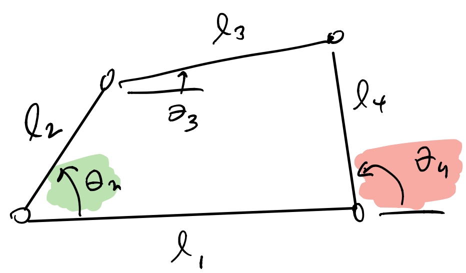
1) Velocity analysis of 4-bar

2) Mechanical advantage

3) Construction of kinematics with transformation matrices

4) Sim Mechanics



<u>Freudenstein's equation</u>

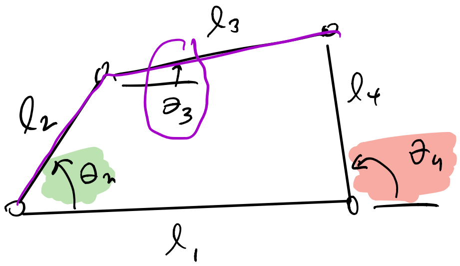
$$\Rightarrow \quad D_3 - \cos(\theta_2 - \theta_4) + D_2 \cos(\theta_4) - D_1 \cos(\theta_2) = 0$$

A) if specify 3 $\theta_2 \to \theta_4$ relationships then equation has 3 unknowns (D_3, D_2, D_1)

Linkage length design problem

B) if link lengths known (D_1, D_2, D_3 known) then can solve $\underline{\underline{\theta_4 = f(\theta_2)}}$

Numerically



x: $l_2 \cos\theta_2 + l_3 \cos\theta_3 = l_1 + l_4 \cos\theta_4$

y: $l_2 \sin\theta_2 + l_3 \sin\theta_3 = 0 + l_4 \sin\theta_4$

$$\rightarrow \frac{d}{dt}\left(D_3 - \cos(\theta_2 - \theta_4) + D_2 \cos(\theta_4) - D_1 \cos(\theta_2) = 0\right)$$

$$\sin(\theta_2 - \theta_4)\cdot(\dot\theta_2 - \dot\theta_4) - D_2 \sin(\theta_4)\dot\theta_4 + D_1 \sin(\theta_2)\dot\theta_2 = 0$$

$$\dot\theta_4 = f(\dot\theta_4)$$

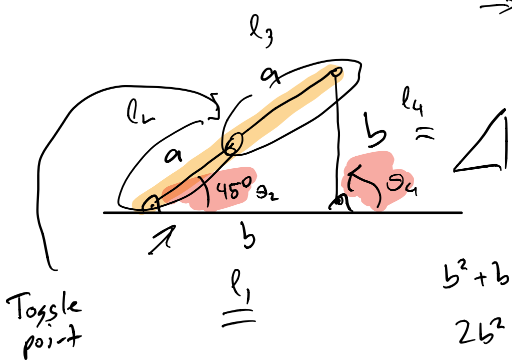
$$\dot\theta_2\underbrace{\left[\sin(\theta_2 - \theta_4) + D_1 \sin(\theta_2)\right]} = \dot\theta_4\underbrace{\left[\sin(\theta_2 - \theta_4) + D_2 \sin(\theta_4)\right]}$$

$$\underbrace{\frac{\dot\theta_2}{\dot\theta_4}}_{\text{Velocity ratio}} = \underbrace{\frac{D_2 \sin(\theta_4) + \sin(\theta_2 - \theta_4)}{D_1 \sin(\theta_2) + \sin(\theta_2 - \theta_4)}}$$

$$\tau_2 \cdot \dot\theta_2 = \tau_4 \cdot \dot\theta_4$$

Power in = Power out

$$\frac{\dot\theta_2}{\dot\theta_4} = \frac{\tau_4}{\tau_2} = \text{Mechanical advantage}$$



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$$\to \frac{\tau_4}{\tau_2} = \frac{\dot{\theta}_2}{\dot{\theta}_4} = \left[\frac{D_2 \sin(\theta_4) + \sin(\theta_2 - \theta_4)}{D_1 \sin(\theta_2) + \sin(\theta_2 - \theta_4)} \right]$$

$$b^2 + b^2 = (2a)^2$$

$$2b^2 = 4a^2$$

$$b = \sqrt{2}\, a$$

$$D_1 = \frac{l_1}{l_4} \qquad D_2 = \frac{l_1}{l_2}$$

$$D_3 = \frac{l_1^2 + l_2^2 - l_3^2}{l_2 \cdot l_4}$$

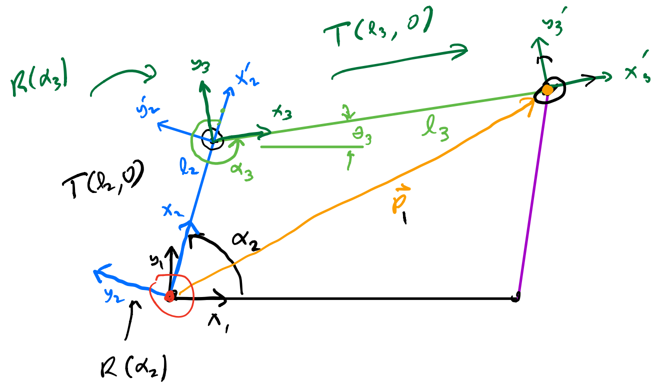
$$\frac{\tau_4}{\tau_2} = \frac{\frac{b}{a} \sin(\pi/2) + \sin(-\pi/4)}{\underbrace{1 \cdot \sin(\pi/4) + \sin(-\pi/4)}_{0}}$$

