6

Design of cams

6.1 Introduction

A *cam* is a mechanical element of a machine that converts a rotary motion of a *driveshaft* into a linear motion of a *follower*, which is in permanent contact with the cam contour. Figure 6.1 shows a typical radial cam/follower mechanism.

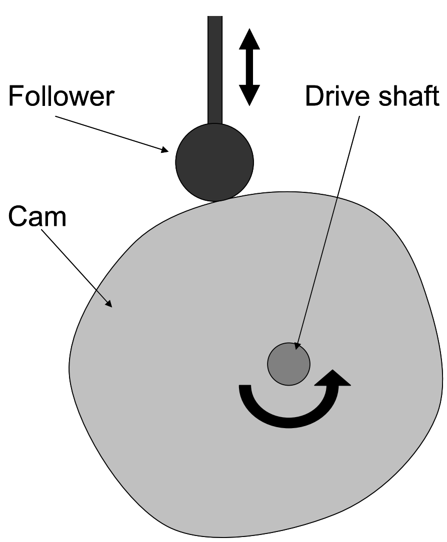

Figure 6.1: An end-on illustration of a typical cam with radial follower.

Cams are a reliable and simple way to convert one type of motion into another type of motion. Cams can be classified by

- the shape of the cam (plate cam, wedge cam, cylindrical cam),
- the shape of the follower (knife edge follower, flat follower, roller follower, spherical follower), and
- the output motion (reciprocal, oscillating).

The cam designer must insure that the follower maintains contact with the cam surface at all times. Gravity, springs, hydraulics, or mechanical restraints may be used. The cam contour determines the displacement of the follower. Examples of cams can be found in an automobile combustion engine, where a camshaft opens and closes the inlet and outlet valves of the cylinders; in paper transport through a printer; in medical equipment; in slot machines; and in locking mechanisms for doors and clamps of all sizes. Fig. 6.2 shows some examples of *plate cams*. Though there are many other kinds of cams, including the following, we will only consider plate cams:

Face cam: The follower engages in a groove cut into the face of a rotating or linearly translating surface of the cam, pushing the follower back and forth. A record player is a good example: the groove cut into the record swings the needle back and forth in a magnetic field to cause an electrical signal used to make sound. The arm carrying the needle is unable to follow the rapid motion of the groove, instead following along slowly to allow the needle to gradually move from the outside of the record to the inside.

Cylindrical cam: The follower engages in a groove cut into a cylindrical surface of the cam as it rotates along the cylinder's axis. An example is the ball-screw mechanism, where the screw is the cam and the ball structure that moves upon it is the follower. Other common places it appears include shifting mechanisms in transmissions, to guide the synchronized motion of several parts within.

Swash plate cam: A disk is mounted with its axis not aligned with the axis of rotation. As the disk is rotated, its edge moves parallel to the rotation axis, and the follower is put in contact with the face of the disk to capture this motion. A common example is the swashplate pump, used to pump slurries.

Linear cam: The follower engages with a surface that moves back and forth in a linear fashion. Helps one to translate one kind of rectilinear motion into another. The machine used to duplicate keys is a good example: the original key serves as the linear cam for the system as it cuts a new key.

The contour of the cam determines the follower motion and the dynamic behavior of the follower. Hence, it is necessary to study the

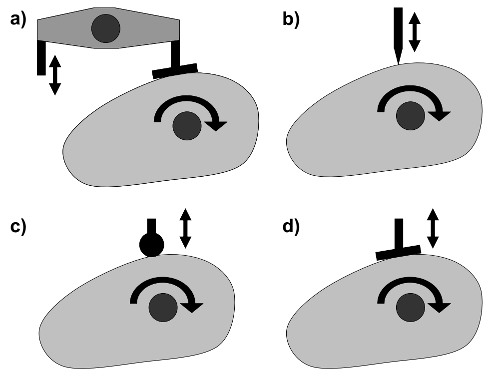

Figure 6.2: Plate cam examples: a) cam with rocker and a flat follower, b) knife edge follower cam, c) cam with roller follower, and d) cam with flat-face follower.

design of cam contours, the displacement diagrams of cams and the dynamics of cams. But first we need to define some of the terms used in describing cams.

6.2 *Terminology*

Fig. 6.3 shows a typical cam contour, with definitions of important terms, including the *base circle*, the *pitch curve* and the *prime circle*.

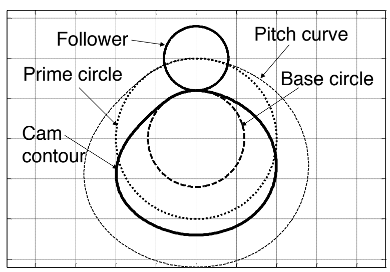

Figure 6.3: Terminology of plate cams and their components.

