Problem Set 5

The problems listed here are from the 3rd edition of the Sipser book, standard version (i.e., not the problematic international version that I had). You can look at your copy and do the problems as they appear in there.

- Chapter 7: Problems 7.23, 7.35, 7.41

- Problem a: For a language $L \subseteq \Sigma^*$ and a function $f : \Sigma^* \to \Sigma^*$, we let

$$L_f = \{x \in \Sigma^* \ : \ f(x) \in L\} \ .$$

  We say that a complexity class is *closed under composition* if, for any L in the class and any polynomial time computable function f, it is the case that the language L_f is also in the class.

  Is the class **NP** closed under composition? Prove your answer.

- Problem b: We say that a language L is *trivial* if either $L = \emptyset$ or $L = \Sigma^*$, and *non-trivial* otherwise. Prove that $\mathbf{P} = \mathbf{NP}$ if and only if every non-trivial language in **NP** is NP-Complete.

Please follow the ground rules as discussed in class.

– Open books. Specifically, you may use any books.

– Closed Internet.

– You must work alone.

– Your solutions must be hand-written unless you obtain permission from me otherwise.

– Be sure to be meticulous in your write-up. Make sure you prove all the necessary components according to the definitions. For example, to show that a language is NP-Complete, you must show that it is in **NP** and is NP-hard.

– Once you finish your write-up, please generate a single PDF file and upload it to gradescope. If you write on pieces of paper (as opposed to writing on an app on a tablet), please scan the pages (or take clear pictures of the pages) and make a single PDF file, then upload the file.