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Mobility

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Abstract

This paper studies short-run wealth mobility in a heterogeneous agents, incomplete-markets model. Wealth mobility has a "hump-shaped" relationship with the persistence of the stochastic process governing labor income: low when shocks are close to IID or close to a random walk, and higher in between. Across all levels of persistence, the standard incomplete markets framework features less wealth mobility than found in the PSID wealth supplements. We include augmentations to income risk commonly used in the literature to capture wealth inequality, and find that they do little to improve the model's performance for wealth mobility. Intuitively, when agents face risk to their income, they self-insure, accumulating assets in order to smooth consumption. This self-insurance motive slows the pace with which agents move through the wealth distribution. Looking more closely at the data, we find that families that make large moves through the wealth distribution over short time periods are more likely to receive shocks directly to their wealth, such as marriages or divorces, inheritances, or capital gains or losses from ownership of stocks or business. Motivated by these observations, we add idiosyncratic *return* risk to the baseline model, and demonstrate that return shocks produce mobility in line with the data.

Keywords: Wealth Mobility; Heterogeneous Agents; Inequality,
JEL classification codes:

1 Introduction

This paper examines wealth mobility in a simple dynamic stochastic general equilibrium model with incomplete markets in the spirit of Bewley (1986), Aiyagari (1994), and Huggett (1993). This model and its many variations has become the workhorse model of macroeconomics in great part because it generates an endogenous distribution of agents across income and wealth. This endogenous distribution is ideal for studying the effects of policy on inequality. Very little is understood in this environment regarding wealth mobility – the frequency with which agents "switch places".

Mobility is distinct from inequality. It should be obvious that inequality is a necessary condition for mobility – if everyone is the same, it makes no sense to talk about households switching places in any distribution – but inequality can arise in the absence of mobility. For example, without risk inequality can be not only present but permanent, depending on how savings choices vary in the population, while mobility may be zero as agents remain frozen in their ordering within the wealth distribution. Thus, inequality per se is not informative about the insurance opportunities that agents can access. Furthermore, given the generally-poor quality of the consumption panel data needed to directly characterize the incompleteness of asset markets, we think it useful to consider whether mobility data can help us characterize the asset markets used by households.

This question has policy implications. As discussed in Carroll and Young (2011), changes in the progressivity of income tax functions have qualitatively different effects depending on whether inequality is driven by time-invariant characteristics like preferences and average labor efficiency as opposed to uninsurable risk (as in Castaneda et al. (2003)). Specifically, a more progressive income tax function leads to lower inequality under idiosyncratic risk, as it effectively reduces the volatility of the idiosyncratic component (labor productivity) and compresses the distribution of returns; in contrast, under no risk but permanent heterogeneity, rich households actually increase their assets and poor households reduce them. It seems important therefore to get a clear picture of what drives inequality, and that requires an understanding of asset market opportunities.

Since inequality is not sufficient, we turn to mobility. In this paper our goal is to characterize how mobility is determined with a given workhorse model and compare it to the numbers found in the Panel Study of Income Dynamics Wealth Supplements. To this end, we first present a battery of different measures of mobility found in the literature. The Shorrocks measure uses the trace of the Markov transition matrix only – a process has higher mobility if the trace is smaller, meaning that households are more likely to leave their current quantile. In contrast, the Bartholomew measure uses a weighted average of transition probabilities, where the weights are the absolute number of quantiles that the agent 'passes through'; under this measure, an economy is more mobile if agents within it make distant moves. The 'second highest eigenvalue' measure is commonly used in macroeconomics; we show for two-state chains the autocorrelation of a chain equals the second highest eigenvalue.[1] Finally, the 'mean first passage time' calculates the expected number of periods before a household exiting an initial quantile reaches any other particular quantile for the first time. We also briefly mention the Cowell-Flachaire measure, which is more general.

We use these measures to interpret the wealth movements generated by our model. Comparing two environments in which the only difference is the persistence of the idiosyncratic shock, we find that mobility is 'hump-shaped' – mobility is low when shocks are close to iid and when they are close to a random walk, and higher at intermediate values. We decompose the change in mobility as the sum of three components, which we label luck, behavior, and structure. First, fix the behavior of all households at a given persistence value, and let one household draw a sample sequence from a process with a higher autocorrelation; all that changes is the particular realizations of the shock, which we label 'luck'. Now let this household realize that the persistence of her shock is different and reoptimize, leaving the behavior of all other households unchanged (including their persistence); we label this change 'behavior', since it captures the effect of different decision rules on mobility. Third, suppose all other households also face the new persistence coefficient, leading to changes in the distribution of wealth against which any particular household will be viewed; we call this the 'structural' effect.[2]

All five mobility measures return a similar decomposition pattern. Structure has a minimal effect on mobility, while behavior has a large negative effect and luck a large positive effect (given the change is to increase the autocorrelation). The negative effect of behav-

[1] This result is not general. We cannot prove anything for chains with more than two states, but a Monte Carlo experiment with random stochastic matrices shows a low correlation between the modulus of the second-highest eigenvalue and the autocorrelation of the chain.

[2] Technically, the behavior effect could also arise under the third change, since equilibrium prices differ. We ignore this distinction as the interest rate turns out to change very little across these experiments.

ior results from the decreased sensitivity of saving to more persistent income shocks; this sensitivity is a reflection of consumption-smoothing, wherein shocks that are permanent are absorbed into consumption since there is no 'better future' to borrow from, while transitory shocks are smoothed away using a buffer-stock of assets. In contrast, increased persistence more often generates longer consecutive strings of high or low productivity draws (luck of the draw), and therefore generates more movement up and down the wealth ordering.

Using a reasonable calibration for the income process (taken from Floden and Lindé (2001)) with a high persistence of shocks, we find that the benchmark model delivers too little short-run mobility relative to the data (over five-year horizons). Specifically, we find that the model implies far too little mobility overall, but in particular fails to deliver the high mobility observed in the lowest and highest quintiles; in the model, households stay in these quintiles on average 38 and 63 years, in contrast to values in the data closer to 15 and 17 years, respectively. Furthermore, households in the model also stay in their initial quintile too frequently, and when they move they move only one quintile at a time; in the data wealth moves more rapidly, with significant numbers of households switching more than one quintile in either direction.

We then move to consider different environments, designed to illuminate what is causing the model to fail. We first look at a variety of changes used to match the extreme wealth inequality observed in the data. We examine the Krusell and Smith (1998) modification that introduces stochastic movements in discount factors that are highly persistent. The discount factor model improves a small amount by increasing mobility at the lower end of the wealth distribution, but actually reduces it at the high end; the reason the model gets high wealth concentration is that it nearly 'freezes' rich households in the top quintile, since high discount factor types will save a significant amount whether their income is high or not, and these discount factor states must be very persistent if they are to match wealth inequality.

We then examine a 'rockstar' model as in Castaneda et al. (2003), in which the earnings process has a rare and transitory state with very high income and a relatively high probability of dropping to the lowest state. The rockstar model works relatively well, as it increases mobility across the board and introduces some households that shift across more than one quintile; nevertheless, mobility at the high end is still substantially too low as households do not choose to let their wealth fall fast enough. This failure can be understood as the result of standard buffer-stock behavior combined with decreasing absolute risk aversion – with high temporary income, households save rapidly to move away from the borrowing constraint but dissave slowly. Furthermore, the rockstar model requires an earnings process unlike anything in the data (see Guvenen et al. (2015)).

We then consider how the span of assets affects mobility – we permit households to purchase some contingent securities, but maintain a borrowing constraint. With contingent claims, wealth mobility can increase if the household takes extreme positions; if transitions are rare, then insurance against those transitions is cheap and permits very large portfolio positions, which are useful due to the borrowing constraint. On the other hand, as shown in Rampini and Viswanathan (2016), poor households hold portfolios that hedge against fewer states than rich households do, meaning that wealth mobility may fall. We find that the hump-shaped pattern from the one asset case holds in an environment with an incomplete set of state-contingent assets. Compared to the one-asset baseline economy, partially completing the market decreases wealth mobility when the underlying income shock persistence is not too high. When the persistence becomes sufficiently large, however, the partial insurance economy has greater wealth mobility, due to this portfolio-composition effect. These effects get stronger as we add more contingent claims.

Having shown that the basic model does not replicate the wealth mobility statistics particularly well, we next drill deeper into the facts – can we learn anything about why these families move up and down the wealth distribution? We run probits to study the determinants of the probability that a family makes a "jump" (a movement of more than two quintiles) over a five-year horizon. First, we see that families that make one jump are significantly more likely to make another jump; that is, some families are just more mobile than others. Second, we find that the portfolio of the family is critical for these jumps – families with stocks are more likely to move up and less likely to move down, and families with private business income are more likely to move up and down as well as more likely to jump up or down.[3] Third, families that experience a marriage, divorce, or inheritance are also more likely to move and jump (in the obvious directions).

We explore simple ways to integrate these "direct wealth" shocks into the basic model; our goal is to give researchers an easy modification that brings the model closer to the

[3] Quadrini (1999) and Quadrini (2000) also point out the connection between private business activity and mobility, but only over the period 1984-1989.

data, so our representations of these shocks is simple and abstract. We find that "marriage/divorce/bequest" shocks, which take the form of large but infrequent shocks to the dynasty's current stock of assets, do not help much – because they are rare, they do not generate enough movement. Introducing iid heterogeneity in returns also does not improve the situation significantly, even if we choose the variance to be unreasonably large (that is, with returns that range from -64 percent to 158 percent). We conclude that simple modifications to the model are not straightforward to construct.

2 Measures of Mobility

The literature on measuring income mobility with transition matrices dates back to at least as early as Prais (1955) who examined transitions between occupational classes in England. There is no standardized measure in part because there are many aspects to mobility.[4] In this paper, we are primarily interested in so-called *relative mobility*. Relative mobility measures how likely it is that a household in wealth quantile n_1 at time s will be in some other quantile n_2 at time $s+t$, where t is a fixed number of periods in the future.

Formally, represent by $x(\Gamma)$ the distribution over each of N wealth quantiles (i.e., $x = [\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N}]$) and by $q(\Gamma)$ the wealth values defining the quantiles. That is,

$$q(\Gamma) = [q_1, q_2, ..., q_N]$$

where $q_1 = \underline{a}$ and $q_i = a_i : \sum_{j=1}^{J} \int d\Gamma(a, \varepsilon_j) \mathbf{1}_{\{q_{i-1} \leq a < q_i\}} = \frac{1}{5}$, for $i = \{1, ..., N\}$. The q_i values define the cutoff wealth values for entering the i^{th} quantile (the lowest wealth value in the quantile). Further, denote by $Q_i = \{a : a \in [q_i, q_{i+1})\}_{i=1,N-1}$ and $Q_N = \{a : a \geq q_N\}$; these sets define the wealth levels that constitute a given quantile. Finally, let $M_{NxN}(\Gamma)$ be a regular transition matrix induced by Γ with the element m_{ij} indicating the probability that a household in quantile Q_i will be in quantile Q_j after some fixed number of periods.[5]

[4]For a broad overview of the literature, see Fields and Ok (1999).

[5]According to Theorem 4.1.2 in Kemeny and Snell (1976)), a transition matrix is regular if and only if for some $t > 0$, M^t has no zero entries. Regularity guarantees that starting from any state in the Markov chain any other state can be visited in a finite amount of periods (that is, all states communicate). This condition is related to the 'monotone mixing condition' (see Hopenhayn and Prescott (1992)) used to prove the existence of a stationary distribution Γ, which Ríos-Rull (1998) labels 'the American Dream and the American Nightmare' condition. This condition is a long-run mobility requirement, whereas we are interested in short-run effects.

We will consider four measures from the literature, discussed at length in Dardanoni (1993). In particular, we highlight how each measure captures somewhat different aspects of mobility (due to the loss of information generated by moving from a matrix to a scalar).

2.0.1 Shorrocks Measure

Shorrocks (1978) measure of mobility focuses on the probability weight along the diagonal of M. One interpretation of the measure is that it reports the 'stickiness' of initial conditions. Formally, Shorrocks' measure is

$$\mu_S(M) = \frac{N - \mathrm{trace}(M)}{N-1}.$$

The Shorrocks measure takes values between 0 and 1, with smaller values indicating a lower likelihood that a household will escape its initial quantile. Importantly, the measure is unaffected a reallocation of mass along off-diagonal elements. The Shorrocks measure makes no distinction between economies where households move immediately from rags to riches and those where the poor become only slightly less poor. For example, the two Markov processes

$$\Pi_A = \begin{bmatrix} 0.5 & 0.5 & 0.0 \\ 0.25 & 0.5 & 0.25 \\ 0.0 & 0.5 & 0.5 \end{bmatrix}$$

and

$$\Pi_B = \begin{bmatrix} 0.5 & 0.0 & 0.5 \\ 0.25 & 0.5 & 0.25 \\ 0.5 & 0.0 & 0.5 \end{bmatrix}$$

would be regarded as equally mobile; however, the second process moves 'faster' since it admits one-period transitions between the lowest and highest wealth states, while the first process requires any movement between extreme states first pass through the middle.

2.0.2 Bartholomew's Immobility Measure

In contrast to the Shorrocks measure, Bartholomew and Bartholomew (1967) deals exclusively with the off-diagonal elements:

$$\mu_B(M) = \frac{1}{N-1} \sum_{i=1}^{N} \sum_{j=1}^{N} m_{ij} |i-j|$$

is the expected number of quantiles a household would cross into each period. The measure puts positive weight only on the off-diagonal probabilities. The term $|i-j|$, the absolute number of quantiles crossed into, places more weight on transitions that cross multiple quantiles; a transition matrix with more probability mass further from the diagonal has greater mobility (like Π_B in the previous subsection). Fields and Ok (1999) point out that Bartholomew's measure can be thought of as capturing total movement; economies in which households oscillate between being very rich and very poor would be measured as much more mobile than those where households transition more slowly through adjacent quantiles, even if the former involved fewer such transitions. To see how this measure works, consider the Markov processes

$$\Pi_A = \begin{bmatrix} 0.5 & 0.5 & 0.0 \\ 0.25 & 0.5 & 0.25 \\ 0.0 & 0.5 & 0.5 \end{bmatrix}$$

and

$$\Pi_B = \begin{bmatrix} 0.75 & 0.0 & 0.25 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.0 & 0.75 \end{bmatrix}$$

According to Bartholomew's measure, these chains are equally mobile:

$$\mu_B(\Pi_A) = 0.75$$
$$\mu_B(\Pi_B) = 0.75.$$

Agents make more frequent, "small" moves in A, and less frequent "large" moves in B.