Base circle: The smallest circle that can be drawn tangent to the cam surface.

Pitch curve: The trace of the center of the follower when it is rotated around the cam contour.

Prime circle: The smallest circle about the cam center through the pitch curve.

Pressure angle: The angle ϕ between the direction of the follower motion and the normal to the cam contour, at the same point along the contour. The pressure angle varies during a revolution of the cam. A nonzero pressure angle implies that the force on the follower is not aligned with the follower axis. This can cause the follower to be jammed, if the pressure angle becomes too large. Figure 6.4 illustrates this, and the topic is covered in more detail on page 89.

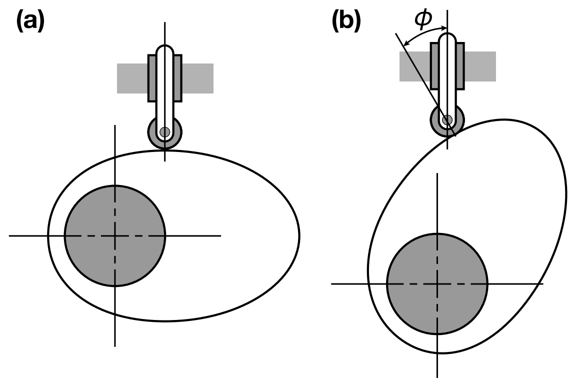

Figure 6.4: The roller follower (a) has a zero pressure angle: the cam-roller contact interface is perpendicular to the axis of the follower's linear motion. However, (b) in another part of the cam, the angle between that contact interface and the follower's linear motion axis is very large, shown here by ϕ. This angle is the pressure angle. The cam pushes the follower leftward and upward; the leftward force could cause the follower to jam in its bearing.

Pitch point: The point where the pressure angle is at its maximum.

Trace point: An arbitrary point on the follower. It is used to generate the pitch curve.

6.3 *Geometry of the radial cam*

The motion of the follower is determined by the shape of the cam contour. In this section, we discuss how to determine the exact cam contour to obtain a specific follower motion. For the sake of simplicity, the cam is assumed to rotate at a constant angular velocity, and the radially translating follower has roller contact with the cam contour. To determine the cam contour, one can apply the principle of inversion, i.e., the cam is kept stationary and the follower is rotated in the opposite direction of the cam rotation. The circumference of the cam contour is subdivided in equal sections, and each section divider line is given an identification number. Figure 6.5 illustrates this graphical method.

The follower displacement diagram, Fig. 6.5(b), is a developed view of one rotation of the cam. The x axis shows the different circumferential positions, θ, at which the follower displacement, y, was evaluated. The y axis shows the actual follower displacement (Fig. 6.5(b)). Notice that the follower displacement y is always positive: the *prime circle* defines $y = 0$; from this circle y becomes positive as the follower is pushed away from the axis of rotation of the cam.

The displacement diagram (Fig. 6.5(b)) is subdivided in the same sections as the cam contour, and uses the same identification numbers. One can now transfer the follower displacement, evaluated at a certain identification number on the displacement diagram, to the cam contour at the corresponding identification number. *Curve fitting* (*see* Ch. 3) a smooth curve through all transferred follower displacements produces the cam contour. This is how one might design a cam to achieve certain desired displacements—to open a set of valves a particular

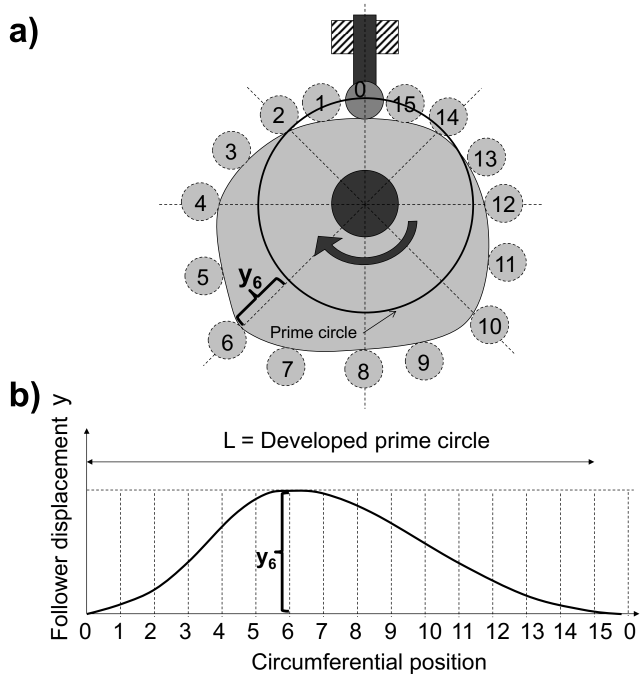

Figure 6.5: Graphically determining the (a) cam contour and the consequent (b) radial displacement of the follower, y, as a function of circumferential position, θ.

distance, for example.

