APPENDIX A

FLOWCHART OF THE MONTE CARLO SIMULATION

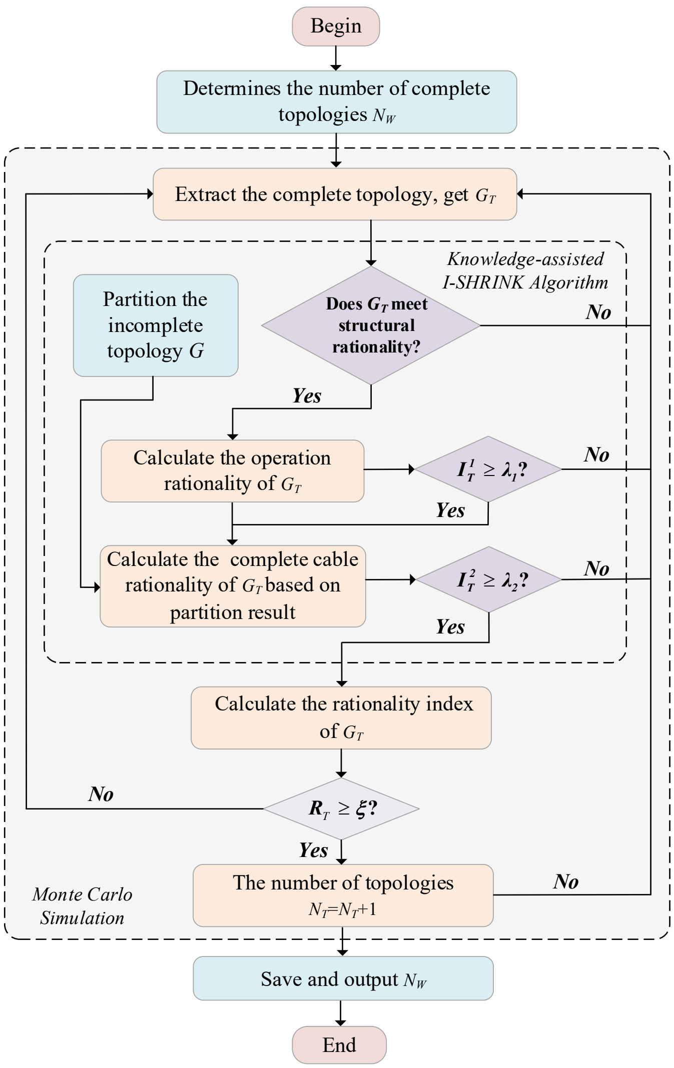

Fig. A1 The process of obtaining Ω_T simulated by Monte Carlo.

APPENDIX B

THE KKT TRANSFORMATION WITH LINEARIZATION PROCESS

For $G_T \in \boldsymbol{\Omega_T}$, the Lagrangian function for the lower-level problem (34)-(38) is:

$$\begin{aligned} L = & \sum_{i\in \boldsymbol{V}} \Delta Pd_i^T - \sum_{l_{ij}\in \boldsymbol{L}_T} \mu_{ij}^T \left(p_{ij}^T - v_{ij}^T \cdot \frac{1}{x_{ij}} \cdot \sum_{i\in \boldsymbol{V}} A_{nl} \cdot \delta_n \right) \\ & - \sum_{i\in \boldsymbol{V}} \lambda_i \left(\sum_{k\in \boldsymbol{V}_G(i)} g_k^T - \sum_{l_{ij}\in \boldsymbol{L}_T} A_{ij}^T \cdot p_{ij}^T + \Delta Pd_i^T - Pd_i \right) \\ & - \sum_{l_{ij}\in \boldsymbol{L}_T} \underline{\omega}_{ij}^T \left(p_{ij}^T + \overline{p_{ij}} \right) - \sum_{l_{ij}\in \boldsymbol{L}_T} \overline{\omega}_{ij}^T \left(\overline{p_{ij}} - p_{ij}^T \right) \\ & - \sum_{k\in \boldsymbol{V_G}} \underline{\theta}_k^T \left(g_k^T - \underline{g_k} \right) - \sum_{k\in \boldsymbol{V_G}} \overline{\theta}_k^T \left(\overline{g_k} - g_k^T \right) \\ & - \sum_{i\in \boldsymbol{V}} \underline{\alpha}_i^T \left(\Delta Pd_i^T \right) - \sum_{i\in \boldsymbol{V}} \overline{\alpha}_i^T \left(Pd_i - \Delta Pd_i^T \right) \end{aligned} \tag{B1}$$

