

RECURSIVE TRACING: NICOLE DELOS REYES

1. Given the method defined below what does the following print:
mystery(4,3)?

```
public static int mystery(int n, int a){
    if (n == 1)
        return a;

    return a * mystery(n-1,a);
}
```

Alternative way:

$$\begin{aligned}
 \text{mystery}(4,3) &= 3 \times \text{mystery}(3,3) \\
 &= 3 \times 3 \times \text{mystery}(2,3) \\
 &= 3 \times 3 \times 3 \times \text{mystery}(1,3) \\
 &= 3 \times 3 \times 3 \times 1 \\
 &= 81
 \end{aligned}$$

$$81 = 3 \times \text{mystery}(3,3)$$

$$\downarrow$$

$$27 = 3 \times \text{mystery}(2,3)$$

$$\downarrow$$

$$9 = 3 \times \text{mystery}(1,3)$$

$$\downarrow$$

$$3$$

2. What happens when we call bunnyEars(5)?

```
public static int bunnyEars(int bunnies){
    if (bunnies == 0)
        return 0;
    else if (bunnies == 1)
        return 2;
    else
        return 2 + bunnyEars(bunnies - 1);
}
```

Alternative way:

bunnyEars(5)

$$= 2 + \text{bE}(4)$$

$$= 2 + 2 + \text{bE}(3)$$

$$= 2 + 2 + 2 + \text{bE}(2)$$

$$= 2 + 2 + 2 + 2 + \text{bE}(1)$$

$$= 2 + 2 + 2 + 2 + 2$$

$$= 10$$

$$\text{bunnyEars}(5) = 10$$

$$\downarrow 2 + \text{bunnyEars}(4) = 10$$

$$\downarrow 2 + \text{bunnyEars}(3) = 8$$

$$\downarrow 2 + \text{bunnyEars}(2) = 6$$

$$\downarrow 2 + \text{bunnyEars}(1) = 4$$

$$\downarrow 2$$

3. Given the method defined below what does the following print:
mystery(1234)?

```
public static void mystery (int x) {
    System.out.print(x % 10);

    if ((x / 10) != 0) {
        mystery(x / 10);
    }
    System.out.print(x % 10);
}
```

completes the rest of each method after $(x/10) \neq 0$

Console : " 4 3 2 1 1 2 3 4 "

$$\text{mystery}(1234) = "4"$$

$$\downarrow \text{mystery}(123) = "3"$$

$$\downarrow \text{mystery}(12) = "2"$$

$$\downarrow \text{mystery}(1) = "1"$$

4. Given the method defined below what does the following return:
mystery("xyzxyxy")? length = 7

```
public static int mystery(String str)
{
    if (str.length() == 1)
        return 0;
    else
    {
        if (str.substring(0,1).equals("y"))
            return 1 + mystery(str.substring(1));
        else
            return mystery(str.substring(1));
    }
}
```

$$\text{mystery}("xyzxyxy") = 2$$

$$\downarrow \text{mystery}("yzxyxy") = 2$$

$$\downarrow 1 + \text{mystery}("zxyxy") = 2$$

$$\downarrow \text{mystery}("xyxy") = 1$$

$$\downarrow \text{mystery}("yxy") = 1$$

$$\downarrow 1 + \text{mystery}("xy") = 1$$

$$\downarrow \text{mystery}("y") = 0$$

5. Given the method defined below what does the following return: a) `starString(4)`? b) `starString(5)`?

```
public String starString(int n)
{
    if (n == 0) {
        return "";
    } else {
        return starString(n - 1) + starString(n - 1);
    }
}
```

MS. XIE'S WAY

a) $\text{starString}(5) = "*" \times 32$

\downarrow
 $\text{starString}(4) + \text{starString}(4) \rightarrow "*" \times 16$

\downarrow
 $\text{starString}(3) + \text{starString}(3) \rightarrow "*" \times 8$

\downarrow
 $\text{starString}(2) + \text{starString}(2) \rightarrow "*" \times 4$

\downarrow
 $\text{starString}(1) + \text{starString}(1) \rightarrow "*" \times 2$

