AIRES — MAS Project Theory, Set Up, and Some Toy Examples

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Overview

We are trying to model a real estate market with n buyers $B = \{b_1, \ldots, b_n\}$ and m sellers $S = \{s_1, \ldots, s_m\}$. The sellers are selling houses to the buyers. The buyers have some set of preferences over the houses, $U(S) = \{\{u_1(s_1), \ldots, u_1(s_m)\}, \ldots, \{u_n(s_1), \ldots, u_n(s_m)\}\}$, where u_i is the utility, or reward, function of buyer b_i. Each seller s_j has some value for their house $v_j \in V$. Because the sellers value their houses, they will only sell the house to a buyer if they will receive from the buyer some amount $p_j \geq v_j$. For now, for simplicity, we will normalize all the values and utilities to be in the same unit, and allow them to be able to take some amounts from 0 to 10.

Now, every house also has some characteristics that lead to them having the value they do to their seller and lead to the utility that a certain buyer will receive from purchasing it. Let's specify four of them, F (for square footage), Y (for yard size), W (for number of windows), and L (for levels).

We will define the utility function for each buyer b_i coming from buying the house sold by s_j as:

$$u_i(F_j, Y_j, W_j, L_j) = f_i \times F_j + y_i \times Y_j + w_i \times W_j + l_i \times L_j - p_j,$$

where $f_i, y_i, w_i, l_i \in [0, 10]$ are some constants specific to buyer b_i. Suppose, for instance that b_i really liked natural light and had 3 kids, one of whom was a soccer player. Then they would probably want a large house with lots of windows and a big yard so their child could practice soccer. Their constants could then be $f_i = 8, y_i = 7, w_i = 9, l_i = 2$.

(The buyers also need to have some sort of wealth to serve as a maximum constraint on how large p_j can be, but I want to get the toy model working first. That can be added in later.)

Next, we will similarly define the value of the house for the seller s_j as:

$$v_j(F_j, Y_j, W_j, L_j) = f_j \times F_j + y_j \times Y_j + w_j \times W_j + l_j \times L_j,$$

where $f_j, y_j, w_j, l_j \in [0, 10]$ are some constants specific to seller s_j. I think, although I guess we can play around with this, that the constants on the sellers should have less variation, just because it feels like when a house is being sold, a site like ZIllow will appraise the house based on some fixed formula. We should think more about how this works, but for now, I will let them vary as much as the buyers do.

Now the sellers have two options. Ideally they would sell their house for the price p_j, but only if p_j is more than their value v_j. At the end of the round, then, the seller's reward is $\max(v_j, p_j)$.

Some basic assumptions on the market's design — the bids being made on the houses are public information and the auction only ends after every buyer has made a bid that is acceptable to some seller. The utility and value functions are private information, but the characteristics of the houses are public. Every buyer can make a bid to every seller, simultaneously. Finally, assume the sellers do not lie about the valuations of their houses.

With the groundwork laid out, we can now look at some examples.

Homogeneity in the Buyers and the Sellers

First, suppose everyone on one side of the market and the other are the same. Specifically, let all the houses have the same properties: $F_1 = \cdots = F_m = F$, etc.

Further, let all of the buyers have the same utility function:

$$u(F_j, Y_j, W_j, L_j) = f_b \times F + y_b \times Y + w_b \times W + l_b \times L - p_j.$$

Finally, we let all of the sellers have the same value function:

$$v(F_j, Y_j, W_j, L_j) = f_s \times F + y_s \times Y + w_s \times W + l_s \times L.$$

Now, it is clear that until $p_j \geq f_s \times F + y_s \times Y + w_s \times W + l_s \times L = v_s$ then the sellers will not sell their houses, as they would be better off keeping them (assuming indifference for the equality). Now, the buyers want some non-negative utility — they are better off not buying a house, getting 0 utility, than buying a house that they value less than they paid for it, which would yield a negative utility. So, the buyers only buy a house if $p_j \leq f_b \times F + y_b \times Y + w_b \times W + l_b \times L = u_b + p_j$.

This result gives us some limits on the possible prices for a house in this market:

$$p_j \in [v_s, u_b + p_j].$$

This will help us later on down the line, where the limits on the price of a house being sold between a buyer b_i and a seller s_j is strictly $p_{i,j} \in [v_j, u_i + p_{i,j}]$.

Finally, just to conclude this trivial case, so long as the buyers value the houses more than the sellers (they will pay more than what the sellers think it is worth), then they will pay some price in the above defined range.

If there are more buyers than sellers ($n > m$), every buyer has an incentive to keep raising the price being bid to a higher amount up until their ceiling, which is $u_i + p_j$. This happens as they try to outbid the other buyers in the market, which will drive the price to its limit. In the end, all of the sellers will sell their houses to m of the buyers, with $n - m$ of the buyers not receiving a house. In this case, the houses are assigned to buyers randomly.

If there are more sellers than buyers ($m > n$), as soon as the buyers offer to pay the value of the houses to the sellers, then the houses will be sold. If the price being offered is higher than the value of the houses to the sellers, one of the sellers will have an incentive to lower their asking price marginally to ensure they sell their house at a profit. This will lead to an equilibrium price of the houses' value to the sellers. Not all of the sellers will sell their houses, but their final utility will be equal (or maybe infinitesimally higher for those who do, depending on how we treat indifference).

Finally, if there are the same number of buyers and sellers ($n = m$), then any price in the range $[v_j, u_i + p_{i,j}]$ has can happen, if initial bids are made at random. This is a bit more of an edge case, but essentially, each of the buyers will eventually bid some amount in this range and the sellers will accept these bids. However, there will be movement among both sides, recognizing that some sellers are making more profit from the sale, other sellers will raise their prices, and recognizing that other buyers are saving money, some buyers will lower their prices. The prices will coalesce to some equilibrium, which, if n, m is large enough, should be $\frac{v_j + u_i + p_{i,j}}{2}$.