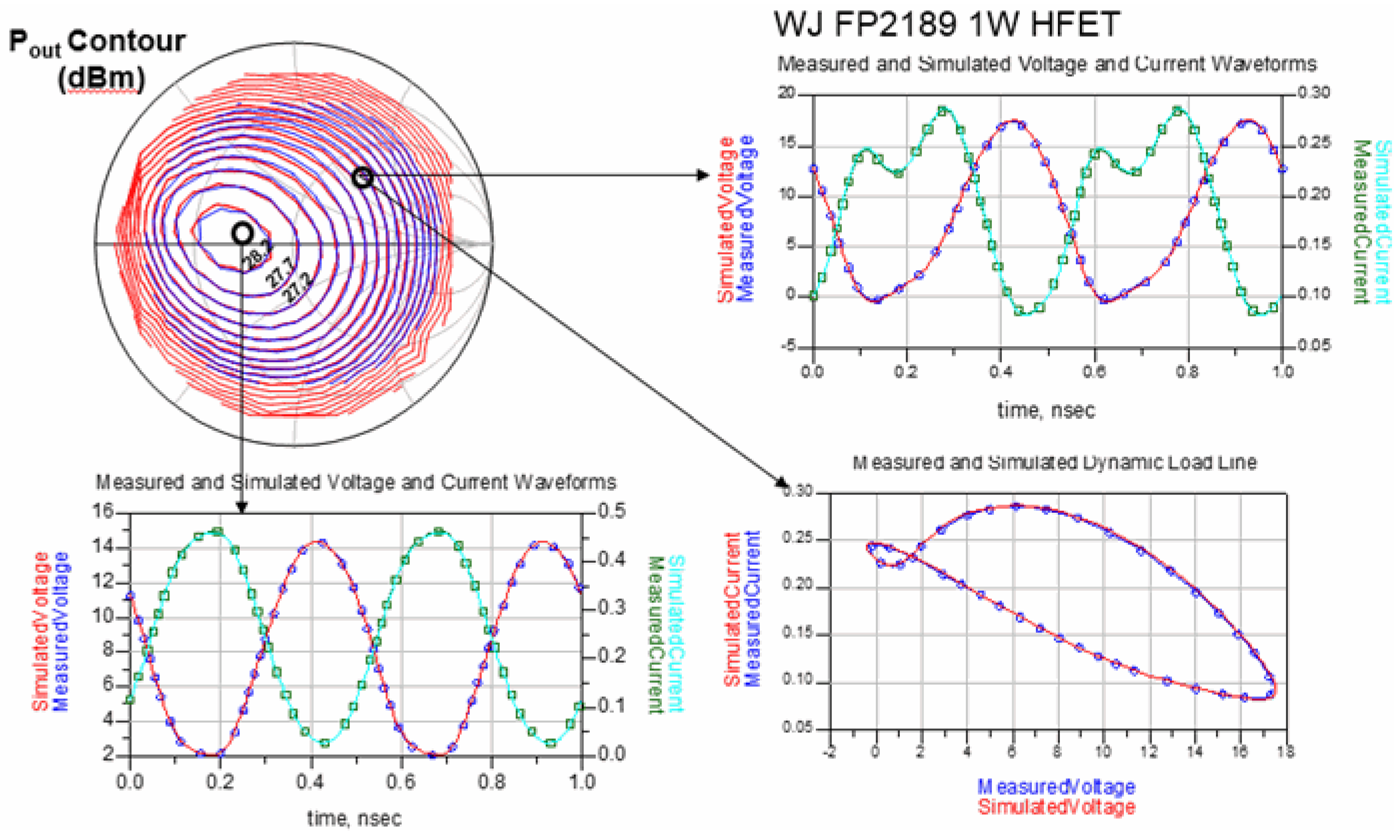
power **@ different harmonics**

Tools:

??

Experimental approaches: *Load pull approach*

The early large-signal nonlinear models and techniques were attempts to bend linear theory to nonlinear applications, were highly approximate, or were attempts at “black box” characterizations that did not include all the variables or parameters necessary to obtain meaningful results.

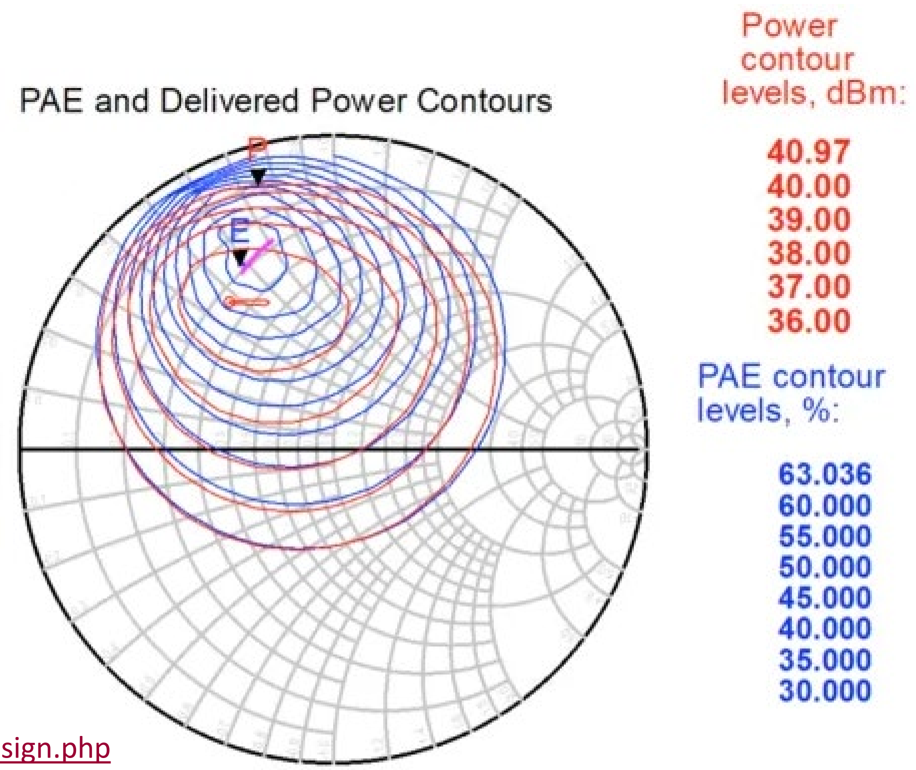


http://na.support.keysight.com/pna/nvna/NVNAWebHelp/Load_Pull_Options.htm

Experimental approaches: *Load pull approach*

One straightforward way to characterize a large-signal circuit such as an amplifier is to graph on a Smith chart the contours of its load impedances that result in prescribed values of gain and output power.

These approximately circular contours can then be used to select output load impedance that represents the best trade-off **gain vs. power** and **Power-Added Efficiency.**



https://www.nwengineeringllc.com/article/load-pull-analysis-for-rf-signal-chain-design.php

Experimental approaches: *Load pull approach*

Varying ("pulling") load impedance Z_L seen by an active device under test:

Measuring performance metrics of DUT at each Z_L point:

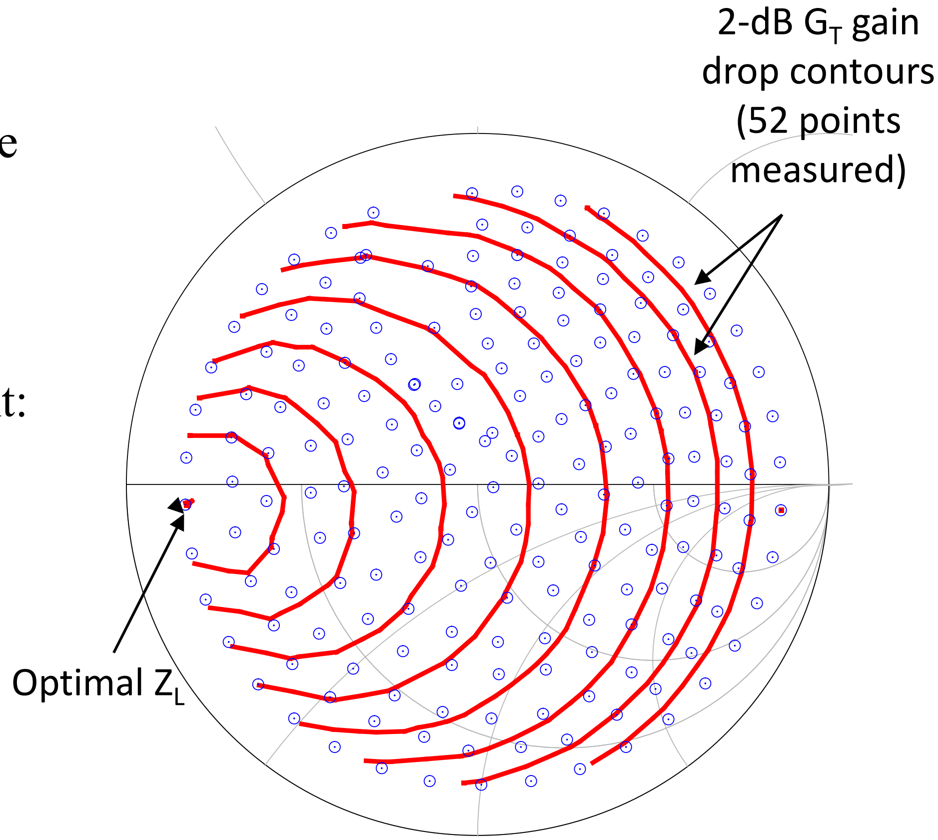
- Transducer Gain (as shown)
- Power-added Efficiency
- 1-dB Gain Compression Point
- Noise Figure

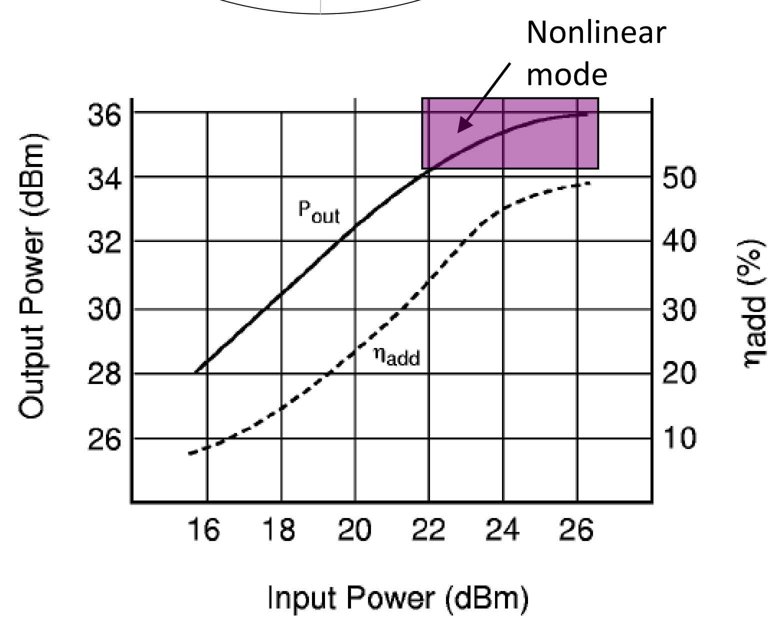
Not needed for linear devices

- Performance with any Z_L determined by measuring small-signal S-parameters

Important for characterizing devices operating in large-signal (nonlinear) mode

- Operating point (and thereby performance) may change for different loads





Experimental approaches: *Load pull approach*

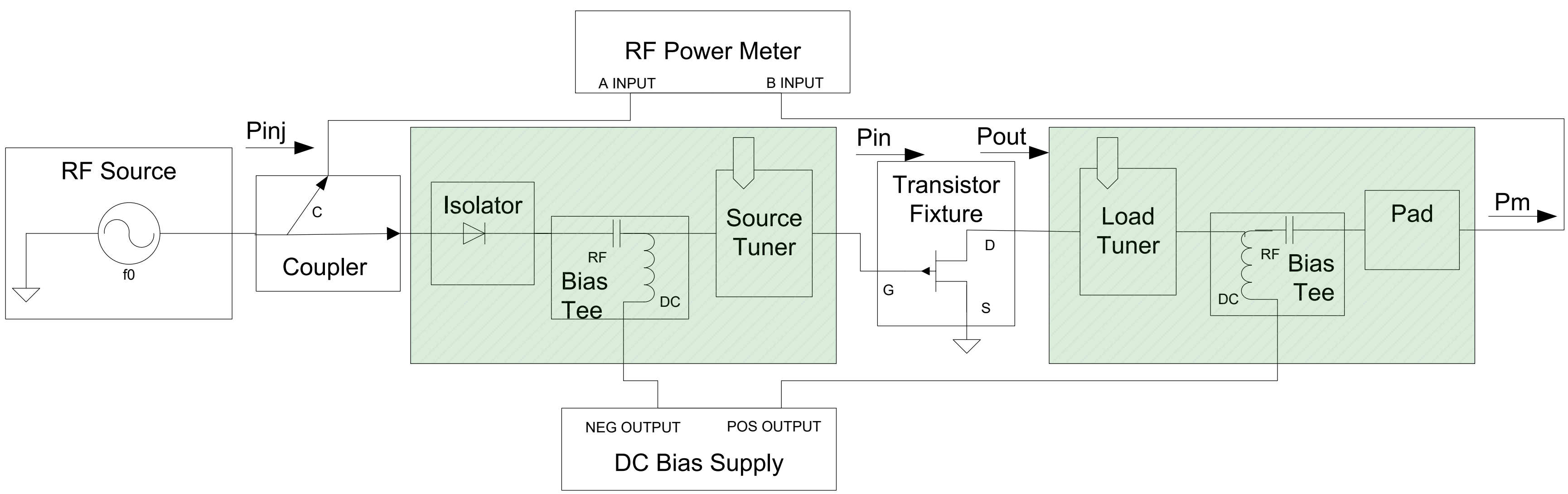
Load-Pull Setup (Measurements are usually automated)

- First set up source tuner for maximum gain, then fix its position
- For each (X,Y) position of the load tuner:

1. Measure injected power P_{inj}, delivered power P_m
2. Measure S-parameters of source and load circuitry (shaded boxes)
3. Move the reference plane to the DUT input and output:

   Calculate input and output power loss (due to mismatch) from measured S-parameters

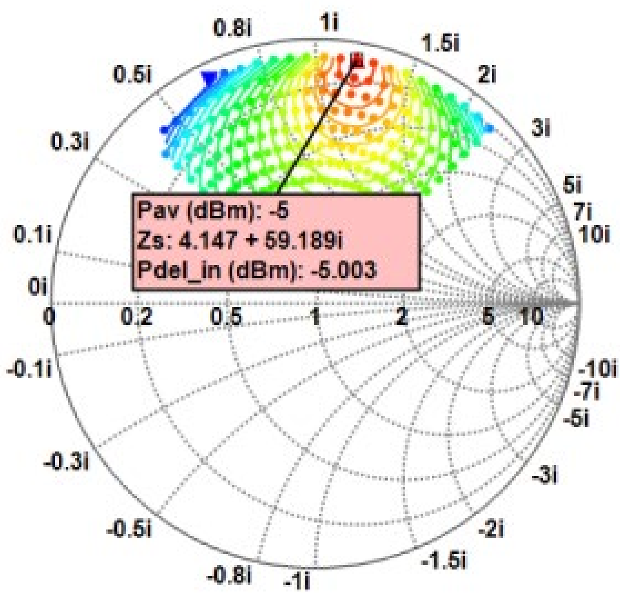
   Obtain P_{in}, P_{out} by correcting P_{inj}, P_m for power losses
4. Solve for transducer gain $G_T=P_{out}/P_{in}$

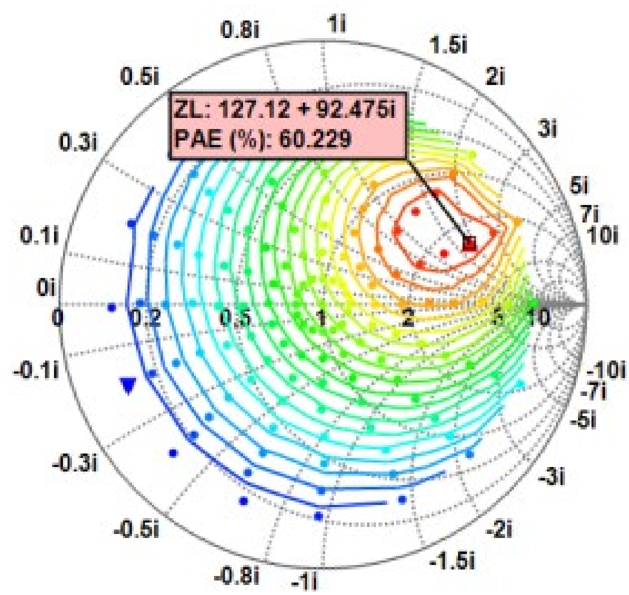


This process, called load pulling, has many limitations:

- The major practical one is the difficulty of measuring the load impedances at the device terminals.
- The major theoretical one is in the fact that the load impedance at harmonics of the excitation frequency can significantly affect circuit performance.

In fact, load pulling is concerned only with the load impedance at the fundamental frequency. Furthermore, load pulling is not useful for determining other important properties of nonlinear circuits (e.g., harmonic levels or effects of multi-tone excitations).





TIME-DOMAIN APPROACHES

Simulation approaches: Time-domain

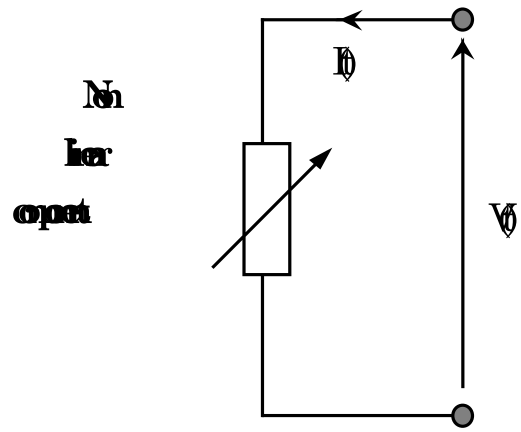
An useful approach is to use time-domain techniques.

This approach is theoretically valid because the use of time-domain differential equations that describe a nonlinear circuit is straightforward in conventional circuit theory. The resulting differential equations in the form of a nonlinear differential system

$$f\left([\mathbf{x}(\mathbf{t})], \left[\frac{\mathbf{d\,x(t)}}{\mathbf{dt}}\right], t\right) = 0 \qquad [\mathbf{x}(\mathbf{0})] = [\mathbf{x_o}]$$

Time-domain analyses are numerical integration methods, which discretize point by point the time variable and then linearize the nonlinear differential system.

TIME-DOMAIN TECHNIQUES



Time Domain

Types

- Shooting Method
- Direct Integration Method
- Finite Integration Method
- Extrapolation Method

The **shooting method** (which allows obtaining the steady state by optimizing the initial conditions and integrating over only one period) and the **successive approximations method** (which starts from an estimate value of the initial point) are the two most used ones in nonlinear time domain analyses.

Shooting Method

The steady state solution of a nonlinear dynamic system can be determined

It integrates over one period and obtains steady state by optimizing initial conditions

It solves time domain boundary solutions initially and at the end of the procedure

- **Main Advantage :**

  This method can be applied in circuits with slow transients

- **Main Disadvantage :**

  Initial conditions are difficult to establish

Direct integration method

The steady state solution is obtained after the calculation of the whole transient cycle

It provides the complete solution including the transient state

The system is iteratively solved in time interval. It gives both the transient and the steady state solution

- **Main Advantage :**

  Transient and steady state are well computed making it very suitable

- **Main Disadvantage :**

  Simulation time is devoted to the transient samples of circuits with very long transients

Finite integration method

The key idea is the discretization of the integral, irrespective of forming the Maxwell Equations

The code to solve the Maxwell Equations is basically called as MAFIA

(Maxwell equations with Finite Integration algorithm)

It explains the Maxwell Equations without approximations/losses in physical properties

- **Main Advantage**

  It is a versatile modeling technique used to solve Maxwell's equations

- **Main Disadvantage**

  There is no way to determine unique values for permittivity and permeability at a material interface

Extrapolation

The problem of solving the periodic steady state response can be done by solving a nonlinear equation of the form z = F (z)

The sequence can be accelerated by extrapolation methods

Convergence defines the iteration algorithm for the formulation of a fixed point of the nonlinear function

- **Main Advantage**

  Steady state problems can be formulated as an algebraic fixed point problem

- **Main Disadvantage**

  The extrapolation algorithm can only be applied to solution vectors

Time-domain methods present several advantages:

- First, they allow the analysis of **highly nonlinear transient** behaviours.
- Second, the input-output characteristics are already in time-domain. Therefore, the system can have **very low dimensions** (the number of equations is equal to the number of nodes in the circuit).

Although time-domain techniques are most practical for analyzing circuits that include only lumped elements, they can be used with a limited variety of distributed elements such as ideal transmission lines. In fact, lossy transmission lines cannot be efficiently modeled in time-domain (even if some physical/electrical "equivalent" models are available in time-domain simulators, e.g., HSpice).

The major limitations of time-domain analysis are:

- Its inability to handle **frequency-domain quantities** (e.g., impedances at different frequencies)
- The difficulty of applying it to circuits having **non-commensurate excitations**.
- The persistent presence of **late time high frequency oscillations** and the **integration step size**.

