


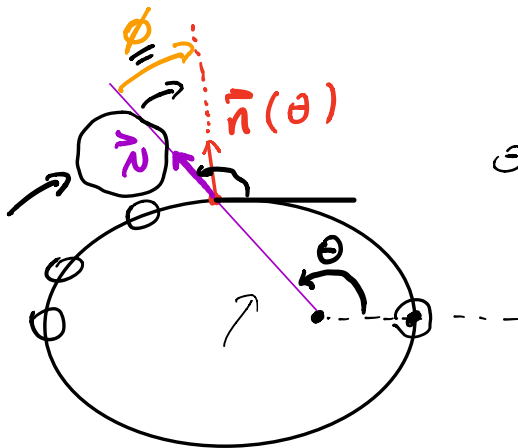
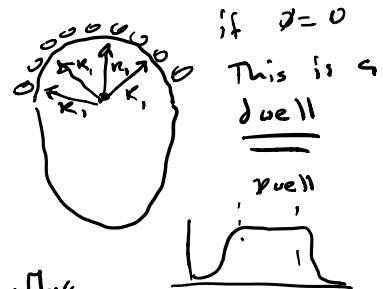
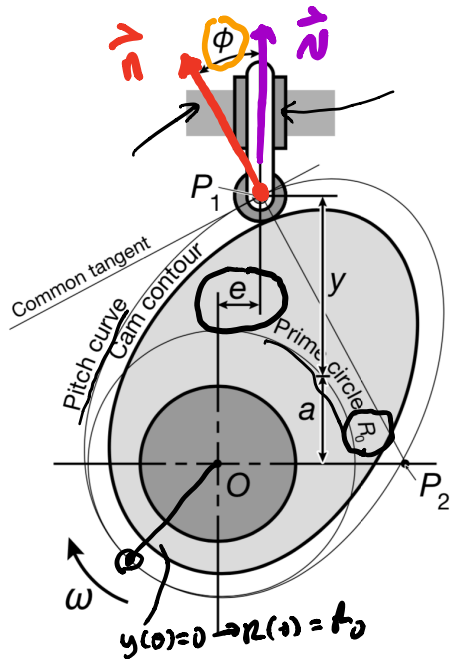
Today:

~~- HW 3 discussion~~  ~~offset curve~~
~~derivs numerically~~

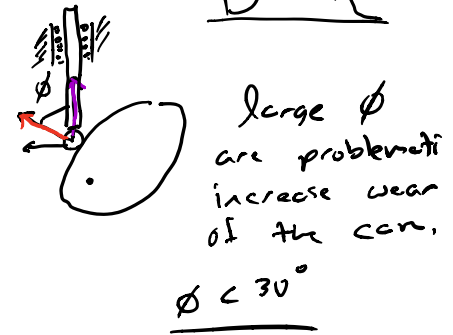
- Cam pressure angle

- Cam radius of curvature

- Mobility equations



$$\theta = \omega t$$



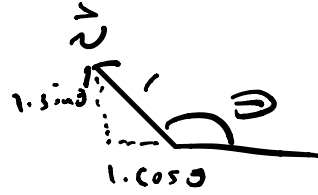
$$\underline{r}(\theta) = \underline{y}(\theta) + \underline{R}_0$$

$$\vec{n}(\theta) = \frac{1}{\sqrt{x'^2 + y'^2}} [-y', x']$$

$$\vec{n}(\theta) = \frac{1}{\sqrt{r'^2 + r^2}} [-r' \sin \theta - r \cos \theta, r' \cos \theta - r \sin \theta]$$

$$\begin{aligned} x &= R \cos \theta \\ y &= R \sin \theta \\ x' &= R' \cos \theta - R \sin \theta \\ y' &= R' \sin \theta + R \cos \theta \end{aligned}$$

$$\vec{n} \cdot \vec{n} = ? \rightarrow \underline{\vec{n} \cdot \vec{n} = 1}$$



$$\vec{n} = [\cos(\theta), \sin(\theta)]$$

$$\underline{\vec{v} \cdot \vec{n}} = |\vec{v}| |\vec{n}| \cos(\phi)$$

↑
pressure angle

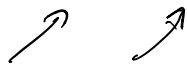
$$\cos \phi = \frac{1}{\sqrt{R'^2 + R^2}} [-R' \sin \theta - R \cos \theta, R' \cos \theta - R \sin \theta] \cdot [\cos(\theta), \sin(\theta)]$$

$$= \frac{1}{\sqrt{R'^2 + R^2}} [-\cancel{R' \sin \theta \cos \theta} - R \cos^2 \theta + \cancel{R' \cos \theta \sin \theta} - R \sin^2 \theta]$$

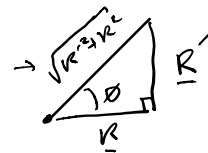
$$\cos \phi = \frac{-R}{\sqrt{R'^2 + R^2}}$$

fine & Done ✓

→ but look for tan φ



$$\tan \phi = \frac{R'}{R}$$



$$b^2 = R'^2 + R^2 - R^2$$

$$b^2 = R'^2$$

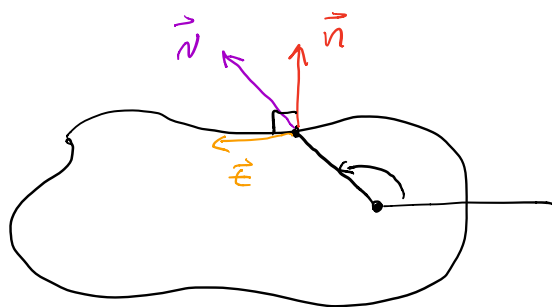
$$\tan \phi = \frac{y' - \cancel{x}}{\sqrt{K_0^2 - \cancel{x^2}} + y}$$

