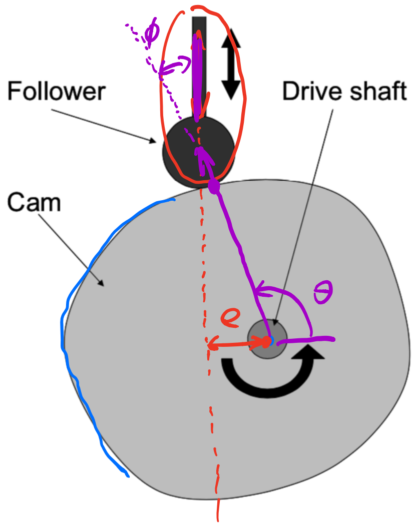
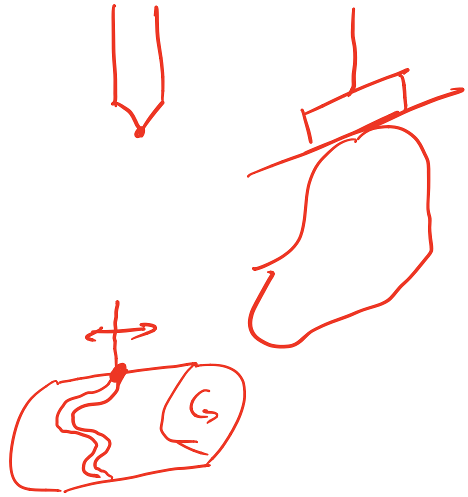
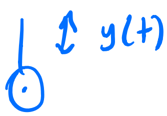
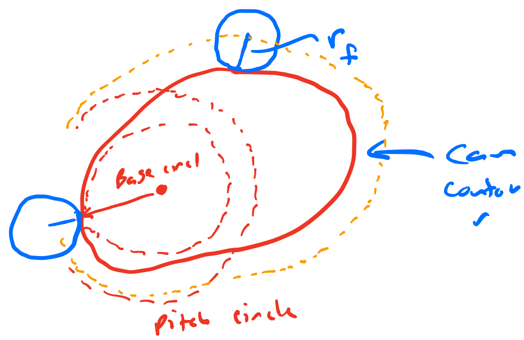
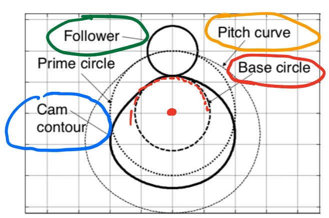








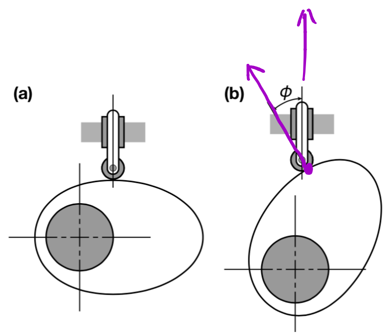


Base circle: The smallest circle that can be drawn tangent to the cam surface.

Pitch curve: The trace of the center of the follower when it is rotated around the cam contour.

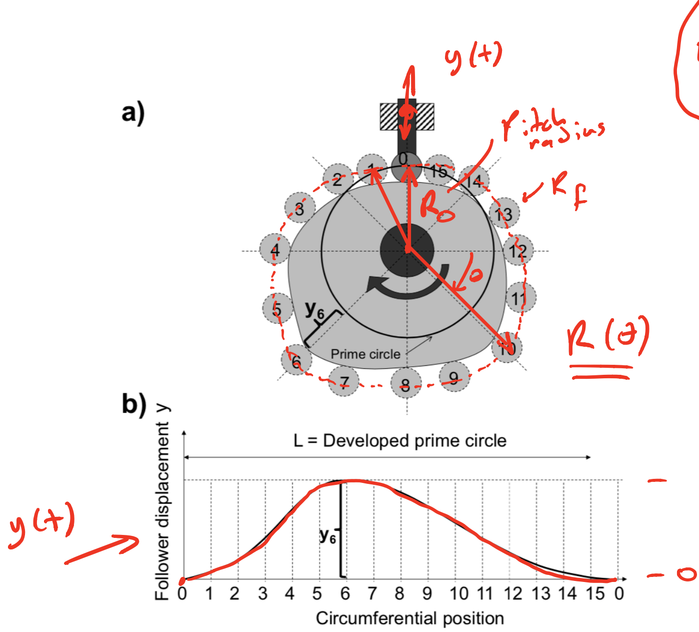
Prime circle: The smallest circle about the cam center through the pitch curve.