where μ_{ij}^T, λ_i^T, $\overline{\omega}_{ij}^T$, $\underline{\omega}_{ij}^T$, $\overline{\theta}_k^T$, $\underline{\theta}_k^T$, $\overline{\alpha}_k^T$, $\underline{\alpha}_k^T$ are the Lagrangian multipliers associated with the DC power flow constraints (34)-(38), respectively. The optimality conditions for the original problem KKT of these constraints are:

$$\frac{\partial L}{\partial \delta_i} = \sum_{l_{ij}\in L_T} \frac{1}{x_{ij}} \cdot A_{ij}^T \cdot \mu_{ij}^T \cdot v_{ij}^T = 0, \quad i \in \boldsymbol{V} \tag{B2}$$

$$\frac{\partial L}{\partial g_k} = -\lambda_i^T \Big|_{k\in \boldsymbol{V}_{G(i)}} - \underline{\theta}_k + \overline{\theta}_k = 0, \quad k \in \boldsymbol{V_G} \tag{B3}$$

$$\frac{\partial L}{\partial p_{ij}^T} = \sum_{y\in \boldsymbol{V}} A_{ij}^T \lambda_y^T - \mu_{ij}^T - \underline{\omega}_{ij}^T + \overline{\omega}_{ij}^T = 0, \quad l_{ij} \in \boldsymbol{L_T} \tag{B4}$$

$$\frac{\partial L}{\partial \Delta Pd_i^T} = 1 - \lambda_i^T - \underline{\alpha}_i^T + \overline{\alpha}_i^T = 0, \quad i \in \boldsymbol{V} \tag{B5}$$

$$\underline{\omega}_{ij}^T > 0, \quad l_{ij} \in \boldsymbol{L_T} \tag{B6}$$

$$\overline{\omega}_{ij}^T > 0, \quad l_{ij} \in \boldsymbol{L_T} \tag{B7}$$

$$\underline{\theta}_k^T > 0, \quad k \in \boldsymbol{V_G} \tag{B8}$$

$$\overline{\theta}_k^T > 0, \quad k \in \boldsymbol{V_G} \tag{B9}$$

$$\underline{\alpha}_i^T \geq 0, \quad i \in \boldsymbol{V} \tag{B10}$$

$$\overline{\alpha}_i^T \geq 0, \quad i \in \boldsymbol{V} \tag{B11}$$

$$\underline{\omega}_{ij}^T \left(p_{ij}^T + \overline{p_{ij}} \right) = 0, \quad l_{ij} \in \boldsymbol{L_T} \tag{B12}$$

$$\overline{\omega}_{ij}^T \left(\overline{p_{ij}} - p_{ij}^T \right) = 0, \quad l_{ij} \in \boldsymbol{L_T} \tag{B13}$$

$$\underline{\theta}_k^T \left(g_k^T - \underline{g}_k \right) = 0, \quad k \in \boldsymbol{V_G} \tag{B14}$$

$$\overline{\theta}_k^T \left(\overline{g}_k - g_k^T \right) = 0, \quad k \in \boldsymbol{V_G} \tag{B15}$$

$$\underline{\alpha}_i^T \left(\Delta Pd_i^T \right) = 0, \quad i \in \boldsymbol{V} \tag{B16}$$

$$\overline{\alpha}_i^T \left(Pd_i - \Delta Pd_i^T \right) = 0, \quad i \in \boldsymbol{V} \tag{B17}$$

where (B2) - (B11) denotes the original lower-level problem dual constraints and (B12) - (B17) denotes the complementary slackness constraints.