2.0.3 Second Largest Eigenvalue

The second largest eigenvalue of a stochastic matrix governs the mixing rate of a Markov chain process, where a larger eigenvalue implying a slower mixing rate. Let $\lambda_i(M)$ be the i^{th} largest eigenvalue of M. A natural measure of mobility is $\mu_{2E}(M) = 1 - |\lambda_2(M)|$. Because M is regular $\lambda_1 = 1$, and $\lambda_i < 1$ for all $i > 1$. Sommers and Conlisk (1979) show that $\mu_{2E}(M)$ measures the total deviation of M from a matrix with perfect mobility.[6]

To understand why this measure captures mobility, we can show for a two-state Markov chain that the second highest eigenvalue is equal to the autocorrelation of the chain. Let the Markov chain transition matrix be

$$\Pi = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

which has invariant distribution

$$\pi^* = \left[\frac{1-q}{2-p-q}, 1 - \frac{1-q}{2-p-q}\right].$$

The autocorrelation is

$$\rho\left(z_t|z_{t-1}\right) = \frac{\left(z_1 - z_2\right)^2 (1-p)(1-q)\frac{p+q-1}{(2-p-q)^2}}{\left(z_1 - z_2\right)^2 (1-p)\frac{1-q}{(p+q-2)^2}} = p + q - 1$$

and the eigenvalues of Π are 1 and $p + q - 1$.

This result is not general, and we were unable to derive any analytical results for chains with more than 2 states. We therefore conducted a Monte Carlo exercise by drawing 5000 random stochastic matrices and computing the sample autocorrelation from a simulation of length $100,000$; it turns out that this autocorrelation is only weakly correlated with the second-highest eigenvalue of the transition matrix. However, if we confine ourselves to only Markov chains generated using the Rouwenhorst method, which preserves the autocorrelation of the two-state process as additional states are introduced, this result does hold.

[6] Perfect mobility for a NxN matrix is one with all elements equal to $1/N$. This concept is related to 'origin independence.'

2.0.4 Mean First Passage Time

The mean first passing matrix $T(M)$ is the expected number of periods until a household initially in quintile i first arrives in quintile j. Set $A = I - M$ and partition as

$$A = I - M = \begin{bmatrix} U & c \\ d^T & \alpha \end{bmatrix}.$$

Meyer (1978) shows that

$$T = (I - K + J \operatorname{diag}(K)) (\operatorname{diag}(K))^{-1} + E$$

where

$$E = \begin{bmatrix} 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \cdots & 0 & -1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix},$$

J is a matrix of all ones, and

$$K = \begin{bmatrix} U & 1^T \\ d^T & 1 \end{bmatrix}^{-1}.$$

Conlisk (1990) proposes using

$$MFP = x'Tx$$

as a measure of mobility; MFP is the expected number of periods before one household enters the quintile of another household when both are drawn at random from Γ. Because x has equal elements that sum to one (recall that x is a vector of quantiles), MFP is just the average value of the elements of T. For ease of comparison to the other measures, we define

$$\mu_{MFP}(M) = \frac{N}{MFP}.$$

If M is "perfectly mobile" $\mu_{MFP} = 1$. As the diagonal elements of M approach one, $\mu_M FP \to 0$.[7]

[7] μ_{MFP} cannot exceed 1 if M is *monotone* (i.e., each row is stochastically dominated by the one below it). Huggett (1993) proves the monotonicity of M in Bewley models with positively-autocorrelated shocks.

2.0.5 Cowell-Flachaire index

Cowell and Flachaire (2018) proposes a superclass of mobility measures which allows for the aggregation of a broad range of mobility concepts. Members of the class take the form

$$\Omega\left(M\right)=\begin{cases}\frac{1}{\alpha\left|1-\alpha\right|}n\sum_{i=1}^{n}\left[\left[\frac{u_i}{U}\right]^{\alpha}\left[\frac{v_i}{V}\right]^{1-\alpha}-1\right] & \alpha\neq0,1\\ -\frac{1}{n}\sum_{i=1}^{n}\frac{v_i}{V}\log\left(\frac{u_i}{U}/\frac{v_i}{V}\right) & \alpha=0\\ \frac{1}{n}\sum_{i=1}^{n}\frac{u_i}{U}\log\left(\frac{u_i}{U}/\frac{v_i}{V}\right) & \alpha=1\end{cases}$$

where n is the number of individuals in the population; u_i and v_i represent the "status" of individual i at the beginning and end, respectively, of the time period under consideration; and U and V are the mean status levels across individuals in each period. The index is amenable to different definitions of status. For example, in the context of wealth mobility, status may be defined as the specific levels of wealth held each individual, as a collection of intervals of over wealth, or as a subsets of a wealth distribution. The parameter α controls the weight given to downward movements relative to upward ones. For $\alpha<(>)0.5$, $\Omega\left(M\right)$ is more sensitive to upward (downward) movements. Cowell and Flachaire (2018) shows that members Ω satisfy many desirable properties in a mobility measure, including independence of population size and preservation of order under scaling.

The member of Ω that is suitable for comparing mobility between $K\times K$ quantile transition matrices is

$$\mu_{CF}\left(M\right)=\begin{cases}\frac{1}{\alpha(\alpha-1)}\left[\frac{2}{K(K+1)}\sum_{k=1}^{K}\sum_{l=1}^{K}m_{kl}k^{\alpha}l^{1-\alpha}-1\right] & \alpha\neq0,1\\ \frac{-2}{K(K+1)}\sum_{k=1}^{K}\sum_{l=1}^{K}m_{kl}l\log\left(\frac{k}{l}\right) & \alpha=0\\ \frac{2}{K(K+1)}\sum_{k=1}^{K}\sum_{l=1}^{K}m_{kl}k\log\left(\frac{k}{l}\right) & \alpha=1\end{cases}$$

So far we have defined these measures generally for any set of evenly-spaced quantiles. In the remainder of this paper, we will restrict attention to quintiles, that is $\dim\left(M\right)=5$.

2.1 Structural vs. Exchange Mobility

We are concerned with how quickly and to what extent agents change their ordering within the stochastic stationary distribution of wealth (known as *relative mobility*). In the steady state, households' wealth positions change, but the wealth distribution itself is time-invariant.

It would be intuitive to presume that relative mobility is just a simple function of the rates at which agents accumulate wealth and that greater relative mobility implies that households transition more quickly through quintiles, by rising and falling over a shorter time span. This *exchange* or *pure* mobility, however, is only one component of relative mobility. Differences in relative mobility can also arise from changes in the shape of the wealth distribution, even if individual savings behavior is the same. This concept is called *structural mobility*, and it can appear in the data when wealth inequality changes over time. In the stochastic steady state of a Bewley model, wealth inequality does not change over time. Nevertheless, structural mobility must still be taken into account when comparing the steady states from two models. Because of general equilibrium effects, changes in the model environment induce changes in the shape of the stationary distribution as well and are likely to alter the cutoffs defining wealth quintiles.

To illustrate, consider two distributions of wealth, Γ_1 and Γ_2, and let Γ_2 be a shape-preserving spread of Γ_1. Take a household from each distribution and label them according to their distribution of origin. Because their is more wealth in equality in Γ_2 than in Γ_1, the cutoffs which define the quantiles will be spread more apart. Even if household 1 and 2 begin with the same initial wealth, have the same optimal saving policies, and experience identical realizations for labor productivity, household 2 will transition across quantiles less frequently over the same amount of time, and so our measures of mobility would rank Γ_2 as less mobile than Γ_1. Figure 1 plots the cutoffs for entering each quintile as defined by the distribution of wealth from our experiments. Notice that there is not much change in the cutoffs until ρ exceeds 0.7. Beyond that, as the productivity process becomes more persistent, the distribution spreads out, and the cutoffs become further apart. In our numerical experiments, we will detail how we use the model to identify exchange mobility from structural mobility.

2.2 Exchange Mobility: Behavior vs. Luck

Once the movement of households through the distribution has been isolated from movements in the distribution itself, exchange mobility can separated further into changes due to differences in productivity shock process and changes in household behavior. Consider two households A and B with two Π matrices. Let $\rho_A = 0$ and $\rho_B = 0.5$ so

$$\Pi_A = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

and

$$\Pi_B = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}.$$

One might initially suppose that household A will have greater mobility than household B. After all, according to any one of the above measures, the earnings mobility of A is considerably greater than that of B. This fact however does not necessarily translate to greater wealth mobility. The reason is that randomness in the household earnings does not wholly determine a household's wealth. Because household utility is strictly concave, they try to smooth consumption over time. Since shocks for A are less persistent, the optimal response of household A to a switch in productivity is to adjust savings. The more persistent the shocks, the more closely earnings resemble permanent income and the less savings adjusts.

3 Wealth Mobility in the Data

3.1 Data

There has been relatively little empirical work on the intragenerational evolution of wealth.[8] We study eight waves of wealth supplements from the Panel Study of Income Dynamics (PSID) from 1984-2015 to measure wealth mobility. Following Hurst et al. (1998), we use identifiers from the family and individual files in the PSID to link families in the wealth supplements[9]. Then, using the population weights from the family files, we construct the distribution of wealth in each year and divide each distribution into quintiles. Finally, we measure the fraction of households that transition between quintile i and quintile j for

[8]Several studies on intragenerational wealth mobility have been conducted using a small number of waves from the PSID. See Castaneda et al. (2003), Hurst et al. (1998), and Díaz-Giménez et al. (2011).

[9]We include only families who have the same head at the beginning and end of the sample period. This would exclude cases where the head becomes deceased or institutionalized. In the case of a divorce, our methodology retains the head, but the non-head spouse is discarded. On average, this restriction removes 8-9 percent of any sample.

$i, j \in \{1, ..., 5\}$, between the starting and ending years.

We study three time horizons: short, medium and long. We define the short horizon as 5-6 years, the medium horizon as 9-10 years, and the long horizon as 19-21 years[10]. Table 1 reports the short, medium, and long-horizon wealth mobility matrices obtained from the PSID data.

Table 1: Mobility Matrices

Short Horizon

1984-1989

$$\begin{bmatrix} 0.70 & 0.23 & 0.05 & 0.02 & 0.00 \\ 0.25 & 0.45 & 0.22 & 0.06 & 0.02 \\ 0.06 & 0.24 & 0.44 & 0.19 & 0.06 \\ 0.02 & 0.06 & 0.22 & 0.47 & 0.23 \\ 0.01 & 0.01 & 0.06 & 0.22 & 0.70 \end{bmatrix}$$

1989-1994

$$\begin{bmatrix} 0.66 & 0.24 & 0.07 & 0.02 & 0.01 \\ 0.27 & 0.45 & 0.18 & 0.07 & 0.02 \\ 0.08 & 0.25 & 0.42 & 0.19 & 0.06 \\ 0.03 & 0.06 & 0.27 & 0.42 & 0.21 \\ 0.01 & 0.03 & 0.05 & 0.24 & 0.66 \end{bmatrix}$$

1994-1999

$$\begin{bmatrix} 0.64 & 0.26 & 0.07 & 0.02 & 0.01 \\ 0.25 & 0.47 & 0.21 & 0.05 & 0.02 \\ 0.10 & 0.22 & 0.43 & 0.21 & 0.04 \\ 0.04 & 0.07 & 0.24 & 0.44 & 0.21 \\ 0.01 & 0.03 & 0.06 & 0.20 & 0.70 \end{bmatrix}$$

2001-2007

$$\begin{bmatrix} 0.62 & 0.25 & 0.10 & 0.03 & 0.01 \\ 0.27 & 0.43 & 0.22 & 0.07 & 0.02 \\ 0.09 & 0.28 & 0.37 & 0.21 & 0.05 \\ 0.03 & 0.08 & 0.24 & 0.43 & 0.22 \\ 0.01 & 0.03 & 0.05 & 0.22 & 0.69 \end{bmatrix}$$

2007-2013

$$\begin{bmatrix} 0.60 & 0.30 & 0.08 & 0.02 & 0.00 \\ 0.26 & 0.44 & 0.24 & 0.06 & 0.01 \\ 0.14 & 0.18 & 0.43 & 0.21 & 0.04 \\ 0.07 & 0.08 & 0.20 & 0.47 & 0.19 \\ 0.02 & 0.02 & 0.05 & 0.21 & 0.71 \end{bmatrix}$$

Medium Horizon

1984-1994

$$\begin{bmatrix} 0.63 & 0.24 & 0.09 & 0.03 & 0.02 \\ 0.23 & 0.41 & 0.21 & 0.10 & 0.05 \\ 0.10 & 0.28 & 0.33 & 0.21 & 0.09 \\ 0.05 & 0.08 & 0.26 & 0.37 & 0.23 \\ 0.02 & 0.03 & 0.09 & 0.25 & 0.61 \end{bmatrix}$$

1994-2003

$$\begin{bmatrix} 0.61 & 0.26 & 0.09 & 0.03 & 0.02 \\ 0.24 & 0.44 & 0.23 & 0.06 & 0.03 \\ 0.11 & 0.25 & 0.35 & 0.23 & 0.06 \\ 0.06 & 0.09 & 0.24 & 0.39 & 0.22 \\ 0.03 & 0.04 & 0.08 & 0.21 & 0.65 \end{bmatrix}$$

2003-2013

$$\begin{bmatrix} 0.57 & 0.29 & 0.10 & 0.03 & 0.01 \\ 0.27 & 0.41 & 0.23 & 0.07 & 0.02 \\ 0.14 & 0.21 & 0.39 & 0.20 & 0.05 \\ 0.06 & 0.08 & 0.21 & 0.44 & 0.22 \\ 0.02 & 0.02 & 0.05 & 0.23 & 0.68 \end{bmatrix}$$

Long Horizon

1984-2003

$$\begin{bmatrix} 0.58 & 0.25 & 0.11 & 0.05 & 0.02 \\ 0.26 & 0.35 & 0.22 & 0.12 & 0.05 \\ 0.09 & 0.29 & 0.27 & 0.22 & 0.13 \\ 0.05 & 0.11 & 0.27 & 0.32 & 0.26 \\ 0.03 & 0.06 & 0.11 & 0.26 & 0.55 \end{bmatrix}$$

1989-2009

$$\begin{bmatrix} 0.56 & 0.28 & 0.10 & 0.04 & 0.03 \\ 0.27 & 0.37 & 0.20 & 0.12 & 0.05 \\ 0.12 & 0.25 & 0.29 & 0.22 & 0.12 \\ 0.08 & 0.11 & 0.29 & 0.32 & 0.20 \\ 0.02 & 0.05 & 0.09 & 0.25 & 0.60 \end{bmatrix}$$

1994-2015

$$\begin{bmatrix} 0.58 & 0.24 & 0.11 & 0.04 & 0.03 \\ 0.28 & 0.38 & 0.20 & 0.10 & 0.04 \\ 0.13 & 0.25 & 0.32 & 0.21 & 0.08 \\ 0.07 & 0.11 & 0.24 & 0.34 & 0.25 \\ 0.03 & 0.05 & 0.09 & 0.25 & 0.58 \end{bmatrix}$$

It is immediately clear from Table 1 that empirical wealth data show quite a bit of mobility. Particularly, we see that although the first and fifth quintiles are the most persistent, families who begin in these quintiles have at least a thirty percent chance of ending elsewhere, at all time horizons. Additionally, families in the middle three quintiles are, in every period and at all horizons, more likely to leave their starting quintile than they are to stay.

