Free Riding in a Military Alliance

Nicholas Cassol-Pawson

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I. Introduction

We develop a limited model of the individual payments of two coalitions of countries providing funding to separate aggressors in a war. The solution concept we propose is a refinement of an all-pay auction to include a free riding scenario.

II. Defining the Game

Players

The framework of this game is a war between two rival countries, which we denote by the index $j \in 1, 2$.

This war consists of a battle that happens one time, and the belligerent that wins the war is the one with the greater number of resources. We assume that both belligerents $j = 1, 2$ start with the same amount of resources, which we denote by C. If at the end of the war, both belligerents have the same amount of resources, then we consider it a draw.

The game we wish to solve is between two coalitions of countries, with each coalition composed of players who support one of the two belligerents. The players are denoted by an individual index i, as well as by an index j denoting which belligerent they support. When referring to player i, we will refer to the belligerent they support with the notation j, the belligerent they oppose with the notation j', the other players with the notation i' and the other players in the coalition with the notation $i'j$.

For simplicity, we make a game with symmetric teams, each with k players, so the players are indexed along $i \in 1, 2, \ldots, 2k$. The first k players are in the coalition supporting belligerent 1 and the second set in the coalition supporting belligerent 2.

Actions

In the game, each player has one action — to send some amount of resources (be this equipment, soldiers, economic aid, etc.) P_{ij} to the belligerent j they support.

P_{ij} can never be less than 0. We assume that every player has the same amount of resources to send to the belligerent that they back.

Payoffs

Now, we've reached the point where we need to define formally how a belligerent j wins the war. This is simple, we've already touched on the concept, but the winning belligerent merely needs to have more resources than the losing one. Since we assume they both start with the same amount of resources, the ending resources they have are merely the sum of the payments they received from their respective backers plus some constant C.

Belligerent j wins the war when:

$$\sum_{i=1}^{2k} P_{ij} s_{ij} + C > \sum_{i=1}^{2k} T_{ij'} s_{ij'} + C$$

where $s_{ij} \in 0, 1$ is a binary indicator of whether player i supports belligerent j, taking the value of 0 if they do not and the value of 1 if they do.

We assume that each player i assigns some value V_{ij} to the belligerent they support, j, winning the war. We assume that Vij is strictly positive and that $V_{ij'}$ is strictly negative — that is, player i certainly wishes to see belligerent j win the war.

We can expand on the value, however, to come up with a more concrete way to calculate it.

We assign each player i some number W_{ij}, which is the benefit they would get if belligerent j won the war. The real world interpretation of this value encompasses a variety of factors, including direct monetary benefit from having the belligerent they support win the war (say from that player gaining an increased ability to trade), an increase in soft or hard power perception by other countries from backing the winning side, and more. We set W_{ij} is strictly greater than 0.

We also give each player i a second parameter L_{ij}, which we define as the loss they would get if belligerent j lost the war. An interpretation of a high L_{ij} would be that player i sees belligerent j' winning the war as a significant threat to themselves, say they are located near belligerent j and are concerned that if belligerent j' wins the war, they will move on to attacking player i. We define L_{ij} to be strictly negative, that is, player i always faces some loss from belligerent j' winning the war.

We can then define the value V_{ij} to player i of being on the winning side of the war as the difference between W_{ij} and L_{ij}, as it accounts for how much they would gain from winning the war and how much they don't want to lose it:

$$V_{ij} = W_{ij} - L_{ij}$$

An important feature of this game is that the value of V_{ij} is private to player i. That is, none of its coalition partners know how much it wants belligerent j to win the war.

We assign V_{ij} from an independent, identical distribution F to each player, $0 \leq V_{ij} \leq 100$. All players know the bounds on the distribution of V_{ij}. The distribution F is symmetric, as we assume that a range of values assigned to their favored side winning the war exist: some players will have high W_{ij} and L_{ij}, corresponding to a large V_{ij}, while a similar number of players will likely have small numbers for both, corresponding to a small V_{ij}. Symmetric distributions to consider for the model of V_{ij} include the uniform and the normal, scaled to include the parameters of interest. We choose to use the uniform distribution as it is easier to work with.

