

APPENDIX A

THE KKT TRANSFORMATION WITH LINEARIZATION PROCESS

The lagrangian function for the original lower-level problem (32)-(37) is:

$$\begin{aligned}
 L = & \sum_{n \in V_T} \Delta P d_n - \sum_{l \in L} \mu_l \left(p_l - v_l \cdot \frac{1}{x_l} \cdot \sum_{n \in V_T} A_{nl} \cdot \delta_n \right) \\
 & - \sum_{n \in V_T} \lambda_n \left(\sum_{g \in G_n} p_g - \sum_{l \in L} A_{nl} \cdot p_l + \Delta P d_n - P d_n \right) \\
 & - \sum_{l \in L} \underline{\omega}_l (p_l + \bar{p}_l) - \sum_{l \in L} \bar{\omega}_l (\bar{p}_l - p_l) \\
 & - \sum_{j \in G} \underline{\theta}_j (g_j - \underline{g}_j) - \sum_{j \in G} \bar{\theta}_j (\bar{g}_j - g_j) \\
 & - \sum_{n \in V_T} \underline{\alpha}_n (\Delta P d_n) - \sum_{n \in V_T} \bar{\alpha}_n (P d_n - \Delta P d_n)
 \end{aligned} \quad (A1)$$

Where μ_l , λ_n , $\underline{\omega}_l$, $\bar{\omega}_l$, $\underline{\theta}_j$, $\bar{\theta}_j$, $\underline{\alpha}_n$, $\bar{\alpha}_n$ are the Lagrangian multipliers associated with the DC power flow constraints (33)-(37), respectively, in addition to the original feasibility constraints (33)-(37), the optimality conditions for the original problem KKT are:

$$\frac{\partial L}{\partial \delta_n} = \sum_{l \in L} \frac{1}{x_l} A_{nl} \mu_l v_l = 0, \quad n \in V_T \quad (A2)$$

$$\frac{\partial L}{\partial g_j} = -\lambda_n - \underline{\theta}_j + \bar{\theta}_j = 0, \quad j \in G \quad (A3)$$

$$\frac{\partial L}{\partial p_l} = \sum_{n \in V_T} A_{nl} \lambda_n - \mu_l - \underline{\omega}_l + \bar{\omega}_l = 0, \quad l \in L \quad (A4)$$

$$\frac{\partial L}{\partial \Delta P d_n} = 1 - \lambda_n - \underline{\alpha}_n + \bar{\alpha}_n = 0, \quad n \in V_T \quad (A5)$$

$$\underline{\omega}_l \geq 0, \quad l \in L \quad (A6)$$

$$\bar{\omega}_l \geq 0, \quad l \in L \quad (A7)$$

$$\underline{\theta}_j \geq 0, \quad j \in G \quad (A8)$$

$$\bar{\theta}_j \geq 0, \quad j \in G \quad (A9)$$

$$\underline{\alpha}_n \geq 0, \quad n \in V_T \quad (A10)$$

$$\bar{\alpha}_n \geq 0, \quad n \in V_T \quad (A11)$$

$$\underline{\omega}_l (p_l + \bar{p}_l) = 0, \quad l \in L \quad (A12)$$

$$\bar{\omega}_l (\bar{p}_l - p_l) = 0, \quad l \in L \quad (A13)$$

$$\underline{\theta}_j (g_j - \underline{g}_j) = 0, \quad j \in G \quad (A14)$$

$$\bar{\theta}_j (\bar{g}_j - g_j) = 0, \quad j \in G \quad (A15)$$

$$\underline{\alpha}_n (\Delta P d_n) = 0, \quad n \in V_T \quad (A16)$$

$$\bar{\alpha}_n (P d_n - \Delta P d_n) = 0, \quad n \in V_T \quad (A17)$$

where (A2)-(A11) denotes the original lower-level problem dual constraints and (A12)-(A17) denotes the complementary slackness constraints.

