

ASSET PRICING - EMPIRICAL APPLICATION 1

FACTORIAL MODEL AND RISK PREMIUM DECOMPOSITION - APT

Luis Miguel Fonseca
Stéphane Eloundou Mvondo
Natalia Cárdenas Frías

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Introduction

Focusing on recent data from the French equity market, we want to better comprehend how the market prices systemic, non-diversifiable risk embedded in the risk premium of stocks, i.e. any expected compensation beyond the risk-free return. We base our analysis on a linear decomposition of said premium on different *factors* of risk in the spirit of the Arbitrage Pricing Theory (APT) pioneered by Ross [1976]. Unlike the CAPM model which considers a unique risk premium in the market, the Ross model gives a more detailed description of the pricing of aggregate risk by decomposing the contributions of different sources of risk. Here, a risky portfolio of j stocks¹ is compensated with k risk premia associated with the k *common factors* that the portfolio is exposed to.

1 Data

1.1 French stock market data

We decided to build a case study of the French market because it is a liquid and matured market, central in Europe. In the case of this analysis, we had trouble getting the data needed to perform it for other countries² and the fact that France has more publicly available data helped us choose it as our market of study.

We built a portfolio with 30 French stocks that we got from Yahoo Finance. For simplicity, the synthetic portfolio is composed of one stock of each company and its composition does not change during the period studied. Table 1 shows the companies that we used to create this portfolio, they are all publicly traded companies in France since the early 2000's in Euronext Paris. Importantly, we tried to have a certain diversity in the sectors

¹Let $j \in \{1, \dots, J\}$ with J sufficiently large so that all idiosyncratic risk can be fully diversified. We better explain the difference between idiosyncratic and aggregate in the context of the Ross model in Section 1.1

²Initially we thought about using data from the German market but we couldn't for instance find data for their inflation-linked bond yield that we use as an endogenous factor in Section 2.1.2

represented to be able to capture some diversification to risk even if the portfolio is too small and we do not reweight it. However, because we try to implement a version of the [Fama and French \[1992\]](#) factor analysis, we restrain ourselves from choosing financial companies as the authors do due to their high leverage. Other than these two conditions, the choice of the companies was mainly restricted to data availability on public 'long' series on firm-level data, notably on market capitalization and book-to-market ratio to be able to incorporate the [Fama and French \[1992\]](#) factors to our analysis.

Because we are interested in the underlying determinants of the risk premia, and due to data availability issues, we decided to have a broad analysis with monthly data for our selection of stocks for the period 2005-2022. Monthly data is for instance used by [Chen et al. \[1986\]](#). While this is not a very long period, it encompasses important moments in the financial markets in particular the Great Financial Crisis, the subsequent European Debt Crisis, and the Covid years. Also importantly, during most of this period (following the GFC), monetary policy fixed interest rates were extremely low driving down the return of sovereign debt for countries like France and Germany³ that could have been assimilated to the risk-free rate. This means that for an investor to get any returns, it had to hold risky assets. Moreover, the period following the GFC and up to 2021 was also characterized by extremely low inflation in the Euro Zone. This is interesting because the APT usually incorporates inflation risk as market risk, yet inflation was nowhere to be found for more than a decade. Seeing how the market incorporated this monetary reality is by itself an intriguing question.

1.2 Data description and sources

The other series that we use are the following and its sources, how they are used in the context of the analysis is described in subsequent sections.

- As a proxy for the free rate of the market we consider two measures:
 - The yield of short-term OAT, i.e. French treasuries taken from [Banque de France's website](#). As for most developed, stable countries, short-term sovereign bonds are taken as the risk-free asset as Governments are supposed to be more solvent than other agents in the economy, after all, they decide their income and could seize resources via taxes to meet their obligations.
 - The spot yield curve spot rate, for 3-month maturity of all government bonds rated triple A in the Euro Area, retrieved from the [ECB webpage](#). On top of the fact that this is a measure for short-term sovereign bonds, we consider this to be a relevant proxy for the French market due to the strong integration within the European capital market. If an investor decides that the French market becomes risky, she can easily move her investments to another European capital market that looks safer.
- To get the market rate, we use the return of the main index of the country, the CAC40 also taken from Yahoo Finance as for the components of our synthetic portfolio. In hindsight, we are not sure of the pertinence of comparing our portfolio to this index. While the composition of our portfolio is not the

³They were negative for certain maturities in real terms for a part of the time frame analyzed, pretty much since the European Debt Crisis until the inflation surge after Covid.

