MAE 292 Spring 2020 Midterm
(Problem clarifications in red)

Assigned April 29, 11am
Due Tuesday May 5, 9:29am

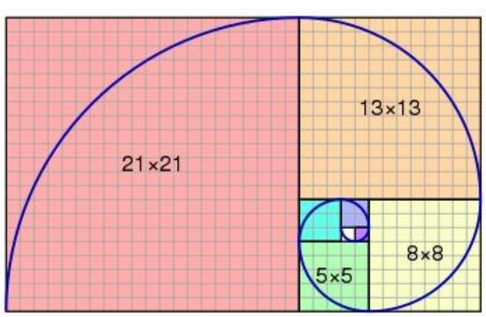
Midterm directions:

This midterm is to be completed individually with no discussion among classmates. You may use your notes, class material, and any other resources. You are to turn in a PDF copy of all written solutions and Matlab outputs, as well as a ZIP file containing all Matlab code you use for your solution. You are to show all steps for derivations.

Problem 1: CAD transformations to generate fractals (20 points)

Many structures in nature are self-similar, formed from repeating patterns of scaling, translation, and rotation operations. In this problem you will use Matlab to construct two fractals. Turn in all plots in your final pdf and all Matlab code.

1. The golden spiral shown below can be constructed from a series of iterative CAD transformations of rotation, scaling, and translation using the Fibonacci sequence to generate the scaling amount (sequence definition below). To generate this shape in Matlab, start with one 1×1 square (side length equal to the first Fibonacci number) and add an arc (quarter of circle). Represent the arc as a set of points. To expand the spiral we scale the box and arc by the next Fibonacci number, and rotate and translate the shape so that the arcs meet. To continue building the spiral we keep adding square and arc shapes with side lengths given by the fibonacci series. **Generate a program to draw the golden spiral up to the 16th Fibonacci number using CAD transformations.** For reference the spiral below is drawn to the 8th Fibonacci number (21). The bounding boxes should be blue and the arcs red in your final plot.



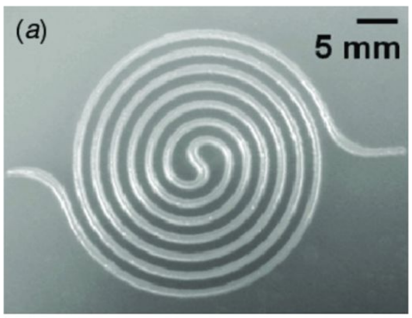
$$F(1) = 1, F(2) = 1$$
$$F(n) = F(n-2) + F(n-1) \text{ for } n > 2$$

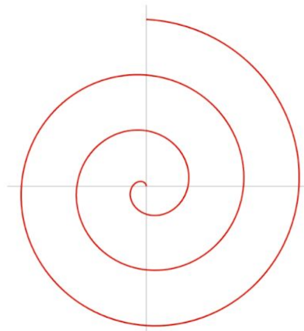
which gives the numbers:

$1, 1, 2, 3, 5, 8, 13, 21 \ldots$

Problem 2: Soft robot sensor design (20 points)

In the following problem we will use the properties of parametric curves to design an optimal sensor for a soft robot. Soft sensors can be made by embedding liquid conducting material in linear patterns or spirals like the shape below. It is important that the conducting liquid in the channel (the silver region) be separated by a minimum offset distance from the next channel. An Archimedes spiral (formula and representative curve below right) has the property that a line pointing out from the origin will pass through the spiral at constant length intervals.





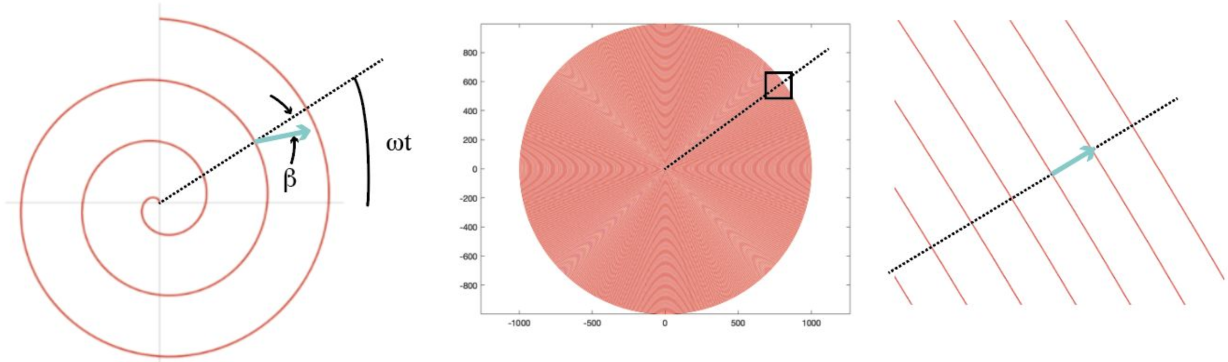
$$r = b(1 + \omega t)$$
$$\theta = \omega t$$

