UTRECHT UNIVERSITY

ENERGY IN THE BUILT ENVIRONMENT
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# Computer Practical Assignment: Utrecht Smart District

*Part B: Home Energy Management*


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# 1 Home Energy Optimisation: Problem Formulation

An optimisation was devised in order to minimise household electricity costs over a sample three day period in summer and winter. The objective was to minimise electricity costs while still satisfying household demand. The optimisation model, including constraints, is elucidated further below.

The summer period modelled was July 7-10 and the winter period was January 21-24, 2018.

## 1.1 Input Parameters and Decision Variables

The continuous input parameters are the residential load, solar PV generation and electricity price for each 15 minute interval across a 3 day period. Each time step (t) therefore represents a 15 minutes for a total of 288 ($4 \cdot 24 \cdot 3$) time steps. Data is provided for 3 days in summer and 3 days in winter. The battery and grid also have discrete parameters. These are summarised in table 1.

*Table 1: Input Parameters*

| **Input Parameters** | **Symbol** | **Value** |
|---|---|---|
| Residential Load [kW] | $\mathrm{P_{dem}}$ | *per time step* |
| PV Generation [kW] | $\mathrm{P_{PV}}$ | *per time step* |
| Electricity Price [€/kWh] | $\mathrm{C_{elec}}$ | *per time step* |
| Maximum Grid Power [kW] | $P_{\mathrm{grid,max}}$ | 3 |
| Battery Efficiency [%] | $\eta_{\mathrm{ch/dis}}$ | 94 |
| Battery Capacity [kWh] | $\mathrm{C_{bat}}$ | 13.5 |
| Maximum Battery Power [kW] | $P_{\mathrm{bat,max}}$ | 4 |
| Minimum SoC [%] | $\mathrm{SoC_{min}}$ | 20 |
| Maximum SoC [%] | $\mathrm{SoC_{max}}$ | 100 |
| Initial SoC [%] | $\mathrm{SoC_0}$ | 50 |

The decision variables are all continuous and are the power supplied to or from the grid, the amount of charging or discharging of the battery and the battery state of charge (SoC). These are summarised in table 2.

*Table 2: Decision Variables*

| **Decision Variables** | Symbol |
|---|---|
| Grid Power [kW] | $\mathrm{P_{grid}}$ |
| Battery Charging Power [kW] | $\mathrm{P_{bat,ch}}$ |
| Battery Discharging Power [kW] | $\mathrm{P_{bat,dis}}$ |
| Battery SoC [%] | SoC |

Note that although the grid power can be negative, to denote surplus power being sold to the grid, battery charging and discharging are considered separate, positive variables to allow for efficiency losses to be correctly modelled. The battery charging and discharging is modelled at the point of the battery, not the grid, hence efficiency losses are accounted for within the SoC Dynamic constraint and not the Power Balance constraint.

## 1.2 Objective Function

The objective function and constraints are stated below. Since t represents a 15 minute interval, $\Delta T = 0.25$, representing a quarter hour. SoC for a given time step is the SoC at the end of that time step.

**minimize** $\sum_{t=1}^{T} C_{\text{elec},t} \cdot P_{\text{grid},t} \cdot \Delta T$

**subject to**

$P_{\text{dem},t} = P_{\text{grid},t} + P_{\text{PV},t} + P_{\text{bat,dis},t} - P_{\text{bat,ch},t}, \ : \forall\, t$ **Power Balance**

$0 \leq P_{\text{bat},t} \leq \ P_{\text{bat,max}}, \ : \forall\, t$ **Battery Power**

$-P_{\text{grid,max}} \leq P_{\text{grid},t} \leq \ P_{\text{grid,max}}, \ : \forall\, t$ **Grid Power**