The opposite is also true. From a given cam contour, the displacement diagram can be determined. The follower position on the cam contour can be obtained with the graphical method shown in Fig. 6.5(a). The follower displacement, evaluated at each identification number on the cam contour, is plotted in the displacement diagram at the corresponding number. By curve fitting a smooth curve through the follower displacements, one obtains the displacement diagram.

The travel of the follower can be separated in the *rise* or *lift*, in which the follower moves "up" and the *return* or *fall*, during which the follower moves "down". The period in which the follower remains at rest, is called *dwell*. Fig.7.6 6.6 illustrates the terminology. It is clear that the inflection points of the displacement diagram, i.e., the points where the curvature of the displacement diagram is maximum, indicate the pitch points on the cam contour.

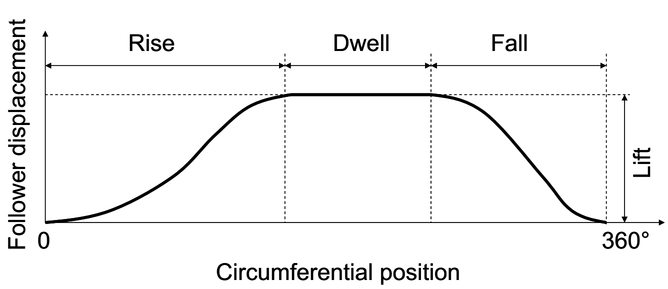

Figure 6.6: More cam terminology, illustrated on a follower's radial displacement plot with respect to the circumferential position around the cam. Also notice how the follower displacement curve is spliced together between the rise, dwell, and fall sections. This is often how cam profiles are designed.

6.4 Basic follower motions

There are many different follower motions which can be used for the follower rise and return. In this section, the *linear*, *parabolic*, *harmonic*, *cycloidal*, and *polynomial* motion will be described. Different graphical methods have been developed to design cam profiles based on these motions and more, and the reader is advised to seek details in the published literature if more information is needed.

It is, however, possible to use the displacement diagram as a rectangular coordinate system and obtain the follower displacement, $y(\theta)$ as a function of the cam angle, θ, plotted along the x axis of the rectangular coordinate system. By relating the cam dimensions to the displacement diagram, a cam contour can be created, corresponding to any curve which can be mathematically described.

The velocity of the follower can be calculated as the first derivative of a given displacement (or *cam*) profile y with respect to time t. The velocity v can then be expressed as

$$v = \frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt} = \frac{dy}{d\theta}\omega \tag{6.1}$$

where θ is the angular position of the cam and ω is the angular velocity of the cam. The acceleration a of the follower is then the derivative of the velocity;

$$a = \frac{dv}{dt} = \frac{d^2y}{dt^2} = \frac{d^2y}{d\theta^2}\omega^2 + \frac{dy}{d\theta}\alpha \tag{6.2}$$

where α is the angular acceleration.

When the angular velocity $\omega = \omega_0$ is constant, the above result can be re-written as

$$v = \frac{dy}{dt} = \frac{dy}{d\theta}\omega_0 \tag{6.3}$$

and

$$a = \frac{dv}{dt} = \frac{d^2y}{d\theta^2}\omega_0^2; \tag{6.4}$$

we also have the *jerk* as the time derivative of the acceleration,

$$\text{jerk} = \frac{da}{dt} = \frac{d^3y}{d\theta^3}\omega_0^3. \tag{6.5}$$

6.4.1 Linear (uniform) motion

The follower displacement y versus the cam angle θ can be expressed as

$$y = C\theta \tag{6.6}$$

where C is a constant. If one assumes that the total rise, L, has to occur within a cam angle, β, then

$$L = C\beta \tag{6.7}$$

or $C = L/\beta$. Substituting the value for C in eqn. (6.6) provides

$$y = \frac{L}{\beta}\theta \tag{6.8}$$

Equation (6.8) describes the displacement profile for a linear or uniform motion. The velocity and acceleration of the follower can be calculated by taking the first and second derivative of eqn. (6.8) with respect to time.

$$\frac{dy}{dt} = \frac{L}{\beta}\frac{d\theta}{dt} = \frac{L}{\beta}\omega \tag{6.9}$$

where $\omega = \frac{d\theta}{dt}$, the rotational speed of the cam.

$$\frac{d^2y}{dt^2} = \frac{L}{\beta}\frac{d\omega}{dt} = 0 \tag{6.10}$$

since it was assumed that the cam rotates at a constant rotational speed, i.e., $\omega =$ constant.

