Speed Round

EMCC

February 2021

1. Evaluate $20 \cdot 21 + 2021$.
2. Let points A, B, C, and D lie on a line in that order. Given that $AB = 5CD$ and $BD = 2BC$, compute $\frac{AC}{BD}$.
3. There are 18 students in Vincent the Bug's math class. Given that 11 of the students take U.S. History, 15 of the students take English, and 2 of the students take neither, how many students take both U.S. History and English?
4. Among all pairs of positive integers (x, y) such that $xy = 12$, what is the least possible value of $x + y$?
5. What is the smallest positive integer n such that $n! + 1$ is composite?
6. How many ordered triples of positive integers (a, b, c) are there such that $a + b + c = 6$?
7. Thomas orders some pizzas and splits each into 8 slices. Hungry Yunseo eats one slice and then finds that she is able to distribute all the remaining slices equally among the 29 other math club students. What is the fewest number of pizzas that Thomas could have ordered?
8. Stephanie has two distinct prime numbers a and b such that $a^2 - 9b^2$ is also a prime. Compute $a + b$.
9. Let $ABCD$ be a unit square and E be a point on diagonal AC such that $AE = 1$. Compute $\angle BED$, in degrees.
10. Sheldon wants to trace each edge of a cube exactly once with a pen. What is the fewest number of continuous strokes that he needs to make? A continuous stroke is one that goes along the edges and does not leave the surface of the cube.
11. In base b, 130_b is equal to $3n$ in base ten, and 1300_b is equal to n^2 in base ten. What is the value of n, expressed in base ten?
12. Lin is writing a book with n pages, numbered $1, 2, \ldots, n$. Given that $n > 20$, what is the least value of n such that the average number of digits of the page numbers is an integer?
13. Max is playing bingo on a 5×5 board. He needs to fill in four of the twelve rows, columns, and main diagonals of his bingo board to win. What is the minimum number of boxes he needs to fill in to win?
14. Given that x and y are distinct real numbers such that $x^2 + y = y^2 + x = 211$, compute the value of $|x - y|$.

15. How many ways are there to place 8 indistinguishable pieces on a 4×4 checkerboard such that there are two pieces in each row and two pieces in each column?

16. The *Manhattan distance* between two points (a, b) and (c, d) in the plane is defined to be $|a - c| + |b - d|$. Suppose Neil, Neel, and Nail are at the points $(5, 3)$, $(-2, -2)$ and $(6, 0)$, respectively, and wish to meet at a point (x, y) such that their Manhattan distances to (x, y) are equal. Find $10x + y$.

17. How many positive integers that have a composite number of divisors are there between 1 and 100, inclusive?

18. Find the number of distinct roots of the polynomial

$$(x-1)(x-2)\cdots(x-90)(x^2-1)(x^2-2)\cdots(x^2-90)(x^4-1)(x^4-2)\cdots(x^4-90).$$

19. In triangle ABC, let D be the foot of the altitude from A to BC. Let P, Q be points on AB, AC, respectively, such that PQ is parallel to BC and $\angle PDQ = 90°$. Given that $AD = 25$, $BD = 9$, and $CD = 16$, compute $111 \cdot PQ$.

20. The simplified fraction with numerator less than 1000 that is closest but not equal to $\frac{47}{18}$ is $\frac{p}{q}$, where p and q are relatively prime positive integers. Compute p.