Appendix A

The Calculation Method for a_l

The constant a_l in function (4) is determined by the number, distribution, and types of observable cable signs on the road l. Different cable signs vary in their importance to indicate the presence of cables. As shown in Table III, cable signs are classified into three classes (class A/B/C) based on the degree of importance from highest to lowest. And a_l can be calculated based on the number of three classes of cable sign near l:

$$a_l = E_l \cdot \left(\varepsilon_1 \cdot m_l^a + \varepsilon_2 \cdot m_l^b + \varepsilon_3 \cdot m_l^c \right) \tag{A1}$$

Where m_l^a, m_l^b and m_l^c represent the quantities of class A/B/C cable sign near l, and $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$ denote the respective weights assigned to class A/B/C cable signs. E_l represents the overall score of experts on the reasonableness of cable laying along l. The higher the value of E_l is, the lower the difficulty of laying cables along l is.

TABLE III

Cable Observable Marker Classification

Class of Cable Sign	Type of Visible Cable sign	Position
A	Open Compress Cable Section	Along the Road
B(Cable Pipe Network)	Markers for Maintenance, Vents, Entrances and Exits, Alarms/Reminders etc	Along the Road or Pavement
C（Utility Tunnel）	Markers for Maintenance, Vents, Entrances and Exits, Alarms/Reminders etc	Along the Road or Pavement

Use the example of Fig.A1 to better understand the presence of concealed cable between i and x.

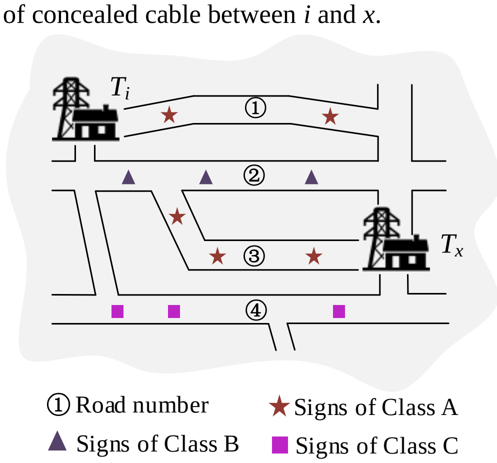

Fig. A1 Road network and cable signs between substation *i* and *x*.

Assuming that $\mathcal{E}_1$=0.6, $\mathcal{E}_2$=0.3 and $\mathcal{E}_3$=0.1. As shown in Fig.A1, there are four road paths between i and x, and the numbers of m_l^a, m_l^b and m_c can be seen in Fig.A1. Suppose E_l of these four roads are 0.75, 0.65, 0.77, 0.35, respectively. Then the a_l of the four roads between i and x is calculated based on function (A1) as 0.9, 0.59, 2.1, 0.11, respectively.

Appendix B

Derivation Process of Line Loss

In the AC power flow model, the active power transmitted at the start and end of the transmission line can be expressed as:

$$\begin{cases} p_{ij}^a = U_i^2 g_{ij} - U_i U_j \left[g_{ij} \cdot \cos \Delta\theta_{ij} + b_{ij} \cdot \sin \Delta\theta_{ij} \right] \\ p_{ij}^a = U_j^2 g_{ij} - U_j U_i \left[g_{ij} \cdot \cos \Delta\theta_{ij} + b_{ij} \cdot \sin \Delta\theta_{ij} \right] \end{cases} \tag{B1}$$

where p_{ij}^a and p_{ji}^a are the active power at the beginning node i and the end node j of line l_{ij} respectively; U_i and U_j are the voltage amplitudes of node i and node j respectively; $\Delta\theta_{ij}$ represents the voltage phase angle difference between the two ends of l_{ij}; g_{ij} represents the electrical conductance. The model variables in the equation are all expressed in per unit values.

The expression for g_{ij} is as follows:

$$g_{ij} = \frac{r_{ij}}{r_{ij} + x_{ij}^2} \tag{B2}$$

where r_{ij} and x_{ij} respectively represent the resistance and reactance of line l_{ij}.