\downarrow
 $\text{starString}(0) + \text{starString}(0) \rightarrow ""$

b) $\text{starString}(5) = "*" \times 64$

\downarrow
 $\text{starString}(4) + \text{starString}(4) = "*" \times 32$

\downarrow
 $\text{starString}(3) + \text{starString}(3) = "*" \times 16$

\downarrow
 $\text{starString}(2) + \text{starString}(2) = "*" \times 8$

\downarrow
 $\text{starString}(1) + \text{starString}(1) = "*" \times 4$

\downarrow
 $\text{starString}(0) + \text{starString}(0) = ""$

6. Given the method defined below what does the following return `multiplyEvens(3)`?

```
public static int multiplyEvens(int n)
{
    if (n == 1) {
        return 2;
    } else {
        return 2 * n * multiplyEvens(n - 1);
    }
}
```

$\text{multiplyEvens}(3) = 48$

\downarrow
 $2 \times 3 \times \text{multiplyEvens}(2) = 24$

\downarrow
 $2 \times 2 \times \text{multiplyEvens}(1) = 8$

\downarrow
 2

7. Given the following method declaration, what value is returned as the result of the call `product(5)`?

```
public static int product(int n)
{
    if (n <= 1)
        return 1;
    else
        return n * product(n - 2);
}
```

$\text{product}(5) = 15$

\downarrow
 $5 \times \text{product}(3) = 15$

\downarrow
 $3 \times \text{product}(1) = 3$

\downarrow
 1

8. Given the following method declaration, what value is returned as the result of the call `f(5)`?

```
public static int f(int n)
{
    if (n == 0)
        return 0;
    else if (n == 1)
        return 1;
    else return f(n-1) + f(n-2);
}
```

$f(5) = 5$

\downarrow
 $f(4) + f(3) = 3 + 2 = 5$

\downarrow
 $f(3) + f(2) = 2 + 1 = 3$

\downarrow
 $f(2) + f(1) = 1 + 1 = 2$

\downarrow
 $f(1) + f(0) = 1$

\downarrow
 1

* DOUBLE CHECK!!!

9. Given the following method declaration, this method will return true if and only if:

```
public static boolean check(String s)
{
    return s.length() >= 2 &&
        (s.charAt(0) == s.charAt(1) ||
        check(s.substring(1)));
}
```

A. The string s contains two or more of the same characters.

B. The string s starts with two or more of the same characters.

C. The string s contains two or more of the same character that are next to each other.

D. The string s ends with two or more of the same characters

10. Given the following method declaration, what will redo(82, 3) return?

```
public static int redo(int i, int j)
{
    if (i==0)
        return 0;
    else
        return redo(i/j, j)+1;
}
```

$$\text{redo}(82, 3) = 5$$

$$\hookrightarrow \text{redo}(27, 3) + 1 = 5$$

$$\hookrightarrow \text{redo}(9, 3) + 1 = 4$$

$$\hookrightarrow \text{redo}(3, 3) + 1 = 3$$

$$\hookrightarrow \text{redo}(1, 3) + 1 = 2$$

$$\hookrightarrow \text{redo}(0, 3) + 1 = 1$$

$$\hookrightarrow 0$$

11. Given the following method declaration, what will mystery ("Hello", 3) return?

```
public static String mystery(String s, int n)
{
    if (n == 0) {
        return "";
    } else {
        return s + " " + mystery(s, n - 1);
    }
}
```

$$\text{mystery}(\text{"Hello"}, 3) = \text{"Hello"} + \text{" " } + \text{"Hello"} + \text{" " } + \text{"Hello"} + \text{" " } + \text{" "}$$

$$\hookrightarrow \text{"Hello"} + \text{" " } + \text{mystery}(\text{"Hello"}, 2) = \text{"Hello"} + \text{" " } + \text{"Hello"} + \text{" " } + \text{"Hello"} + \text{" " } + \text{" "}$$