$SoC_{\text{min}} \ \leq SoC_t \leq \ SoC_{\text{max}}, \ : \forall\, t$ **SoC Static**

$$SoC_t = \begin{cases} SOC_0 + \frac{P_{\text{bat,ch},t} \cdot \Delta T \cdot \eta_{\text{ch}}}{C_{\text{bat}}} - \frac{P_{\text{bat,dis},t} \cdot \Delta T}{C_{\text{bat}} \cdot \eta_{\text{dis}}} & : t = 1 \\ SOC_{t-1} + \frac{P_{\text{bat,ch},t} \cdot \Delta T \cdot \eta_{\text{ch}}}{C_{\text{bat}}} - \frac{P_{\text{bat,dis},t} \cdot \Delta T}{C_{\text{bat}} \cdot \eta_{\text{dis}}} & : t = 1 - T \end{cases}$$ **SoC Dynamic**

## 1.3 Constraint Definitions

The Power Balance constraint shows that for every time step, the supply must match the demand. This means that the residual load for the household must always be matched by the sum of purchased/sold grid power, PV generation, and any battery charging or discharging.

The Battery Power constraint shows that the battery charging or discharging rate cannot exceed the rated power of the battery. Note again that battery power is calculated at the battery, not at the grid, and that both are modelled as positive variables, with battery discharging being added and charging being subtracted in the Power Balance constraint.

The Grid Power constraint shows that the grid power cannot exceed the maximum grid capacity either when electricity is being sold to (represented as negative) or bought from (represented as positive) the grid.

The SoC Static constraint shows that the SoC must be between a minimum of 20 and maximum of 100% for each time step.

The SoC Dynamic constraint shows that for each time step, the SoC must be equal to the previous time step plus any battery charging or discharging effects of the previous time.

## 1.4 Optimisation Class

This optimisation problem can be classified as a linear optimization problem. An optimisation problem can be either linear, mixed-integer linear, quadratic or non-convex. Since all variables are continuous and both the objective function and the constraints are linear, this is a linear optimisation problem.

# 2 Optimisation Results

## 2.1 Summer and Winter Minimum Costs

In the summer, the minimal electricity costs for July 7 to 10 is -€0.50. This means that for the 3 day period the minimum possible 'cost' is to earn €0.50 by selling electricity to the grid.

In the winter, the investigated period of January 21 to 24 has a minimum electricity cost of €0.44.

## 2.2 Decision Variable Outputs and Active Constraint Assessment

In order to better present the battery behaviour graphically, $P_{\text{bat}}$ is determined as per 2.

$$P_{\text{bat}} = P_{\text{bat,ch}} - P_{\text{bat,dis}} \tag{2}$$

As per equation 2, a positive value for $P_{\text{bat}}$ now represents the battery power when the battery is charging and a negative value represents the batter power when the battery is discharging. The battery power is represented alongside the grid power and electricity price in figure 1.