Company Name	Ticker	Industry
Accor	AC.PA	Hospitality
Air Liquide	AI.PA	Industrial Gases
Air France-KLM	AF.PA	Airlines
Airbus	AIR.PA	Aerospace
Biomerieux	BIM.PA	Biotechnology
BIC	BB.PA	Consumer Goods
Bouygues	EN.PA	Construction
Capgemini	CAP.PA	Information Technology
Carrefour	CA.PA	Retail
Casino	CO.PA	Retail
Dassault Aviation	AM.PA	Aerospace
Danone	BN.PA	Food and Beverage
Hermes International	RMS.PA	Fashion and Luxury
JCDecaux	DEC.PA	Advertising
Kering	KER.PA	Fashion and Luxury
L'Oreal	OR.PA	Cosmetics
LVMH	MC.PA	Fashion and Luxury
Michelin	ML.PA	Automotive
Nexans	NEX.PA	Electrical Equipment
Orange	ORA.PA	Telecommunications
Renault	RNO.PA	Automotive
Saint-Gobain	SGO.PA	Manufacturing
Sanofi	SAN.PA	Pharmaceuticals
Sodexo	SW.PA	Food Services
TF1	TFL.PA	Broadcasting
Thales	HO.PA	Aerospace and Defense
TotalEnergies	TTE.PA	Energy
Ubisoft	UBI.PA	Video Games
Vinci	DG.PA	Construction
Vivendi	VIV.PA	Entertainment

Table 1: Synthetic portfolio: Companies, Tickers, and Industries

same as the CAC40⁴, due to the data availability issues we've been mentioning, we see that our choices are heavily biased towards 'big name' companies that are those belonging to the index.

- We got the series of the exchange rate between the Euro and the US dollar from Yahoo Finance. It is read as the amount of USD needed to get one euro.
- The GDP series is taken from the [ECB webpage](#). It is available at a quarterly frequency and is available at market prices.
- The harmonized headline inflation rate is taken from the [INSEE webpage](#).
- For the market inflation expectation in a 10-year horizon, we use the break-even inflation rate published by [Agence France Trésor](#) online. Sadly, data is only available from 2013.
- For the implementation of the two additional [Fama and French \[1992\]](#) factors, we took different routes
 - We found the estimation of the factors published by K. French in his [online Data Library](#) that are

⁴Not all the stocks we chose are necessarily part of the CAC40 at every period studied, and the CAC40 is a weighted index that evolves over time.

constantly updated. Their estimations start in the 1990s and are made for different markets using the comprehensive CRSP dataset that is not freely available. He has an estimation for the European market that we downloaded to use but it is not clear which stocks are used to replicate their portfolio.

- To try to build these estimations ourselves for our portfolio meaning that a minima we need data on the market capitalization of each company during the time frame studied and its *book*. This information is hardly available without having access to platforms like Bloomberg or CRSP. The best information that we could find comes from [this](#) website that publishes the market capitalization and the price-to-book (the inverse of the book-to-market ratio) annual series for several stocks. The data is however cannot be directly downloaded from the site so we scrap it to get the series (see Code Appendix B). Our biggest fear with this source is that it is not clear at all where the information comes from even if they mention several quality [data providers](#) as their partners. Since it is the only source that resembles what is needed for this part we used it but we are not confident on it.

2 Empirical strategy and implementation

We implement a minimal approach to [Ross \[1976\]](#), namely using fewer factors than in [Chen et al. \[1986\]](#). We include both exogenous and endogenous macroeconomic risk factors as well as an approximation to implement [Fama and French \[1993\]](#) three-factor model which used stock-specific data.

Let the return R_j of the j -th component of her portfolio can be described by the following expression $\forall j \in \{1, \dots, N\}$:

$$R_j = \mathbb{E}[R_j] + \underbrace{\sum_{k=1}^K \beta_{j,k} f_k}_{\text{Systemic risk}} + \underbrace{u_j}_{\text{Idiosyncratic risk}} \quad (1)$$

Where $\mathbb{E}[R_j]$ is the expected return of asset j and R_j its return without dividend i.e. the first difference of the stock price. This is the so-called *factorial model* that includes two sources of risk:

- The investor faces centered idiosyncratic risks u_j , $\mathbb{E}[u_j] = 0$ that are assumed to be completely diversifiable with a portfolio "large enough" (N big) because they are independent of each other $u_j \perp u_{j'} \forall j \neq j'$.
- She also faces k different sources of aggregate risk, modeled by the linear combination of f_k centered *shocks* that influence all R_j with a sensitivity $\beta_{j,k}$. By definition, these risks cannot be diversified because they affect the returns of all asset and thus has to be compensated which is the focal point of our study.

