APPENDIX A

THE KKT TRANSFORMATION WITH LINEARIZATION PROCESS

For $G_T \in \boldsymbol{\Omega_T}$, the Lagrangian function for the lower-level problem (34)-(38) is:

$$L=\sum_{i\in \boldsymbol{V}}\Delta Pd_i^T-\sum_{l_{ij}\in \boldsymbol{L}_T}\mu_{ij}^T\left(p_{ij}^T-v_{ij}^T\cdot\frac{1}{x_{ij}}\cdot\sum_{i\in \boldsymbol{V}}A_{nl}\cdot\delta_n\right)$$
$$-\sum_{i\in \boldsymbol{V}}\lambda_i\left(\sum_{k\in \boldsymbol{V}_G(i)}g_k^T-\sum_{l_{ij}\in \boldsymbol{L}_T}A_{ij}^T\cdot p_{ij}^T+\Delta Pd_i^T-Pd_i\right)$$
$$-\sum_{l_{ij}\in \boldsymbol{L}_T}\underline{\omega}_{ij}^T\left(p_{ij}^T+\overline{p_{ij}}\right)-\sum_{l_{ij}\in \boldsymbol{L}_T}\overline{\omega}_{ij}^T\left(\overline{p_{ij}}-p_{ij}^T\right) \quad (A1)$$
$$-\sum_{k\in \boldsymbol{V_G}}\underline{\theta}_k^T\left(g_k^T-\underline{g_k}\right)-\sum_{k\in \boldsymbol{V_G}}\overline{\theta}_k^T\left(\overline{g_k}-g_k^T\right)$$
$$-\sum_{i\in \boldsymbol{V}}\underline{\alpha}_i^T\left(\Delta Pd_i^T\right)-\sum_{i\in \boldsymbol{V}}\overline{\alpha}_i^T\left(Pd_i-\Delta Pd_i^T\right)$$

where μ_{ij}^T, λ_i^T, $\overline{\omega}_{ij}^T$, $\underline{\omega}_{ij}^T$, $\overline{\theta}_k^T$, $\underline{\theta}_k^T$, $\overline{\alpha}_k^T$, $\underline{\alpha}_k^T$ are the Lagrangian multipliers associated with the DC power flow constraints (34)-(38), respectively. The optimality conditions for the original problem KKT of these constraints are:

$$\frac{\partial L}{\partial\delta_i}=\sum_{l_{ij}\in L_T}\frac{1}{x_{ij}}\cdot A_{ij}^T\cdot\mu_{ij}^T\cdot v_{ij}^T=0,\qquad i\in \boldsymbol{V} \quad (A2)$$
$$\frac{\partial L}{\partial g_k}=-\lambda_i^T\Big|_{k\in \boldsymbol{V}_{G(i)}}-\underline{\theta}_k+\overline{\theta}_k=0,\qquad k\in \boldsymbol{V_G} \quad (A3)$$
$$\frac{\partial L}{\partial p_{ij}^T}=\sum_{y\in \boldsymbol{V}}A_{ij}^T\lambda_y^T-\mu_{ij}^T-\underline{\omega}_{ij}^T+\overline{\omega}_{ij}^T=0,\quad l_{ij}\in \boldsymbol{L_T} \quad (A4)$$
$$\frac{\partial L}{\partial\Delta Pd_i^T}=1-\lambda_i^T-\underline{\alpha}_i^T+\overline{\alpha}_i^T=0,\qquad i\in \boldsymbol{V} \quad (A5)$$
$$\underline{\omega}_{ij}^T>0,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A6)$$
$$\overline{\omega}_{ij}^T>0,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A7)$$
$$\underline{\theta}_k^T>0,\qquad k\in \boldsymbol{V_G} \quad (A8)$$
$$\overline{\theta}_k^T>0,\qquad k\in \boldsymbol{V_G} \quad (A9)$$
$$\underline{\alpha}_i^T\ge 0,\qquad i\in \boldsymbol{V} \quad (A10)$$
$$\overline{\alpha}_i^T\ge 0,\qquad i\in \boldsymbol{V} \quad (A11)$$
$$\underline{\omega}_{ij}^T\left(p_{ij}^T+\overline{p_{ij}}\right)=0,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A12)$$
$$\overline{\omega}_{ij}^T\left(\overline{p_{ij}}-p_{ij}^T\right)=0,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A13)$$
$$\underline{\theta}_k^T\left(g_k^T-\underline{g}_k\right)=0,\qquad k\in \boldsymbol{V_G} \quad (A14)$$
$$\overline{\theta}_k^T\left(\overline{g}_k-g_k^T\right)=0,\qquad k\in \boldsymbol{V_G} \quad (A15)$$
$$\underline{\alpha}_i^T\left(\Delta Pd_i^T\right)=0,\qquad i\in \boldsymbol{V} \quad (A16)$$
$$\overline{\alpha}_i^T\left(Pd_i-\Delta Pd_i^T\right)=0,\qquad i\in \boldsymbol{V} \quad (A17)$$

