MAE 292 Spring 2020 Homework 1

Due on Thursday April 9, 2020, at 11:59 PM

Problem 1: Conceptual Questions. (10 points)

In a 2D plane, the homogeneous transformation matrix composed of a rotation and translation transforms coordinates $(X,Y)_0$ to new coordinates $(X,Y)_1$

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

(a) What are the dimensions of R and d?
(b) In this operation what happens first, the rotation or the translation?
(c) Find the homogeneous transformation matrix that takes the transformed point $(X,Y)_1$ back to $(X,Y)_0$.

Problem 2: Rotation Matrix (20 points)

Rotation matrices belong to special orthogonal group [1].

(a) Consider a 2D rotation matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Calculate the eigenvalues and corresponding eigenvectors of the matrix R for $0 < \theta < 2\pi$. Express your results in terms of θ. What's the relationship of the eigenvalues and the rotation angle?

(b) Consider a 3D rotation matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Calculate the eigenvalues and corresponding eigenvectors of the matrix R for $\theta = \frac{\pi}{3}$. What's the relationship of the eigenvalues and the rotation angle? Determine the rotation axis.
(Hint: you may use MATLAB function *eig*.)

(c) Consider the following two matrices

$$A = \begin{bmatrix} 0.7500 & -0.6124 & -0.2500 \\ 0.2500 & 0.6124 & -0.7500 \\ 0.6124 & 0.5000 & 0.6124 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0975 & 0.9575 & 0.9706 \\ 0.2785 & 0.9649 & 0.9572 \\ 0.5469 & 0.1576 & 0.4854 \end{bmatrix}$$

Determine which is/are rotation matrices. Calculate the eigenvalues and corresponding eigenvectors of the rotation matrices you find. Determine the rotation axis and rotation angle.
(Hint: you may use MATLAB function *eig*.)

Problem 3: CAD Operations in 2D (25 points)

Consider a star-shaped object defined by a set of points whose (x, y) coordinate pairs are $(4, 0), (1, 1), (0, 6), (-1, 1), (-5, 0), (-1, -1), (0, -7), (1, -1)$.

(a) Draw the star-shaped object in a MATLAB figure by plotting line segments which connect the object points in appropriate fashion. Set the axis limits of your plot to be [-30, 30], and set your axis to equal.
(b) Rotate the star shaped object counter-clockwise by 60º about the origin and plot it on the same figure.
(c) Rotate the star object clockwise by 30º about the point (3,3) and plot it on the same figure.
(d) Scale the object by a factor of 2 in the x-direction and a factor of 4 in the y direction and plot it on the same figure.
(e) Reflect the star object about the x axis and plot it on the same figure.
(f) Show the homogeneous transformation matrices for parts (b), (c), (d) and (e). Plot all the figures for all the parts from (a) - (e) in the same plot. Provide legends and use different colors/markers/lines to differentiate your plots. All the operations are performed on the original star shape.

Problem 4: CAD Operations in 3D (25 points)

(a) For the body defined by the points $(4,6,0), (11,5,0), (3,14,0), (4,6,2), (11,5,4), (3,14,6)$, plot the original position of the body. Then, determine and plot the position of the body after reflection on the y-z plane and after subsequent uniform scaling by a factor of 4. Give the CAD operators and plot the original, the reflected and the scaled body using MATLAB.
(b) Rotate the body defined above by $120°$ in the clockwise direction. The axis of rotation goes through the origin $(0, 0, 0)$, and the following angles are given: $\varphi_x = 60°$, $\varphi_y = 60°$. Plot the original body and the rotated body using MATLAB.

(Hint: For problem 4a and 4b, make sure that all surfaces are plotted and that only those lines are shown that belong to the body. Try to color code each surface individually.)

Problem 5: Computational Design of a Particle-Size Sorting Sieve (20 points)

We want to design a particle size sorting sieve on an acrylic board with a series of holes of diameters $[1, 6, 11, 16, 21, 26]$ cm, as shown in the figure below. The size of the acrylic board is 50×100 cm. The holes in each column are the same size and the diameters increase from left to right columns. We want to fill in as many holes as possible while also meet some design requirements listed below.

x edge	y edge	x spacing	y spacing
$\geq$ 1cm	$\geq$ 1cm	= 2 cm	= 1 cm

Use MATALB to draw the design pattern of the particle size sorting sieve. It is important that your design be parameterized through the use of one or more loops. We do not want you to just hand plot each circle in a script. Your final figure should look similar to the one present below.

Please submit a picture of the plotted geometry and your code.