[10]While it would be ideal to have a fixed length for each horizon, the irregular timing of PSID releases does not permit it.

A final key observation is the nonzero masses of families that make large transitions, crossing multiple quintiles in one period. For example, we find that of the families who began in the first quintile in 2003, about three percent end in the fourth quintile, and about one percent end in the fifth. We also see large movements in the other direction with a nontrivial frequency: over the same period, for example, about two percent of families who began in the uppermost quintile ended in the second, and about the same proportion ended in the first quintile.

Figures 17 through 19 show the evolution of our mobility measures over time for each horizon. There is no apparent trend at the short horizon. However, over the medium and long horizons, our measures show a decline in wealth mobility since 1984.

3.2 Confidence Intervals for Mobility

A question that arises when measuring economic mobility is the degree of certainty associated with mobility matrices, and the measures applied to them. For example, as shown in Table 1, we find that the proportion of families who begin in the first quintile and end in the fifth fell from 1.2% between 1994 and 1999, to about 0.7% between 2001 and 2007. This observation raises the question of certainty: can we say that this decrease is *statistically significant*? The same question can be raised in relation to the observation that, for example, the Shorrocks measure of mobility rose between these two periods: can we say that mobility increased significantly?

In order to address these questions, we estimate standard errors associated with wealth mobility using a bootstrapping procedure. In each iteration, we draw a new panel sample with replacement, using the same number of observations as our original sample. For each such sample, we re-estimate the wealth distributions in the starting and ending years, and construct a mobility matrix using the same procedure as before. Using this procedure, we construct 95% confidence intervals around each entry in the mobility matrix, as well as the same intervals around the mobility exhibited by these matrices, as measured by each of the aforementioned measures. In Table 2 we show an example of the results of this process for the period 1994-1999. The large entries in the matrix represent our point estimates for the transition rates over this period, as reported in table 1. The smaller, parenthetical entries below represent a 95% confidence interval for each element of the matrix.

From this table, we can see that, for example, between 61 and 67 percent of families in the

Table 2: Wealth Transition Matrix with 95% Confidence Intervals

<u>1994-1999</u>

$$\begin{bmatrix} 0.64 & 0.26 & 0.07 & 0.02 & 0.01 \\ (0.61,0.67) & (0.24,0.29) & (0.05,0.08) & (0.01,0.02) & (0.01,0.02) \\ 0.25 & 0.47 & 0.21 & 0.05 & 0.02 \\ (0.22,0.28) & (0.44,0.50) & (0.19,0.24) & (0.04,0.07) & (0.01,0.03) \\ 0.10 & 0.22 & 0.43 & 0.21 & 0.04 \\ (0.08,0.12) & (0.19,0.24) & (0.40,0.46) & (0.18,0.23) & (0.03,0.06) \\ 0.04 & 0.07 & 0.24 & 0.44 & 0.21 \\ (0.02,0.05) & (0.05,0.09) & (0.21,0.27) & (0.41,0.47) & (0.18,0.24) \\ 0.01 & 0.03 & 0.06 & 0.20 & 0.70 \\ (0.00,0.02) & (0.02,0.05) & (0.04,0.08) & (0.17,0.22) & (0.67,0.73) \end{bmatrix}$$

first quintile in 1994 ended in the first quintile in 2003, while between one and two percent of these families transitioned to the fifth quintile. The interval for the latter figure over the period 2001 to 2007 is about 0.2% to 1.2%, so to answer an earlier question, we cannot conclude that the proportion of families transitioning from the first to the fifth quintile did not fall significantly between the periods 1994-1999 and 2001-2007.

Our bootstrapping procedure also gives us the opportunity to reassess time trends in mobility. In each bootstrapping iteration, we apply our four measures of mobility to the resultant matrix. By doing so, we can compute bootstrapped standard errors for measures of wealth mobility for each sample. Figures 20 through 22 show the measures of short, medium, and long-horizon wealth mobility as reported before, with shading indicating 95% confidence intervals. Broadly, we see that the size of these intervals depends on both the time horizon (which influences the number of observations available), and on the measure used. Once again, it is difficult to extrapolate a trend in the measures of short-horizon wealth mobility[11]. At the medium horizon, we find that, at the 5% level, only one measure rates any given period as having significantly less mobility than the preceding period. However, by three of the four measures used, wealth mobility from 2003-2013 was significantly lower than from 1984-1994. Thus, we can still safely say that medium-horizon wealth mobility has significantly declined over our entire sample period. At the long horizon, we cannot say that mobility declined significantly over our sample period.

[11] Although, to answer a question posed earlier, the increase in mobility from 1994-1999 to 2001-2007 was in fact significant at the five percent level.

3.3 Structural and Exchange Mobility in the Data

As was mentioned in section 3.1, wealth mobility arises from two sources: *structural* mobility and *exchange* mobility. Again, structural mobility refers to mobility that arises from changes in the shape of the wealth distribution, while exchange mobility is mobility arising from households changing their wealth position relative to other households. We aim decompose mobility in the PSID wealth data into structural and exchange mobility. Using the same samples that we use to measure overall mobility, we estimate exchange mobility by recalculating the mobility matrix for each period, holding fixed the cutoff values for wealth quintiles. That is, for each sample we divide the families into quintiles based on their starting wealth as before, and keep track of the wealth values that demarcate those quintiles. We then take that family's wealth in the ending year and record the quintile in which that wealth value would have fallen, using the quintile cutoffs from the starting year. In this way, we hold the distribution fixed, and any mobility is purely the result of households' changing their relative wealth. In order to estimate structural mobility, then, we subtract exchange mobility from total mobility over the sample period, as calculated by the procedure outlined above.

Figures 23 through 25 show the time patterns in structural, exchange, and total mobility over our sample period. In most cases, total mobility lies between exchange and structural mobility, and the contribution of structural mobility to total mobility is negative. This observation is consistent with the well-documented fact that wealth inequality in the US has increased over the past 30 years. That being said, in most cases, the mobility lost to structural changes in the wealth distribution is minimal. This decomposition shows that the large majority of empirical wealth mobility is exchange mobility.

4 Factors Influencing Mobility in Data

Comparing matrices generated by our model to those generated by the PSID wealth data shows that our models fall short of approximating the level of wealth mobility seen in the data. In this section, we use regression analysis to find suggestive evidence of the type of shocks that, if added to our model, would help us better approximate real-world mobility. In particular, we will focus on factors that affect wealth directly, such as a payoff from a risky asset, a divorce, or the receipt of a large inheritance.

We address this question in two ways. Following Jianakoplos and Menchik (1997), we estimate regressions using data from our samples to determine the effect of the factors mentioned above on wealth movements. First, we regress a family's change in percentile ranking on a vector X of factors that may influence changes in a family's relative ranking in the wealth distribution:

$$\Delta p_{i,t} = X_{i,t}^{\top}\beta + \alpha_t + \varepsilon_{i,t}$$

Here, $\Delta p_{i,t} = p_{i,t} - p_{i,t-1}$, that is, the change in a family's percentile ranking in the wealth distribution over a given period. The vector $X_{i,t}$ includes control variables such as a family's income and the head's age at the beginning of period $[t-1,t]$, as well as indicators for the holding of certain assets, or the head's possession of a college degree at the start of period $[t-1,t]$. We also include period fixed effects, α_t, in order to control for unobserved factors that may influence wealth mobility[12].

Then, using the same vector of explanatory variables X_{it}, we estimate four Probit regressions. First, we define $\Delta q_{i,t} = q_{i,t} - q_{i,t-1}$, where $q_{i,t}$ represents the family's quintile in the wealth distribution in year t. We estimate two Probit models using this outcome:

$$\Pr(\Delta q_{i,t} < 0|X) = \Phi(X^{\top}\beta) \tag{1}$$

$$\Pr(\Delta q_i > 0|X) = \Phi(X^{\top}\beta) \tag{2}$$

Here, model (1) measures the probability that a family *fell* one or more quintiles over the period $[t-1,t]$, model (2) measures the probability that a family *rose* one or more quintiles, and Φ is the Cumulative Distribution Function of a standard normal distribution.

Our final two models aim to measure factors that may influence the likelihood that a family makes a large movement through the distribution. We update models (1) and (2) to focus on families who rise or fall *two* or more quintiles over a given sample period, movements that we refer to as "jumps:"

[12]Such as a recession, or a change in tax policy.

$$\Pr(\Delta q_{i,t} \leq -2|X) = \Phi(X^\top \beta) \quad (3)$$

$$\Pr(\Delta q_i \geq 2|X) = \Phi(X^\top \beta) \quad (4)$$

All of the aforementioned regressions are estimated using data on wealth mobility over the short, medium, and long time horizons.

One note should be made on these large movements. Due to the high reinterview rates in the PSID, families often appear in our samples for multiple time periods. Thus, we are able to follow some families through most or all of our thirty-year sample horizon. Doing so suggests that some families have a higher tendency than others to make "jumps," movements of two or more quintiles in the wealth distribution. At any given short-horizon period in our sample horizon, the probability that a family moves two or more quintiles is between nine and twelve percent. However, the probability that a family makes such a "jump" conditional on that family having made a "jump" in the preceding period is substantially higher–between 20 and 30 percent, depending on the period in question.

Additionally, using our panel samples taken from the PSID data, we calculate mobility matrices for subsets of our samples. Calculating these matrices for different time horizons gives us a better sense of the ways in which some of the factors in our analysis contribute to overall mobility.

4.1 Risk, Return, and Entrepreneurship

Evidence from the PSID suggests that a contributing factor in wealth mobility may be heterogeneity in risk preferences and returns among families. Using questions from the wealth survey, we can study the movements of families who hold at least some portion of their net worth in assets with large variance in returns, such as stocks, real estate outside of their main residence, and entrepreneurial ventures such as farms and self-owned businesses. Broadly, our Probit models do suggest that holding such assets does make a family more likely to move throughout the distribution. For example, at each time horizon, we find that ownership of stocks makes a family more likely to move up one or more quintile, and less likely to fall.

Perhaps most notably, we find that ownership of a farm or business increases both the likelihood that a family will *fall* in the distribution, and the likelihood that a family will *rise* in the distribution. The symmetric effect of these entrepreneurial activities also holds when we look at the likelihood that a family moves two or more quintiles in a given period; a family's holding assets in this category increased the likelihood of "jumps" in both directions. This dichotomy can also be seen in Table 3, which shows the mobility matrices for families who respectively did (Π_B) and did not own a business (Π_{NB}) between 2003 and 2013:

Table 3: Mobility With and Without Business Ownership, 2003-2013

$$\Pi_B = \begin{bmatrix} 0.45 & 0.27 & 0.14 & 0.10 & 0.04 \\ 0.20 & 0.31 & 0.28 & 0.16 & 0.04 \\ 0.10 & 0.18 & 0.35 & 0.24 & 0.14 \\ 0.09 & 0.04 & 0.14 & 0.44 & 0.29 \\ 0.03 & 0.02 & 0.04 & 0.18 & 0.73 \end{bmatrix}, \Pi_{NB} = \begin{bmatrix} 0.58 & 0.29 & 0.10 & 0.02 & 0.00 \\ 0.28 & 0.43 & 0.22 & 0.05 & 0.01 \\ 0.16 & 0.22 & 0.40 & 0.19 & 0.03 \\ 0.04 & 0.09 & 0.24 & 0.45 & 0.18 \\ 0.02 & 0.02 & 0.05 & 0.27 & 0.64 \end{bmatrix}$$

Clearly, households who owned a farm or business were more likely to leave their starting quintiles, as well as more likely to make large movements in the distribution. Notice, for example, the different patterns in movements made by families who began in the first quintile: those who owned a business were about as likely as those who did not to move to the second quintile, but were far more likely to move to the third, fourth, and fifth quintiles. Similarly, families that owned a business were about twice as likely as those who did not to fall from the fourth quintile to the first.

These results suggest that a key influence in wealth movements is the opportunity for a person to invest time and resources into a project that is at least partially self-funded, and face the potential for both large gains and large losses from this project.

4.2 Other Shocks to Wealth

We document evidence that wealth mobility in data may be driven by other shocks directly to wealth, outside of those resulting from the realization of a return on an asset. We study two such shocks: marriages/divorce (wherein assets are combined and divided, respectively) and the receipt of inheritances. In order to capture the effect of these shocks, we include in our Probit specifications binary variables indicating whether the head went through a marriage or divorce or received an inheritance, in any of the intervening years between the start and end of the time given time horizon.

Not surprisingly, we find that the occurrence of a marriage and a divorce have symmetric effects: marriages increase the likelihood of a family rising in the distribution, while divorces make it more likely that a family will slip at least one quintile. Additionally, the occurrence of a marriage is a strong predictor of a family making an upward "jump" of two or more quintiles, and a divorce is a strong predictor of a family making a downward "jump." Importantly, the explanatory power of these events holds at the short time horizon.

We also find evidence that inheritances are strong predictors of large upward movements, particularly over short time horizons. Although this is hardly surprising, it does give us further insight into the type of features that could augment the model in order to better match wealth mobility in the data. The PSID provides us with evidence that incorporating shocks that affect wealth directly–rather than indirectly, through the labor income or savings process–may be a key component in producing realistic mobility.

5 Model

As a starting point, we study the long run properties of Aiyagari (1994) with no borrowing.[13] There is a unit measure of *ex ante* identical households. Every period, each household receives an idiosyncratic labor productivity shock, ε, from a finite set $\mathcal{E} = [\varepsilon_1, \varepsilon_2, ..., \varepsilon_J]$ with $\varepsilon_1 < \varepsilon_2 < ... < \varepsilon_J$. The process for productivity shocks be Markov with stochastic transition matrix $\Pi = \Pr(\varepsilon_j|\varepsilon_i)$ for $j, i \in 1, .., J$. Every household supplies the same fixed number of hours, $\overline{h}$, and earns total labor income equal to $\omega\overline{h}\varepsilon$, where ω is a market-wide wage. Because the wage and hours supplied do not change across periods, labor productivity shocks are equivalent to random labor income endowments. As in the standard incomplete-markets model, there is only one asset, a, which is a claim to the capital stock K. Because no state contingent claims exist, households have a motive to self-insure through precautionary savings.

A stand-in firm combines capital and effective labor through a constant-returns-to-scale production technology $F : \Re^+ \times \Re^+ \to \Re^+$ to produce a final good which may be consumed or invested in capital for next period. The firm manages the capital stock from household's saving, pays an interest rate r on assets, hires labor, and invests in new capital. Capital depreciates at a constant rate δ each period. We assume that the firm behaves competitively.

[13]Because we are concerned with mobility in the stochastic steady state, we omit time subscripts.

