APPENDIX A

THE KKT TRANSFORMATION WITH LINEARIZATION PROCESS

For $G_T \in \boldsymbol{\Omega_T}$, the Lagrangian function for the lower-level problem (25)-(30) is:

$$
\begin{aligned}
L = & \sum_{i\in \boldsymbol{V}} \Delta Pd_i^T - \sum_{l_{ij}\in \boldsymbol{L}_T} \mu_{ij}^T \left(p_{ij}^T - v_{ij}^T \cdot \frac{1}{x_{ij}} \cdot \sum_{i\in \boldsymbol{V}} A_{nl}\cdot \delta_n \right) \\
& - \sum_{i\in \boldsymbol{V}} \lambda_i \left(\sum_{k\in \boldsymbol{V}_G(i)} g_k^T - \sum_{l_{ij}\in \boldsymbol{L}_T} A_{ij}^T \cdot p_{ij}^T + \Delta Pd_i^T - Pd_i \right) \\
& - \sum_{l_{ij}\in \boldsymbol{L}_T} \underline{\omega}_{ij}^T \left(p_{ij}^T + \overline{p_{ij}} \right) - \sum_{l_{ij}\in \boldsymbol{L}_T} \overline{\omega}_{ij}^T \left(\overline{p_{ij}} - p_{ij}^T \right) \\
& - \sum_{k\in \boldsymbol{V_G}} \underline{\theta}_k^T \left(g_k^T - \underline{g_k} \right) - \sum_{k\in \boldsymbol{V_G}} \overline{\theta}_k^T \left(\overline{g_k} - g_k^T \right) \\
& - \sum_{i\in \boldsymbol{V}} \underline{\alpha}_i^T \left(\Delta Pd_i^T \right) - \sum_{i\in \boldsymbol{V}} \overline{\alpha}_i^T \left(Pd_i - \Delta Pd_i^T \right)
\end{aligned}
\tag{A1}
$$

where μ_{ij}^T, λ_i^T, $\overline{\omega}_{ij}^T$, $\underline{\omega}_{ij}^T$, $\overline{\theta}_k^T$, $\underline{\theta}_k^T$, $\overline{\alpha}_k^T$, $\underline{\alpha}_k^T$ are the Lagrangian multipliers associated with the DC power flow constraints (26) - (30), respectively. The optimality conditions for the original problem KKT of these constraints are:

$$\frac{\partial L}{\partial \delta_i} = \sum_{l_{ij}\in L_T} \frac{1}{x_{ij}} \cdot A_{ij}^T \cdot \mu_{ij}^T \cdot v_{ij}^T = 0, \qquad i\in \boldsymbol{V} \tag{A2}$$

$$\frac{\partial L}{\partial g_k} = -\lambda_i^T \Big|_{k\in \boldsymbol{V}_{G(i)}} - \underline{\theta}_k + \overline{\theta}_k = 0, \qquad k\in \boldsymbol{V_G} \tag{A3}$$

$$\frac{\partial L}{\partial p_{ij}^T} = \sum_{y\in \boldsymbol{V}} A_{ij}^T \lambda_y^T - \mu_{ij}^T - \underline{\omega}_{ij}^T + \overline{\omega}_{ij}^T = 0, \quad l_{ij}\in \boldsymbol{L_T} \tag{A4}$$

$$\frac{\partial L}{\partial \Delta Pd_i^T} = 1 - \lambda_i^T - \underline{\alpha}_i^T + \overline{\alpha}_i^T = 0, \qquad i\in \boldsymbol{V} \tag{A5}$$

$$\underline{\omega}_{ij}^T > 0, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A6}$$

$$\overline{\omega}_{ij}^T > 0, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A7}$$

$$\underline{\theta}_k^T > 0, \qquad k\in \boldsymbol{V_G} \tag{A8}$$

$$\overline{\theta}_k^T > 0, \qquad k\in \boldsymbol{V_G} \tag{A9}$$

$$\underline{\alpha}_i^T \geq 0, \qquad i\in \boldsymbol{V} \tag{A10}$$

$$\overline{\alpha}_i^T \geq 0, \qquad i\in \boldsymbol{V} \tag{A11}$$

$$\underline{\omega}_{ij}^T \left(p_{ij}^T + \overline{p_{ij}} \right) = 0, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A12}$$

$$\overline{\omega}_{ij}^T \left(\overline{p_{ij}} - p_{ij}^T \right) = 0, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A13}$$