It can be seen that the original function (34) in lower-level problem, dual constraints (B2) and complementary slackness constraints (B12) - (B17) are all non-linear terms. The above constraints are linearized by the following method:

(1) For the linearization of formulation (34)

In function (34), there are two non-linear terms-v_{ij}^T multiplied by the phase angle δ_i^T at the beginning of the line and v_{ij}^T multiplied by the phase angle δ_j^T at the end of the line, so continuous variables s_i^T and s_j^T are introduced to represent $v_{ij}^T \delta_i^T$ and $v_{ij}^T \delta_j^T$ respectively. Further intermediate variables z_i^T and z_j^T are introduced to equate the non-linear term (34) with the following linear terms:

$$p_{ij}^T = \frac{1}{x_l} \cdot \left(z_i^T - z_j^T \right), \quad l_{ij} \in \boldsymbol{L_T} \tag{B18}$$

$$z_i^T = \delta_i^T - \delta_i^T, \quad l_{ij} \in \boldsymbol{L_T} \tag{B19}$$

$$z_j^T = \delta_j^T - \delta_j^T, \quad l_{ij} \in \boldsymbol{L_T} \tag{B20}$$

$$\underline{\delta} \cdot v_{ij}^T \leq z_i^T \leq \overline{\delta} \cdot v_{ij}^T, \quad l_{ij} \in \boldsymbol{L_T} \tag{B21}$$

$$\underline{\delta} \cdot v_{ij}^T \le z_j^T \le \overline{\delta} \cdot v_{ij}^T, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B22)}$$

$$\underline{\delta} \cdot \left(1 - v_{ij}^T\right) \le z_i^T \le \overline{\delta} \cdot \left(1 - v_{ij}^T\right), \quad l_{ij} \in \boldsymbol{L_T} \quad \text{(B23)}$$

$$\underline{\delta} \cdot \left(1 - v_{ij}^T\right) \le z_j^T \le \overline{\delta} \cdot \left(1 - v_{ij}^T\right), \quad l_{ij} \in \boldsymbol{L_T} \quad \text{(B24)}$$

The formulations (B18)-(B24) represents the linearized expression for the calculation of the line DC power flow, if line l_{ij} is destroyed (v_{ij}=0), then according to the formulations (B21), (B22) can be obtained: z_i^T=0, z_j^T=0, so the line power flow p_{ij}^T=0, $s_i^T = \delta_i^T$, $s_j^T = \delta_j^T$; if l_{ij} is not destroyed (v_{ij}^T=1), then according to the functions (B23), (B24) can be obtained: s_i^T=0, s_j^T=0, so $z_i^T = \delta_i^T$, $z_j^T = \delta_j^T$, the line plow flow p_{ij}^T is determined by the phase angle difference between the two ends of the line.

(2) Linearization for the dual constraint (B2)

Similarly, by introducing the continuous variables t_l and h_l, the nonlinear dual constraint (B2) is equivalently represented by the following set of linearization constraints:

$$\sum_{l_{ij} \in L_T} \frac{1}{x_{ij}} \cdot A_{ij}^T \cdot t_{ij}^T = 0, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B25)}$$

$$t_{ij}^T = u_{ij}^T - h_{ij}^T, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B26)}$$

$$\underline{u_{ij}} \cdot v_{ij}^T \le t_{ij}^T \le \overline{u_{ij}} \cdot v_{ij}^T, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B27)}$$

$$\underline{u_{ij}} \cdot \left(1 - v_{ij}^T\right) \le z_i^T \le \overline{u_{ij}} \cdot \left(1 - v_{ij}^T\right), \quad l_{ij} \in \boldsymbol{L_T} \quad \text{(B28)}$$

(3) Linearization for the complementary slackness constraints (B12) - (B17)

