ALGORITHM EFFICIENCY

LECTURE 12-1

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BINARY SEARCH TREES

- Binary search trees are a way of keeping track of a ***sorted*** collection.
- Here, we are using them as an *ordered dictionary*.
- For our dictionaries, there is at most one entry per key.
- The link structure sorts the entries; maintains a sorted order.
  - The keys are usually organized alphabetically when strings.
  - The keys are usually sorted smaller/larger if numbers.
- (Generally, in binary search trees, keys might appear more than once; have multiple entries.)
- (Generally, in binary search trees, the nodes might only contain keys without associated values.)

A BST CLASS

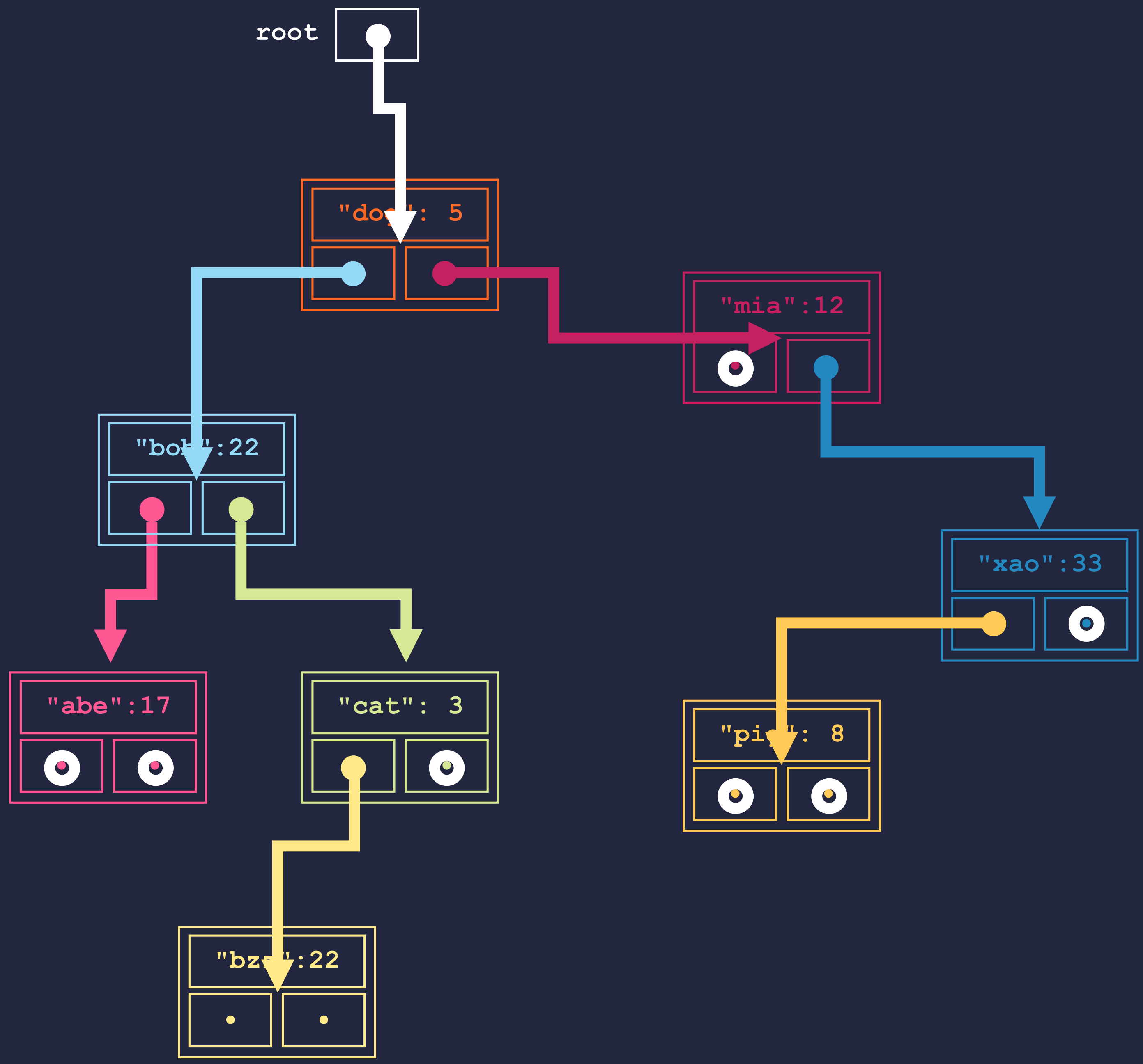
- **Operations**:
  - Searching for an entry by key.
  - Adding or updating an entry, ordered according to key, storing the value.
  - Removing an entry.
  - Visiting all the entries in sorted order.
- The first three operations rely on a *search*.
  - This works from the root, moving left or right.