Finally, we define the payoffs to player i as the following:

If belligerent j wins the war, player i gets the benefit of winning the war less how much they invested in belligerent j:

$$\Pi_{ij}^{W} = W_{ij} - P_{ij}$$

However, if the belligerent they support loses the war, they get the negative payoff of the loss amount they have less the amount they paid into the war:

$$\Pi_{ij}^{L} = L_{ij} - P_{ij}$$

If there is a draw at the end of the war (both belligerents j, j' have the same amount of resources), then we assume that every player i gets a benefit of $-\varepsilon < 0$ — the status quo holds, but due to the conflict everyone loses slightly. Then, the payoffs to each player i are:

$$\Pi_{ij}^{D} = -\varepsilon - P_{ij}$$

Strategies

We now consider the strategies a player i has.

Each player i has one action — how much P_{ij} they should pay to the belligerent j they back. We have already restricted $0 \leq P_{ij} \leq 100$. However, because the player i is in a coalition backing belligerent j, they can still win the war and reap the payoff W_{ij} without contributing any P_{ij} as long as their coalition partners contribute more resources to the war than the enemy coalition does.

Therefore, there is a risk of free-riding present in this game. We wish to formally determine the cutoff for a player to be a free rider.

III. Solving the Game

Solving the Game for Large k

We first look at the case where $k \to \infty$. This is a generalization of the problem from the real world, as there are not a nearly infinite number of countries, but we will develop some solution concepts here.

We first make the assumption that the amount of resources that a player i will send to the belligerent j that they back is defined as a strictly increasing function of V_{ij}. This makes intuitive sense, as we'd imagine that a player i with a higher value for belligerent j to win the war would put more resources to the cause.

Now, we wish to find an upper bound on the P_{ij}.

For the player i, the best case scenario would be to win the war without having to spend any money to do so, so they would get a payoff of:

$$\Pi_{ij}^{W} = W_{ij}$$

However, every player would love for this situation to happen and would follow it, resulting in a draw where everyone gets the payoff of $-\varepsilon$.

The next best situation is to win the war while contributing money. Then all payoffs are:

$$\Pi_{ij}^{W} = W_{ij} - P_{ij}$$

A worse case scenario would be for a country to pay money into the war and then lose it anyways. So we have that:

$$\Pi_{ij}^{L} = L_{ij} - P_{ij}$$

In a situation where there is a risk of this happening, the country would rather provide no resources and get the payoff of:

$$\Pi_{ij}^{L} = L_{ij}$$

Then, a player i will make a payment P_{ij} if winning the war after making the payment provides a greater net benefit to them than losing the war without providing resources to country j (we assume that they make the payment if the payoffs are the same between paying and winning and not paying and losing):

$$W_{ij} - P_{ij} \geq L_{ij} \iff W_{ij} - L_{ij} \geq P_{ij} \iff V_{ij} \geq P_{ij}$$

The conclusion is that at most, a country i will provide resources to belligerent j so long as the amount of resources they provide is less than the value of winning the war is to them.

A Small Problem of Free Riding

Now, we take the point of view of some country I. They support belligerent J and are trying to determine how much resources to give to that belligerent. Crucially, however, their value V_{ij} of belligerent J winning the war is $V_{Ij} < 50$.

Player I sits back and thinks for a bit:

They know that everyone's value for their respective countries to win the war is derived from a uniform distribution, with a maximum value of 100 and a minimum value of 0, which we denote by $U(0, 100)$. The expected value of a uniform distribution, of course, is:

$$E(U(l, h)) = \frac{h - l}{2}$$

so in this game we have

$$E(U(0, 100)) = \frac{100 - 0}{2} = 50$$

Now player I thinks about the payments every country will make. They know that, at most, each country will pay their maximum value, so I assumes they do — if they pay less, they run the risk of losing the war.