The nonlinear complementary slackness constraints (B12) - (B17) are equivalently represented by the following set of linearization constraints by introducing the 0-1 variables $\omega_{ij}^{\underline{\omega}T}$, $\omega_{ij}^{\overline{\omega}T}$, $\omega_k^{\underline{\theta}T}$, $\omega_k^{\overline{\theta}T}$, $\omega_i^{\underline{\alpha}T}$, $\omega_i^{\overline{\alpha}T}$:

$$\underline{\omega}_{ij}^T \le M \cdot \omega_{ij}^{\underline{\omega}T}, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B29)}$$

$$p_{ij}^T + \overline{p}_{ij} \le M \cdot \left(1 - \omega_{ij}^{\underline{\omega}T}\right), \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B30)}$$

$$\overline{\omega}_{ij}^T \le M \cdot \omega_{ij}^{\overline{\omega}T}, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B31)}$$

$$\underline{p}_{ij} - p_{ij}^T \le M \cdot \left(1 - \omega_{ij}^{\overline{\omega}T}\right), \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B32)}$$

$$\overline{g}_k - g_k^T \le M \cdot \left(1 - \omega_k^{\overline{\theta}T}\right), \qquad k \in \boldsymbol{V_G} \quad \text{(B33)}$$

$$\underline{\theta}_k^T \le M \cdot \omega_k^{\underline{\theta}T}, \qquad k \in \boldsymbol{V_G} \quad \text{(B34)}$$

$$g_k^T - \underline{g}_k \le M \cdot \left(1 - \omega_k^{\underline{\theta}T}\right), \qquad k \in \boldsymbol{V_G} \quad \text{(B35)}$$

$$\overline{\theta}_k^T \le M \cdot \omega_k^{\overline{\theta}T}, \qquad k \in \boldsymbol{V_G} \quad \text{(B36)}$$

$$\underline{\alpha}_i^T \le M \cdot \omega_k^{\underline{\alpha}T}, \qquad i \in \boldsymbol{V} \quad \text{(B37)}$$

$$Pd_i \le M \cdot \left(1 - \omega_i^{\underline{\alpha}T}\right), \qquad i \in \boldsymbol{V} \quad \text{(B38)}$$

$$\overline{\alpha}_i^T \le M \cdot \omega_i^{\overline{\alpha}T}, \qquad i \in \boldsymbol{V} \quad \text{(B39)}$$

$$Pd_i - \Delta Pd_i^T \le M \cdot \left(1 - \omega_i^{\overline{\alpha}T}\right), \qquad i \in \boldsymbol{V} \quad \text{(B40)}$$

$$\omega_{ij}^{\underline{\omega}T} + \omega_{ij}^{\overline{\omega}T} < 1, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(B41)}$$

$$\omega_k^{\underline{\theta}T} + \omega_k^{\overline{\theta}T} < 1, \qquad k \in \boldsymbol{V_G} \quad \text{(B42)}$$

$$\omega_i^{\underline{\alpha}T} + \omega_i^{\overline{\alpha}T} < 1, \qquad i \in \boldsymbol{V} \quad \text{(B43)}$$

where (B29) - (B30), (B31) - (B32), (B33) - (B34), (B35) - (B36), (B37) - (B38) and (B39) - (B40) are linearised equivalent representations of the constraints (B12), (B13), (B14), (B15), (B16) and (B17) respectively. The intermediate variables introduced satisfies formulations (B41) - (B43).

In summary, the original lower-level problem after transformation by KKT is expressed as formulations (B18)-(B24), formulations (34) - (38), (B25) - (B28), (B3) - (B11) and (B29) - (B43). Therefore, the transformed vulnerability identification problem is as follows:

$$\max_{a} \sum_{T \in \boldsymbol{\Omega_T}} \pi_T \left(\sum_{i \in \boldsymbol{V}} \Delta Pd_i^T \right) \quad \text{(A44)}$$

s.t. (21) - (24), (29) - (32), (B18) - (B43).

