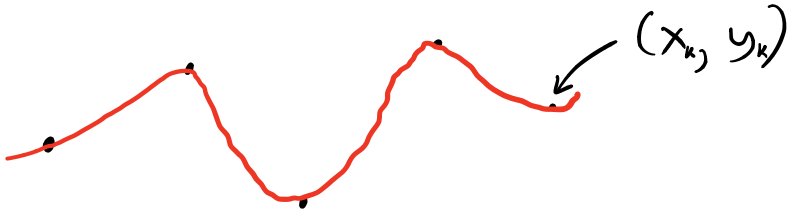
Lagrange polynomials :

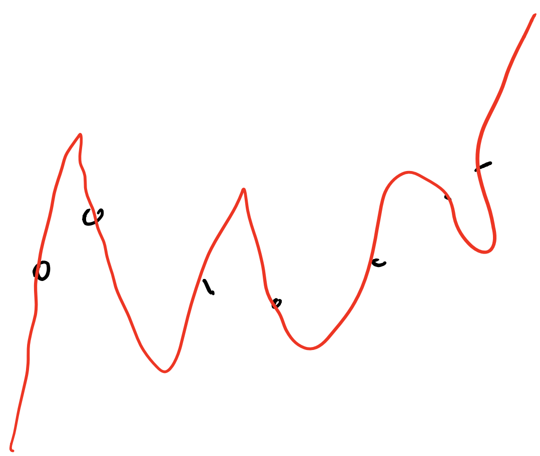


$$y(x) = \sum_{k=0}^{N} \ell_k(x)\, y_k$$

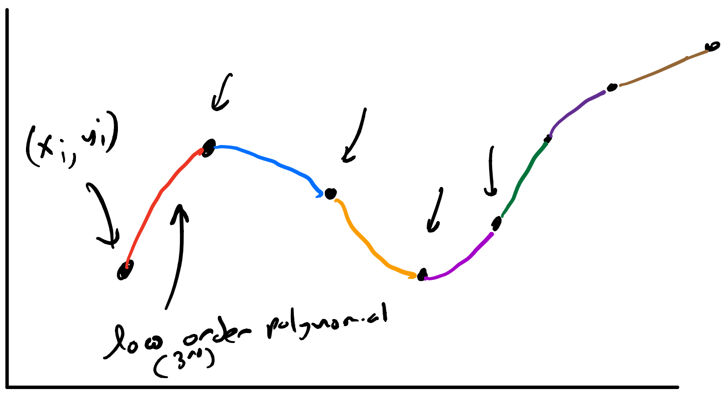
$$\ell(k) = \begin{cases} 1 & \text{if } x = x_k \\ 0 & \text{if } x \neq x_k \end{cases}$$

$$\ell_k(x) = \prod_{\substack{0 \le m \le k \\ m \neq k}} \frac{(x - x_m)}{(x_k - x_m)}$$

$$= \frac{(x-x_0)(x-x_1)\ldots(x-x_{k-1})\cdot(x-x_{k+1})\cdots(x-x_N)}{(x_k-x_0)(x_k-x_1)\ldots(x_k-x_{k-1})\cdot\underbrace{\,}_{\substack{k^{th}\text{ term}\\ \text{is not in}\\ \text{product}}}(x_k-x_{k+1})\cdots(x_k-x_N)}$$



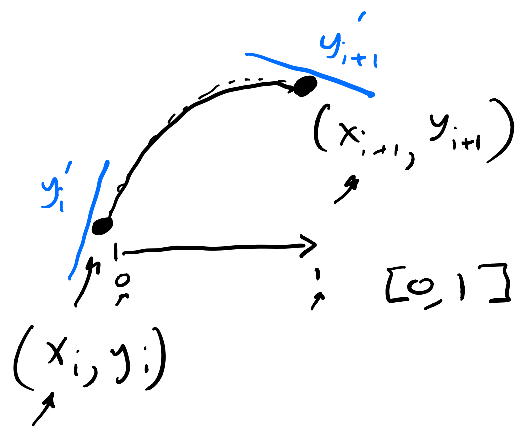
Polynomial fits have tendency towards large oscillations and in general provide a poor fit above 4^{th} order ...