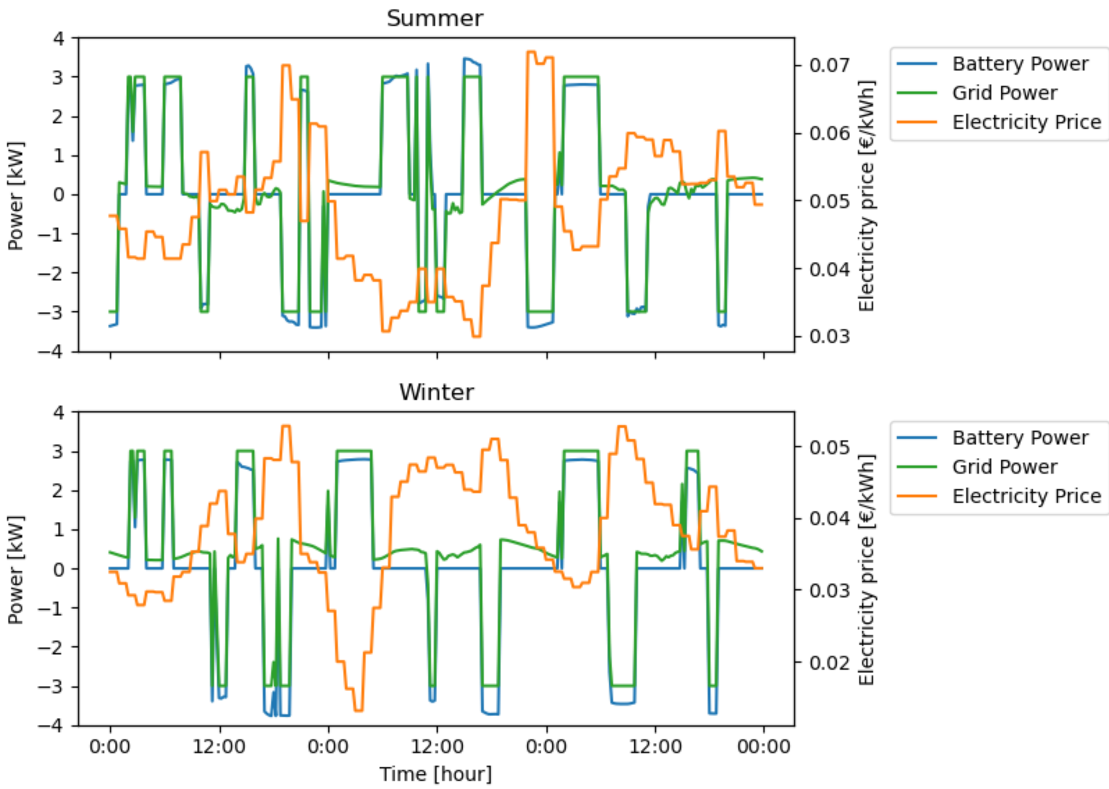


*Figure 1: Battery and Grid Power with Electricity Price across the 3 day period in Summer and Winter*

From the figure it can be seen that the battery power constraints are never reached, since $P_{\text{bat,max}}$ is equal to 4, and the maximum battery power is seen in the winter period on January

21 at 17:45. The power at this time is - 3.77 kW representing 3.77 kW of discharging. On the other hand, the maximum grid constraint of +/- 3 kW is frequently reached, the first occurrence in summer being the first time step, where 3 kW are sold to the grid, followed shortly by 3 kW being bought from the grid 2 hours later, at 02:00.

Figure 1 also demonstrates the relationship between electricity price and power flow from the battery and grid. There is a clear relationship between low electricity prices leading to battery charging and electricity being bought from the grid, and high electricity prices leading to battery discharging and electricity being sold to the grid. Graphically, you see this as the orange line being an inverse pattern to the green and blue.

At times, you would expect deviation from this correlation between battery power and electricity prices, when the SoC constraints are met and therefore further charging/discharging is not possible. It does not occur here so much since the model assumes perfect forecasting, as such, it can predict when it is best to charge or discharge fully for maximum cost benefit. Figure 2 shows the activation of the SoC static constraint, which occurs frequently in summer and winter, whenever the SoC is 20% or 100%, the constraint is active.