APPENDIX C

DERIVATION PROCESS OF LINE LOSS

In the AC power flow model, the active power transmitted at the start and end of the transmission line can be expressed as:

$$\begin{cases} p_{ij}^a = U_i^2 g_{ij} - U_i U_j \left[g_{ij} \cdot \cos\Delta\theta_{ij} + b_{ij} \cdot \sin\Delta\theta_{ij} \right] \\ p_{ij}^a = U_j^2 g_{ij} - U_j U_i \left[g_{ij} \cdot \cos\Delta\theta_{ij} + b_{ij} \cdot \sin\Delta\theta_{ij} \right] \end{cases} \quad \text{(C1)}$$

where p_{ij}^a and p_{ji}^a are the active power at the beginning node i and the end node j of line l_{ij} respectively; U_i and U_j are the voltage amplitudes of node i and node j respectively; $\Delta\theta_{ij}$ represents the voltage phase angle difference between the two ends of l_{ij}; g_{ij} represents the electrical conductance. The model variables in the equation are all expressed in per unit values.

The expression for g_{ij} is as follows:

$$g_{ij} = \frac{r_{ij}}{r_{ij} + x_{ij}^2} \quad \text{(C2)}$$

where r_{ij} and x_{ij} respectively represent the resistance and reactance of line l_{ij}.

The active power loss L_{ij} on line l_{ij} is calculated as:

$$L_{ij} = p_{ij}^a + p_{ji}^a = \left(U_i^2 + U_j^2\right) g_{ij} - 2U_i U_j \cdot g_{ij} \cos\Delta\theta_{ij} \quad \text{(C3)}$$

Under the normal operation of the UPN, the node voltage amplitude is close to 1, the voltage phase angle difference between the two ends of the line is very small, and the active power loss L_{ij} of line l_{ij} can be approximated according to the Taylor expansion formula as follows:

$$L_{ij} \approx g_{ij} \cdot \left(\Delta\theta_{ij}\right)^2 \quad \text{(C4)}$$

In the DC power flow model, the DC power flow on line l_{ij} is given by:

$$p_{ij} = \frac{\theta_i - \theta_j}{x_{ij}} \quad \text{(C5)}$$

Therefore, combine B4 and B5, in the DC power flow, the active power loss L_{ij} can be expressed as:

$$L_{ij} = g_{ij}\left(\theta_i - \theta_j\right)^2 = g_{ij} \cdot p_{ij}^2 \cdot x_{ij}^2 = \frac{r_{ij} \cdot x_{ij}^2}{r_{ij} + x_{ij}^2} \cdot p_{ij}^2 \quad \text{(C6)}$$

Multiplying L_{ij} by the line capacity base value H_B gives the final line loss:

$$L_{ij}^{'} = \frac{r_{ij} \cdot x_{ij}^2}{r_{ij} + x_{ij}^2} \cdot p_{ij}^2 \cdot H_B \tag{C7}$$

APPENDIX D

PARAMETERS OF UTN AND ROAD NETWORK

Table IV contains information of the road lengths and cable signs, Table V contains the node parameters of the case study, the number of cables on the roads, and E_{ij} for each road in the case study are shown in Figure D1.

