CS120: Intro. to Algorithms and their Limitations Hesterberg & Vadhan

Sender–Receiver Exercise 1: Reading for Receivers

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The goals of this exercise are:

- to develop your skills at understanding, distilling, and communicating proofs and the conceptual ideas in them
- to practice reasoning about updates to dynamic data structures and binary search trees in particular

In the previous class (Thursday 9/15), we saw that insert operations can be performed on a binary search tree (BST) in time $O(h)$, where h is the height of the tree. As an in-class exercise, some of you saw that a variety of different operations (search, min/max, next-smaller/next-bigger) can also be done in time $O(h)$; pseudocode for those operations is in the detailed lecture notes. Here you will see how *deletions* can be done in time $O(h)$:

Theorem 0.1. *Given a binary search tree T of height h and a key K stored in the tree, we can delete a matching key-value pair (K, V) from T in time $O(h)$. Deletion means that we produce a new binary search tree that contains all of the key-value pairs in T except for one less occurrence of a pair with key K.*

To prepare for the exercise, we recommend reviewing the lecture notes from Thursday 9/16 to make sure you are comfortable with BSTs and the simpler operations on them (insert, search, min/max, and next-smaller/next-bigger)

Proof.