1) $(x(t), y(t))$?

$$r(t) = b(1+\omega t)$$
$$\theta(t) = \omega t$$

parametric polar form

$$x(t) = r(t) \cdot \cos(\theta(t))$$
$$y(t) = r(t) \cdot \sin(\theta(t))$$

$$\boxed{\begin{aligned} x(t) &= b(1+\omega t) \cdot \cos(\omega t) \\ y(t) &= b \cdot (1+\omega t) \cdot \sin(\omega t) \end{aligned}}$$

2) Solve for $x'(t)$, $y'(t)$ and $s = \sqrt{x'^2 + y'^2}$

$$x'(t) = \frac{\partial x(t)}{\partial t} = \underline{b\omega \cdot \cos(\omega t) - b(1+\omega t) \cdot \sin(\omega t)}$$

$$y'(t) = \frac{\partial y(t)}{\partial t} = \underline{b\omega \sin(\omega t) + b(1+\omega t)\cos(\omega t)}$$

$$x'(t)^2 = \left(b\omega \cdot \cos(\omega t) - b(1+\omega t) \cdot \sin(\omega t)\right)^2$$
$$= (b\omega)^2 \cos^2(\omega t) + b^2(1+\omega t)^2 \sin^2(\omega t)$$
$$- 2b^2\omega(1+\omega t)\cos(\omega t)\sin(\omega t)$$

$$y'(t)^2 = b\omega \sin(\omega t) + b(1+\omega t)\cos(\omega t)$$

$$= (b\omega)^2 \sin^2(\omega t) + b^2(1+\omega t)^2 \cos^2(\omega t)$$

$$+ 2b^2\omega(1+\omega t)\sin(\omega t)\cos(\omega t)$$

$$\left(x'(t)^2 + y'(t)^2\right)^{1/2} = (b\omega)^2 \underline{\cos^2(\omega t)} + b^2(1+\omega t)^2 \underline{\sin^2(\omega t)}$$

$$\cancel{-\,2b^2\omega(1+\omega t)\cos(\omega t)\sin(\omega t)}$$

$$+ (b\omega)^2 \underline{\sin^2(\omega t)} + b^2(1+\omega t)^2 \underline{\cos^2(\omega t)}$$

$$\cancel{+\,2b^2\omega(1+\omega t)\sin(\omega t)\cos(\omega t)}$$

$$\boxed{s = \sqrt{(b\omega)^2 + b^2(1+\omega t)^2}}$$

3. Derive offset equation and show as $t \to \infty$ for the right offset distance the offset points to the point $t + \frac{2\pi}{\omega}$

Archimedes spiral has property that a line drawn from the origin will cross the spiral at equal radial intervals. We can prove this from the polar form.

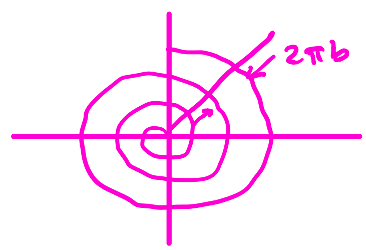
$$\left[\begin{array}{l} \theta_1 = \omega t \\ r_1 = b(1+\omega t) \end{array}\right.$$

at t

at $t + \frac{2\pi}{\omega}$

$$\theta_2 = \omega\left(t + \frac{2\pi}{\omega}\right)$$

$$= \omega t + 2\pi = \underline{\underline{\theta_2}}$$



$$r_2 = b\left(1 + \omega\left(t + \frac{2\pi}{\omega}\right)\right)$$

$$= b(1 + \omega t + 2\pi)$$

$$= r_1 + 2\pi b$$

So the offset distance of $d = 2\pi b$ at $(x(t), y(t))$ should approach $(x(t+2\pi), y(t+2\pi))$ as $t \to \infty$

Offset equation:

$$x_{off}(t) = x(t) - d\,\frac{y'(t)}{s}$$

$$y_{off}(t) = y(t) + d\,\frac{x'(t)}{s}$$

$$x_{off}(t) = b(1+\omega t)\cos(\omega t) - d\,\frac{b\omega\sin(\omega t) + b(1+\omega t)\cos(\omega t)}{\sqrt{(b\omega)^2 + b^2(1+\omega t)^2}}$$

$$y_{off}(t) = b(1+\omega t)\sin(\omega t) + d\,\frac{b\omega\cos(\omega t) - b(1+\omega t)\sin(\omega t)}{\sqrt{(b\omega)^2 + b^2(1+\omega t)^2}}$$

a) Show $x_{off}(t) - x(t+\frac{2\pi}{\omega}) \to 0$ as $t \to \infty$

$$x_{off}(t) - x(t+\tfrac{2\pi}{\omega}) = b(1+\omega t)\cos(\omega t) - d\,\frac{b\omega\sin(\omega t) + b(1+\omega t)\cos(\omega t)}{\sqrt{(b\omega)^2 + b^2(1+\omega t)^2}}$$

$$- b(1+\omega t + 2\pi)\cos(\omega t + 2\pi)$$

$$= 2\pi b\cos(\omega t) - d\left[\frac{b\omega\sin(\omega t) + b(1+\omega t)\cos(\omega t)}{\sqrt{(b\omega)^2 + b^2(1+\omega t)^2}}\right]$$

so if $d = 2\pi b$ and ↑ equals $\cos(\omega t)$ in limit $t \to \infty$ then $x_{off}(t) - x(t+\frac{2\pi}{\omega}) = 0$

$$\frac{b\omega\sin(\omega t) + b(1+\omega t)\cos(\omega t)}{\sqrt{(b\omega)^2 + b^2(1+\omega t)^2}} = \frac{b\omega\sin(\omega t)}{\sqrt{(b\omega)^2 + b^2(1+\omega t)^2}} + \frac{b(1+\omega t)\cos(\omega t)}{\sqrt{(b\omega)^2 + b^2(1+\omega t)^2}}$$

$$\text{as } t \to \infty \approx \frac{1}{t} + \frac{t}{t}\cos(\omega t)$$

$$\approx \cos(\omega t)$$

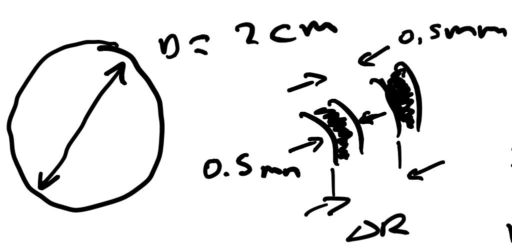
So as $t \to \infty$ if $\partial = 2\pi b$ then

$$x_{off}(t) - x(t + \tfrac{2\pi}{\omega}) \approx 2\pi b\cos(\omega t) - 2\pi b\cos(\omega t) = \underline{\underline{0}}$$

A similar justification can be shown for

$$y_{off}(t) - y(t + \tfrac{2\pi}{\omega}) = \underline{\underline{0}}$$

4)



so dist from center to center in channels is 1 mm, so should be able to fit ≈ 10 rings because

$R = 10\ m$, $\Delta r = 1\ mm$

$10 \cdot \Delta r \approx R$

5) Many methods