TABLE IV

ROAD NETWORK CONNECTION RELATION AND PARAMETER

l_{ij}	d_{ij}(km)	mra_{ij}	mrb_{ij}	mrc_{ij}	E_{ij}
1-2	1.8	0	0	0	0
1-3	1.6	0	0	0	0
2-51	1.7	0	0	0	0
2-52	1.7	0	0	0	0
3-4	1.3	0	0	0	0
3-12	1.9	0	0	0	0
4-5	1.7	0	0	0	0
4-12	1.7	0	0	1	0.43
5-6	1.8	0	0	0	0
6-9	1.5	3	0	0	0.93
6-10	1.5	2	0	0	0.95
6-13	1.5	2	0	0	0.78
7-8	1.3	2	0	0	0.87
7-11	1.3	4	0	0	0.95
8-10	1.32	3	0	0	0.97
8-30	1.2	3	0	0	0.92
9-13	1.5	3	0	0	0.83
9-16	1.5	2	0	0	0.84
9-49	1.6	2	0	0	0.86
10-16	1.6	3	0	0	0.97
11-17	1.3	0	0	0	0
11-30	1.1	3	0	0	0.96
12-13	1.3	0	1	0	0.6
12-52	1.8	0	0	0	0
13-15	1.5	2	0	0	0.75
14-32	1	0	0	0	0
14-50	1	0	0	0	0
15-49	1.3	3	0	0	0.97
15-54	1.27	3	0	0	0.98
16-31	1.2	2	0	0	0.9
16-50	1.12	1	0	0	0.97
17-18	0.9	4	0	0	0.36
18-19	1	0	2	0	0
18-21	1.28	3	0	0	0.55
18-30	1.22	2	0	0	0
19-20	1.12	3	0	0	0
19-22	1.15	1	0	0	0.57
20-23	1.3	4	0	0	0
20-24	1.35	3	0	0	0
21-22	1.4	0	0	1	0.63
21-31	1.39	0	0	0	0.92
21-33	1.29	0	2	0	0.92
22-29	1.23	0	0	0	0
23-25	2.1	0	0	0	0
24-25	1.4	0	0	2	0
24-26	1.6	0	0	0	0
25-27	1.5	0	0	0	0
26-27	1.57	0	0	2	0.76
26-29	1.05	4	0	0	0.75
26-37	4	3	0	0	0
27-28	1.9	0	0	0	0
28-37	3	0	0	0	0
28-38	3.5	0	0	0	0
29-35	2	0	0	0	0.93
30-31	2.3	3	0	0	0.95
31-32	1.8	3	0	0	0.93
32-33	2	5	0	0	0.93
32-45	2.2	0	0	0	0
33-34	1.7	3	0	0	0.89
34-35	1.82	4	0	0	0.89
34-44	2	2	0	0	0.68
35-36	1.5	2	0	0	0.89
36-37	1.4	3	0	0	0.9
36-41	2.7	0	0	0	0
36-44	1.7	2	0	0	0.87
37-40	3.7	0	0	0	0
38-39	1.9	0	0	0	0
39-40	1.7	1	0	0	0.56
39-85	2.1	0	0	0	0
40-41	1.82	1	0	0	0.6
40-65	1.95	0	0	0	0
41-42	1.67	3	0	0	0.96
41-43	1.32	2	0	0	0.87
42-61	1.65	4	0	0	0.95
42-65	0.89	0	0	0	0
43-44	1.32	3	0	0	0.93
44-46	1.35	2	0	0	0.89
45-46	1.4	0	0	0	0
45-47	1.6	3	0	0	0.79
46-61	1.62	4	0	0	0.95
47-48	1.45	3	0	0	0.9
47-50	1.7	0	0	0	0
47-60	1.89	0	0	0	0
48-49	1.43	4	0	0	0.97
48-58	1.05	3	0	0	0.89
49-50	0.89	2	0	0	0.88

