CSCI/MATH 387: Computability and Complexity | Fall 2022
Reed College | Department of Computer Science
Handout 2: Notes on Pumping Lemma for Regular Languages | **Instructor:** Chanathip Namprempre

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Interpreting Pumping Lemma for Regular languages

Here is one way to interpret the Pumping Lemma for regular languages. Credit is due to Shai Halevi, who taught this in a recitation I was in at MIT in 1995.

Pumping Lemma

If L is regular	$\Longrightarrow$	then it has a DFA D recognizing it
then there exists $p \geq 0$	$\Longrightarrow$	where p is the number of states of D
so that for all $s \in L$	$\Longrightarrow$	which defines a path in D
with $\lvert s\rvert \geq p$	$\Longrightarrow$	so this path contains a cycle
there is a parse of $s = xyz$	$\Longrightarrow$	x is everything *before* the first cycle
with $\lvert y\rvert \geq 1$ and $\lvert xy\rvert \leq p$		y is everything *on* the first cycle
		z is everything *after* the first cycle
such that for any $i \geq 0$	$\Longrightarrow$	no matter how many times we repeat the cycle
$xy^iz \in L$	$\Longrightarrow$	we end up in a final state of D

Showing that L is not regular

For any integer $p \geq 0$	$\Longrightarrow$	show that L is not recognizable by any DFA with p states
		suppose toward a contradiction that there was a DFA D
		having p states and recognizing L
there exists a string $s \in L$ such that $\lvert s\rvert \geq p$	$\Longrightarrow$	so in processing s we must visit more than p states
	$\Longrightarrow$	so it defines a path with a cycle
so that for any parse $s = xyz$	$\Longrightarrow$	no matter where this cycle is
with $\lvert y\rvert \geq 1$ and $\lvert xy\rvert \leq p$	$\Longrightarrow$	
there exists some $i \geq 0$	$\Longrightarrow$	we can repeat the cycle i times
so that $xy^iz \notin L$	$\Longrightarrow$	and end up in a non-final state of D

Example

Let the alphabet be $\{a, b, c\}$. Show that $L = \{a^n w \mid w \in \{b, c\}^* \text{ and } |w| = 2n\}$ is not regular.

Proof

Let $p \geq 0$. Consider $s = a^p b^p c^p \in L$.
Let $s = xyz$ with $|y| \geq 1$ and $|xy| \leq p$.
Let i, j be integers such that $i \geq 0$ and $j \geq 1$ and $i + j \leq p$ and

$$x = a^i \quad \text{and} \quad y = a^j \quad \text{and} \quad z = a^{p-i-j} b^p c^p \ .$$

Consider xy^2z. We have

$$\begin{aligned} xy^2z &= a^i a^{2j} a^{p-i-j} b^p c^p \\ &= a^{i+2j+p-i-j} b^p c^p \\ &= a^{p+j} b^p c^p \end{aligned}$$

Since $j \geq 1$, we have that $2(p + j) \neq p + p$. So, $xy^2z \notin L$. Thus, L is not regular.