SUSTech
Southern University of Science and Technology

神经计算与控制实验室
Neural Computing & Control Lab

Machine Learning and Medical Engineering Applications

机器学习与医学工程应用

Lecture 5 - Ridge and LASSO for modelling the brain

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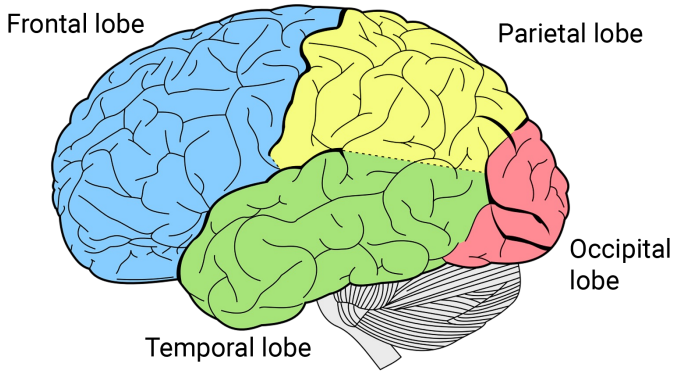
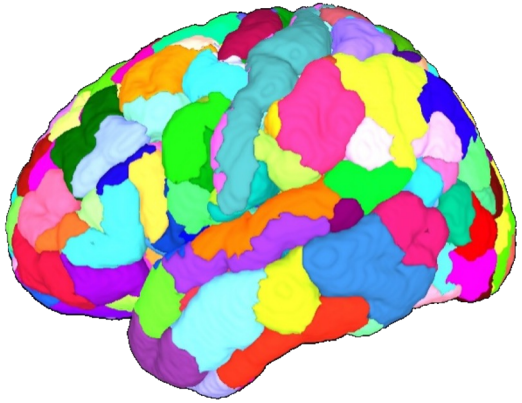
The human brain

a brain

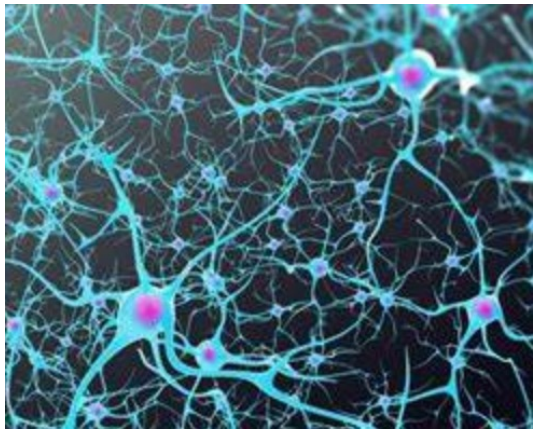
few lobes



hundreds of regions



100 billions of neurons

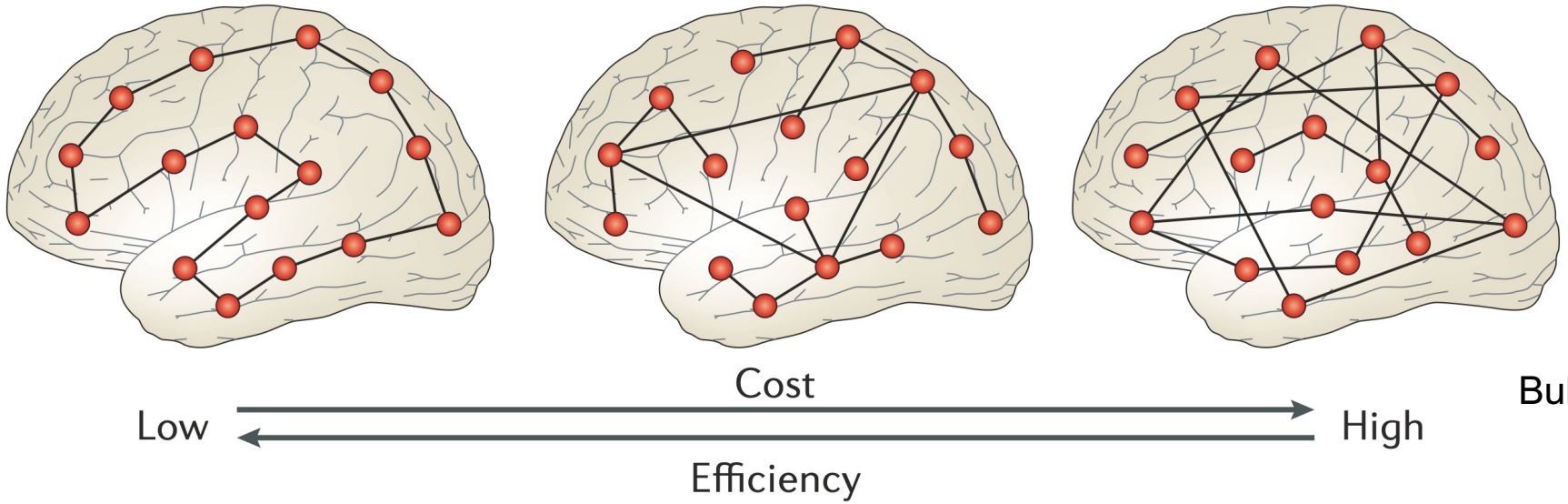


The neurons (regions) are connected by synapses (white matter fibers).

The activity of the connected neurons (regions) are coupled.

The economy of brain network organization

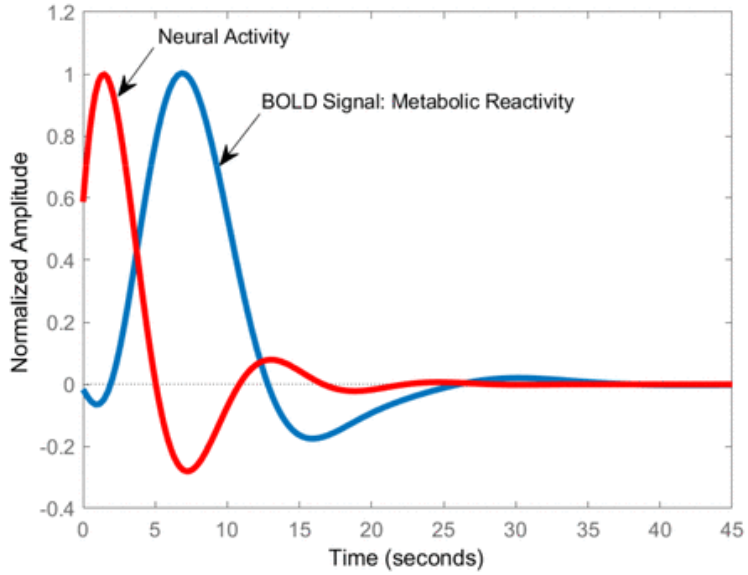
- Efficiency and economy tradeoff in brain
  - The brain can **increase connection strength** to increase **communication efficiency**, but it **compromises the economy** as wiring cost increases.
  - How the brain evolves to its topography remains unknown.