Player I expects that the opposing coalition (who support J') will wind up paying the sum total:

$$E\left(\sum_{i=1}^{k} V_{ij'}\right) = \sum_{i=1}^{k} E(V_{ij'}) = kE(V_{ij}) = k * 50$$

Then player I looks at their own coalition and expects that, without them, they pay:

$$E\left(\sum_{I \notin i} V_{ij}\right) = \sum_{I \notin i} E(V_{ij}) = (k-1)E(V_{ij}) = (k-1) * 50$$

However, since player I has a value of $V_{ij} < 50$, they see that if they were to do their full payment, it would be the case that their side would pay:

$$\sum_{i=1}^{k} V_{ij} = (k-1) * 50 + V_{Ij}$$

while the opposition pays:

$$\sum_{i=1}^{k} V_{ij'} = (k-1) * 50 + 50$$

Comparing, it is clear that

$$\sum_{i=1}^{k} V_{ij} < \sum_{i=1}^{k} V_{ij'}$$

Player I then realizes that if they contribute $V_{Ij} \geq P_{Ij} > 0$ to belligerent J their side would lose the war regardless, so they would choose $P_{Ij} = 0$, to minimize the loss they would make.

This assessment is made by all players with $V_{ij} < 50$, so we can conclude that all these players will free ride off of the players with values $V_ij > 50$.

But Wait, There's More (or: The Short Tale of How All Wars Should End in a Draw)

Now that we have eliminated the players with $V_{ij} < 50$, we've wound up in a subset of the game. All of the players who would contribute resources in the first stage of the game have values $V_{ij} > 50$. Now, we essentially have the same game as before, except the values V_{ij} are randomly generated from a uniform distribution with a minimum of 50. All of the players with V_{ij} lower than the expected value of this uniform distribution will free ride, in this case, they are players with $V_{ij} < 75$. But then, the game shrinks again until players with $V_{ij} < 87.5$ free ride, and this continues ad infinitum until everyone free rides — we wind up in a perfect tragedy of the commons situation, where everyone expects everyone else to carry the coalition (in a continuous distribution, the probability of having any specific value is 0, so in this case, the player with the highest value V_{ij} would have expectations based on some other player having the value of 100). The war ends in a draw, causing all sides to get the payoff of $-\varepsilon$.

IV. An Assessment of the Model

This model is a bit fishy. In the real world, entire coalitions of countries do not not provide resources and backing to their allies. So what went wrong?

Several assumptions compounded to create the final result.

The key one was that there is a very large number of players. For a smaller number of players, say $k = 3$, there is a lot more variance in the uniform distribution, so a player with a high value V_{ij} would be quite confident that they could be a deciding factor in the war and would thus choose to not free ride.

The assumption of a uniform distribution of V_{ij} is not highly limiting, it makes the model more easily solvable. Choosing instead some other symmetric distribution, like the normal, would allow for a higher percentage of the V_{ij}'s to be clustered near the center value, but would still eventually converge to no one providing any resources.

A third assumption that causes the model to break is that of perfect rationalizability. We assume that every player on every turn looks at the payment that would be made without them and makes a judgement call as to whether they should provide resources to the belligerent they support. In reality, this is not the case: players will not think through all the subsets of the game until they decide they wouldn't play — likely after the second or third iteration of the game they would stick with their decision. This can be seen in a very obvious deviation: for any country, paying even a tiny amount of money would mean that their side wins the war because no participant will provide any funding.

V. Adjusting the Model to Reduce Free Riding Capacity

We can adjust the model to try to reduce free riding. The critical component is to make the model dynamic — which is more reflective of real wars, where funding isn't provided once, but rather over the course of the entire war. In this case, players in a coalition can see how much resources other players provide and pressure them to not free ride.