Financial Econometrics 1 - M2 FTD

Empirical Applications

Luis Miguel Fonseca
Stéphane Eloundou Mvondo
Natalia Cárdenas Frías

December 10, 2023

Contents

Introduction

something, probably describe how all applications make sense one after the other and what is the research question we could have made ourselves when doing the applications, try to give a coherent look to the whole thing.

This document compiles all our applications of the Financial Econometrics course. Each section represents a specific application, but we tried to make them coherent across them around a broad question:

1 Series Dynamics

Note: Depending on each exercise along these applications we might use different series. In this first section, we performed the stationarity and component analysis of all of them to be able to use them rapidly without having to worry about seasonality or the presence of UR. Therefore, this section encompasses more than the 3 series that were asked in the exercise.

In this work, we focus on the US market. We use the following series extracted for the most part from FRED with its Python API :

Inflation Expectation

GDP deflator

Unemployment rate

FED func rate

S&P 500 price

Corporate Debt All (monthly[1]) time series de can be decomposed in the following elements:

$$X_t = \underbrace{\alpha}_{\text{drift}} + \underbrace{\beta \times t}_{\text{deterministic trend}} + \underbrace{\gamma T t}_{\text{stochastic trend}} + \underbrace{\Sigma_{i=1}^{11} \rho_i \, \mathbb{1}_{m_i}}_{\text{seasonality}} + \underbrace{c_i}_{\text{cyclical component}}$$

Remark that according to definitions, the deterministic might as well include the drift/constants α. Also, distinguish the presence of a deterministic and stochastic trend needs to be done jointly and with specific procedures (see Section 1.1 for the implementation of the ADF unit root tests).

Using R's built in function `stl`[2], we can decompose the time series. The results are plotted in Figure 1.

[1]One should adapt the number of indicators in the seasonal components according to the data frequency.
[2]Seasonal Decomposition of Time Series by Loess