Bullmore et al., 2012

Linear system of the large-scale fMRI dynamics

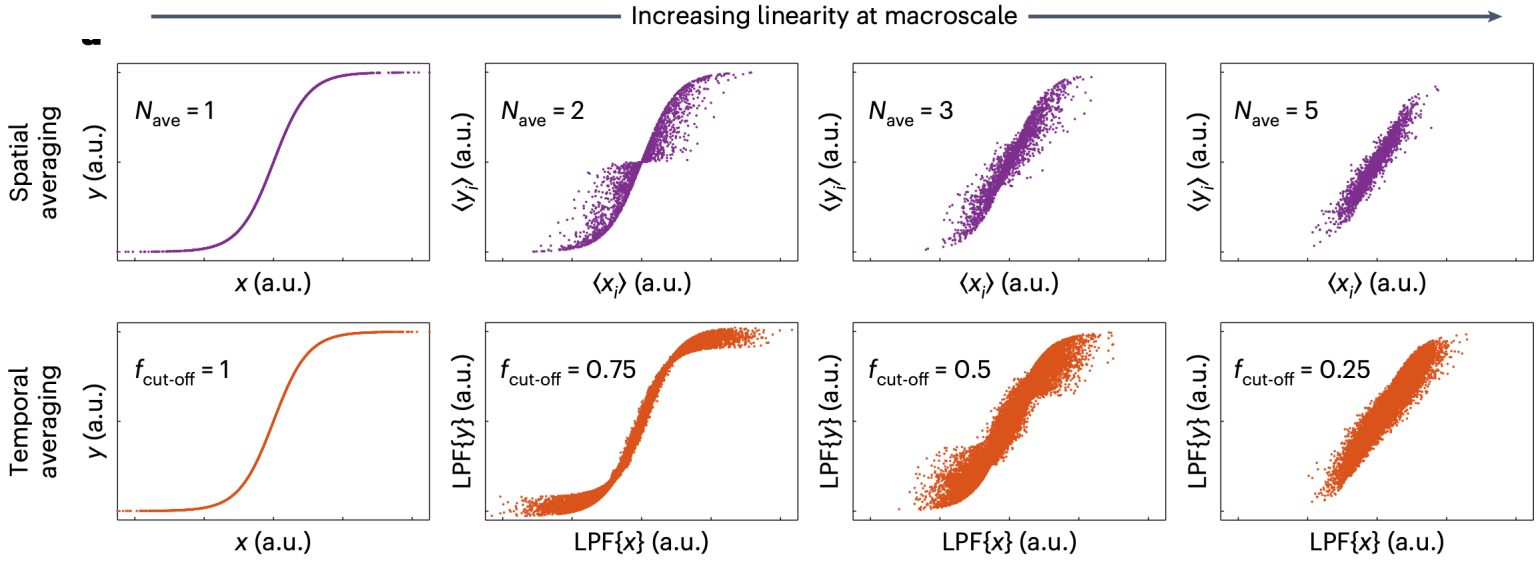
- Macroscopic resting-state fMRI (rsfMRI)
  - Blood-oxygen-level-dependent (BOLD) in fMRI is a method measuring **brain activity** by detecting changes in **blood oxygenation and flow**.
  - Hemodynamic Response Function (HRF) is a model of the **blood flow changes** in the brain following **neural activity**.



Linear system of the large-scale fMRI dynamics

- Macroscopic resting state fMRI (rsfMRI)
  - In large scale fMRI paradigm, single neuron's non-linear activity is averaged spatially and temporally.
  - Macroscopic rsfMRI shows **linear effect**, which can be described by **linear models**.

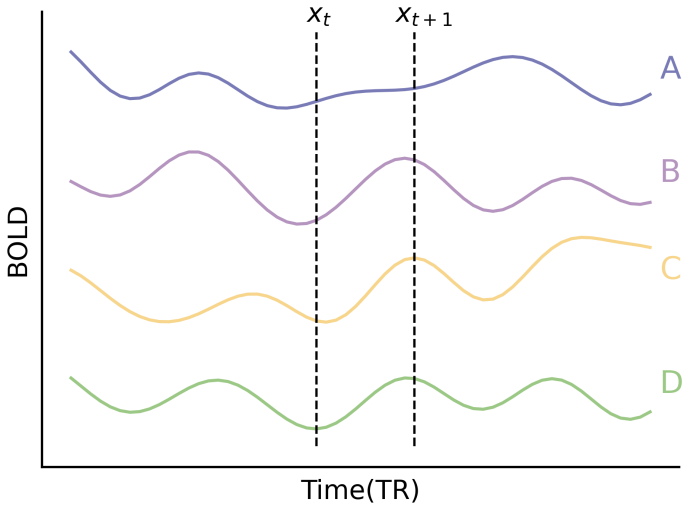
$$x_{t+1} = Wx_t$$

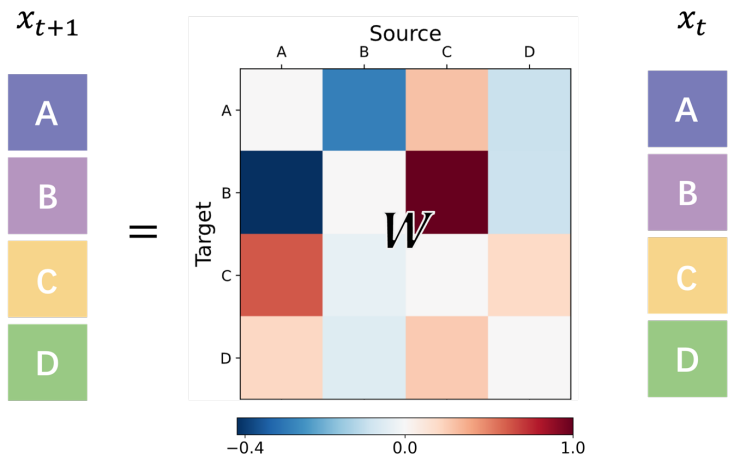


Nozari et al., 2023, Nature BME

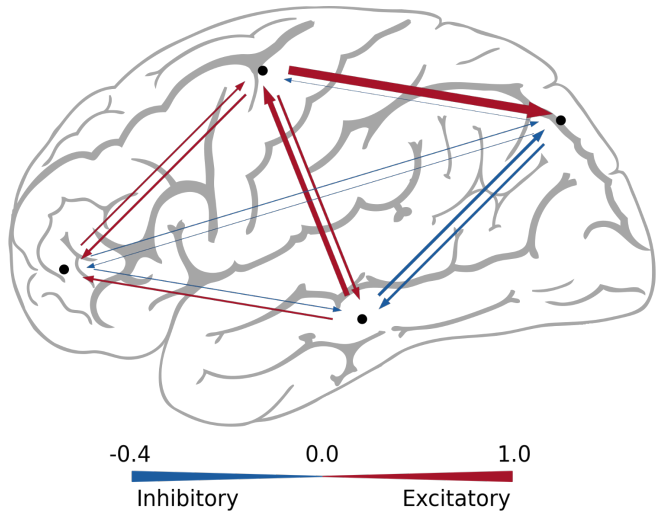
Modeling of the brain dynamics by linear model

- fMRI ridge regression schematic diagram
  - Use activity levels in **each region** to predict the levels in **other regions** at **next sampling point**.
  - The W explicitly reveals the **effective connectivity** in brain.





