一、选择题

1.如果$F(x)$是$f(x)$ 的一个原函数，C 为常数，那么(　　)也是$f(x)$的原函数

$A.F(Cx), B.F(\frac{x}{C}), C.CF(x), D.C+F(x)$

2. $\frac{1}{\sqrt{x^2-1}}$的原函数是(　　)

$A.\arcsin x, B.-\arcsin x, C.\ln\left|x+\sqrt{x^2-1}\right|, D.\ln\left|x-\sqrt{x^2-1}\right|$

3.设$\int F'(x)dx=\int G'(x)dx$,则下列结论中错误的是(　　)

$A.F(x)=G(x), B.F(x)=G(x)+C, C.F'(x)=G'(x), D.d\int F'(x)dx=d\int G'(x)dx$

4.若$F'(x)=\frac{1}{\sqrt{1-x^2}}, F(1)=\frac{3}{2}\pi$, 则$F(x)$ 为(　　)

$A.\arcsin x, B.\arcsin x+C, C.\arccos x+\pi, D.\arcsin x+\pi$

5.如果等式$\int f(x)e^{-\frac{1}{x}}dx=-e^{-\frac{1}{x}}+C$,则函数$f(x)=$(　　)

$A.-\frac{1}{x}, B.-\frac{1}{x^2}, C.\frac{1}{x}, D.\frac{1}{x^2}$

6.设$f'(\cos^2 x)=\sin^2 x$,且$f(0)=0$, 则$f(x)=$(　　)

$A.\cos x+\frac{1}{2}\cos^2 x, B.\cos^2 x-\frac{1}{2}\cos^4 x, C.x+\frac{1}{2}x^2, D.x-\frac{1}{2}x^2$

7.下列函数中，不是$e^{2x}-e^{-2x}$的原函数是(　　)

$A.\frac{1}{2}(e^{2x}+e^{-2x}), B.\frac{1}{2}(e^x+e^{-x})^2, C.\frac{1}{2}(e^x-e^{-x})^2, D.\frac{1}{2}(e^{2x}-e^{-2x})$

8. 设$f(x)$ 有原函数$x\ln x$, 则$\int xf(x)dx=$(　　)

$A.x^2(\frac{1}{2}+\frac{1}{4}\ln x)+C, B.x^2(\frac{1}{4}+\frac{1}{2}\ln x)+C$

$B.x^2(\frac{1}{4}-\frac{1}{2}\ln x)+C, D.x^2(\frac{1}{2}-\frac{1}{4}\ln x)+C$

9. 设$F(x)$是$f(x)$的一个原函数，则$\int e^{-x}f(e^{-x})dx=$(　　)

$A.F(e^{-x})+C, B.-F(e^{-x})+C, C.F(e^{x})+C, D.-F(e^{x})+C$

10. 如果$f(x)=e^{-x}$,则$\int \frac{f'(\ln x)}{x}dx=$(　　)

$A.-\frac{1}{x}+C, B.\frac{1}{x}+C, C.-\ln x+C, D.\ln x+C$

二、解答题

1.求下列不定积分

(1) $\int \frac{dx}{\sqrt{x+1}+\sqrt{x-1}}$

(2) $\int \frac{e^{3x}+e^{x}}{e^{4x}-e^{2x}+1}dx$

(3) $\int \sqrt{\frac{e^{x}-1}{e^{x}+1}}dx$

(4) $\int \frac{\arctan \frac{1}{x}}{1+x^2} dx$

(5) $\int \frac{1+\cos x}{1+\sin^2 x} dx$

(6) $\int \frac{1}{1-x^2} \ln \frac{1-x}{1+x} dx$

(7) $\int \left[\frac{f(x)}{f'(x)} - \frac{f^2(x) f''(x)}{f'^3(x)} \right] dx$

(8) $\int \sqrt{\frac{1-x}{1+x}} \frac{dx}{x}$

(9) $\int \frac{\arcsin x}{\sqrt{(1-x^2)^3}} dx$

(10) $\int \frac{xdx}{(x+2)\sqrt{x^2+4x-12}}$

(11) $\int \frac{dx}{x^2\sqrt{x^2-1}}$

(12) $\int \frac{dx}{x^4(1+x^2)}$

(13) $\int \frac{x^3}{(1+x^8)^2}dx$

(14) $\int \frac{x^5}{\sqrt[4]{x^3+1}}dx$

(15) $\int \frac{\sqrt[3]{x}}{x(\sqrt{x}+\sqrt[3]{x})}dx$

(16) $\int x^2 \cos^2 x dx$

(17) $\int (x^3 + 2x + 5)e^{-x} dx$

(18) $\int \frac{xe^x}{(e^x + 1)^2} dx$

(19) $\int \frac{\tan \frac{x}{2}}{1 + \sin x + \cos x} dx$

(20) $\int (|x| + 2) dx$

2.设$f'(\cos x) = \sin x (0 < x < \pi)$, 试证: $f(x) = \frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2} \arcsin x C(|x| < 1)$.

三、选做题

1.求不定积分$\int \frac{1}{(x+1)^3\sqrt{x^2+2x}}dx$.

2.求不定积分$\int \frac{1}{1+x^4}dx$.

3.求不定积分$\int \frac{xe^x}{\sqrt{e^x-2}}dx$.

4.设$f(x^2-1)=\ln\frac{x^2}{x^2-2}$,且$f[\varphi(x)]=\ln x$,求$\int \varphi(x)dx$.

5.设$I_n=\int \frac{1}{\sin^n x}dx(n\geqslant 2,\text{ 整数})$,试证:

$I_n = \frac{n-2}{n-1} I_{n-2} - \frac{\cos x}{(n-1)\sin^{n-1} x}$.

书上习题选

P208 (36) $\int \frac{x^2 dx}{\sqrt{a^2 - x^2}} (a > 0)$

P208 (38) $\int \frac{dx}{\sqrt{(x^2+1)^3}}$

P208 (40) $\int \frac{dx}{1 + \sqrt{2x}}$

P208 (42) $\int \frac{dx}{x + \sqrt{1 - x^2}}$

P208 (44) $\int \frac{x^3+1}{(x^2+1)^2}dx$

P213 (20) $\int \cos \ln x dx$

P213(22) $\int e^x \sin^2 x dx$

P213(24) $\int e^{\sqrt{3x+9}} dx$

P218 (14) $\int \frac{dx}{3+\sin^2 x}$

P218(18) $\int \frac{dx}{2\sin x - \cos x + 5}$

P218(22) $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$