Figure 6.7 shows the displacement and velocity diagram for the case of a linear follower motion. It was shown that the acceleration of the

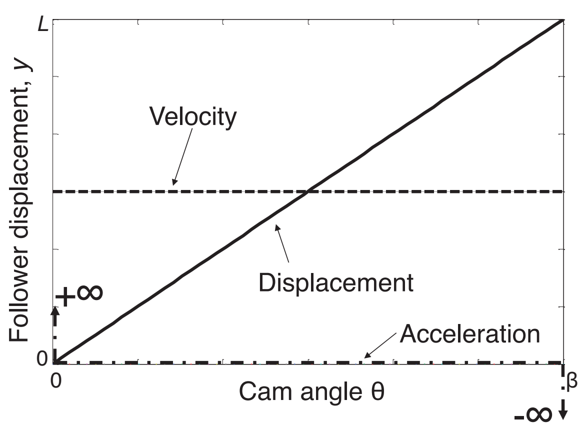

Figure 6.7: Linear follower motion, illustrating the cam angle, θ, and the displacement, velocity, and acceleration.

follower will be zero during the linear motion. During the transition from linear, uniform motion into dwell, however, the acceleration will be infinite, as can be seen from Fig. 6.8. It is clear that for fast moving cams, discontinuities are harmful and need to be avoided, because dynamic effects are important. Discontinuities may lead to noise, wear and even failure of the cam.

6.4.2 *Parabolic motion*

The follower displacement y versus the cam angle θ can be expressed as

$$y = C\theta^2, \tag{6.11}$$

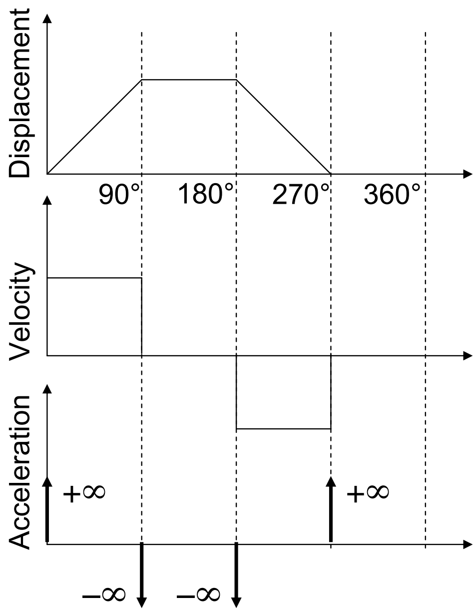

Figure 6.8: Uniform cam motion, illustrating the displacement, velocity, and acceleration of the follower. Note how the acceleration becomes infinite at the discontinuities in the displacement with respect to the circumference: this can cause damage.

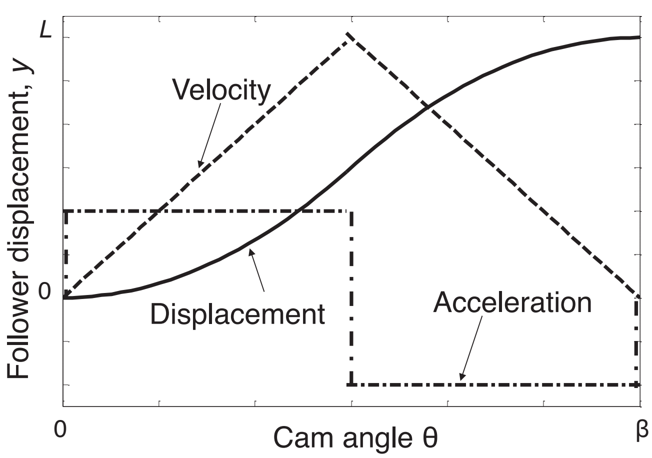

Figure 6.9: Parabolic cam motion.

