Context-Free Grammars

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Outline

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Pushdown Automata

A pushdown automata is a non-deterministic, finite state machine with access to a stack.

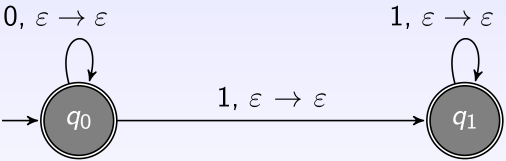
Definition

A pushdown automaton is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$ where Q, Σ, Γ, and F are all finite sets, and

1. Q is a set of states
2. Σ is the input alphabet
3. Γ is the stack alphabet
4. $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to \mathcal{P}(Q \times \Sigma_\varepsilon)$ is the transition function
5. $q_0 \in Q$ is the start state
6. $F \subseteq Q$ is the set of accept states

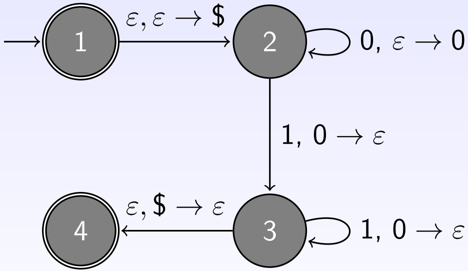
Example 1

$$L_1 = L(0^*1^*)$$



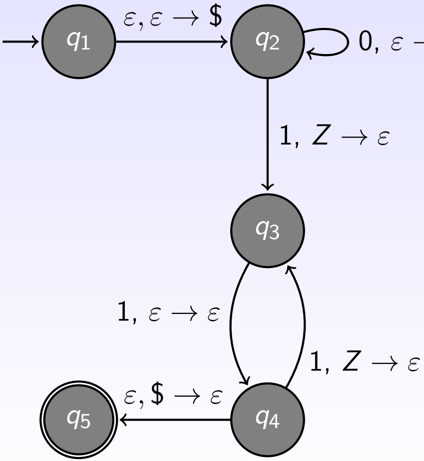
Example 2

$$L_2 = \{0^n 1^n \mid n \geq 0\}$$



Example 3

$$L_3 = \{0^n 1^{2n} \mid n \geq 0\}$$



Formal definition of computation

Let $m \in Z^+$, $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a pushdown automaton.

Definition

M accepts w if w can be written as $y_1 y_2 \ldots y_m$ where each $y_i \in \Sigma_\varepsilon$, and a sequence of states $r_0, r_1, \ldots, r_m \in Q$ and string $s_o, s_1, \ldots, s_m \in \Gamma^*$ exist with the following three conditions:

1. [M starts in the start state.] $r_0 = q_0$ and $s_0 = \varepsilon$,
2. [M moves according to δ, state, stack, and input symbol.] For $i = 0, \ldots, m-1$, we have $(r_{i+1}, b) \in \delta(r_i, y_{i+1}, a)$ where $s_i = at$ and $s_{i+1} = bt$ for some $a, b \in \Gamma_\varepsilon$ and $t \in \Gamma^*$,
3. [M ends up in an accept state.] $r_m \in F$.

Equivalence of PDA and CFG

Theorem: PDA and CFG are equivalent

A language is context free if and only if some pushdown automaton recognizes it.

There are two directions that need to be proved:

[$\Longrightarrow$] If a language is a context free language, i.e., has a CFG generating it, then there is a PDA recognizing it.

(Easy)

[$\Longleftarrow$] If a language has a PDA recognizing it, then it has a CFG generating it.

(Hairy)