

## Problem 2

(a)  $M = 3(1-1) - 2j \quad l=5 \quad j=5$

$M = 3 \times 4 - 10 = 2 \quad \therefore \text{DOF} = 2$

(b)  $\begin{cases} x: l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_0 - l_3 \cos(\theta_3) - l_4 \cos(\theta_3 + \theta_4) = 0 \\ y: l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) - l_3 \sin(\theta_3) - l_4 \sin(\theta_3 + \theta_4) = 0 \end{cases}$

(c) Unlike 4-bar (DOF=1), the 5-bar has 2 degrees of freedom, so we should certain two variables, for example the  $\theta_1$  and  $\theta_3$  as independent variables

① Define the range of  $\theta_1$  and  $\theta_3$

② Two for loop eg. to create 2 dimensional ranges, by the constrain with fsolve,  $x_e, y_e$  can be certain. And the

Outlink of matlab

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for  $\theta_1$ 
  for  $\theta_3$ 
     $F = \text{constraint equation with } \theta_1 \text{ and } \theta_3$ 
    and fsolve(F, 0)
  end
end

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whole movement can be simulated.

(d) ① certain the relationship, such as  $x_e, y_e$ , and also  $v_{x_e}, v_{y_e}$ , ( $\dot{x}_e^2 + \dot{y}_e^2 = 0$ ) as the constraint  $F$  we need, with all variables and parameters as Problem 2c. However, the length through  $l_0$  to  $l_4$  change, which indeed increase the DOF of the whole system.

We can use more layers "for loop" to "fsolve" the extremum, but we also can use "fminsearch" and "fmincon" to optimize.

② "fmincon" is a gradient based function to find minimum of constrained nonlinear multivariable.

$$\min f(x) \text{ such that } \begin{cases} c(x) \leq 0 \\ c_{eq}(x) = 0 \\ A x \leq b \\ A_{eq} \cdot x = b_{eq} \end{cases}$$

"fminsearch" find minimum of unconstrained multivariable function using derivative-free method.

$$\min_x f(x)$$

③ Fminsearch do not need to know the gradient b/c it can't deal with explicit boundaries, and for large numbers of parameters, it gets inefficient.

In this question, Fmincon is very suitable for bounded optimization, and thus are better given the leg requirements

(a).

$$AB: (x - \frac{a}{2})^2 + (y + \frac{\sqrt{3}}{3} \frac{a}{2})^2 = a^2$$

$$x^2 + y^2 - ax + \frac{\sqrt{3}}{3} ay - \frac{2}{3} a^2 = 0$$

$$\Rightarrow \rho^2 - a \cos \theta \rho + \frac{\sqrt{3}}{3} a \sin \theta \rho - \frac{2}{3} a^2 = 0 \quad \theta \in (\frac{\pi}{6}, \frac{7}{6} \pi)$$

$$AC: (x + \frac{a}{2})^2 + (y + \frac{\sqrt{3}}{3} \frac{a}{2})^2 = a^2 \quad \theta \in (-\frac{\pi}{6}, \frac{\pi}{2})$$

$$x^2 + y^2 + ax + \frac{\sqrt{3}}{3} ay - \frac{2}{3} a^2 = 0$$

$$\Rightarrow \rho^2 + a \cos \theta \rho + \frac{\sqrt{3}}{3} a \sin \theta \rho - \frac{2}{3} a^2 = 0$$

$$BC: (x - \frac{2\sqrt{3}}{3} \cdot \frac{a}{2})^2 + (y - \frac{2}{3} \cdot \frac{a}{2})^2 = a^2 \quad \theta \in (-\frac{5\pi}{6}, -\frac{\pi}{6})$$

$$x^2 + y^2 - \frac{2\sqrt{3}}{3} ax - \frac{2}{3} ay - a^2 = 0$$

$$\Rightarrow \rho^2 - \frac{2\sqrt{3}}{3} a \rho \cos \theta - \frac{2}{3} a \rho \sin \theta - a^2 = 0$$

(b).