Letting F be Cobb-Douglas, the optimal choice of the firm implies that each factor is paid its marginal product:

$$\omega = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

and

$$r = \alpha\left(\frac{K}{N}\right)^{\alpha-1} - \delta.$$

The state vector of the household has two elements: current wealth, a, and current labor productivity, ε. Let period utility be represented by a continuous, strictly concave function $u : \Re^{+} \to \Re$, and assume that u is continuously differentiable as many times as necessary. The household problem in recursive form is

$$V(a,\varepsilon) = \max_{c,a'}\left\{u(c) + \beta E_{\varepsilon'|\varepsilon}\left[V(a',\varepsilon')\right]\right\}$$

subject to the budget constraint

$$c + a' \leq w\varepsilon + (1+r)\,a$$

and lower bound constraints

$$c > 0; a' \geq \underline{a}.$$

Denote by $\Gamma(a,\varepsilon)$ the distribution of households over $\mathcal{A} \times \mathcal{E}$.

Definition 1. *A **steady-state recursive competitive equilibrium** is a set of value functions $V(a,\varepsilon)$, policy functions $g_a(a,\varepsilon)$, $g_c(a,\varepsilon)$), pricing functions, r and w, and a distribution $\Gamma(a,\varepsilon)$ such that*

1. Given prices, V, g_a and g_c solve the household's problem.

2. Firms maximize profits

   $$\omega = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

   and

   $$r = \alpha\left(\frac{K}{N}\right)^{\alpha-1} - \delta.$$

3. Markets clear:

$$K = \sum_{j=1}^{J} \int a d\Gamma \left(a, \varepsilon_j\right)$$

$$N = \sum_{j=1}^{J} \int \overline{h} \varepsilon_j d\Gamma \left(a, \varepsilon_j\right).$$

4. Γ is consistent with the saving decisions of households and the process for ε.

5. The joint distribution of wealth and productivity $\Gamma(a, \varepsilon)$ is stationary.

6 Numerical Experiments

6.1 Baseline

We choose fairly standard values for our structural parameters: we let utility be logarithmic, we choose $\beta = 0.99$ and $\delta = 0.025$ as roughly consistent with quarterly aggregates for the capital/output and investment/output ratios, and we set $\alpha = 0.36$ to match capital's share of income. We also choose a zero borrowing limit.

We follow Floden and Lindé (2001) who estimate an earnings process of $\rho = 0.92$ and $\sigma_\varepsilon = 0.21$ (annual) from the PSID. The resulting 5-year wealth transition matrix is

$$\begin{bmatrix} 0.87 & 0.14 & 0.00 & 0.00 & 0.00 \\ 0.13 & 0.73 & 0.14 & 0.0 & 0.00 \\ 0.00 & 0.14 & 0.74 & 0.12 & 0.00 \\ 0.00 & 0.00 & 0.12 & 0.81 & 0.07 \\ 0.00 & 0.00 & 0.00 & 0.08 & 0.92 \end{bmatrix}$$

which features far less wealth mobility than any of the transition matrices above. Because the underlying source of both inequality and mobility in this model is the stochastic earnings process, we examine how the transition matrix above responds to different assumptions about the Markov process.

6.1.1 Earnings Process

The fundamental force driving the distribution of wealth in the economy is the labor productivity process. We assume the Markov process above approximates

$$\log(\varepsilon') = \rho \log(\varepsilon) + \nu', \quad \nu' \sim N\left(0, \sigma^2\right).$$

We set J, the number of individual productivity states, to 2.[14] Given this and the parameters ρ and σ, we use the Rouwenhorst method to construct the Markov chain process. Under the Rouwenhorst method, the Markov chain depends upon ρ and σ. The states are equally-space over the interval $[-\psi, \psi]$, where

$$\psi = \frac{\sqrt{(J-1)}}{\sqrt{(1-\rho^2)}}\sigma.$$

The transition matrix, Π, depends on two parameters, p and q. Following Kopecky and Suen (2010), we set

$$p = q = \frac{1+\rho}{2};$$

note that Π only depends upon the persistence parameter ρ.

A consequence of generating a Markov chain in this manner is that if one only varies ρ and keeps σ fixed, the vector of states will be different for each value of ρ. This dependence will cause the marginal distribution of effective labor to vary across experiments due solely to the approximation procedure, which could mess up our comparisons. To prevent this contamination, we make σ a function ρ. Given a baseline ρ_0 and σ_0, we define

$$\sigma(\rho) = \sigma_0 \sqrt{\frac{1-\rho^2}{1-\rho_0^2}}.$$

This procedure guarantees that the ε state vector of productivity remains the same across ρ experiments and, because labor is supplied inelastically, so does N. Moreover, because Π depends solely on ρ, we can isolate changes to the transition probabilities without altering the states. In this way, ρ will increase the probability of earning the same (by construction)

[14] We have run our experiments with 7 productivity states as well. In general, the qualitative results do not change significantly. One issue that arises when there are more than 2 values for productivity is for very low values of ρ the transition matrix is no longer monotone (i.e., the conditional probability of moving from $\varepsilon = \varepsilon_i$ to $\varepsilon' = \varepsilon_j$, $j \neq i$, does not monotonically decrease as the distance between j and i increases). Since monotonicity of the transition matrix is important for understanding the mobility measures and this failure is simply an approximation error, we concentrate on the two-state case.

current labor income in the next period (it increases the weight along the diagonal of the transition matrix).

6.2 The three factors affecting wealth mobility as ρ changes

We conduct a series of computational experiments to identify the fundamental ingredients governing individual wealth mobility within the model. Specifically, we vary ρ, compute the stochastic steady state, approximate the quintile wealth transition matrix via simulation, and calculate mobility. Figure 2 plots the relationship between ρ and several measures of mobility. Mobility is hump-shaped across persistence with mobility being low when ρ is near 0 and when ρ is near 0.9, and reaches its peak for $\rho \in (0.75, 0.80)$. Because for each value of ρ the model is solved in general equilibrium, the market clearing interest rate and the wealth distribution itself will differ in each case. Thus, our results are the combination of changes in structure, behavior, and luck. In a later section, we describe our strategy for identifying the portion of mobility arising from each of these components, but first we will discuss persistence affects each in turn.

Structure Figure 3 plots the steady state wealth distribution under different values of the persistence of the productivity process. There are two things to note about the distribution as ρ increases. First, the wealth becomes more unequally distributed as the right tail stretches out. Because there are only two productivity states, in equilibrium households with the high (low) productivity are savers (dissavers). The closer ρ is to 1, the more likely households with high ε are to draw high ε'. As a consequence, some households will receive a very long string of good productivity shocks, allowing them to amass a considerable amount of wealth. In the same way, households that draw a low productivity will be more likely to draw low productivity in the future, leading to the second feature of a larger ρ: more households are borrowing constrained. These changes in the structure of the wealth distribution affect the boundaries between quintiles. Figure 1 plots these boundaries for different values of ρ. The cutoffs move apart gradually as ρ approaches 0.7. As the productivity process becomes more even persistent, however, the distribution spreads out rapidly, and the boundaries become further apart. When $\rho = 0.99$, the entire first quintile is at the borrowing limit.[15]

[15] Under some measures, the narrowness of the first quintile can lead to 'spurious' mobility because households will very frequently transition between the first and second quintiles despite almost no change in

Behavior Optimal household behavior changes responds to the persistence of the shocks as well. The more sensitive is the saving policy to ε, the larger the wealth movements will be across periods, which in turn implies more rapid resorting. Here we state a proposition about the relationship between ρ and the saving policy function $g_a(a, \varepsilon)$ when the wealth distribution is fixed.

Proposition 2. *Consider two households, A and B, from the same steady state wealth distribution, and without loss of generality, let $\rho_A > \rho_B$. For $a > \underline{a}$, the distance between saving functions across productivity draws is larger for the household with a higher probability of switching productivity states,*

$$(i.e., \left|g_a^B(a, \varepsilon_2) - g_a^B(a, \varepsilon_1)\right| > \left|g_a^A(a, \varepsilon_2) - g_a^A(a, \varepsilon_1)\right|).$$

Proof. Consider two households in the same wealth distribution Denote by π_{ij} the conditional probability that $\varepsilon' = \varepsilon_j$ given $\varepsilon = \varepsilon_i$. The corresponding conditional probability that $\varepsilon' = \varepsilon_{-j}$ is $1 - \pi_j$. Because $\rho^A > \rho^B$, $\pi_{11}^A > \pi_{11}^B$, and $\pi_{21}^B > \pi_{21}^A$.

We will show that $g_a^B(a, \varepsilon_1) < g_a^A(a, \varepsilon_1) < g_a^A(a, \varepsilon_2) < g_a^B(a, \varepsilon_2)$. It follows from the conditions on u and on the compactness of the budget set that $g_a^i(a, \varepsilon)$ is strictly increasing both arguments, so the inner most inequality is immediate. Next we will prove that $g_a^B(a, \varepsilon_1) < g_a^A(a, \varepsilon_1)$.

Assume not so $g_a^A(a, \varepsilon_1) \leq g_a^B(a, \varepsilon_1)$. Then by the budget constraint $c^B \leq c^A$, where c^i is consumption of household i. By the strict concavity of u,

$$u'\left(c^A\right) \leq u'\left(c^B\right)$$

which from the Euler equation implies

$$\pi_{11}^A V_1^A\left(g_a^A(a, \varepsilon_1), \varepsilon_1\right) + \left(1 - \pi_{11}^A\right) V_1^A\left(g_a^A(a, \varepsilon_1), \varepsilon_2\right) \leq \pi_{11}^B V_1^B\left(g_a^B(a, \varepsilon_1), \varepsilon_1\right) + \left(1 - \pi_{11}^B\right) V_1^B\left(g_a^B(a, \varepsilon_1), \varepsilon_2\right)$$

where V_1 is the derivative of V with respect to wealth.

We can use Theorem 6.8 from Acemoglu (2009) to establish that V is strictly concave in a.

wealth.

The strict concavity of V in a leads to a contradiction since

$$\begin{aligned} V_1^A\left(g_a^A(a,\varepsilon_1),\varepsilon_1\right) &< \pi_{11}^A V_1^A\left(g_a^A(a,\varepsilon_1),\varepsilon_1\right) + \left(1-\pi_{11}^A\right) V_1^A\left(g_a^A(a,\varepsilon_1),\varepsilon_2\right) \\ &\leq \pi_{11}^B V_1^B\left(g_a^B(a,\varepsilon_1),\varepsilon_1\right) + \left(1-\pi_{11}^B\right) V_1^B\left(g_a^B(a,\varepsilon_1),\varepsilon_2\right) \\ &< V_1^B\left(g_a^B(a,\varepsilon_1),\varepsilon_1\right) \end{aligned}$$

which implies

$$g_a^A(a,\varepsilon_1) > g_a^B(a,\varepsilon_1).$$

Finally, we will show that $g_a^A(a,\varepsilon_2) < g_a^B(a,\varepsilon_2)$. Once again, assume not. Then

$$\begin{aligned} g_a^B(a,\varepsilon_2) &\leq g_a^A(a,\varepsilon_2) \\ u'\left(c^B\right) &\leq u'\left(c^A\right) \\ \pi_{21}^B V_1^B\left(g_a^B(a,\varepsilon_2),\varepsilon_1\right) + \left(1-\pi_{21}^B\right) V_1^B\left(g_a^B(a,\varepsilon_2),\varepsilon_2\right) &\leq \pi_{21}^A V_1^A\left(g_a^A(a,\varepsilon_2),\varepsilon_1\right) + \left(1-\pi_{21}^A\right) V_1^A\left(g_a^A(a,\varepsilon_2),\varepsilon_2\right) \\ V_1^B\left(g_a^B(a,\varepsilon_2),\varepsilon_1\right) &< V_1^A\left(g_a^A(a,\varepsilon_2),\varepsilon_2\right) \end{aligned}$$

Again by strict concavity of V in a,

$$g_a^B(a,\varepsilon_2) > g_a^A(a,\varepsilon_2)$$

which is a contradiction. □

Intuitively, Proposition 2 is the permanent income hypothesis. If household A and household B have the same assets today and each draws the good shock, but A believes that its shock comes from a more persistent process than B does, then A's consumption will be more responsive and so A's saving will move less than B's will. The consequence is that, all else equal, mobility due to behavior should decrease as ρ increases.

Luck Finally, a household's mobility will be affected by the particular sequence of productivity draws. Within a given measurement window, if a household, beginning from a low wealth level, happens by chance to get higher productivity than would be expected, then that household will have high wealth mobility. The effect on mobility of more persistence in good and bad luck is not monotone. Generally, mobility will be low when persistence is either very low or very high. At very high ρ, households that start with good fortune will tend to continue having good productivity, increasing their saving and moving further away from other less fortunate households. At very low ρ, mobility is low because households switch too frequently. If the household starts in a low quintile and receives a good shock, it saves and moves up a bit in the wealth ordering, but in order to move even further up and transition through multiple quintiles over time, the household needs to get *a string* of positive shocks that is well above average. The probability of getting such a string however increases in ρ. The result is that for low ρ households tend to move around only a small region of their initial wealth position. Luck will tend to push up mobility if ρ lies in some intermediate range. In that region, households will tend to get sufficiently long strings of positive shocks to transition across quintiles, but switch between states frequently enough to support mixing.

Total mobility With these three factors in mind, the inverted U-shape of mobility over ρ can now be understood more easily. As ρ increases, agents experience longer sequences of above (below) average productivity, leading to longer strings of saving (dissaving) and a wider distribution of wealth. The expansion of the distribution should reduce mobility since it increases the distance between quintile boundaries (with the possible exception of the one between the first and second quintiles). More autocorrelated shocks should increase mobility since it allows households to experience longer strings of movement in the same direction, whether up or down; however, this effect is somewhat offset by the reduction in the sensitivity of savings to the shocks. While at higher ρ, households move in the same direction longer, they in smaller steps.

The above proposition explains the hump-shape in mobility. At low ρ, a move from state (k, ε_1) to (k, ε_2) induces a large change in k'. In itself, this would increase mobility, but because ρ is low, the probability of returning to the lower $g_k(k, \varepsilon_1)$ rule is high. Thus, it is likely that such a household will not experience a long enough string of high productivities to accumulate a lot more wealth and move up into other quintiles. By a similar logic, a household that just drew ε_1 after having been ε_2 is unlikely to move down quintiles. On average households in a low-ρ environment, are very unlikely to move far away from their initial wealth level, k, though they will move very frequently within a small neighborhood of k.

As ρ increases, the distance of between savings functions does not fall much but the

Table 4: Correlation between mobility measures

Correlation Coefficients			
	μ_B	μ_{2E}	μ_S
μ_{MFP}	0.9991	0.9997	0.9991
μ_S	1.0000	0.9991	—
μ_{2E}	0.9991	—	—

likelihood of experiencing a long string of consecutive ε_2 productivities rises. This allows households to move greater distances within the wealth distribution over a fixed amount of time. At some point however, ρ becomes so large that households switch productivities very infrequently, and the distance between savings rules gets very small. A household that starts on the savings path implied by $g(k, \varepsilon_2)$ is likelihood to continue building up wealth for a long time but very slowly so that it takes many periods to transition between quintiles. In our numerical experiments, we find a ρ near 0.7 returns the highest measure of mobility over quintiles.