The active power loss L_{ij} on line l_{ij} is calculated as:

$$L_{ij} = p_{ij}^a + p_{ji}^a = \left(U_i^2 + U_j^2 \right) g_{ij} - 2U_i U_j \cdot g_{ij} \cos \Delta\theta_{ij} \tag{B3}$$

Under the normal operation of the UPN, the node voltage amplitude is close to 1, the voltage phase angle difference between the two ends of the line is very small, and the active power loss L_{ij} of line l_{ij} can be approximated according to the Taylor expansion formula as follows:

$$L_{ij} \approx g_{ij} \cdot \left(\Delta\theta_{ij} \right)^2 \tag{B4}$$

In the DC power flow model, the DC power flow on line l_{ij} is given by:

$$p_{ij} = \frac{\theta_i - \theta_j}{x_{ij}} \tag{B5}$$

Therefore, combine B4 and B5, in the DC power flow, the active power loss L_{ij} can be expressed as:

$$L_{ij} = g_{ij} \left(\theta_i - \theta_j \right)^2 = g_{ij} \cdot p_{ij}^2 \cdot x_{ij}^2 = \frac{r_{ij} \cdot x_{ij}^2}{r_{ij} + x_{ij}^2} \cdot p_{ij}^2 \tag{B6}$$

Multiplying L_{ij} by the line capacity base value H_B gives the final line loss:

$$L_{ij}^{'} = \frac{r_{ij} \cdot x_{ij}^2}{r_{ij} + x_{ij}^2} \cdot p_{ij}^2 \cdot H_B \tag{B6}$$

Appendix C

The KKT Transformation with Linearization Process

The lagrangian function for the original lower-level problem (25)-(30) is:

$$L=\sum_{n\in \boldsymbol{V_T}}\Delta Pd_n-\sum_{l\in \boldsymbol{L}}\mu_l\left(p_l-v_l\cdot\frac{1}{x_l}\cdot\sum_{n\in \boldsymbol{V_T}}A_{nl}\cdot\delta_n\right)$$
$$-\sum_{n\in \boldsymbol{V_T}}\lambda_n\left(\sum_{g\in \boldsymbol{G_n}}p_g-\sum_{l\in \boldsymbol{L}}A_{nl}\cdot p_l+\Delta Pd_n-Pd_n\right)$$
$$-\sum_{l\in \boldsymbol{L}}\underline{\omega}_l\left(p_l+\overline{p}_l\right)-\sum_{l\in \boldsymbol{L}}\overline{\omega}_l\left(\overline{p}_l-p_l\right) \quad \text{(C1)}$$
$$-\sum_{j\in \boldsymbol{G}}\underline{\theta}_j\left(g_j-\underline{g}_j\right)-\sum_{j\in \boldsymbol{G}}\overline{\theta}_j\left(\overline{g}_j-g_j\right)$$
$$-\sum_{n\in \boldsymbol{V_T}}\underline{\alpha}_n\left(\Delta pd_n\right)-\sum_{n\in \boldsymbol{V_T}}\overline{\alpha}_n\left(Pd_n-\Delta Pd_n\right)$$

Where μ_l, λ_n, $\underline{\omega}_l$, $\overline{\omega}_l$, $\underline{\theta}_j$, $\overline{\theta}_j$, $\underline{\alpha}_n$, $\overline{\alpha}_n$ are the Lagrangian multipliers associated with the DC power flow constraints (26) - (30), respectively. The optimality conditions for the original problem KKT of these constraints are:

$$\frac{\partial L}{\partial \delta_n}=\sum_{l\in \boldsymbol{L}}\frac{1}{x_l}A_{nl}\mu_l v_l=0, \qquad n\in \boldsymbol{V_T} \quad \text{(C2)}$$
$$\frac{\partial L}{\partial g_j}=-\lambda_n-\underline{\theta}_j+\overline{\theta}_j=0, \qquad j\in \boldsymbol{G} \quad \text{(C3)}$$
$$\frac{\partial L}{\partial p_l}=\sum_{n\in \boldsymbol{V_T}}A_{nl}\lambda_n-\mu_l-\underline{\omega}_l+\overline{\omega}_l=0, \quad l\in \boldsymbol{L} \quad \text{(C4)}$$
$$\frac{\partial L}{\partial \Delta Pd_n}=1-\lambda_n-\underline{\alpha}_n+\overline{\alpha}_n=0, \qquad n\in \boldsymbol{V_T} \quad \text{(C5)}$$
$$\underline{\omega}_l\ge 0, \qquad l\in \boldsymbol{L} \quad \text{(C6)}$$
$$\overline{\omega}_l\ge 0, \qquad l\in \boldsymbol{L} \quad \text{(C7)}$$
$$\underline{\theta}_j\ge 0, \qquad j\in \boldsymbol{G} \quad \text{(C8)}$$
$$\overline{\theta}_j\ge 0, \qquad j\in \boldsymbol{G} \quad \text{(C9)}$$
$$\underline{\alpha}_n\ge 0, \qquad n\in \boldsymbol{V_T} \quad \text{(C10)}$$
$$\overline{\alpha}_n\ge 0, \qquad n\in \boldsymbol{V_T} \quad \text{(C11)}$$
$$\underline{\omega}_l\left(p_l+\overline{p}_l\right)=0, \qquad l\in \boldsymbol{L} \quad \text{(C12)}$$
$$\overline{\omega}_l\left(\overline{p}_l-p_l\right)=0, \qquad l\in \boldsymbol{L} \quad \text{(C13)}$$
$$\underline{\theta}_j\left(g_j-\underline{g}_j\right)=0, \qquad j\in \boldsymbol{G} \quad \text{(C14)}$$
$$\overline{\theta}_j\left(\overline{g}_j-g_j\right)=0, \qquad j\in \boldsymbol{G} \quad \text{(C15)}$$
$$\underline{\alpha}_n\left(\Delta Pd_n\right)=0, \qquad n\in \boldsymbol{V_T} \quad \text{(C16)}$$
$$\overline{\alpha}_n\left(Pd_n-\Delta Pd_n\right)=0, \qquad n\in \boldsymbol{V_T} \quad \text{(C17)}$$

where (C2) - (C11) denotes the original lower-level problem dual constraints and (C12) - (C17) denotes the complementary slackness constraints.

It can be seen that the original lower-level problem line flow formulation (26), dual constraints (C2) and complementary slackness constraints (C12) - (C17) are all non-linear terms. The above constraints are linearized by the following method:

(1) For the linearization of formulation ((33)

In function (26), there are two non-linear terms-- v_l multiplied by the phase angle δ_l^f at the beginning of the line and v_l multiplied by the phase angle δ_l^t at the end of the line, so continuous variables s_l^f and s_l^t are introduced to represent $v_l\delta_l^f$ and $v_l\delta_l^t$ respectively. Further intermediate variables z_l^f and z_l^t are introduced to equate the non-linear term (26) with the following linear terms:

$$p_l=\frac{1}{x_l}\cdot\left(z_l^f-z_l^t\right), \qquad l\in \boldsymbol{L} \quad \text{(C18)}$$
$$z_l^f=\delta_l^f-s_l^f, \qquad l\in \boldsymbol{L} \quad \text{(C19)}$$
$$z_l^t=\delta_l^t-s_l^t, \qquad l\in \boldsymbol{L} \quad \text{(C20)}$$
$$\underline{\delta}\cdot v_l\le z_l^f\le\overline{\delta}\cdot v_l, \qquad l\in \boldsymbol{L} \quad \text{(C21)}$$
$$\underline{\delta}\cdot v_l\le z_l^t\le\overline{\delta}\cdot v_l, \qquad l\in \boldsymbol{L} \quad \text{(C22)}$$
$$\underline{\delta}\cdot\left(1-v_l\right)\le s_l^f\le\overline{\delta}\cdot\left(1-v_l\right), \qquad l\in \boldsymbol{L} \quad \text{(C23)}$$
$$\underline{\delta}\cdot\left(1-v_l\right)\le s_l^t\le\overline{\delta}\cdot\left(1-v_l\right), \qquad l\in \boldsymbol{L} \quad \text{(C24)}$$

The formulations (C18)-(C24) represents the linearized expression for the calculation of the line DC power flow, if line l is destroyed ($v_l=0$), then according to the formulations (C21), (C22) can be obtained: z_l^f=0, z_l^t=0, so the line power flow p_l=0, $s_l^f=\delta_l^f$, $s_l^t=\delta_l^t$; if l is not destroyed (v_l=1), then according to the formulations (C23), (C24) can be obtained: s_l^f=0, s_l^t=0, so $z_l^f=\delta_l^f$, $z_l^t=\delta_l^t$, and the line plow flow p_l is determined by the phase angle difference between the two ends of the line.