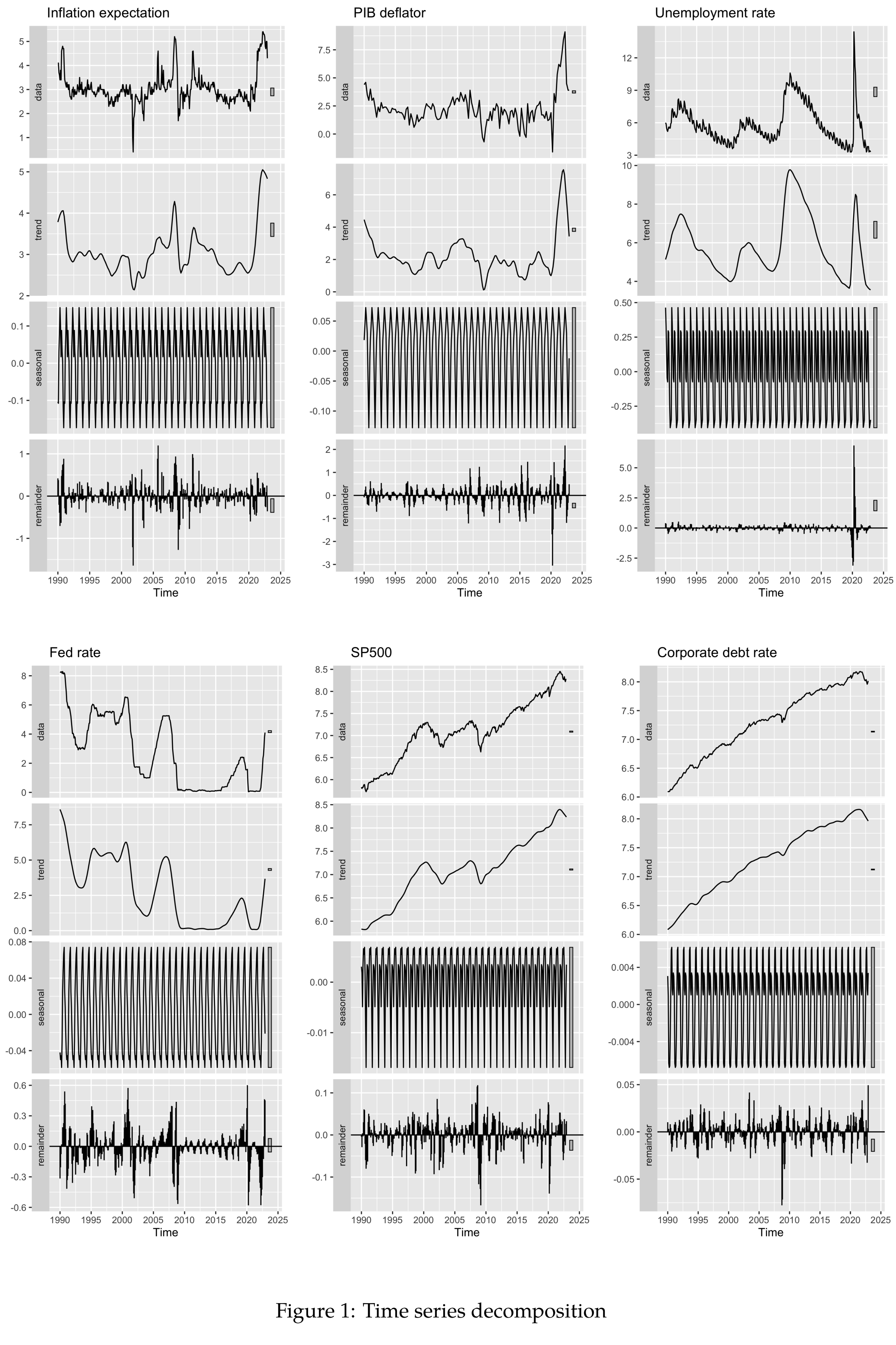

Figure 1: Time series decomposition

1.1 Seasonality

From the decompositions made in the previous section, in Figure 1 we can see that R picks up small seasonal variations in each series (the values on the y-axis are rather very small). The estimations of teh seasonal coefficients are reported in Table 1, which indeed shows very small coefficients. We nonetehless decided to work with deseasonalized series even if the chages are minimal.

Table 1: Estimation of the seasonality of each series

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
infl_e	-0.104	-0.108	0.031	0.078	0.149	0.1	0.017	0.088	0.031	0.006	-0.116	-0.17
deflator	0.018	0.035	0.053	0.072	0.06	0.045	0.03	-0.024	-0.078	-0.128	-0.072	-0.01
unempl	0.464	0.327	0.132	-0.045	-0.074	0.296	0.284	0.007	-0.257	-0.406	-0.379	-0.34
rate	-0.042	-0.05	-0.046	-0.059	-0.03	0.025	0.048	0.067	0.074	0.025	0.008	-0.02
splong	0.003	0.002	-0.005	0.003	0.007	0.006	0.007	0.002	-0.007	-0.017	-0.003	0.003
corp_debt	0.003	0.001	-0.006	-0.007	-0.007	-0.004	0.001	0.006	0.006	0.002	0.001	0.003

1.2 Unit root and trends

As for any time series analysis, the first analysis to perform is regarding the presence of unit roots in the series that would make them non-stationary. To do so, we perform the Augmented Dickey-Fuller tests that evaluate the presence of a stochastic trend (a unit root), a deterministic trend, and an intercept or drift. Importantly, this test requires to estimate three equations/specifications because it requires to investigate the joint presence of both types of trends and drift, for them discar elements one by one. The inference with this test is non-standard and requires to use corrected critical values to assess significance with the t-statistics.

1.2.1 ADF - Test jointly for deterministic and stochastic trend (with drift)

We first run the following specification to the ADF test to *jointly* investigate the presence of a stochastic and a determinist trend for each series $(X_t)_t$:

$$\Delta X_t = \alpha + \beta t + \gamma X_{t-1} + \sum_{i=1,2,..} \rho_i \Delta X_{t-i} + \varepsilon_t \tag{1}$$

As per usual, the ADF test assumes H0: $\gamma = 0$ i.e. a unit root exists and the series is non-stationary. We use R's built-in function `ur.df` with `type='trend'` to get this estimation. This function gives us (i) a regression table per series and (ii) a summary table with the following test statistics:

- tau3 refers to the t-statistic associated to $\gamma = 0$
- phi2 refers to the F-statistic associated to $\alpha = \beta = \gamma = 0$
- phi3 is also an F-statistic, now associated to $\beta = \gamma = 0$

Remark that the critical value in both tables can be a little different. This is because they are sensitive to the number of observations in each series. In Table 2, the critical values correspond to those provided directly by R and are associated to $N = 500$, while in Table 3 we give the values for $N = 250$. Since we have 396 data points per series we prefer to refer to the higher critical values but it does not change the analysis done.