where C is a constant. This equation is only valid up to the inflection point, i.e., where the curvature changes sign. If one assumes that the inflection point is located at a rise $L/2$ for a cam angle $\beta/2$, one finds that

$$C = \frac{2L}{\beta^2} \tag{6.12}$$

or,

$$y = 2L\left(\frac{\theta}{\beta}\right)^2. \tag{6.13}$$

The velocity and acceleration are then

$$\frac{dy}{dt} = \frac{4L\omega}{\beta^2}\theta \tag{6.14}$$

and

$$\frac{d^2y}{dt^2} = \frac{4L\omega^2}{\beta^2}. \tag{6.15}$$

From eqn. (6.14) it can be seen that the velocity is maximum at the inflection point $\beta/2$ and equal to $\left(\frac{dy}{dt}\right)_{\max} = \frac{2L\omega}{\beta}$. From eqn. (6.15), one observes that the acceleration is constant. For the second half of the displacement equation, i.e., past the inflection point, the displacement may be described as

$$y = C_1 + C_2\theta + C_3\theta^2. \tag{6.16}$$

The velocity is then

$$\frac{dy}{dt} = C_2\omega + 2C_3\omega\theta \tag{6.17}$$

To determine the constants, one assumes that the total rise L is reached at a cam angle β and that the velocity at that point is zero. Hence,

$$\begin{aligned} L &= C_1 + C_2\beta + C_3\beta^2 \\ 0 &= C_2\omega + 2C_3\omega\beta. \end{aligned} \tag{6.18}$$

Since the maximum velocity occurs at $\beta/2$, one knows that

$$\frac{2L\omega}{\beta} = C_2\omega + 2C_3\omega\frac{\beta}{2} \tag{6.19}$$

Simultaneously solving eqns. (6.18) and (6.19) provides definitions for the constants

$$\begin{aligned} C_1 &= -L \\ C_2 &= \frac{4L}{\beta} \\ C_3 &= -\frac{2L}{\beta^2}. \end{aligned} \tag{6.20}$$

Substituting eqns. (6.20) in eqn. (6.16) gives the displacement for the second half of the parabolic motion as

$$y = L\left[1 - 2\left(1 - \frac{\theta}{\beta}\right)^2\right]. \tag{6.21}$$

The velocity and acceleration may finally be calculated to be

$$\frac{dy}{dt} = \frac{4L\omega}{\beta}\left(1 - \frac{\theta}{\beta}\right) \tag{6.22}$$

and

$$\frac{d^2y}{dt^2} = -\frac{4L\omega^2}{\beta^2}. \tag{6.23}$$

The term *jerk* is often used to describe the third derivative of the displacement with respect to time[1].[2] Hence, it is a measure for how the acceleration changes with time. Figure 6.9 shows the displacement and velocity diagram for the case of a parabolic follower motion. The jerk (third derivative of displacement with respect to time) will become infinite at the middle of the cam angle plot since the acceleration has a discontinuity here.

[1] James Kilner, Antonia F de C Hamilton, and Sarah-Jayne Blakemore. Interference effect of observed human movement on action is due to velocity profile of biological motion. *Social Neuroscience*, 2(3-4):158–166, 2007; Kaan Erkorkmaz and Yusuf Altintas. High speed cnc system design. part i: jerk limited trajectory generation and quintic spline interpolation. *International Journal of machine tools and manufacture*, 41(9):1323–1345, 2001; D Hrovat and M Hubbard. Optimum vehicle suspensions minimizing rms rattlespace, sprung-mass acceleration and jerk. *Journal of Dynamic Systems, Measurement, and Control*, 103(3):228–236, 1981; and Sonja Macfarlane, Elizabeth Croft, et al. Jerk-bounded manipulator trajectory planning: design for real-time applications. *Robotics and Automation, IEEE Transactions on*, 19(1):42–52, 2003

[2] And yes, there's more: beyond jerk, there's jounce, snap, crackle and pop, pretty useless with the possible exception of cosmology.

6.4.3 *Harmonic motion*

We can also drive the cam-follower system in a harmonic motion. The follower displacement y versus the cam angle θ can be expressed in this case as

$$y = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right). \tag{6.24}$$

Using the same approach as used in the linear and parabolic motion, one can find the velocity and acceleration diagram:

$$\frac{dy}{dt} = \frac{\pi L\omega}{2\beta}\sin\frac{\pi\theta}{\beta} \tag{6.25}$$

and

$$\frac{d^2y}{dt^2} = \frac{L}{2}\left(\frac{\pi\omega}{\beta}\right)^2\cos\frac{\pi\theta}{\beta} \tag{6.26}$$

Figure 6.10 shows the displacement and velocity diagram for the case of a harmonic follower motion. Unlike in the case of a parabolic motion, there is no discontinuity at the inflection point.