In order to implement a regression analysis and estimation as the one that follows in this section, we shall also assume that the idiosyncratic risks are uncorrelated with aggregate risk $\text{corr}(u_j, f_k) = 0, \forall j, k$.

This section is organized as follows. Section 2.1 identifies the aggregate market risks $(f_k)_k$ that we are going to consider as the ones priced by the market. Section 2.2 runs a first regression analysis to identify the sensitivity of each return to each factor of risk, i.e. to identify the $(\beta_{j,k})_{j,k}$ from the factorial model (Eq. 1). It also implements an approximation to a parallel model that decomposes the return of stocks: the [Fama and French \[1992\]](#) that

also will lead us to estimate the sensitivity of the return of each return to a series of factors. Finally, section 2.3 uses the series of $(\hat{\beta}_{j,k})_{j,k}$ estimated before to implement an estimation of the *multibeta relationship* which is the regression that will allow us to get how much the market is remunerating the exposition to a market factor risk.

2.1 Identify the risk factors

We decided to explore the role of the following sources of risk.

- The activity risk, measured by changes in GDP
- Inflation risk, measured both by the HICP of France for the short term and by the market inflation expectation in a 10-y horizon.
- Devaluation risk measured by the exchange rate between the Euro and the US dollar.

2.1.1 Exogeneous factors

These are risk factors that do not depend directly on the financial markets or more precisely that are not deduced from a linear combination of the returns of financial assets. In our case, it is mainly the activity risk, the short-term interest rate (measured by the HICP), and the devaluation risk. Importantly, the measure of these risks is not directly measured by changes or by the level of the underlying variables as markets are informational efficient [Fama, 1970], and they have already priced in all relevant information conveyed by prices. In particular, all *predictable* movements, say in inflation, have already been incorporated by the market. This means that the factors of risk are actually the *surprises* in the movements of these variables.

To extract this unpredictable part of these variables we need to use the residuals of some time series model of the (stationary) variables⁵, in our case ARIMA(p, d, q) models. We used the function *auto.arima* from the R *forecasts* package that set de parameters optimally by minimizing the BIC between different specifications. We ended up with:

- ARIMA(0,1,0) (i.e. a random walk⁶) for the series of log GDP.
- ARIMA(0,1,1) for the exchange rate series.
- ARIMA(2,1,2) for the HICP series.

We then stored the residuals of each one of these models and merge it in the data frame with the stock return

2.1.2 Endogeneous factors

These factors are linear combinations of the returns of financial assets. We used the

⁵Evidently, we need that the series used are stationary before conducting any TS analysis. We tested for stationarity of our series using the ADP test and then differentiate the series that were non-stationary (see Appendix A). We didn't apply the ADF test to the series in logs afterward as the R function *auto.arima* automatically differentiates the series d times until they are stationary.

⁶We do find this result 'weird' but didn't find any mistake in the code

2.1.3 French-Fama factors

2.2 Estimate the sensitivities of the factorial model

2.3 Estimate the remuneration of risk from the multibeta relationship

2.4 Test the validity of the multibeta relationship

Conclusion

Appendix A Additional tables and figures

Macroeconomic factors

Table 2: Results ADF with trend and drift

	EuroUSD	CAC40	Inflation	PIB	RF AAA	OAT	1pct	5pct	10pct
tau3	-2.945	-2.028	-0.565	-4.716	-1.011	-0.720	-3.990	-3.430	-3.130
phi2	3.012	1.652	1.243	7.665	0.800	0.555	6.220	4.750	4.070
phi3	4.421	2.220	1.522	11.121	1.198	0.683	8.430	6.490	5.470

Table 3: Results ADF with drift

	EuroUSD	CAC40	Inflation	PIB	RF AAA	OAT	1pct	5pct	10pct
tau2	-1.841	-1.428	-0.444	-2.117	-1.545	-1.142	-3.460	-2.880	-2.570
phi1	1.791	1.277	0.438	2.589	1.195	0.803	6.520	4.630	3.810

Table 4: Results ADF with no trend nor drift

	EuroUSD	CAC40	Inflation	PIB	RF AAA	OAT	1pct	5pct	10pct
tau1	-0.618	0.414	0.353	0.721	-1.437	-0.995	-2.580	-1.950	-1.620