Pressure angle: The angle ϕ between the direction of the follower motion and the normal to the cam contour, at the same point along the contour. The pressure angle varies during a revolution of the cam. A nonzero pressure angle implies that the force on the follower is not aligned with the follower axis. This can cause the follower to be jammed, if the pressure angle becomes too large. Figure 6.4 illustrates this, and the topic is covered in more detail on page 89.



Pitch point: The point where the pressure angle is at its maximum.

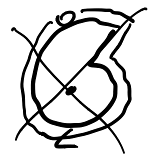
Trace point: An arbitrary point on the follower. It is used to generate the pitch curve.

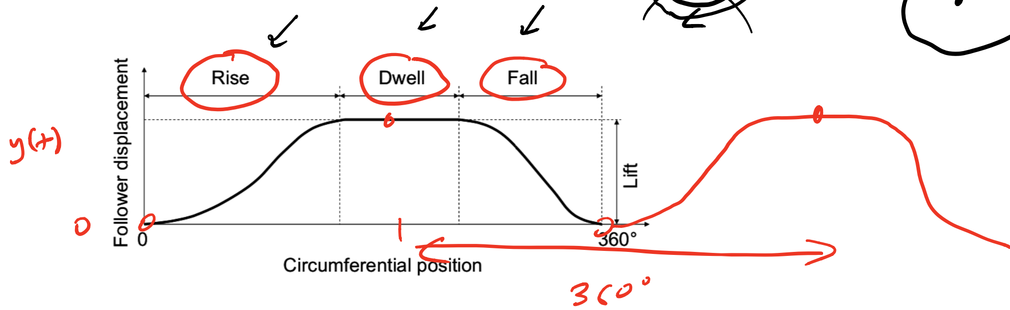


Polar representation of cam surface and pitch curve

y(t) is relative height change of follower

$y(t) + R_0$ = Absolute dist from center of rotation





Motion profile $\left\{ \begin{array}{l} y(t) \\ \dot{y}(t) \\ \ddot{y}(t) \end{array} \right.$ $\longrightarrow$

$y(\theta)$ $\qquad \theta = \omega t \rightarrow \frac{\partial \theta}{\partial t} = \omega$

$$\frac{dy}{dt} = \frac{dy}{d\theta}\frac{\partial\theta}{\partial t} = y'(\theta)\,\omega$$

y'

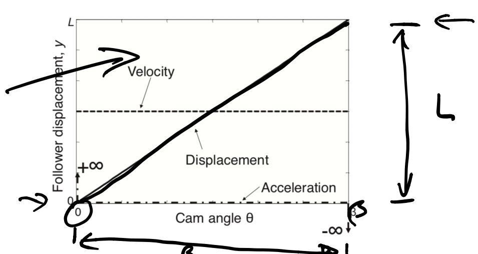
$$\frac{\partial}{\partial t}\left(y'(\theta)\,\omega\right) \rightarrow \frac{\partial}{\partial t} y' \cdot \omega + y' \frac{\partial \omega}{\partial t}$$

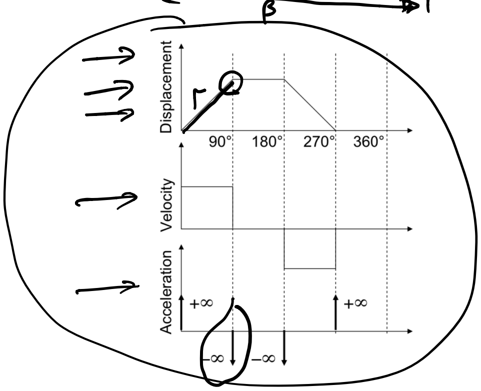
$$\frac{\partial}{\partial t}\frac{dy}{d\theta} \cdot \frac{\partial\theta}{\partial t} + y'\alpha$$

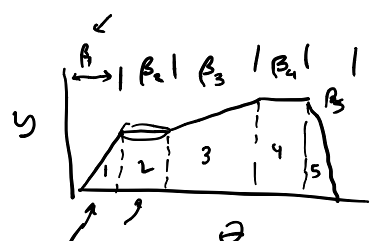
$$\frac{d^2y}{d\theta^2} \cdot \frac{\partial\theta^2}{\partial t^2}$$