(2) Linearization for the dual constraint (C2)

Similarly, by introducing the continuous variables t_l and h_l, the nonlinear dual constraint (C2) is equivalently represented by the following set of linearization constraints:

$$\sum_{l\in L}\frac{1}{x_l}\cdot A_{nl}\cdot t_l=0, \qquad n\in \boldsymbol{V_T} \quad \text{(A25)}$$
$$t_l=u_l-h_l, \qquad l\in \boldsymbol{L} \quad \text{(A26)}$$
$$\underline{u}_l\cdot v_l\le t_l\le v_l\cdot\overline{u}_l, \qquad l\in \boldsymbol{L} \quad \text{(A27)}$$
$$\underline{u}_l\cdot\left(1-v_l\right)\le h_l\le\overline{u}_l\cdot\left(1-v_l\right), \quad l\in \boldsymbol{L} \quad \text{(A28)}$$

(3) Linearization for the complementary slackness constraints (C12) - (C17)

The nonlinear complementary slackness constraints (C12) - (C17) are equivalently represented by the following set of linearization constraints by introducing the 0-1 variables $\omega_l^{\underline{\omega}}$, $\omega_l^{\overline{\omega}}$, $\omega_g^{\underline{\theta}}$, $\omega_g^{\overline{\theta}}$, $\omega_n^{\underline{\alpha}}$, $\omega_n^{\overline{\alpha}}$:

$$\underline{\omega}_l\le M\cdot\omega_l^{\underline{\omega}}, \qquad l\in \boldsymbol{L} \quad \text{(C29)}$$
$$p_l+\overline{p}_l\le M\cdot\left(1-\omega_l^{\underline{\omega}}\right), \qquad l\in \boldsymbol{L} \quad \text{(C30)}$$
$$\overline{\omega}_l\le M\cdot\omega_l^{\overline{\omega}}, \qquad l\in \boldsymbol{L} \quad \text{(C31)}$$
$$\overline{p}_l-p_l\le M\cdot\left(1-\omega_l^{\overline{\omega}}\right), \qquad l\in \boldsymbol{L} \quad \text{(C32)}$$
$$\underline{\theta}_j\le M\cdot\omega_j^{\underline{\theta}}, \qquad j\in \boldsymbol{G} \quad \text{(C33)}$$

$$g_j - \underline{g}_j \le M \cdot \left(1-\omega_j^{\underline{\theta}}\right), \qquad j \in \boldsymbol{G} \tag{C34}$$

$$\overline{\theta}_j \le M \cdot \omega_j^{\overline{\theta}}, \qquad j \in \boldsymbol{G} \tag{C35}$$

$$\overline{g}_j - g_j \le M \cdot \left(1-\omega_j^{\overline{\theta}}\right), \qquad j \in \boldsymbol{G} \tag{C36}$$

$$\underline{\alpha}_n \le M \cdot \omega_n^{\underline{\alpha}}, \qquad n \in \boldsymbol{V_T} \tag{C37}$$

$$\Delta p_n^d \le M \cdot \left(1-\omega_n^{\underline{\alpha}}\right), \qquad n \in \boldsymbol{V_T} \tag{C38}$$

$$\overline{\alpha}_n \le M \cdot \omega_n^{\overline{\alpha}}, \qquad n \in \boldsymbol{V_T} \tag{C39}$$

$$p_n^d - \Delta p_n^d \le M \cdot \left(1-\omega_n^{\overline{\alpha}}\right), \qquad n \in \boldsymbol{V_T} \tag{C40}$$

$$\omega_l^{\underline{\omega}} + \omega_l^{\overline{\omega}} \le 1, \qquad l \in \boldsymbol{L} \tag{C41}$$

$$\omega_j^{\underline{\theta}} + \omega_j^{\overline{\theta}} \le 1, \qquad j \in \boldsymbol{G} \tag{C42}$$

$$\omega_n^{\underline{\alpha}} + \omega_n^{\overline{\alpha}} \le 1, \qquad n \in \boldsymbol{V_T} \tag{C43}$$

where (C29) - (C30), (C31) - (C32), (C33) - (C34), (C35) - (C36), (C37) - (C38) and (C39) - (C40) are linearised equivalent representations of the constraints (C12), (C13), (C14), (C15), (C16) and (C17) respectively. The intermediate variables introduced satisfies formulations (C41) - (C43).

In summary, the original lower-level problem after transformation by KKT is expressed as formulations (C18)-(C24), formulations (33)- (37), (C25) - (C28), (C3) - (C11) and (C29) - (C43). Therefore, the transformed vulnerability identification problem is as follows:

$$\max_{a} \sum_{T \in \Omega_T} \pi_T \left(\sum_{i \in \boldsymbol{V}} \Delta P d_{i*}^T \right) \tag{C44}$$

s.t. (26) - (29), (33) - (37), (C18) - (C43).