FREQUENCY-DOMAIN APPROACHES

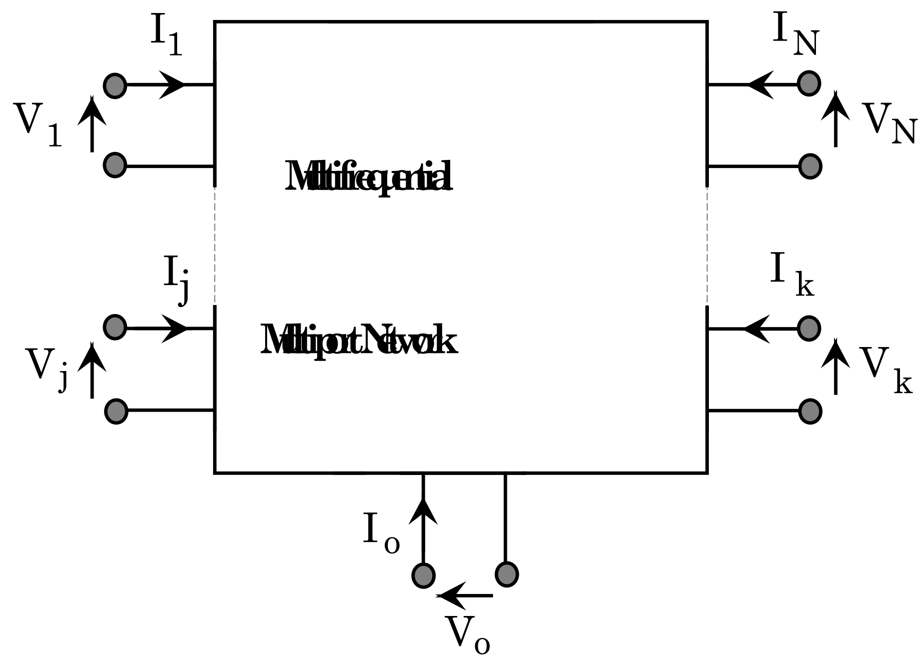
Frequency domain approaches

Frequency-domain techniques are efficient methods to analyze the nonlinear behaviours of weakly nonlinear circuits having multiple non-commensurate small-signal excitations.

The nonlinearities in these circuits are often so weak that they have a negligible effect on their linear response. However, the nonlinear phenomena in such quasi-linear circuits can affect the whole system performance (in terms of intermodulation distortions).

The problem of analyzing such circuits is called the *small signal nonlinear problem.*

FREQUENCY-DOMAIN TECHNIQUES



Frequency Domain → Types:

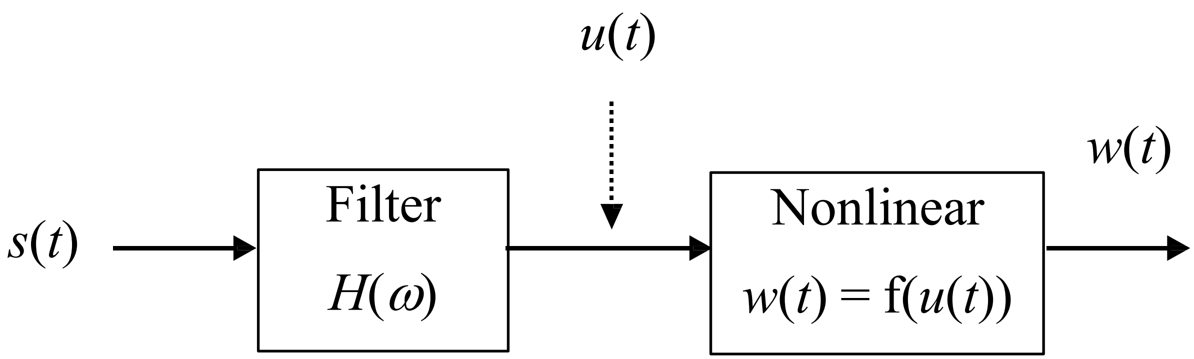
- Power series
- Volterra series
- Arithmetic Operator

The **power series** (which give a good intuitive sense of the behaviour of many types of nonlinear circuits) and the **Volterra series** (similar to the power series, except that is no separation between the memory-less and the reactive parts of the circuit (linear selective circuit)).

Frequency domain approaches

Power series analysis

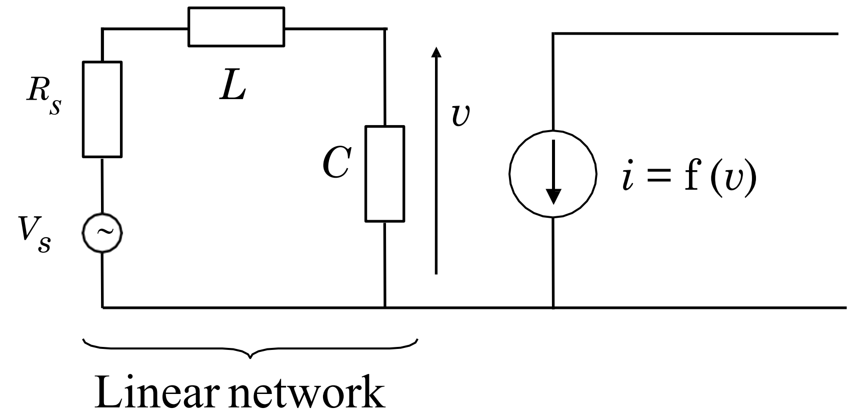
This technique is easy to use but requires a simplifying assumption that is often unrealistic: the circuit should contain only ideal **memory-less transfer nonlinearities** (e.g., no hysteresis effect).



Many nonlinear systems are modeled as a filter, or other linear frequency-selective network $H(\omega)$, followed by a memory-less transfer nonlinearity. The transfer function of the can be written as

$$w(t)=\sum_{k=1}^{N} w_k(t)=f(u(t))=\sum_{k=1}^{N} a_k\, u^k(t)=a_1\, u(t)+a_2\, u^2(t)+a_3\, u^3(t)+\cdots$$

For illustration, let us consider a simplified circuit of a FET.



The input linear transfer function is

$$H(\omega)=\frac{V(\omega)}{V_s(\omega)}=\frac{1}{R_s C\, j\omega - LC\omega^2 + 1}$$

and the nonlinear function { $i = \mathrm{f}(v)$ } can be expended in Taylor series around the dc voltage V_{gs0} across C

$$f(v)=F\left(V_{gs0}+v\right)-F\left(V_{gs0}\right)=v\left.\frac{dF(V)}{dV}\right|_{V=V_{gs0}}+\frac{1}{2}v^2\left.\frac{d^2F(V)}{dV^2}\right|_{V=V_{gs0}}+\frac{1}{6}v^3\left.\frac{d^3F(V)}{dV^3}\right|_{V=V_{gs0}}+\ldots$$

In the case where the excitation contains at least two non-commensurate frequencies

$$s(t) = v_s(t) = \frac{1}{2}\sum_{q=1}^{Q}\left\{V_{s,q}\exp\left(j\,\omega_q\,t\right) + V_{s,q}^{*}\exp\left(-\,j\,\omega_q\,t\right)\right\}$$

$$s(t) = v_s(t) = \frac{1}{2}\sum_{\substack{q=-Q\\ q\neq 0}}^{Q} V_{s,q}\exp\left(j\,\omega_q\,t\right) \quad \Rightarrow \quad u(t) = v(t) = \frac{1}{2}\sum_{\substack{q=-Q\\ q\neq 0}}^{Q} V_{s,q}\,H\left(\omega_q\right)\exp\left(j\,\omega_q\,t\right)$$

In these equations $\{\omega_{-q} = -\omega_q\}$, $\{\, V_{s,-q} = (V_{s,q})^{*} \,\}$ and $\{\, H(\omega_{-q}) = H^{*}(\omega_q)\,\}$.

The output of the nonlinear stage is found by substituting $v(t)$

$$a_n\, v^n(t) = a_n\left[\frac{1}{2}\sum_{\substack{q=-Q\\ q\neq 0}}^{Q} V_{s,q}\,H\left(\omega_q\right)\exp\left(j\,\omega_q\,t\right)\right]^n = \sum_{k=1}^{N} a_k\left\{\sum_{q1=-Q}^{Q}\cdots\sum_{qk=-Q}^{Q} V_{q1}\ldots V_{qk}\,H(\omega_{q1})\ldots H(\omega_{qk})\,e^{\left(\omega_{q1}+\ldots+\omega_{qk}\right)t}\right\}$$

leading to $$w(t) = i(t) = \sum_{n=1}^{N} a_n\, v^n(t)$$

Power series similar to

$$V_o = A V_i + B\, V_i^2 + C\, V_i^3 + \ldots$$

Note that such power series will generate intermodulations:

For instance, *the* combined frequency $\{ 2\omega_1 - \omega_2 \}$ appears at first to be the third order, that is

$$2\omega_1 - \omega_2 = \omega_1 + \omega_1 - \omega_2$$

Therefore, in complicated circuits, a nonlinearity of degree *n* can generate mixing products of order equal to or greater than *n*

Power series

Assume that circuits should contain only ideal memoryless transfer nonlinearities.

This techniques gives a glimpse of behavior of nonlinear circuits.

Some nonlinear circuits can be modeled as filters or frequency selective circuits by a memoryless nonlinear transfer function

- **Main Advantage**

  It allows multi frequency analysis of intermodulation products.

- **Main Disadvantage**

  The mathematical process to expand a given current voltage relationship into power series is difficult, leading to a system of equations of high dimensions.

Volterra series

This technique does not require restrictive assumptions.

It allows multiple frequency analysis and effective evaluation of intermodulation products

The orders of various intermodulation products called as mixing products can be evaluated

- **Main Advantage**

  Steady state: the solution is given directly

- **Main Disadvantage**

  Multiple excitations are small and non-commensurate

Arithmetic operator

Arithmetic operator method is generally seen as the extension to the power series method.

Two basic spectral analysis: multiplication and division.

Combined with spectral addition and subtraction, these operators perform their respective arithmetic operator method entirely in frequency method.

- **Main Advantage**

  Nonlinear elements described by arbitrary method can be easily simulated

- **Main Disadvantage**

  Has varying calculation efficiencies

Frequency-domain methods present several advantages:

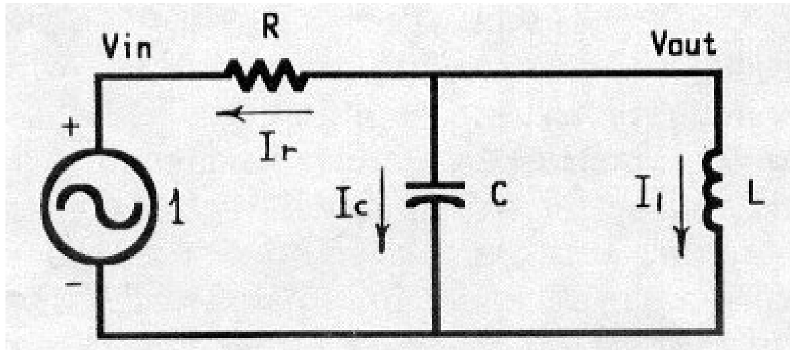
- First, they allow the analysis of circuits with **large-signal excitations**.
- Second, the **spectrum of the output** can be calculated directly if the spectrum of the input is given
- Third, they help formulating the time-domain differential equations into **algebraic frequency-domain** equations.
- Fourth, the **harmonics**' magnitude and phase are **naturally obtained**.

However,

- Signals are represented using the FFT. So **transients cannot be taken into account**.
- The **mathematical process** to expand a given I-V relationship is **difficult**, leading to a **large** system of **nonlinear equations**. Therefore, the analysis should be limited to **weak** nonlinearities.

TIME-DOMAIN VS. FREQUENCY-DOMAIN

LINEAR CASE



FREQUENCY-DOMAIN

$$\left.\begin{aligned} I_r &= \frac{(V_{out} - V_{in})}{R} \\ I_c &= V_{out} \times j\omega C \\ I_d &= \frac{V_{out}}{j\omega L} \end{aligned}\right\} I_r + I_c + I_l = 0$$

$$\Rightarrow \frac{(V_{out} - V_{in})}{R} + V_{out} \times j\omega C + \frac{V_{out}}{j\omega L} = 0$$

TIME-DOMAIN

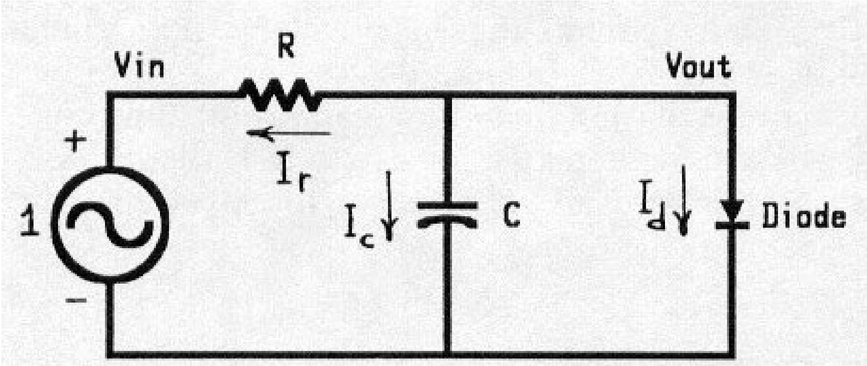
$$\left.\begin{aligned} I_r &= \frac{(V_{out} - V_{in})}{R} \\ I_c &= C\frac{dV_{out}}{dt} \\ I_l &= L\int V_{out}\,dt \end{aligned}\right\} I_r + I_c + I_l = 0$$

$$\Rightarrow \frac{(V_{out} - V_{in})}{R} + C\frac{dV_{out}}{dt} + L\int V_{out}\,dt = 0$$

EASIER IN FREQUENCY-DOMAIN

TIME-DOMAIN VS. FREQUENCY-DOMAIN

NONLINEAR CASE



FREQUENCY-DOMAIN

$$\left.\begin{aligned} I_r &= \frac{(V_{out} - V_{in})}{R} \\ I_c &= V_{out} \times j\omega C \\ I_d &= ???? \end{aligned}\right\} I_r + I_c + I_l = 0$$

$$\Rightarrow \frac{(V_{out} - V_{in})}{R} + V_{out} \times j\omega C + ???? = 0$$

TIME-DOMAIN

$$\left.\begin{aligned} I_r &= \frac{(V_{out} - V_{in})}{R} \\ I_c &= C\frac{dV_{out}}{dt} \\ I_d &= I_s\{\exp(\frac{V_{out}}{V_t}) - 1\} \end{aligned}\right\} I_r + I_c + I_d = 0$$

$$\Rightarrow \frac{(V_{out} - V_{in})}{R} + C\frac{dV_{out}}{dt} + I_S\{\exp(\frac{V_{OUT}}{V_T}) - 1\} = 0$$

EASIER IN TIME-DOMAIN

HYBRID APPROACHES

Hybrid domain approaches

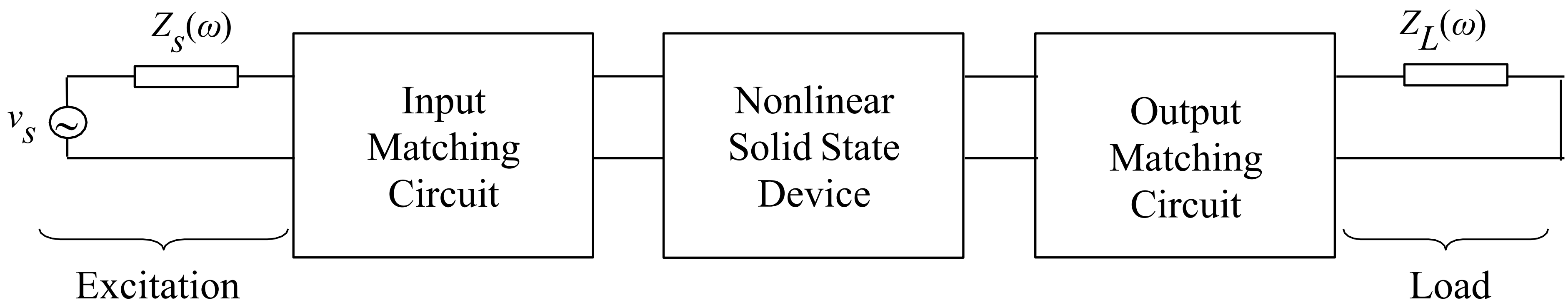
To overcome the limitations imposed by the above all time- or frequency-domain approaches, two important hybrid techniques are used.

The first, the *harmonic balance*, is most useful for both strongly or weakly nonlinear circuits that have **single-tone excitation**. It can be applied to a wide variety of circuits as power amplifiers, frequency multipliers and mixers subjected to local oscillator drive.

The second technique, the *large-signal small-signal analysis*, is used for nonlinear circuits that are **excited by two tones**, **one** of which is **very large** and the **other** is **vanishingly small**. This situation is encountered most frequently in **mixers** wherein a nonlinear element is excited by a large-signal local oscillator and a much smaller received RF signal.

The circuit is first analyzed via harmonic balance, under local oscillator excitation alone, and it converted into a small-signal linear, time-varying equivalent. The time-varying circuit is then analyzed as a quasi-linear circuit under small-signal RF excitation.

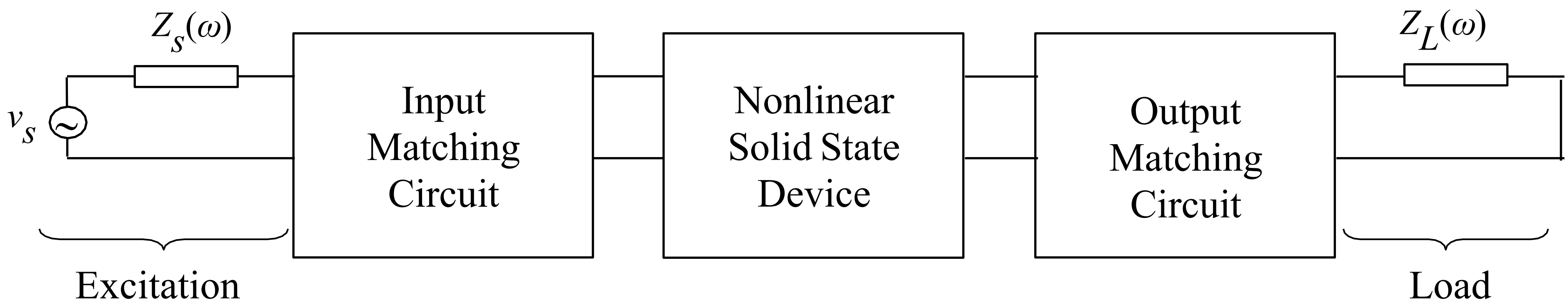
HYBRID APPROACHES



Let consider a basic nonlinear circuit. It consists of:

- nonlinear solid-state devices (often transistor(s) or diode(s)) - the core of the circuit -,
- a load,
- a source of signal frequency large-signal excitation,
- and matching input/output networks.

HYBRID APPROACHES

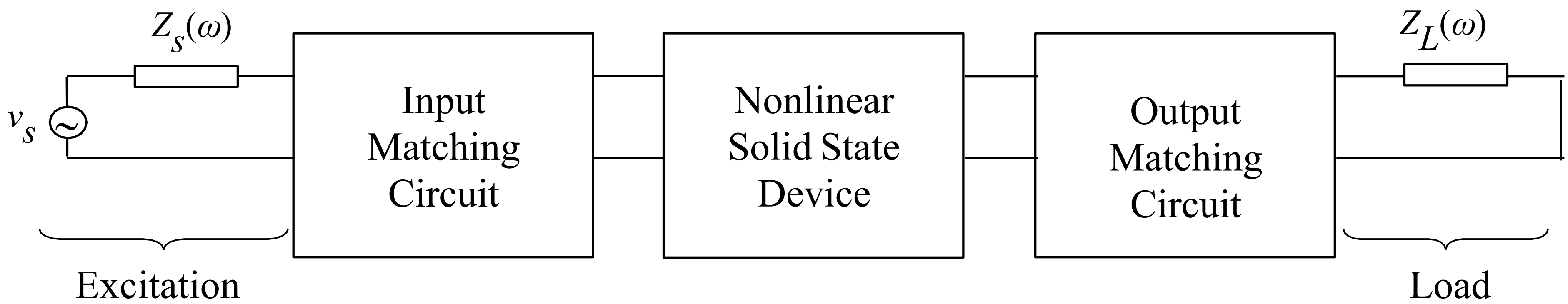


Matching networks are used at the input and output to optimize the circuit performance, to couple bias voltages to the device, to filter and terminate various harmonics appropriately (filters, matching cells, bias circuits, …). The matching networks are invariably linear and the problem of analyzing this type of circuit does not seem difficult (these circuits are usually LC circuits).

solving them in ***Frequency domain*** (Phasors)

Why not in **time-domain**?