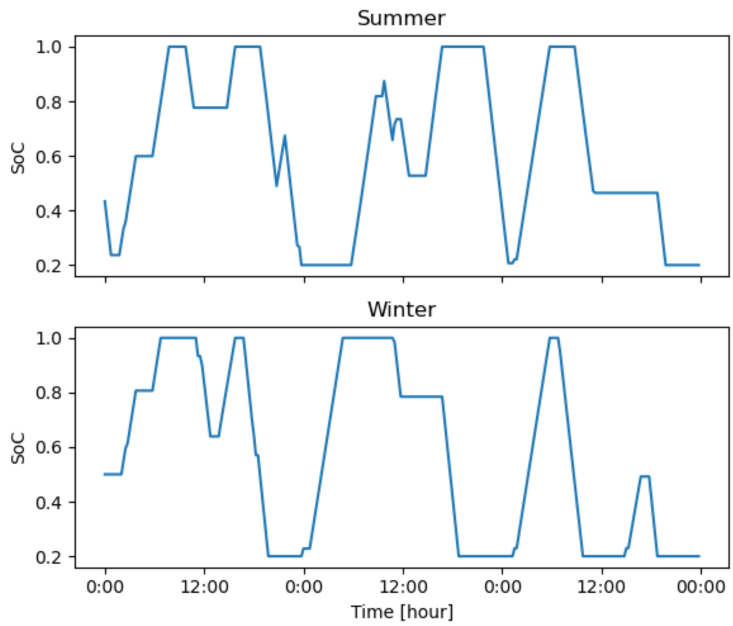


*Figure 2: Battery State of Charge through Summer and Winter periods*

The reason the battery and grid power decisions are based so closely on the electricity prices is demonstrated in figure 3. As can be seen, the household power demand and the PV generation are marginal compared with the battery and grid power, therefore, will have little affect on the battery and grid in comparison with the electricity price.

As expected, greater PV generation is seen in summer than winter, and conversely, greater demand in seen in winter than summer. When PV generation is high, it can be used to satisfy some or all of demand, this occurs more so in summer where there are comparable magnitudes of demand and generation, and the battery storage can account for some of the temporal mismatch. In winter on the other hand, the magnitudes of power being bought from the grid are greater due to the greater demand compared with generation.

Finally, note that both the Power Demand and SoC Dynamic constraints must be active for all time steps since they are equality rather than inequality constraints.

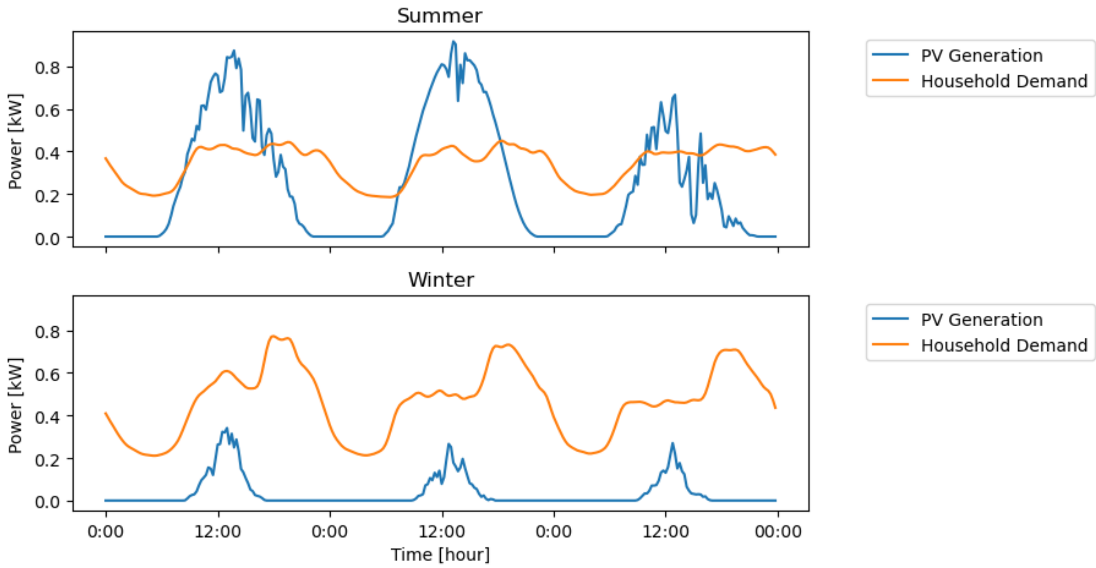


*Figure 3: Household power demand and PV generation*

# 3 Multi-Objective Optimisation

## 3.1 Pareto Frontiers

To create the Pareto Frontier, the optimisation problem is run for both summer and winter without additional constraints, to find the cost minimum. This minimum is then recorded. Next, a new objective function is created, with the same variables and constraints as previously described, but with the electricity cost being substituted for emissions in kg $CO_2$-eq. In this way, the minimum emissions are obtained.