$$\underline{\theta}_k^T \left(g_k^T - \underline{g}_k \right) = 0, \qquad k\in \boldsymbol{V_G} \tag{A14}$$

$$\overline{\theta}_k^T \left(\overline{g}_k - g_k^T \right) = 0, \qquad k\in \boldsymbol{V_G} \tag{A15}$$

$$\underline{\alpha}_i^T \left(\Delta Pd_i^T \right) = 0, \qquad i\in \boldsymbol{V} \tag{A16}$$

$$\overline{\alpha}_i^T \left(Pd_i - \Delta Pd_i^T \right) = 0, \qquad i\in \boldsymbol{V} \tag{A17}$$

where (A2) - (A11) denotes the original lower-level problem dual constraints and (A12) - (A17) denotes the complementary slackness constraints.

It can be seen that the original lower-level problem line flow formulation (33), dual constraints (A2) and complementary slackness constraints (A12) - (A17) are all non-linear terms. The above constraints are linearized by the following method:

(1) For the linearization of formulation (33)

In function (33), there are two non-linear terms-v_{ij}^Tmultiplied by the phase angle δ_i^T at the beginning of the line and v_{ij}^T multiplied by the phase angle δ_j^T at the end of the line, so continuous variables s_i^T and s_j^T are introduced to represent $v_{ij}^T\delta_i^T$ and $v_{ij}^T\delta_j^T$ respectively. Further intermediate variables z_i^T and z_j^T are introduced to equate the non-linear term (33) with the following linear terms:

$$p_{ij}^T = \frac{1}{x_l} \cdot \left(z_i^T - z_j^T \right), \qquad l_{ij}\in \boldsymbol{L_T} \tag{A18}$$

$$z_i^T = \delta_i^T \text{-} \delta_i^T, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A19}$$

$$z_j^T = \delta_j^T \text{-} \delta_j^T, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A20}$$

$$\underline{\delta}\cdot v_{ij}^T \leq z_i^T \leq \overline{\delta}\cdot v_{ij}^T, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A21}$$

$$\underline{\delta}\cdot v_{ij}^T \leq z_j^T \leq \overline{\delta}\cdot v_{ij}^T, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A22}$$

$$\underline{\delta}\cdot \left(1 - v_{ij}^T\right) \leq z_i^T \leq \overline{\delta}\cdot \left(1 - v_{ij}^T\right), \quad l_{ij}\in \boldsymbol{L_T} \tag{A23}$$

$$\underline{\delta}\cdot \left(1 - v_{ij}^T\right) \leq z_j^T \leq \overline{\delta}\cdot \left(1 - v_{ij}^T\right), \quad l_{ij}\in \boldsymbol{L_T} \tag{A24}$$

The formulations (A18)-(A24) represents the linearized expression for the calculation of the line DC power flow, if line l_{ij} is destroyed (v_{ij}=0), then according to the formulations (A21), (A22) can be obtained: z_i^T=0, z_j^T=0, so the line power flow p_{ij}^T=0, s_i^T=δ_i^T, s_j^T=δ_j^T; if l_{ij} is not destroyed (v_i^T=1), then according to the functions (A23), (A24) can be obtained: s_i^T=0, s_j^T=0, so z_i^T=δ_i^T, z_j^T=δ_j^T, the line plow flow p_{ij}^T is determined by the phase angle difference between the two ends of the line.

(2) Linearization for the dual constraint (A2)

Similarly, by introducing the continuous variables t_l and h_l, the nonlinear dual constraint (A2) is equivalently represented by the following set of linearization constraints:

$$\sum_{l_{ij}\in L_T} \frac{1}{x_{ij}} \cdot A_{ij}^T \cdot t_{ij}^T = 0, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A25}$$

$$t_{ij}^T = u_{ij}^T - h_{ij}^T, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A26}$$

$$\underline{u_{ij}}\cdot v_{ij}^T \leq t_{ij}^T \leq \overline{u_{ij}}\cdot v_{ij}^T, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A27}$$

$$\underline{u_{ij}}\cdot \left(1 - v_{ij}^T\right) \leq z_i^T \leq \overline{u_{ij}}\cdot \left(1 - v_{ij}^T\right), \quad l_{ij}\in \boldsymbol{L_T} \tag{A28}$$

(3) Linearization for the complementary slackness constraints (A12) - (A17)