Figure 2 plots these mobility measures as functions of ρ (again where σ is normalized). While the levels of the mobility measures differ, the orderings are very similar. For instance, the correlations are nearly 1 as shown in Table 4.

6.2.1 Ghost households

In order to isolate the effects of structure, behavior and luck to mobility, we introduce 'ghost' households into the computed steady state wealth distributions. A ghost is single, zero-measure agent that differs from the other households in the economy in some way. Because a ghost is atomistic, its presence does not alter either equilibrium prices or the quintile boundaries of the wealth distribution. By changing the ghost's environment, policy rules, or labor productivity we can control for each of the other factors. In the first step toward constructing our decomposition, we introduce ghosts with different labor income processes into each of the steady wealth distributions found in the baseline. For exposition, we will draw a distinction between the ρ value of the process faced by normal households (that is, the value which gave rise that particular wealth distribution) and the ρ value of the ghost. Denote the first, ρ_{GE}, and the second, ρ_G. We then simulate and construct a 5×5 mobility matrix for each ghost. We will perform this exercise for two types of ghost households.

The first ghosts understand that their process has a different autocorrelation than that of the other households around them. As a result, their saving decision rules will differ from those of the standard households in the economy, as will the realization of their productivity shocks. The second type of ghosts, believe that they have the same process as the standard households but experience the productivity sequence of a household of with a different ρ. These households do not have different savings rules, only different shock realizations.

Informed ghosts The informed ghost understands the true value of its ρ. It takes prices as given and solves the household problem. The ghost differs from the standard households in its economy in both how it responds to shocks conditional on current wealth and the shock sequence it faces. We calculate the ghost's mobility matrix under the wealth distribution generated by $\rho_{GE} \neq \rho_G$ and compare it to the mobility matrix generated by the $\rho_{GE} = \rho_G$ economy and attribute the differences to structure. Figures 4-7 plot contours of the surface generated by the (ρ_{GE}, ρ_G) pairs. The 45 degree line running the through the contour is general equilibrium mobility measures from our baseline experiments. Starting at a point on the that line, mobility declines as we move along.

On Figure 4, we draw an example of the structural vs. exchange mobility calculation. Comparing mobility at point A to mobility at point B, our method first picks out point C where ρ_{GE} is the same as in B but ρ_G is equal to the persistence in A. Any differences in mobility between C and A must come from facing a different distribution of wealth (i.e., structure). Movement from A to C then is 'structure' and movement from C to B is 'exchange'.

Figure 8 plots the savings decision rules of three households with different when the economy-wide ρ is 0.73. First notice Proposition 2 at play. Ghosts with low ρ have savings decisions that are much more distant across ε realizations, while those with ε near 1 have policy rules near the 45 degree line. Agents with $\rho = 0.05$ will experience relative large and frequent changes in wealth across one period, while those with $\rho = 0.98$ will switch infrequently but their wealth will also change very little each period. Importantly, notice that the change in distance from $\rho = 0.05$ to $\rho = 0.73$ is much smaller than it is from $\rho = 0.73$ to $\rho = 0.98$. This is a key factor for the hump-shape in total mobility. Depending upon the measure used, the trade off between persistent shocks and smaller step sizes reaches maximum mobility value somewhere between $\rho = 0.7$ and $\rho = 0.8$. For values below 0.7, mobility is reduced because agents are switching from savers to dissavers too frequently. For values above 0.8, households are accumulating (decumulating) wealth too slowly.

We find an analogy to driving helpful for explaining how mobility works in this model. Think of the support of wealth as a highway that runs east and west. Take any location on that highway and call all locations to the west of it 'poorer' and all locations to the east 'richer'. 'Checkpoints' along the highway correspond to quintiles of wealth (also called 'class boundaries'). Household decision rules are lanes on a highway. Some lanes move east (toward higher wealth) and others move west (toward lower wealth); and the fastest lanes are one the outside of the highway. The fastest westbound lane corresponds to the lowest labor income value, and the fastest eastbound lane to the highest value. The further the saving decision is from the 45 degree line, the faster it moves. Changing ρ alters how likely one is to switch out of their current lane and into another one. In the two ε case, there is only one westbound and one eastbound lane. If ρ is high, than a household will likely stay in its lane continuing to move up or down in the wealth ordering. As Figure 8 shows however, the more persistent the Markov process the closer the decision rules are to the 45 degree line and so the more slowly will be the pace of the lane in our analogy. If ρ is low the lane speeds will be faster, but the households will switch directions frequently, moving up and the moving down the ordering. Maximum wealth mobility is achieved where lanes move quickly enough to allow for distant movement, but also where they are likely not to switch too often, allowing for a sufficiently long chain of movements in the same direction.

Uninformed ghosts To decompose exchange mobility from between behavior and luck, we run the same type of experiment as above, but now the ghost does not realize that its labor productivity process has a different autocorrelation. This ghost uses the same decision rules as the other households in the distribution, but it realizes a different sequence of shocks. Figure 9-12 plot mobility of these agents as a function of (ρ_G, ρ_{GE}). As before with the informed ghosts, we draw path to highlight one of the three components, here being luck. We have a similar breakdown on figure 9. Moving from A to B is a combination of all three components, but movement between A and C is entirely due to luck because the ghosts in both cases reside in the same distribution and have the same decision rules. The only difference is that a ghost at C has a more persistent shock process (identical to the ghost at B).

The differences in the measures are also notable. The Bartholomew and Shorrocks measures show mobility increasing as the ghost's persistence parameter increases. For the mean first passage measure, the relationship has a similar hump-shaped pattern. Holding ρ_{GE} constant, mobility increases in ρ_G until it reaches a maximum somewhere between 0.70 and 0.80; then it declines rapidly. Oddly, the 2^{nd} largest eigenvalue measure actually decreases in ρ_G.

Here the we see the hump-shaped pattern in mobility. When the economy-wide ρ is low, the savings rules are far apart so non-phantom agents in a fast lane but change often. Mobility is low. The non-optimizing phantom agents with higher ρ share the same fast lanes but are much less likely to switch. They have longer chains of wealth accumulations and decumulations, and so their mobility is higher. One again, when ρ gets too high, the ghost agents remain in their lane for a very long time. They will move through the distribution but only very infrequently, and they will usually just 'pass through' one intermediate quintile. Those with low ε will spend a large number of periods in the bottom quintile before finally drawing a good shock and making a transition back through the distribution toward the top quintile where they will once again remain for a large number of periods.

6.2.2 Decomposing changes in mobility

We have identified three sources for the differences in mobility as the labor income process becomes more persistent. In order to disentangle the contributions of each source to the total change in steady state mobility, we will run several counterfactual experiments. Consider the steady states of two economies, one with $\rho = \rho_x$ and one with $\rho = \rho_y$; and without loss of generality, let $\rho_y > \rho_x$. Denote by $\mu_{[j,j,j]}$, the measured mobility induced by an agent acting in a distribution produced by agents with $\rho = \rho_j$, having optimal policy rules consistent with $\rho = \rho_j$, and experiencing a realized sequence of labor productivity shocks generated according to $\rho = \rho_j$. For ease of exposition, let $\mu_{[J,J,J]} = \mu_J$. Finally, let $\Delta\mu_{xy} = \mu_y - \mu_x$. $\Delta\mu_{xy}$ is the total change in mobility between the economy with a labor income persistence of ρ_x and ρ_y.

We decompose $\Delta\mu_{xy}$ in the following manner:

$$\begin{aligned}\Delta\mu_{xy} &= \Delta structure + \Delta behavior + \Delta luck \\ &= \left(\mu_y - \mu_{[x,y,y]}\right) + \left(\mu_{[x,y,y]} - \mu_{[x,x,y]}\right) + \left(\mu_{[x,x,y]} - \mu_x\right).\end{aligned}$$

Each component removes one conflating factor in the relative mobility difference, starting with the structures of the ρ_x and ρ_y distributions, moving to differences in the decision rules (behavior) of the agents, and finishing with differences in the realized sequence of productivity shocks.

Figure 13 decomposes the total change in mobility as ρ rises into these three components. Across all four measures, the decomposition is qualitatively the same. Structure has a small negative effect on mobility, while behavior and luck make larger contributions, negative and positive, respectively. At low levels of ρ, mobility rises in the shock persistence because luck offsets behavior. Past a certain point, however, behavior becomes more powerful and pulls total mobility down.

6.2.3 Borrowing Limits

So far we have imposed a strict borrowing limit of zero. A large fraction of households can find this constraint binding, particularly when the labor income shocks are very persistent. As a result, the steady state wealth level separating the first and second quintiles can be very close to 0 so that even a small movement away from the borrowing limit can move a household into the second quintile. In this case, households in the first (second) quintile would appear to be very upwardly (downwardly) mobile. We have run cases with high persistence and exogenous borrowing limits near the natural borrowing limit and found that while it has little effect on our mobility measures. Therefore, we do not think that our assumption of no borrowing is restricting our findings.

7 Mobility and other features

7.1 Increased skewness in the wealth distribution

It is well-known that a Bewley model with idiosyncratic labor income risk alone does a poor job matching the high concentration of wealth in the right tail.[16] The fundamental issue is that the sufficient amount of wealth to self-insure is low when agents are very patient and shocks are relatively small. Once a household can adequately smooth its consumption, it has no other incentive to continue saving, since interest rates are necessarily lower than time

[16]See Quadrini and Ríos-Rull (1997) and Carroll (1998) for discussion.

rates of preference. Several approaches have been used to generate longer right tails in the wealth distribution.

Krusell and Smith (1998) replace the scalar household discount factor with a 3-state, highly persistent Markov chain. The three values are $[0.9763, 0.9812,\ 0.9861]$, and the transition matrix is

$$\begin{bmatrix} 0.99654 & 0.00346 & 0 \\ 0.00043 & 0.999135 & 0.00043 \\ 0 & 0.00346 & 0.99654 \end{bmatrix};$$

these choices deliver a Gini coefficient of wealth equal to 0.78. The invariant distribution of β is $[0.1, 0.8, 0.1]$ and the average duration in either extreme-β state is 200 quarters. The 5-year wealth mobility matrix for the stochastic-β environment is

$$\begin{bmatrix} 0.84 & 0.16 & 0.00 & 0.00 & 0.00 \\ 0.16 & 0.70 & 0.15 & 0.0 & 0.00 \\ 0.00 & 0.15 & 0.73 & 0.11 & 0.00 \\ 0.00 & 0.00 & 0.11 & 0.84 & 0.05 \\ 0.00 & 0.00 & 0.00 & 0.05 & 0.95 \end{bmatrix}$$

The stochastic-β model makes the mobility match worse – the top quintiles get even more persistent, since drawing a high discount factor leads even agents with temporarily low income to save, and discount factor shocks are very persistent. It is this immobility that delivers the high wealth concentration that was the goal of Krusell and Smith (1998), but it does not come for free.[17]

Castaneda et al. (2003) add a very high productivity state with relatively low persistence and a high probability of transitioning immediately to the lowest productivity [18]. The transitory nature of this 'rockstar' state combined with the increased risk motivates households in this state to build up a substantial amount of precautionary savings. When a house-

[17] Carroll (2001) shows that a permanent 'two-β' model looks very much like the stochastic-β model, so the fact that the discount factors mean-revert does not seem important provided they do so slowly.

[18] Specifically, the state vector for labor productivity is $\mathcal{E} = [1.0, 3.15, 9.78, 250]$. The Markov process for labor earnings in Castaneda et al. (2003) has a stochastic aging component. Here, we abstract from this by isolating the submatrix associated with the worker-to-worker transition and then renormalizing the rows so that Π is stochastic. The resulting transition matrix is $\Pi = \begin{bmatrix} 0.984 & 0.012 & 0.004 & 0.000 \\ 0.031 & 0.965 & 0.004 & 0.000 \\ 0.015 & 0.004 & 0.980 & 0.000 \\ 0.109 & 0.005 & 0.062 & 0.823 \end{bmatrix}$

hold draws the rockstar state, it takes advantage of its temporary good fortune by saving rapidly. This 'burst of saving' produces the matrix below which has considerably more upward mobility than the benchmark:

$$\begin{bmatrix} 0.73 & 0.17 & 0.05 & 0.04 & 0.00 \\ 0.24 & 0.52 & 0.19 & 0.05 & 0.00 \\ 0.00 & 0.34 & 0.52 & 0.14 & 0.00 \\ 0.00 & 0.00 & 0.24 & 0.59 & 0.17 \\ 0.00 & 0.00 & 0.00 & 0.17 & 0.83 \end{bmatrix}.$$

Nevertheless, the rockstar model still has too little downward mobility. The consumption-smoothing motive implies that while households save rapidly, they dissave slowly – staying away from the borrowing constraint is the reason they save, after all. And furthermore the resulting labor earnings process looks nothing like we find in the data (see Guvenen et al. (2015)).

8 Mobility and Market Incompleteness

We know that market incompleteness is a necessary condition for permanent mobility – mobility may be present along a transition path if agents have different preferences, but eventually it will disappear as the economy transitions to a steady state (see Caselli and Ventura (2000) and Carroll and Young (2011)). We now take up the question of how mobility is connected to incompleteness, in the sense of the spanning of assets.

We consider two experiments. First, suppose there exist two assets, one of which pays off if $\varepsilon \geq E[\varepsilon]$ and one that pays off if $\varepsilon < E[\varepsilon]$. Second, suppose there exist three assets, which pay off if $\varepsilon > E[\varepsilon]$, $\varepsilon = E[\varepsilon]$, and $\varepsilon < E[\varepsilon]$. In each case, asset markets are 'more complete', but mobility could easily go either way. Since the price of these assets is smaller than the price of a risk-free security, portfolios that 'lever up' in certain states can lead to large changes in wealth should those states realize; the results inRampini and Viswanathan (2016) show that agents in our economy will in fact choose to endogenously hold a skewed portfolio if they are sufficiently poor.[19]

[19] It is straightforward conceptually to permit an arbitrary number of state-contingent claims, but the high autocorrelation of the states means that some of these assets will have essentially zero price; prices that are

We compare the mobility results from these partial insurance cases to the baseline model. In each case, we set the number of productivity states to 7. We will mainly discuss the two-asset case; for simplicity, denote the productivity states where $\varepsilon \geq E[\varepsilon]$ 'good' states, and the other 'bad' states.