$$\frac{d^2y}{dt^2} = \left(y'' \cdot \omega^2 + y'\alpha\right)$$

Linear motion







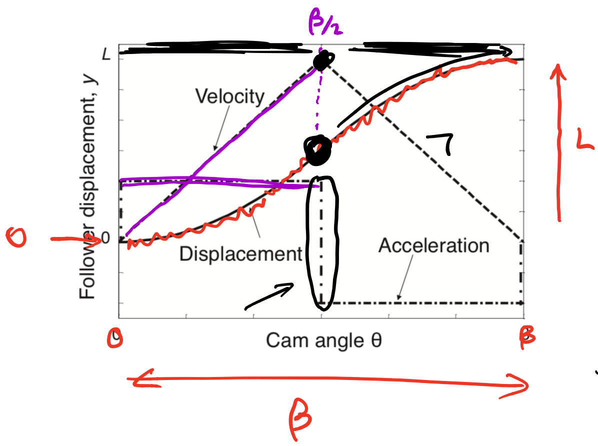
$$\boxed{y(\theta) = L \cdot \frac{\theta}{\beta_1}}$$

$y(0) = 0$

$y(\beta_1) = L$

$$y'(\theta) = \frac{L}{\beta} \rightarrow \dot{y}(\theta) = \frac{L}{\beta}\omega$$

Parabolic motion



$$y(\theta) = C\theta^2$$

?

$$y(0) = 0$$

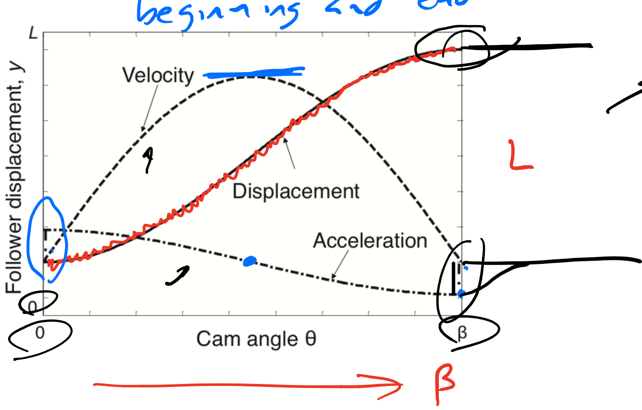
$$y(\beta/2) = L/2 \rightarrow L/2 = C\cdot\left(\frac{\beta}{2}\right)^2 = C\frac{\beta^2}{4}$$

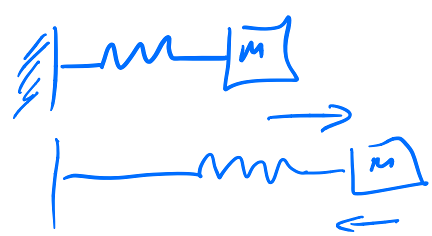
$$C = \frac{2L}{\beta^2}$$

$$y(\theta)\begin{cases} \frac{2L}{\beta^2}\cdot\theta & 0 \le \theta \le \beta/2 \\ L\left[1-2\cdot\left(1-\frac{\theta}{\beta}\right)^2\right] & \beta/2 < \theta < \beta \end{cases}$$

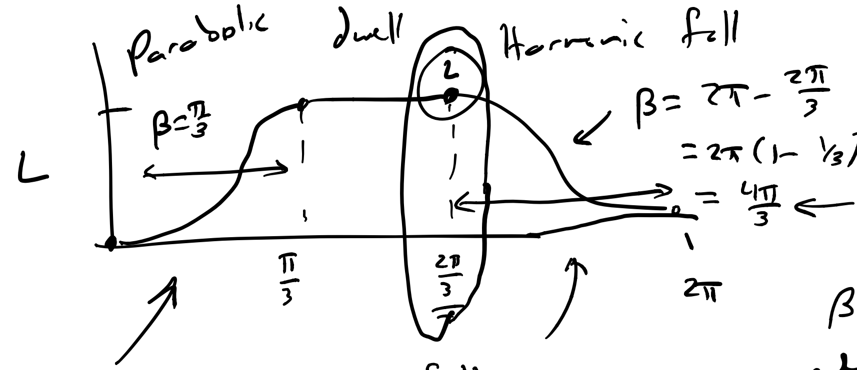
Harmonic motion

Produces continuous y, y'
but y'' discontinuous at
beginning and end





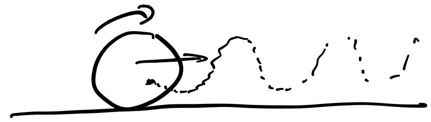
$$y(\theta) = \frac{L}{2}\left[1-\cos\left(\frac{\pi}{\beta}\theta\right)\right]$$