$$arg\min_{W}(1-\lambda)\ \frac{1}{n}\sum||x_{t+1}-Wx_t||_2^2+\lambda||W||_2^2$$



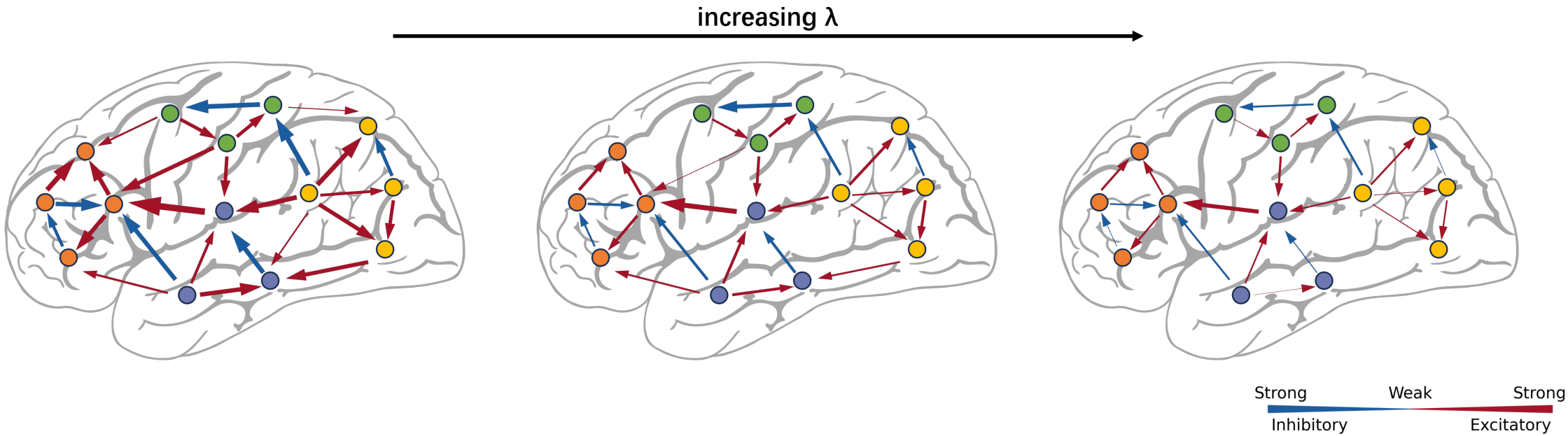
Modeling of the brain dynamics by linear model

卓著

- Constrain the brain wiring by L2 regularization

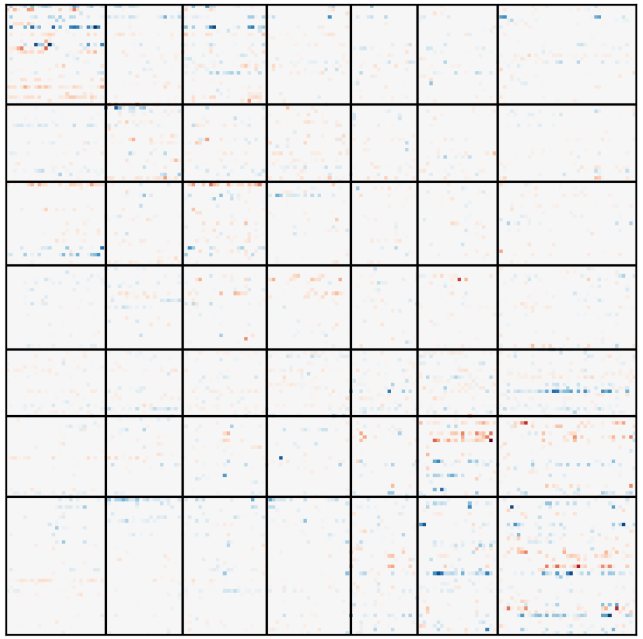
  - **Higher λ** results in constrained connection, thus **increasing the economy** while **decreasing the efficiency**.

$$arg\min_{W}(1-\lambda)\ \frac{1}{n}\sum||x_{t+1}-Wx_t||_2^2+\lambda||W||_2^2$$



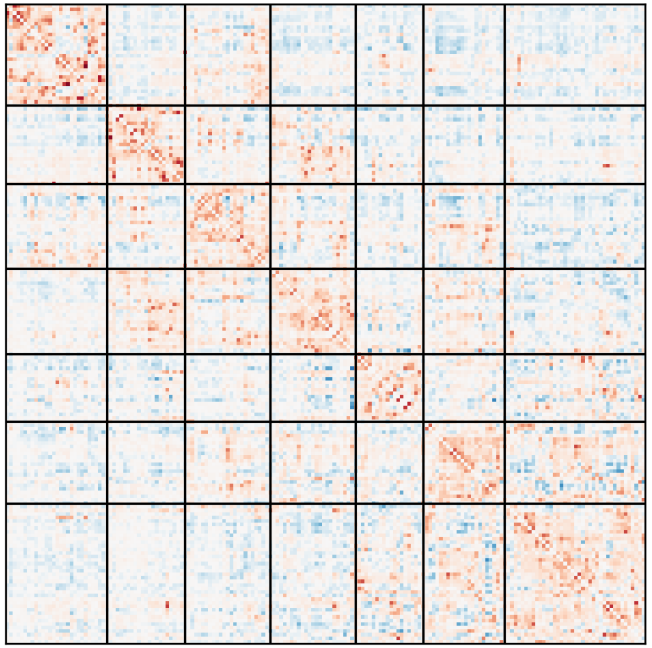
Comparison between Lasso and Ridge

Lasso



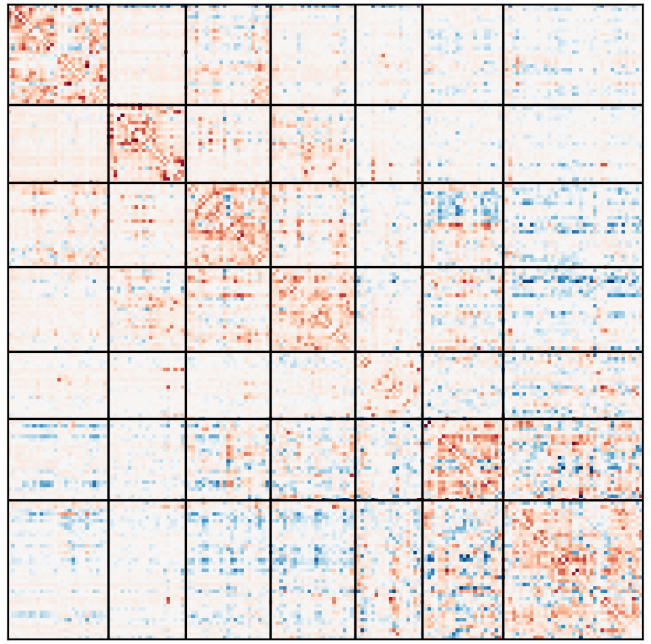
$$arg\min_{W}(1-\lambda)\ \frac{1}{n}\sum||x_{t+1}-Wx_t||_2^2+\lambda||W||_1^1$$