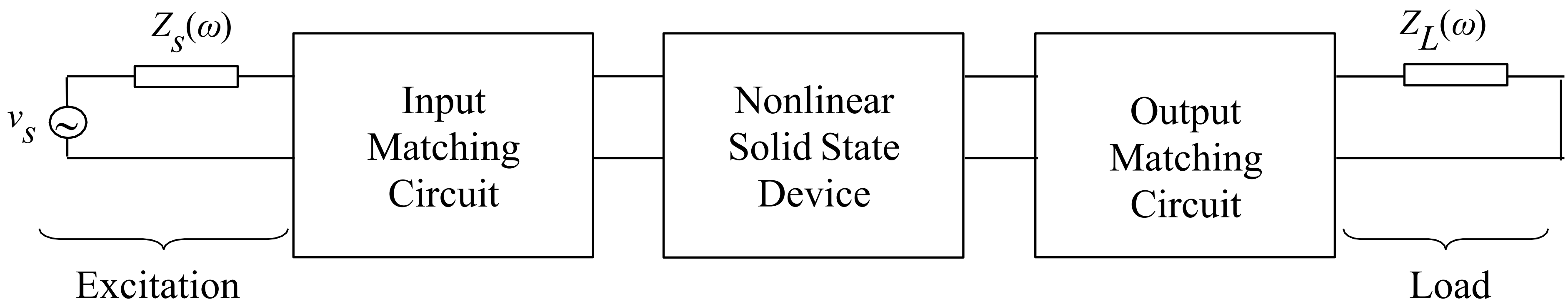
HYBRID APPROACHES



In fact, one could conceive of writing a set of time domain differential equations describing the combined nonlinear device and matching networks, solving them to obtain the steady-state voltage waveform across the load, and Fourier-transforming to obtain the frequency component corresponding to the desired output harmonic.

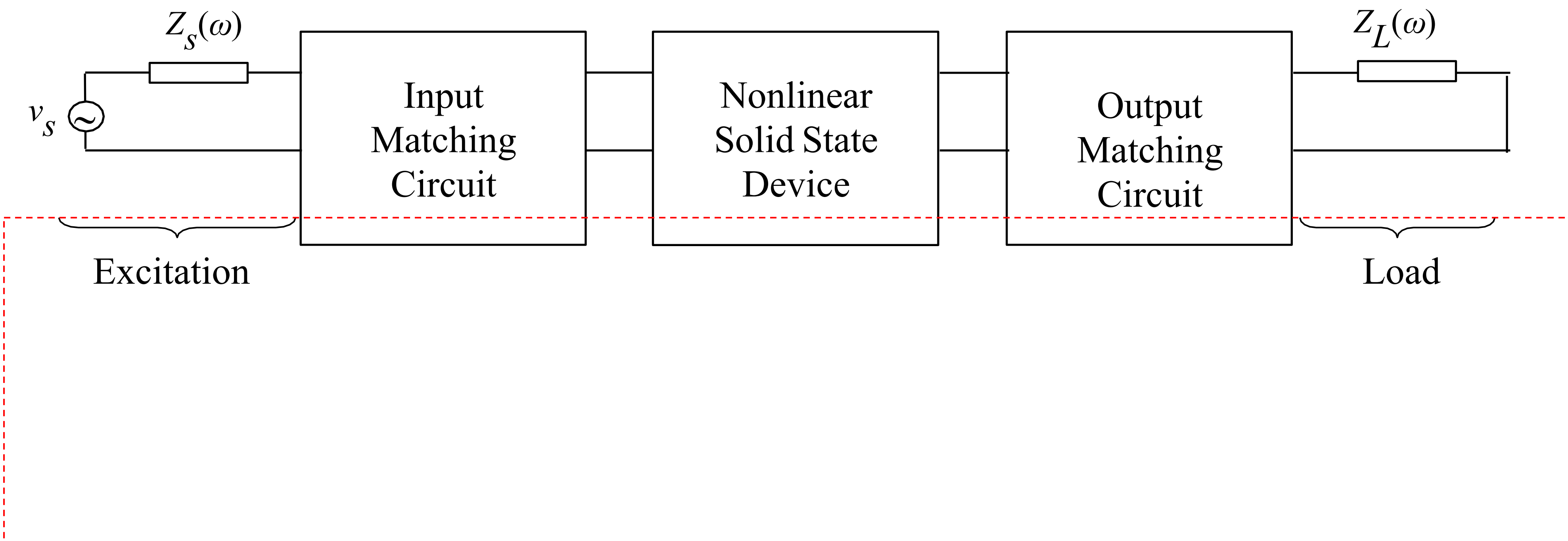
As the differential equations would be nonlinear and have to be solved numerically, **three problems** often occur that can make such time-domain techniques impractical

HYBRID APPROACHES



- The matching circuits may contain dispersive transmission lines and transmission-line discontinuities that are difficult to analyze in time domain.
- The circuit may have time constants that are large compared to the inverse of the fundamental excitation frequency. When these exist, it becomes necessary to continue the numerical integration of the equations through many excitation cycles until the transient part of the response has decayed and **only the steady-state part remains**. As the number of iterations can be very high, this long integration is an extravagant use of computer time and a source of large numerical errors that reduce the accuracy of the solution.
- Each linear and nonlinear reactive element in the circuit adds a **differential equation** to the set of equations that describes the circuit. A large system can have many reactive elements, so the set of equations that must be solved may be very large.

HYBRID APPROACHES

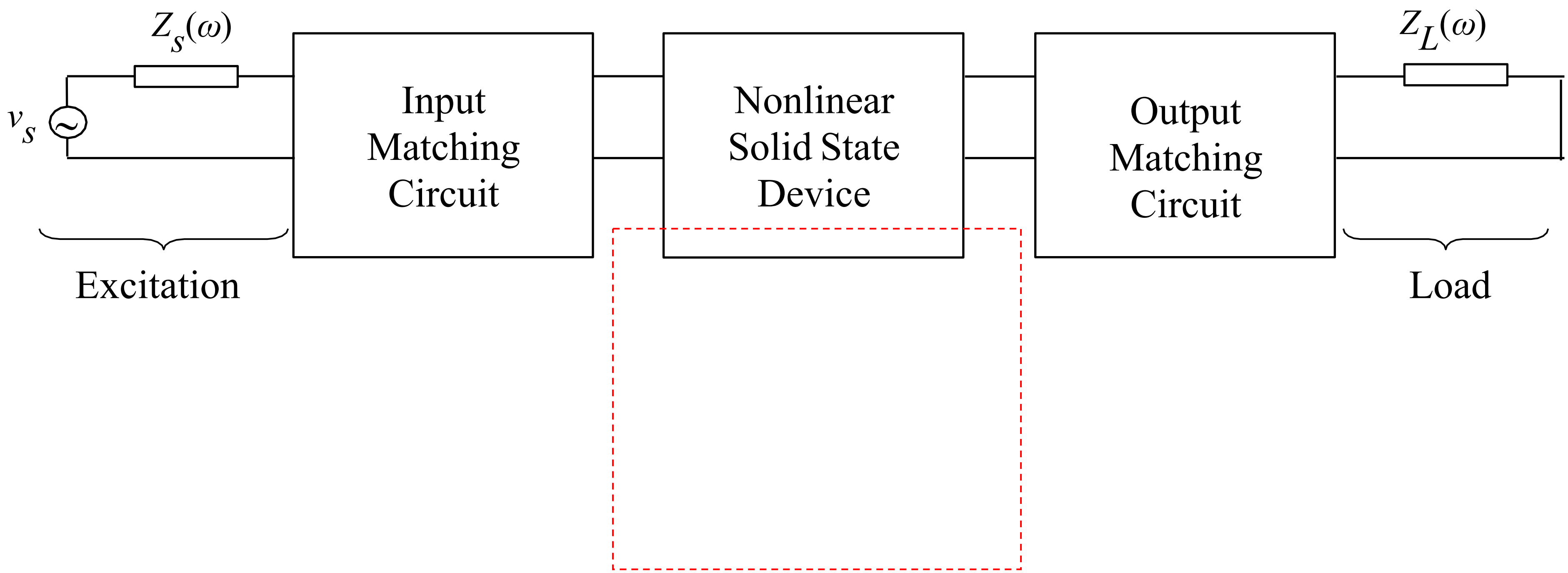


- Matching networks: filters, bias networks, RLC-based circuits
- Load impedance: harmonic dependent
- Source impedance: harmonic dependent

Grouping them in a LINEAR SUB-NETWORK !!

Better/easier to simulate them in Frequency domain !!

HYBRID APPROACHES

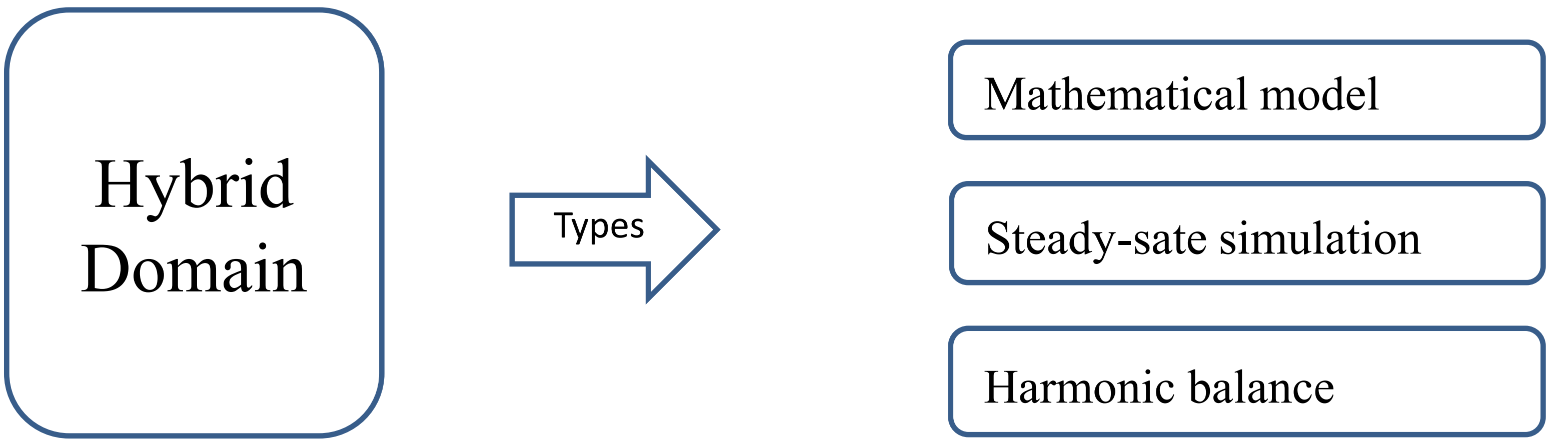


- Nonlinear sub-circuit

Renaming it as NONLINEAR SUB-NETWORK !!

Better/easier to simulate it in Time domain !!

HYBRID TECHNIQUES



The **harmonic balance** is the **most widely** used in the RF/microwave area for **both** linear and nonlinear circuit simulation and design.

HYBRID DOMAIN

- It is highly heterogeneous when operating in multiple time scales.
- It handles the response in time by time step basis.
- Aperiodic information dimension is treated in discrete time domain and periodic carrier dimension is treated in frequency domain.

❑ Main Advantage

Circuits dependent on the number of harmonics for convenient representation in frequency domain.

❑ Main Disadvantage

Knowledge in several numerical methods is needed and the coupling of different methods may lead to unexpected side effects

Mathematical Model

The equations can be extracted from nodal analysis which involves Kirchhoff's Law at each node of the circuit and using with each branch or element in the circuit.

The simulation uses the parameters that can be used by both linear and non linear elements.

- **Main Advantage**

  Used for the formulation of lumped problems

- **Main Disadvantage**

  Mostly used when the size of the circuit is smaller than the wavelength.

Steady State solution

Uses ordinary differential system of equations.

Core of SPICE (simulation program with integrated circuit emphasis).

In the process, no element should be changing with time.

- **Main Advantage**

  All the derivatives with respect to time are equal to zero

- **Main Disadvantage**

  Slow: integrates the whole circuit system from the lowest frequency to the highest frequency

Harmonic Balance

- Most widely used technique for evaluating periodic steady state solution of RF and microwave circuits
- Descriptive method: starts with frequency domain Kirchhoff's current laws chosen by the number of frequencies.
- Represents the coefficients of sinusoid signals in trigonometric series.

❑ **Main Advantage**

Because of the trigonometric series, even a small term of steady-state can be represented

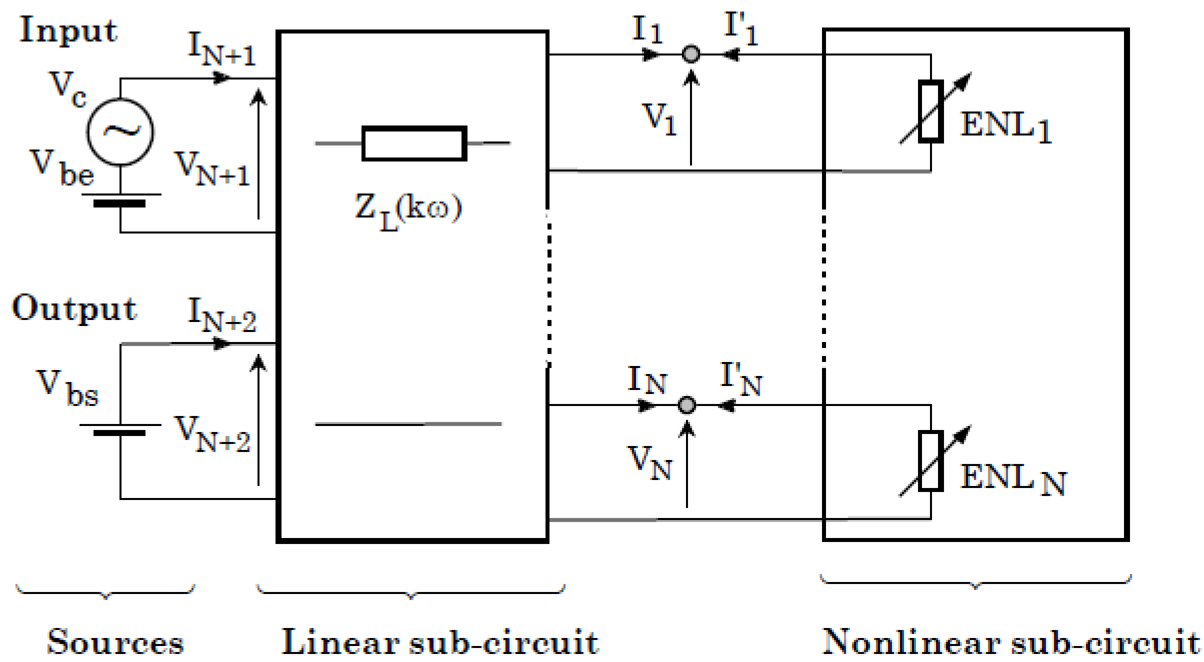
❑ **Main Disadvantage**

Basically operates in frequency domain, no transient behaviour.

MOST USED FREQUENCY- AND TIME-DOMAIN APPROACHES IN NONLINEAR MICROWAVE CAD

		NONLINEAR NETWORK	
		Time Domain	Frequency Domain
LINEAR NETWORK	Time Domain	Extrapolation methods Direct integration methods	
	Frequency Domain	Harmonic Balance [Direct Methods, relaxation and optimum methods, ...]	Volterra Series Power Series Descriptive Functions

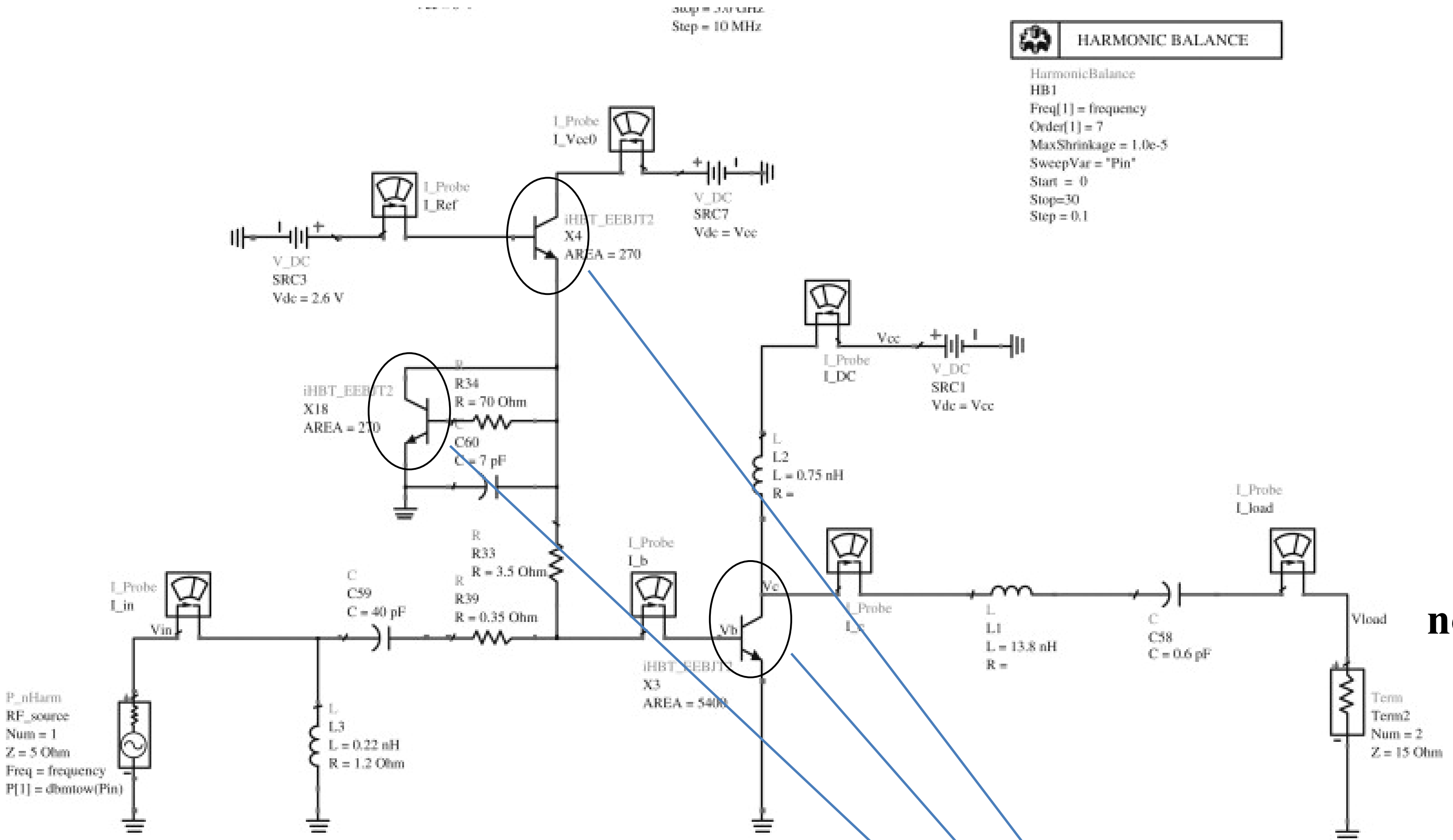
HARMONIC BALANCE



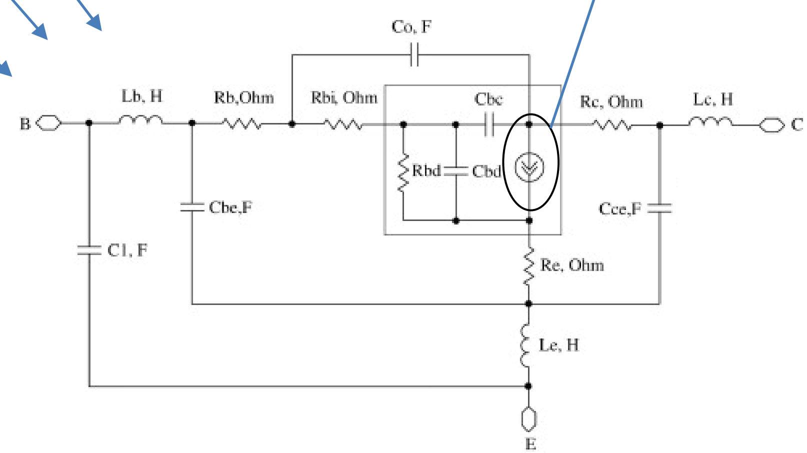
The "Harmonic Balance" is an hybrid technique . based on the principle of dividing the nonlinear circuit into two sub-circuits:

- The first sub-circuit called **linear sub-network** will contain all linear elements (bias networks, filters, matching networks as well as all linear elements of active device equivalent circuits).

- The second called **nonlinear sub-network** will contain all nonlinear elements.