SEARCHING FOR AN ENTRY IN A BST

```
class BSTNode:
    def __init__(self,k,v):
        self.key = k
        self.value = v
        self.left = None
        self.right = None

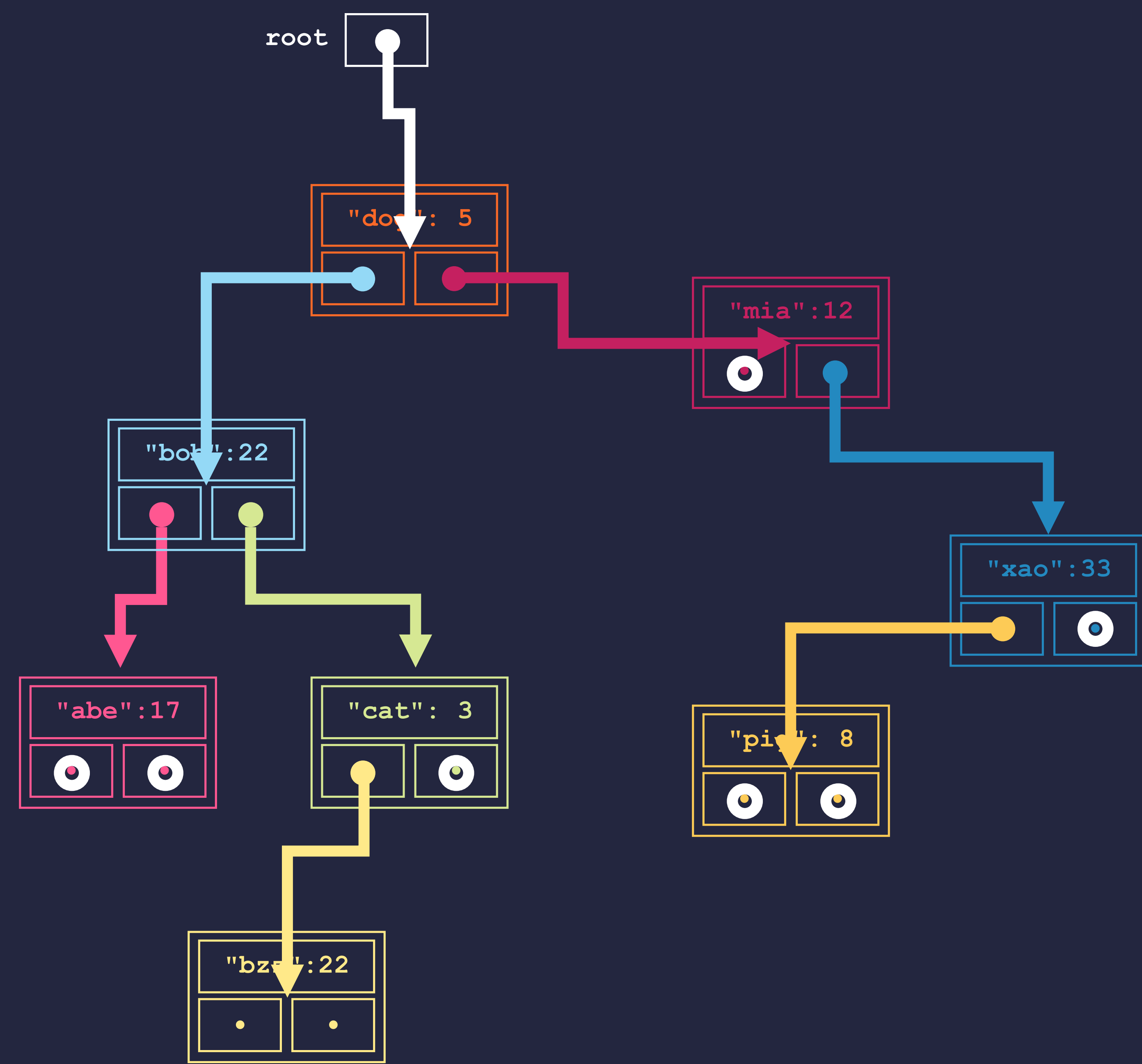
class BSTree:
    def __init__(self):
        self.root = None

    def contains(self,k):
        curr = self.root
        while curr is not None:
            if k == curr.key:
                return True
            if k < curr.key:
                curr = curr.left
            if k > curr.key:
                curr = curr.right
        return False
```



