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Game Theory, Spring 2020 A
Assignment 4

1. In the job market signaling model, workers are of type $\theta \in \{\theta_L, \theta_H\} = \{0.2, 1\}$, each with equal one-half probability. In order to obtain education level e as a signal to the firms, the workers pay cost e/θ. The firm sets its wages such that $w(e) = \mathbb{E}(\theta|e)$. Thus, a worker of type θ receives utility $w(e) - \frac{e}{\theta}$. The firms form beliefs of the type $\mu(\theta|e)$, the probability that the worker is of type θ given her level of education.

   a) In a separating equilibrium, workers of each type choose different levels of education e^*, where $e^*(\theta_L) < e^*(\theta_H)$. For simplicity, I denote these equilibrium levels of education e_L and e_H, respectively. The firms, knowing this, can differentiate, and thus for the low worker, they set

   $$w(e_L) = \mathbb{E}(\theta_L|e_L) = \mu(e_L)\theta_L + \big(1 - \mu(e_l)\theta_H\big) = \theta_L$$

   The low-type worker then solves
   $$\max_e \theta_L - \frac{e_L}{0.2}$$
   and thus chooses $e_L = 0$.

   The firm can set a range of wage schedules $w(e)$ to support a range of education levels for the high type. As with the low type, the firm will set $w(e_H) = \theta_H = 1$. What remains to be determined is the level of education at or above which a worker can earn the wage of 1. To show the result visually, Figure () plots indifference curves for the high and low types:

   The firm can sustain any level of education for the high type $e_H \in [0.16, 0.8]$ by offering $w(e) = 1$ for any level of education $e \in [e_H, \infty]$. If they attempt to sustain $e_H < 0.16$, then the high-type workers are better off getting zero education. If the firm attempts to sustain $e_H > 0.8$, meanwhile, the low-type workers will also choose e_H, and thus this level cannot be a separating equilibrium. Thus, the equilibrium which sustains $e_L = 0$ and $e_H = \hat{e}$ is characterized by firm beliefs

   $$\mu(\theta_H|e) = \begin{cases} 0 & e \in [0, \hat{e}) \\ 1 & e \in [\hat{e}, \infty) \end{cases}$$

   and wage schedule
   $$w(e) = \begin{cases} 0.2 & e \in [0, \hat{e}) \\ 1 & e \in [\hat{e}, \infty) \end{cases}$$

   b) In the pooling equilibrium, both types of workers choose the same level of education, the firm sets one wage, equal to the unconditional expectation of θ:

   $$w(e) = 0.5\theta_L + 0.5\theta_H = 0.6$$

   Figure () shows this wage along with indifference curves for the high and low type workers:

The firm can sustain any pooling equilibrium level of education $\hat{e}^* \in [0, 0.08]$. If they set $\hat{e} > 0.08$, the low-type workers are better off choosing $e_L = 0$. The pooling equilibrium that sustains $\hat{e}^* \in [0, 0.08]$ is fully characterized by firm beliefs

$$\mu(e) = \begin{cases} 0 & e \in [0, \hat{e}) \\ \frac{1}{2} & e \in [\hat{e}, \infty) \end{cases}$$

and wage schedule

$$w(e) = \begin{cases} 0.2 & e \in [0, \hat{e}) \\ 0.6 & e \in [\hat{e}, \infty) \end{cases}$$

2. The tree for the extensive form of the game is as follows:

   There are no separating or pooling Perfect Bayesian equilibria in pure strategies. In order to find the mixed-strategy PBE, I define variables for the players' respective strategies. Let T_R and L_R denote Anna's actions, respectively, of taking and leaving her umbrella when it rains, and T_S and L_S the actions of taking and leaving when it is sunny. Anna's corresponding mixed strategies are γ_R and γ_S, indicating the probability of her taking her umbrella when it rains and is sunny, respectively. Note that because he cannot observe the weather forecast,