Ridge



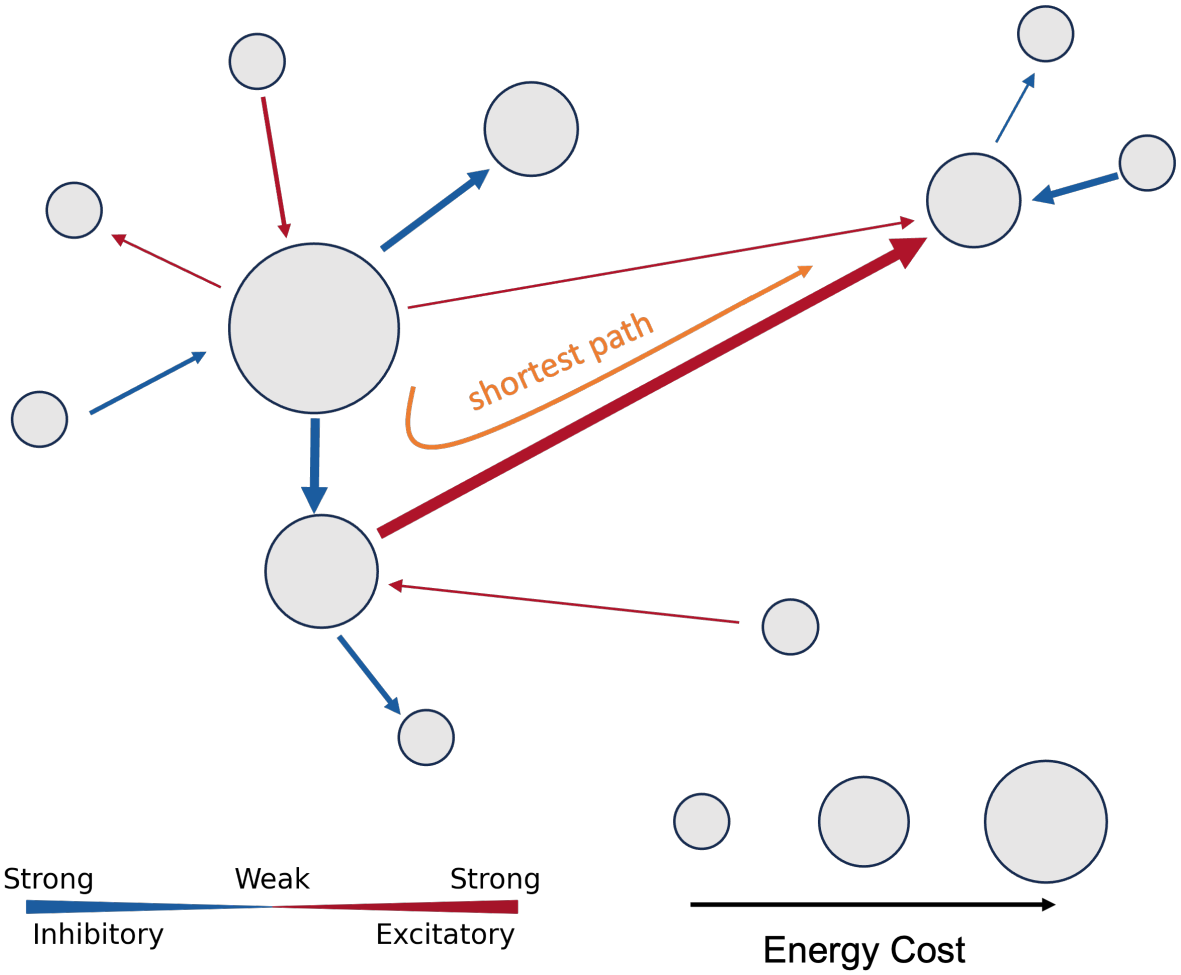
$$arg\min_{W}(1-\lambda)\ \frac{1}{n}\sum||x_{t+1}-Wx_t||_2^2+\lambda||W||_2^2$$

Emprical Effective Connectivity



- Connectivity revealed by Ridge resembles empirical data more than that by Lasso.
- According to the prior, it may indicate that the connections in brain are smooth rather than sparse.

Quantify the Efficiency and Economy

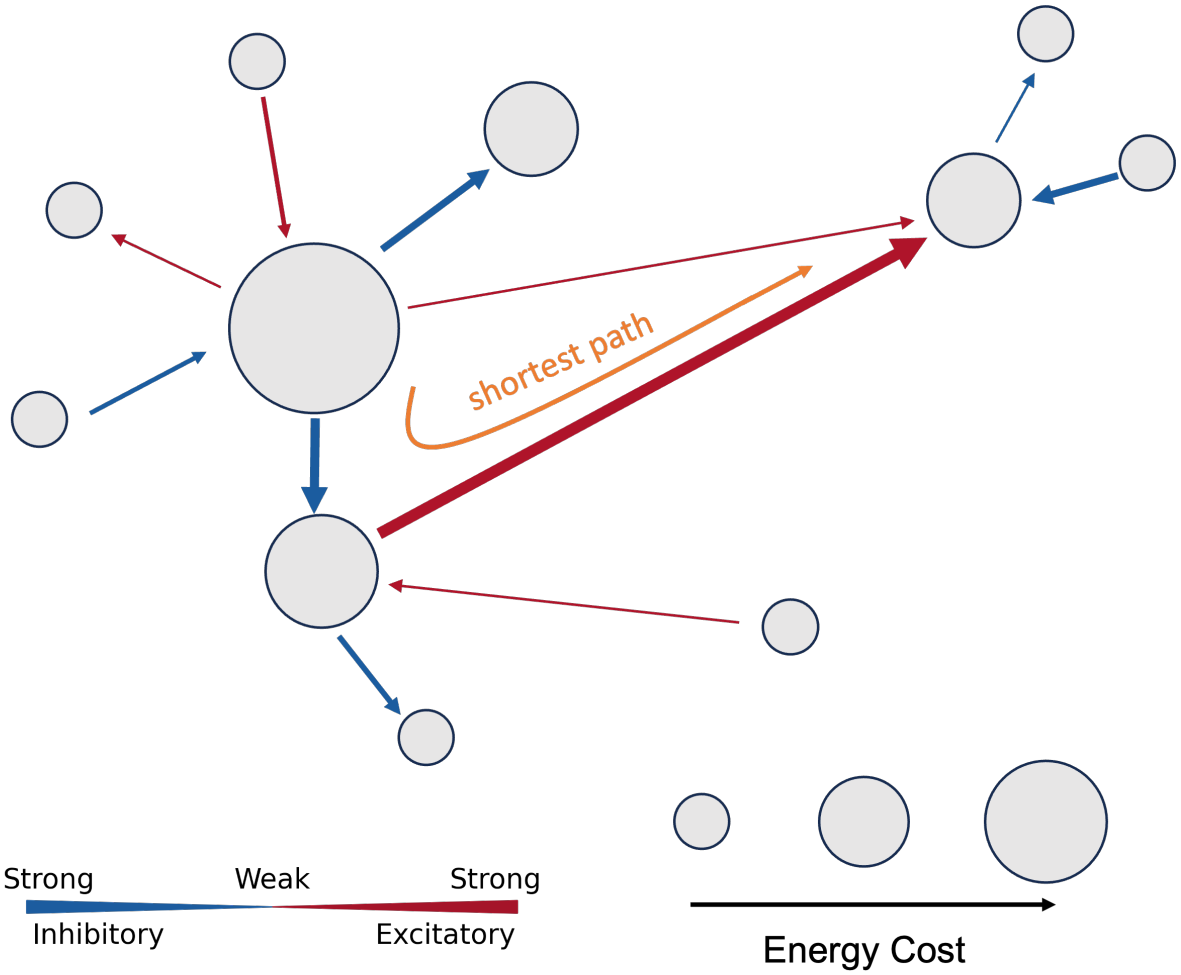


- Quantify the network efficiency
  - If the absolute connection strength between two nodes is high, it will be easier for them to do communication, thus they are shorter in topological space.
  - The efficiency is defined as the **average shortest path** from one node to another in the network.