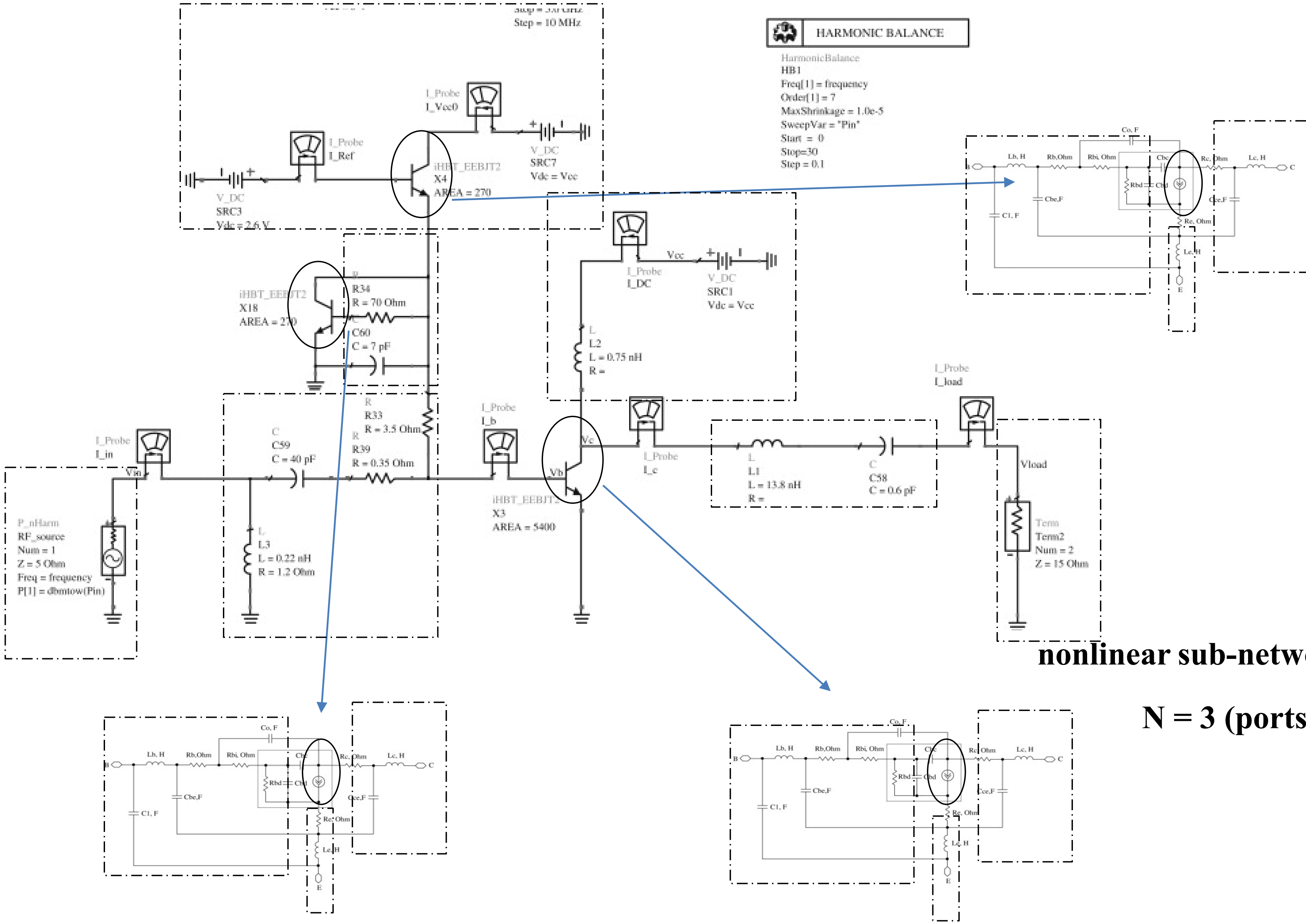
nonlinear sub-network



The rest of the circuit: linear sub-network

??

https://www.sciencedirect.com/topics/engineering/harmonic-balance

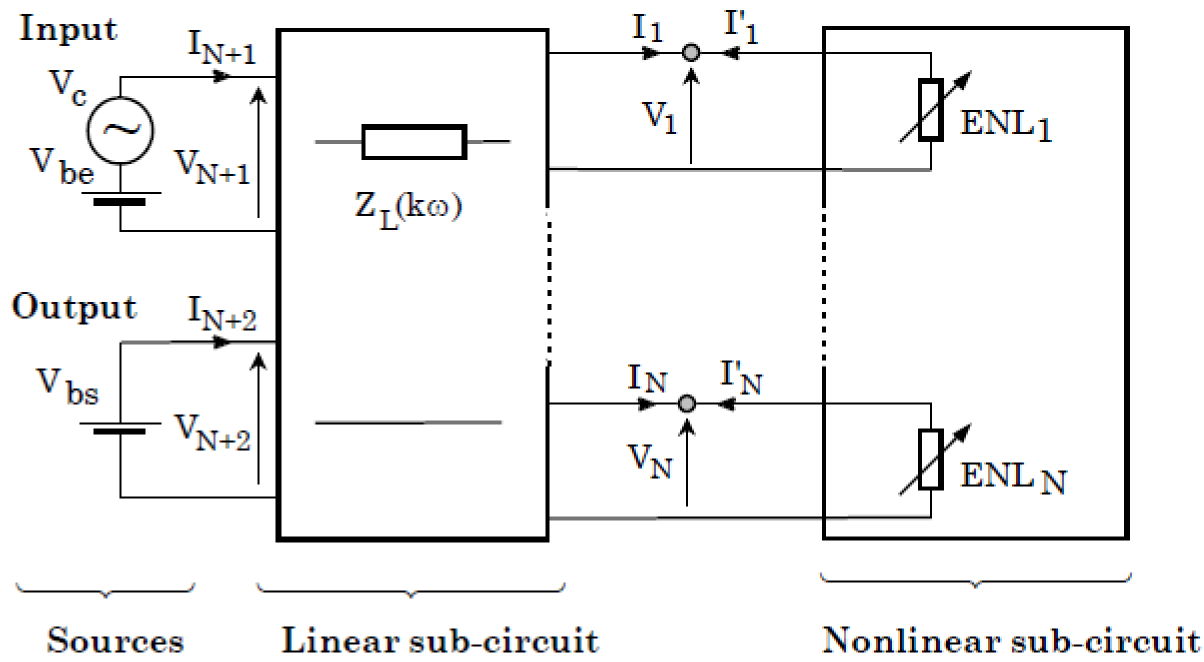


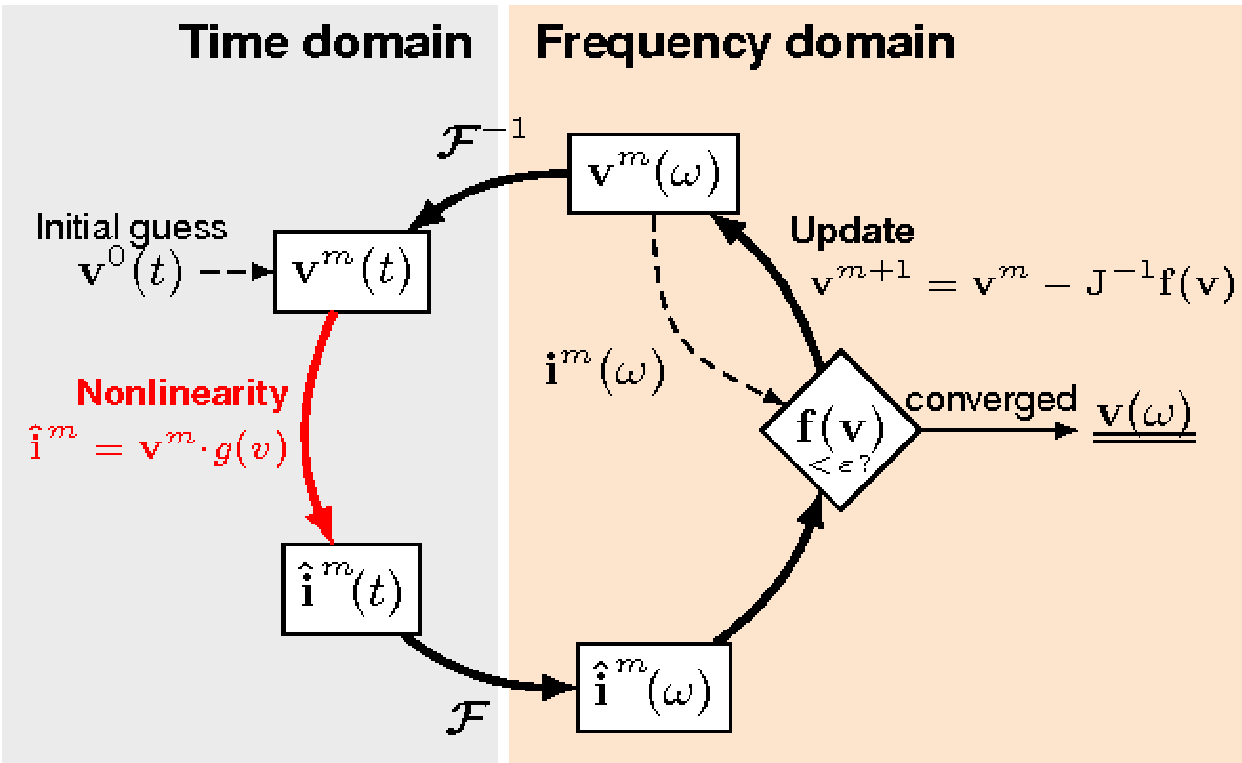
nonlinear sub-network

N = 3 (ports)

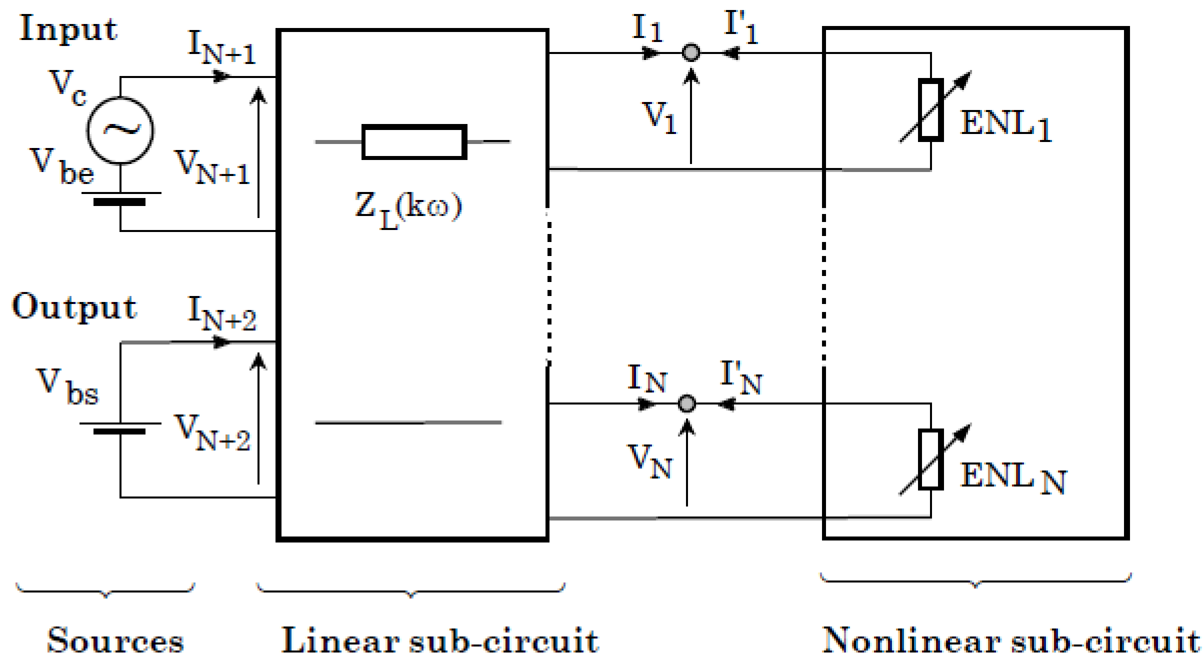
The rest of the circuit: linear sub-network!

https://www.sciencedirect.com/topics/engineering/harmonic-balance http://www.hade.ch/docs/report_HB.pdf





The idea of *harmonic balance* is to find sets of port voltage waveforms (or alternatively, the harmonic voltage components) that give the same currents in both the linear and nonlinear sub-circuits.

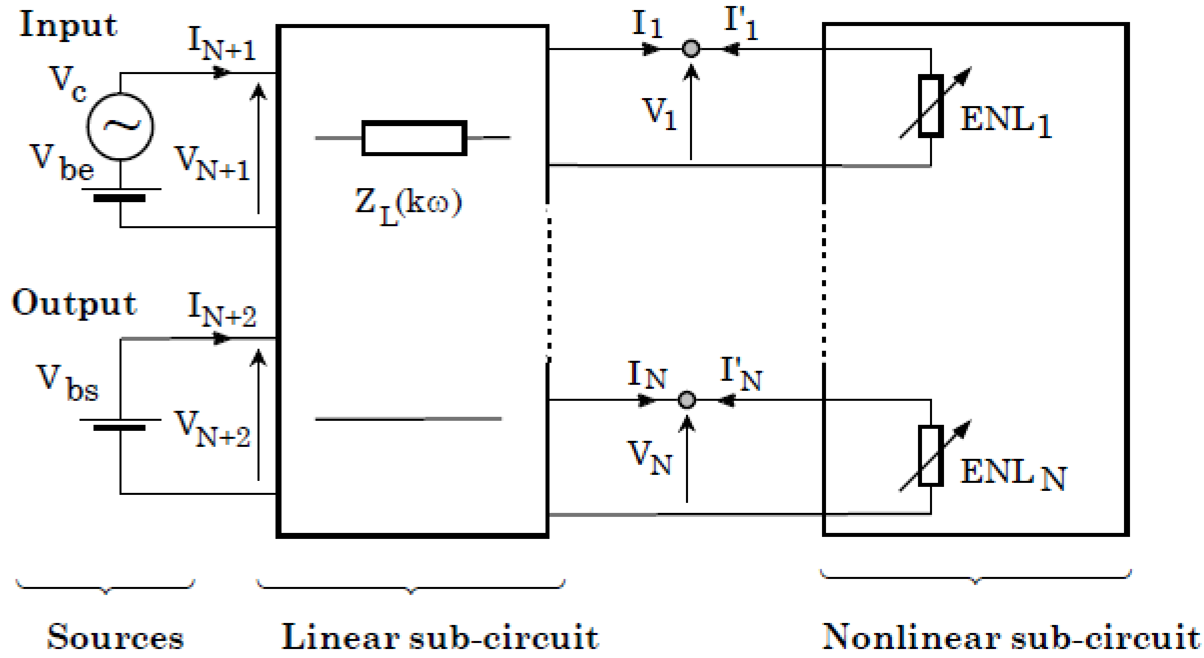


Kirchhoff's current law gives:

$$\begin{bmatrix} I_{1,0} \\ \vdots \\ I_{1,M} \\ \vdots \\ \vdots \\ I_{N,0} \\ \vdots \\ I_{N,M} \end{bmatrix} + \begin{bmatrix} I'_{1,0} \\ \vdots \\ I'_{1,M} \\ \vdots \\ \vdots \\ I'_{N,0} \\ \vdots \\ I'_{N,M} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The vectors include only positive-frequency components because the negative-frequency components, being the complex conjugates of the positive-frequency components, can be found immediately if needed.

First, the admittance matrix of the linear sub-circuit should satisfy the following equation:



$$
\begin{bmatrix} [\mathbf{I}_1] \\ [\mathbf{I}_2] \\ [\mathbf{I}_3] \\ \vdots \\ [\mathbf{I}_N] \\ [\mathbf{I}_{N+1}] \\ [\mathbf{I}_{N+2}] \end{bmatrix}
=
\begin{bmatrix}
[\mathbf{Y}_{1,1}] & [\mathbf{Y}_{1,2}] & \cdots & [\mathbf{Y}_{1,N}] & [\mathbf{Y}_{1,N+1}] & [\mathbf{Y}_{1,N+2}] \\
[\mathbf{Y}_{2,1}] & [\mathbf{Y}_{2,2}] & \cdots & [\mathbf{Y}_{2,N}] & [\mathbf{Y}_{2,N+1}] & [\mathbf{Y}_{2,N+2}] \\
[\mathbf{Y}_{3,1}] & [\mathbf{Y}_{3,2}] & \cdots & [\mathbf{Y}_{3,N}] & [\mathbf{Y}_{3,N+1}] & [\mathbf{Y}_{3,N+2}] \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\
[\mathbf{Y}_{N,1}] & [\mathbf{Y}_{N,2}] & \cdots & [\mathbf{Y}_{N,N}] & [\mathbf{Y}_{N,N+1}] & [\mathbf{Y}_{N,N+2}] \\
[\mathbf{Y}_{N+1,1}] & [\mathbf{Y}_{N+1,2}] & \cdots & [\mathbf{Y}_{N+1,N}] & [\mathbf{Y}_{N+1,N+1}] & [\mathbf{Y}_{N+1,N+2}] \\
[\mathbf{Y}_{N+2,1}] & [\mathbf{Y}_{N+2,2}] & \cdots & [\mathbf{Y}_{N+2,N}] & [\mathbf{Y}_{N+2,N+1}] & [\mathbf{Y}_{N+2,N+2}]
\end{bmatrix}
\begin{bmatrix} [\mathbf{V}_1] \\ [\mathbf{V}_2] \\ [\mathbf{V}_3] \\ \vdots \\ [\mathbf{V}_N] \\ [\mathbf{V}_{N+1}] \\ [\mathbf{V}_{N+2}] \end{bmatrix}
$$

$$
[\mathbf{I}_k] = \begin{bmatrix} I_{k,0} \\ I_{k,1} \\ \vdots \\ I_{k,M} \end{bmatrix}
$$

$$
[\mathbf{V}_k] = \begin{bmatrix} V_{k,0} \\ V_{k,1} \\ \vdots \\ V_{k,M} \end{bmatrix}
$$

The elements of the admittance matrix **[Y]** are all matrices; each sub-matrix is a diagonal, whose elements are the values $Y_{m,n}$ are the admittances $\{ Z_L(k\omega_p) \}^{-1}$ between ports m and n at each harmonic k of the fundamental frequency ω_p

$$\left[\mathbf{Y_{m,n}}\right] = \text{diag}\left\{Y_{m,n}\left(k\omega_p\right)\right\} \qquad k = 0,\ldots,M$$

$$\left[\mathbf{Y_{m,n}}\right] = \begin{bmatrix} Y_{m,n}(0) & 0 & 0 & \cdots & 0 \\ 0 & Y_{m,n}(\omega_p) & 0 & \cdots & 0 \\ 0 & 0 & Y_{m,n}(2\omega_p) & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & Y_{m,n}(M\omega_p) \end{bmatrix}$$

$$\begin{bmatrix} [\mathbf{I_1}] \\ [\mathbf{I_2}] \\ [\mathbf{I_3}] \\ \vdots \\ [\mathbf{I_N}] \\ [\mathbf{I_{N+1}}] \\ [\mathbf{I_{N+2}}] \end{bmatrix} = \begin{bmatrix} [\mathbf{Y_{1,1}}] & [\mathbf{Y_{1,2}}] & \cdots & [\mathbf{Y_{1,N}}] & [\mathbf{Y_{1,N+1}}] & [\mathbf{Y_{1,N+2}}] \\ [\mathbf{Y_{2,1}}] & [\mathbf{Y_{2,2}}] & \cdots & [\mathbf{Y_{2,N}}] & [\mathbf{Y_{2,N+1}}] & [\mathbf{Y_{2,N+2}}] \\ [\mathbf{Y_{3,1}}] & [\mathbf{Y_{3,2}}] & \cdots & [\mathbf{Y_{3,N}}] & [\mathbf{Y_{3,N+1}}] & [\mathbf{Y_{3,N+2}}] \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ [\mathbf{Y_{N,1}}] & [\mathbf{Y_{N,2}}] & \cdots & [\mathbf{Y_{N,N}}] & [\mathbf{Y_{N,N+1}}] & [\mathbf{Y_{N,N+2}}] \\ [\mathbf{Y_{N+1,1}}] & [\mathbf{Y_{N+1,2}}] & \cdots & [\mathbf{Y_{N+1,N}}] & [\mathbf{Y_{N+1,N+1}}] & [\mathbf{Y_{N+1,N+2}}] \\ [\mathbf{Y_{N+2,1}}] & [\mathbf{Y_{N+2,2}}] & \cdots & [\mathbf{Y_{N+2,N}}] & [\mathbf{Y_{N+2,N+1}}] & [\mathbf{Y_{N+2,N+2}}] \end{bmatrix} \begin{bmatrix} [\mathbf{V_1}] \\ [\mathbf{V_2}] \\ [\mathbf{V_3}] \\ \vdots \\ [\mathbf{V_N}] \\ [\mathbf{V_{N+1}}] \\ [\mathbf{V_{N+2}}] \end{bmatrix}$$

$[\mathbf{V_{N+1}}]$ and $[\mathbf{V_{N+2}}]$, the excitation vectors, have the form

$$[\mathbf{V_{N+1}}] = \begin{bmatrix} V_{be} \\ V_c \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad [\mathbf{V}_{N+2}] = \begin{bmatrix} V_{bs} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} [\mathbf{I_1}] \\ [\mathbf{I_2}] \\ [\mathbf{I_3}] \\ \vdots \\ [\mathbf{I_N}] \\ [\mathbf{I_{N+1}}] \\ [\mathbf{I_{N+2}}] \end{bmatrix} = \begin{bmatrix} [\mathbf{Y_{1,1}}] & [\mathbf{Y_{1,2}}] & \cdots & [\mathbf{Y_{1,N}}] & [\mathbf{Y_{1,N+1}}] & [\mathbf{Y_{1,N+2}}] \\ [\mathbf{Y_{2,1}}] & [\mathbf{Y_{2,2}}] & \cdots & [\mathbf{Y_{2,N}}] & [\mathbf{Y_{2,N+1}}] & [\mathbf{Y_{2,N+2}}] \\ [\mathbf{Y_{3,1}}] & [\mathbf{Y_{3,2}}] & \cdots & [\mathbf{Y_{3,N}}] & [\mathbf{Y_{3,N+1}}] & [\mathbf{Y_{3,N+2}}] \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ [\mathbf{Y_{N,1}}] & [\mathbf{Y_{N,2}}] & \cdots & [\mathbf{Y_{N,N}}] & [\mathbf{Y_{N,N+1}}] & [\mathbf{Y_{N,N+2}}] \\ [\mathbf{Y_{N+1,1}}] & [\mathbf{Y_{N+1,2}}] & \cdots & [\mathbf{Y_{N+1,N}}] & [\mathbf{Y_{N+1,N+1}}] & [\mathbf{Y_{N+1,N+2}}] \\ [\mathbf{Y_{N+2,1}}] & [\mathbf{Y_{N+2,2}}] & \cdots & [\mathbf{Y_{N+2,N}}] & [\mathbf{Y_{N+2,N+1}}] & [\mathbf{Y_{N+2,N+2}}] \end{bmatrix} \begin{bmatrix} [\mathbf{V_1}] \\ [\mathbf{V_2}] \\ [\mathbf{V_3}] \\ \vdots \\ [\mathbf{V_N}] \\ [\mathbf{V_{N+1}}] \\ [\mathbf{V_{N+2}}] \end{bmatrix}$$