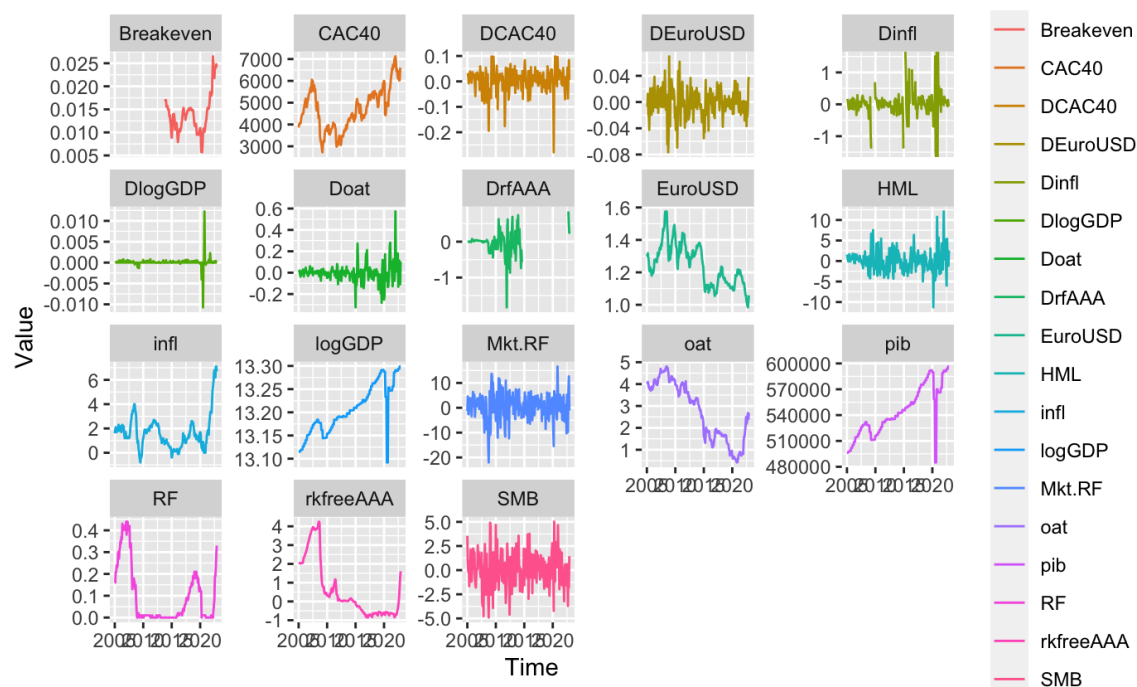


Figure 1: Factors: Series in level and in deltas

Appendix B Code - Data gathering and cleaning

```

1 #!/usr/bin/env python3
2 # -*- coding: utf-8 -*-
3 """
4 APi - Data gathering and Data Cleaning
5
6 Scrapping - Firm level data for French and Fama factors
7 Use Yahoo Finance API to get the financial data for all the stocks
8 Use Eurostat API to get the macro data
9 Two data sets (French-Fama factors and long term inflation expectation) are found online and
   have been downloaded in CSV file beforehang
10
11 Merge and clean the dataset
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13 @author: nataliacardenasf
14 """
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16 import pandas as pd
17 import numpy as np
18 import os
19
20 import requests
21 from bs4 import BeautifulSoup
22
23 #import pandas_datareader.data as web