Spline fitting

Why 3^{rd} order?

$$y = ax^3 + bx^2 + cx + d$$



4 b.c.'s to solve 3^{rd} order polynomial coeffs

$$\underline{y_1,\ y_2,\ y'_1,\ y'_2}$$

$$y_1 = f(0)$$
$$y_2 = f(1)$$
$$y'_1 = f'(0)$$
$$y'_2 = f'(1)$$

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f'(x) = 3ax^2 + 2bx + c$$

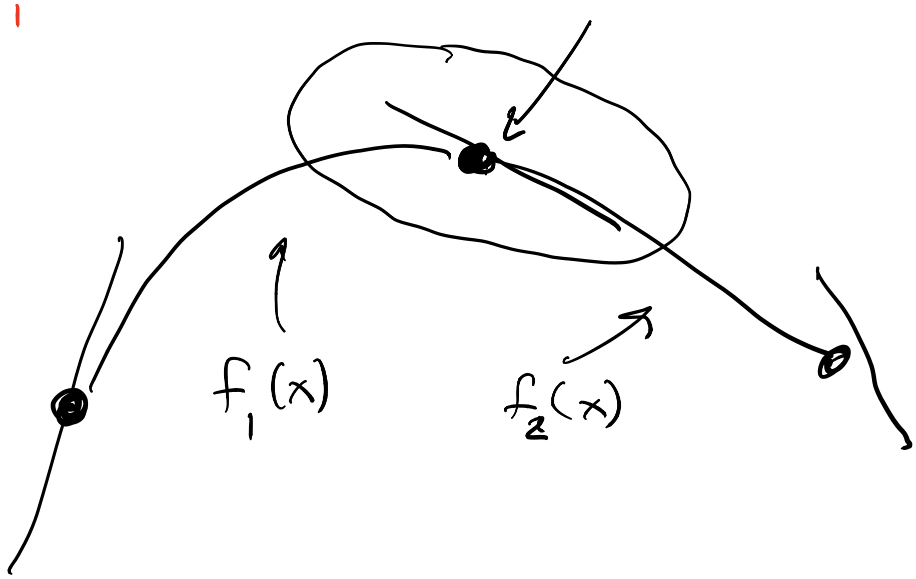
$$x \in [0, 1]$$

$$\begin{bmatrix} x^3 & x^2 & x & 1 \\ 3x^2 & 2x & 1 & 0 \\ x^3 & x^2 & x & 1 \\ 3x^2 & 2x & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} y_1 \\ y_1' \\ y_2 \\ y_2' \end{bmatrix}$$

$$\boxed{\begin{aligned} f(x) = {} & y_1(2x^3 - 3x^2 + 1) + y_2(-2x^3 + 3x^2) + \\ & y_1'(x^3 - 2x^2 + x) + y_2'(x^3 - x^2) \end{aligned}}$$

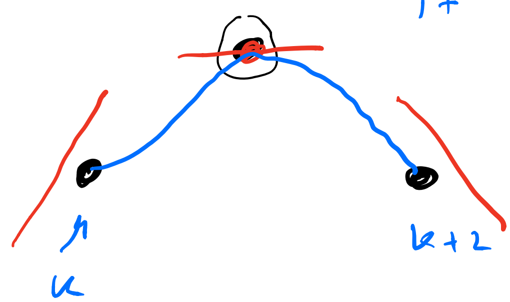
$$f(0) = y_1$$

$$f(1) = y$$



Spline : piecewise polynomials across data

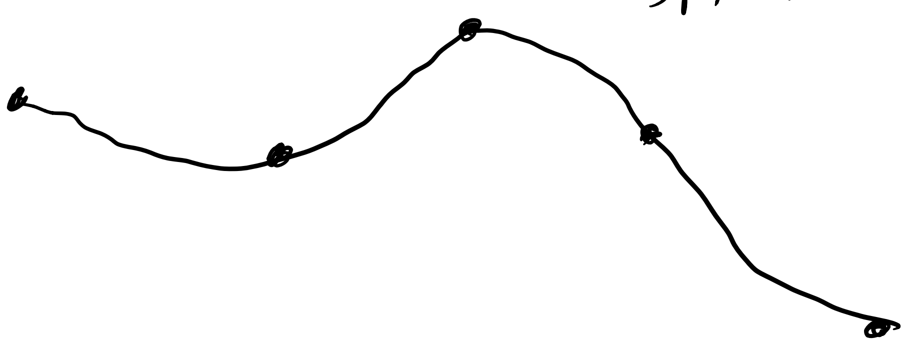
pchip : Routine tries to minimize overshoots



