EMCC 2025

Team Test

January 25, 2025

Do not open the booklet until you are instructed to do so.

This is the Team Round of the EMCC. There are 15 problems, worth 20 points each, to be solved in 60 minutes. There is no penalty for guessing. As with all other rounds, calculators, graph paper, lined paper, rulers, protractors and compasses are prohibited.

The answer to a problem may not necessarily be an integer. See the provided *Acceptable Forms of Answers* sheet for a breakdown of correct and incorrect ways to express an answer.

The opposite side of this page contains the answer form that your team will turn in. Once you are instructed to begin the test, tear this page off of the booklet. At the conclusion of the Team Round, only this page will be collected. Anything written elsewhere on any of the team's four booklets will not be read or scored. In particular, the three other copies of the team test are marked "unofficial", and do not have an answer sheet attached.

Best of luck!

Team name: ________________________________

Team ID #: ____ ____ ____

Team Test Answer Form

Tear this page off the rest of the booklet; this is the only sheet of paper that will be collected. Make sure that all identifying information has been filled in on the other side of this page.

Please write legibly!

1. ______________________
2. ______________________
3. ______________________
4. ______________________
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12. ______________________
13. ______________________
14. ______________________
15. ______________________

Team Test

January 25, 2025

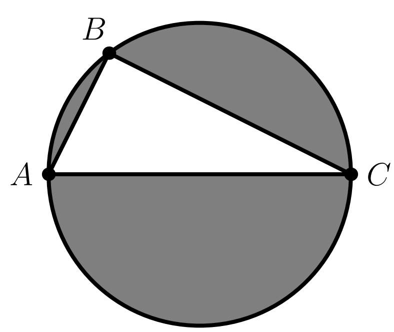
There are 15 problems, worth 20 points each, to be solved in 60 minutes. Answers must be simplified and exact unless otherwise specified. There is no penalty for guessing.

1. Students and chaperones board a school bus with a capacity of 50. Each student is assigned to a chaperone; each chaperone is assigned up to 7 students. At most how many students may board the bus?

2. How many ordered triples (x, y, z) of real numbers satisfy the system of equations shown below?

$$\begin{cases} x = y \cdot z \\ y = z \cdot x \\ z = x \cdot y \end{cases}$$

3. Triangle ABC with $\angle B = 90^\circ$ is inscribed in a circle, as shown in the diagram below. The area of the shaded region is 176π and the area of the triangle is 80π. What is the distance from B to line AC?



4. A billboard reads

Perfect squares such as $\boxed{2}\boxed{0}\boxed{2}\boxed{5}$ are cool.

A prankster wants to pick exactly one digit on the billboard and replace it with any one of the nine other digits, such that the number on the billboard is no longer a perfect square. How many ways can the prankster do this?

5. Phillips Exeter and Phillips Andover pit their finest racehorses against each other in a derby on Front Street. The results of the top horses are as follows:

#	Name	Team
1	Blaze	Andover
2	Storm	Exeter
3	Comet	Exeter
4	Dash	Exeter
5	Echo	Exeter
6	Nova	Andover
7	Spirit	Andover

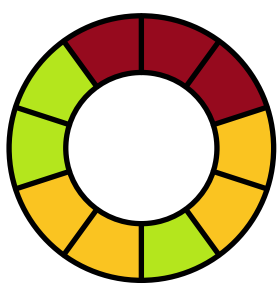
#	Name	Team
8	Jett	Andover
9	Shadow	Andover
10	Vortex	Exeter
11	Zephyr	Exeter
12	Ember	Exeter
13	**Yash**	Andover
14	Titan	Andover

$\boxed{\textbf{Exeter: 24}}$ **Andover: 31**

As shown above, the score of each team is the sum of the ranks of their top five horses (so, Exeter is the winner here since they have a lower score). At least how many positions faster would Yash have had to have placed so that Andover's score was lower than Exeter's score?

6. Suppose n is a positive integer such that the sum of the digits of n is 100 and the sum of the digits of $11n$ is 2. How many nonzero digits does n have?

7. A ring is divided into ten equal sectors. Each sector is randomly shaded with one of ten possible colors, selected uniformly and independently. What is the expected number of connected regions in the resulting figure?
(For example, in the diagram below, three colors are used, resulting in five connected regions.)



8. In $\triangle ABC$, point D lies on side $\overline{BC}$ such that $\overline{AD} \perp \overline{BC}$. Suppose that the lengths AD, AB, AC and BC form an increasing geometric sequence with common ratio r. What is r^2?

9. Let $\mathcal{P}$ be a regular 40 sided polygon. What fraction of the diagonals of $\mathcal{P}$ are longer than the circumradius of $\mathcal{P}$?

10. Julie draws a square with an area of 6. How many ways can she split it into three triangles with areas 1, 2 and 3?
(Two ways that differ by a rotation and/or reflection are considered distinct.)

11. Define the function $\text{mod}(x, y) = x - y\lfloor x/y \rfloor$. Bryan randomly and uniformly picks a real number x in the interval $[0, 20)$. What is the probability that

$$\lfloor \text{mod}(x, 2.5) \rfloor = \text{mod}(\lfloor x \rfloor, \lfloor 2.5 \rfloor)?$$

12. In an infinite grid of squares, each square is initially colored white. Oron paints n squares red and n squares blue such that:

- No square is painted both colors.
- Any two red squares are connected by a path of red squares. (Any two consecutive squares in the path must be vertically or horizontally adjacent. In particular, they may not be diagonally adjacent.)
- Any two blue squares are connected by a path of blue squares.
- No red square is adjacent to a white square.

What is the minimum possible value of n?

13. Let $\triangle ABC$ be an acute triangle with incenter I such that $AB = AC = 7$. Let D and E be points on sides AB and AC, respectively, such that $AD = AE = 3$. Given that the circumcircle of $\triangle DIE$ is tangent to $\overline{AB}$ and $\overline{AC}$, what is BC?

14. What is the sum of the three smallest positive integers n for which $n^n \times 20^{25}$ is a perfect cube?

15. How many ordered triples (x, y, z) of positive real numbers satisfy the system of equations shown below?

$$\begin{cases} xyz &= 100 \\ xy\lfloor z \rfloor &= 99 \\ x\lfloor yz \rfloor &= 98 \end{cases}$$