The nonlinear complementary slackness constraints (A12) - (A17) are equivalently represented by the following set of linearization constraints by introducing the 0-1 variables $\omega_{ij}^{\underline{\omega}T}$, $\omega_{ij}^{\overline{\omega}T}$, $\omega_k^{\underline{\theta}T}$, $\omega_k^{\overline{\theta}T}$, $\omega_i^{\underline{\alpha}T}$, $\omega_i^{\overline{\alpha}T}$:

$$\underline{\omega}_{ij}^T \leq M\cdot \omega_{ij}^{\underline{\omega}T}, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A29}$$

$$p_{ij}^T + \overline{p}_{ij} \leq M\cdot \left(1 - \omega_{ij}^{\underline{\omega}T}\right), \qquad l_{ij}\in \boldsymbol{L_T} \tag{A30}$$

$$\overline{\omega}_{ij}^T \leq M\cdot \omega_{ij}^{\overline{\omega}T}, \qquad l_{ij}\in \boldsymbol{L_T} \tag{A31}$$

$$\underline{p}_{ij} - p_{ij}^{T} \le M \cdot \left(1-\omega_{ij}^{\bar{\omega}T}\right), \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(A32)}$$

$$\underline{\theta}_{k}^{T} \le M \cdot \omega_{k}^{\underline{\theta}T}, \qquad k \in \boldsymbol{V_G} \quad \text{(A33)}$$

$$g_{k}^{T} - \underline{g}_{k} \le M \cdot \left(1-\omega_{k}^{\underline{\theta}T}\right), \qquad k \in \boldsymbol{V_G} \quad \text{(A34)}$$

$$\overline{\theta}_{k}^{T} \le M \cdot \omega_{k}^{\bar{\theta}T}, \qquad k \in \boldsymbol{V_G} \quad \text{(A35)}$$

$$\overline{g}_{k} - g_{k}^{T} \le M \cdot \left(1-\omega_{k}^{\bar{\theta}T}\right), \qquad k \in \boldsymbol{V_G} \quad \text{(C36)}$$

$$\underline{\alpha}_{i}^{T} \le M \cdot \omega_{k}^{\underline{\alpha}T}, \qquad i \in \boldsymbol{V} \quad \text{(A37)}$$

$$Pd_{i} \le M \cdot \left(1-\omega_{i}^{\underline{\alpha}T}\right), \qquad i \in \boldsymbol{V} \quad \text{(A38)}$$

$$\overline{\alpha}_{i}^{T} \le M \cdot \omega_{i}^{\bar{\alpha}T}, \qquad i \in \boldsymbol{V} \quad \text{(A39)}$$

$$Pd_{i} - \Delta Pd_{i}^{T} \le M \cdot \left(1-\omega_{i}^{\bar{\alpha}T}\right), \qquad i \in \boldsymbol{V} \quad \text{(A40)}$$

$$\omega_{ij}^{\underline{\omega}T} + \omega_{ij}^{\bar{\omega}T} < 1, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(A41)}$$

$$\omega_{k}^{\underline{\theta}T} + \omega_{k}^{\bar{\theta}T} < 1, \qquad k \in \boldsymbol{V_G} \quad \text{(A42)}$$

$$\omega_{i}^{\underline{\alpha}T} + \omega_{i}^{\bar{\alpha}T} < 1, \qquad i \in \boldsymbol{V} \quad \text{(A43)}$$

where (A29) - (A30), (A31) - (A32), (A33) - (A34), (A35) - (A36), (A37) - (A38) and (A39) - (A40) are linearised equivalent representations of the constraints (A12), (A13), (A14), (A15), (A16) and (A17) respectively. The intermediate variables introduced satisfies formulations (A41) - (A43).

In summary, the original lower-level problem after transformation by KKT is expressed as formulations (A18)-(A24), formulations (33)- (37), (A25) - (A28), (A3) - (A11) and (A29) - (A43). Therefore, the transformed vulnerability identification problem is as follows:

$$\max_{a} \sum_{T \in \boldsymbol{\Omega_T}} \pi_T \left(\sum_{i \in V} \Delta Pd_i^T \right) \quad \text{(A44)}$$

s.t. (26) - (29), (A18) - (A43).

APPENDIX B

PARAMETERS OF UTN AND ROAD NETWORK

Table IV contains information of the road lengths and cable signs, Table V contains the node parameters of the case study, the number of cables on the roads, and E_{ij} for each road in the case study are shown in Figure B1.