$$= \frac{y'}{R_0 + y} = \boxed{\frac{R'}{R}}$$

$$R(\theta) = R_0 + y$$

$$R'(\theta) = y'$$

Radius of curvature:



$$|\vec{n}| = 1$$

$$|\vec{t}| = 1$$

$$\vec{n} \cdot \vec{n} = 1$$

$$\vec{t} \cdot \vec{t} = 1$$

units $\frac{1}{\text{length}}$ length

$$\vec{t}' = \underbrace{K(\theta)}_{\text{Curvature}} \underbrace{v(\theta)}_{v(\theta) = \sqrt{x'^2 + y'^2}} \vec{n}$$

$$\frac{d}{d\theta} (\vec{t} \cdot \vec{t}) = 0$$

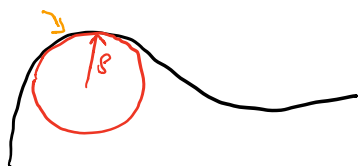
$$\vec{t}' \cdot \vec{t} = 0$$

$$\vec{t}' \text{ is } \perp \vec{t}$$

$$K(\theta) = \frac{1}{\rho(\theta)}$$

Radius of curvature

$$\vec{n} = \frac{1}{\sqrt{x'^2 + y'^2}} \begin{bmatrix} -y' \\ x' \end{bmatrix}$$



$$K(\theta) \gg 1$$

$$\rho(\theta) \ll 1$$

$$K(\theta) \ll 1$$

$$\rho(\theta) \gg 1$$



$$(\vec{n} \cdot \vec{t}' = \hat{n}(k(\vartheta) \underline{v(\vartheta)} \vec{n}) \quad \vec{n} \cdot \vec{n} = 1$$

$$\left\{ \frac{\vec{n} \cdot \vec{t}'}{v(\vartheta)} = \underline{k(\vartheta)} \right\} \left\{ \frac{v(\vartheta)}{\vec{n} \cdot \vec{t}'} = \underline{\rho(\vartheta)} \right\}$$

Curvature relationships

$$v = \sqrt{x'^2 + y'^2}$$

$$x(\vartheta) = R(\vartheta) \cos \vartheta$$

$$\vec{n} = \frac{1}{v} [-y', x']$$

$$\vec{t}' = ?$$

$$\vec{t} = \frac{1}{v} [x', y']$$

$$\frac{\partial}{\partial \vartheta} \frac{x'}{v} = \frac{x''v - v'x}{v^2}$$

$$\left[\frac{\partial}{\partial \vartheta} \frac{x'}{v}, \frac{\partial}{\partial \vartheta} \frac{y'}{v} \right]$$

$\uparrow \quad \quad \uparrow$
 $v(\vartheta) \quad \quad v(\vartheta)$

$$\vec{n} = \frac{1}{v} [-y', x']$$

$$\vec{t}' = \frac{1}{v^2} [x''v - v'x, y''v - v'y]$$

$$k(\vartheta) = \frac{\vec{n} \cdot \vec{t}'}{v}$$

$$[-x''vy' + \cancel{y'v'x} + y''vx' - \cancel{x'vy'}]$$

$$= \frac{1}{v} \frac{1}{v} \frac{1}{v^2} [-x''vy' + y''vx']$$

$$k(\vartheta) = \rightarrow \frac{y''x' - x''y'}{v^3} \rightarrow \rho(\vartheta) = \frac{v^3}{\underline{y''x'} - \underline{x''y'}}$$

$$\begin{aligned} x &= R \cos \theta \\ y &= R \sin \theta \end{aligned} \quad \rightarrow \quad \rho(\theta) = \sqrt{x'^2 + y'^2} = \sqrt{R'^2 + R^2}$$

$$x' = R' \cos \theta - R \sin \theta$$

$$y' = R' \sin \theta + R \cos \theta$$

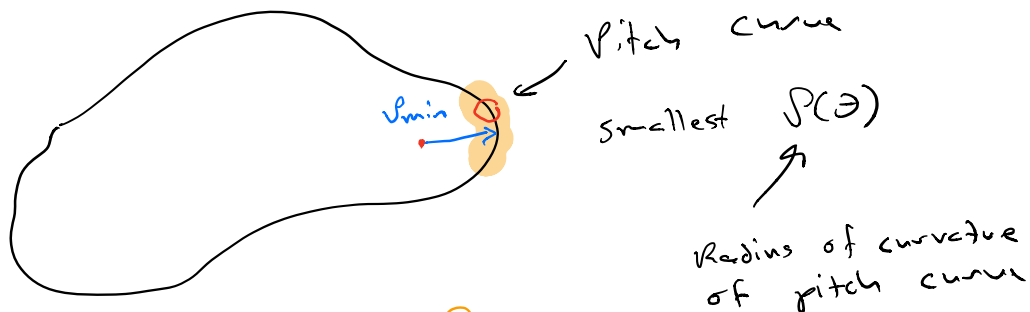
$$x'' = R'' \cos \theta - 2R' \sin \theta - R \cos \theta$$

$$y'' = R'' \sin \theta + 2R' \cos \theta - R \sin \theta$$

$$y''x' - x''y' \rightarrow R^2 + 2R'^2 - RR'$$

$$\rho(\theta) = R_p + y(\theta)$$

$$\rho(\theta) = \frac{(R'^2 + R^2)^{3/2}}{R^2 + 2R'^2 - RR'}$$



$$R_F < \rho_{min}$$