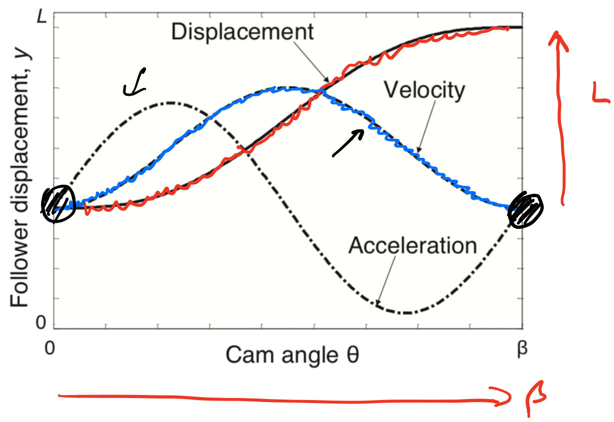


Harmonic fall

$$y(\theta) = \begin{cases} L - \frac{L}{2}\left[1 - \cos\left(\frac{\pi}{\beta}\overbrace{\left(\theta - \frac{2\pi}{3}\right)}^{\beta \text{ when } \theta = 2\pi}\right)\right] \end{cases}$$

Cycloidal motion

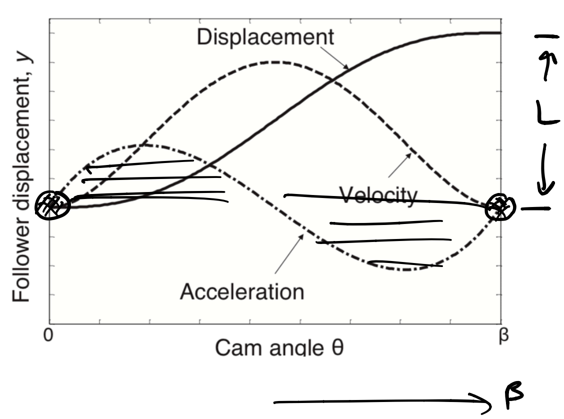




$$y = L\left[\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\beta}\right)\right]$$

3-4-5 polynomial

$$y = L\left(\frac{10}{\beta^3}\theta^3 - \frac{15}{\beta^4}\theta^4 + \frac{6}{\beta^5}\theta^5\right)$$



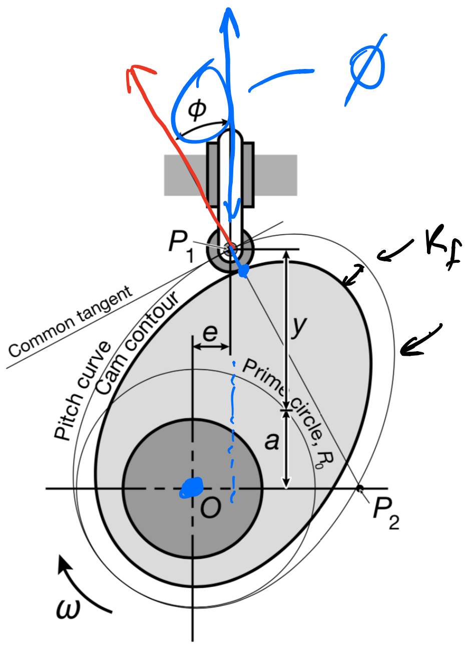
Motion profiles:

1) Pay attention to boundary conditions of motion sections $y'(0), y'(\beta)$ $\underline{y''(0), y''(\beta)}$

2) Continuity in y, y'

3) minimize y'''

4)

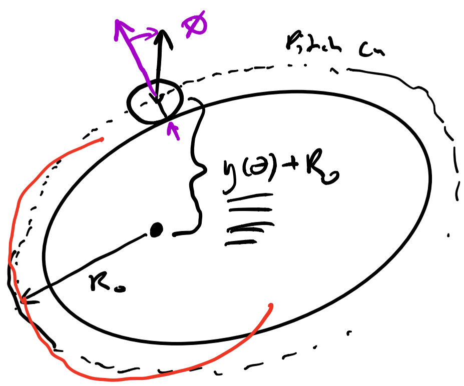


Cam contour?

Pressure angle?

$y(\theta) \rightarrow$ follower pitch curve

~~Cam contour = $y(\theta) - R_{follower}$~~



Cam contour is the offset curve of the pitch curve

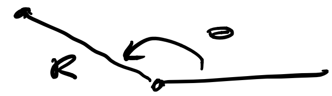
follower traj

$\rightarrow$ Pitch curve $= y(\theta) + R_0$

Prime circle

$\theta \in [0, 2\pi]$

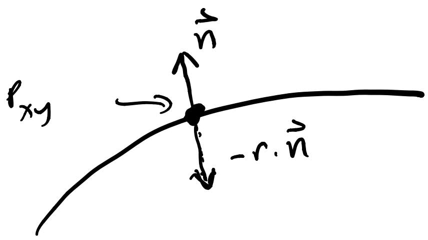
$$R(\theta) = y(\theta) + R_0$$



$$\theta = [0, 2\pi]$$

$$P_{xy} = \Big(\overset{x}{R(\theta)\cos\theta},\ \overset{y}{R(\theta)\sin\theta} \Big)$$