Let us examine each series' results, summarized in the following tables:

Table 2: ADF test - 1st regression with drift, deterministic trend and stochastic trend

	infl_e	deflator	unempl	rate	splong	corp_debt	CV 1pct	CV 5pct	CV 10pct
tau3	-4.490	-5.166	-3.346	-1.672	-1.949	-1.759	-3.980	-3.420	-3.130
phi2	6.945	8.984	3.786	2.022	3.926	10.602	6.150	4.710	4.050
phi3	10.416	13.475	5.665	2.927	1.953	4.228	8.340	6.300	5.360

Table 3: ADF test - 1st regression t statistics

	infl_e	deflator	unempl	rate	splong	corp_debt
alpha	3.829	2.584	3.016	0.649	2.078	1.882
gamma	-4.490	-5.166	-3.346	-1.672	-1.949	-1.759
beta	1.366	1.154	-0.488	-0.127	1.696	1.407
rho	-0.558	14.221	1.072	15.797	4.100	6.757

Notes: With N=396, critical values at 5%: alpha = 3.09 ; gamma= -3.43 ; beta = 2.79

Inflation expectation We find $t_\gamma = -4.490 < -3.43$ we reject H0 ie we can't say that the series has a UR. Note that the F-statistic corresponding to the nullity of all the coefficients of interes (phi2) leads us to reject H0: $\alpha = \beta = \gamma = 0$ (same for phi3), leading us to believe that the series has either a drift and/or a deterministic trend. We therefore compare the t-statistics associated to α and β to the standard interest threshold (the critical values below Table 3 are conditional on having an UR). Since $|t_\alpha| > 1.96$ and $|t_\beta| > 1.96$, we fail to reject the absence of a drift and of a determinsitic trend. We conclude that the series is *stationnary with a constant and a deterministic trend*.

GDP deflator With $t_\gamma = -5.166 < -3.43$, as before we can reject H0 indicating that the series is stationary in levels. Since $|t_\alpha| = 2.584 > 1.96$, we reject the nullity of the drift. Finally, since $|t_\beta| = 1.154 < 1.96$ we cannot reject the absence of a deterministic trend. We conclude that the series is *stationary with a constant*

Unemployment rate Since $t_\gamma = -3.346 > -3.43$, we fail to reject H0. We then test the significance of β with respect to the non-standard threshold 2.79 (bilateral test). Once again, the values of the F-statiss of the other two tests phi2 and phi3, lead us to believe that the series has a constant and/or a deterministic trend. Since, $|t_\beta| = 0.488 < 2.79$, the ADF methodology leads us to use the second specification for this series and discard the existence of a deterministic trend.

Fed fund rate $t_\gamma = -1.672 > -3.43$, we are in the same situation as we the previous series where we cannot reject the existence of a UR. Because $|t_\beta| = 0.127 < 2.79$ (not significant) and because we cannot reject phi2 nor phi3 ($F_{phi2} = 2.022 < 4.71,\ F_{phi3} = 2.927 < 6.3$) we need to continue testing this series with the other specifications as we don't reject the existence of a stochastic trend.

SP500 Without much surprise for price series, $t_\gamma = -1.949 > -3.43$ and we cannot reject the existence of a UR. Given that $t_\beta = 1.696 < 2.79$ we cannot reject the nullity of β.

Moreover, $F_{phi2} = 3.926 < 4.71$ and $F_{phi3} = 1.953 < 6.3$ (non-rejection of the null for both tests) lead to conclude that at leat one of these coefficients are non-null. Given the shape of the series, we decided to keep testing the presence of a UR in the second step of the ADF test.

Corporate debt Finally, this series has $t_\gamma = -1.759 > -3.43$, and $|t_\beta| = 1.696 < 2.79$. We cannot reject the presence of UR nor a deterministic trend and we need to continue to test this series too with the second specification of the ADF test.