6.4.4 *Cycloidal motion*

Next we consider *cycloidal motion*; a cycloid is the curve traced by a point on the circumference of a circle as that circle rolls along a straight surface without slippage. The follower displacement y versus the cam angle θ can be expressed as

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right). \tag{6.27}$$

The velocity and acceleration diagram are then given by

$$\frac{dy}{dt} = \frac{L\omega}{\beta}\left(1 - \cos\frac{2\pi\theta}{\beta}\right) \tag{6.28}$$

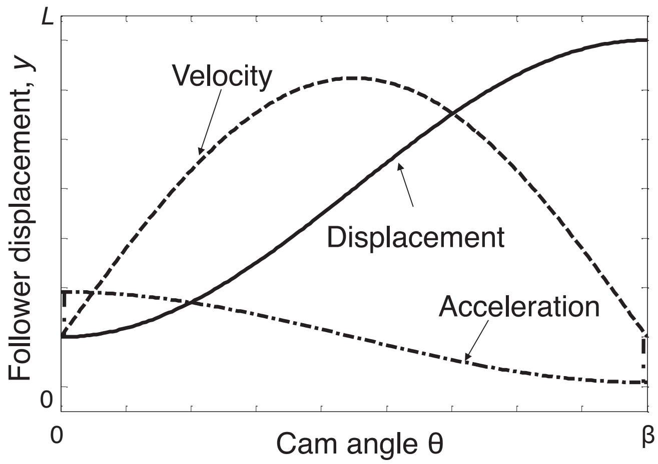

Figure 6.10: Harmonic cam motion.

and

$$\frac{d^2y}{dt^2} = 2L\pi\left(\frac{\omega}{\beta}\right)^2 \sin\frac{2\pi\theta}{\beta}. \tag{6.29}$$

Figure 6.11 shows the displacement and velocity diagram for the case of a cycloidal follower motion.

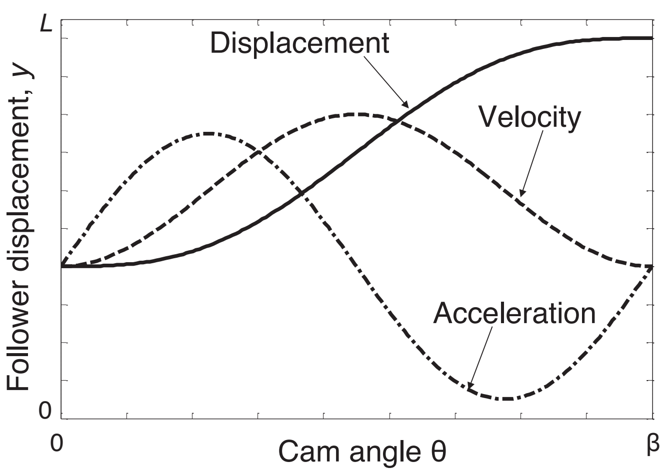

Figure 6.11: Cycloidal cam motion.

6.4.5 *Polynomial motion*

We can move beyond the linear and parabolic motions to general polynomials; here we will consider polynomials in θ up to order 5 for $y = y(\theta)$, though this could fairly easily be extended to even higher order polynomials if needed. The follower displacement y versus the cam angle θ can be expressed as

$$y = L\left(\frac{10}{\beta^3}\theta^3 - \frac{15}{\beta^4}\theta^4 + \frac{6}{\beta^5}\theta^5\right); \tag{6.30}$$

the zeroth, first, and second order terms in θ are also not included—because they would lead to discontinuities in the acceleration. The velocity and acceleration are given by

$$\frac{dy}{dt} = L\left(\frac{30\omega}{\beta^3}\theta^2 - \frac{60\omega}{\beta^4}\theta^3 + \frac{30\omega}{\beta^5}\theta^4\right) \tag{6.31}$$

and

$$\frac{d^2y}{dt^2} = L\left(\frac{60\omega^2}{\beta^3}\theta - \frac{180\omega^2}{\beta^4}\theta^2 + \frac{120\omega^2}{\beta^5}\theta^3\right) \tag{6.32}$$

Figure 6.12 shows the displacement and velocity diagram for the case of a 3–4–5 polynomial follower motion.

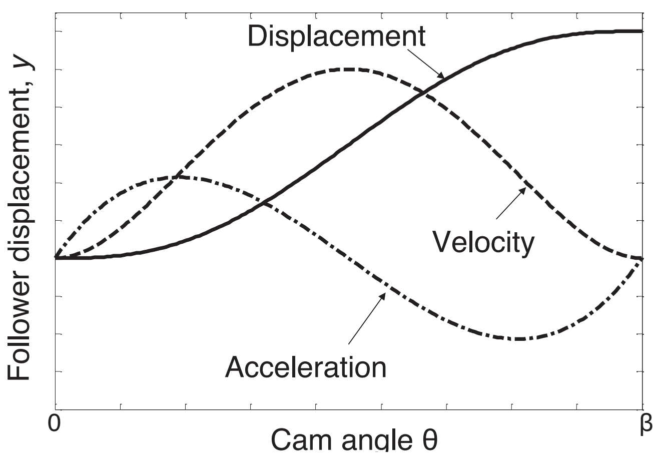

Figure 6.12: Polynomial cam motion.