Cam contour $\rightarrow$ $P_{xy} + r_f \cdot \left[\dfrac{-y'}{\sqrt{x'^2+y'^2}},\ \dfrac{x'}{\sqrt{x'^2+y'^2}} \right]$



$$x' = R'(\theta)\cos\theta - R(\theta)\sin\theta$$

$$y' = R'(\theta)\sin\theta + R(\theta)\cos\theta$$

$$\sqrt{x'^2+y'^2} = R'^2 + R^2$$

$$\text{Cam surface} = \begin{bmatrix} R(\theta)\cos\theta \\ R(\theta)\sin\theta \end{bmatrix} + \frac{r_f}{\sqrt{R'^2+R^2}} \begin{bmatrix} -R'\sin\theta + R\cos\theta \\ R'\cos\theta - R\sin\theta \end{bmatrix}$$

$$\underline{\underline{\text{Cam surface}}} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} R\cos\theta - r_f \dfrac{R'\sin\theta + R\cos\theta}{\sqrt{R'^2+R^2}} \\[2ex] R\sin\theta + r_f \dfrac{R'\cos\theta - R\sin\theta}{\sqrt{R'^2+R^2}} \end{bmatrix} & \begin{array}{c} x \\[4ex] y \end{array} \end{array}$$

$R \rightarrow R(\theta)$

$\rightarrow y(\theta) + R_0$

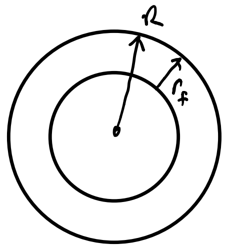
$\theta \rightarrow [0, 2\pi]$

Lets check this result for a circle ($R'=0$)

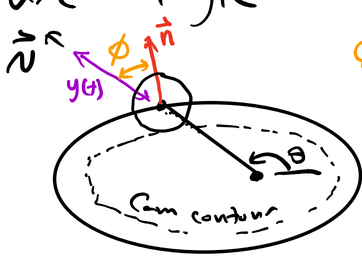
$$R'=0 \rightarrow \text{circle} \begin{bmatrix} R\cos\theta - r_f\cos\theta \\ R\sin\theta + r_f \cdot -\sin\theta \end{bmatrix}$$

$$= \left((R-r_f)\cos\theta, (R-r_f)\sin\theta\right)$$

equation for a circle of radius $(R-r_f)$



Pressure angle:



$\phi \rightarrow$ Pressure angle is the angle between surface normal of the pitch curve and the follower motion [y(t)]

Again we represent the pitch curve as

$$PC_{xy} = \begin{bmatrix} R(\theta)\cos\theta \\ R(\theta)\sin\theta \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} \frac{-y'}{\sqrt{x'^2+y'^2}} \\ \frac{x'}{\sqrt{x'^2+y'^2}} \end{bmatrix} \quad |\vec{n}| = 1$$

$$= \begin{bmatrix} \frac{-R'\sin\theta - R\cos\theta}{\sqrt{R'^2+R^2}} \\ \frac{R'\cos\theta - R\sin\theta}{\sqrt{R'^2+R^2}} \end{bmatrix}$$

Define vector $\vec{v}$ that points in direction of follower motion

$$\vec{v} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \quad |v| = 1$$

Pressure angle

$$\cos\phi = \vec{n} \cdot \vec{v}$$

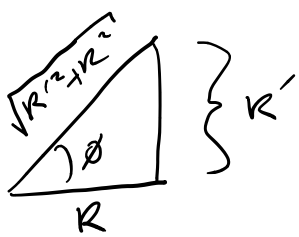
$$\cos\phi = \frac{1}{\sqrt{R^2+R'^2}}\left(-\cancel{R'\sin\theta\cos\theta} - R\cos^2\theta + \cancel{R'\cos\theta\sin\theta} - R\cos^2\theta\right)$$

$$\boxed{\cos\phi = \frac{-R}{\sqrt{R'^2+R^2}}}$$

$$R = R_0 + y(\theta)$$

$$R' = y'(\theta)$$

to get same answer as book



because $a^2+b^2=c^2$

$$\tan\phi = \frac{R'}{R}$$

$$\boxed{\tan\phi = \frac{y'(\theta)}{R_0+y(\theta)}}$$

Same as reader