Accuracy Round

EMCC

February 2021

1. Evaluate $1^2 - 2^2 + 3^2 - 4^2 + \cdots + 19^2 - 20^2 + 21^2$.

2. Kevin is playing in a table-tennis championship against Vincent. Kevin wins the championship if he wins two matches against Vincent, while Vincent must win three matches to win the championship. Given that both players have a 50% chance of winning each match and there are no ties, the probability that Vincent loses the championship can be written in the form $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

3. For how many positive integers n less than 2000 is n^{3n} a perfect fourth power?

4. Given that a coin of radius $\sqrt{3}$ cm is tossed randomly onto a plane tiled by regular hexagons of side length 14 cm, the chance that it lands strictly inside of a hexagon can be written in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

5. Given that A, C, E, I, P, and M are distinct nonzero digits such that $\overline{EPIC} + \overline{EMCC} + \overline{AMC} = \overline{PEACE}$, what is the least possible value of $\overline{PEACE}$?

6. A *palindrome* is a number that reads the same forwards and backwards. Call a number *palindrome-ish* if it is not a palindrome but we can make it a palindrome by changing one digit (we cannot change the first digit to zero). For instance, 4009 is palindrome-ish because we can change the 4 to a 9. How many palindrome-ish four-digit numbers are there?

7. Given that the heights of triangle ABC have lengths $\frac{15}{7}$, 5, and 3, what is the square of the area of ABC?

8. Suppose that cubic polynomial $P(x)$ has leading coefficient 1 and three distinct real roots in the interval $[-20, 2]$. Given that the equation $P(x + \frac{1}{x}) = 0$ has exactly two distinct real solutions, the range of values that $P(3)$ can take is the open interval (a, b). Compute $b - a$.

9. Vincent the Bug has 17 students in his class lined up in a row. Every day, starting on January 1, 2021, he performs the same series of swaps between adjacent students. One example of a series of swaps is: swap the 4th and the 5th students, then swap the 2nd and the 3rd, then the 3rd and the 4th. He repeats this series of swaps every day until the students are in the same arrangement as on January 1. What is the greatest number of days this process could take?

10. The summation
$$\sum_{i=1}^{18} \frac{1}{i}$$
can be written in the form $\frac{a}{b}$, where a and b are relatively prime positive integers. Compute the number of divisors of b.


MATH
EXETER
CLUB