

# ASSET PRICING - EMPIRICAL APPLICATION 1

## FACTORIAL MODEL AND RISK PREMIUM DECOMPOSITION - APT

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### 1 Data and Framework

Our goal is to better comprehend how the market price systemic, non-diversifiable risk embedded in the risk premium of stocks, i.e. any expected compensation beyond the risk-free return. We base our analysis in a linear decomposition of said premium on different *factors* of risk in the spirit of the Arbitrage Pricing Theory (APT) pioneered by Ross [1976]. Unlike the CAPM model that considers an risk premium, the factorial model considers that investors holding risk in their portfolios<sup>1</sup> by holding a stock  $j$ , are compensated with  $k$  different risk premia associated to  $k$  common factors.

That is, the return  $R_j$  of the  $j$ -th component of her portfolio can be described by the following expression  $\forall j \in \{1, \dots, N\}$ :

$$R_j = \mathbb{E}[R_j] + \underbrace{\sum_{k=1}^K \beta_{j,k} f_k}_{\text{Systemic risk}} + \overbrace{u_j}^{\text{Idiosyncratic risk}} \quad (1)$$

Where  $\mathbb{E}[R_j]$  is the expected return of asset  $j$ . The sources of risk are two-fold. The investor faces a centered idiosyncratic risks  $u_j$ ,  $\mathbb{E}[u_j] = 0$  that are assumed to be completely diversifiable with a portfolio "large enough" ( $N$  big) because they are independent from each other  $u_j \perp u_{j'} \forall j \neq j'$ , and uncorrelated with aggregate risk  $\text{corr}(u_j, f_k) = 0, \forall j, k^2$ . She also faces  $k$  different sources of aggregate risk, modelled by the linear combination of  $f_k$  centered *shocks* that influence all  $R_j$  with a sensitivity  $\beta_{jk}$ . By definition, these risks cannot be diversified because they affect the returns of all asset and thus has to be compensated which is the focal point of our study.

#### 1.1 DAX 40

We consider the

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<sup>1</sup>We assume taht said protafolios are sufficiently large so that any source of idiosyncratic risk can be diversified so that only aggregate risk is remunerated.

<sup>2</sup>Required assumption to perform the estimations that will follow.

## 2 Estimation of the Factors

We need to choose what factors we are going to consider generate risk premia that affects the return for the investor. In this section we examine the role of different types of factors (i) exogenous, (ii) endogenous and we examine more closely the three-factor model proposed by [Fama and French \[1993\]](#) in (iii).

### 2.1 Exogeneous Factors

These are risk factors that are supposed to be orthogonal to the portfolio itself. In particular it is interesting to consider the role of

### 2.2 Endogeneous Factors

### 2.3 French-Fama Factors

[Fama and French \[1993\]](#) can be seen as an extension of the CAPM model. The authors show that the variation of the returns of an asset can be explained not only by the exposure to market risk as in the CAPM represented by the difference of the market return and the risk-free rate  $[R_M - R_f]$ , but also by a size and value premium in the following model.

$$R_j = \alpha_j + R_f + \beta_{m,j}[R_M - R_f] + \beta_S SMB + \beta_V HML + \varepsilon_j \quad (2)$$

The size premium refers to the observation that stocks with small market capitalizations tend to outperform stocks with larger ones and it is captured by the factor SMB, *small minus big*. It is computed as the difference in average returns of the 30% stocks with the smallest market capitalization and the average returns of the 30% stocks associated to the firms with the largest market capitalization. The value premium refers to the outperformance of "value stocks" i.e. those that have high book-to-market (B/M) and it is represented by the difference in average return of the 50% of stocks with highest B/M ratio (value stocks) and the 50% with lowest B/M ratio (growth stocks).

[See K. French Data Library](#)

## 3 Estimation of the exposure

## 4 Estimation of the market price of risk(s)

## References

Eugene F. Fama and Kenneth R. French. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56, February 1993. ISSN 0304-405X. doi: 10.1016/0304-405X(93)90023-5.

Stephen A Ross. The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, 13(3):341–360, December 1976. ISSN 0022-0531. doi: 10.1016/0022-0531(76)90046-6.