Leading to

$$\begin{bmatrix} [I_1] \\ \vdots \\ [I_N] \end{bmatrix} = \begin{bmatrix} [Y_{1,N+1}] & [Y_{1,N+2}] \\ \vdots & \vdots \\ [Y_{N,N+1}] & [Y_{N,N+2}] \end{bmatrix} \begin{bmatrix} [V_{N+1}] \\ [V_{N+2}] \end{bmatrix} + \begin{bmatrix} [Y_{1,1}] & & [Y_{1,N}] \\ \vdots & & \vdots \\ [Y_{N,1}] & & [Y_{N,N}] \end{bmatrix} \begin{bmatrix} [V_1] \\ \vdots \\ [V_N] \end{bmatrix}$$

$$\Rightarrow \quad [I] = [I_s] + [Y_{NxN}][V]$$

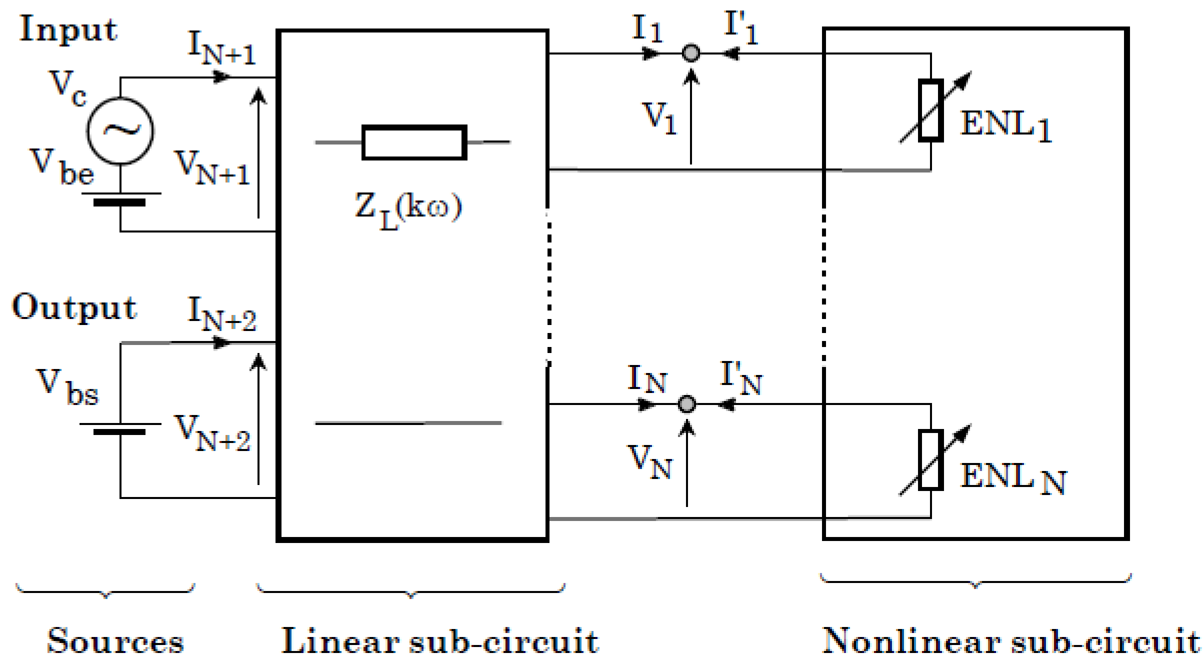
$$\begin{bmatrix} [I_1] \\ [I_2] \\ [I_3] \\ \vdots \\ [I_N] \\ [I_{N+1}] \\ [I_{N+2}] \end{bmatrix} = \begin{bmatrix} [Y_{1,1}] & [Y_{1,2}] & \cdots & [Y_{1,N}] & [Y_{1,N+1}] & [Y_{1,N+2}] \\ [Y_{2,1}] & [Y_{2,2}] & \cdots & [Y_{2,N}] & [Y_{2,N+1}] & [Y_{2,N+2}] \\ [Y_{3,1}] & [Y_{3,2}] & \cdots & [Y_{3,N}] & [Y_{3,N+1}] & [Y_{3,N+2}] \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ [Y_{N,1}] & [Y_{N,2}] & \cdots & [Y_{N,N}] & [Y_{N,N+1}] & [Y_{N,N+2}] \\ [Y_{N+1,1}] & [Y_{N+1,2}] & \cdots & [Y_{N+1,N}] & [Y_{N+1,N+1}] & [Y_{N+1,N+2}] \\ [Y_{N+2,1}] & [Y_{N+2,2}] & \cdots & [Y_{N+2,N}] & [Y_{N+2,N+1}] & [Y_{N+2,N+2}] \end{bmatrix} \begin{bmatrix} [V_1] \\ [V_2] \\ [V_3] \\ \vdots \\ [V_N] \\ [V_{N+1}] \\ [V_{N+2}] \end{bmatrix}$$

$$[\mathbf{I}] = [\mathbf{I_s}] + [\mathbf{Y_{NxN}}][\mathbf{V}]$$

Here, $[\mathbf{Y_{NxN}}]$ is the $N*N$ sub-matrix of $[\mathbf{Y}]$ corresponding to its first N rows and columns. $[\mathbf{I_s}]$ represents a set of current sources in parallel with the first N ports.

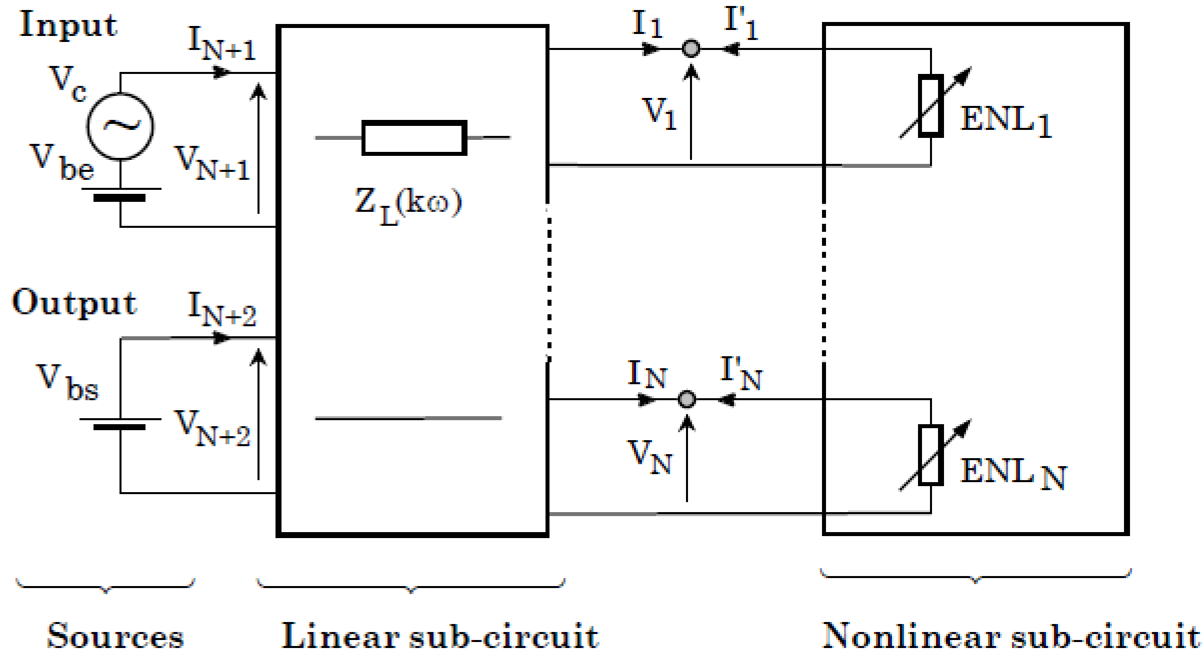
The first matrix term transforms the input- and output-port excitations into this set of current sources, so the N+1th and N+2th ports need not to be considered further.

$$[\mathbf{I}] = [\mathbf{I}_s] + [\mathbf{Y}_{NxN}][\mathbf{V}]$$



The nonlinear element currents, represented by the current vector **[I']** can result from nonlinear capacitors or resistors. Let { $v_o(t)$, ..., $v_N(t)$ } be the time voltage waveforms given by the inverse Fourier transforming $\mathbf{F}^{-1}$ of the voltages at each port of the nonlinear sub-circuit:

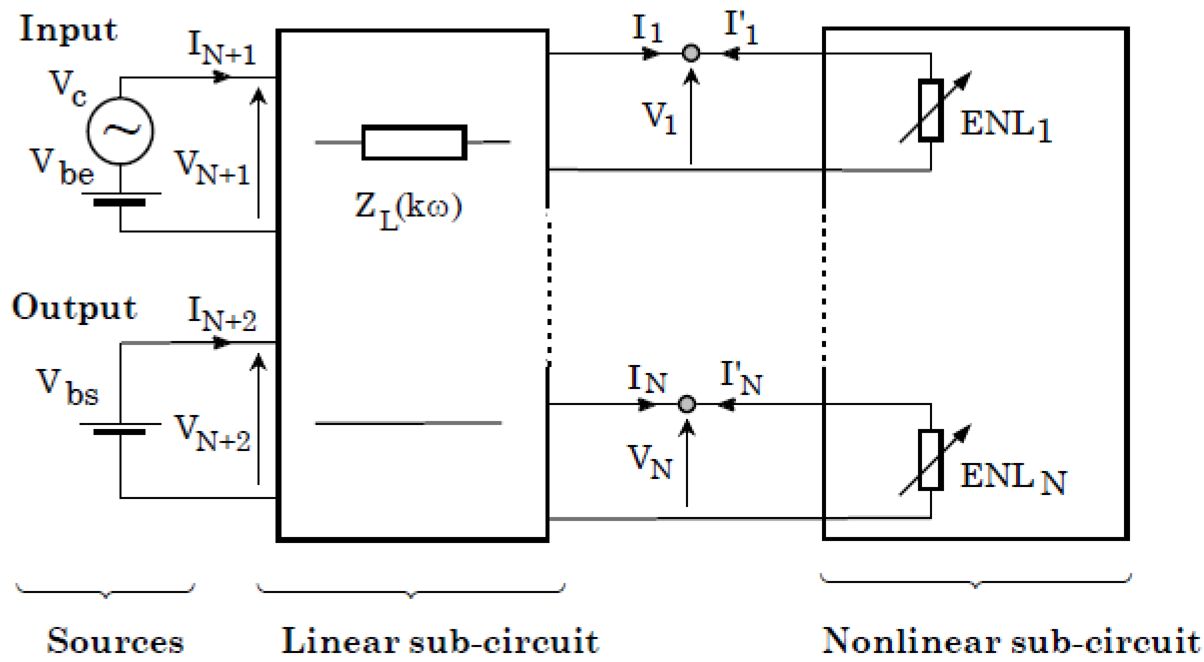
$$\{ \mathbf{F}^{-1}([\mathbf{V}_k]) \rightarrow v_k(t) \}$$



The nonlinear capacitors will be treated first. Because the port voltages uniquely determine all voltages in the network, a capacitor's charge waveforms { $q_o(t)$, ..., $q_N(t)$ } can be expressed as a function of these voltages:

$$q_k(t) = f_{qk}\,(v_o(t), \ldots, v_N(t))$$

$$q_k(t) = f_{qk}\,(v_o(t), \ldots, v_N(t))$$



Fourier transforming the charge waveform at each port gives the charge vectors for the capacitors at each port:

$$\{\ \mathbf{F}\,(q_k(t)) \rightarrow [\mathbf{Q_k}]\ \}$$

$$[\mathbf{Q}] = \begin{bmatrix} [\mathbf{Q_1}] \\ [\mathbf{Q_2}] \\ [\mathbf{Q_3}] \\ \vdots \\ [\mathbf{Q_N}] \end{bmatrix} = \begin{bmatrix} [\mathbf{Q_{1,0}}] \\ \vdots \\ [\mathbf{Q_{1,M}}] \\ \vdots \\ \vdots \\ [\mathbf{Q_{N,0}}] \\ \vdots \\ [\mathbf{Q_{N,M}}] \end{bmatrix}$$

The nonlinear capacitor current is the time derivative of the charge waveform.

Taking the time derivative corresponds to multiplying by $\{ jw_p \}$ in the frequency domain, so

$$i_{c,k}(t) = \frac{dq_k(t)}{dt} \quad \leftrightarrow \quad j\, m\, \omega_p\, Q_{k,m}$$

This equation can be written as

$$[\mathbf{I_c}] = j\,[\boldsymbol{\Omega}]\,[\mathbf{Q}]$$

where **[Ω]** is a diagonal matrix where elements are the pulsations kw_p:

$$[\boldsymbol{\Omega}] = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \omega_p & 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & 2\omega_p & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M\,\omega_p & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \omega_p & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 2\,\omega_p & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & M\,\omega_p \end{bmatrix}$$

This matrix **has *N* cycles** of

$\{ 0, \omega_p, \ldots, M\,\omega_p \}$

along the main diagonal.

N * (M + 1)

frequencies !

Similarly, the current in a nonlinear conductance is

$$i_{g,k}(t) = f_k\,(v_o(t),\,\ldots,\,v_N(t))$$

Fourier transforming these gives $\mathbf{F}\,(i_{g,k}(t)) \rightarrow [\mathbf{I_{G,k}}]$

and substituting the vector $[\mathbf{I_G}]$

$$[\mathbf{I_G}] = \begin{bmatrix} [\mathbf{I_{G1}}] \\ [\mathbf{I_{G2}}] \\ [\mathbf{I_{G3}}] \\ \vdots \\ [\mathbf{I_{GN}}] \end{bmatrix}$$

gives the expression

$$[B\,[\mathbf{V}]] = [\mathbf{I_s}] + [\mathbf{Y_{NxN}}][\mathbf{V}] + [\mathbf{I_g}] + j\,[\Omega][\mathbf{Q}] = [\mathbf{0}]$$

$$\left[B\left[\mathbf{V}\right]\right] = \left[\mathbf{I_s}\right] + \left[\mathbf{Y_{NxN}}\right]\left[\mathbf{V}\right] + \left[\mathbf{I_g}\right] + j\left[\Omega\right]\left[\mathbf{Q}\right] = \left[\mathbf{0}\right]$$

This equation, *called the harmonic balance equation*, represents a test to determine whether a trail set of port voltage components is the correct one; that is if { **[B[V]]** = **[0]** }, then **[V]** is a valid solution. **[B[V]]**, also *called the current-error vector*, represents the difference between the current calculated from the linear and nonlinear sub-circuits, at each port and at each harmonic, for a trial-solution vector **[V]**.

Comparing Harmonic Balance and Time Domain Simulators

In the context of high-frequency circuit and system simulation, harmonic balance has a number of advantages over conventional time-domain transient analysis:

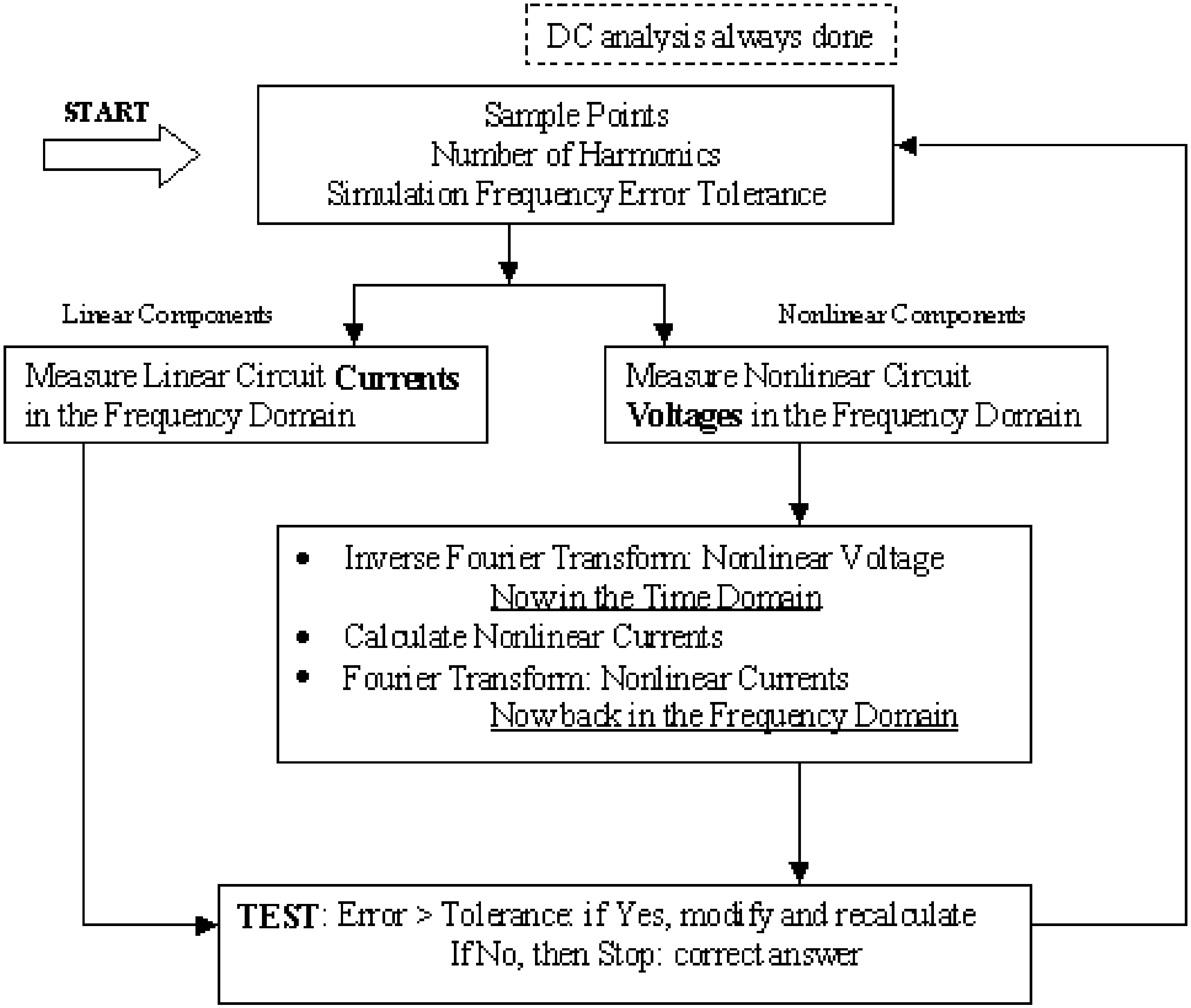
- Designers are usually most interested in a **system's steady-state behavior**.
- Harmonic balance is **faster** at solving typical high-frequency problems that transient analysis can't solve accurately or can only do so at prohibitive costs.
- The applied voltage sources are typically multi-tone sinusoids that may have very narrowly or very widely spaced frequencies. It is not uncommon for the highest frequency present in the response to be many orders of magnitude greater than the lowest frequency. Transient analysis would require an integration over an **enormous** number of periods of the highest-frequency sinusoid. The time involved in carrying out the integration is prohibitive in many practical cases.
- At high frequencies, many linear models are **best represented** in the **frequency domain**. Simulating such elements in the time domain by means of convolution can result in problems related to accuracy, causality, or stability.

http://www.ece.uci.edu/eceware/ads_docs/cktsimhb/ckhb014.html#1104539

THE SIMULATION PROCESS

The harmonic balance method is iterative. It is based on the assumption that for a given sinusoidal excitation there exists a steady-state solution that can be approximated to satisfactory accuracy by means of a finite Fourier series.