ADDING AN ENTRY TO A BST

```
class BSTree:
    ...
    def update(self,k,v):
        parent = None
        curr = self.root
        while curr is not None:
            parent = curr
            if k == curr.key:
                curr.value = v
                return
            if k < curr.key:
                curr = curr.left
            if k > curr.key:
                curr = curr.right
        newNode = BSTNode(k,v)
        if parent is None:
            self.root = newNode
        elif k < prnt:
            parent.left = newNode
        else:
            parent.right = newNode
```



A BST CLASS

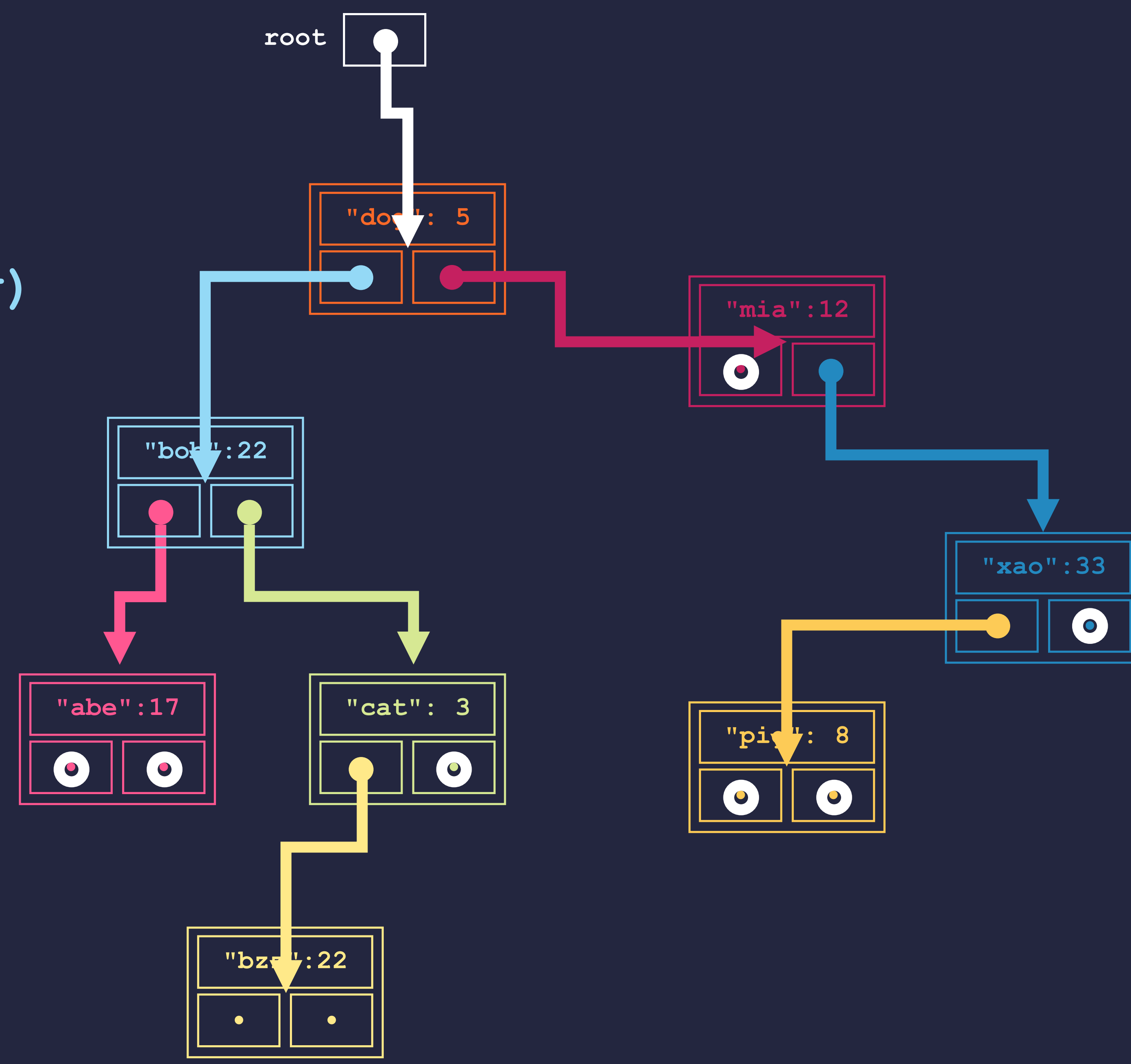
- **Operations**:
  - Searching for an entry by key.
  - Adding or updating an entry, ordered according to key, storing the value.
  - Removing an entry.
  - Visiting all the entries in sorted order.
- The first three operations rely on a *search*.
  - This works from the root, moving left or right.
- The last is *in-order traversal*. It is a recursive. **Example:** printing all the entries
  - You print all of the entries left of the root entry.
  - Then you print the root entry.
  - And then you print all of the entries right of the root entry.

TRAVERSING A BST

```
def printBST(node):
    if node is not None:
        printBst(node.left)
        entry = str(k)+":"+str(v)
