$$u(x,t) = U(z),\ z = x - ct,\ U'' + cU' + f(U) = 0$$

Phase plane system for this equation is

$$U' = V,\ V' = -cV - f(U) \quad = ru(1-u) - \frac{u^2}{1+u^2}$$

which has four steady states

$$(0,0),\ (u_1,0),\ (u_2,0),\ (u_3,0).$$

We want to solve the eigenvalue problem for c and linear phase plane analysis gives the following singular point classifications,

$(0,0)$: $f'(0) > 0 \Rightarrow$ stable spiral if $c^2 < 4f'(0)$ or stable node if $c^2 > 4f'(0)$ for $c > 0$

$(u_2,0)$: $f'(u_2) > 0 \Rightarrow$ stable spiral if $c^2 < 4f'(u_2)$ or stable node if $c^2 > 4f'(u_2)$ for $c > 0$

$(u_i,0)$: $f'(u_i) < 0 \Rightarrow$ saddle point for all c , $i = 1, 3$.

If $c < 0$ then $(0,0)$ and $(u_2,0)$ become unstable and the type of singularity is the same. There are clearly several possible phase plane trajectories depending on the size of $f'(u_i)$ where u_i has $i = 1,2,3$ plus $u_i = 0$. plagiarism

Assume $c^2 > 4\max(4f'(0), 4f'(u_2))$ and looking for whether there is any population dynamics which has a connection with u_1 and u_3, (saddle-saddle connection).

The eigenvalues λ_1 andλ_2 are found for $i = 1,3$ as

$$\lambda_{1,2} = \frac{-c \mp \sqrt{c^2 - 4f'(u_i)}}{2}, \tag{40}$$

where $f'(u_i) < 0$. The corresponding eigenvectors are

$$e_{i,1} = \begin{pmatrix} 1 \\ \lambda_{i,1} \end{pmatrix} \quad e_{i,2} = \begin{pmatrix} 1 \\ \lambda_{i,2} \end{pmatrix}$$

which vary as c varies either positive or negative where there is a heteroclinic connection between u_1 and u_3. eigenvectors vary w/c

There exist c^* so that there is a heteroclinic connection between U_1 and U_3

Looking at T1 we need to find out the following: does this wave is going and which direction $z = x - ct$: if z is positive the wave moves to the right, and if z is negative the wave moves to the left.

Let's find out what the sign for C We have the following ODE: and break start at $u = u_1$ and go to u_3

$$U'' + CU' + f(u) = 0$$

Let's multiply both sides by U' and integrate

$$\int_{-\infty}^{\infty} (U'(U'' + CU' + f(u)) = 0$$

move c^* break saddle