6.5 *Practical aspects to consider in cam design*

To design a cam, especially for high-speed machinery, it is *very* important to carefully study the acceleration and jerk diagrams. Indeed, infinite jerk or acceleration may cause serious problems in cam mechanisms even if operated at slow speeds.

When using unity for the parameters L, β, and ω, one can compare the maximum velocity, acceleration, and jerk for the different motions provided above. Table 7.1 shows the results. In conclusion one can say that the following four requirements must be met to have a good kinematic design of a cam:

1. The prescribed motion requirements are met.

2. There is a continuous displacement, velocity and acceleration.

3. Jerk diagrams may be discontinuous, but not infinite.

4. The maximum amplitudes of velocity and acceleration peaks are low.

Beyond these aspects, there is the matter of trying to decide how large or small the cam itself should be. The smaller the cam, the smaller the mass and rotational inertia it has, which can improve its response at the potential expense of stiffness and strength. Smaller cams tend to be less expensive to produce. However, as the cam is reduced in size, the pressure angle tends to increase, and the *radius of curvature* is reduced, to a point in some cases where it becomes impossible to obtain the desired motion: as an extreme example, a 1-mm diameter cam would not be able to produce a 10-cm rise in a follower. This can give rise to *undercutting*. The following pages cover these aspects in more detail.

6.5.1 *Pressure angle*

It is straightforward to derive a mathematical relationship between the pressure angle and the cam design parameters. Figure 6.13 shows a typical cam/follower. The radius of the prime circle is denoted by R_0 and the pressure angle is given by φ and is measured between the direction of motion and the direction of the force through the trace point P_1. The eccentricity of the cam is denoted by e, while ω represents the rotational speed of the cam. The displacement of the follower is given by y.

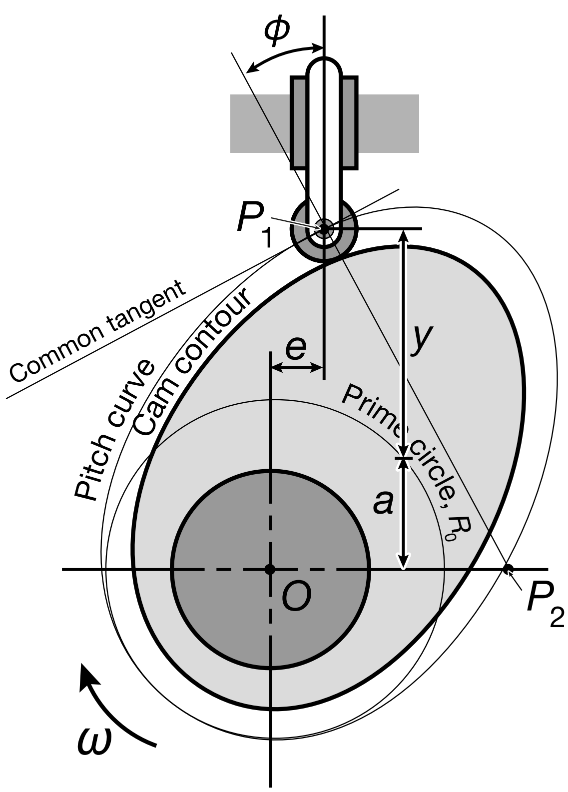

Figure 6.13: Determining the pressure angle, φ. The *common tangent* is the line tangent to the pitch curve at the location of the follower along the cam. The prime circle has a radius R_0.