It can be seen that the original lower-level problem line flow formulation (33), dual constraints (A2) and complementary slackness constraints (A12)-(A17) are all non-linear terms. The above constraints are linearized by the following method:

(1) For the linearization of formulation ((33))

In formulation (33), there are two non-linear terms-- v_l multiplied by the phase angle δ_l^f at the beginning of the line and v_l multiplied by the phase angle δ_l^t at the end of the line, so continuous variables s_l^f and s_l^t are introduced to represent $v_l \delta_l^f$ and $v_l \delta_l^t$ respectively. Further intermediate variables z_l^f and z_l^t are introduced to equate the non-linear term (33) with the following linear terms:

$$p_l = \frac{1}{x_l} \cdot (z_l^f - z_l^t), \quad l \in L \quad (A18)$$

$$z_l^f = \delta_l^f - s_l^f, \quad l \in L \quad (A19)$$

$$z_l^t = \delta_l^t - s_l^t, \quad l \in L \quad (A20)$$

$$\underline{\delta} \cdot v_l \leq z_l^f \leq \bar{\delta} \cdot v_l, \quad l \in L \quad (A21)$$

$$\underline{\delta} \cdot v_l \leq z_l^t \leq \bar{\delta} \cdot v_l, \quad l \in L \quad (A22)$$

$$\underline{\delta} \cdot (1 - v_l) \leq s_l^f \leq \bar{\delta} \cdot (1 - v_l), \quad l \in L \quad (A23)$$

$$\underline{\delta} \cdot (1 - v_l) \leq s_l^t \leq \bar{\delta} \cdot (1 - v_l), \quad l \in L \quad (A24)$$

The formulations (A18)-(A24) represents the linearized expression for the calculation of the line DC power flow, if line l is destroyed ($v_l = 0$), then according to the formulations (A21), (A22) can be obtained: $z_l^f = 0$, $z_l^t = 0$, so the line power flow $p_l = 0$, $s_l^f = \delta_l^f$, $s_l^t = \delta_l^t$; if l is not destroyed ($v_l = 1$), then according to the formulations (A23), (A24) can be obtained: $s_l^f = 0$, $s_l^t = 0$, so $z_l^f = \delta_l^f$, $z_l^t = \delta_l^t$, and the line power flow p_l is determined by the phase angle difference between the two ends of the line.

(2) Linearization for the dual constraint (A2)

Similarly, by introducing the continuous variables t_l and h_l , the nonlinear dual constraint (A2) is equivalently represented by the following set of linearization constraints:

$$\sum_{l \in L} \frac{1}{x_l} \cdot A_{nl} \cdot t_l = 0, \quad n \in V_T \quad (A25)$$

$$t_l = u_l - h_l, \quad l \in L \quad (A26)$$

$$\underline{u}_l \cdot v_l \leq t_l \leq v_l \cdot \bar{u}_l, \quad l \in L \quad (A27)$$

$$\underline{u}_l \cdot (1 - v_l) \leq h_l \leq \bar{u}_l \cdot (1 - v_l), \quad l \in L \quad (A28)$$

(3) Linearization for the complementary slackness constraints (A12)-(A17)

The nonlinear complementary slackness constraints (A12)-(A17) are equivalently represented by the following set of linearization constraints by introducing the 0-1 variables ω_l^{ω} ,

ω_l^{θ} , ω_g^{θ} , $\omega_g^{\bar{\theta}}$, ω_n^{α} , $\omega_n^{\bar{\alpha}}$:

$$\underline{\omega}_l \leq M \cdot \omega_l^{\omega}, \quad l \in L \quad (A29)$$

$$p_l + \bar{p}_l \leq M \cdot (1 - \omega_l^{\omega}), \quad l \in L \quad (A30)$$

$$\bar{\omega}_l \leq M \cdot \bar{\omega}_l^{\bar{\omega}}, \quad l \in \mathbf{L} \quad (\text{A31})$$