        print(entry,end=' ')

class BSTree:
    ...
    def output(self):
        printBST(self.root)
        print()
```

```
>>> t.print()
abe:17 bob:22 bzz:22 cat:3 dog:5
mia:12 pig:8 xao:33
>>>
```



A TOUR OF THE BST CLASS CODE

- **Look at `BSTree.py`**

WRITING BETTER CODE

- As you become a more sophisticated programmer, you'll be driven to write good code. Some measures of "goodness":
  - Is it ***correct***?
  - Is it ***readable***?
  - Is it ***maintainable***?
  - And sometimes writing ***efficient*** code is important, too.

EFFICIENT CODE

- Efficient code is code that uses fewer resources when run. Examples:
  - It makes fewer calculations and/or takes fewer steps.
  - It uses less memory with its data structures.

- For both of these, a program will typically compute its answer faster.
  - It runs faster.

- Today we'll focus on ***running time***.

MEASURING RUNNING TIME

- Suppose you have two programs that compute the same result:
  - **program A** and **program B**.
- **Q**: How do we determine which one is faster?
- **A:** Run the code on typical inputs, measure the time it takes.

```
>>> import timeit
>>> i = 'import pow2'
>>> s = 'pow2.pow2(20)'
>>> timeit.timeit(stmt=s,setup=i,number=100)
0.00022750999977850914
```

- This will time 100 evaluations of `pow2(20)` then report the elapsed time in seconds.

