Pseudorandom Generator

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Agenda: Pseudorandom Generator

1. What is a PRG?
2. PRG-based stream cipher
3. Next-bit unpredictability
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Pseudorandom Generator

Let $n > s$.

$$G : \{0,1\}^s \longrightarrow \{0,1\}^n$$

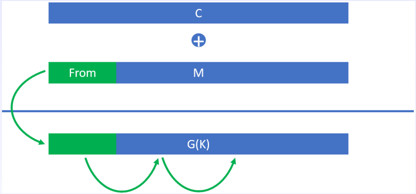
Use PRG to approximate OTP.
We call this a PRG-based stream cipher.

$$C \leftarrow G(K) \oplus M$$

Unpredictability is important

Sendmail: fixed format e.g. email messages begin with "From:"

1. Snoop ciphertext C
2. $X \leftarrow C \oplus$ "From:"
3. X is the first part of the output of $G(K)$



Bottom line: If G is predictable, then a small prefix reveals entire message.

Source: Dan Boneh's Coursera

PRG security notion

Definition (PRG)

Let s, n be positive integers.

Subroutines

Subroutine $\texttt{Initialize}$
$b \xleftarrow{\$} \{0,1\}$
If $b = 1$
then $x \xleftarrow{\$} \{0,1\}^s$; $y \leftarrow G(x)$
else $y \xleftarrow{\$} \{0,1\}^n$
Return y

Subroutine $\texttt{Finalize}(d)$
Return (d = b)

Experiment

Experiment $\mathbf{Exp}_G^{\mathrm{prg}}(A)$

$y \xleftarrow{\$} \texttt{Initialize}$
$d \xleftarrow{\$} A(y)$
Return $\texttt{Finalize}(d)$

We define the **prg advantage** of an adversary A attacking G as

$$\mathbf{Adv}_G^{\mathrm{prg}}(A) = 2 \cdot \Pr\left[\mathbf{Exp}_G^{\mathrm{prg}}(A) \Rightarrow \mathrm{true}\right] - 1 \ .$$

Next-Bit Unpredictability

Definition (Unpredictability)

Let s, n, i be positive integers.

Subroutines

Subroutine `Initialize`
$x \xleftarrow{\$} \{0,1\}^s$; $y \leftarrow G(x)$
Return $y[1, \ldots, i]$

Subroutine `Finalize`(z)
Return (z = y[i+1])

Experiment

Experiment $\mathbf{Exp}_G^{\mathrm{unp}}(A)$

$y' \xleftarrow{\$}$ `Initialize`
$z \xleftarrow{\$} A(y')$
Return `Finalize`(z)

We define the **next-bit unpredictability advantage** of an adversary A attacking G as

$$\mathbf{Adv}_G^{\mathrm{unp}}(A) = 2 \cdot \Pr\left[\,\mathbf{Exp}_G^{\mathrm{unp}}(A) \Rightarrow \mathrm{true}\,\right] - 1 \ .$$

Examples

1. A generator G such that, for all K,

$$\text{XOR}(G(K)) = 1\ .$$

2. MS-PPTP in Windows NT
3. 802.11b WEP
4. eStream

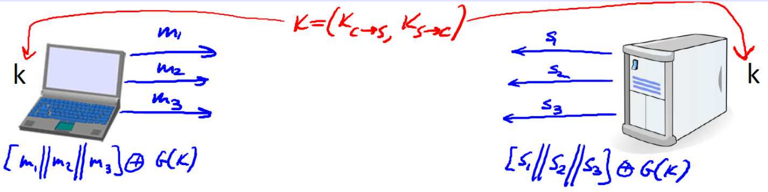
Toy Example

Consider a generator G such that, for all K,

$$\mathrm{XOR}(G(K)) = 1\ .$$

Is G secure under the PRG notion?

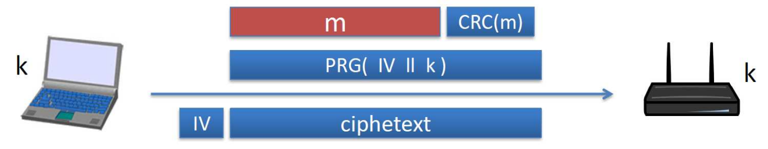
Two-Time Pad is insecure



Bottom line: The secret key is being used twice, one for each direction. This is a two-time pad.

Source: Dan Boneh's Coursera

WEP is insecure



- For WEP-40, |IV|= 24 bits. |k|= 40 bits.
- For WEP-104, |IV|= 24 bits. |k|= 104 bits.
- For WEP-104, PRG: $\{0,1\}^{128} \rightarrow 2048$ is RC4.
- IV is incremented by 1 per frame.
- IV repeats after $2^{24} \approx 16M$ frames.
- On some 802.11 cards, IV resets to 0 after power cycle.

Bottom line:

The pad is being used twice whenever IV is reset.

Source: Dan Boneh's Coursera

WEP issues

Actually, the picture is misleading. An 802.11 packet is usually 2000 bytes long. So, WEP breaks up each packet into frames.

frame #	key stream
1	PRG(1 ‖ k)
2	PRG(2 ‖ k)
3	PRG(3 ‖ k)
...	...

- The seeds are very much related. This breaks security assumption underlying PRG security definition.
- Attacks exploiting this using 1M frames were found in 2001.
- Recent attacks use about 40K frames.

The Big Question remains: How do we use a PRG to encrypt a long stream of data?

Source: Dan Boneh's Coursera

Modern PRGs

Question: How do we use a PRG to encrypt a long stream of data?
Answer: Cheat! Add a "nonce."

$\boxed{\text{Old:}}$

$$\mathsf{PRG} : \{0,1\}^s \to \{0,1\}^n$$

$\boxed{\text{New:}}$

$$\mathsf{PRG} : \{0,1\}^s \times \mathcal{N} \to \{0,1\}^L$$

- $\mathcal{N}$ is the nonce space, e.g., $\{0,1\}^{64}$ (Salsa and Chacha).
- L is much larger than n, e.g., 2^{73} bits (Salsa and Chacha).

Restriction: Nonces are not allowed to repeat.

Nonce-Based Stream Cipher

$$\mathsf{Enc}(K, N, M) = M \oplus \mathsf{PRG}(K, N)$$
$$\mathsf{Dec}(K, N, C) = C \oplus \mathsf{PRG}(K, N)$$

PRG security vs. Unpredictability

They are equivalent!

Theorem

Let G be a PRG. Then, it is secure if and only if it is unpredictable.

[$\Longrightarrow$] Easy.

[$\Longleftarrow$] Hybrid proof, e.g., Boneh-Shoup Theorem 3.6.