```



```

24 import yfinance as yf
25 #from eurostatapidclient import EurostatAPIClient
26 import datetime
27
28
29 os.chdir('/Users/nataliacardenasf/Documents/GitHub/PROJECTS_AP_FE/AP 1')
30
31 company_names_lower = [
32     'air-liquide', 'airbus', 'bouygues', 'capgemini', 'carrefour', 'casino-guichard-perrachon',
33     , 'vivendi',
34     'kering', 'l-oreal', 'lvmh', 'michelin', 'orange', 'renault', 'sanofi', 'thales',
35     'totalenergies', 'vinci', 'compagnie-de-saint-gobain', 'ubisoft', 'tf1', 'danone',
36     'dassault-aviation', 'air-france-klm', 'accor', 'bic', 'hermes-international',
37     'jcdecaux', 'nexans', 'sodexo', 'biomerieux', "CAC40", "EuroUSD"]
38
39 tickers = [
40     'AI.PA', 'AIR.PA', 'EN.PA', 'CAP.PA', 'CA.PA', 'CO.PA', 'VIV.PA', 'KER.PA', 'OR.PA', 'MC.
41     PA',
42     'ML.PA', 'ORA.PA', 'RNO.PA', 'SAN.PA', 'HO.PA', 'TTE.PA', 'DG.PA', 'SGO.PA', 'UBI.PA', '
43     TFI.PA',
44     'BN.PA', 'AM.PA', 'AF.PA', 'AC.PA', 'BB.PA', 'RMS.PA', 'DEC.PA', 'NEX.PA', 'SW.PA', 'BIM.
45     PA', "^FCHI", 'EURUSD=X']
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66         data.append({'Year': year, 'MarketCap': market_cap, 'Company':
        company_name})
67         return pd.DataFrame(data)
68     return None
69
70 def scrape_price_book(url, company_name):
71     response = requests.get(url)
72     if response.status_code == 200:
73         soup = BeautifulSoup(response.content, 'html.parser')
74         table_body = soup.find('table', class_='table').find('tbody')
75         if table_body:
76             data = []
77             rows = table_body.find_all('tr')
78             for row in rows:
79                 cols = row.find_all('td')
80                 if len(cols) >= 2:
81                     year = cols[0].text.strip()
82                     pricebook = cols[1].text.strip()
83                     #variation = cols[2].text.strip()
84                     data.append({'Year': year, 'PriceBook': pricebook, 'Company': company_name
        })
85         return pd.DataFrame(data)
86     return None
87
88
89 # Scraping market cap data
90 dfmktcap = pd.DataFrame()
91 for url, company in zip(mktcap_urls, company_names_lower[:-2]):
92     data = scrape_market_cap(url, company)
93     if data is not None:
94         dfmktcap = pd.concat([dfmktcap, data])
95
96 #Scap price book
97 dfpricebook = pd.DataFrame()
98 for url, company in zip(pricebook_urls, company_names_lower[:-2]):
99     data = scrape_price_book(url, company)
100    if data is not None:
101        dfpricebook = pd.concat([dfpricebook, data])
102
103 #indexes
104 dfmktcap['Year'] = pd.to_datetime(dfmktcap['Year'])
105 dfmktcap['Year'] = pd.DatetimeIndex(dfmktcap['Year']).year
106
107 dfpricebook['Year'] = pd.to_datetime(dfpricebook['Year'])
108 dfpricebook['Year'] = pd.DatetimeIndex(dfpricebook['Year']).year
109
110
111 ##Merge datasets

```

```

112 final_firm = dfmktcap.copy()
113 final_firm = final_firm.merge(dfpricebook, how='outer', on=['Year', 'Company'])
114
115
116 del dfmktcap, dfpricebook, mktcap_urls, pricebook_urls, url, data, company
117
118
119 missing = final_firm[final_firm.isna().any(axis=1)]
120 missing = missing.sort_values(by=['Year'])
121 missing = missing.reset_index()
122 #have both data points for all firms for 2010-2022
123 # in 09 only missing data is from BIC, Carrefour, Ubisolft, AirFrance
124
125 #remove 2023
126 final_firm = final_firm[final_firm.Year != 2023]
127
128 #Get book to market ratio = inverse of price-book ratio
129 final_firm['PriceBook'] = pd.to_numeric(final_firm['PriceBook'], errors='coerce')
130 final_firm['PriceBook'].replace('nan', np.nan, inplace=True)
131 final_firm['BookMarket'] = final_firm['PriceBook'].apply(lambda x: x ** -1 if not pd.isnull(x)
    ) else np.nan)
132
133 #Clean MarketCap
134 final_firm['MarketCap'] = (final_firm['MarketCap'].replace({'\$': '', ' B': ''}, regex=True).
    astype(float) * 1_000) # Clear the letters, convert to float and scale to millions
135
136 #Date format
137 final_firm['Year'] = pd.to_datetime(final_firm['Year'], format='%Y')
138
139 #====Get monthly data
140 monthly_data = pd.DataFrame()
141 # Repeat the yearly data for each month and each firm
142 for index, row in final_firm.iterrows():
143     firm_data = pd.DataFrame()
144     monthly_year = pd.date_range(start=row['Year'], periods=12, freq='MS')
145     firm_data['Date'] = monthly_year
146     firm_data['Company'] = row['Company']
147     firm_data['MarketCap'] = row['MarketCap']
148     firm_data['BookMarket'] = row['BookMarket']
149     firm_data['PriceBook'] = row['PriceBook']
150     monthly_data = pd.concat([monthly_data, firm_data])
151
152 del index, monthly_year, row, firm_data
153
154 firms_year = final_firm.copy()
155 firms_month = monthly_data.copy()
156
157 del final_firm, monthly_data, missing

