Asset Pricing - Empirical Application 1

Factorial Model and Risk Premium Decomposition - APT

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Introduction

Focusing on recent data from the German equity market, we want to better comprehend how the market prices systemic, non-diversifiable risk embedded in the risk premium of stocks, i.e. any expected compensation beyond the risk-free return. We base our analysis in a linear decomposition of said premium on different *factors* of risk in the spirit of the Arbitrage Pricing Theory (APT) pioneered by Ross [1976]. Unlike the CAPM model which considers a unique risk premium in the market, the Ross model gives a more detailed description of the pricing of aggregate risk by decomposing the contributions of different sources of risk. Here, a risky portfolio of j stocks[1] is compensated with k risk premia associated with the k *common factors* that the portfolio is exposed to.

1 Data and Framework

1.1 German stock market data

1.2 Factorial model and multibeta relationship

In the name of completeness, this section summarizes the lecture on the Ross [1976] model and the multibeta relationship.

[1] Let $j \in \{1, ..., J\}$ with J sufficiently large so that all idiosyncratic risk can be fully diversified. We better explain the difference between idiosyncratic and aggregate in the context of the Ross model in Section 1.1

2 Empirical strategy and implementation

2.1 Identify the risk factors

2.1.1 Exogeneous factors

2.1.2 Endogeneous factors

2.1.3 French-Fama factors

2.2 Estimate the β_k coefficients from the factorial model

2.3 Estimate the λ_k parameters from the multibeta relationship

2.4 Test the validity of the multibeta relationship

Conslusion

3 Data and Framework

That is, the return R_j of the j-th component of her portfolio can be described by the following expression $\forall j \in \{1, ..., N\}$:

$$R_j = \mathbb{E}[R_j] + \underbrace{\sum_{k=1}^{K} \beta_{j,k} f_k}_{\text{Systemic risk}} + \overbrace{u_j}^{\text{Idiosyncratic risk}} \tag{1}$$

Where $\mathbb{E}[R_j]$ is the expected return of asset j. The sources of risk are two-fold. The investor faces centered idiosyncratic risks u_j, $\mathbb{E}[u_j] = 0$ that are assumed to be completely diversiable with a portfolio "large enough" (N big) because they are independent of each other $u_j \perp\!\!\!\perp u_{j'} \forall j \neq j'$, and uncorrelated with aggregate risk $corr(u_j, f_k) = 0, \forall j, k$ which is a required assumption to perform the estimations that will follow. She also faces k different sources of aggregate risk, modeled by the linear combination of f_k centered *shocks* that influence all R_j with a sensitivity β_{jk}. By definition, these risks cannot be diversified because they affect the returns of all asset and thus has to be compensated which is the focal point of our study.

3.1 German Stock Market

We decided to consider the German stock market for this analysis because it is a major liquid stock market in Europe. It is also the biggest economy in the Europe, with mayor

3.2 Estimation of the Factors

We need to choose what factors we are going to consider to generate risk premia that affect the return for the investor. In this section we examine the role of different types of factors (i) exogenous, (ii) endogeneous and we examine more closely the three-factor model proposed by Fama and French [1993] in (iii).

3.3 Exogeneous Factors

These are risk factors that are supposed to be orthogonal to the portfolio itself. In particular, it is interesting to consider the role of

3.4 Endogeneous Factors

3.5 French-Fama Factors

Fama and French [1993] can be seen as an extension of the CAPM model. The authors show that the variation of the returns of an asset can be explained not only by the exposure to market risk as in the CAPM represented by the difference of the market return and the risk-free rate $[R_M - R_f]$, but also by a size and value premium in the following model.

$$R_j = \alpha_j + R_f + \beta_{m,j}[R_M - R_f] + \beta_S SMB + \beta_V HML + \varepsilon_j \tag{2}$$

The size premium refers to the observation that stocks with small market capitalizations tend to outperform stocks with larger ones and it is captured by the factor SMB, *small minus big*. It is computed as the difference in average returns of the 30% stocks with the smallest market capitalization and the average returns of the 30% stocks associated with the firms with the largest market capitalization. The value premium refers to the outperformance of "value stocks" i.e. those that have high book-to-market (B/M) and it is represented by the difference in an average return of the 50% of stocks with the highest B/M ratio (value stocks) and the 50% with lowest B/M ratio (growth stocks).

See K. French Data Library

4 Estimation of the exposure

5 Estimation of the market price of risk(s)

Consider a series of returns for different stock prices of at least 30 over a given period of time and frequency. The goal is to estimate rk premium by choosing a relevant so-called risk-free asset obtained as the return of treasury bond with relevant maturity

Develop econometric analysis which provides the multi-beta relationship 1. Identify the series for the risk factors (endo and exo) and justify choices + including 2 factors proposed by French and Fama 2. Estimate beta coeds or different stocks with relevant linear regression 3. Estimate market price of different sources of risk retained in analysis with appropriate linear reg

Comment the results from a financial point of view: are the estimated exposures of the different stocks to the different factors in line with expectations

References

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