1.2.2 ADF - Test jointly for stochastic trend and drift

The second specification of the test models $\forall (X_t)_t$:

$$\Delta X_t = \alpha + \gamma X_{t-1} + \sum_{i=1,2,..} \rho_i \Delta X_{t-i} + \varepsilon_t \tag{2}$$

The null hypothesis still refers to H0: $\gamma = 0$ the presence of a unit root. We use now `type='trend'` in the `ur.df` function to get this estimation. The output to the test is similar to the previous specification and the same remarks on the critical values apply here. Now the test statistics reported refer to:

- tau2 refers to the t-statistic associated to $\gamma = 0$
- phi1 refers to the F-statistic associated to $\alpha = \gamma = 0$

Table 4: ADF test - 2nd regression with drift and stochastic trend

	unempl	rate	splong	corp_debt	CV 1pct	CV 5pct	CV 10pct
tau2	-3.334	-2.419	-1.013	-2.542	-3.440	-2.870	-2.570
phi1	5.570	3.032	4.431	14.876	6.470	4.610	3.790

Table 5: ADF test - 2nd regression t statistics

	unempl	rate	splong	corp_debt
alpha	3.153	1.513	1.270	2.934
gamma	-3.334	-2.419	-1.013	-2.542
rho	1.079	16.076	3.982	6.605

Notes: With N=396, critical values at 5%: alpha = 2.53 ; gamma= -2.88

Unemployment rate With $t_\gamma = -1.013 > -2.88$ we cannot reject H0. We thus test the t-statistic associated with α to its non standard critical value 2.53 (bilateral test). Since $|t_\alpha| = 3.153 > 2.53$, we reject the nullity of the drift term. Fianlly, as an extra check, we chek the F-statistic associated to $H0 : \gamma = \alpha = 0$, i.e. the test *phi1* in the default R test. Given that $F_{phi1} = 5.57 > 4.61$, we reject the null and conclude that the *series is* $I(0)$ at 5%.

However, the null is not rejected at the one percent level, and by looking at the series it does not look very stationary. Also R's built in function `order_integration` concludes that the series is actually non-stationay which made us doubt our results. We decided to continue the test just in case and compare

Fed fund rate Given that $t_\gamma = -2.419 > -2.88$, we cannot reject the null hypothesis. Similarly as before, we check the significance of the drift term: here it is not significantly different from zero as $|t_\alpha| = 1.513 < 1.96$. We then check the F-stistic of the joint test *phi1*: $F_{phi1} = 3.032 < 4.61$ and we cannot reject leading us to use the third specification of the test.

SP500 Since $t_\gamma = -1.013 > -2.87$, we cannot reject H0. We then check $|t_\alpha| = 1.270 < 1.96$, leading to the conclusion that the constant is not significantly different from zero. By checking $F_{phi1} = 4.431 < 4.61$ we fall in the same case as before where we need to continue testing the series.

Corporate debt For the last series we find $t_\gamma = -2.542 > -2.87$,leading to a non-rejection of the null hypothesis. Once again we check the other two dtstistics: $|t_\alpha| = 2.934 > 1.96$ driving the hypothesis taht the series has a constant term. Finally, since $F_{phi1} = 14.876 > 4.61$ we reject the joint nullity of γ and α (pretty much in line with α's t-statistic.)

1.2.3 ADF - Test for stochastic trend only

The last specification of the test keeps only the stochastic trend, $\forall (X_t)_t$:

$$\Delta X_t = \gamma X_{t-1} + \sum_{i=1,2,..} \rho_i \Delta X_{t-i} + \varepsilon_t \tag{3}$$

The null hypothesis still refers to H0: $\gamma = 0$ the presence of a unit root and we use `type='none'`. The output of the test is similar to the previous ones but now there is only one test statistic reported referring to the null (tau1).

Table 6: ADF test - 3rd regression with stochastic trend

	infl_e	deflator	unempl	rate	splong	corp_debt	CV 1pct	CV 5pct	CV 10pct
tau1	-0.826	-2.642	-1.083	-1.940	2.690	4.554	-2.580	-1.950	-1.620

Table 7: ADF test - 3rd regression t statistics

	infl_e	deflator	unempl	rate	splong	corp_debt
gamma	-0.826	-2.642	-1.083	-1.940	2.690	4.554
rho	-1.585	13.581	0.557	16.044	3.982	7.167

