

Interpreting Pumping Lemma for Regular languages

Here is one way to interpret the Pumping Lemma for regular languages. Credit is due to Shai Halevi, who taught this in a recitation I was in at MIT in 1995.

Pumping Lemma

If L is regular	\implies	then it has a DFA D recognizing it
then there exists $p \geq 0$	\implies	where p is the number of states of D
so that for all $s \in L$	\implies	which defines a path in D
with $ s \geq p$	\implies	so this path contains a cycle
there is a parse of $s = xyz$	\implies	x is everything <i>before</i> the first cycle
with $ y \geq 1$ and $ xy \leq p$	\implies	y is everything <i>on</i> the first cycle
		z is everything <i>after</i> the first cycle
such that for any $i \geq 0$	\implies	no matter how many times we repeat the cycle
$xy^iz \in L$	\implies	we end up in a final state of D

Showing that L is not regular

For any integer $p \geq 0$	\implies	show that L is not recognizable by any DFA with p states
		suppose toward a contradiction that there was a DFA D
		having p states and recognizing L
there exists a string $s \in L$ such that $ s \geq p$	\implies	so in processing s we must visit more than p states
	\implies	so it defines a path with a cycle
so that for any parse $s = xyz$	\implies	no matter where this cycle is
with $ y \geq 1$ and $ xy \leq p$	\implies	
there exists some $i \geq 0$	\implies	we can repeat the cycle i times
so that $xy^iz \notin L$	\implies	and end up in a non-final state of D