It is worth noting here that the cost associated with the minimum possible emissions will not necessarily be the same as when we later use this minimum emissions value to minimise for cost. This is because there are multiple possible pathways to the minimum emissions value, therefore, the cost given when optimising for minimum emissions is not necessarily the minimum possible cost for that amount of emissions.

Eight more emissions values are then produced between the two extremes of the emissions associated with minimum cost, and the minimum possible emissions to form a total of ten equally spaced bins. These values are used as the emissions constraints to be used for 10 schedules varying from cheapest to cleanest. The schedules 2 to 9 will be examples where either costs and emissions are minimised to varying degrees, therefore, to find a balance between cost and emissions, the median emissions value was also obtained, and this is used as the constraint for a 'balanced' optimisation schedule. These are summarised in table 3.

The cost optimisation is run with the emissions constraints at each bin value and the results are plotted in figure 4.

*Table 3: Emissions Constraints in kg $CO_2$-eq*

| **Optimisation** | **Bin** | **Summer** | **Winter** |
|---|---|---|---|
| **Cheapest** | **1** | 12.89 | 26.78 |
| | **2** | 9.86 | 23.88 |
| | **3** | 6.84 | 20.98 |
| | **4** | 3.81 | 18.07 |
| | **5** | 0.78 | 15.17 |
| **Balanced** | **5.5** | - 0.73 | 13.72 |
| | **6** | -2.24 | 12.27 |
| | **7** | -5.27 | 9.37 |
| | **8** | -8.30 | 6.47 |
| | **9** | -11.32 | 3.57 |
| **Cleanest** | **10** | -14.35 | 0.67 |

Figure 4 shows the key results of the trade-offs between the costs and emissions for the summer and winter. Where the upper left ends of the lines represent the cleanest schedules and the lower right ends of the lines are the cheapest schedules. As expected, there is a balance between minimum cost and minimum emissions whereby as you constrain emissions further, costs increase more. This is the same for both summer and winter.

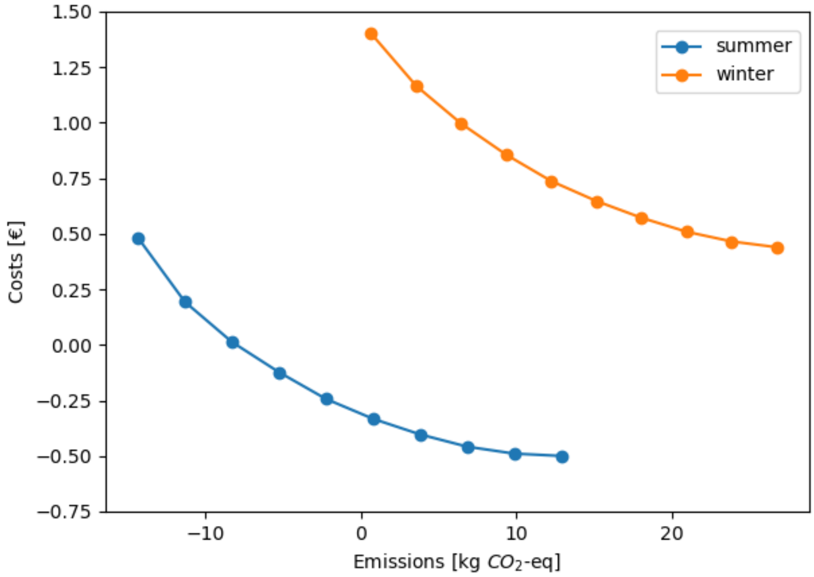


*Figure 4: Pareto frontier for emissions versus costs for the summer and the winter*