MEASURING RUNNING TIME

- Suppose you have two programs that compute the same result:
  - **program A** and **program B**.
- But maybe...
  - You don’t have an exact sense of the typical inputs.
  - The size of typical inputs increases over the lifetime of the algorithms’ use.
  - The size and typicality of inputs might vary widely, depending on the application of the algorithm.
  - The computer might get upgraded in some near future. Or the programs might be rewritten for some unknown system.

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- We then also work to ***estimate*** running times.
  ⇒ We use ***running time analysis***.

RUNNING TIME ANALYSIS

- Typical major concerns of running time estimation:

  - How does the running time scale (roughly) with input complexity?
    E.g. searching for an item in a list of size `n`
    - We will estimate "limiting" or *asymptotic* running time.

  - For a particular input sizer, what are the trickiest inputs the code will face?
    E.g. the search might have to scan the whole list.
    - We sometimes give *bounds* on the *worst cases*.

RUNNING TIME ANALYSIS

- Typical lesser concerns of running time estimation:
  - e.g., Something that runs 11% faster on one machine over another.
  - e.g., **Program A** runs a little slower on small inputs well on large inputs (0.2sec versus 0.15sec for **Program B**), even though **Program A** runs must faster on large inputs (20sec versus 1000sec for **Program B**).

ASYMPTOTIC EQUIVALENCE

- Let's formalize some of these ideas:
  - Two algorithms' running times are ***asymptotically equal*** if, for large inputs, which algorithm is faster *depends on the relative speed of their executing computers*.

- Example scenario:
  - Suppose **algorithm A** takes $n^3 - 4n^2$ steps on an n-bit input.
  - Suppose **algorithm B** takes $10n^3+15$steps on an n-bit input.
    - If **A** and **B** run on the same computer, A runs faster.
    - If **B** runs on a 100x speedier machine, it beats **A** on large inputs.

ASYMPTOTIC EFFICIENCY

- Let's formalize some of these ideas:
  - Two algorithms' running times are ***asymptotically equal*** if, for large inputs, which algorithm is faster depends on the relative speed of their executing computers.

- We define $\Theta(\, g(n)\,)$, the set of functions asymptotically equal to g, with:

Definition: $f\,(n)$ is in the set $\Theta(\, g(n)\,)$ whenever there exist positive constants L and U, and a positive constant m where

$$L\, g(n) \leq f\,(n) \leq U\, g(n)$$

for all $n \geq m$.

BIG THETA

Definition: $f(n)$ is in the class $\Theta(g(n))$ whenever there exist positive constants L and U, and a positive constant m where

$$L\, g(n) \leq f(n) \leq U\, g(n)$$

for all $n \geq m$.

Examples:

$n^3 - 4n^2$ is in the class $\Theta(10n^3+15)$

$10n^3+15$ is in the class $\Theta(n^3 - 4n^2)$

$n^3 - 4n^2$ is in the class $\Theta(n^3)$

$10n^3+15$ is in the class $\Theta(n^3)$

NOTE: All these functions grow as ***cubic functions*** of n.

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for all $n \geq m$.

Examples from the last lecture:

Searching... an entire list of length n takes $\Theta(n)$ time.

...a balanced BST of size n to discover that a key is missing is $\Theta(\log_2(n))$ time.

A nested pair of loops that sum the products $i*j$ takes $\Theta(n^2)$ time.

Computing `pow2(n)` using repeated squaring takes $\Theta(\log_2(n))$ time.

Computing `pow2(n)` by multiplying 2 of n times takes $\Theta(n)$ time.

Computing `pow2(n)` by summing *1*s takes $\Theta(2^n)$ time.

BIG OH

Definition: $f(n)$ is in the class $\mathbf{O}(g(n))$ whenever there are positive U and m such that

$$0 \leq f(n) \leq U\, g(n)$$

for all $n \geq m$.