where (A2) - (A11) denotes the original lower-level problem dual constraints and (A12) - (A17) denotes the complementary slackness constraints.

It can be seen that the original function (34) in lower-level problem, dual constraints (A2) and complementary slackness constraints (A12) - (A17) are all non-linear terms. The above constraints are linearized by the following method:

(1) For the linearization of formulation (34)

In function (34), there are two non-linear terms-v_{ij}^Tmultiplied by the phase angle δ_i^T at the beginning of the line and v_{ij}^T multiplied by the phase angle δ_j^T at the end of the line, so continuous variables s_i^T and s_j^T are introduced to represent $v_{ij}^T\delta_i^T$ and $v_{ij}^T\delta_j^T$ respectively. Further intermediate variables z_i^T and z_j^T are introduced to equate the non-linear term (34) with the following linear terms:

$$p_{ij}^T=\frac{1}{x_l}\cdot\left(z_i^T-z_j^T\right),\qquad l_{ij}\in \boldsymbol{L_T} \quad (A18)$$
$$z_i^T=\delta_i^T\text{-}\delta_i^T,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A19)$$
$$z_j^T=\delta_j^T\text{-}\delta_j^T,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A20)$$
$$\underline{\delta}\cdot v_{ij}^T\le z_i^T\le\overline{\delta}\cdot v_{ij}^T,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A21)$$
$$\underline{\delta}\cdot v_{ij}^T\le z_j^T\le\overline{\delta}\cdot v_{ij}^T,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A22)$$
$$\underline{\delta}\cdot\left(1-v_{ij}^T\right)\le z_i^T\le\overline{\delta}\cdot\left(1-v_{ij}^T\right),\quad l_{ij}\in \boldsymbol{L_T} \quad (A23)$$
$$\underline{\delta}\cdot\left(1-v_{ij}^T\right)\le z_j^T\le\overline{\delta}\cdot\left(1-v_{ij}^T\right),\quad l_{ij}\in \boldsymbol{L_T} \quad (A24)$$

The formulations (A18)-(A24) represents the linearized expression for the calculation of the line DC power flow, if line l_{ij} is destroyed (v_{ij}=0), then according to the formulations (A21), (A22) can be obtained: z_i^T=0, z_j^T=0, so the line power flow p_{ij}^T=0, $s_i^T=\delta_i^T$, $s_j^T=\delta_j^T$; if l_{ij} is not destroyed (v_{ij}^T=1), then according to the functions (A23), (A24) can be obtained: s_i^T=0, s_j^T=0, so $z_i^T=\delta_i^T$, $z_j^T=\delta_j^T$, the line plow flow p_{ij}^T is determined by the phase angle difference between the two ends of the line.

(2) Linearization for the dual constraint (A2)

Similarly, by introducing the continuous variables t_l and h_l, the nonlinear dual constraint (A2) is equivalently represented by the following set of linearization constraints:

$$\sum_{l_{ij}\in L_T}\frac{1}{x_{ij}}\cdot A_{ij}^T\cdot t_{ij}^T=0\ ,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A25)$$
$$t_{ij}^T=u_{ij}^T-h_{ij}^T,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A26)$$
$$\underline{u_{ij}}\cdot v_{ij}^T\le t_{ij}^T\le\overline{u_{ij}}\cdot v_{ij}^T,\qquad l_{ij}\in \boldsymbol{L_T} \quad (A27)$$
$$\underline{u_{ij}}\cdot\left(1-v_{ij}^T\right)\le z_i^T\le\overline{u_{ij}}\cdot\left(1-v_{ij}^T\right),\ l_{ij}\in \boldsymbol{L_T} \quad (A28)$$

(3) Linearization for the complementary slackness constraints (A12) - (A17)