*Table 4: Optimisation Schedules for Summer and Winter*

| Optimisation | Emissions [kg $CO_2$-eq] | Electricity Costs [€] |
|---|---|---|
| **Summer** | | |
| **Cheapest** | 12.89 | - 0.50 |
| **Balanced** | - 0.73 | - 0.29 |
| **Cleanest** | - 14.35 | 0.48 |
| **Winter** | | |
| **Cheapest** | 26.78 | 0.44 |
| **Balanced** | 13.72 | 0.69 |
| **Cleanest** | 0.67 | 1.44 |

Table 4 shows the corresponding electricity costs and emission values for the cleanest, balanced and cheapest schedules in summer and winter. As can be seen from the table, running the cheapest schedule in summer would enable the household to make money from selling the locally produced energy back to the grid. On the other hand, when running the cleanest schedule the household reduces their $CO_2$-eq emissions by 211.44%. While this leads to avoided $CO_2$-eq emissions, it also comes at the cost of losing the potential to make revenue from selling electricity back to the grid. Fortunately, the household would still able to create revenue while also preventing $CO_2$-eq emissions when running a balanced schedule.

For the winter, the cheapest schedule leads to a cost reduction of 69.44% compared to cleanest schedule. While the cleanest schedule leads to an emission reduction of 97.5% in relation to the cheapest schedule. However, both the cheapest and the cleanest schedule are not able to generate revenue or result in avoided $CO_2$-eq emissions. Since the more that emissions are reduced, the greater the cost increase will be, the balanced schedule provides a good option since the emissions can be halved for only a 57% increase in cost, compared with a 277% cost increase to achieve the cleanest schedule.

From figure 4, it is also clear that winter is more expensive and produces more emissions than in summer. This is to be expected, as there is greater PV generation and lower energy demand in the summer than in the winter, as demonstrated in figure 3, for large parts of the day generation exceeded consumption so cost and emission-free power can be supplied to the house and stored in the battery, leading to reduced emissions and reduced costs.

# 4 Reflection on the Model

In the current model, it is assumed that the battery will not degrade. In reality the performance of a battery will decline over its lifetime (Xu et al., 2016). When a battery is used more, charging/discharging efficiencies and the capacity will decrease. The battery degradation per cycle can be determined, note this is a non-linear function but can be incorporated into the linear optimisation problem as piece-wise linear functions using multiple segments (Brinkel et al., 2020) Testing a reduced battery capacity in the current model to crudely mimic the effects of battery degradation, we find that emissions decrease and costs increases as the battery capacity decreases. This is to be expected when minimising for cost, since less battery capacity gives less storage capacity for clean solar PV generation as well as electricity to buy and sell at strategic times for cost minimisation.

Another aspect which is not incorporated in this model, is the fact that the battery does not always charge at the same rate. A battery which with a SoC of 50% has a higher electricity inflow than a battery with a SoC of 90%. Currently, it is assumed that the battery charges and discharges at constant rates, independent of SoC. Implementing the charge rates as a function of SoC into the model would therefore produce more realistic battery schedules. The SoC needs to be updated for each time step and this SoC should then be used to define the actual charge/discharge rate of the battery at that time step. If this were incorporated, the model results would probably be less favourable, since currently the battery can charge/discharge at maximum rates at any SoC as required. With this incorporated the charge/discharge rate will be less than its optimum for some time steps. As a result, the model becomes more realistic.

# 5 References

Brinkel, N. B. G., Schram, W. L., AlSkaif, T. A., Lampropoulos, I., & van Sark, W. G. J. H. M. (2020). Should we reinforce the grid? Cost and emission optimization of electric vehicle charging under different transformer limits. Applied Energy, 276.
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Xu, B., Oudalov, A., Ulbig, A., Andersson, G., Kirschen, D. S. (2016). Modelling of lithium-ion battery degradation for cell life assessment. IEEE Transactions on Smart Grid, 9(2), 1131-1140