```

```

158
159 %%Get return data with Yahoo finance
160
161 start = datetime.datetime(2002, 1, 1)
162 end = datetime.datetime(2022, 12, 31)
163
164 #Get all data
165 data = yf.download(tickers, start=start,
166                     end=end)
167
168 #Focus on adjusted closed values only
169 adjclose=data['Adj Close']
170 adjclose = adjclose.set_axis(company_names_lower, axis=1)
171
172 #Use monthly data: mean of the months value
173 adjclose = adjclose.resample('1M').mean(numeric_only=True)
174 adjclose_y = adjclose.resample('1Y').mean(numeric_only=True)
175
176 ## extract CAC40 and exchange rate
177 cac_xrate_month = adjclose.loc[:, ["CAC40", "EuroUSD"]]
178 cac_xrate_year = adjclose_y.loc[:, ["CAC40", "EuroUSD"]]
179 adjclose = adjclose.drop(columns=["CAC40", "EuroUSD"])
180 adjclose_y = adjclose_y.drop(columns=["CAC40", "EuroUSD"])
181
182 #Reshape
183 prices_monthly = pd.melt(adjclose, value_vars=company_names_lower[0:-2], ignore_index=False)
184 prices_yearly = pd.melt(adjclose_y, value_vars=company_names_lower[0:-2], ignore_index=False)
185
186
187 del data, adjclose, adjclose_y
188
189 #I'm not getting right values for xrate when downloading in bulk
190 data = yf.download(['EURUSD=X', '^FCHI'], start=start, end=end)
191 adjclose=data['Adj Close']
192 adjclose = adjclose.set_axis(['EuroUSD', 'CAC40'], axis=1)
193 #adjclose = pd.DataFrame(adjclose, columns=['Date', 'EuroUSD'])
194
195 cac_xrate_month = adjclose.resample('1M').mean(numeric_only=True)
196 cac_xrate_year = adjclose.resample('1Y').mean(numeric_only=True)
197
198 del data, adjclose, start, end
199
200
201 %% Endogeneous factor: long term inflation expectation from external file
202
203 #upload the Agence France Tresor data
204 pi_endo= pd.read_excel('2023_11_01_rend_tit_ref_oatei.xls', skiprows=[0,1,2,3,4], usecols
                    =[0,3])

```

```

205 pi_endo.columns = ['Date', "Breakeven"]
206
207 pi_endo["Date"] = pd.to_datetime(pi_endo["Date"])
208 pi_endo= pi_endo.set_index(pi_endo["Date"])
209 pi_endo.drop(columns=['Date'])
210
211 #get monthly data
212 piendo_month = pi_endo.resample('1M').mean(numeric_only=True)
213
214 #get yearly data
215 piendo_year = pi_endo.resample('1Y').mean(numeric_only=True)
216
217 del pi_endo
218
219 ### French and Fama - their data
220
221 df = pd.read_csv('Europe_3_Factors.csv',skiprows=[0,1,2])
222
223 #montly data, need to fix dates
224 frenchfama_month = df.iloc[:399,:]
225
226 frenchfama_month['Unnamed: 0'] = frenchfama_month['Unnamed: 0'].astype(str) # Convert to
    string for manipulation
227 frenchfama_month['Year'] = frenchfama_month['Unnamed: 0'].str[:4] # Extract year from the
    encoded date
228 frenchfama_month['Month'] = frenchfama_month['Unnamed: 0'].str[4:] # Extract month from the
    encoded date
229 frenchfama_month['Date'] = pd.to_datetime(dict(year=frenchfama_month['Year'], month=
    frenchfama_month['Month'], day=1))
230 frenchfama_month.drop(['Year', 'Month', 'Unnamed: 0'], axis=1, inplace=True)
231 frenchfama_month = frenchfama_month.set_index(frenchfama_month['Date'])
232 frenchfama_month = frenchfama_month.drop(columns=["Date"])
233 frenchfama_month = frenchfama_month.loc['2002-01-01':]
234 frenchfama_month = frenchfama_month.astype(float)
235
236
237 #yearly data
238 frenchfama_year = df.iloc[402:,:]
239 frenchfama_year["Unnamed: 0"] = pd.to_datetime(frenchfama_year['Unnamed: 0'])
240 frenchfama_year.rename(columns={"Unnamed: 0":'Date'}, inplace=True)
241 frenchfama_year = frenchfama_year.set_index(frenchfama_year['Date'])
242 frenchfama_year = frenchfama_year.drop(columns=["Date"])
243 frenchfama_year = frenchfama_year.loc['2002-01-01':]
244 frenchfama_year = frenchfama_year.astype(float)
245
246 del df
247
248

