Round 1

1. [6] Andrew eats

$$20^{5/2} \cdot 25^{5/20}$$

erasers. How many erasers is this?

2. [6] While eating erasers, Andrew imagines two rectangles with diagonals of lengths 3 and 4, respectively. Their heights are the same and the sum of their widths is 5. What is the sum of their areas?

3. [6] Andrew repeatedly rolls a fair six-sided die, keeping track of the product of all the numbers he has rolled thus far. Whenever this product is prime, he eats an eraser. What is the probability that Andrew never eats an eraser?

Round 2

4. [7] The number 136,279,845 has all digits from 1 to 9, but it is not prime. What digit must be erased and replaced in order to make the number prime?

5. [7] If m and b are real numbers such that the line $y = mx+b$ passes through the points $(-9876, 54321)$ and $(9878, 12345)$, what is $m + b$?

6. [7] Sylvia writes down a string of digits, containing the sub-strings "121", "412", "1234". Interpreted as a base 10 number, what is the smallest possible positive value that Sylvia's string could take?

Round 3

7. [9] Stanley has a cube with a side length of $n > 2$ units. He paints the surface red and cuts the cube into n^3 unit cubes. If seven times as many cubes have exactly one face painted compared to exactly two faces painted, what is n?

8. [9] An escalator moves upwards at 1 step per second, and half of its steps are visible at any time. If Stanley's sister walks up the escalator, taking 3 steps every second, what fraction of the total steps in the escalator does she step on?

9. [9] Stanley's cat draws a convex hexagon $\mathcal{H}$. The area of $\mathcal{H}$ is equal to each of its side lengths. If there exists a circle tangent to each side of $\mathcal{H}$, what is this circle's area?

Round 4

10. [11] A frog is at the point $(1, 1)$. Each second, the frog jumps one unit up, left, down or right at random. It is bounded by the square with vertices $(0, 0)$, $(0, 3)$, $(3, 3)$ and $(3, 0)$. What is the probability that it will touch an edge of the square within the next three jumps?

11. [11] Clara stands at the point $(0, 5)$. She picks a random point with integer coordinates on the circle $x^2 + y^2 = 25$, possibly where she is standing. If d is the distance between Clara and her lattice point, what is the average value of d^2?

12. [11] Benny draws a quadrilateral Q on the coordinate plane. If he stretches the plane vertically (about the x-axis) by a factor of 2, the image of Q is a square; if he instead stretches the plane horizontally (about the y-axis) by a factor of 2, the image of Q is a rhombus. What is the ratio of the side length of the square to the side length of the rhombus?

Round 5

13. [13] Tess plays a gambling game on an 8 by 8 chessboard. Each second, a rook is placed on a randomly chosen empty cell of the chessboard that does not share a row or column with any existing rooks. Once no more rooks can be placed, Tess wins if every rook is on a black square. What is the probability that she wins?

14. [13] Danny is booking several 7-day vacations at a luxury resort on the Strip. He knows that there, the weather cycles between thunderstorms for 60 consecutive days and sunshine for 10 consecutive days. However, he does not currently know the weather there. What is the fewest number of vacations that Danny must book, at this moment, to guarantee that he experiences at least one vacation with sunshine every day? (No two of his booked vacations may overlap.)

15. [13] Terry builds a casino in the shape of a pyramid. The base of this pyramid is a regular pentagon with an area of 2025. The five other faces are congruent isosceles triangles with an area of A. If the angle between one of the triangular faces and the pentagonal base is $60°$, what is A?

Round 6

16. [15] Kevin draws a Christmas tree in the shape of $\triangle ABC$, and draws two point-sized ornaments D and E on segments AB and AC, respectively. Triangle ADE has a perimeter of 11 and an inradius of 1, and quadrilateral $DBCE$ has a perimeter of 31 and an incircle with a radius of 3. What is DE?

17. [15] Harry writes down a three digit positive integer with a nonzero leading digit. Marv notices that if he erases any one of the three digits, the remaining two digits, when read from left to right, form a positive integer that divides Harry's original integer. How many possible integers could Harry have written down?
(It is okay for the remaining two digits to begin with a 0.)

18. [15] A list of positive integers satisfies the following properties:

    (A) The mean of the list is 8.

    (2) The median of the list is 13.

    (D) The mode of the list is 15.

    Moreover, the range of the list is 27. What is the fewest possible number of elements that could be in the list?

Round 7

19. [18] In a concert, 6 singers will perform. Each singer randomly chooses a (possibly empty) subset of the other singers, and requests to perform later than all the singers from that set. Let N be the number of orders of the singers such that all of their wishes are satisfied. What is the expected value of N?

20. [18] How many subsets of $\{1, 2, \ldots, 12\}$ with 5 elements contain no two elements differing by 1, 6 or 11?

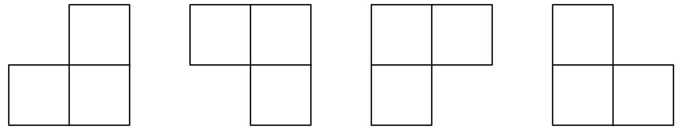
21. [18] Call a positive integer *chromatic* if its leftmost digit is a 1, and each subsequent digit is 0 or 1 greater than the digit immediately to its left. Let N be the sum of all *chromatic* integers less than one billion. What is the largest odd divisor of N?

Round 8

22. [21] Evan doodles a triangle ABC with side lengths $AB = 9$, $BC = 16$ and $AC = 15$. Points D and E lie on $\overline{BC}$ with $BD = 4$ and $EC = 7$. Circle ω passes through D and E and is tangent to $\overline{AB}$. If ω intersects $\overline{AC}$ at X and Y, what is XY?

23. [21] Let k be a rational number. In the coordinate plane, Celine draws a line of slope k through every lattice point. If exactly 1000 distinct lines pass through the interior of the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, and $(0, 1)$, how many possible values of k are there? (The interior of a square does not contain its boundary.)

24. [21] Anika draws a 4 by 6 rectangle. How many ways can she completely tile this rectangle with L-shaped triominoes (shown below) and color each triomino red, green or blue, such that any two neighboring triominoes are different colors? (Two triominoes neighbor if they share a positive amount of perimeter.)



Estimation Round

25. [10] Estimate the highest tentative score S that a team will earn on this problem. The graders will choose the smallest possible correct answer, accurate to the nearest hundredth. An estimate of E will earn a *tentative score* of $10 \cdot 2^{-|S-E|}$. We will finalize scores by rounding your *tentative score* to the nearest integer.

26. [10] Suppose each pet owned by someone on the EMCC Problem Selection Committee is weighed in kilograms. Estimate r, the real number such that the product of all these numerical masses is 10^r. An estimate of E will earn a score of the closest integer to $10 \cdot 3^{-|r-E|}$.

27. [10] Let $s(n)$ denote the sum of the digits of n. Estimate the smallest positive integer N such that among the list

$$s(1^2), s(2^2), \ldots, s(N^2),$$

there are 2025 more odd numbers than even numbers. An estimate of E will earn a score of the closest integer to $10\left(\min\left(\frac{N}{E}, \frac{E}{N}\right)\right)^2$.