The nonlinear complementary slackness constraints (A12) - (A17) are equivalently represented by the following set of linearization constraints by introducing the 0-1 variables $\omega_{ij}^{\underline{\omega}T}$, $\omega_{ij}^{\overline{\omega}T}$, $\omega_k^{\underline{\theta}T}$, $\omega_k^{\overline{\theta}T}$, $\omega_i^{\underline{\alpha}T}$, $\omega_i^{\overline{\alpha}T}$:

$$\underline{\omega}_{ij}^T\le M\cdot\omega_{ij}^{\underline{\omega}T},\qquad l_{ij}\in \boldsymbol{L_T} \quad (A29)$$
$$p_{ij}^T+\overline{p}_{ij}\le M\cdot\left(1-\omega_{ij}^{\underline{\omega}T}\right),\qquad l_{ij}\in \boldsymbol{L_T} \quad (A30)$$
$$\overline{\omega}_{ij}^T\le M\cdot\omega_{ij}^{\overline{\omega}T},\qquad l_{ij}\in \boldsymbol{L_T} \quad (A31)$$
$$\underline{p}_{ij}-p_{ij}^T\le M\cdot\left(1-\omega_{ij}^{\overline{\omega}T}\right),\qquad l_{ij}\in \boldsymbol{L_T} \quad (A32)$$

$$\underline{\theta}_k^T \le M \cdot \omega_k^{\underline{\theta}T}, \qquad k \in \boldsymbol{V_G} \quad \text{(A33)}$$

$$g_k^T - \underline{g}_k \le M \cdot \left(1 - \omega_k^{\underline{\theta}T}\right), \qquad k \in \boldsymbol{V_G} \quad \text{(A34)}$$

$$\overline{\theta}_k^T \le M \cdot \omega_k^{\overline{\theta}T}, \qquad k \in \boldsymbol{V_G} \quad \text{(A35)}$$

$$\overline{g}_k - g_k^T \le M \cdot \left(1 - \omega_k^{\overline{\theta}T}\right), \qquad k \in \boldsymbol{V_G} \quad \text{(C36)}$$

$$\underline{\alpha}_i^T \le M \cdot \omega_k^{\underline{\alpha}T}, \qquad i \in \boldsymbol{V} \quad \text{(A37)}$$

$$Pd_i \le M \cdot \left(1 - \omega_i^{\underline{\alpha}T}\right), \qquad i \in \boldsymbol{V} \quad \text{(A38)}$$

$$\overline{\alpha}_i^T \le M \cdot \omega_i^{\overline{\alpha}T}, \qquad i \in \boldsymbol{V} \quad \text{(A39)}$$

$$Pd_i - \Delta Pd_i^T \le M \cdot \left(1 - \omega_i^{\overline{\alpha}T}\right), \qquad i \in \boldsymbol{V} \quad \text{(A40)}$$

$$\omega_{ij}^{\underline{\omega}T} + \omega_{ij}^{\overline{\omega}T} < 1, \qquad l_{ij} \in \boldsymbol{L_T} \quad \text{(A41)}$$

$$\omega_k^{\underline{\theta}T} + \omega_k^{\overline{\theta}T} < 1, \qquad k \in \boldsymbol{V_G} \quad \text{(A42)}$$

$$\omega_i^{\underline{\alpha}T} + \omega_i^{\overline{\alpha}T} < 1, \qquad i \in \boldsymbol{V} \quad \text{(A43)}$$

where (A29) - (A30), (A31) - (A32), (A33) - (A34), (A35) - (A36), (A37) - (A38) and (A39) - (A40) are linearised equivalent representations of the constraints (A12), (A13), (A14), (A15), (A16) and (A17) respectively. The intermediate variables introduced satisfies formulations (A41) - (A43).

In summary, the original lower-level problem after transformation by KKT is expressed as formulations (A18)-(A24), formulations (34) - (38), (A25) - (A28), (A3) - (A11) and (A29) - (A43). Therefore, the transformed vulnerability identification problem is as follows:

$$\max_{a} \sum_{T \in \mathbf{\Omega_T}} \pi_T \left(\sum_{i \in V} \Delta Pd_i^T \right) \quad \text{(A44)}$$

s.t. (27) - (30), (A18) - (A43).

APPENDIX B

PARAMETERS OF UTN AND ROAD NETWORK

Table IV contains information of the road lengths and cable signs, Table V contains the node parameters of the case study, the number of cables on the roads, and E_{ij} for each road in the case study are shown in Figure B1.