$$\bar{p}_l - p_l \leq M \cdot (1 - \bar{\omega}_l^{\bar{\omega}}), \quad l \in \mathbf{L} \quad (\text{A32})$$

$$\bar{\theta}_j \leq M \cdot \bar{\omega}_j^{\bar{\theta}}, \quad j \in \mathbf{G} \quad (\text{A33})$$

$$g_j - \underline{g}_j \leq M \cdot (1 - \bar{\omega}_j^{\bar{\theta}}), \quad j \in \mathbf{G} \quad (\text{A34})$$

$$\bar{\theta}_j \leq M \cdot \bar{\omega}_j^{\bar{\theta}}, \quad j \in \mathbf{G} \quad (\text{A35})$$

$$\bar{g}_j - g_j \leq M \cdot (1 - \bar{\omega}_j^{\bar{\theta}}), \quad j \in \mathbf{G} \quad (\text{A36})$$

$$\underline{\alpha}_n \leq M \cdot \bar{\omega}_n^{\underline{\alpha}}, \quad n \in \mathbf{V}_T \quad (\text{A37})$$

$$\Delta p_n^d \leq M \cdot (1 - \bar{\omega}_n^{\underline{\alpha}}), \quad n \in \mathbf{V}_T \quad (\text{A38})$$

$$\bar{\alpha}_n \leq M \cdot \bar{\omega}_n^{\bar{\alpha}}, \quad n \in \mathbf{V}_T \quad (\text{A39})$$

$$p_n^d - \Delta p_n^d \leq M \cdot (1 - \bar{\omega}_n^{\bar{\alpha}}), \quad n \in \mathbf{V}_T \quad (\text{A40})$$

$$\omega_l^{\bar{\omega}} + \bar{\omega}_l^{\bar{\omega}} \leq 1, \quad l \in \mathbf{L} \quad (\text{A41})$$

$$\omega_j^{\bar{\theta}} + \bar{\omega}_j^{\bar{\theta}} \leq 1, \quad j \in \mathbf{G} \quad (\text{A42})$$

$$\omega_n^{\bar{\alpha}} + \bar{\omega}_n^{\bar{\alpha}} \leq 1, \quad n \in \mathbf{V}_T \quad (\text{A43})$$

where (A29)-(A30), (A31)-(A32), (A33)-(A34), (A35)-(A36), (A37)-(A38) and (A39)-(A40) are linearised equivalent representations of the constraints (A12), (A13), (A14), (A15), (A16) and (A17) respectively. The intermediate variables introduced satisfies formulations (A41)-A43).

In summary, the original lower-level problem after transformation by KKT is expressed as formulations (A18)-(A24), formulations (33)-(37), (A25)-(A28), (A3)-(A11) and (A29)-(A43). Therefore, the transformed vulnerability identification problem is as follows:

$$\max_a \sum_{T \in \Omega_T} \pi_T \left(\sum_{i \in V} \Delta P d_{i*}^T \right) \quad (\text{A44})$$

s.t. (26)-(29), (33)-(37), (A18)-(A43).

APPENDIX B

PARAMETERS OF UTN AND ROAD NETWORK

Table VI contains information such as the road lengths, the number of cables on the roads, and E_{ij} for each road in the case of Figure B1.

TABLE VI

ROAD NETWORK CONNECTION RELATION AND PARAMETER

l_{ij}	$d_{ij}(\text{km})$	mra_{ij}	mrb_{ij}	mrc_{ij}	E_{ij}
1-2	1.8	0	0	0	0
1-3	1.6	0	0	0	0
2-51	1.7	0	0	0	0
2-52	1.7	0	0	0	0
3-4	1.3	0	0	0	0
3-12	1.9	0	0	0	0
4-5	1.7	0	0	0	0
4-12	1.7	0	0	1	0.43
5-6	1.8	0	0	0	0