if $y'_k \cdot y'_{k+2} < 0$

$y'_{k+1} = 0$

Natural spline : Minimize 2^{nd} derivative of curve across spline

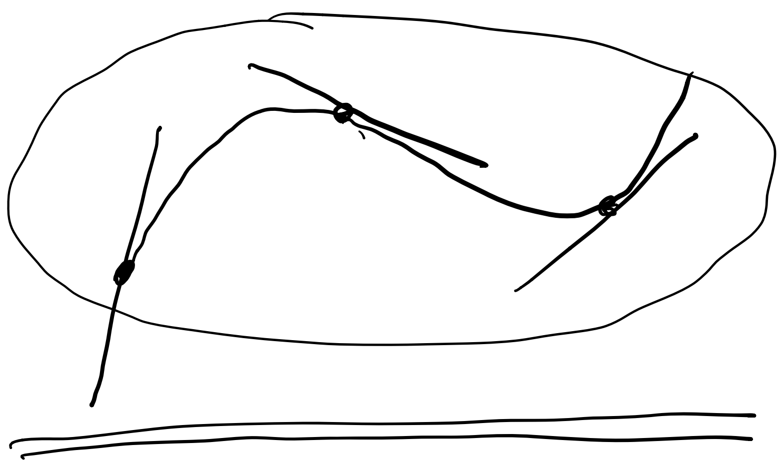


Curves from data:

- LLS fits
- Lagrange exact fit
- Splim piecewise fit

Designed curves:

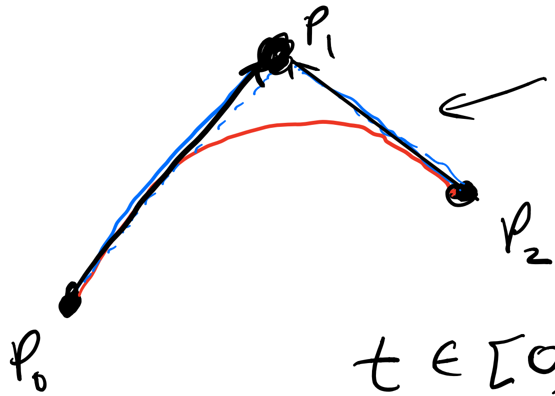
- Spline



- Bezier curves

Parametric representation of curves using n anchor points

Quadratic Bezier curve



$t \in [0, 1]$

$$\vec{B}(t) = (1-t^2)\vec{P}_0 + 2(1-t)\,t\vec{P}_1 + t^2\vec{P}_2$$

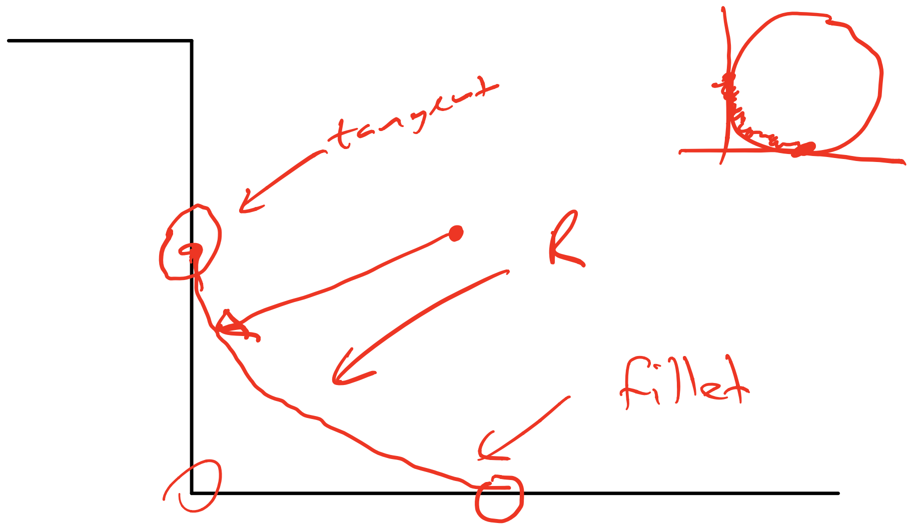
$t=0 \quad B(0) = P_0$

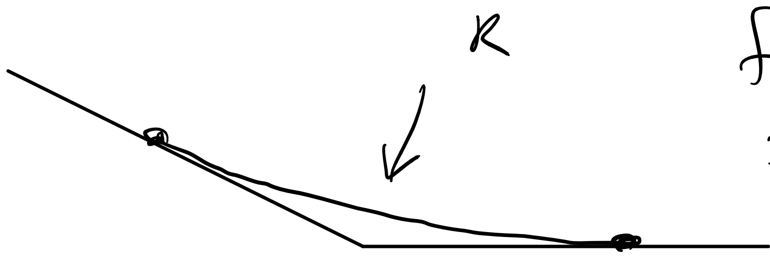
$B(1) = P_2$

$$B'(t) = (2t-2)(\vec{P}_1 - \vec{P}_0) + 2t(\vec{P}_2 - \vec{P}_1)$$

$$B'(1) = 2(\vec{P}_2 - \vec{P}_1)$$

$$B'(0) = 2(\vec{P}_1 - \vec{P}_0)$$

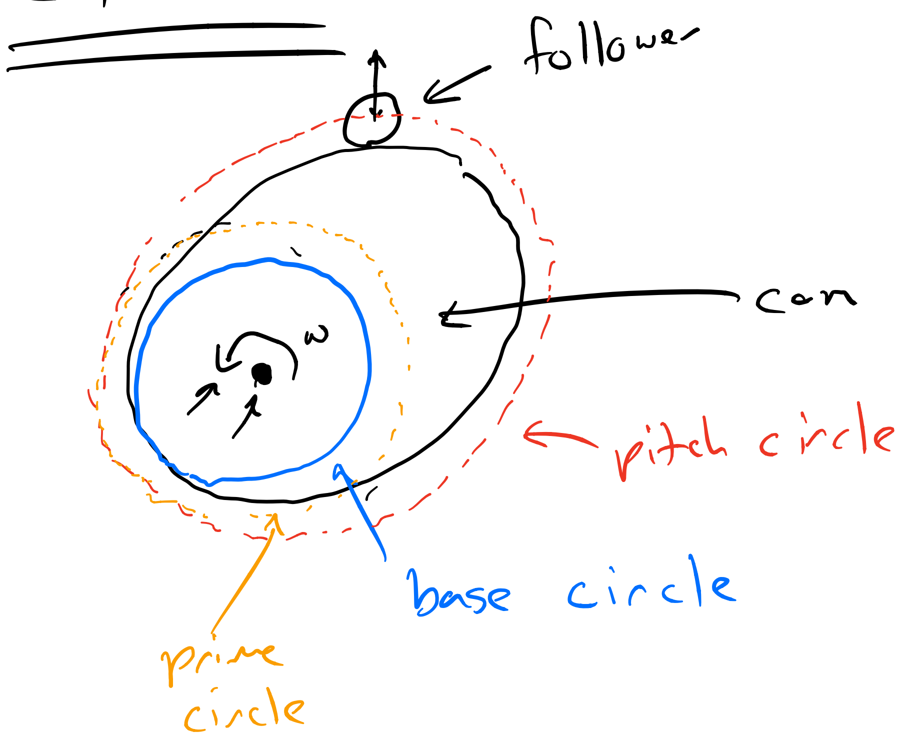




$$f(x,y)=0$$

$$x^2+y^2+k^2=0$$

CAMs



follower described by vertical motion $y(t)$

$y(t)$ $\qquad \dfrac{dy}{dt} = v$

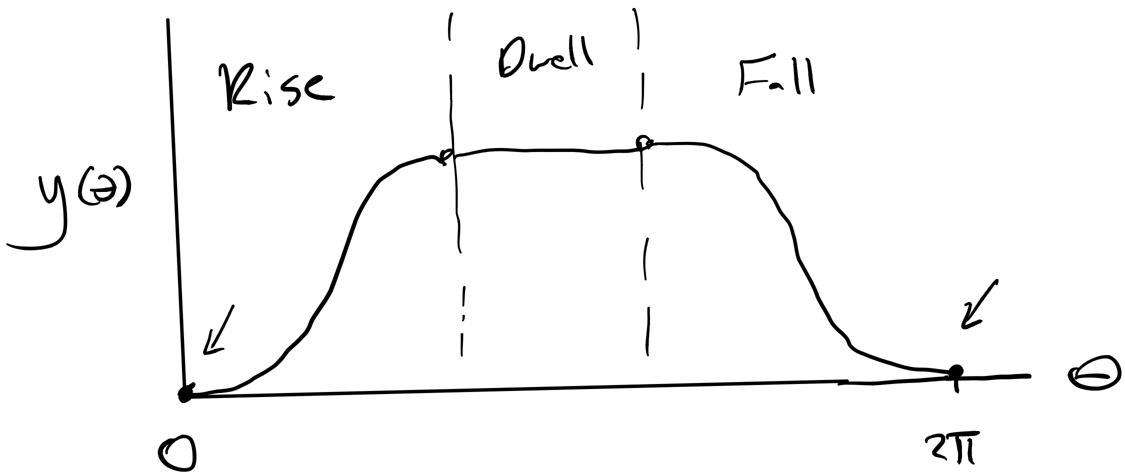
$$\frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt}$$

$$v = \frac{dy}{d\theta}\,\omega$$ velocity follower

$$\frac{\partial^2 y}{\partial t^2} = \underline{\underline{\frac{\partial^2 y}{\partial \theta^2}}} \omega^2 + \underline{\underline{\frac{\partial y}{\partial \theta}}} \dot{\omega}$$

design $\underline{\underline{y(t)}} \longleftrightarrow y(\theta)$

$\omega t = \theta$



$$\underline{y(\theta = 0) = 0}$$

What is important for a follower trajectory?

$y(\theta)$ continuous

$y'(\theta)$ smooth

$y''(\theta)$ smooth