Topological shortest path: $d_{ij}^{W} = \sum_{a_{uv} \in g, i \leftrightarrow j} f(w_{uv})$

Efficiency: $Eff^{W} = \frac{1}{n} \sum_{i \in N} \frac{\sum_{j \in N, j \neq i} (d_{ij}^{W})^{-1}}{n-1}$

Quantify the Efficiency and Economy



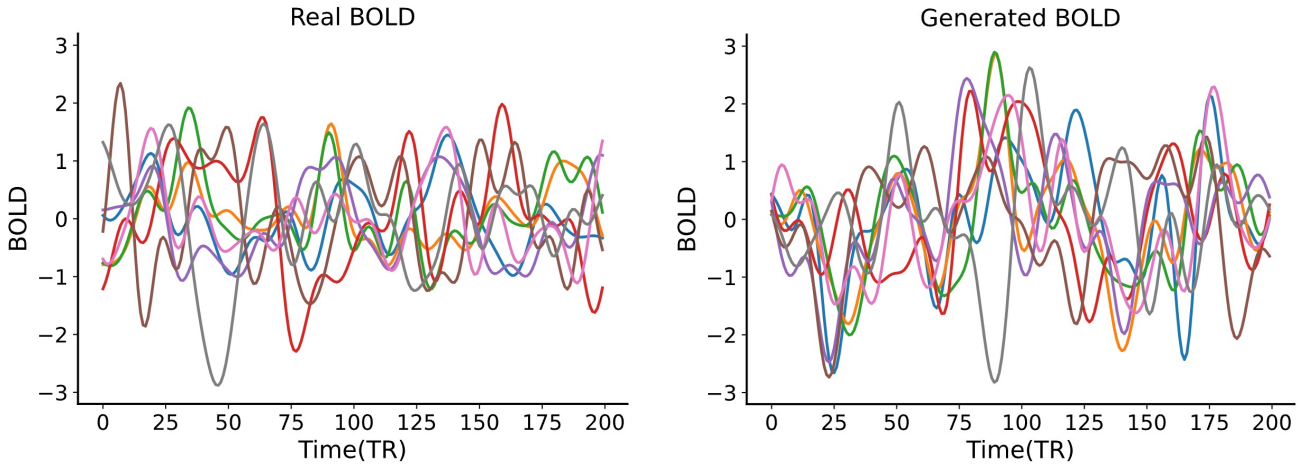
- Quantify the network economy
  - The metabolic rate is logarithmically proportional to the global degree of the node. (Tomasi et. al., 2013)
  - The economy is defined as average of each node's **reciprocal of global degree**.

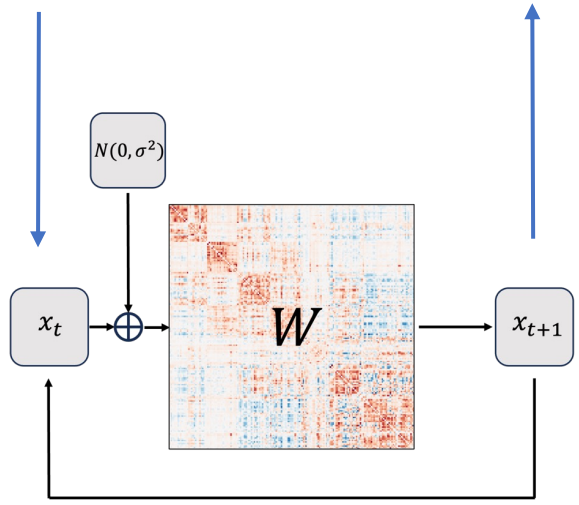
Global Degree: $k_i^W = \sum_{i \in N} W_{ij}$

Cerebral metabolic rate: $\ln(CMR_i) \propto \ln(k_i^W)$

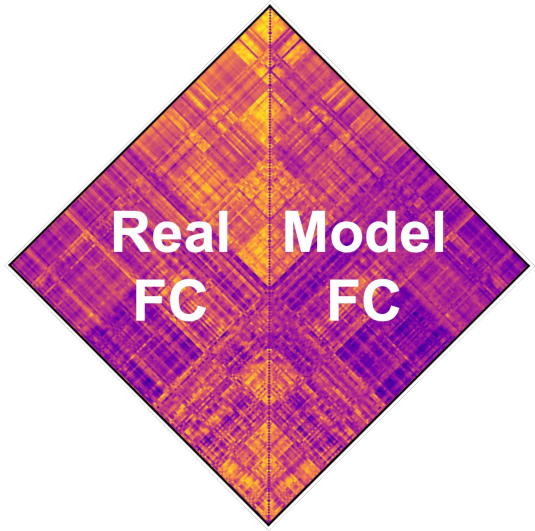
Economy : $Eco^W = \frac{1}{n} \sum_{i \in N} \frac{1}{\ln(CMR_i)}$

