Pearl Curve Growth

Nina Chaichi 2017-02-25

"Pearl Curve" also known as "Logistic Curve" is a growth curve frequently used to forecast improvements in technological approaches, the share of market, the share of total installations, or the rate of technology adoption. Pearl curve function is the appropriate function in the case where the progress depends on both distance to go and distance already come.

Formula

Formula for the Pearl Curve is:

$$y = \frac{L}{1 + ae^-bt}$$

- y is the growth variable
- L is the upper limit to the growth of the variable y
- *e* is the base of the natural logarithms
- t is the time
- a is growth curve coefficient
- \bullet b is the growth curve coefficient

Curve Analysis

The curve is symmetrical about the inflection point, with the upper half being a reflection of the lower half. However, since we work with positive time we will get the upper half of the curve.

The reflection point is at

 $t = \frac{\ln(a)}{b}$

when

$$y = \frac{L}{2}$$

In Pearl curve the steepness and location of curve can be controlled independently. Changes in the coefficient a affect the location only, and changes in the coefficient b affect the steepness only. In this package, a and b coefficient can be provided by user or can be estimated from historical data. The following formula illustrate how to estimate a and b based on historical data:

$$Y = \log(\frac{y}{L - y}) = -A + Bt$$

Analogous Pearl Curve

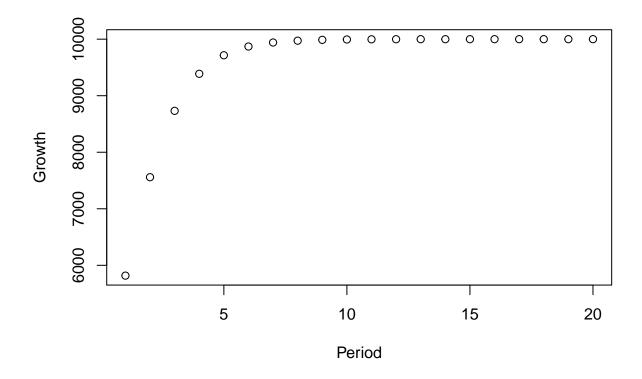
Analogous Preal Curve can provide growth curve based on provided the upper limit of growth, t the period of estimation, and the a and b coefficient according of analogous case. Following is an example of Analogous Pearl Curve. Pearl_AC(1.6, 0.8, 10000, 20) calculates the growth over 20 periods, where the highest growth can be reached is 10000. And, a and b coefficients are 1.6 and 0.8 respectively. Ouput shows the growth for a given period from 1 to 20.

```
library(pander)
pander(Pearl_AC(1.6, 0.8, 10000, 20))
```

period	growth
1	5818
2	7558
3	8732
4	9388
5	9715
6	9870
7	9941
8	9973
9	9988
10	9995
11	9998
12	9999
13	10000
14	10000
15	10000
16	10000
17	10000
18	10000
19	10000
20	10000

 $\label{eq:pearl_AC_Plot} Pearl_AC_Plot \ function \ plots \ the \ result \ of \ Pearl_AC. \ For \ example, \ Pearl_AC_Plot(1.6, \ 0.8, \ 10000, \ 20) \\ plots \ the \ result \ of \ Pearl_AC(1.6, \ 0.8, \ 10000, \ 20).$

Pearl_AC_Plot(1.6, 0.8, 10000, 20)



NULL

Historical Pearl Curve

Following illustrates the growth of cable TV adoption from 1952 through 1989. In 1989, 47800000 housholds in US have been adopted cable tv which is considered 49.2% of all households.

data("CATV")
pander(CATV)

year	$cumulative_growth$	$annual_growth$
1953	14000	16000
1954	30000	35000
1955	65000	85000
1956	150000	150000
1957	3e+05	50000
1958	350000	1e + 05
1959	450000	1e + 05
1960	550000	1e + 05
1961	650000	75000
1962	725000	125000
1963	850000	1e + 05
1964	950000	135000
1965	1085000	190000
1966	1275000	3e + 05

year	cumulative_growth	annual_growth
1967	1575000	525000
1968	2100000	7e + 05
1969	2800000	8e + 05
1970	3600000	9e + 05
1971	4500000	8e + 05
1972	5300000	7e + 05
1973	6e + 06	1300000
1974	7300000	1400000
1975	8700000	1100000
1976	9800000	1e+06
1977	10800000	1100000
1978	11900000	1100000
1979	1.3e + 07	1100000
1980	14100000	1400000
1981	15500000	2800000
1982	18300000	2700000
1983	2.1e+07	4e + 06
1984	2.5e + 07	5e + 06
1985	3e+07	1300000
1986	31300000	6200000
1987	37500000	3500000
1988	4.1e + 07	2800000
1989	43800000	4e+06

Coefficients of pearl curve can be estimated based on the historical data. Pearl_HC function estimates the coefficients from historical data and calculates the growth. Pearl_HC(CATV,89024390,20) estimates the a and b from historical cable tv data. Then it calculates the growth from 1952 through 20 years above last available data - in this case 1989. The upper growth for cable tv is when all the households in U.S. (89024390).

pander(Pearl_HC(CATV,89024390,20))

period	growth
1953	85528
1954	104582
1955	127873
1956	156342
1957	191136
1958	233654
1959	285598
1960	349046
1961	426520
1962	521090
1963	636478
1964	777193
1965	948683
1966	1157516
1967	1411584
1968	1720326
1969	2094981
1970	2548847
1971	3097537

period	growth
1972	3759209
1973	4554730
1974	5507723
1975	6644429
1976	7993280
1977	9584097
1978	11446793
1979	13609488
1980	16096012
1981	18922846
1982	22095721
1983	25606277
1984	29429337
1985	33521478
1986	37821512
1987	42253223
1988	46730294
1989	51162822
1990	55464426
1991	59558771
1992	63384516
1993	66898071
1994	70074096
1995	72904087
1996	75393662
1997	77559220
1998	79424536
1999	81017705
2000	82368632
2001	83507146
2002	84461697
2003	85258547
2004	85921344
2005	86470981
2006	86925640
2007	87300956
2008	87610246
2009	87864768

Pearl_HC_plot(CATV,89024390,20)