TABLE IV

ROAD NETWORK CONNECTION RELATION AND PARAMETER

l_{ij}	d_{ij}(km)	mra_{ij}	mrb_{ij}	mrc_{ij}	E_{ij}
1-2	1.8	0	0	0	0
1-3	1.6	0	0	0	0
2-51	1.7	0	0	0	0
2-52	1.7	0	0	0	0
3-4	1.3	0	0	0	0
3-12	1.9	0	0	0	0
4-5	1.7	0	0	0	0
4-12	1.7	0	0	1	0.43
5-6	1.8	0	0	0	0
6-9	1.5	3	0	0	0.93
6-10	1.5	2	0	0	0.95
6-13	1.5	2	0	0	0.78
7-8	1.3	2	0	0	0.87
7-11	1.3	4	0	0	0.95
8-10	1.32	3	0	0	0.97
8-30	1.2	3	0	0	0.92
9-13	1.5	3	0	0	0.83
9-16	1.5	2	0	0	0.84
9-49	1.6	2	0	0	0.86
10-16	1.6	3	0	0	0.97
11-17	1.3	0	0	0	0
11-30	1.1	3	0	0	0.96
12-13	1.3	0	1	0	0.6
12-52	1.8	0	0	0	0
13-15	1.5	2	0	0	0.75
14-32	1	0	0	0	0
14-50	1	0	0	0	0
15-49	1.3	3	0	0	0.97
15-54	1.27	3	0	0	0.98
16-31	1.2	2	0	0	0.9
16-50	1.12	1	0	0	0.97
17-18	0.9	4	0	0	0.36
18-19	1	0	2	0	0
18-21	1.28	3	0	0	0.55
18-30	1.22	2	0	0	0
19-20	1.12	3	0	0	0
19-22	1.15	1	0	0	0.57
20-23	1.3	4	0	0	0
20-24	1.35	3	0	0	0
21-22	1.4	0	0	1	0.63
21-31	1.39	0	0	0	0.92
21-33	1.29	0	2	0	0.92
22-29	1.23	0	0	0	0
23-25	2.1	0	0	0	0
24-25	1.4	0	0	2	0
24-26	1.6	0	0	0	0
25-27	1.5	0	0	0	0
26-27	1.57	0	0	2	0.76
26-29	1.05	4	0	0	0.75
26-37	4	3	0	0	0
27-28	1.9	0	0	0	0
28-37	3	0	0	0	0
28-38	3.5	0	0	0	0
29-35	2	0	0	0	0.93
30-31	2.3	3	0	0	0.95
31-32	1.8	3	0	0	0.93
32-33	2	5	0	0	0.93
32-45	2.2	0	0	0	0
33-34	1.7	3	0	0	0.89
34-35	1.82	4	0	0	0.89