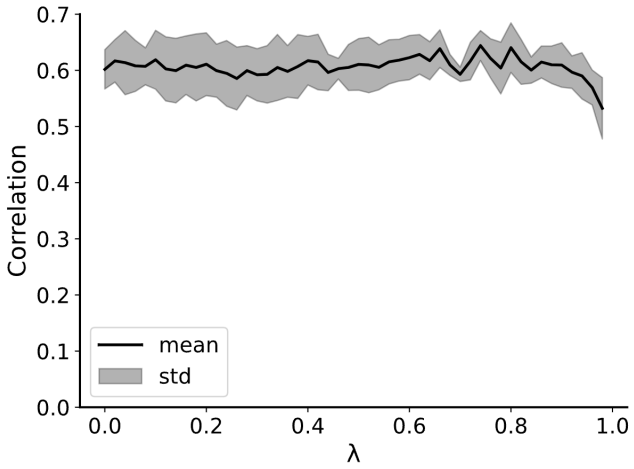
EC Ridge Model Validation





BOLD generator

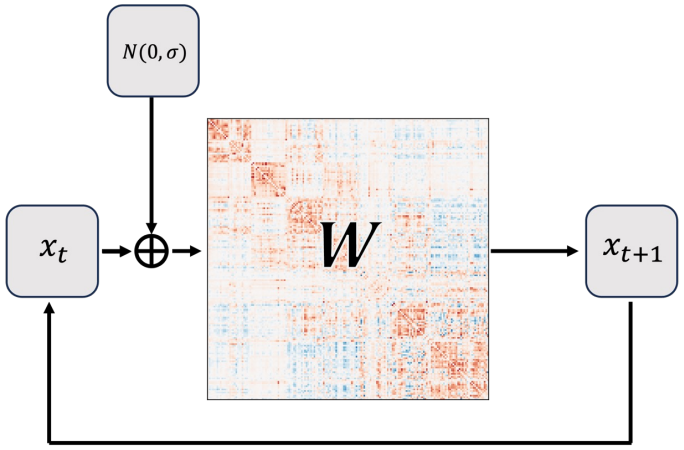


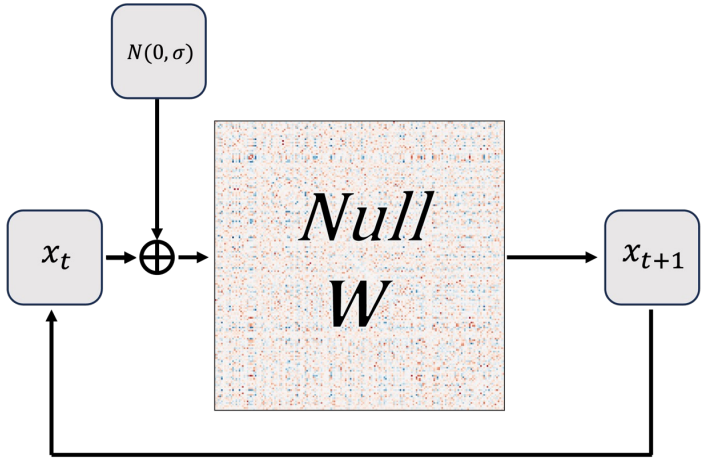


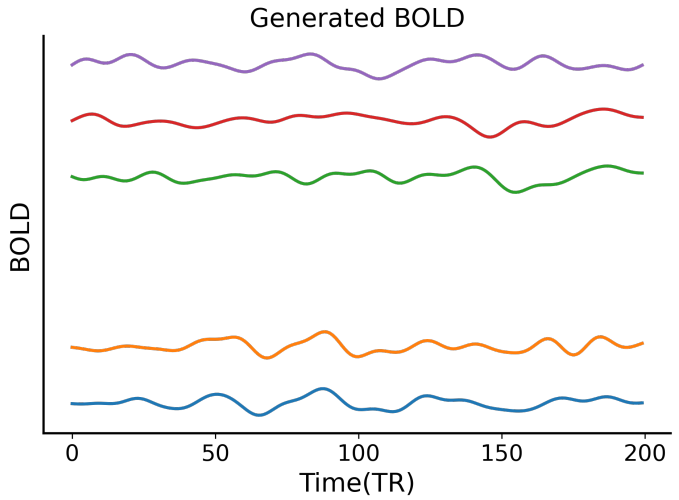
Correlation (Real FC, Model FC)

Model Validation by Generating BOLD

- Use trained model to generate BOLD driven by noise
  - The causality model should be able to generate time series with similar property compared to brain dynamic.
  - The trained model should perform better than null model.