51-55	1.97	0	0	0	0
52-53	1.9	2	0	0	0.89
52-54	1.9	2	0	0	0.93
53-55	1.82	0	0	0	0
53-56	1.89	4	0	0	0.94
54-56	2.2	2	0	0	0.9
54-57	0.93	4	0	0	0.92
55-66	1.1	0	0	1	0.5
56-66	1.25	0	0	0	0
56-70	1.19	0	0	0	0
57-58	0.65	2	0	0	0.78
57-70	1.8	0	0	0	0
58-59	1.1	0	1	0	0.78
58-72	1.4	0	2	0	0.57
59-60	1.4	0	0	0	0
59-79	1.62	0	0	3	0.78
60-61	1.63	0	0	0	0
60-62	1.4	0	0	0	0
62-63	1.3	0	0	0	0
62-80	1.6	0	2	0	0.76
63-64	1	0	0	0	0
64-65	0.9	0	0	0	0
64-82	1.2	2	0	0	0.9
64-87	1.3	0	0	0	0
65-84	1.7	0	0	0	0
66-67	1.02	0	0	0	0
66-69	1.02	0	1	0	0.7
67-68	1.9	0	0	0	0
68-76	1.79	0	0	0	0
69-70	1.76	0	0	2	0.67
69-71	1.69	0	0	0	0
69-75	1.95	4	0	0	0.89
70-72	1.05	0	1	0	0.78
71-73	2.3	0	0	0	0
72-73	1.21	0	2	0	0.74
73-74	1.25	0	0	1	0.32
74-75	2.52	0	0	0	0
74-77	2.1	0	0	2	0.56
74-80	2.9	0	0	3	0.67
75-77	1.92	4	0	0	0.97
76-77	1.68	0	0	0	0
77-78	2.6	4	0	0	0.96
78-81	2	2	0	0	0.92
79-80	1.97	0	2	0	0.59
80-81	2.01	4	0	0	0.95
81-82	2	4	0	0	0.94
81-83	2.32	4	0	0	0.96
82-83	1.87	3	0	0	0.89
83-87	1.62	0	0	0	0
84-85	1.82	0	0	0	0
84-86	1.3	0	0	0	0
86-87	2.05	0	0	0	0

TABLE V
NODE PARAMETERS

Node Number	Node Types	Load/MW	Active Output/MW
1	pq	0	0
2	pq	0	0
3	pq	342	0
4	pq	385	0
5	pq	0	0
6	pq	0	0
7	pq	233.8	0
8	pq	268.5	0
9	pq	0	0
10	pq	0	0
11	pq	0	0
12	pq	287.5	0
13	pq	0	0
14	pq	0	0
15	pq	220	0
16	pq	329	0
17	pq	0	0
18	pq	258	0
19	pq	0	0
20	pq	228	0
21	pq	274	0
22	pq	385	0
23	pq	247.5	0
24	pq	368.6	0
25	pq	324	0
26	pq	339	0
27	pq	261.5	0
28	pq	306	0
29	pq	583.5	0
30	pq	0	600
31	vθ	0	0
32	pv	9.5	750
33	pv	0	850
34	pv	0	508
35	pv	0	650
36	pv	0	250
37	pv	0	540
38	pv	0	830
39	pv	0	868

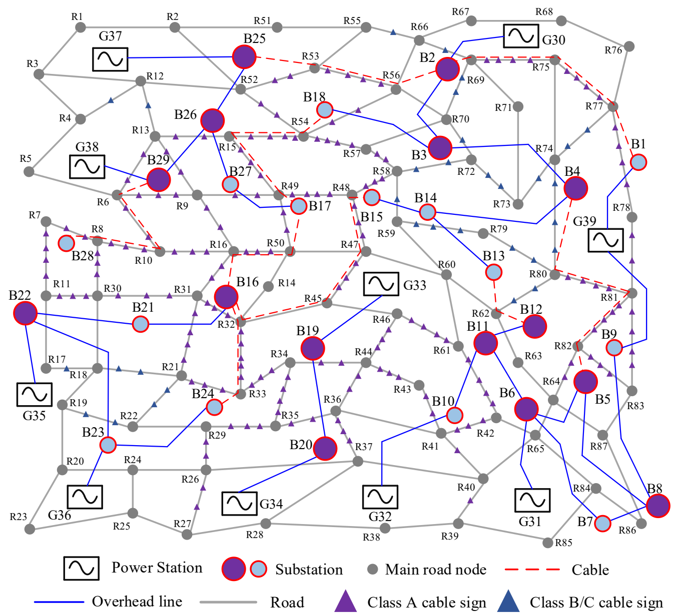

Fig. D1 The coupling network between UPN and the road network of the case.