Examples:

$n^3 - 4n^2$ is in the class $\mathbf{O}(10n^3+15)$

$10n^3+15$ is in the class $\mathbf{O}(n^3 - 4n^2)$

n^2 is in the class $\mathbf{O}(n^3)$

$100000n+987987987$ is in the class $\mathbf{O}(n)$

- We use "big Oh" to say “asymptotically grows no faster than…”

BIG OH

Definition: $f(n)$ is in the class **O**$(g(n))$ whenever there are positive U and m such that

$$0 \le f(n) \le U\, g(n)$$

for all $n \ge m$.

Examples:

Searching a list of length n takes **O**(n) time.

Searching a balanced BST of size n takes **O**$(\log_2(n))$ time.

Searching a BST of size n takes **O**(n) time.

- We use "big Oh" to say “asymptotically grows no faster than…”

A CASE STUDY: SEARCHING A LIST

SEARCHING A LIST

```
def search(item, someList):
    i, n = 0, len(someList)
    while i < n:
        if someList[i] == item: return True
        i += 1
    return False
```

SEARCHING A SORTED LIST

Can we do better if a list is sorted?

- Suppose that

`someList[0]` ≤ `someList[1]` ≤ … ≤ `someList[n-1]`

SEARCHING A SORTED LIST

```
def binarySearch(item,someList):
    left, right = 0, len(someList)-1
    while left <= right:
        middle = (left + right) // 2
        if item == someList[middle]:
            return True
        elif item < someList[middle]:
            right = middle-1
        else:
            left = middle+1
    return False
```

SEARCHING A SORTED LIST

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def binarySearch(item,someList):
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    while left <= right:
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        if item == someList[middle]:
            return True
        elif item < someList[middle]:
            right = middle-1
        else:
            left = middle+1
    return False
```

- With each **`someList[middle]`** check, we eliminate half the undetermined list items from consideration.
- This means we inspect the list $\mathbf{O}(\log_2(n))$ times.

ANOTHER CASE STUDY: SORTING A LIST

BUBBLE SORT

- With bubble sort we make several left-to-right scans over the list.
  - We swap out-of-order values at neighboring locations
  - This “bubbles up” larger values so they “rise” to the right.

```
def bubbleSort(aList):
    n = len(aList)
    for scan in range(1,n):
        i = 0
        while i < n - scan:
            if aList[i+1] < aList[i]: # Out of order? Swap!
                aList[i],aList[i+1] = aList[i+1],aList[i]
            i += 1
```

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- This means we only need to make *n* -1 scans.

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```

- This means we only need to make *n* -1 scans.
- This means we can stop the scan earlier for later passes.

BUBBLE SORT ANALYSIS

- What is the running time of bubble sort?

```
def bubbleSort(aList):
    n = len(aList)
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            i += 1
```

The if statement runs $n - 1$ times on the first scan, then $n - 2$ times on the second scan, then $n - 3$ times on the third scan, ...

BUBBLE SORT ANALYSIS

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            i += 1
```

The if statement runs n - 1 times on the first scan, then n - 2 times on the second scan, then n - 3 times on the third scan, …

→ The total number of swaps is

$$n\,(n - 1)\,/\,2 = (n - 1) + (n - 2) + \ldots + 3 + 2 + 1$$

- Its running time scales ***quadratically*** with n.

SUMMARY

- In running time analysis use asymptotic notation to describe efficiency.
  - We use **Big Theta** for asymptotic equivalence.
  - We use **Big Oh** for asymptotic guarantees, i.e. *upper bounds*.

- Two classic searching and sorting algorithms:
  - Binary search is a ***logarithmic time*** algorithm. It works on sorted lists.
  - Bubble sort is a ***quadratic time*** algorithm. It sorts a list.
- Can we sort faster than in quadratic time?