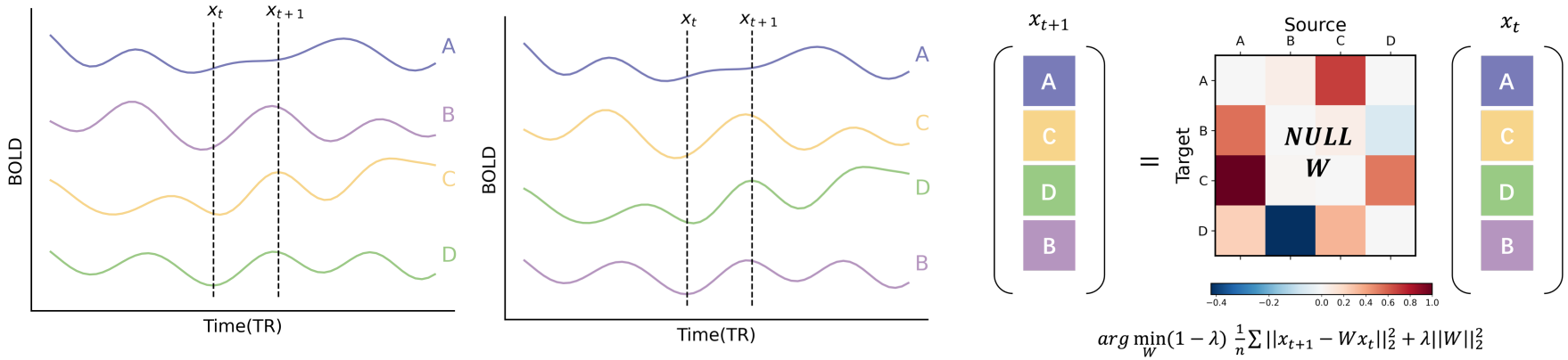




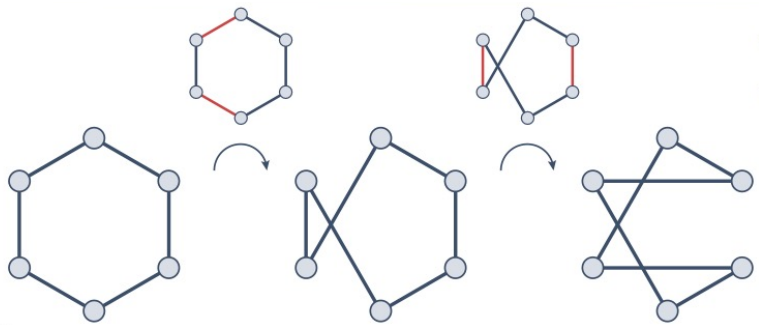


Model Validation by Generating BOLD

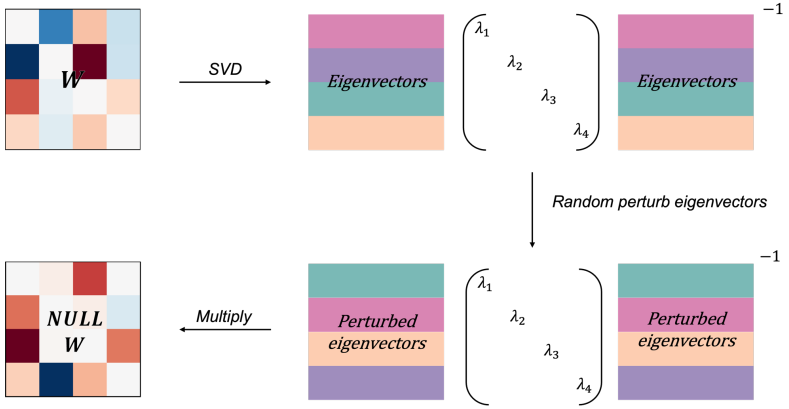
Random perturb regions



Random perturb connections



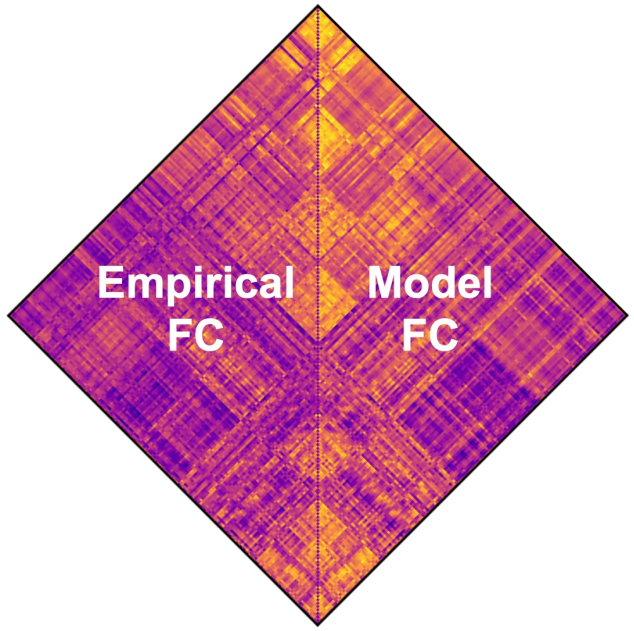
Random perturb eigenvectors

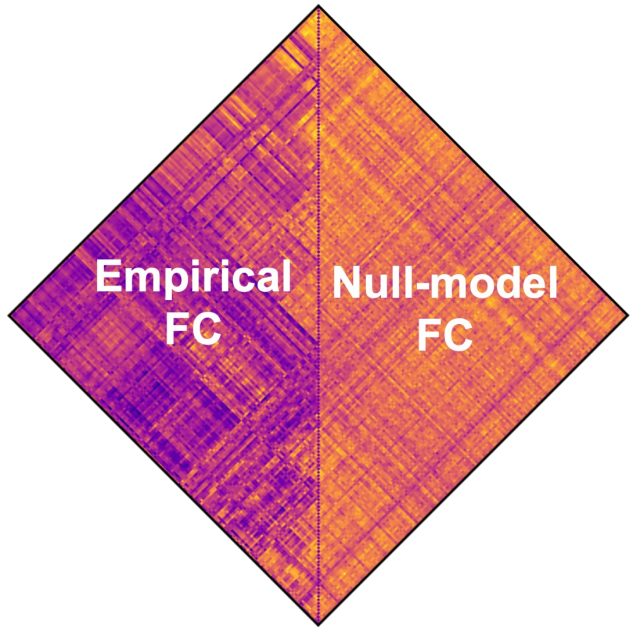


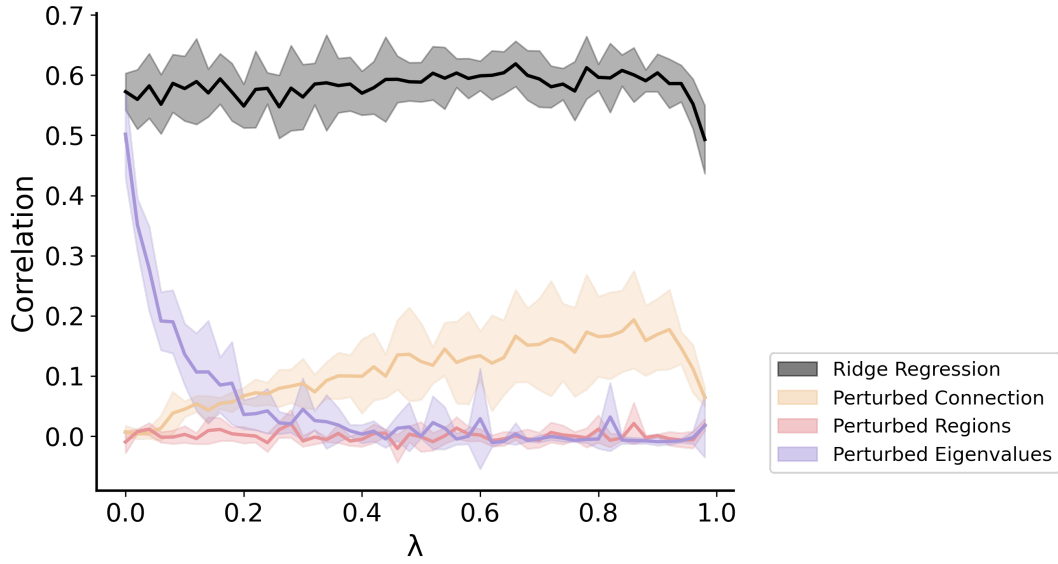
- Null models are a **baseline** for comparing against observed data to identify significant effects.
- Null models are generated by **perturbing specific property** in the original model.

Model Validation by Generating BOLD

- Compare the generated BOLD and real BOLD
  - **Functional connectivity** revealed by model generated BOLD shows **high correlation** with empirical data, while the one from null model not.
  - Regularized regression model successfully capture the brain dynamic.





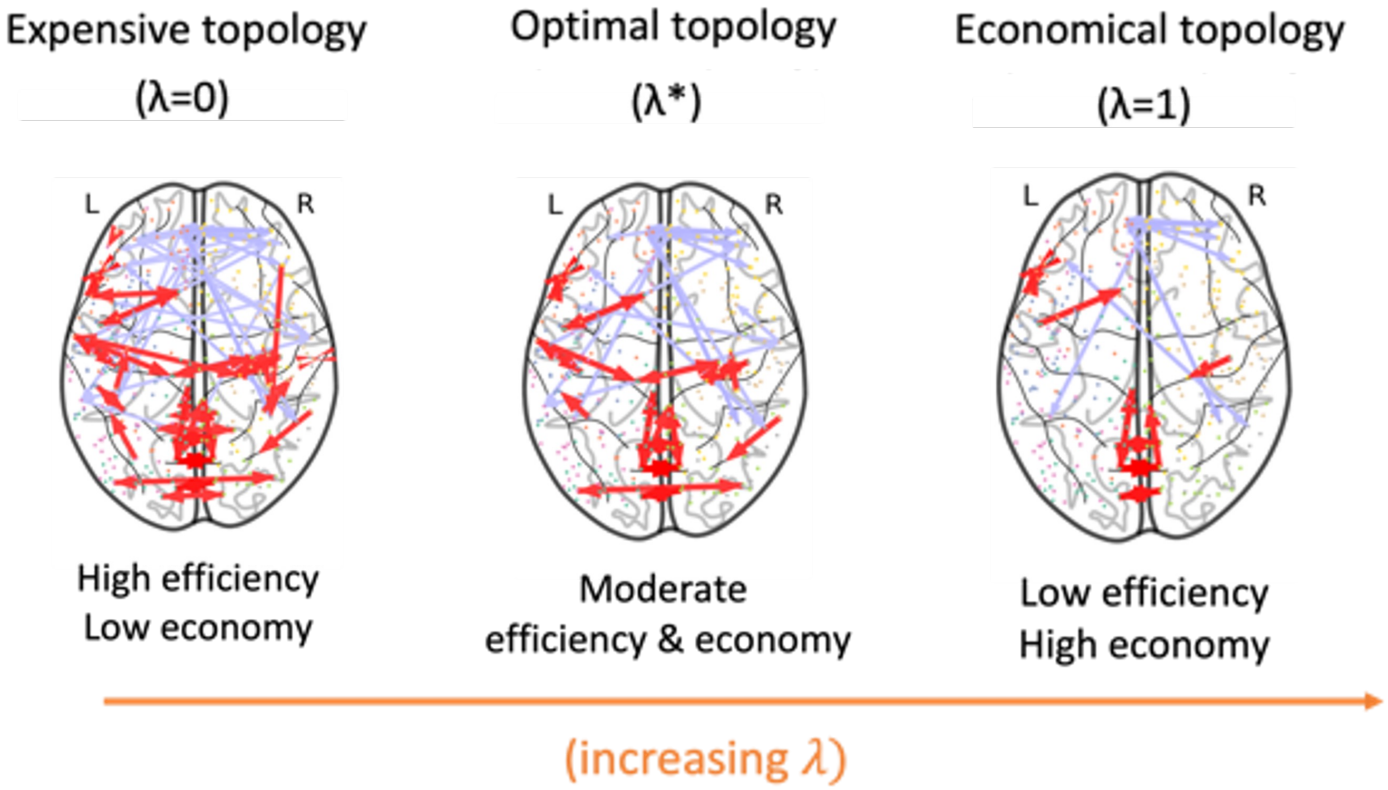


Efficiency-Economy Tradeoff

$$arg\min_{W}(1-\lambda)\ \frac{1}{n}\sum||x_{t+1}-Wx_t||_2^2+\lambda||W||_2^2$$

Term1: Recover dynamics

Term2: Constrain on wiring



- Brain pursues a balance between and **efficiency** and **economy**. (Ed Bullmore, 2012)
- The model can be trained to optimize for either **accuracy of cognitive task** or **wiring cost**.