Figure 14 plots the portfolio decisions of several informed ghost households. In each case, the ρ value of the underlying economy is 0.73. Each subplot shows the decisions of two ghosts with the same persistence value, one with $\varepsilon = \varepsilon_{\min}$ and one with $\varepsilon = \varepsilon_{\max}$. The solid lines represent the number of claims purchased which pay off if the next period's productivity belongs to the same state as today's productivity. The dashed lines are the claims which pay off in the opposite state from today's. For example, for the $\varepsilon = \varepsilon_{\min}$ household, the solid line is the stock of claims that pay off if one of the bad states is realized next period, and the dashed line is those that pay off if the good state is realized instead.

First notice that the a household currently in the bad (good) state purchases contingent claims against the bad (good) state near the 45 degree line. In fact, the household's decision rules in this regard are similar in appearance to those in the one asset case. Just as in the baseline case, these saving rules become closer as the probability of remaining in the same state increases. Again, households consume a larger fraction of income from more persistent shocks. This feature of the portfolio induces more mobility as it allows for long strings of consistent wealth accumulation and decumulation, as we illustrated in the section above.

The other side of the portfolio, that is the holding of claims which pay off only if the household's state switches (from good to bad or bad to good) in the next period, is quite different, and it can have a big effect on mobility, particularly in the ghost household cases. Households currently in a good productivity state purchase considerably more claims against switching to a bad productivity state. These claims compensate both for the low labor income from a bad state and provide additional precautionary savings the likely recurrence of bad state shocks. Moreover, because the probability of switching between good and bad states is low (especially for an $\varepsilon_{\max}$ or $\varepsilon_{\min}$ household), this insurance is very cheap. Naturally then, as ρ increases the good state household's claims against bad states rises, causing the balance of the portfolio to tilt more and more.

The portfolio of household's currently in a bad productivity depends on their wealth level. At sufficiently high wealth, the portfolio looks like a mirror image of the good

too low lead to instability in our solution algorithm.

state household's portfolio. The purchase of claims against a bad state lie close to the 45 degree line, while the purchases of claims against the good state are much lower. At lower levels of current wealth, households would like to short the claim against good states, since consumption in the bad states is very valuable. Since this shorting is not allowed, these households simply do not participate in that asset market. With the exception of the wealth region where the non-negativity constraint binds, the response of any household portfolios can be generalized in the following way: as ρ increases, the demand for claims that pay off if the current state continues become less sensitive to income shocks, while the demand for assets that payoff if the state switches becomes more sensitive.

The consequence of this portfolio behavior for mobility across ρ is that as households become less and less likely to switch states, their wealth path is characterized by small, gradual movements interspersed with infrequent large shifts. Figure 15 plots mobility in the partial insurance cases against the single asset baseline. Notice that mobility is lower in the partial insurance environment unless the labor income process is quite persistent. Regardless of the type of measure, mobility under partial insurance peaks at a higher ρ and may even reach a higher (absolute) level before quickly descending again as shocks approach being permanent. Figure 15 also shows that the pattern is strengthened by the addition of the third asset.

Although the partial insurance environment features more wealth mobility at high values of ρ, there is still less mobility than in data for our chosen value of ρ. The five-year wealth transition matrix is

$$\begin{bmatrix} 0.75 & 0.25 & 0.00 & 0.00 & 0.00 \\ 0.24 & 0.56 & 0.19 & 0.05 & 0.00 \\ 0.01 & 0.19 & 0.66 & 0.14 & 0.00 \\ 0.00 & 0.00 & 0.15 & 0.77 & 0.08 \\ 0.00 & 0.00 & 0.00 & 0.08 & 0.92 \end{bmatrix}$$

9 Reconciling the data and the model

Benhabib et al. (2015) use in a partial equilibrium OLG model with deterministic, heterogeneous earnings profiles and rates of return on saving to match aspects of inequality and intergenerational mobility in the US wealth distribution (see also Hubmer et al. (2017)). They argue that three factors are critical for modeling wealth inequality and wealth mobility:

stochastic earnings, capital income risk, and differential saving and bequest motives. While we focus on wealth mobility over shorter time horizons than a generation, our probit estimates suggest that heterogeneous capital income risk may still be an important contributing factor.

9.1 Heterogeneous Return Risk

We adapt the baseline model of section 5 to incorporate heterogeneous return shocks: if r_t is the equilibrium interest rate in the capital market, a household with shock z_t earns return $(1 + r_t)z_t$ on their prior savings a_t. The budget constraint for an agent with state $(a_t, \varepsilon_t, z_t)$ is given by[20]

$$c_t + a_{t+1} \leq (1 + r_t)\, z_t a_t + \omega_t \overline{h} \varepsilon_t$$

As with labor income shocks (ε_t), we assume that return shocks z_t follow an autoregressive process:

$$z_{t+1} = \rho_z z_t + \nu_t$$
$$\nu_t \sim \mathcal{N}\left(0, \sigma_z^2\right)$$

Our assumption on return shocks is parsimonious, and yet has several features that aid in reconciling wealth mobility in the model with that in the data. First, note that this return risk is *uninsurable*: agents have no alternative asset, and thus all consumption smoothing is done via saving in the now-risky asset. This assumption has empirical support: data on household holdings suggest that households who hold risky assets are under-diversified, and highly exposed to these risky assets (see, for instance, Moskowitz and Vissing-Jørgensen (2002)). Additionally, these shocks affect wealth, and thus mobility, in both levels and at the margin of intertemporal substitution. The process for z_t allows for households to experience large capital income shocks, which scale with their wealth. These households will also alter their savings behavior: because returns are persistent, households with high z shocks will increase their savings rate in order to capture higher returns, and households with poor z shocks will dissave more rapidly. The combination of behavioral and mechanical

[20] With heterogeneity in returns, individual wealth will evolve according to a random growth process, and as such the wealth distribution will have a Pareto tail; see e.g. Gabaix (2009). To avoid underestimating the capital stock, we employ the "Pareto extrapolation" technique of Gouin-Bonenfant and Toda (2019).

changes to wealth enables our model to match the frequency of large transitions in the wealth distribution over short horizons that we observe in the data.

We are, of course, not the first to employ heterogeneous return risk in explaining trends in inequality and social mobility. Benhabib et al. (2019) use capital income risk in an estimated model to target *intergenerational* wealth mobility. By contrast, we demonstrate that return shocks are also important in matching short-run wealth mobility. Pugh (2018) argues in a calibrated model that return shocks are key drivers of transitions into and out of the top 1% of wealthiest households. We show that such shocks are critical to capturing mobility throughout the entirety of the distribution, including into and out of the bottom quintiles. As these bins are comprised of households with low consumption (and thus high marginal utilities), it is important to understand how wealth dynamics play out in these ranges of the distribution, rather than just at the top.

Having introduced return risk, we must now take a stand on when exactly in the period wealth is "recorded" for the purposes of measuring mobility. In the data, when households report their wealth, we assume that they include all returns earned on each of their assets. For instance, when reporting the value of retirement accounts, household include not just their contribution, but the full appreciated value of these accounts. When measuring mobility in our model, we record agents assets after the z_t return shock has been realized. The state vector for an agent is now $(w_t, \varepsilon_t, z_t)$, where $w_t = z_t a_t$. Agents now solve

$$\max_{\{c_t, w_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

subject to

$$w_{t+1} = \left(1 + r_{t+1}\right) z_{t+1} \left(w_t - c_t\right)$$

The rest of the model remains the same as in Section 5. In order to pin down the process for z_t, we add an additional calibration target: we require that the Gini coefficient of wealth in the steady state is $G = 0.85$, as in the US data. Note that this leaves us with a degree of freedom: we have added one additional calibration target, but two extra parameters: ρ_z and σ_z. As data on household-level returns for the US is sparse, our strategy is to consider a range of calibrations. For each value of ρ_z, we calibrate σ_z (along with β and aggregate TFP Z) in order to match wealth inequality. We then measure mobility across each of the alternative parameterizations.

Figure 26 shows how ρ_z and σ_z relate to one another across the alternate calibrations that we consider. A clear pattern emerges here: the more persistent agents' returns, the smaller the shocks need to be in order to generate the wealth Gini in the data. This relationship is intuitive: if return shocks are persistent, then agents who enjoy good shocks will save out of them, accumulating wealth over time. Similarly, agents locked into a long stream of persistently low returns will run down their wealth. These forces stretch out the wealth distribution, allowing for a good fit to inequality with smaller shocks than are needed with less persistent returns.

9.2 Mobility with Uninsured Return Risk

Broadly speaking, we find that the inclusion of capital income risk–both in agents' problems, and in our measurement of wealth–improves the fit of the model to empirical mobility substantially. Figure 27 shows wealth mobility according to our four measures, normalized to the average of their values in the data. In contrast to the baseline model of Section 5, here we *overshoot* wealth mobility relative to the data. Notably, the overshoot is less pronounced at higher values of persistence (ρ_z). A sample transition matrix from this exercise with $\rho_z = 0.9$ and $\sigma_z = 0.064$ is:

$$\begin{bmatrix} 0.32 & 0.3 & 0.25 & 0.12 & 0.01 \\ 0.25 & 0.34 & 0.27 & 0.13 & 0.01 \\ 0.04 & 0.33 & 0.4 & 0.21 & 0.02 \\ 0.0 & 0.04 & 0.35 & 0.51 & 0.1 \\ 0.0 & 0.0 & 0.01 & 0.26 & 0.73 \end{bmatrix}$$

Relative to earlier models, the most marked difference is that now, we are able to generate downward mobility similar to that in the data. As in the data, capital income risk increases the propensity with which households make large movements in both directions through the wealth distribution. In particular, note the final row in the matrix above, as compared to those in sections 5 (baseline model) and 7 (those augmented with additional features), the top quintile of wealth is very persistent; households who enter this quintile tend to dissave slowly and remain there. With uninsurable return risk, however, this quintile is far less persistent: of the households who begin there, less than three quarters remain there five years later. Over this period, some wealthy agents draw unfavorable return shocks, and fall downward in the wealth distribution as they rapidly deaccumulate wealth.

In Figure 27, higher values of persistence in the return shocks produce mobility more in line with the data. These shocks also generate a wealth distribution more in line with its empirical counterpart, especially at the very top. Figure 28 shows the share of aggregate wealth held by the wealthiest 50%, 10%, and 1% of households in the model, along with empirical counterparts, across values of persistence ρ_z. The most noted change is in the wealth share of the top 1%: although we overestimate this share across all ρ_z values, the fit at the top is better with more persistent shocks.[21] None of the moments in Figure 28 are targeted, and so we take the good fit here as a welcome sign that our model with return shocks, parsimonious though it is, captures important features of both inequality and social mobility throughout the wealth distribution, including at the very top.

9.3 Policy Implications: Risk and Returns to Wealth

Having established a way in which to augment the standard incomplete-markets model in order to generate realistic wealth mobility, we now turn to the normative implications of such an augmentation. Why is it important to match mobility? How does optimal policy change when we allow for agents to move through the wealth distribution at the rate they do in reality?

To answer these questions, we update the budget constraint of agents in our model to allow for a lump-sum transfer T, financed by a linear tax on capital income τ_k:

$$c_t + a_{t+1} \leq (1 + (1 - \tau_k) r_t)\, z_t a_t + \omega_t \overline{h} \varepsilon_t + T_t \tag{5}$$

For each value of ρ_z, we calculate the optimal linear capital income tax τ_k^*, shown in Figure 29. As the persistence ρ_z increases, bringing wealth mobility generated by the model in line with that in the data, the optimal capital income tax falls.

The intuition behind these results is as follows. Because individuals cannot save in an asset other than now-risky claims to the capital stock a, they value the insurance provided through the tax-and-transfer system. Effectively, a capital income tax here acts as a savings

[21] Recall that we target the Gini coefficient in calibrating this version of the model, so all economies here reproduce this summary measure of the wealth distribution.

vehicle, and agents with low z_t shocks now prefer that capital income be taxed so that they can receive a higher transfer (T), which now allows them to smooth consumption. The capital income tax also "squeezes" returns as in Guvenen et al. (2015), reducing the variance in post-tax returns and thereby ameliorating risk. The tradeoff, of course, is that the capital income tax discourages investment, lowering the capital stock and therefore the wage.

Recall from Figure 26 that when calibrating to US wealth inequality, a higher persistence of return shocks ρ_z implies that the shocks themselves need to be smaller. As such, when shocks are more persistent, risk is inherently lower: agents receive return shocks that are smaller, and today's return shock is more informative in predicting tomorrow's. Therefore, as we increase the persistence of returns, keeping inequality constant, the risk-reduction motive for capital income taxes shrinks, inducing a fall in the agents' preferred rate of capital income taxation. The relationship between mobility and capital income taxes is easy to infer: as wealth mobility rises, so too does the optimal tax rate. The predominant force seems to be an insurance motive: the more mobile agents are, the more their wealth fluctuates from period to period. All else being equal, agents value insurance against these movements, as swings in wealth make it difficult for agents to smooth consumption. The tax code provides this insurance, reducing returns but equalizing incomes via the lump-sum transfer.

10 Conclusion

We have studied wealth mobility in a Bewley model. In particular, we have shown how assumptions about the underlying process driving long run wealth inequality affect relative mobility. As labor income shocks become more persistent, relative mobility displays a hump-shape, starting low growing monotonically to a maximum around $\rho = 0.75$ and then declining sharply towards 0 as the process becomes closer to permanent. Using 'ghost' households, we run several counterfactuals in order to decompose the pattern in mobility into the change in the structure of the wealth distribution, the change in optimal savings behavior in the face of different income risk, and changes in sequence of labor income itself (i.e., luck). We find that the hump-shape is generally attributable to the mixture of behavior and luck. The first contributes negatively to mobility as household's saving is less sensitive to more persistent shocks. The second contributes positively by generating longer strings of low or high income allowing wealth to accumulate or decline for longer over a fixed amount of time.

We document that the baseline Bewley model generates a stationary wealth distribution with lower short-run wealth mobility than has been found empirically. In the data, a non-trivial fraction of households experience large movements across wealth quintiles, even over fairly short horizons, while these movements do not occur in the model. We extend the baseline model in several ways commonly used in the literature to better match wealth inequality. While the inclusion of a very high income state with low persistence as in Castaneda et al. (2003) improves the model's predictions for upward mobility somewhat, it does not match the observed downward mobility. In all versions of the model studied, households move down in wealth too slowly, a natural result of the precautionary saving motive present in the incomplete markets model.

We examine the relationship between market completeness and wealth mobility. We find that replacing the non-contingent capital asset with two state-contingent claims (i.e., partial insurance) may reduce or increase mobility depending upon the underlying persistence of the income shock process. If ρ is sufficiently high, the more complete markets economy has higher mobility. Nevertheless, the model still fails to quantitatively match the observed mobility.

Finally, we study wealth mobility when agents are subject not just to uninsurable income risk, but also uninsurable risk to their rates of return. In contrast to the other models considered, the addition of return risk generates a frequency of downward movements in the distribution over short horizons that we observe in the data. Our results here suggest that highly persistent, moderately-sized shocks to rates of return perform well in replicating wealth inequality and mobility. We explore the positive implications of this feature of idiosyncratic risk, finding that the higher is wealth mobility, the higher is the optimal capital income tax, as this tax provides insurance against large swings in returns and thus wealth.