```

```

249 ### Macro data
250 #APIs didn't work as planned
251
252 ## PIB Q
253 pib = pd.read_csv("ECB_PIB.csv")
254 pib.columns = ['Date', 'Q', 'pib']
255 pib['Date'] = pd.to_datetime(pib['Date'])
256 pib = pib.drop(columns=['Q'])
257 pib = pib.set_index(pib['Date'])
258 pib = pib.loc['2002-01-01':]
259 pib = pib.drop(columns=['Date'])
260
261 #monthly
262 pib_monthly = pib.resample('MS').ffill()
263 #yearly
264 pib_year = pib.resample('1Y').last()
265
266 del pib
267
268 ##risk free AAA
269 rkfreeAAA = pd.read_csv('ECB_yield.csv')
270 rkfreeAAA.columns=['Date', "time", "rkfreeAAA"]
271 rkfreeAAA['Date'] = pd.to_datetime(rkfreeAAA['Date'])
272 rkfreeAAA = rkfreeAAA.set_index(rkfreeAAA['Date'])
273 rkfreeAAA = rkfreeAAA.drop(columns=['time', "Date"])
274
275 rkfreeAAA_monthly = rkfreeAAA.resample("1M").mean(numeric_only=True)
276 rkfreeAAA_year = rkfreeAAA.resample("1Y").mean(numeric_only=True)
277
278 del rkfreeAAA
279
280 ##HICP
281 pi_month = pd.read_csv("HICP.csv", sep=';', encoding = 'latin1', skiprows=[0,1,2,3], usecols
    = [0,1], header=None)
282 pi_month.columns= ['Date', "infl"]
283 pi_month["Date"] = pd.to_datetime(pi_month['Date'], format = "%Y-%m")
284 pi_month=pi_month.set_index(pi_month["Date"])
285 pi_month = pi_month.drop(columns=['Date'])
286
287 #yearly
288 pi_year = pi_month.resample("1Y").mean(numeric_only=True)
289
290
291 ##OAT
292 oat = pd.read_csv('OAT.csv', sep=';', skiprows=[0,1,2,3,4,5], usecols=[0,5], header=None)
293 oat.columns = ['Date', 'oat']
294 oat['Date'] = pd.to_datetime(oat["Date"])
295 oat = oat.set_index(oat["Date"])

```

```

296 oat = oat.drop(columns=["Date"])
297 oat = oat.loc["2002-01-01":]
298
299 oat['oat'] = oat['oat'].replace("-", np.nan)
300 oat['oat'] = oat['oat'].str.replace(',', '.').astype(float)
301
302 oat_month = oat.resample('M').mean(numeric_only=True)
303 oat_year = oat.resample('1Y').mean(numeric_only=True)
304
305 del oat
306
307
308 ### Merge the dataframes
309
310 ===MONTHLY
311 #macro stuff, only date index matter
312 monthly = pd.concat([cac_xrate_month, frenchfama_month, oat_month, pi_month, pib_monthly,
313                     piendo_month, rkfreeAAA_monthly], axis=1)
314 monthly= monthly.resample('M').last() #some tables encoded end of month, others on the 1st
315 monthly.to_csv('Monthly_series.csv')
316
317
318 #firm specific data
319 firms_month['Date'] = firms_month['Date'] + pd.offsets.MonthEnd(0) #all other df have eomonth
320                                     date
321 firms_month = firms_month.set_index(['Date'])
322 firms_month = firms_month.set_index('Company', append=True)
323
324 prices_monthly.columns = ['Company', 'value']
325 prices_monthly.set_index('Company', append=True)
326
327 monthly_stock = pd.merge(prices_monthly.reset_index(), firms_month.reset_index(), on =["Date",
328                                     "Company"], how='outer').set_index(["Date", "Company"])
329 monthly_stock.to_csv("Firm_monthly.csv")
330
331 #merge the two
332 merged_monthly = pd.merge(monthly, monthly_stock, left_index=True, right_index=True, how='
333                                     right')
334
335 #Export df in csv
336 merged_monthly.to_csv("DATA_month.csv")
337
338 === YEARLY
339 yearly = pd.concat([cac_xrate_year, frenchfama_year, oat_year, pi_year, pib_year, piendo_year,
340                     rkfreeAAA_year], axis=1)