- Consequently, the circuit node voltages take on a set of amplitudes and phases for all frequency components.
    - Currents flowing from nodes into **linear elements** are calculated by means of a straightforward **frequency-domain** linear analysis.
    - Currents from nodes into **nonlinear elements** are calculated in the **time-domain**.
    - Generalized Fourier analysis is used to **transform** from the time-domain to the frequency-domain.



http://literature.cdn.keysight.com/litweb/pdf/ads2003a/adshbapp/adshbapp.html

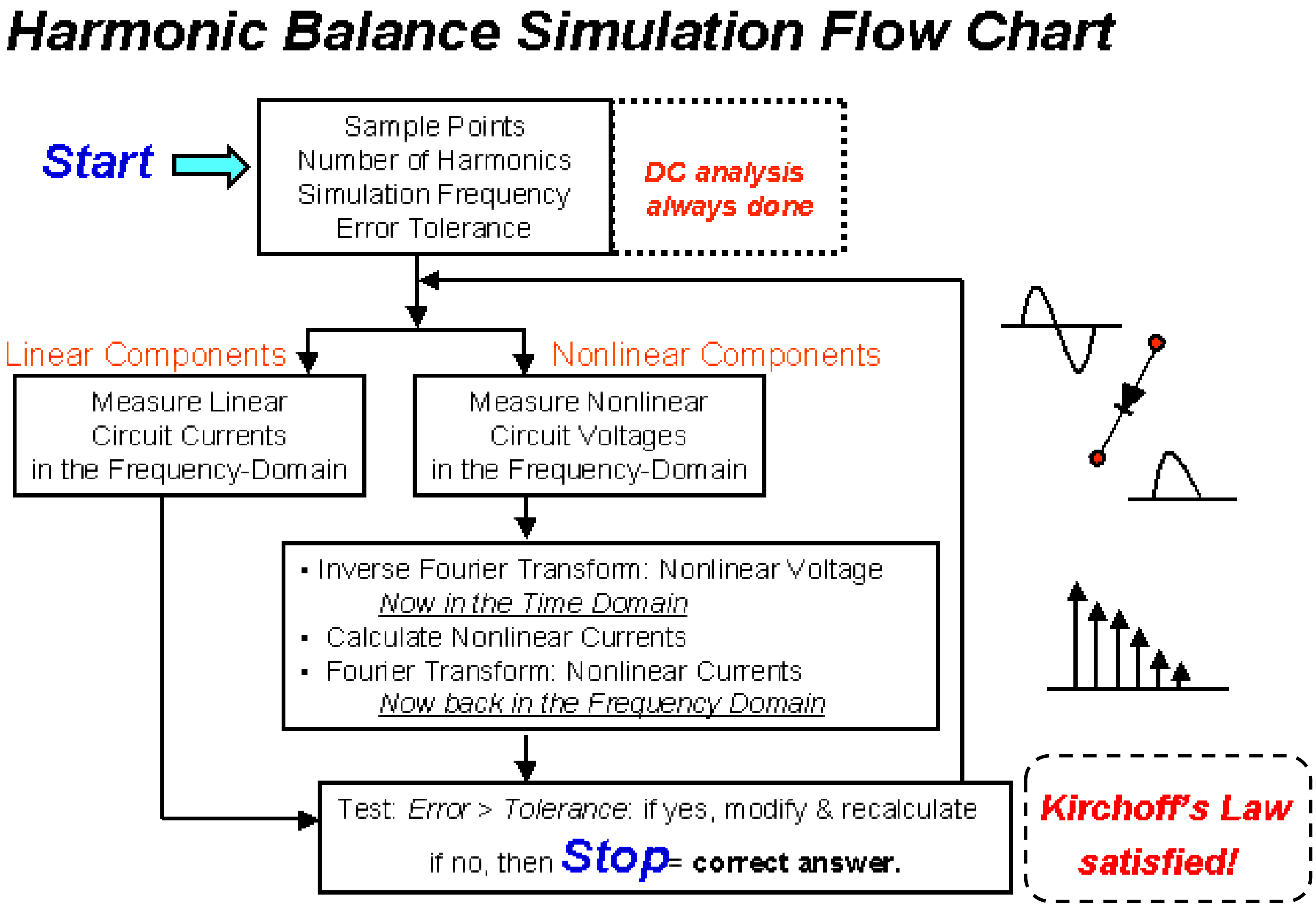
http://www.ece.uci.edu/eceware/ads_docs/cktsimhb/ckhb014.html#1104539

THE SIMULATION PROCESS

The Harmonic Balance solution is approximated by truncated Fourier series and this method is **inherently incapable of representing transient behavior**.

- A frequency-domain representation of all currents flowing away from all nodes is available.
- According to Kirchoff's Current Law (KCL), **these currents should sum to zero at all nodes.**
- Therefore, **an error function is formulated** by calculating the sum of currents at all nodes.

THE SIMULATION PROCESS



Selecting a Solver

- Many harmonic balance simulators rely on the **Newton-Raphson** technique to solve the nonlinear systems of algebraic equations that arise in large-signal frequency-domain circuit simulation problems. Each iteration of Newton-Raphson requires an **inversion of the Jacobian matrix** associated with the nonlinear system of equations. When the matrix is factored by direct methods, memory requirements climb as $O(H^2)$, where H is the number of harmonics. Thus, the factorization of a Jacobian at H=500 will require 2500 times as much RAM as one at H=10.

- An alternate approach to solving the linear system of equations associated with the Jacobian is to use a **Krylov subspace** iterative method that offers substantial savings in memory requirements for large harmonic-balance problems. Similar arguments show that even larger increases in computational speed can be obtained.

Note For circuits involving large numbers of frequencies, consider using the Circuit Envelope simulator.

Selecting a Solver

Use the following guidelines when selecting a solver:

- Direct Solver

  The *Direct Solver* option is recommended for the majority of small problems (circuit with relatively few nonlinear components). In such cases the Direct Solver is **not only faster, but also exhibits superior convergence**.

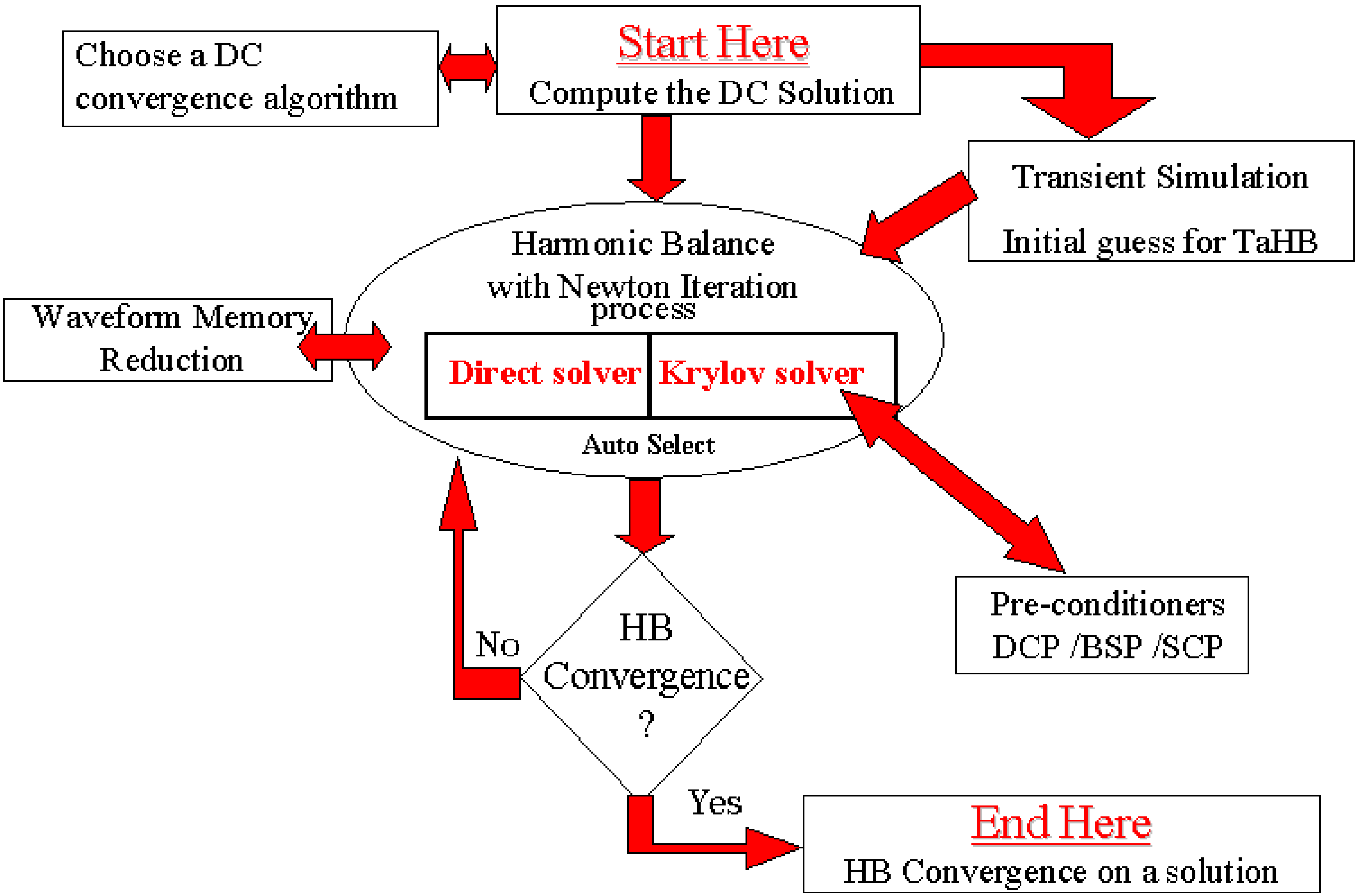
- Krylov Solver

  The *Krylov Solver* option should be used when solving ***large problems*** (large number of nonlinear components and/or large number of harmonics required for simulation). Krylov is **less robust** than the Direct Solver method because it uses iterative algorithms to solve the matrix equations.

- Auto Select

  Selecting this option allows the simulator to choose which solver to use. The simulator analyzes factors such as circuit or spectral complexity and compares memory requirements for each solver against the available memory.

HB Frequency-domain Circuit Simulation Flow



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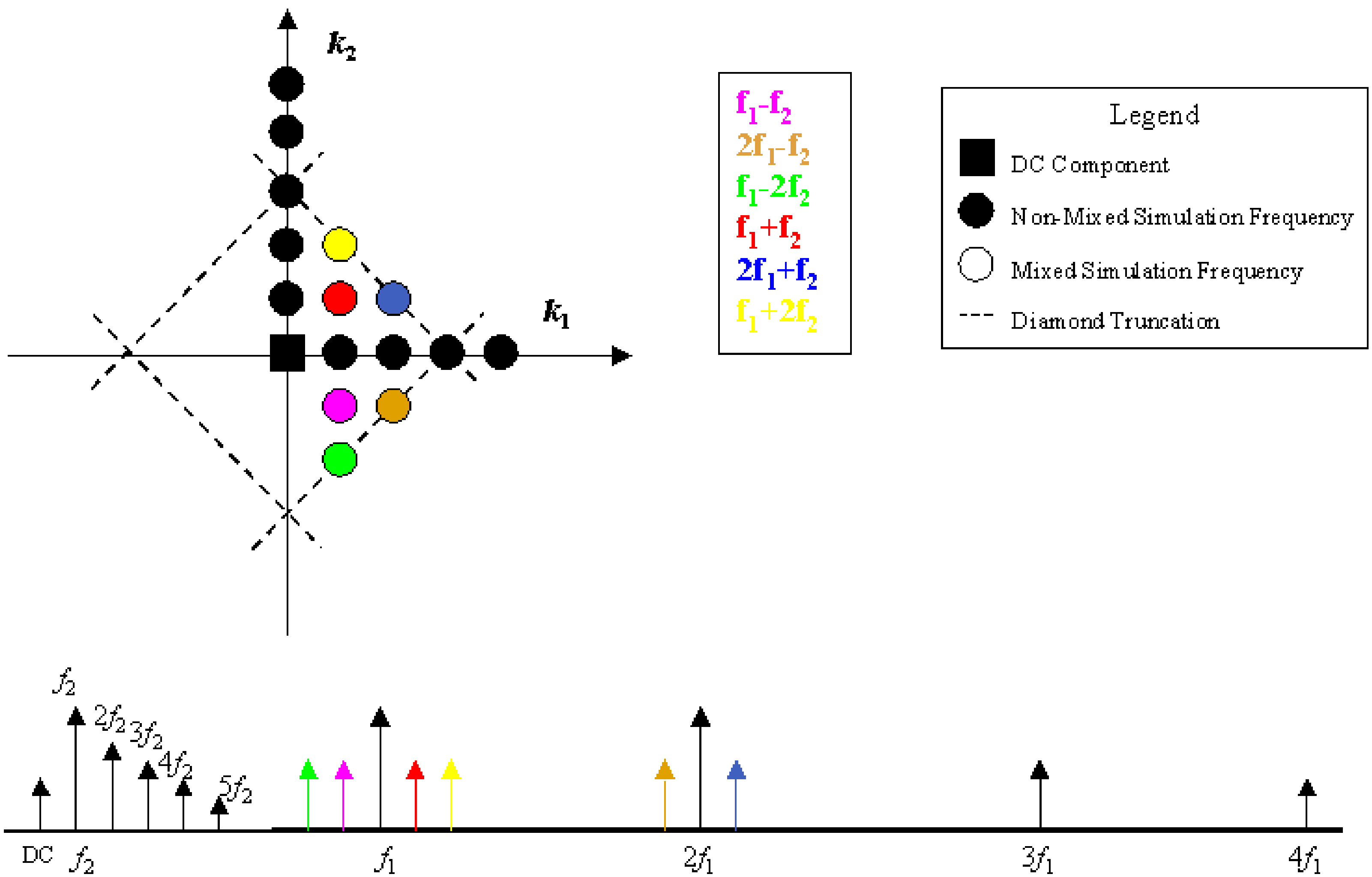
Intermodulation Products Involved and Truncation



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Solutions algorithms

A number of algorithms have been proposed for solving HB equation. Initiated by Kryloff and Bogoliuboff, the harmonic balance technique was applied to microwave circuit CAD by Egami who utilized an iterative technique, namely the Newton algorithm, to minimize the error. Afterwards, many authors like Kerr, Hicks and Khan, and Camacho-Penalosa developed several relaxation methods to solve the harmonic balance equation.

Optimization methods

Since solving the HB equation looks like an optimization problem, it is possible to solve it by minimizing the magnitude of the current-error function. An advantage of this approach is that optimization routines are widely used by designers. However, most optimization routines are relatively slow and may have convergence problems.

Solutions algorithms

Newton's method

Newton's method (or its variants the Newton-Raphson or the Quasi-Newton method) is a very powerful algorithm for finding the zeros of a multivariate function. It is an iterative technique: it finds the zero of a function by using its first derivative to extrapolate to the axis of the independent variable and repeating the process until the zero is found.

Splitting methods

A number of relaxation methods, which are both simple to implement and intuitively satisfying, have been proposed. Two of the most popular are those of Hicks and Khan, and Kerr (also called the reflection algorithm).

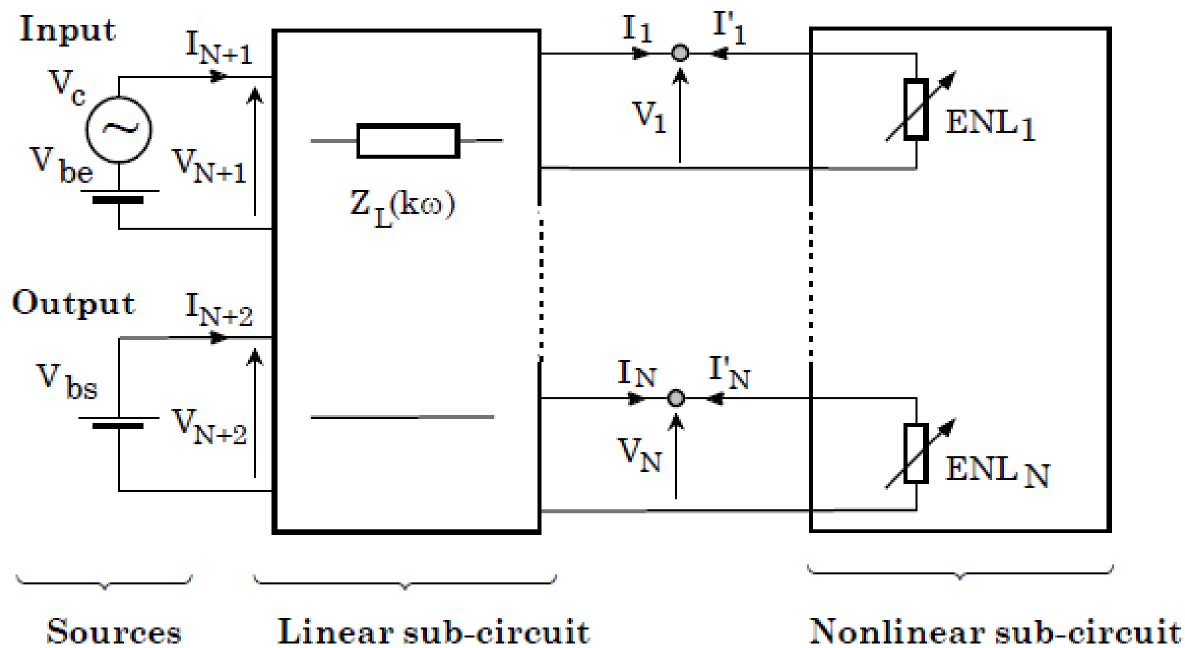
Splitting methods: Method of Hicks and Khan

- Estimate the initial voltage vector **[V°]** of the linear sub-circuit,
- Inverse-Fourier transform **[V°]** to get its time-domain waveforms : { $\mathbf{F^{-1}}$ (**[V°]**) → **[V(t)°]** }
- Deduce the time-domain waveforms of the current vector **[I'(t)°]** for the nonlinear sub-circuit,
- Fourier transform **[I'(t)°]** to obtain the frequency domain current vector **[I'°]**: { **F** (**[I'(t)°]**) → **[I'°]** }
- Assume the current vector **[I°]** of the linear sub-circuit is equal to { - **[I'°]** },
- The linear sub-circuit having an impedance matrix $[\mathbf{Z_{NxN}}]$, the voltage vector **[V']** of the nonlinear sub-circuit is then $$[\mathbf{V'}] = [\mathbf{Z_{NxN}}]\left\{[\mathbf{I^o}] - [\mathbf{I_s}]\right\}$$
- Form the new estimate voltage vector $[\mathbf{V^1}]$ of the linear sub-circuit as $$[\mathbf{V^1}] = s\,[\mathbf{V'}] + (1-s)\,[\mathbf{V^o}]$$

  where s is a real positive constant ($0.0 < s < 1.0$). The vector **[V°]** is then replaced by $[\mathbf{V^1}]$ and the process is repeated until minimal change in the voltage vector is observed between iterations.

The method is simple, but the variable s, called the splitting coefficient, is a constant that must be determined empirically. Small values of s favor slow but monotonic convergence; increasing s gives faster convergence up to the point at which oscillation begins. **Typically, s = 0.2.**

Splitting methods: Reflection algorithm

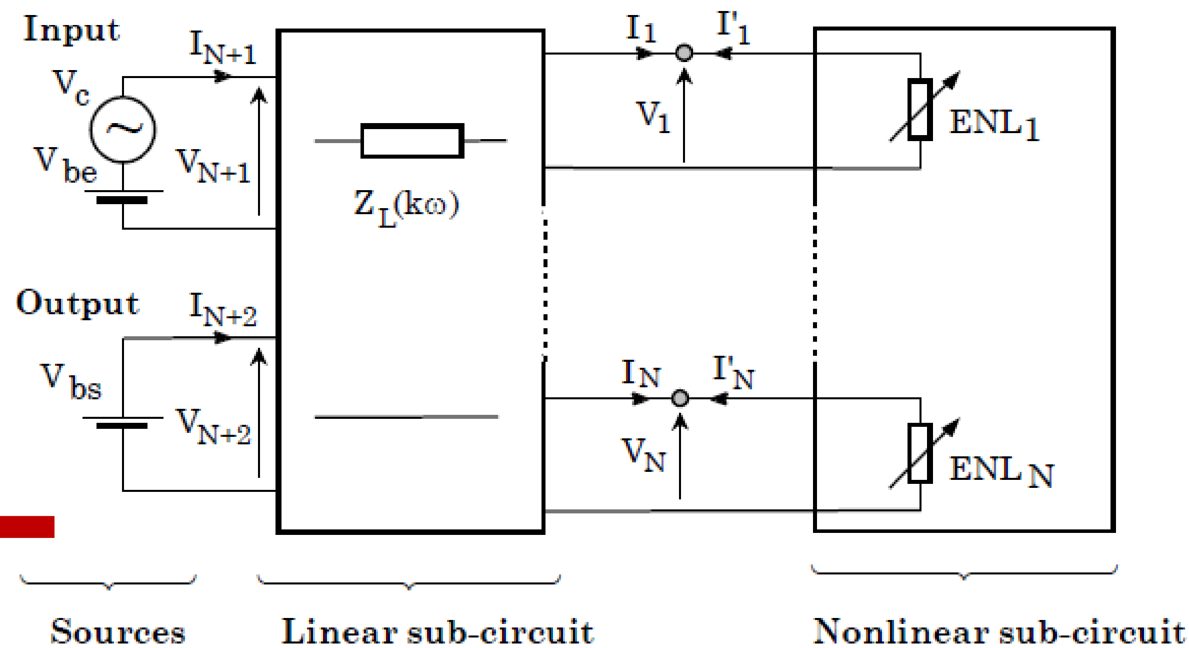


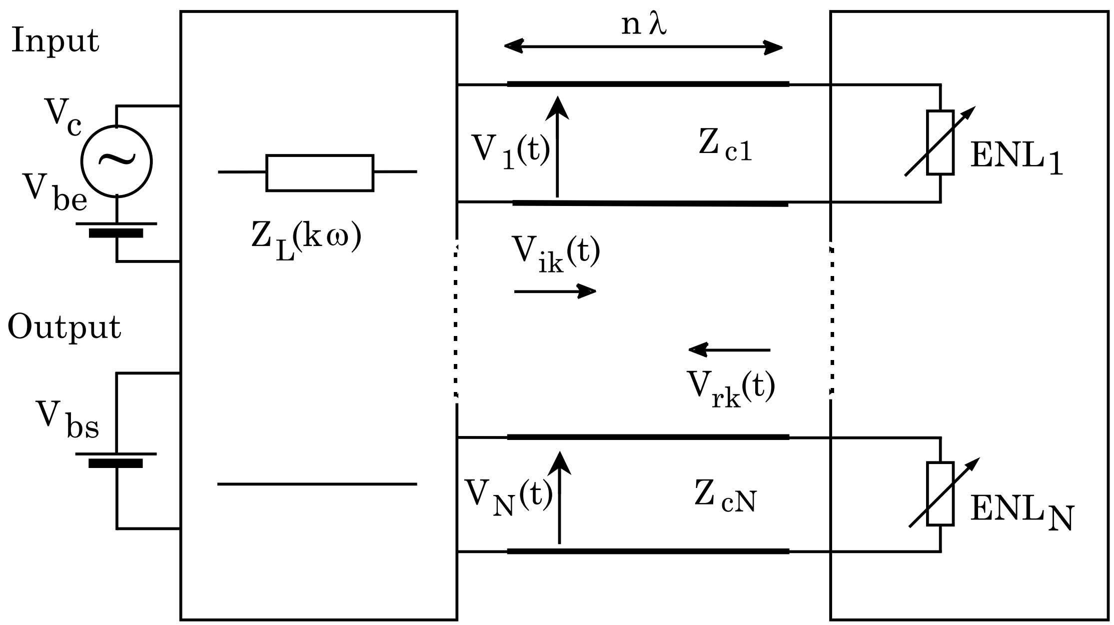
All the above methods are based on mathematical aspects, without any direct relation with the physical meaning of the problem. The reflection algorithm solves the harmonic balance equations via a process that mimics the turn-on process of a real circuit. To implement the reflection algorithm, the equivalent circuit is redrawn.