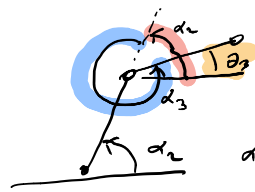
$$D_1 = 1$$

$$D_2 = b/a$$

$$\theta_2 = \pi/4 \qquad \theta_4 = \pi/2$$

Construction of kinematics through reference frame transformations





$\alpha_2 + \alpha_3 = \theta_3 + 2\pi$

$\cos(\alpha_2 + \alpha_3) = \cos(\theta_3)$

$\sin(\alpha_2 + \alpha_3) = \sin(\theta_3)$

$$R(\alpha_2) = \begin{bmatrix} \cos\alpha_2 & -\sin\alpha_2 & 0 \\ \sin\alpha_2 & \cos\alpha_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

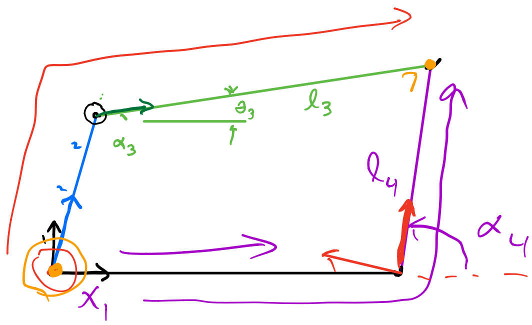
$$T(l, 0) = \begin{bmatrix} 1 & 0 & l \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= R(\alpha_2)\, T(l_1, 0)\, R(\alpha_3)\, T(l_3, 0)$$

$$= \begin{bmatrix} \cos(\alpha_2+\alpha_3) & -\sin(\alpha_2+\alpha_3) & l_3\cos(\alpha_2+\alpha_3) + l_2\cos(\alpha_2) \\ \sin(\alpha_2+\alpha_3) & \cos(\alpha_2+\alpha_3) & l_3\sin(\alpha_2+\alpha_3) + l_2\sin(\alpha_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{p}_1 = \begin{bmatrix} \cos(\alpha_2+\alpha_3) & -\sin(\alpha_2+\alpha_3) & l_2\cos(\alpha_2+\alpha_3) + l_1\cos(\alpha_2) \\ \sin(\alpha_2+\alpha_3) & \cos(\alpha_2+\alpha_3 & l_2\sin(\alpha_2+\alpha_3) + l_1\sin(\alpha_2) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} l_2\cos(\alpha_2) + l_3\cos(\alpha_2+\alpha_3) \\ l_2\sin(\alpha_2) + l_3\sin(\alpha_2+\alpha_3) \\ 1 \end{bmatrix}$$

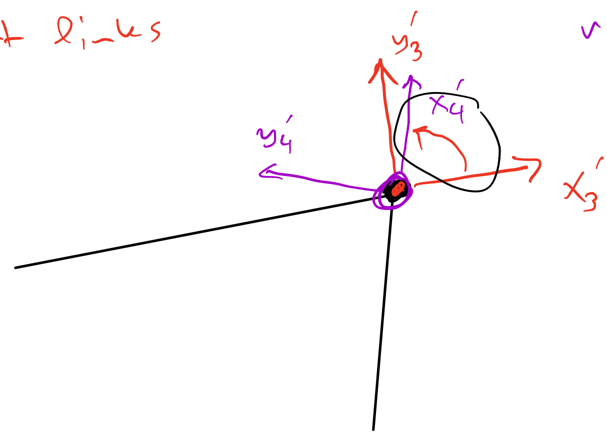


Not true because orientation is different

$$\cancel{R(\alpha_2)\,T(l_2,0)\,R(\alpha_3)\,T(l_3,0) = T(l_1,0)\,R(\alpha_4)\,T(l_4,0)}$$

Transformation from left links

Transformation right links



$$\underline{R(\alpha_2)}\,\underline{T(l_2,0)}\,\underline{R(\alpha_3)}\,\underline{T(l_3,0)}\begin{bmatrix}0\\0\\1\end{bmatrix} = T(l_1,0)\,R(\alpha_4)\,T(l_4,0)\begin{bmatrix}0\\0\\1\end{bmatrix}$$

- But the origins are the same
- produces two constraint equations

Simscape Multibody