```

```

339 yearly = yearly.resample('Y').last()
340
341 yearly.to_csv('Yearly_series.csv')
342
343 # =====
344 # #firm specific
345 # firms_year.rename(columns={"Year":'Date'}, inplace=True)
346 # firms_year['Date'] = firms_year['Date'] + pd.offsets.MonthEnd(0)
347 # firms_year = firms_year.set_index(['Date'])
348 # firms_year = firms_year.set_index('Company', append=True)
349 #
350 # prices_yearly.columns = ['Company', 'value']
351 # prices_yearly = prices_yearly.set_index('Company', append=True)
352 #
353 # yearly_stock = pd.merge(prices_yearly.reset_index(), firms_year.reset_index(), on=["Date",
354 #     "Company"], how='outer').set_index(["Date", "Company"])
355 # =====
356
357 #yearly_stock = monthly_stock.groupby('Company').resample('Y').mean()
358
359 monthly_data_reset = monthly_stock.reset_index()
360 yearly_stock = monthly_data_reset.groupby('Company').resample('Y', on='Date').mean()
361 #yearly_stock = yearly_stock.set_index(["Date", "Company"])
362
363
364
365 #merge the two
366 merged_year = pd.merge(yearly, yearly_stock, left_index=True, right_index=True, how='right')
367
368 merged_year.to_csv("DATA_yearly.csv")

```


Appendix C Code - Analysis

```
1 ### AP 1
2 ### ncardenasfrias
3
4 pacman::p_load(data.table, tidyverse, gplots, xts, stargazer, plm)
5
6 setwd('/Users/nataliacardenasf/Documents/GitHub/PROJECTS_AP_FE/AP 1')
7
8 df <- fread("DATA_month.csv")
9 as.data.table(df)
10
11
12 ## Identify the risk factors
13 #Need to remove the predicable part to the endogeneous and exogeneous series
14
15 #upload the monthly data with the yearly factors and transform into a list of TS
16 data = read.csv('Monthly_series.csv')
17 data$Date <- as.Date(data$Date)
18
19 # Filter rows between 2005 and 2022
20 filtered_data <- data %>%
21   filter(Date >= as.Date("2005-01-01") & Date <= as.Date("2022-12-31"))
22
23 time_series_cols <- filtered_data %>%
24   select(-Date)
25
26 time_series_list <- lapply(time_series_cols, function(col) {
27   ts_values <- ts(col, start = c(year(min(filtered_data$Date)), month(min(filtered_data$Date))
28     ), frequency = 12)
29   return(ts_values)
30 })
31
32 names(time_series_list) <- names(time_series_cols)
33
34
35
36 ## Estimate the beta coefficients
37
38
39 ## Estimate the lamdas
40
41
42 #Test the validity of the multi-beta relationship
```

3 Data and Framework

3.1 Estimation of the Factors

We need to choose what factors we are going to consider to generate risk premia that affect the return for the investor. In this section we examine the role of different types of factors (i) exogenous, (ii) endogenous and we examine more closely the three-factor model proposed by [Fama and French \[1993\]](#) in (iii).

3.2 Exogeneous Factors

These are risk factors that are supposed to be orthogonal to the portfolio itself. In particular, it is interesting to consider the role of

3.3 Endogeneous Factors

3.4 French-Fama Factors

[Fama and French \[1993\]](#) can be seen as an extension of the CAPM model. The authors show that the variation of the returns of an asset can be explained not only by the exposure to market risk as in the CAPM represented by the difference of the market return and the risk-free rate $[R_M - R_f]$, but also by a size and value premium in the following model.

$$R_j = \alpha_j + R_f + \beta_{m,j}[R_M - R_f] + \beta_S SMB + \beta_V HML + \varepsilon_j \quad (2)$$

The size premium refers to the observation that stocks with small market capitalizations tend to outperform stocks with larger ones and it is captured by the factor *SMB, small minus big*. It is computed as the difference in average returns of the 30% stocks with the smallest market capitalization and the average returns of the 30% stocks associated with the firms with the largest market capitalization. The value premium refers to the outperformance of "value stocks" i.e. those that have high book-to-market (B/M) and it is represented by the difference in an average return of the 50% of stocks with the highest *B/M* ratio (value stocks) and the 50% with lowest *B/M* ratio (growth stocks).

4 Estimation of the exposure

5 Estimation of the market price of risk(s)

Consider a series of returns for different stock prices of at least 30 over a given period of time and frequency. The goal is to estimate risk premium by choosing a relevant so-called risk-free asset obtained as the return of treasury bond with relevant maturity

Develop econometric analysis which provides the multi-beta relationship 1. Identify the series for the risk factors (endo and exo) and justify choices + including 2 factors proposed by French and Fama 2. Estimate beta coeds or different stocks with relevant linear regression 3. Estimate market price of different sources of risk retained in analysis with appropriate linear reg

Comment the results from a financial point of view: are the estimated exposures of the different stocks to the different factors in line with expectations

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