TABLE IV
ROAD NETWORK CONNECTION RELATION AND PARAMETER

l_{ij}	d_{ij}(km)	mra_{ij}	mrb_{ij}	mrc_{ij}	E_{ij}
1-2	1.8	0	0	0	0
1-3	1.6	0	0	0	0
2-51	1.7	0	0	0	0
2-52	1.7	0	0	0	0
3-4	1.3	0	0	0	0
3-12	1.9	0	0	0	0
4-5	1.7	0	0	0	0
4-12	1.7	0	0	1	0.43
5-6	1.8	0	0	0	0
6-9	1.5	3	0	0	0.93
6-10	1.5	2	0	0	0.95
6-13	1.5	2	0	0	0.78
7-8	1.3	2	0	0	0.87
7-11	1.3	4	0	0	0.95
8-10	1.32	3	0	0	0.97
8-30	1.2	3	0	0	0.92
9-13	1.5	3	0	0	0.83
9-16	1.5	2	0	0	0.84
9-49	1.6	2	0	0	0.86
10-16	1.6	3	0	0	0.97
11-17	1.3	0	0	0	0
11-30	1.1	3	0	0	0.96
12-13	1.3	0	1	0	0.6
12-52	1.8	0	0	0	0
13-15	1.5	2	0	0	0.75
14-32	1	0	0	0	0
14-50	1	0	0	0	0
15-49	1.3	3	0	0	0.97
15-54	1.27	3	0	0	0.98
16-31	1.2	2	0	0	0.9
16-50	1.12	1	0	0	0.97
17-18	0.9	4	0	0	0.36
18-19	1	0	2	0	0
18-21	1.28	3	0	0	0.55
18-30	1.22	2	0	0	0
19-20	1.12	3	0	0	0
19-22	1.15	1	0	0	0.57
20-23	1.3	4	0	0	0
20-24	1.35	3	0	0	0
21-22	1.4	0	0	1	0.63
21-31	1.39	0	0	0	0.92
21-33	1.29	0	2	0	0.92
22-29	1.23	0	0	0	0
23-25	2.1	0	0	0	0
24-25	1.4	0	0	2	0
24-26	1.6	0	0	0	0
25-27	1.5	0	0	0	0
26-27	1.57	0	0	2	0.76
26-29	1.05	4	0	0	0.75
26-37	4	3	0	0	0
27-28	1.9	0	0	0	0
28-37	3	0	0	0	0
28-38	3.5	0	0	0	0
29-35	2	0	0	0	0.93
30-31	2.3	3	0	0	0.95
31-32	1.8	3	0	0	0.93
32-33	2	5	0	0	0.93
32-45	2.2	0	0	0	0
33-34	1.7	3	0	0	0.89
34-35	1.82	4	0	0	0.89
34-44	2	2	0	0	0.68
35-36	1.5	2	0	0	0.89

36-37	1.4	3	0	0	0.9
36-41	2.7	0	0	0	0
36-44	1.7	2	0	0	0.87
37-40	3.7	0	0	0	0
38-39	1.9	0	0	0	0
39-40	1.7	1	0	0	0.56
39-85	2.1	0	0	0	0
40-41	1.82	1	0	0	0.6
40-65	1.95	0	0	0	0
41-42	1.67	3	0	0	0.96
41-43	1.32	2	0	0	0.87
42-61	1.65	4	0	0	0.95
42-65	0.89	0	0	0	0
43-44	1.32	3	0	0	0.93
44-46	1.35	2	0	0	0.89
45-46	1.4	0	0	0	0
45-47	1.6	3	0	0	0.79
46-61	1.62	4	0	0	0.95
47-48	1.45	3	0	0	0.9
47-50	1.7	0	0	0	0
47-60	1.89	0	0	0	0
48-49	1.43	4	0	0	0.97
48-58	1.05	3	0	0	0.89
49-50	0.89	2	0	0	0.88
51-55	1.97	0	0	0	0
52-53	1.9	2	0	0	0.89
52-54	1.9	2	0	0	0.93
53-55	1.82	0	0	0	0
53-56	1.89	4	0	0	0.94
54-56	2.2	2	0	0	0.9
54-57	0.93	4	0	0	0.92
55-66	1.1	0	0	1	0.5
56-66	1.25	0	0	0	0
56-70	1.19	0	0	0	0
57-58	0.65	2	0	0	0.78
57-70	1.8	0	0	0	0
58-59	1.1	0	1	0	0.78
58-72	1.4	0	2	0	0.57
59-60	1.4	0	0	0	0
59-79	1.62	0	0	3	0.78
60-61	1.63	0	0	0	0
60-62	1.4	0	0	0	0
62-63	1.3	0	0	0	0
62-80	1.6	0	2	0	0.76
63-64	1	0	0	0	0
64-65	0.9	0	0	0	0
64-82	1.2	2	0	0	0.9
64-87	1.3	0	0	0	0
65-84	1.7	0	0	0	0