Splitting methods: Reflection algorithm

Adding a set of transmission lines between the ports of the linear and nonlinear sub-circuits.

These lines are assumed to be ideal and very long at the fundamental excitation frequency [large integer number of wavelengths long ($n*\lambda$)].

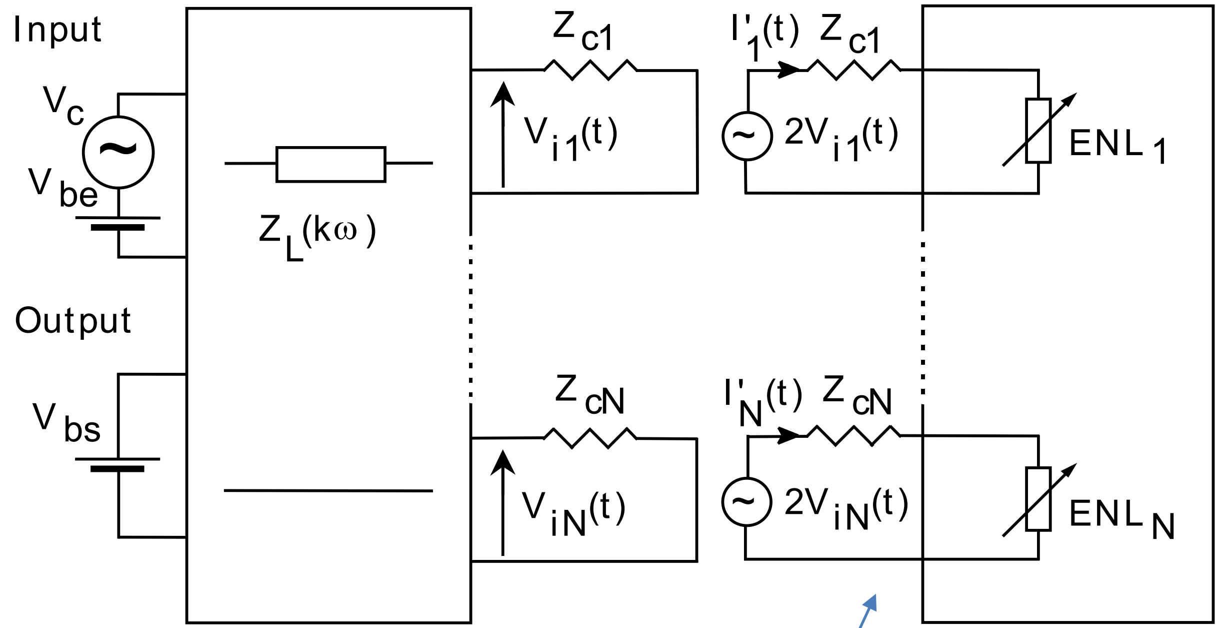


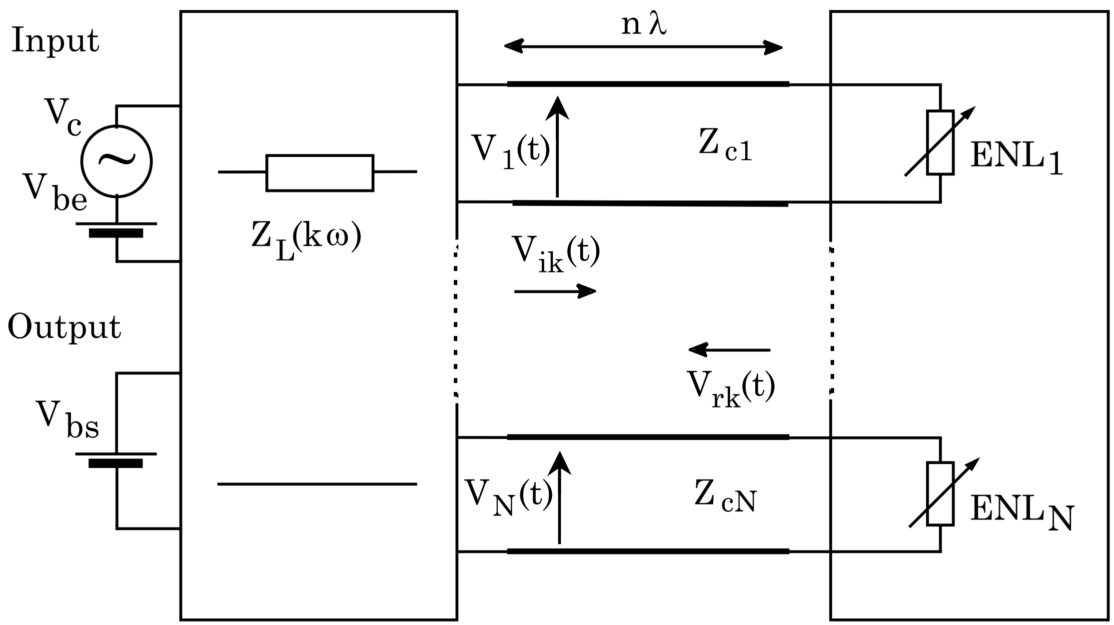


Thus, they do not affect the circuit's steady-state response (infinite ideal transmission lines).

The ruse of **long** ideal transmission lines allows the circuit to be separated into two parts.

At the instant the circuit is turned on, no reflected wave exists, so the input impedance of the transmission line is equal to its characteristic impedance.





When the incident wave reaches the nonlinear sub-circuit, the transmission line interface has the equivalent circuit shown on the right side of the Figure.

LARGE-SIGNAL SMALL-SIGNAL ANALYSIS

Large-signal - small-signal analysis, or conversion matrix analysis, is the second nonlinear approach for analyzing nonlinear circuits. It is useful for a large class of problems wherein a nonlinear device is pumped by a single large sinusoidal signal and another signal, which is assumed to be much smaller, is also applied.

The most common application of this technique is in the design of microwave mixers, but it is also applicable to such circuits as modulators, parametric amplifiers, and parametric up-converters.

The process involves *first* analyzing the nonlinear device under large-signal excitation *only*, usually via the harmonic-balance method.

This is called the large signal analysis.

The nonlinear element or elements in the device's equivalent circuit are then converted to small-signal, linear, time-varying elements and the small-signal analysis is performed, without further consideration of the large-signal excitation.

This is called the small signal analysis.

LARGE-SIGNAL SMALL-SIGNAL ANALYSIS

The assumption of small-signal linearity requires that the quasi-linear response be of primary interest, but these techniques are also useful, with significant modifications, to circuits wherein quasi-linearity is not assumed.

Mixing process

In general, a nonlinear element excited by two tones supports currents and voltages at the mixing frequencies

$$\omega_{m,n} = \pm m * \omega_1 \pm n * \omega_2$$

where m and n can be positive and negative integers, including zero.

In the present case, one of the tones, ω_{RF}, is at such a low level that is does not generate harmonics, and the other is a large-signal sinusoid at ω_p. Then, the mixing frequencies are

$$\omega = \pm \omega_{RF} + n\omega_p$$

This equation represents the set of frequency components which consists of two tones on either side of each large-signal harmonic frequency, separated by $\{ \omega_o = | \omega_p - \omega_{RF} | \}$.

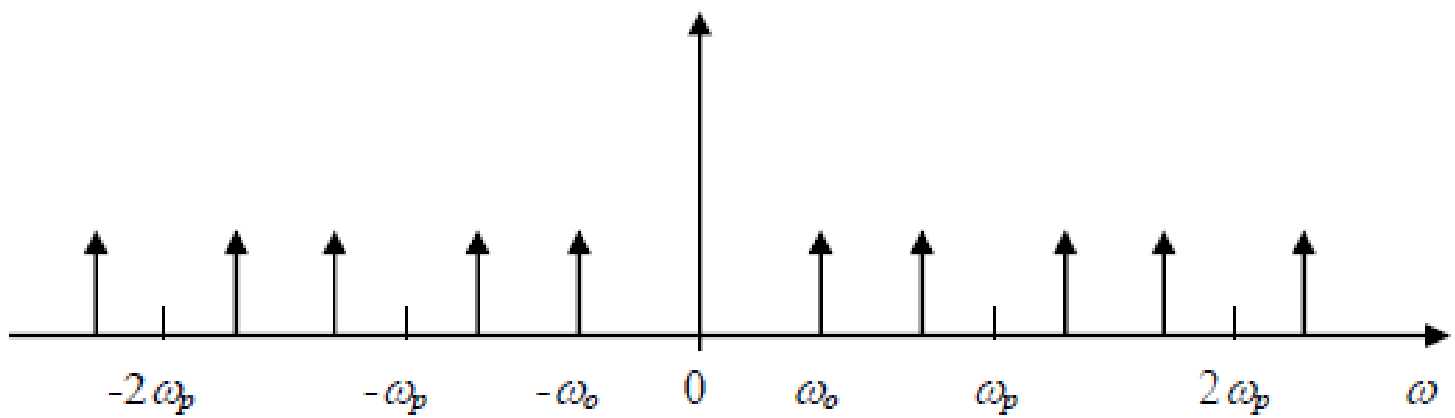
A more convenient representation of the mixing frequencies is

$$\omega_n = \omega_o + n\omega_p$$

The frequency domain currents and voltages in a time varying circuit element are related by a conversion matrix.

The small-signal voltage and current can be expressed in the frequency notation as follows

$$V(t) = \sum_{k=-\infty}^{\infty} V_k e^{jk\omega t} = \sum_{k=-\infty}^{\infty} V_k e^{j\omega_k t} \qquad I(t) = \sum_{k=-\infty}^{\infty} I_k e^{jk\omega t} = \sum_{k=-\infty}^{\infty} I_k e^{j\omega_k t}$$



SMALL/LARGE SIGNAL ASSUMPTIONS

- The small-signal excitation does not perturb the time-varying operating point of the circuit;
- The transfer function from the small-signal excitation to every node in the circuit is linear.

Harmonic balance useful for single tone strongly or weakly nonlinear circuit

Small-signal/Large-signal useful for two tone nonlinear circuits (if one signal is much larger than the second one)

HARMONIC BALANCE FEATURES

- The simulation time is relatively insensitive to the numerical values of the excitation frequencies
- Accurate and fast when all signals in the steady- state solution can be approximated using a small number of Fourier coefficients.
- Frequency-dependent distributed elements are handled with no relative difficulty
- Parameter sweep can be performed very efficiently.

SMALL/LARGE SIGNAL FEATURES

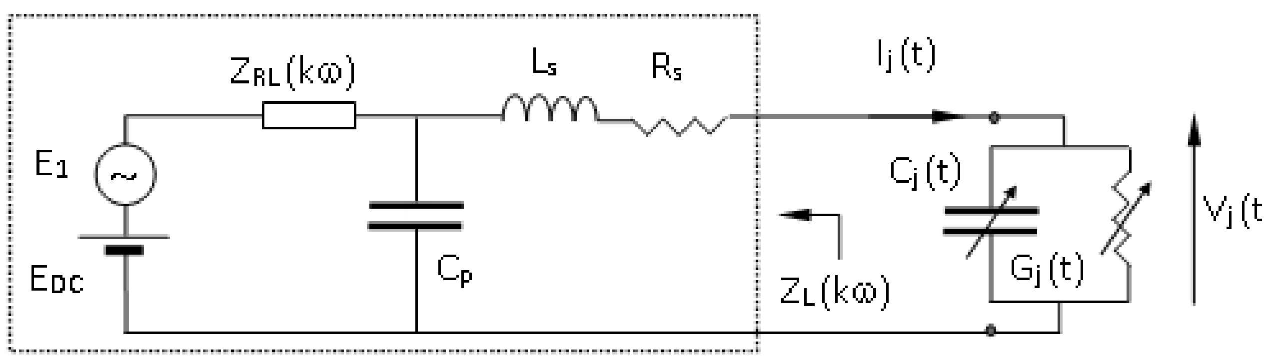
- Useful for non linear devices pumped by both a large signal (pump) and a small signal (input excitation)
- Large signal: analyzing the nonlinear device under large-signal excitation only.
- Small signal: convert circuit elements to small signal elements without large signal consideration.

EXAMPLE : DIODE CIRCUIT ANALYSIS

In order to illustrate this process, we will consider the case of a mixing diode, e.g. the Schottky diode.

1– Large signal analysis

The goal of the large signal analysis is to determine the Fourier coefficients of the nonlinear elements. Assuming that the parasitic elements R_s, L_s and C_p are constant up to the millimeter range, they can be included in the linear sub-circuit. Therefore, the nonlinear elements are the junction capacitance C_j and the junction conductance G_j.



1– Large signal analysis

For each harmonic $k\omega_p$ of the excitation signal, the junction voltage $V_j(t)$ and current $I_j(t)$ can be found by expending

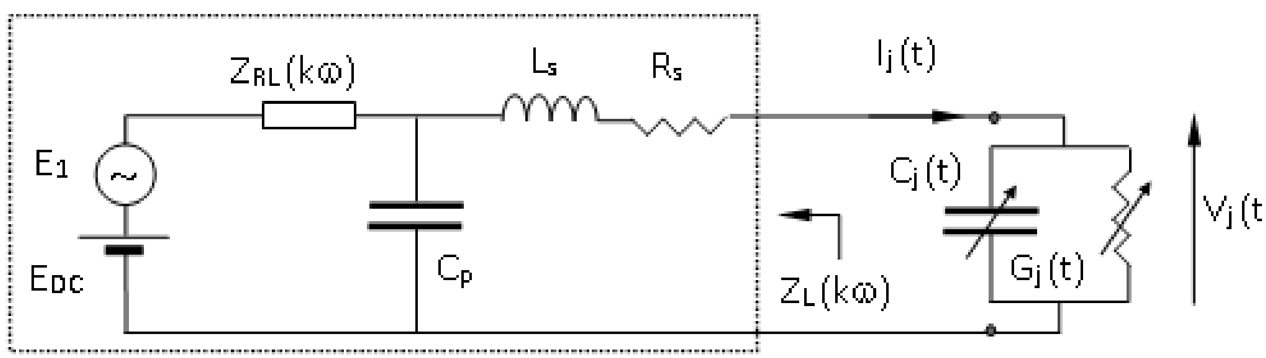
$$V_j(t) = \sum_{k=-\infty}^{\infty} V_{jk} e^{jk\omega_p t} \qquad I_j(t) = \sum_{k=-\infty}^{\infty} I_{jk} e^{jk\omega_p t}$$

Let us define $Z_L(k\omega_p)$ as the impedance of Thevenin seen by the junction and constituted by the impedance $Z_{RL}(k\omega_p)$ of the linear sub-circuit and the parasitic elements of the diode. We have then the following system

$$V_{j0} = E_{DC} - I_{j0}\, Z_L(0) \quad \text{for} \quad k = 0$$

$$V_{j1} = E_1 - I_{j1}\, Z_L(\pm\,\omega) \quad \text{for} \quad k = \pm 1$$

$$V_{jk} = -I_{jk}\, Z_L(k\omega) \quad \text{for} \quad k = \pm 2, \pm 3, \ldots$$



Its resolution allows characterizing the nonlinear elements

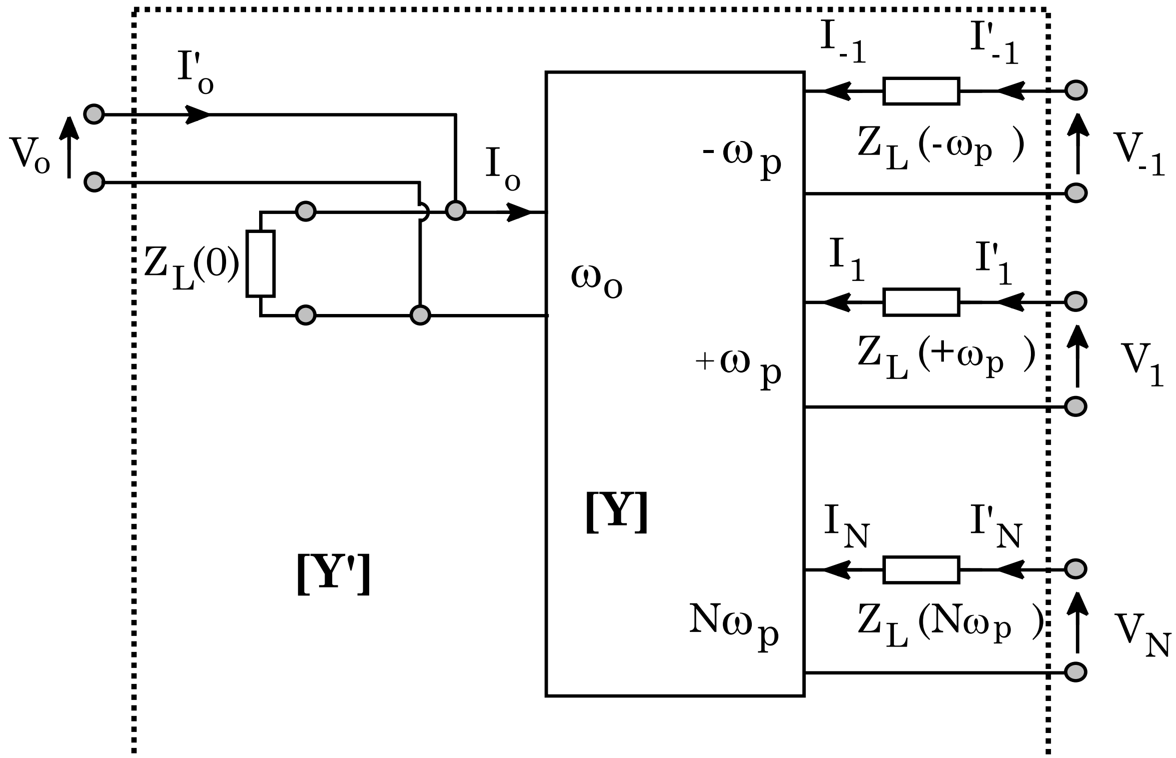
$$C_j(t) = \sum_{k=-\infty}^{\infty} C_{jk} e^{jk\omega_p t} \qquad G_j(t) = \sum_{k=-\infty}^{\infty} G_{jk} e^{jk\omega_p t}$$

The Fourier coefficients represent the element values at the different harmonics. They allow obtaining the transfer matrix of the diode at each harmonic of the excitation signal. Therefore, the circuit parameters can be determined.

2– Small signal analysis

In the small-signal analysis, the circuit is represented by a multifrequency multiport network

(see the definition in Chapter I).



2– Small signal analysis

By **adding** the small input signal (RF excitation) to the large signal of the pump, we obtain

$$V_j(t) = \sum_{k=-\infty}^{\infty} V_{jk} e^{j(\omega_o + k\omega_p)t} \qquad I_j(t) = \sum_{k=-\infty}^{\infty} I_{jk} e^{j(\omega_o + k\omega_p)t}$$

SUPERPOSITION !!!

As the Fourier representation of a time-varying resistor, $R_j(t)$ is defined by

$$R_j(t) = \frac{V_j(t)}{I_j(t)} = \sum_{q=-N}^{N} R_{jq} e^{jq\omega_p t}$$

$$\Rightarrow \quad \sum_{k=-N}^{N} V_k e^{j(\omega_o + k\,\omega_p)t} = \sum_{q=-N}^{N} \sum_{m=-N}^{N} R_q I_m e^{j(\omega_o + (q+m)\omega_p)t}$$

2– Small signal analysis

$$\sum_{k=-N}^{N} V_k e^{j(\omega_o + k\omega_p)t} = \sum_{q=-N}^{N} \sum_{m=-N}^{N} R_q I_m e^{j(\omega_o + (q+m)\omega_p)t}$$

This equation can be reordered to show the nonlinear conversion matrix **[R]** of a resistor:

$$\begin{bmatrix} V_{-N} \\ \vdots \\ V_0 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} R_0 & \cdots & R_{-N} & \cdots & R_{-2N} \\ \vdots & & \vdots & & \vdots \\ R_N & \cdots & R_0 & \cdots & R_{-N} \\ \vdots & & \vdots & & \vdots \\ R_{2N} & \cdots & R_N & \cdots & R_0 \end{bmatrix} \begin{bmatrix} I_{-N} \\ \vdots \\ I_0 \\ \vdots \\ I_N \end{bmatrix}$$

The elements are the Fourier components of the resistance.

