Problem 1 rotation + translation

$$(X,Y)_0 \xrightarrow[H=\begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}]{} (X,Y)_1$$

(a) R 2×2 matrix d 2×1 matrix

(b) Rotation first.

(c) $$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = H \cdot \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix}$$

① $$T_B^{-1} \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix}$$

$$\begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} = T_B^{-1} \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow T_B^{-1} = \begin{bmatrix} 1 & -d \\ 0 & 1 \end{bmatrix}$$

② $$R^{-1} T_B \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix}$$

$$\Rightarrow R^{-1} \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} = E$$

$$R^{-1} = \begin{bmatrix} R^{T} & 0 \\ 0 & 1 \end{bmatrix}$$

Problem 2 (a) $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$$\det(R-\lambda I) = \begin{bmatrix} \cos\theta-\lambda & -\sin\theta \\ \sin\theta & \cos\theta-\lambda \end{bmatrix} = 0$$

$$\cos^2\theta + \lambda^2 - 2\lambda\cos\theta + \sin^2\theta = 0$$

$$\lambda^2 - 2\lambda\cos\theta + 1 = 0$$

$$a=1 \quad b=-2\cos\theta \quad c=1$$

$$\lambda = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \frac{2\cos\theta \pm \sqrt{-4\sin^2\theta}}{2} \quad (\theta=0?)$$

$$= \frac{2\cos\theta \pm 2i|\sin\theta|}{2} = \cos\theta \pm |\sin\theta| i.$$

$$0<\theta<2\pi$$

$$\theta = \pi \Rightarrow \lambda = -1$$

(b) $$\det(R-\lambda I) = \begin{bmatrix} \cos\theta-\lambda & -\sin\theta & 0 \\ \sin\theta & \cos\theta-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)[(\cos\theta-\lambda)^2+\sin^2\theta] = (1-\lambda)(\lambda^2-2\lambda\cos\theta+1) = 0$$

when $\theta = \frac{\pi}{3}$ $\quad (1-\lambda)(\lambda^2-\lambda+1)=0$

$$\lambda_1 = 1 \quad \lambda_{2,3} = \frac{1\pm\sqrt{1-4}}{2} = \frac{1\pm\sqrt{3}i}{2}$$

$\lambda_1 = 1, \rightarrow$ eigenvetor $(0, 0 1)$.

$\therefore$ The rotation axis is z

(c)

```
clc;clear;

% [V,D] = eig(A)

A = [ 0.75 -0.6124 -0.25; 0.25 0.6124 -0.75; 0.6124 0.5 0.6124]
%B=[0.0975 0.9575 0.9706; 0.2785 0.9649 0.9572; 0.5469 0.1576 0.4854]
[v,d] = eig(A)
```

$A \Rightarrow \lambda_1 = 1 \quad \lambda_{2,3} = 0.4874 \pm 0.87321 i$

These outcomes have same forms of λ in Problem 2b

Thus A is rotation matrices

$B \Rightarrow \lambda_1 = 1.7219 \quad \lambda_2 = -0.3024 \quad \lambda_3 = 0.1283$

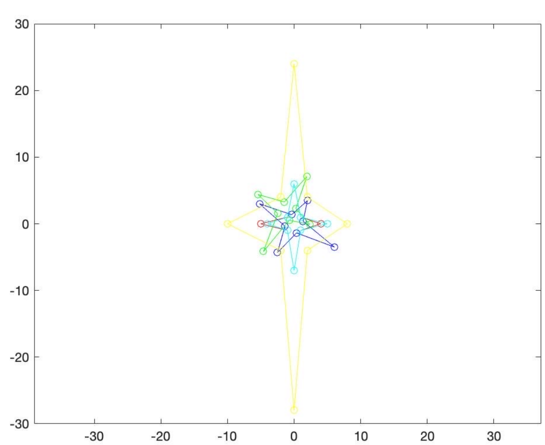
B is not rotation matrices

```
v =

  0.7157 + 0.0000i  -0.2874 + 0.4015i  -0.2874 - 0.4015i
 -0.4938 + 0.0000i   0.1983 + 0.5820i   0.1983 - 0.5820i
  0.4938 + 0.0000i   0.6149 + 0.0000i   0.6149 + 0.0000i
```

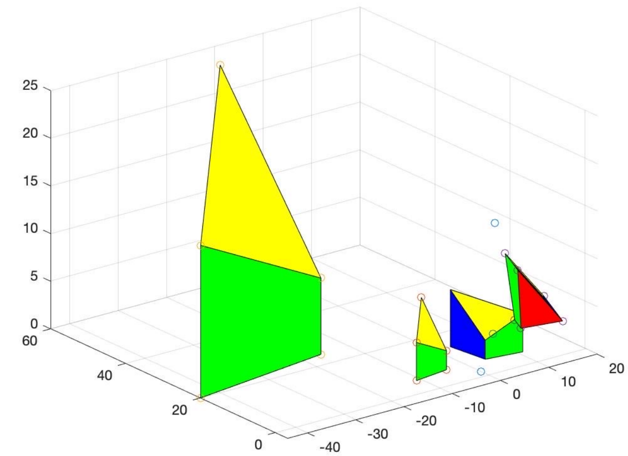
Rotation axis.

```
d =

  1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.4874 + 0.8732i   0.0000 + 0.0000i
  0.0000 + 0.0000i   0.0000 + 0.0000i   0.4874 - 0.8732i
```

Problem 3



Problem 4



Problem 5