From Fig. 6.13 it can be observed that

$$a = \sqrt{R_0^2 - e^2} \tag{6.33}$$

Point P_2 is the velocity pole, i.e., the point about which the system is performing an instantaneous rotation. The velocity pole can be found as the intersection of two lines, perpendicular to the velocity vectors of follower and cam (at the trace point). Hence, the velocity of the follower $\dot{y} = \frac{dy}{dt}$ is equal to the product of the distance $|OP_2|$ and the rotational velocity ω. Thus,

$$\dot{y} = \omega\,[e + (a + y)\tan\varphi] \tag{6.34}$$

$$\tan\varphi = \frac{(\dot{y}/\omega) - e}{a + y} \Rightarrow \varphi = \tan^{-1}\left[\frac{(\dot{y}/\omega) - e}{\sqrt{R_0^2 - e^2} + y}\right] \tag{6.35}$$

Equation (6.35) shows that the pressure angle is only a function of the cam geometry. In particular, increasing R_0 reduces the pressure angle. Furthermore, increasing the eccentricity e reduces the pressure angle during the rise but increases the pressure angle during the fall. Since the spring force of the cam follower is pointing down, the pressure angle during the return motion can be made larger then during the rise. *In general, the pressure angle should never exceed 30 degrees, regardless whether it is during a rise or fall.* If one assumes a zero-offset cam, i.e., a cam without eccentricity e, eqn. (6.35) is reduced to

$$\tan\varphi = \frac{(\dot{y}/\omega)}{a + y} \tag{6.36}$$

6.5.2 *Radius of curvature*

Although a cam might be well-designed in terms of kinematics and pressure angle, a smooth operation is still not guaranteed. An important design constraint is the radius of the pitch curve. Analogous to the pressure angle, the radius of curvature of the pitch curve might change from point to point. The radius of curvature of the pitch curve must always be larger than the radius of the follower, i.e.,

$$\rho_P > r_R \tag{6.37}$$

Figure 6.14 shows both a large and a small diameter follower, each of which follow the same pitch curve and hence, "create" a cam contour.

From Fig. 6.14 it can be observed that the large follower creates a pointed cam with an external cusp—an *undercut*—since it "doubles back" over the cam during its motion. This would create an undesirable discontinuity in velocity, acceleration and jerk. The smaller follower, on the other hand, creates a relatively smooth cam without an

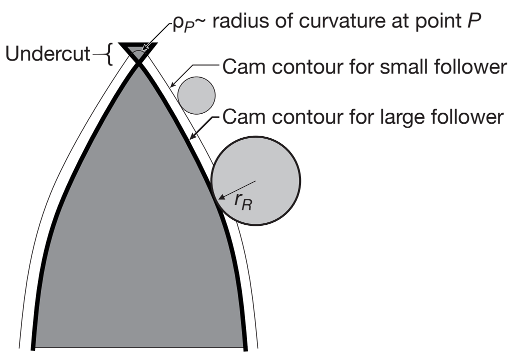

Figure 6.14: Determining the pressure angle, φ. The *common tangent* is the line tangent to the pitch curve at the location of the follower along the cam. The prime circle has a radius R_0.

undercut. It is clear that in the case of the large follower, the condition expressed in eqn. (6.37) has not been satisfied.

So what is the minimum diameter of the follower that avoids this outcome? An analytical expression for the minimum radius of curvature of the pitch curve can be obtained as follows. If the radius of the prime circle is denoted by R_0 and the displacement of the follower by y, then the pitch curve can be expressed as

$$r = R_0 + y. \tag{6.38}$$

From calculus, the general expression of the radius of curvature ρ for a curve given in polar coordinates can be found to be

$$\rho = -\frac{\left[r^2 + (dr/d\theta)^2\right]^{3/2}}{r^2 + 2\,(dr/d\theta)^2 - r\,(d^2r/d\theta^2)} \tag{6.39}$$

Calculating the first derivative with respect to the angle θ of eqn. (6.38), one finds that

$$\frac{dr}{d\theta} = \frac{dy}{d\theta} \tag{6.40}$$

Hence, substituting eqn. (6.38) in eqn. (6.39) produces

$$\rho_P = -\frac{\left[(R_0 + y)^2 + \left(\frac{dr}{d\theta}\right)^2\right]^{3/2}}{(R_0 + y)^2 + 2\left(\frac{dr}{d\theta}\right)^2 - (R_0 + y)\left(\frac{d^2r}{d\theta^2}\right)} \tag{6.41}$$

If the minimum value $\rho_{P,\min}$ of the radius of curvature of the pitch curve is less or equal to the radius of the roller follower r_R, the resulting cam will be pointed. To prevent a pointed cam, the minimum radius of curvature of the pitch curve should be larger than r_R, i.e.,

$$\rho_{P,\min} = \min\left\{\frac{\left[(R_0 + y)^2 + (dr/d\theta)^2\right]^{3/2}}{(R_0 + y)^2 + 2\,(dr/d\theta)^2 - (R_0 + y)\,(d^2r/d\theta^2)}\right\} \geq \rho_{\min} + r_R$$

where $\rho_{\min}$ is the value by which the radius of curvature of the pitch profile should exceed the situation where a pointed cam occurs.