Notes: With N=396, critical values at 5%: gamma= -1.95

DECOMPOSITION SERIES IN DELTAS

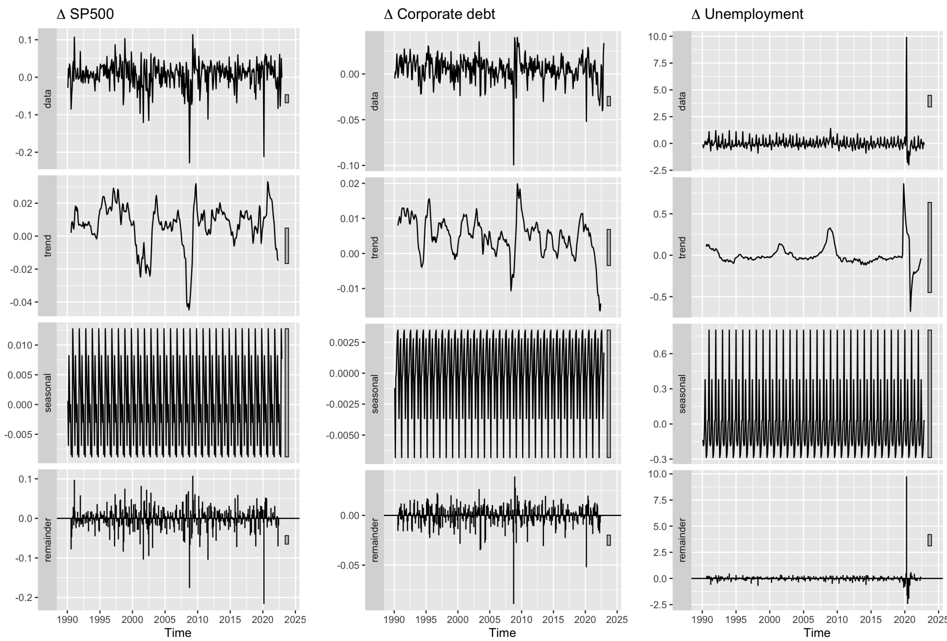

Figure 2: Decomposition of the series in deltas

1.3 Cyclical component

2 Canonical VAR model application

Table 8: Canonical VAR in levels - Identify order

	1	2	3	4	5	6	7	8	9	10
AIC(n)	-10.01	-10.56	-10.59	-10.73	-10.88	-10.87	-10.93	-11.01	-11.00	-10.98
HQ(n)	-9.96	-10.48	-10.47	-10.57	-10.68	-10.64	-10.66	-10.71	-10.66	-10.60
SC(n)	-9.89	-10.35	-10.29	-10.33	-10.38	-10.29	-10.25	-10.25	-10.14	-10.03
FPE(n)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 9: Level VAR - Estimation

	Dependent variable:		
	deflator	unempl	splong
deflator.l1	1.830^{***}	-0.199	0.026^{***}
	(0.051)	(0.124)	(0.008)
unempl.l1	0.120^{***}	0.956^{***}	0.003
	(0.022)	(0.053)	(0.004)
splong.l1	0.556^{*}	-4.968^{***}	1.174^{***}
	(0.321)	(0.786)	(0.053)
deflator.l2	-0.819^{***}	-0.317	-0.042^{**}
	(0.101)	(0.246)	(0.017)
unempl.l2	-0.117^{***}	-0.131^{*}	-0.004
	(0.031)	(0.075)	(0.005)
splong.l2	-0.553	5.816^{***}	-0.246^{***}
	(0.506)	(1.237)	(0.083)
deflator.l3	-0.894^{***}	1.008^{***}	0.010
	(0.108)	(0.264)	(0.018)
unempl.l3	0.014	0.182^{**}	0.002
	(0.031)	(0.076)	(0.005)
splong.l3	0.149	-0.992	0.137
	(0.521)	(1.275)	(0.085)
deflator.l4	1.526^{***}	-0.716^{**}	0.019
	(0.113)	(0.277)	(0.019)
unempl.l4	-0.013	-0.150^{**}	0.003
	(0.031)	(0.076)	(0.005)
splong.l4	0.428	-0.516	-0.012
	(0.523)	(1.278)	(0.086)
deflator.l5	-0.653^{***}	-0.130	-0.021
	(0.111)	(0.271)	(0.018)
unempl.l5	0.001	0.092	-0.008^{*}
	(0.031)	(0.076)	(0.005)
splong.l5	-0.027	1.417	0.063
	(0.527)	(1.288)	(0.086)
deflator.l6	-0.498^{***}	0.609^{**}	-0.003
	(0.107)	(0.262)	(0.018)
unempl.l6	0.004	-0.105	0.006
	(0.031)	(0.075)	(0.005)
splong.l6	-1.048^{**}	-1.069	-0.302^{***}
	(0.527)	(1.290)	(0.086)
deflator.l7	0.834^{***}	-0.476^{*}	0.019
	(0.103)	(0.252)	(0.017)
unempl.l7	0.081^{***}	0.088	-0.0004
	(0.030)	(0.073)	(0.005)
splong.l7	0.954^{*}	-0.455	0.261^{***}
	(0.526)	(1.287)	(0.086)
deflator.l8	-0.350^{***}	0.153	-0.010
	(0.054)	(0.131)	(0.009)
unempl.l8	-0.075^{***}	0.039	-0.001
	(0.021)	(0.052)	(0.004)
splong.l8	-0.428	0.746	-0.078
	(0.340)	(0.832)	(0.056)
const	-0.268^{*}	0.490	0.015
	(0.149)	(0.365)	(0.024)
Adjusted R^2	0.977	0.913	0.997
Residual Std. Error (df = 363)	0.210	0.515	0.034
F Statistic (df = 24; 363)	684.697^{***}	170.695^{***}	$5{,}592.560^{***}$
Note:			*p<0.1; **p<0.05; ***p<0.01