6-9	1.5	3	0	0	0.93
6-10	1.5	2	0	0	0.95
6-13	1.5	2	0	0	0.78
7-8	1.3	2	0	0	0.87
7-11	1.3	4	0	0	0.95
8-10	1.32	3	0	0	0.97
8-30	1.2	3	0	0	0.92
9-13	1.5	3	0	0	0.83
9-16	1.5	2	0	0	0.84
9-49	1.6	2	0	0	0.86
10-16	1.6	3	0	0	0.97
11-17	1.3	0	0	0	0
11-30	1.1	3	0	0	0.96
12-13	1.3	0	1	0	0.6
12-52	1.8	0	0	0	0
13-15	1.5	2	0	0	0.75
14-32	1	0	0	0	0
14-50	1	0	0	0	0
15-49	1.3	3	0	0	0.97
15-54	1.27	3	0	0	0.98
16-31	1.2	2	0	0	0.9
16-50	1.12	1	0	0	0.97
17-18	0.9	4	0	0	0.36
18-19	1	0	2	0	0
18-21	1.28	3	0	0	0.55
18-30	1.22	2	0	0	0
19-20	1.12	3	0	0	0
19-22	1.15	1	0	0	0.57
20-23	1.3	4	0	0	0
20-24	1.35	3	0	0	0
21-22	1.4	0	0	1	0.63
21-31	1.39	0	0	0	0.92
21-33	1.29	0	2	0	0.92
22-29	1.23	0	0	0	0
23-25	2.1	0	0	0	0
24-25	1.4	0	0	2	0
24-26	1.6	0	0	0	0
25-27	1.5	0	0	0	0
26-27	1.57	0	0	2	0.76
26-29	1.05	4	0	0	0.75
26-37	4	3	0	0	0
27-28	1.9	0	0	0	0
28-37	3	0	0	0	0
28-38	3.5	0	0	0	0
29-35	2	0	0	0	0.93
30-31	2.3	3	0	0	0.95
31-32	1.8	3	0	0	0.93
32-33	2	5	0	0	0.93
32-45	2.2	0	0	0	0

33-34	1.7	3	0	0	0.89
34-35	1.82	4	0	0	0.89
34-44	2	2	0	0	0.68
35-36	1.5	2	0	0	0.89
36-37	1.4	3	0	0	0.9
36-41	2.7	0	0	0	0
36-44	1.7	2	0	0	0.87
37-40	3.7	0	0	0	0
38-39	1.9	0	0	0	0
39-40	1.7	1	0	0	0.56
39-85	2.1	0	0	0	0
40-41	1.82	1	0	0	0.6
40-65	1.95	0	0	0	0
41-42	1.67	3	0	0	0.96
41-43	1.32	2	0	0	0.87
42-61	1.65	4	0	0	0.95
42-65	0.89	0	0	0	0
43-44	1.32	3	0	0	0.93
44-46		2	0	0	0.89
45-46	1.4	0	0	0	0
45-47	1.6	3	0	0	0.79
46-61	1.62	4	0	0	0.95
47-48	1.45	3	0	0	0.9
47-50	1.7	0	0	0	0
47-60	1.89	0	0	0	0
48-49	1.43	4	0	0	0.97
48-58	1.05	3	0	0	0.89
49-50	0.89	2	0	0	0.88
51-55	1.97	0	0	0	0
52-53	1.9	2	0	0	0.89
52-54	1.9	2	0	0	0.93
53-55	1.82	0	0	0	0
53-56	1.89	4	0	0	0.94
54-56	2.2	2	0	0	0.9
54-57	0.93	4	0	0	0.92
55-66	1.1	0	0	1	0.5
56-66	1.25	0	0	0	0
56-70	1.19	0	0	0	0
57-58	0.65	2	0	0	0.78
57-70	1.8	0	0	0	0
58-59	1.1	0	1	0	0.78
58-72	1.4	0	2	0	0.57
59-60	1.4	0	0	0	0
59-79	1.62	0	0	3	0.78
60-61	1.63	0	0	0	0
60-62	1.4	0	0	0	0
62-63	1.3	0	0	0	0
62-80	1.6	0	2	0	0.76
63-64	1	0	0	0	0