Answer the following questions to design and select the parameters for our sensor:

1. Represent the curve of the spiral in parametric form (x(t), y(t)).

2. The sensor is built by extruding the conductive liquid from a moving syringe that is following the (x(t), y(t)) trajectory at a speed of (x'(t), y'(t)). Solve for x'(t), y'(t) and the magnitude of the speed, *s*.

3. Using the above parametric form of the equations prove by hand that for the right choice of offset distance, *d*, the offset of the curve at location (x(t), y(t)) is exactly equal to (x(t + 2 π /ω), y(t + 2 π /ω)) as t approaches infinity. (Hint: compare the angle of the offset vector to the angle of a line pointing outward from the origin which is at angle ωt, see diagram below). Describe in words why this is a "good" spiral for this sensor.



4. If we want to build a larger sensor that has an outer diameter of 2 cm and channel width of 0.5 mm, and channel spacing of 0.5 mm, approximately how many “rings” of the sensor can we fit into it? You can use Matlab, Fusion 360, or pen/paper to answer this question. (For reference the sensor above has approximately 8 “rings” counting from the outside to the center).

5. Design and sketch a sensor as shown in figure (a) above using Fusion 360. Choose an equal channel width and channel separation distance to generate the sensor above. Include seven concentric rings and the appropriate input/output channels as shown on the left and right side of the figure. Include a link to your Fusion 360 sketch and in the pdf you turn in clearly describe the steps you took to design this sensor and what programs you used. If you use Matlab to generate the spirals, include the code in the zip file.

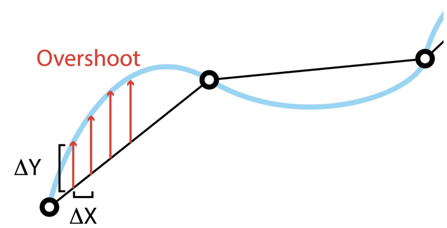
Problem 3: Curve fitting (20 points)

Use the following data for this problem in Matlab.
x = [0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19]
y = [0.13 3.02 6.29 6.52 6.52 7.21 8.20 11.39 12.84 14.66 15.50 15.43 15.05 13.52 10.71 8.96 8.50 8.27 8.20 8.42]

1. Describe the difference between a polynomial curve fit versus a spline fit.

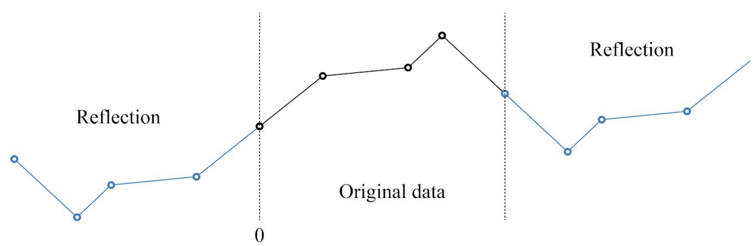
2. Construct a lagrange polynomial fit to this data and generate a plot with the data points and the lagrange polynomial.

3. One method to estimate the "overshoot" of a polynomial fit is to compare the difference between the fit function and a line interpolated between each point. Compute the sum of squares difference between the polynomial and a line interpolation using an interval spacing of $\Delta X = 0.01$.



4. Generate a spline fit in Matlab to the data using any method you choose. Provide a plot and compute the overshoot using the same function as above.

5. The overshoot of polynomial fit typically occurs at the boundaries. We can sometimes improve a fit at the boundaries by reflecting the data both horizontally and vertically about each endpoint thus constructing a new data set as shown below. Perform this boundary reflection in Matlab algorithmically (don't just type the points by hand) and plot the new data set, color the reflected data blue and the original data black.