3 Cointegration theory

4 Impulse Response Analysis

4.1 Canonical IRF

4.2 Structural IRF

5 Introduce non-linearities

5.1 Markov-switching model

5.2 STR model

6 Difference-in-Difference

https://www.tidy-finance.org/r/difference-in-differences.html

Appendix A Code - Data Cleaning

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Financial Econometrics - Empirical Applications d
Data Gathering and Data Cleaning

@author: nataliacardenasf
"""

import pandas as pd
import os
import datetime

from fredapi import Fred

os.chdir('/Users/nataliacardenasf/Documents/GitHub/PROJECTS_AP_FE/FinancialEconometrics1')

### Initialize FRED API
fred = Fred(api_key='23edc2b1b61e17c07b83a97e7abfc02b')

### Import all the data

# S&P 500
sp500 = pd.DataFrame(fred.get_series('SP500')) #daily close, NSA, Index
sp500.columns= ['sp500']

#Inflation expectations from survey UMich
infl_e = pd.DataFrame(fred.get_series('MICH')) #monthly, NSA, median expected in % over next
    12 mo
infl_e.columns= ['infl_e']


#ICE BofA US Corporate Index Total Return Index
corp_debt = pd.DataFrame(fred.get_series('BAMLCC0A0CMTRIV')) #daily, close, NSA, Index
corp_debt.columns= ['corp_debt']


#MP rate
rate = pd.DataFrame(fred.get_series('DFF')) #daily, 7-Day, NSA, %
rate.columns = ['rate']

#Deflator
deflator = pd.DataFrame(fred.get_series('A191RI1Q225SBEA')) #Q, SA Annual Rate
deflator.columns = ['deflator']

#Unemployment
```

```
unempl = pd.DataFrame(fred.get_series('UNRATENSA')) #monthly, NSA, %
unempl.columns=['unempl']

fred.search("BAMLCC0A0CMTRIV").T #this function gives of the info on every series


### Resample into monthly data
corp_debt = corp_debt.resample('1M').mean(numeric_only=True)
rate = rate.resample('1M').mean(numeric_only=True)
sp500 = sp500.resample('1M').mean(numeric_only=True)


dta = [infl_e, rate, sp500, corp_debt, deflator, unempl]

### Slice the df to relevant period
#Find common time span
min_date = max([min(i.index) for i in dta])
max_date = min([max(i.index) for i in dta])
print(min_date, max_date)

#Let us work on monthly data for the 1990-2022 period
start = datetime.datetime(1990,1,1)
end= datetime.datetime(2022,12,31)

## SP500 series is too short, I am taking it from Yahoo Finance
import yfinance as yf
splong = yf.download('^GSPC', start=start,end=end)['Adj Close'].resample('M').mean(numeric_only=True)
splong = pd.DataFrame(splong)
type(splong)


##Get a single DF
dta.append(splong)
for i in range(len(dta)): #we had some indexes at end of month, others at 1st of month: harmonize to 1st each month
    df = dta[i]
    df.index = [pd.datetime(x.year, x.month, 1) for x in df.index.tolist()]
    dta[i] = df.loc[start:end,:]
dta# we're good now
#merge into 1 df, 1 series per column
monthly = pd.concat(dta, axis=1)
#interpolate missing months for deflatior data (Q): uses midpoints ie assumes that each month in the quarter contributes in the same fashion to the increase QoQ
m1 = monthly.interpolate(method ='linear', limit_direction ='forward')


m1.to_csv("DATA/data.csv")
```