Software Verification Homework 4

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1 BDD constructions

a. Denote the set of vertices: $V = \{\overline{x}_i \mid i \in [n]\}$
Let $E(\overline{v}) := E_0(\overline{v}) \wedge E_1(\overline{v})$.

$$A'(\overline{v}) := A(\overline{v}) \wedge \left(\bigwedge_{i=0}^{n} (E(\overline{v}, \overline{x}_i) \Rightarrow B(\overline{x}_i)) \right)$$

The idea is that $A(\overline{v})$ means the 'accepted' node has to be from A, and rest of the expression means that all of it's neighbors have to be in B. It is equivalent to satisfying the following formula:

$$(\overline{v} \in V) \wedge (\forall \overline{x} \in V, E(\overline{x}, \overline{v}) \to B(\overline{x}))$$

b.

$$V_{1,2}(\overline{v}, \overline{v}') := \left(\bigvee_{i=0}^{n} E_1(\overline{v}, \overline{x}_i) \wedge E_0(\overline{x}_i, \overline{v}') \right) \vee \left(\bigvee_{i=0}^{n} E_0(\overline{v}, \overline{x}_i) \wedge E_1(\overline{x}_i, \overline{v}') \right)$$

For the vertices $\overline{v}, \overline{v}'$ to have a path of length 2 and weight 1 between them, there must either be a path of length 2 with weight 1 were the edge connected to $\overline{v}$ is 1 and the other edge is 0, or the other way around.
The primary operator of the expression above describes this fact; to be more specific - the left side of the expression describes the case where there is a path of length 2 were the node connected to $\overline{v}$ has weight 1, and so on.

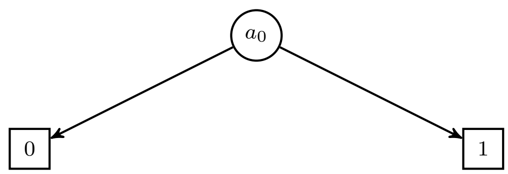
c.

2 BDD operations

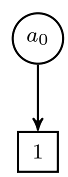
a. **1.** True. $D \subseteq D'$.
Proof:
Let $x \in D$. Denote the path of x on B with $a_1, a_2, ..., a_n$.

Since $x \in D$ then $B(x) = 1$, meaning the path must end with 1. If $\exists i \in [n] : a_i = u$. then on the evaluation of x on B', the path will be $a_1, a_2, ..., a_{i-1}, u, 1$. Thus $B'(x) = 1$ and so $x \in D'$ in this case.
Otherwise, $x \notin D$. Thus the path $a_1, ..., a_n$ is unchanged in B' w.r. to B. Hence $B'(x)$ evaluates on the exact same path - which we know ends with 1. Hence $B'(x) = 1$ too, so $x \in D'$.
Meaning in all cases $x \in D'$.
So $D \subseteq D'$.

2. False. Counter example:
Consider kripke structure $(S = \{0, 1\}, R = \{(s, s) \mid s \in S\}, L = \{(s, \emptyset) \mid s \in S\})$. Consider $D = \{1\}$.
Let B be the BDD representing D:



Now consider u as a_0. This would mean B' is: aa asdas



(Technically the a_0 would be reduced...).
So now $B'(0) = 1$ also, hence $D' = \{0, 1\} \not\subseteq D$.

3 D&D