2– Small signal analysis

Note that the vectors have been truncated to a limit $\pm N$ for both current and voltage, and $\pm 2N$ for the resistor to prevent the occurrence of infinite vectors and matrices (V_k, I_k and R_k are assumed to be negligible beyond this limit).

$$\begin{bmatrix} V_{-N} \\ \vdots \\ V_0 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} R_0 & \cdots & R_{-N} & \cdots & R_{-2N} \\ \vdots & & \vdots & & \vdots \\ R_N & \cdots & R_0 & \cdots & R_{-N} \\ \vdots & & \vdots & & \vdots \\ R_{2N} & \cdots & R_N & \cdots & R_0 \end{bmatrix} \begin{bmatrix} I_{-N} \\ \vdots \\ I_0 \\ \vdots \\ I_N \end{bmatrix}$$

For N harmonics, we deal with a matrix of dimensions **(2N + 1) * (2N + 1)** !!

So it involves harmonics up to the order **2*N** for a **simple resistor !!**

Similarly, for a **capacitor**, we start from the basic equation:

$$I(t) = \frac{dQ(t)}{dt} = C(t)\frac{dV(t)}{dt} + V(t)\frac{dC(t)}{dt}$$

$$\Rightarrow \sum_{k=-N}^{N} I_k e^{j(\omega_o + k\,\omega_p)t} = \sum_{q=-N}^{N} \sum_{m=-N}^{N} j\left(\omega_o + (q+m)\,\omega_p\right) V_q C_m e^{j(\omega_o + (q+m)\,\omega_p)t}$$

Therefore, we can deduce the conversion matrix **[C]** of a capacitor

$$\begin{bmatrix} I^*_{-N} \\ \vdots \\ I_0 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \omega_{-N} C_o & \cdots & \omega_{-N} C_{-N} & \cdots & \omega_{-N} C_{-2N} \\ \vdots & & \vdots & & \vdots \\ \omega_o C_N & \cdots & \omega_o C_o & \cdots & \omega_o C_{-N} \\ \vdots & & \vdots & & \vdots \\ \omega_N C_{2N} & \cdots & \omega_N C_N & \cdots & \omega_N C_o \end{bmatrix} \begin{bmatrix} V^*_{-N} \\ \vdots \\ V_0 \\ \vdots \\ V_N \end{bmatrix}$$

$$\Rightarrow \mathbf{[I]} = j\,[\mathbf{\Omega}]\,\mathbf{[C]}\,\mathbf{[V]}$$

where **[Ω]** is a diagonal matrix whose elements are the **frequencies from ω_{-N} to ω_N.**

Conversion matrices can be treated as in all ways like multiport admittance or impedance matrices. So, we can determine the admittance matrix **[Y]** whose elements should be on the following form ($\omega_m = \omega_o + m\omega_p$)

$$
\begin{array}{cc}
\textit{Line\#} & \\
\begin{array}{c} \vdots \\ 1 \\ 0 \\ -1 \\ \vdots \end{array} &
[\mathbf{Y}] = \left[\begin{array}{ccccc}
 & \vdots & \vdots & \vdots & \\
\cdots & Y_{11} & Y_{10} & Y_{1-1} & \cdots \\
\cdots & Y_{01} & Y_{00} & Y_{0-1} & \cdots \\
\cdots & Y_{-11} & Y_{-10} & Y_{-1-1} & \cdots \\
 & \vdots & \vdots & \vdots &
\end{array}\right] \\
 & \begin{array}{cccccc} \cdots & 1 & 0 & -1 & \cdots & \textit{Column\#} \end{array}
\end{array}
$$

The dimensions are 2N x 2N and each k^{th} line (or column) refers to the k^{th} harmonic of the excitation

$$Y_{mn} = G_{m-n} + j\,(\omega_o + m\omega_p)C_{m-n}$$

$$\begin{array}{c} \text{Line\#} \\ \vdots \\ 1 \\ 0 \\ -1 \\ \vdots \\ \end{array} \quad [\mathbf{Y}] = \begin{bmatrix} & \vdots & \vdots & \vdots & \\ \cdots & Y_{11} & Y_{10} & Y_{1-1} & \cdots \\ \cdots & Y_{01} & Y_{00} & Y_{0-1} & \cdots \\ \cdots & Y_{-11} & Y_{-10} & Y_{-1-1} & \cdots \\ & \vdots & \vdots & \vdots & \end{bmatrix}$$

$$\begin{array}{cccccc} \cdots & 1 & 0 & -1 & \cdots & \text{Column\#} \end{array}$$

From this matrix, we can deduce the overall admittance matrix **[Y']** of the circuit by including the equivalent impedances $Z_L(k\omega_p)$. These impedances are $Z_{RL}(k\omega_p)$ of the linear sub-circuit *plus* the parasitic diode elements:

$$[\mathbf{Y'}] = [\mathbf{Y}] + \operatorname{diag}\left\{\frac{1}{Z_L(k\omega_p)}\right\}$$

This circuit matrix allows obtaining the circuit parameters. Therefore, the conversion loss from an input frequency *nf* (e.g. the RF input signal) to an output frequency *mf* (e.g. the IF frequency) and the corresponding input-output impedances are

$$L_{mn} = 4|Y'_{mn}|^2 \operatorname{Re}\{Z_L(m\omega_p)\}\operatorname{Re}\{Z_L(n\omega_p)\}$$

$$Z_{input} = 1/Y_{mm} - Z_L(m\omega_p)$$

$$Z_{output} = 1/Y_{nn} - Z_L(n\omega_p)$$

TRANSISTOR CIRCUIT ANALYSIS

For nonlinear purposes, the optimized performances are, usually, achieved when the RF signal and the pump are applied to the gate of the pumped transistor. In this case, the capacitances and drain-resistance variations are minimized while the excursion over the trans-conductance is maximized.

These conditions lead to a bias point closed to the saturation region during the pump signal cycle. This saturation is assured by quasi-constant value of the drain voltage (by shunting the drain at the frequency pump and its harmonics).

1 - Large signal analysis

For a transistor (e.g. a FET), the large signal analysis purpose is to find the Fourier coefficients of the input port (noted V_i and I_i respectively), namely the gate, and the output port or drain (V_o and I_o).

$$V_i(t) = \sum_{k=-N}^{N} V_{ik} e^{jk\omega_p t} \qquad V_o(t) = \sum_{k=-N}^{N} V_{ok} e^{jk\omega_p t}$$

$$I_i(t) = \sum_{k=-N}^{N} I_{ik} e^{jk\omega_p t} \qquad I_o(t) = \sum_{k=-N}^{N} I_{ok} e^{jk\omega_p t}$$

Then, similarly to the diode, the coefficients of all nonlinear elements can be deduced (drain current source, capacitances, ..).

2 - Small signal analysis

The process is similar to the one used for the diode circuit. The only difference is that the multifrequency multiport will have { 2(N+1) } ports instead of { N+1 }. Let **[R_a], [G_m], [C_s] and [C_d]** be the conversion matrices of R_{ds}, G_m, C_{gs} and C_{ds} respectively.

The admittance matrix of the transistor circuit is given by

$$[\mathbf{Y}] = \Big[\{ [\mathbf{Z_1}] + [\mathbf{Z_C}][\mathbf{Z_6}] \} \Big\{ [\mathbf{Z_A}]^{-1} [\mathbf{Z_B}] \Big\} + [\mathbf{Z_C}][\mathbf{Z_5}] + [\mathbf{Z_2}] \Big]^{-1}$$

with

$$[\mathbf{Z_A}] = [\mathbf{Z_3}][\mathbf{Z_7}]^{-1}[\mathbf{Z_6}] + [\mathbf{Z_4}]$$

$$[\mathbf{Z_B}] = [\mathbf{R_a}] - [\mathbf{Z_3}][\mathbf{Z_7}]^{-1}[\mathbf{Z_5}]$$

$$[\mathbf{Z_C}] = [\mathbf{R_a}][\mathbf{Z_7}]^{-1}$$

$$[\mathbf{Z_1}] = [\mathbf{Z_{ni}}] + [\mathbf{C_s}]^{-1} + j\,[\mathbf{\Omega}]\,L_s + (R_s + R_i)\,[\Im]$$

$$[\mathbf{Z_2}] = +j\,[\mathbf{\Omega}]\,L_s + R_s\,[\Im]$$

$$[\mathbf{Z_3}] = [\mathbf{C_d}]^{-1} + R_i\,[\Im] + \{[\Im] + [\mathbf{R_a}][\mathbf{G_m}]\}[\mathbf{C}_s]^{-1} + [\mathbf{R_a}]$$

$$[\mathbf{Z_5}] = [\mathbf{Z_{no}}] + j\,[\mathbf{\Omega}]\,L_s + [\mathbf{R_a}] + R_s\,[\Im]$$

$$[\mathbf{Z_4}] = R_i\,[\Im] + \{[\Im] + [\mathbf{R_a}][\mathbf{G_m}]\}[\mathbf{C}_s]^{-1}$$

$$[\mathbf{Z}_7] = [\mathbf{R_a}]\left\{[\mathbf{G_m}][\mathbf{C}_s]^{-1} + [\Im]\right\}$$

$$[\mathbf{Z_6}] = j\,[\mathbf{\Omega}]\,L_s + R_s\,[\Im] - [\mathbf{R_a}][\mathbf{G_m}][\mathbf{C_s}]^{-1}$$

$$[\mathbf{Z_{ni}}] = \mathrm{diag}\,\{Z_{Le}(k\omega_p)\}$$

$$[\mathbf{Z_{no}}] = \mathrm{diag}\,\{Z_{Ls}(k\omega_p)\}$$

$$[\mathbf{Y}] = \left[\{[\mathbf{Z_1}] + [\mathbf{Z_C}][\mathbf{Z_6}]\}\left\{[\mathbf{Z_A}]^{-1}[\mathbf{Z_B}]\right\} + [\mathbf{Z_C}][\mathbf{Z_5}] + [\mathbf{Z_2}]\right]^{-1}$$

where **[$\Im$]** is the identity matrix.

From this formulation, it is possible to deduce the circuit gain G_{mn} from an input frequency nf to an output frequency mf,

$$G_{mn} = 4\left|Y_{mn}\right|^2 \operatorname{Re}\left\{ Z_{Le}\left(m\omega_p\right) \right\} \operatorname{Re}\left\{ Z_{Ls}\left(n\omega_p\right)\right\}$$

the corresponding input impedance while the output is matched:

$$[\mathbf{Z}_{in}] = [\mathbf{Z_1}] + \{[\mathbf{Z_2}] + [\mathbf{Z_C}][\mathbf{Z_5}]\}\{[\mathbf{Z_B}]^{-1}[\mathbf{Z_A}]\} + [\mathbf{Z_C}][\mathbf{Z_6}] - [\mathbf{Z_{ni}}]$$

and the output impedance while the input port is matched

$$[\mathbf{Z}_{out}] = [\mathbf{Z}_{out1}] + \{[\mathbf{Z}_{out2}][\mathbf{Z}_{out3}]^{-1} [\mathbf{Z}_{out4}]\} - [\mathbf{Z_{no}}]$$

with

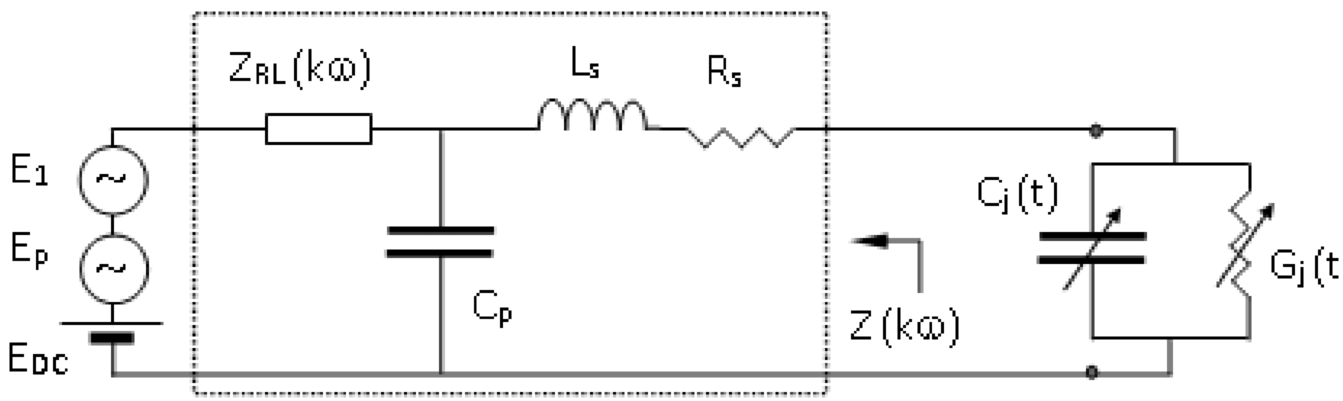
$$[\mathbf{Z}_{out1}] = [\mathbf{Z_5}] + [\mathbf{Z_7}][\mathbf{R_a}]^{-1} [\mathbf{Z_2}] \qquad [\mathbf{Z}_{out2}] = [\mathbf{Z_6}] + [\mathbf{Z_7}][\mathbf{R_a}]^{-1} [\mathbf{Z_1}]$$

$$[\mathbf{Z}_{out3}] = [\mathbf{Z_4}] - [\mathbf{Z_3}][\mathbf{R_a}]^{-1} [\mathbf{Z_1}] \qquad [\mathbf{Z}_{out4}] = [\mathbf{R_d}] + [\mathbf{Z_3}][\mathbf{R_a}]^{-1} [\mathbf{Z_2}]$$

PRACTICAL EXAMPLE : 1-DIODE MIXER

To complete this overview, let us consider the 1-diode mixer proposed by Maas.

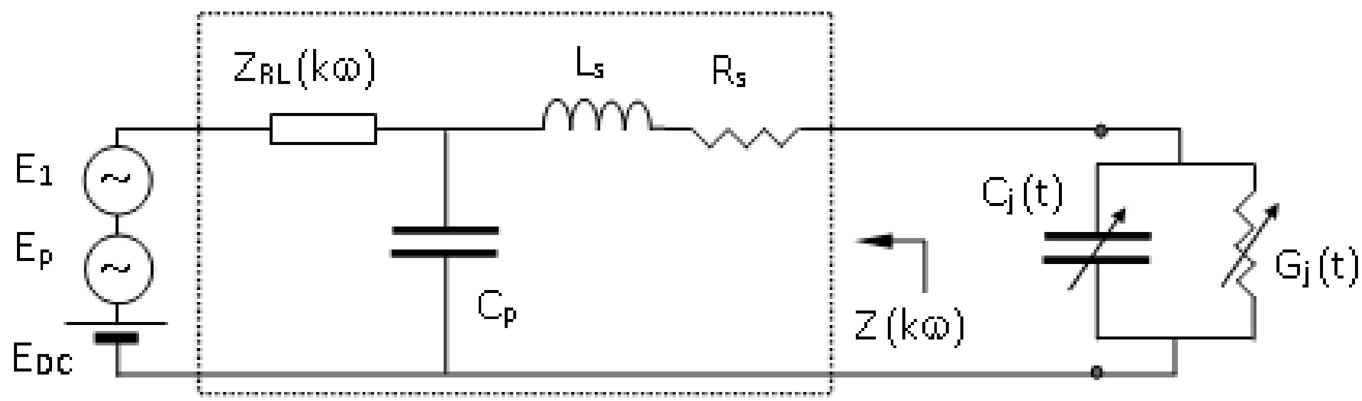
After the design step, the mixer conversion gain is optimized.



PRACTICAL EXAMPLE : 1-DIODE MIXER

By varying the loads impedances $Z(k\omega_p)$, we can notice that the optimization loop reduces the conversion loss as shown in the table below,

		Before Optimization	After Optimization
Conversion Loss L (dB)		4.33 dB	3.48 dB
Large-signal impedances (Ω)	$Z(\omega_p)$	50.0 + j 20.0	49.9 + j 28.0
	$Z(2\omega_p)$	10.0 + j 0.0	9.98 + j 0.01
Small-signal impedances (Ω)	$Z(\omega_{FI}+\omega_p)$	35.0 + j 20.0	48.0 + j 56.0
	$Z(\omega_{FI}+2\omega_p)$	1.0 + j 0.0	1.01 + j 0,0

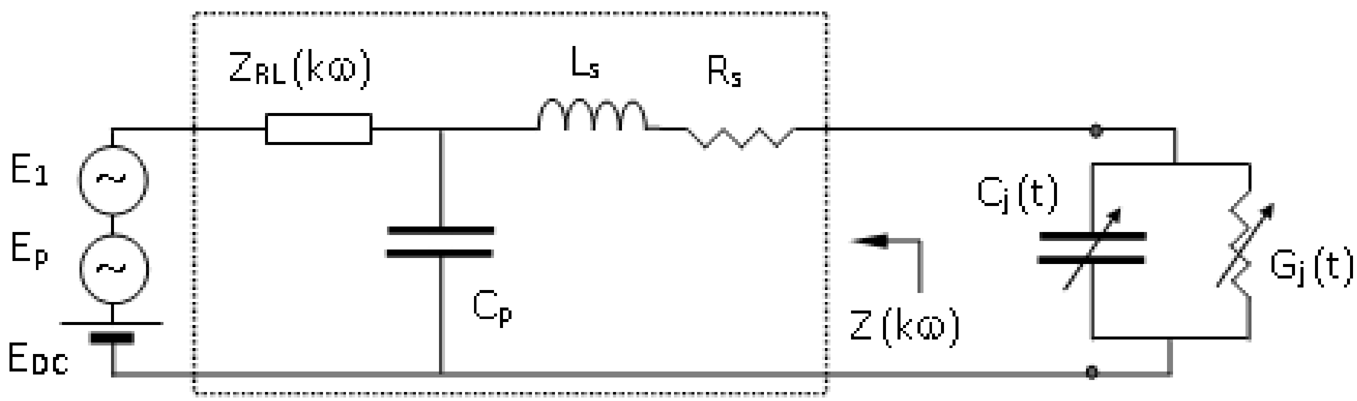


PRACTICAL EXAMPLE : 1-DIODE MIXER

The question now is: What would be the new circuit topology with the new impedance values?

Should we change the matching cells topology, the filters input-output impedances, the bias network …?

		Before Optimization	After Optimization
Conversion Loss L (dB)		4.33 dB	3.48 dB
Large-signal impedances (Ω)	$Z(\omega_p)$	50.0 + j 20.0	49.9 + j 28.0
	$Z(2\omega_p)$	10.0 + j 0.0	9.98 + j 0.01
Small-signal impedances (Ω)	$Z(\omega_{FI}+\omega_p)$	35.0 + j 20.0	48.0 + j 56.0
	$Z(\omega_{FI}+2\omega_p)$	1.0 + j 0.0	1.01 + j 0,0



PRACTICAL EXAMPLE : 1-DIODE MIXER

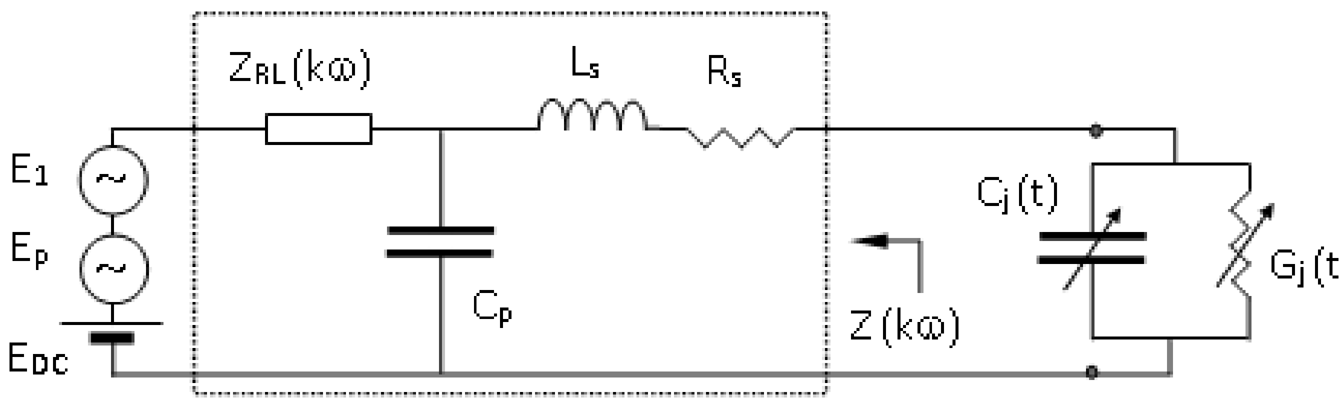
AFTER OPTIMIZATION:

HOW TO BUILD

THE "NEW" MIXER TOPOLOGY CONFIGURATION

(SCHEMATIC)?

IT IS STILL AN OPEN CHALLENGE !!



Thank you !

End of Chapter 3