Appendix D

Parameters of UTN and Road Network

Table IV contains information such as the road lengths, Table V contains the node parameters of the case study, the number of cables on the roads, and E_{ij} for each road in the case study are shown in Figure D1.

TABLE IV

Road Network Connection Relation and Parameter

l_{ij}	d_{ij}(km)	mra_{ij}	mrb_{ij}	mrc_{ij}	E_{ij}
1-2	1.8	0	0	0	0
1-3	1.6	0	0	0	0
2-51	1.7	0	0	0	0
2-52	1.7	0	0	0	0
3-4	1.3	0	0	0	0
3-12	1.9	0	0	0	0
4-5	1.7	0	0	0	0
4-12	1.7	0	0	1	0.43
5-6	1.8	0	0	0	0
6-9	1.5	3	0	0	0.93
6-10	1.5	2	0	0	0.95
6-13	1.5	2	0	0	0.78
7-8	1.3	2	0	0	0.87
7-11	1.3	4	0	0	0.95
8-10	1.32	3	0	0	0.97
8-30	1.2	3	0	0	0.92
9-13	1.5	3	0	0	0.83
9-16	1.5	2	0	0	0.84
9-49	1.6	2	0	0	0.86
10-16	1.6	3	0	0	0.97
11-17	1.3	0	0	0	0
11-30	1.1	3	0	0	0.96
12-13	1.3	0	1	0	0.6
12-52	1.8	0	0	0	0
13-15	1.5	2	0	0	0.75
14-32	1	0	0	0	0
14-50	1	0	0	0	0
15-49	1.3	3	0	0	0.97
15-54	1.27	3	0	0	0.98
16-31	1.2	2	0	0	0.9
16-50	1.12	1	0	0	0.97
17-18	0.9	4	0	0	0.36
18-19	1	0	2	0	0
18-21	1.28	3	0	0	0.55
18-30	1.22	2	0	0	0
19-20	1.12	3	0	0	0
19-22	1.15	1	0	0	0.57
20-23	1.3	4	0	0	0
20-24	1.35	3	0	0	0
21-22	1.4	0	0	1	0.63
21-31	1.39	0	0	0	0.92
21-33	1.29	0	2	0	0.92
22-29	1.23	0	0	0	0
23-25	2.1	0	0	0	0
24-25	1.4	0	0	2	0
24-26	1.6	0	0	0	0
25-27	1.5	0	0	0	0
26-27	1.57	0	0	2	0.76
26-29	1.05	4	0	0	0.75
26-37	4	3	0	0	0
27-28	1.9	0	0	0	0
28-37	3	0	0	0	0
28-38	3.5	0	0	0	0
29-35	2	0	0	0	0.93
30-31	2.3	3	0	0	0.95
31-32	1.8	3	0	0	0.93
32-33	2	5	0	0	0.93
32-45	2.2	0	0	0	0
33-34	1.7	3	0	0	0.89
34-35	1.82	4	0	0	0.89
34-44	2	2	0	0	0.68
35-36	1.5	2	0	0	0.89