6. Perform your lagrange polynomial and spline fits on the new data set. Compute the overshoot quantity again but only evaluated over the original x-range. Did this reflection improve the quality of either fit? Describe why or why not.

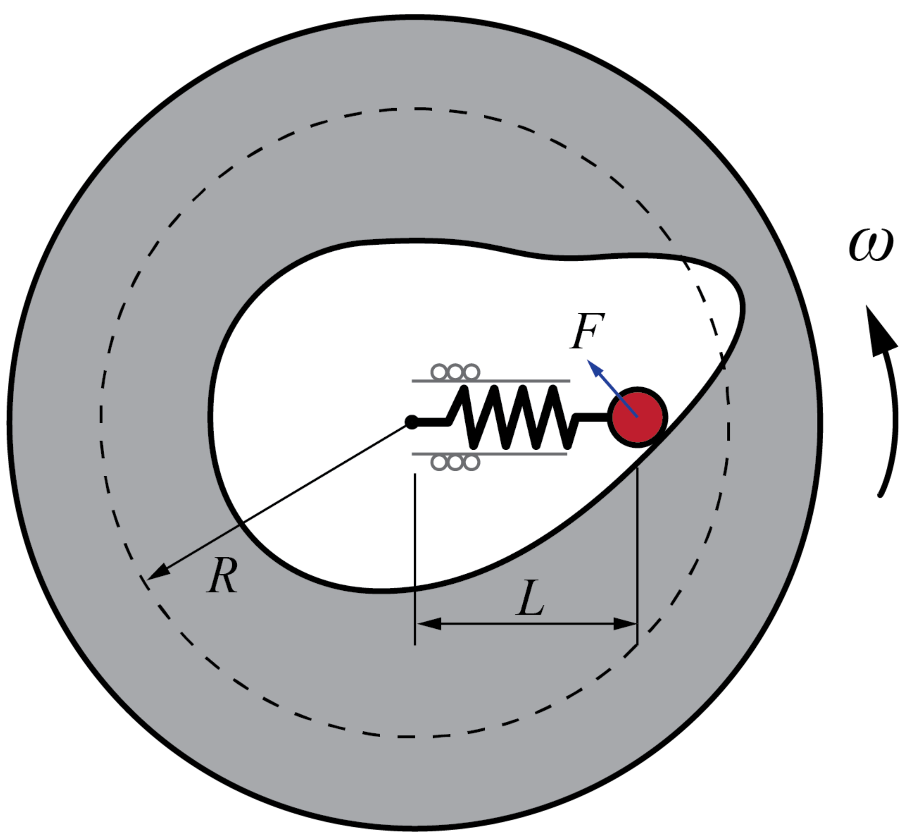
Problem 4: CAM (20 points)

Consider an internal cam as follows. The cam is rotating CCW with constant speed ω. The displacement from the cam center to the center of the follower is denoted as L. The supporting spring has a stiffness of k. The spring reaches its neutral position when the follower extends freely to $L = 2R$. The contact force between the follower, of mass m, and the cam surface is F. The motion of the follower can be described as

$$L(\theta) = R - A\,[1 - \cos(b\theta)]$$

Where θ is the rotation angle, $R > 2A$, $b \in Z^{+}$.

1. Find out the maximum acceleration and jerk of the follower. Identify where these maximums happen.
2. Suppose the follower has a radius of r, derive the equations for the cam surface for $b=1$. What's the maximum follower size that can be used in this internal cam?
3. Find out the maximum follower size for $b \geq 2$.
4. Calculate the pressure angle of the cam for $b = 3$. Find out the maximum pressure angle and calculate the off axis force on the follower bearing.



Problem 5: Free-form CAD (20 points)

1. For the following three objects describe using words what steps you would perform to design 3D models of them in Fusion 360, turn these descriptions in on your pdf solutions.

2. In Fusion 360 construct 3D models of each of these objects. Use dimensions of your choice, they are not expected to be exact. Turn in a link to your objects in your solution pdf. Note, we will not penalize at all for dimensions, but all objects need to be topologically correct and should accurately represent the final objects (if your coffee cup has a hole in the bottom of it you will lose points). Color and material choice is not important, and objects should be modeled as a single solid body.

(a) A cup

(b) A phone case

(c) A ping-pong paddle