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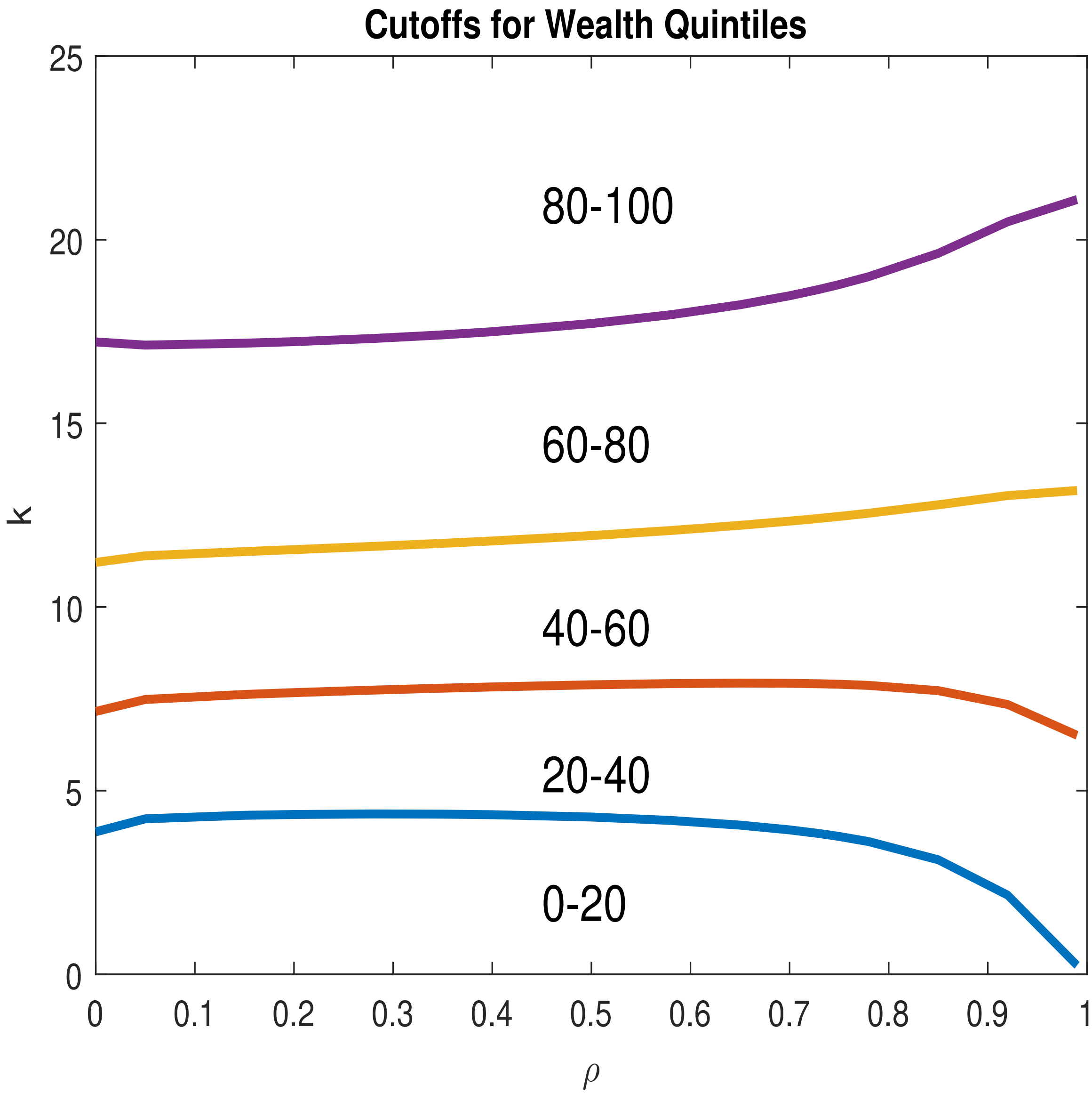