36-37	1.4	3	0	0	0.9
36-41	2.7	0	0	0	0
36-44	1.7	2	0	0	0.87
37-40	3.7	0	0	0	0
38-39	1.9	0	0	0	0
39-40	1.7	1	0	0	0.56
39-85	2.1	0	0	0	0
40-41	1.82	1	0	0	0.6
40-65	1.95	0	0	0	0
41-42	1.67	3	0	0	0.96
41-43	1.32	2	0	0	0.87
42-61	1.65	4	0	0	0.95
42-65	0.89	0	0	0	0
43-44	1.32	3	0	0	0.93
44-46		2	0	0	0.89
45-46	1.4	0	0	0	0
45-47	1.6	3	0	0	0.79
46-61	1.62	4	0	0	0.95
47-48	1.45	3	0	0	0.9
47-50	1.7	0	0	0	0
47-60	1.89	0	0	0	0
48-49	1.43	4	0	0	0.97
48-58	1.05	3	0	0	0.89
49-50	0.89	2	0	0	0.88
51-55	1.97	0	0	0	0
52-53	1.9	2	0	0	0.89
52-54	1.9	2	0	0	0.93
53-55	1.82	0	0	0	0
53-56	1.89	4	0	0	0.94
54-56	2.2	2	0	0	0.9
54-57	0.93	4	0	0	0.92
55-66	1.1	0	0	1	0.5
56-66	1.25	0	0	0	0
56-70	1.19	0	0	0	0
57-58	0.65	2	0	0	0.78
57-70	1.8	0	0	0	0
58-59	1.1	0	1	0	0.78
58-72	1.4	0	2	0	0.57
59-60	1.4	0	0	0	0
59-79	1.62	0	0	3	0.78
60-61	1.63	0	0	0	0
60-62	1.4	0	0	0	0
62-63	1.3	0	0	0	0
62-80	1.6	0	2	0	0.76
63-64	1	0	0	0	0
64-65	0.9	0	0	0	0
64-82	1.2	2	0	0	0.9
64-87	1.3	0	0	0	0
65-84	1.7	0	0	0	0
66-67	1.02	0	0	0	0
66-69	1.02	0	1	0	0.7
67-68	1.9	0	0	0	0
68-76	1.79	0	0	0	0
69-70	1.76	0	0	2	0.67
69-71	1.69	0	0	0	0
69-75	1.95	4	0	0	0.89
70-72	1.05	0	1	0	0.78
71-73	2.3	0	0	0	0
72-73	1.21	0	2	0	0.74
73-74	1.25	0	0	1	0.32
74-75	2.52	0	0	0	0
74-77	2.1	0	0	2	0.56
74-80	2.9	0	0	3	0.67
75-77	1.92	4	0	0	0.97
76-77	1.68	0	0	0	0
77-78	2.6	4	0	0	0.96
78-81	2	2	0	0	0.92
79-80	1.97	0	2	0	0.59
80-81	2.01	4	0	0	0.95
81-82	2	4	0	0	0.94
81-83	2.32	4	0	0	0.96
82-83	1.87	3	0	0	0.89
83-87	1.62	0	0	0	0
84-85	1.82	0	0	0	0
84-86	1.3	0	0	0	0
86-87	2.05	0	0	0	0

TABLE V
Node PARAMETERS

Node Number	Node Types	Load/MW	Active Output/MW
1	pq	0	0
2	pq	0	0
3	pq	342	0
4	pq	385	0
5	pq	0	0
6	pq	0	0
7	pq	233.8	0
8	pq	268.5	0
9	pq	0	0
10	pq	0	0
11	pq	0	0
12	pq	287.5	0
13	pq	0	0
14	pq	0	0
15	pq	220	0
16	pq	329	0
17	pq	0	0
18	pq	258	0
19	pq	0	0

20	pq	228	0
21	pq	274	0
22	pq	385	0
23	pq	247.5	0
24	pq	368.6	0
25	pq	324	0
26	pq	339	0
27	pq	261.5	0
28	pq	306	0
29	pq	583.5	0
30	pq	0	600
31	vθ	0	0
32	pv	9.5	750
33	pv	0	850
34	pv	0	508
35	pv	0	650
36	pv	0	250
37	pv	0	540
38	pv	0	830
39	pv	0	868

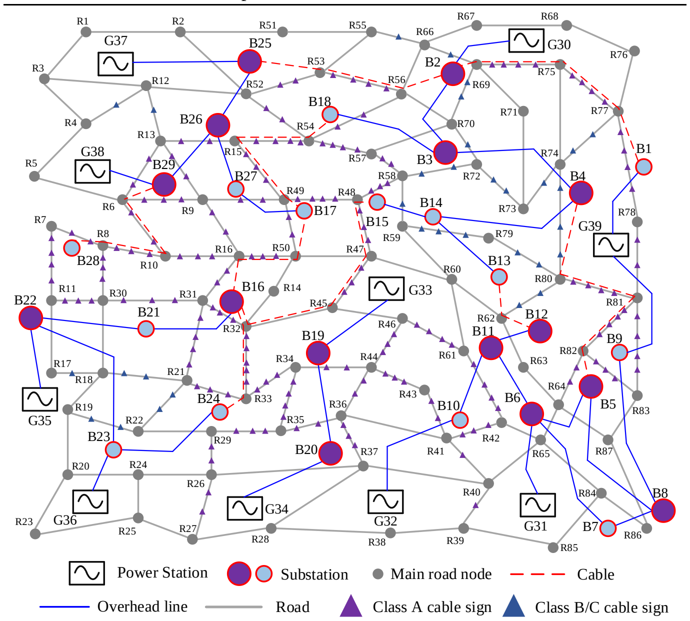

Fig. D1 The coupling network between UPN and the road network of the case