$$\hookrightarrow \text{"Hello"} + \text{" " } + \text{mystery}(\text{"Hello"}, 1) = \text{"Hello"} + \text{" " } + \text{"Hello"} + \text{" " } + \text{" "}$$

$$\hookrightarrow \text{"Hello"} + \text{" " } + \text{mystery}(\text{"Hello"}, 0) = \text{"Hello"} + \text{" " } + \text{" " } + \text{" "}$$

$$\hookrightarrow \text{" "}$$

12. Given the following method declaration, what will mystery(6) return?

```
public static String mystery(int n)
{
    if (n == 0) {
        return "0...";
    } else {
        return mystery(n - 1) + n + "...";
    }
}
```

$$\text{mystery}(6) = \text{"0...1...2...3...4...5...6..."}'$$

$$\hookrightarrow \text{mystery}(5) + 6 + \text{"..."} = \text{"0...1...2...3...4...5...6..."}'$$

$$\hookrightarrow \text{mystery}(4) + 5 + \text{"..."} = \text{"0...1...2...3...4...5..."}'$$

$$\hookrightarrow \text{mystery}(3) + 4 + \text{"..."} = \text{"0...1...2...3...4..."}'$$

$$\hookrightarrow \text{mystery}(2) + 3 + \text{"..."} = \text{"0...1...2...3..."}'$$

$$\hookrightarrow \text{mystery}(1) + 2 + \text{"..."} = \text{"0...1...2..."}'$$

$$\hookrightarrow \text{mystery}(0) + 1 + \text{"..."} = \text{"0...1..."}'$$

$$\hookrightarrow \text{"0..."}'$$

13. Consider the following recursive function.

```
public static int mystery(int a, int b) {
    if (b == 0) return 0;
    if (b % 2 == 0) return mystery(a*a, b/2);
    return mystery(a*a, b/2) + a;
}
```

What are the values of mystery(2, 25) and mystery(3, 11)? Given positive integers a and b, describe what value mystery(a, b) computes. Answer the same question, but replace + with * and replace return 0 with return 1.

```
public static int mystery(int a, int b) {
    if (b == 0) return 1;
    if (b % 2 == 0) return mystery(a*a, b/2);
    return mystery(a*a, b/2) * a;
}
```

PART B:

PART 1)

$$\text{mystery}(2, 25) = 50$$

$$\hookrightarrow \text{mystery}(4, 12) + 2 = 50$$

$$\hookrightarrow \text{mystery}(8, 6) = 48$$

$$\hookrightarrow \text{mystery}(16, 3) = 48$$

$$\hookrightarrow \text{mystery}(32, 1) + 16 = 48$$

$$\hookrightarrow \text{mystery}(64, 0) + 32 = 32$$

$$\hookrightarrow 0$$

$$\text{mystery}(3, 11) = 33$$

$$\hookrightarrow \text{mystery}(6, 5) + 3 = 33$$

$$\hookrightarrow \text{mystery}(12, 2) + 6 = 30$$

$$\hookrightarrow \text{mystery}(24, 1) = 24$$

$$\hookrightarrow \text{mystery}(48, 0) + 24 = 24$$

$$\hookrightarrow 0$$

$$\text{mystery}(2, 25) = 33564432$$

$$\hookrightarrow \text{mystery}(4, 12) \times 2 = 33564432$$

$$\hookrightarrow \text{mystery}(16, 6) = 16777216$$

$$\hookrightarrow \text{mystery}(256, 3) = 16777216$$

$$\hookrightarrow \text{mystery}(65536, 1) \times 256 = 16777216$$

$$\hookrightarrow \text{mystery}(4294967296, 0) \times 65536 = 65536$$

$$\hookrightarrow 1$$

$$\text{mystery}(3, 11) = 177147$$

$$\hookrightarrow \text{mystery}(9, 5) \times 3 = 177147$$

$$\hookrightarrow \text{mystery}(81, 2) \times 9 = 59049$$

$$\hookrightarrow \text{mystery}(6561, 1) = 6561$$

$$\hookrightarrow \text{mystery}(43046721, 0) \times 6561 = 6561$$

$$\hookrightarrow 1$$