Efficiency-Economy Tradeoff

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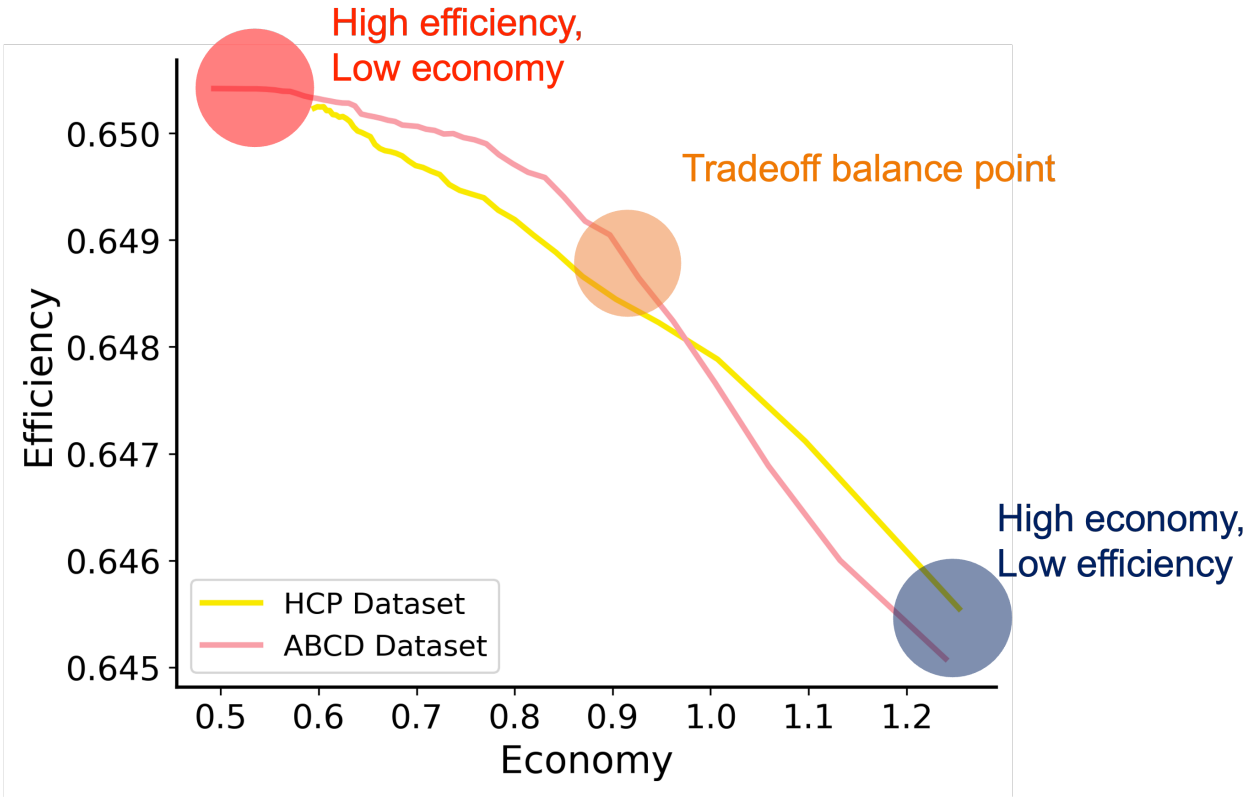
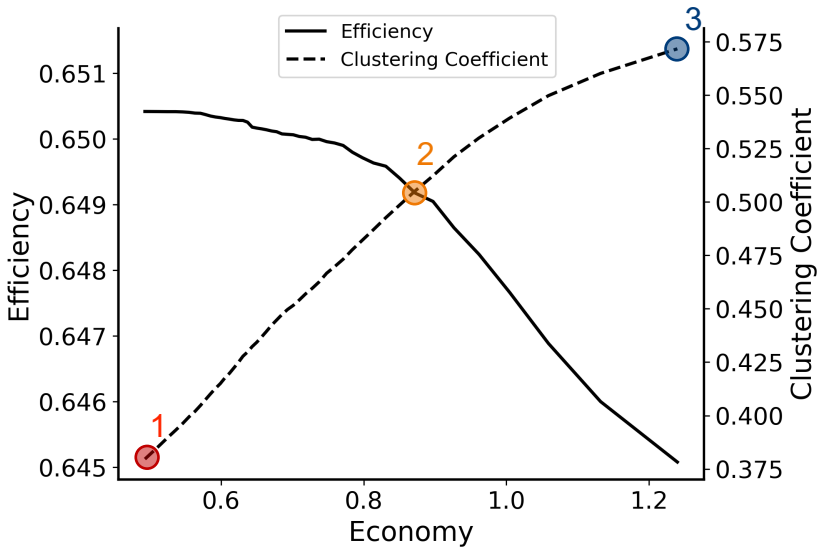
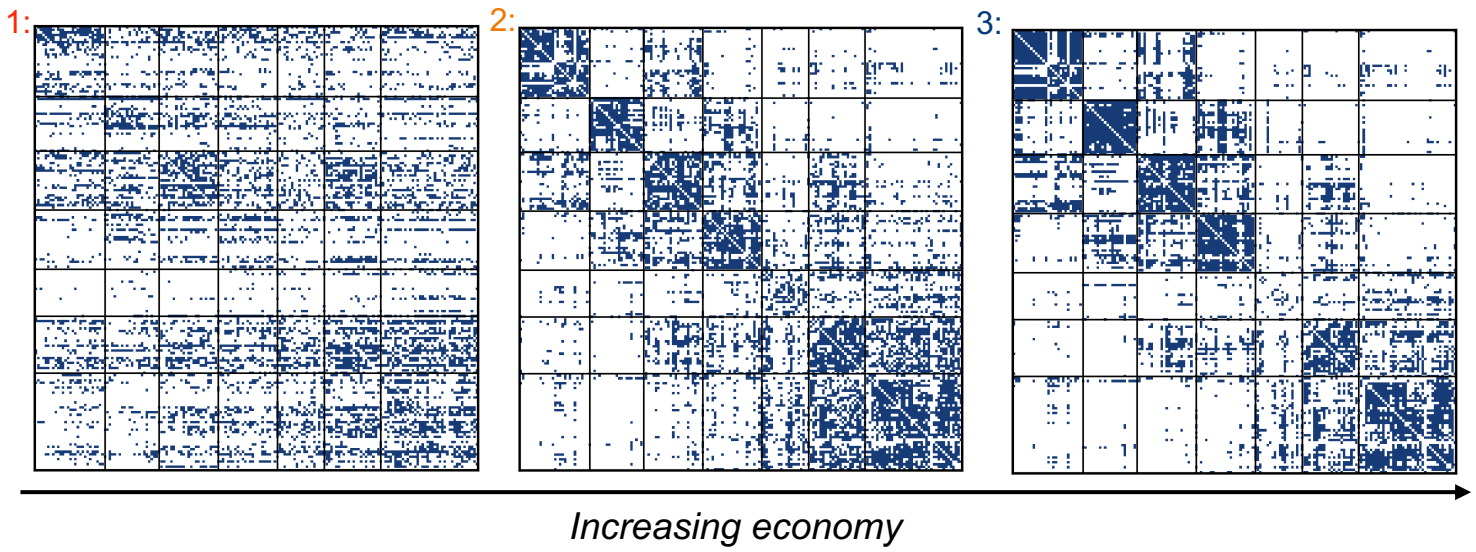


- Brain pursues a balance between and **efficiency** and **economy**. (Ed Bullmore, 2012)
- The model can be trained to optimize for either **accuracy of cognitive task** or **wiring cost**.

Efficiency-Economy Tradeoff in the brain

- Modularity emerges with constrained energy
  - As the economy increases, the **connectivity topology** shows up modular structure.





Efficiency-Economy Tradeoff in the brain

- Topological structure resembles empirical connectivity
  - Network with moderate efficiency and economy shares **similar topological structure** with empirical EC and SC.