64-65	0.9	0	0	0	0
64-82	1.2	2	0	0	0.9
64-87	1.3	0	0	0	0
65-84	1.7	0	0	0	0
66-67	1.02	0	0	0	0
66-69	1.02	0	1	0	0.7
67-68	1.9	0	0	0	0
68-76	1.79	0	0	0	0
69-70	1.76	0	0	2	0.67
69-71	1.69	0	0	0	0
69-75	1.95	4	0	0	0.89
70-72	1.05	0	1	0	0.78
71-73	2.3	0	0	0	0
72-73	1.21	0	2	0	0.74
73-74	1.25	0	0	1	0.32
74-75	2.52	0	0	0	0
74-77	2.1	0	0	2	0.56
74-80	2.9	0	0	3	0.67
75-77	1.92	4	0	0	0.97
76-77	1.68	0	0	0	0
77-78	2.6	4	0	0	0.96
78-81	2	2	0	0	0.92
79-80	1.97	0	2	0	0.59
80-81	2.01	4	0	0	0.95
81-82	2	4	0	0	0.94
81-83	2.32	4	0	0	0.96
82-83	1.87	3	0	0	0.89
83-87	1.62	0	0	0	0
84-85	1.82	0	0	0	0
84-86	1.3	0	0	0	0
86-87	2.05	0	0	0	0

TABLE VII
Node PARAMETERS OF UTN

Node Number	Node Types	Load/MW	Active Output/MW
1	pq	0	0
2	pq	0	0
3	pq	342	0
4	pq	385	0
5	pq	0	0
6	pq	0	0
7	pq	233.8	0
8	pq	268.5	0
9	pq	0	0
10	pq	0	0
11	pq	0	0
12	pq	287.5	0
13	pq	0	0
14	pq	0	0
15	pq	220	0

16	pq	329	0
17	pq	0	0
18	pq	258	0
19	pq	0	0
20	pq	228	0
21	pq	274	0
22	pq	385	0
23	pq	247.5	0
24	pq	368.6	0
25	pq	324	0
26	pq	339	0
27	pq	261.5	0
28	pq	306	0
29	pq	583.5	0
30	pq	0	600
31	vθ	0	0
32	pv	9.5	750
33	pv	0	850
34	pv	0	508
35	pv	0	650
36	pv	0	250
37	pv	0	540
38	pv	0	830
39	pv	0	868

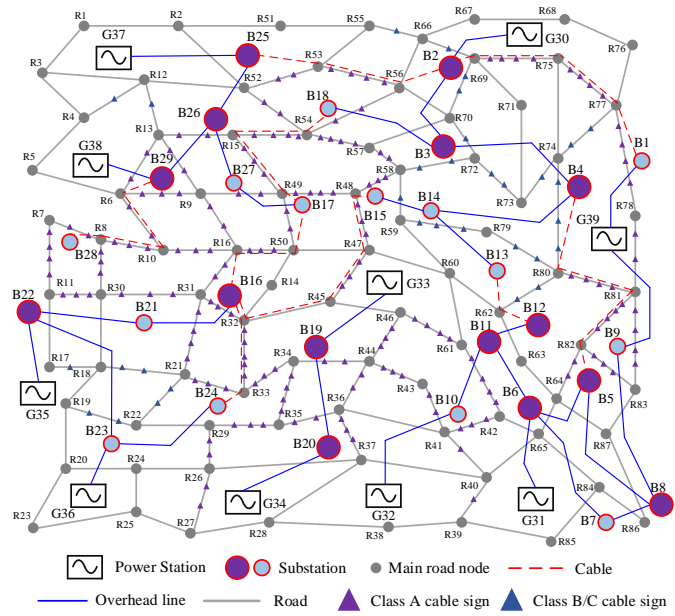


Fig. B1 The coupling network between UTn and the road network of the case