

EMCC 2025

Speed Test



January 25, 2025

Do not open the booklet until you are instructed to do so.

This is the Speed Round of the EMCC. There are 20 problems, worth 3 points each, to be solved in 25 minutes. There is no penalty for guessing. As with all other rounds, calculators, graph paper, lined paper, rulers, protractors and compasses are prohibited.

The answer to a problem may not necessarily be an integer. See the provided *Acceptable Forms of Answers* sheet for a breakdown of correct and incorrect ways to express an answer.

The opposite side of this page contains the answer form. Once you are instructed to begin the test, tear this page off of the booklet. At the conclusion of the Speed Round, only this page will be collected. Anything written elsewhere on the booklet will not be read or scored.

Best of luck!

Name: _____ Team: _____

ID #: ____ - ____

Speed Test Answer Form

Tear this page off the rest of the booklet; this is the only sheet of paper that will be collected. Make sure that all identifying information has been filled in on the other side of this page.

Please write legibly!

1. _____

11. _____

2. _____

12. _____

3. _____

13. _____

4. _____

14. _____

5. _____

15. _____

6. _____

16. _____

7. _____

17. _____

8. _____

18. _____

9. _____

19. _____

10. _____

20. _____

Speed Test

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There are 20 problems, worth 3 points each, to be solved in 25 minutes. Answers must be simplified and exact unless otherwise specified. There is no penalty for guessing.

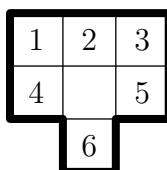
1. What real number $x \geq -1$ satisfies the equation below?

$$1 + \sqrt{x+1} = x + \sqrt{1+x}$$

2. What is

$$\sqrt{20+25} \times \sqrt{20 \times 25}?$$

3. A polygon is made from seven squares. One of the labeled squares can be cut out to increase the polygon's perimeter. What is this square's label?



4. The average of two numbers is half of one of the numbers. What is the product of the two numbers?

5. Andrew splits the six numbers

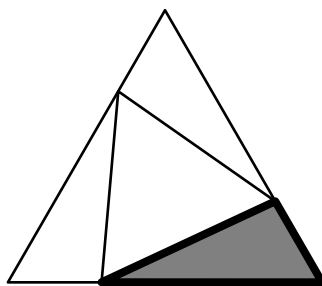
$$1, 1, 2, 2, 3, 3$$

into three pairs so that no pair contains two of the same number. For each pair, he multiplies the two numbers in the pair together. What is the sum of his three products?

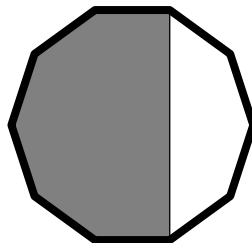
6. What is the value of

$$\gcd(1, 9) \times \gcd(2, 8) \times \gcd(3, 7) \times \cdots \times \gcd(9, 1)?$$

7. For Ms. Jefferson's spelling test, students will be quizzed on 25 words from a 100-word bank. What is the fewest number of words a student must learn from the bank to guarantee at least 15 correct answers on the quiz?
8. A rectangle has an area of 26. Each side of this rectangle is decreased by 2 units, forming a new rectangle with positive side lengths. What is the sum of the area and perimeter of this new rectangle?
9. What positive integer has exactly 5 positive divisors and a units digit of 5?
10. An equilateral triangle with an edge length of 5 is inscribed inside an equilateral triangle with an edge length of 7, as shown in the diagram below. What is the perimeter of the shaded triangle?



11. Let a and b be integers such that $a < 2025 < b$. If 2025 is closer to b than a , what is the maximum possible value of $a + b$?
12. An analog clock is stopped at a random time in the day. To the nearest integer percent, what is the chance that the angle between the minute hand and hour hand is obtuse?
13. In the diagram below, what fraction of the regular 10-sided polygon is shaded?



14. At the Las Olas Taqueria, customers may order a bowl with one of 6 proteins and three different toppings from a choice of 24 toppings total. Albert is allergic to some of the proteins and toppings; there are only 2800 distinct bowls he may order under his dietary restrictions. What is the combined number of proteins and toppings that Albert can safely eat?
15. Triangle ABC satisfies $\angle A = 110^\circ$. Points D and E lie on side \overline{BC} such that $AB = BD$ and $AC = CE$. What is the measure of $\angle DAE$, in degrees?
16. Harini chooses a geometric sequence of six nonzero real numbers. The sum of these numbers is 20 and the sum of their reciprocals is 25. What is the product of these numbers?
17. Let x and y be positive reals such that

$$\begin{cases} x^x = \sqrt[y]{2^{20}}, \\ y^y = \sqrt[x]{2^4}. \end{cases}$$

What is $x + y$?

18. Michael rolls n fair 6-sided dice. What should n be as to maximize the probability that the product of the numbers he rolls is 2025?
19. Quadrilateral $ABCD$ is inscribed in a circle with center O , such that O lies inside the quadrilateral. If the distances from O to sides \overline{AB} , \overline{BC} , \overline{CD} and \overline{DA} are 5, 4, 3 and 4, respectively, what is the area of $ABCD$?

20. How many ways are there to partition a four by four chessboard into polyominoes, each of which contains at most one white square? Partitions which differ by a rotation or reflection are considered distinct.

(A *polyomino* is any connected region consisting of unit squares. One example of a valid partition is shown below.)