66-67	1.02	0	0	0	0
66-69	1.02	0	1	0	0.7
67-68	1.9	0	0	0	0
68-76	1.79	0	0	0	0
69-70	1.76	0	0	2	0.67
69-71	1.69	0	0	0	0
69-75	1.95	4	0	0	0.89
70-72	1.05	0	1	0	0.78
71-73	2.3	0	0	0	0
72-73	1.21	0	2	0	0.74
73-74	1.25	0	0	1	0.32
74-75	2.52	0	0	0	0
74-77	2.1	0	0	2	0.56
74-80	2.9	0	0	3	0.67
75-77	1.92	4	0	0	0.97
76-77	1.68	0	0	0	0
77-78	2.6	4	0	0	0.96
78-81	2	2	0	0	0.92
79-80	1.97	0	2	0	0.59
80-81	2.01	4	0	0	0.95
81-82	2	4	0	0	0.94
81-83	2.32	4	0	0	0.96
82-83	1.87	3	0	0	0.89
83-87	1.62	0	0	0	0
84-85	1.82	0	0	0	0
84-86	1.3	0	0	0	0
86-87	2.05	0	0	0	0

TABLE V
NODE PARAMETERS

Node Number	Node Types	Load/MW	Active Output/MW
1	pq	0	0
2	pq	0	0
3	pq	342	0
4	pq	385	0
5	pq	0	0
6	pq	0	0
7	pq	233.8	0
8	pq	268.5	0
9	pq	0	0
10	pq	0	0
11	pq	0	0
12	pq	287.5	0
13	pq	0	0
14	pq	0	0
15	pq	220	0
16	pq	329	0
17	pq	0	0
18	pq	258	0
19	pq	0	0

20	pq	228	0
21	pq	274	0
22	pq	385	0
23	pq	247.5	0
24	pq	368.6	0
25	pq	324	0
26	pq	339	0
27	pq	261.5	0
28	pq	306	0
29	pq	583.5	0
30	pq	0	600
31	vθ	0	0
32	pv	9.5	750
33	pv	0	850
34	pv	0	508
35	pv	0	650
36	pv	0	250
37	pv	0	540
38	pv	0	830
39	pv	0	868

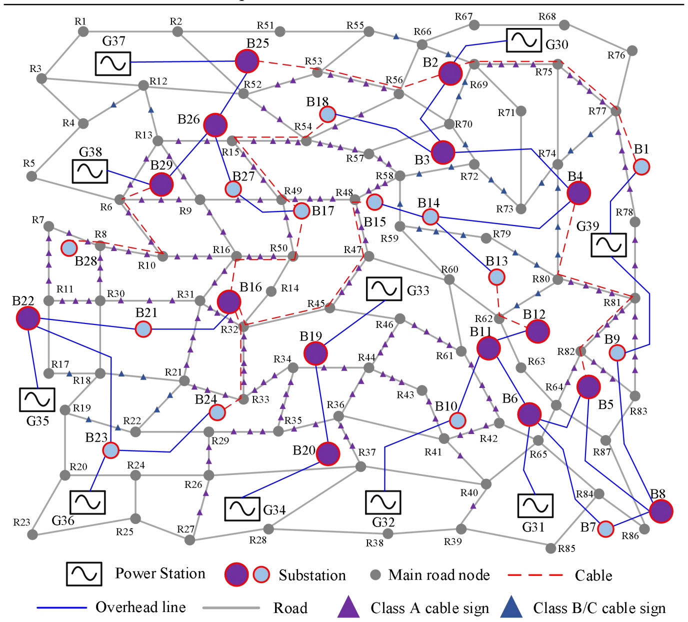

Fig. B1 The coupling network between UPN and the road network of the case.