34-44	2	2	0	0	0.68
35-36	1.5	2	0	0	0.89
36-37	1.4	3	0	0	0.9
36-41	2.7	0	0	0	0
36-44	1.7	2	0	0	0.87
37-40	3.7	0	0	0	0
38-39	1.9	0	0	0	0
39-40	1.7	1	0	0	0.56
39-85	2.1	0	0	0	0
40-41	1.82	1	0	0	0.6
40-65	1.95	0	0	0	0
41-42	1.67	3	0	0	0.96
41-43	1.32	2	0	0	0.87
42-61	1.65	4	0	0	0.95
42-65	0.89	0	0	0	0
43-44	1.32	3	0	0	0.93
44-46	1.35	2	0	0	0.89
45-46	1.4	0	0	0	0
45-47	1.6	3	0	0	0.79
46-61	1.62	4	0	0	0.95
47-48	1.45	3	0	0	0.9
47-50	1.7	0	0	0	0
47-60	1.89	0	0	0	0
48-49	1.43	4	0	0	0.97
48-58	1.05	3	0	0	0.89
49-50	0.89	2	0	0	0.88
51-55	1.97	0	0	0	0
52-53	1.9	2	0	0	0.89
52-54	1.9	2	0	0	0.93
53-55	1.82	0	0	0	0
53-56	1.89	4	0	0	0.94
54-56	2.2	2	0	0	0.9
54-57	0.93	4	0	0	0.92
55-66	1.1	0	0	1	0.5
56-66	1.25	0	0	0	0
56-70	1.19	0	0	0	0
57-58	0.65	2	0	0	0.78
57-70	1.8	0	0	0	0
58-59	1.1	0	1	0	0.78
58-72	1.4	0	2	0	0.57
59-60	1.4	0	0	0	0
59-79	1.62	0	0	3	0.78
60-61	1.63	0	0	0	0
60-62	1.4	0	0	0	0
62-63	1.3	0	0	0	0
62-80	1.6	0	2	0	0.76
63-64	1	0	0	0	0
64-65	0.9	0	0	0	0
64-82	1.2	2	0	0	0.9
64-87	1.3	0	0	0	0
65-84	1.7	0	0	0	0
66-67	1.02	0	0	0	0
66-69	1.02	0	1	0	0.7
67-68	1.9	0	0	0	0
68-76	1.79	0	0	0	0
69-70	1.76	0	0	2	0.67
69-71	1.69	0	0	0	0
69-75	1.95	4	0	0	0.89
70-72	1.05	0	1	0	0.78
71-73	2.3	0	0	0	0
72-73	1.21	0	2	0	0.74
73-74	1.25	0	0	1	0.32
74-75	2.52	0	0	0	0
74-77	2.1	0	0	2	0.56
74-80	2.9	0	0	3	0.67
75-77	1.92	4	0	0	0.97
76-77	1.68	0	0	0	0
77-78	2.6	4	0	0	0.96
78-81	2	2	0	0	0.92
79-80	1.97	0	2	0	0.59
80-81	2.01	4	0	0	0.95
81-82	2	4	0	0	0.94
81-83	2.32	4	0	0	0.96
82-83	1.87	3	0	0	0.89
83-87	1.62	0	0	0	0
84-85	1.82	0	0	0	0
84-86	1.3	0	0	0	0
86-87	2.05	0	0	0	0

TABLE V
NODE PARAMETERS

Node Number	Node Types	Load/MW	Active Output/MW
1	pq	0	0
2	pq	0	0
3	pq	342	0
4	pq	385	0
5	pq	0	0
6	pq	0	0
7	pq	233.8	0
8	pq	268.5	0
9	pq	0	0
10	pq	0	0
11	pq	0	0
12	pq	287.5	0
13	pq	0	0
14	pq	0	0
15	pq	220	0
16	pq	329	0
17	pq	0	0

18	pq	258	0
19	pq	0	0
20	pq	228	0
21	pq	274	0
22	pq	385	0
23	pq	247.5	0
24	pq	368.6	0
25	pq	324	0
26	pq	339	0
27	pq	261.5	0
28	pq	306	0
29	pq	583.5	0
30	pq	0	600
31	vθ	0	0
32	pv	9.5	750
33	pv	0	850
34	pv	0	508
35	pv	0	650
36	pv	0	250
37	pv	0	540
38	pv	0	830
39	pv	0	868

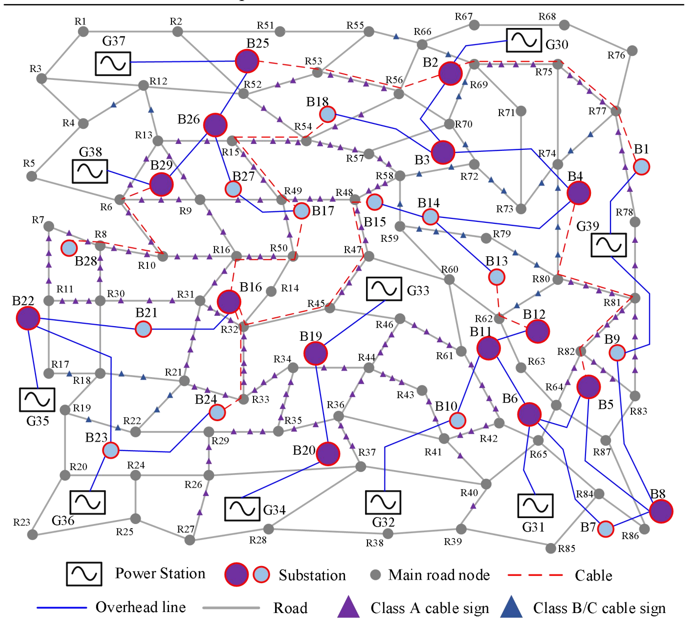

Fig. B1 The coupling network between UPN and the road network of the case.