Figure 1: Cutoffs for wealth quintiles across persistence

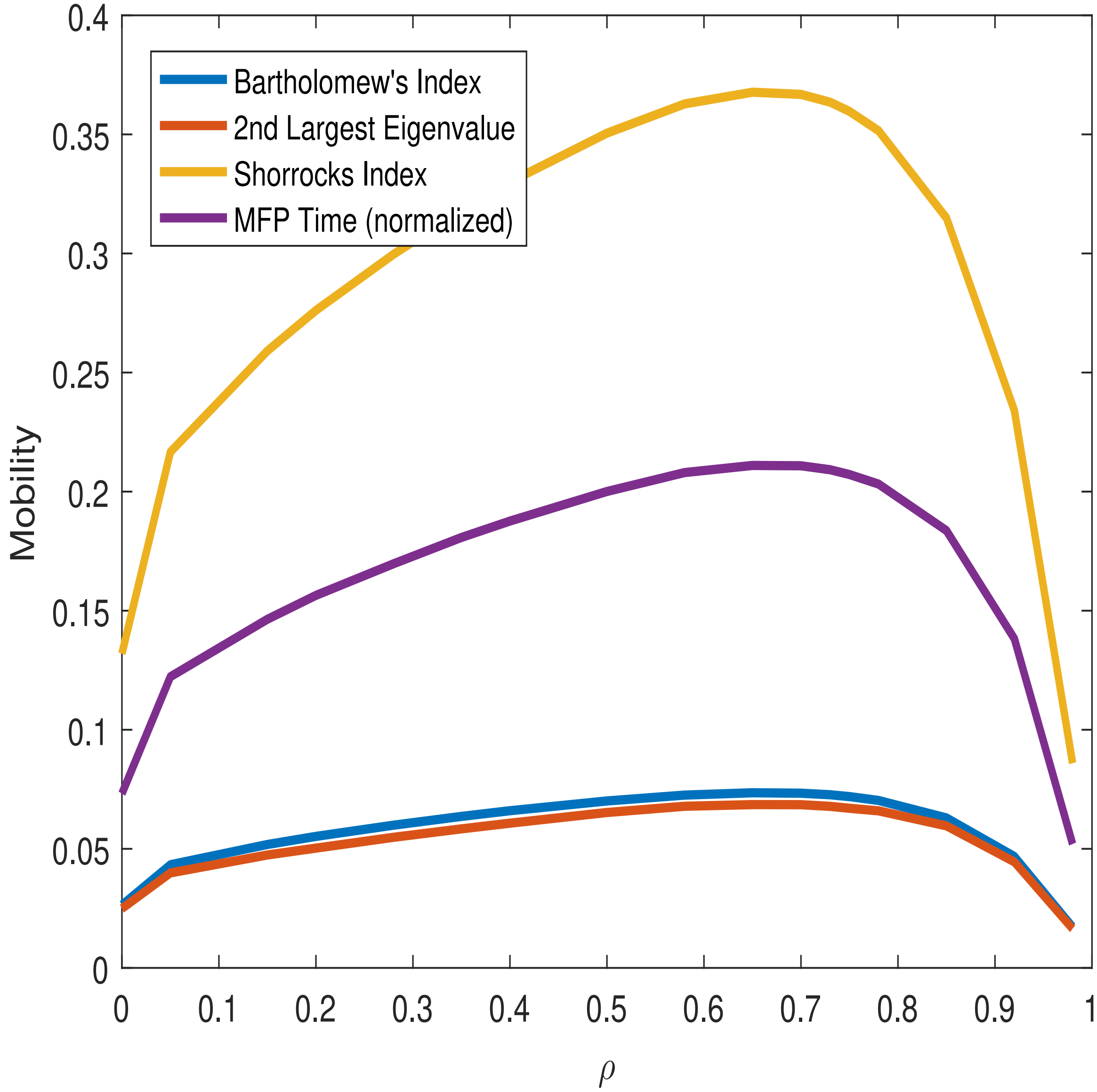


Figure 2: Mobility across persistence

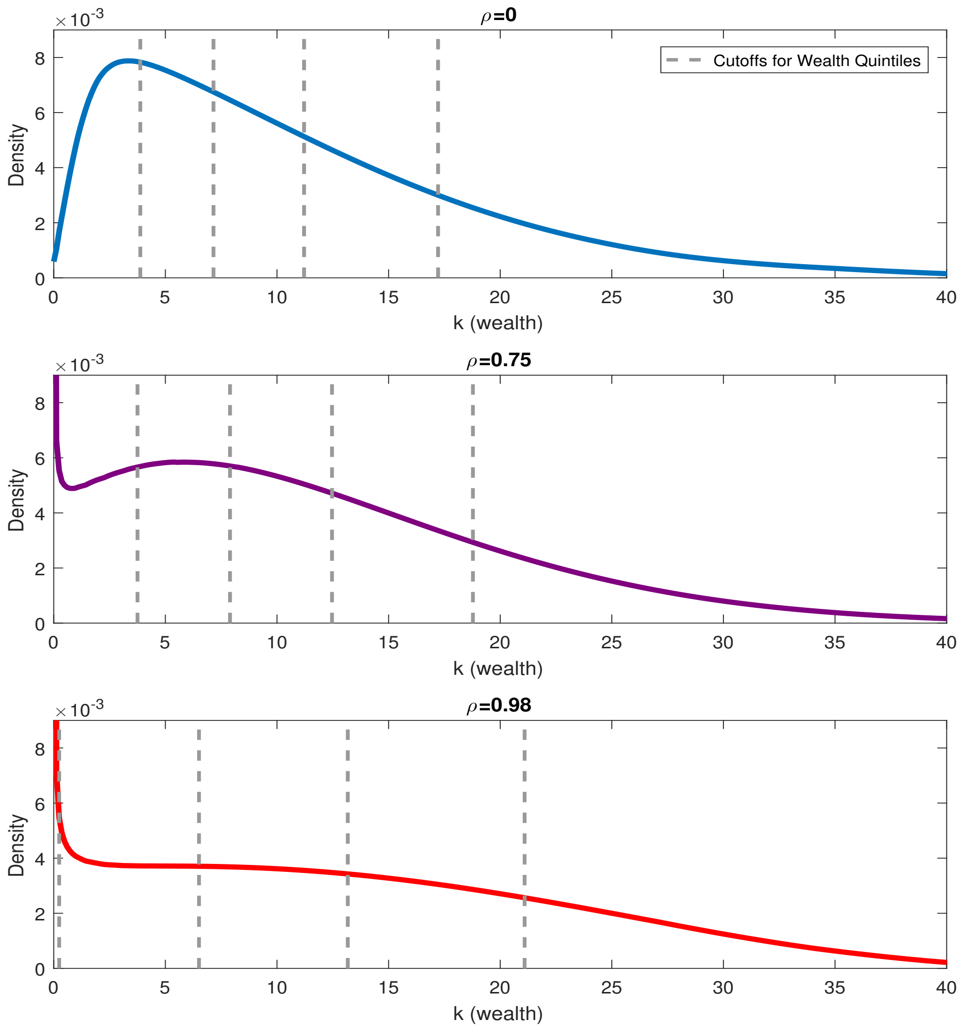


Figure 3: Boundaries between quintiles for different values of ρ

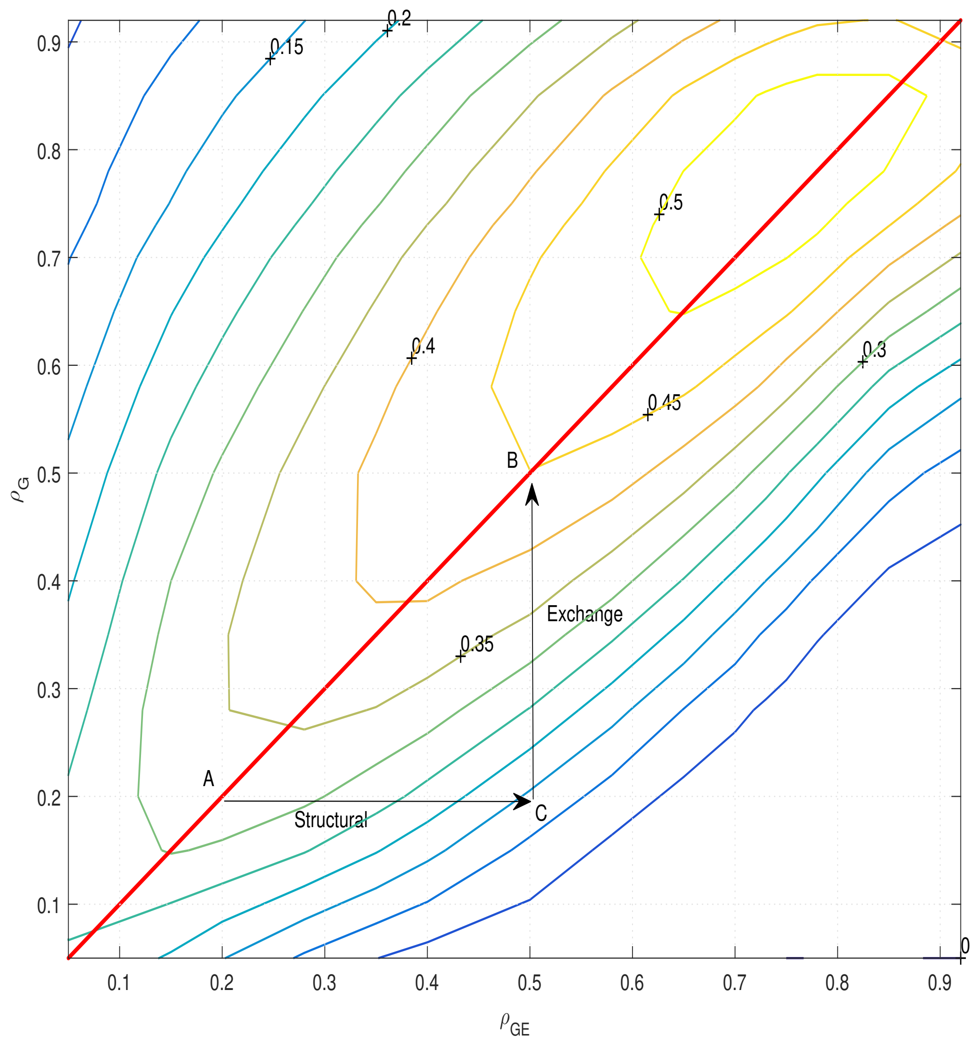


Figure 4: Mobility of optimizing ghost: μ_{MFP}

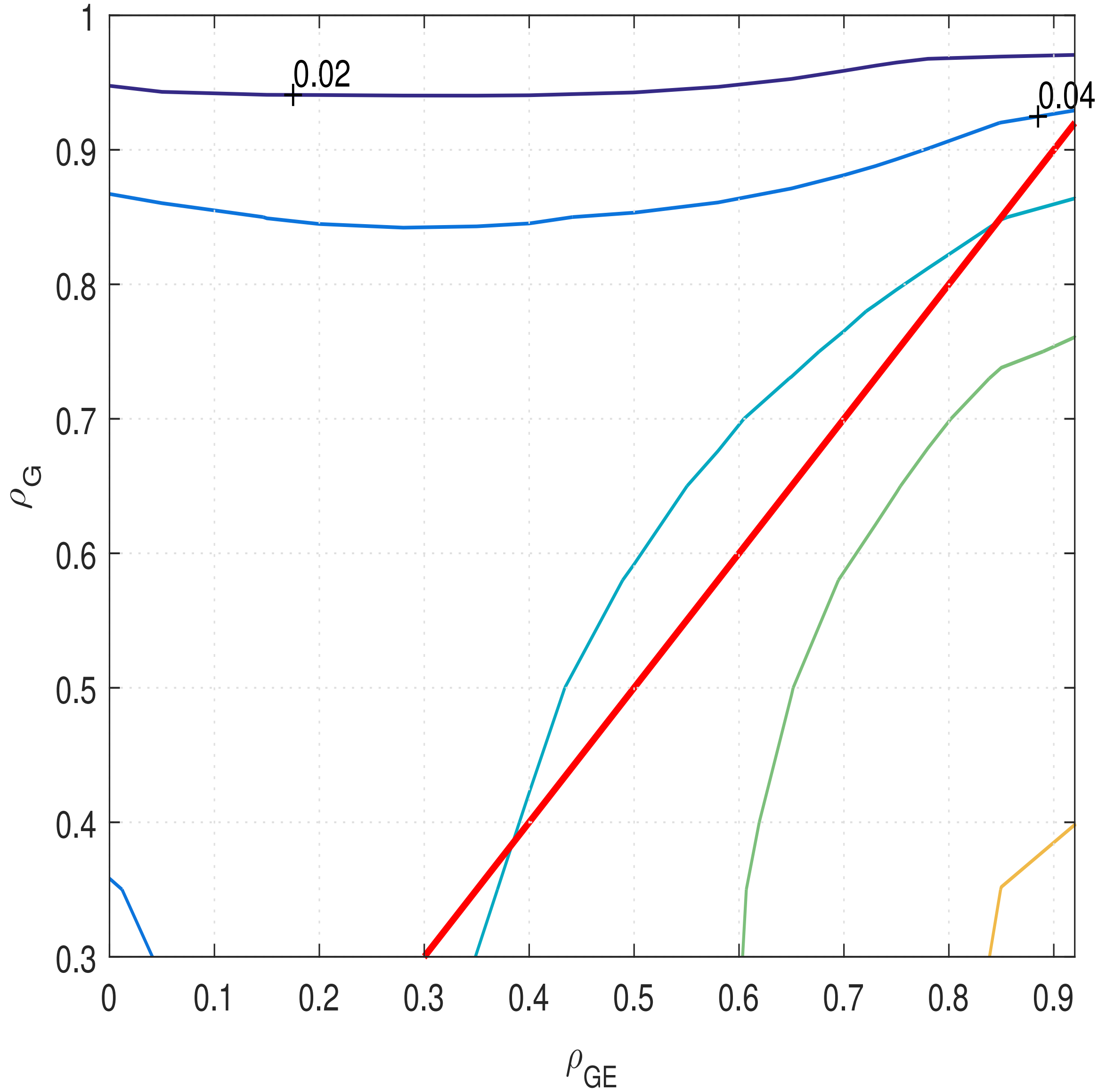


Figure 5: Mobility of optimizing ghost: μ_{2E}

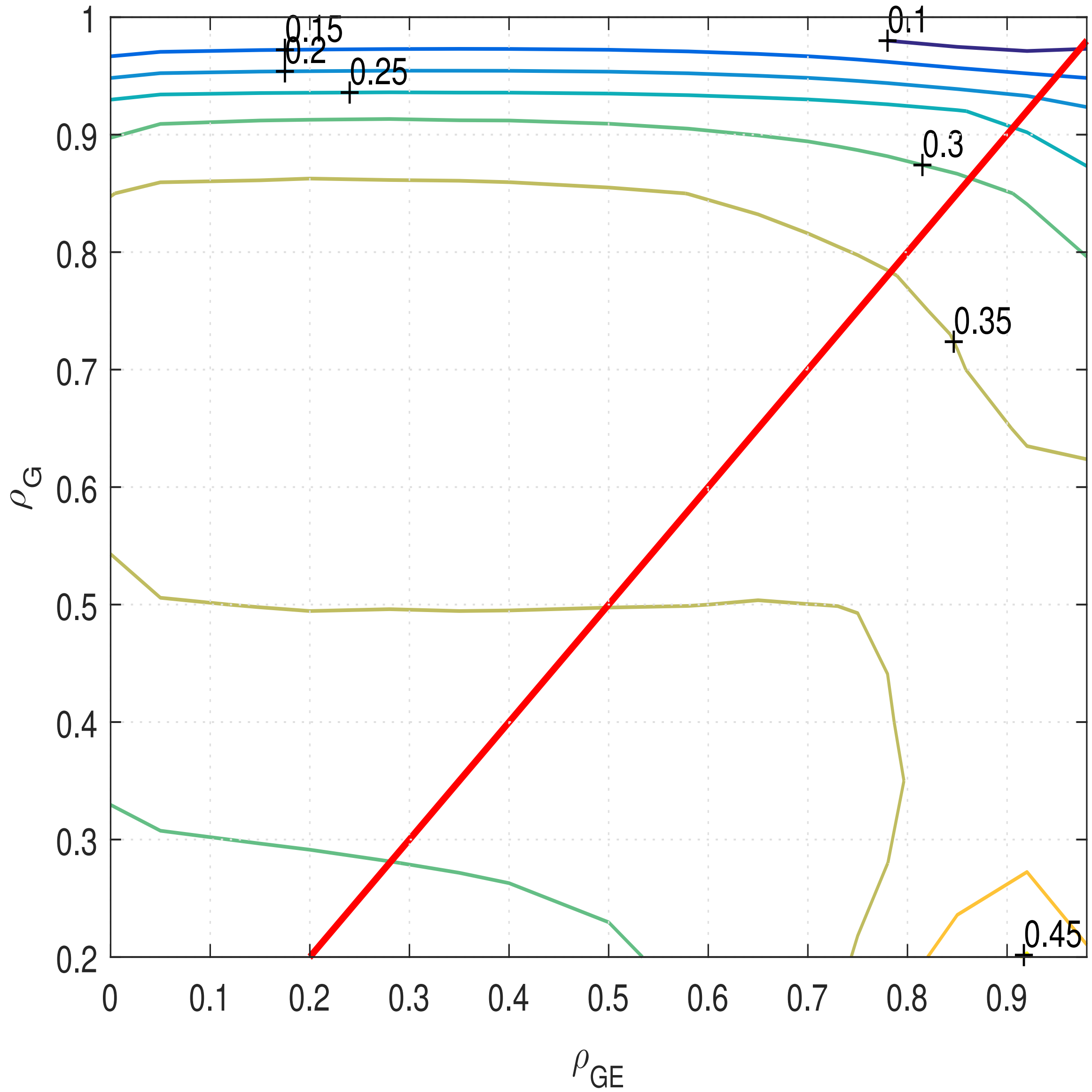


Figure 6: Mobility of optimizing ghost: μ_S

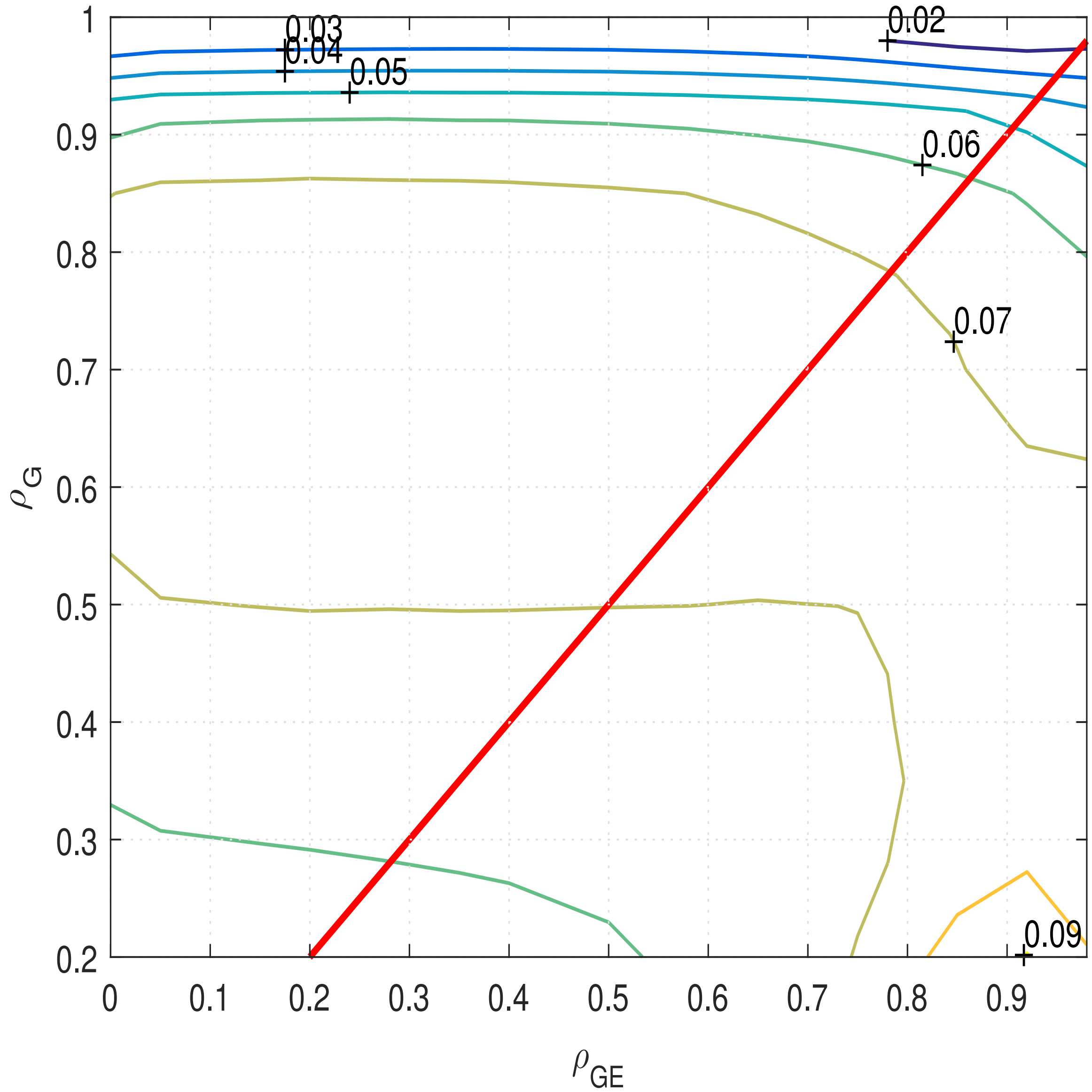


Figure 7: Mobility of optimizing ghost: μ_B

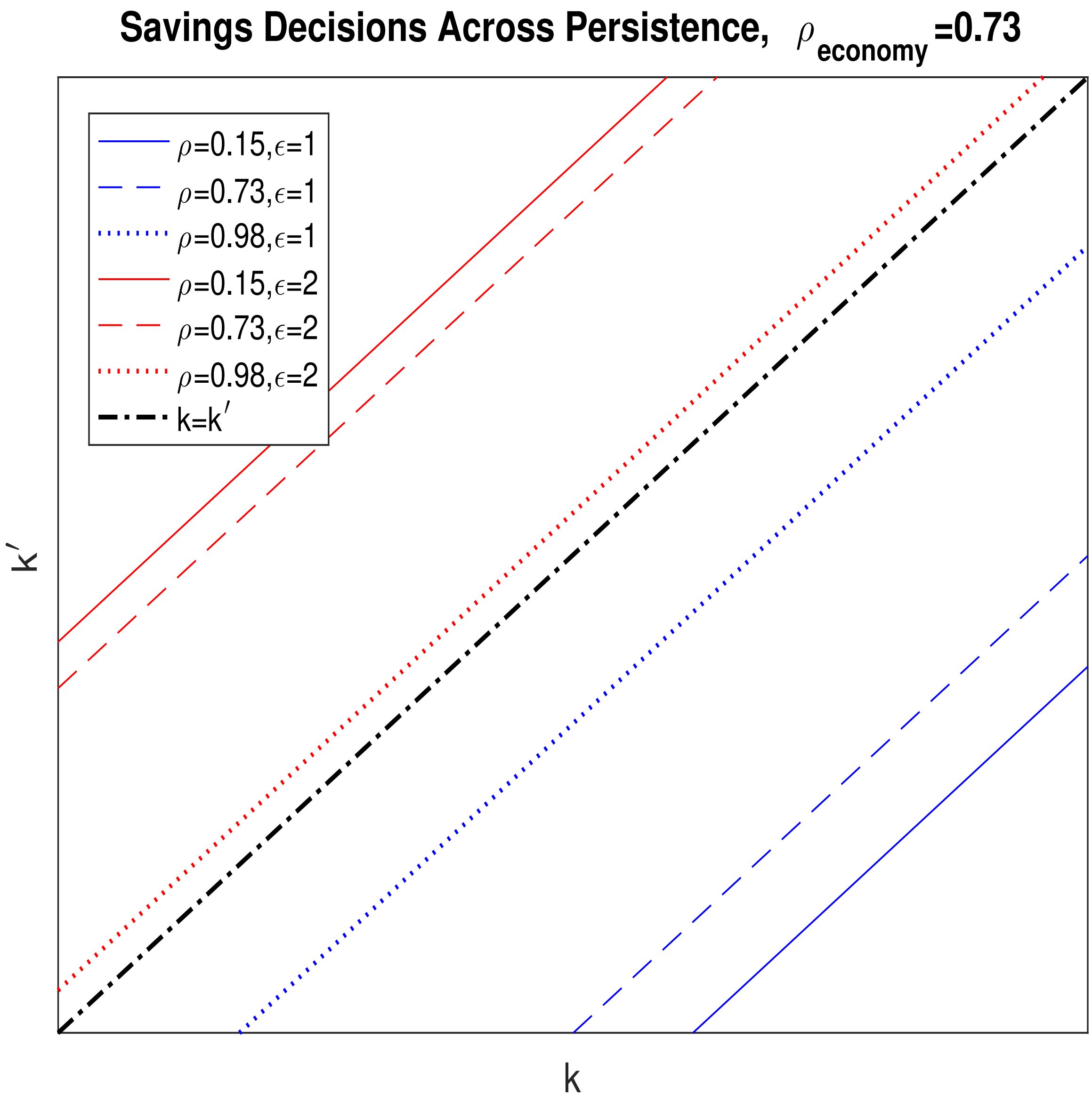


Figure 8: Saving Decisions across persistence

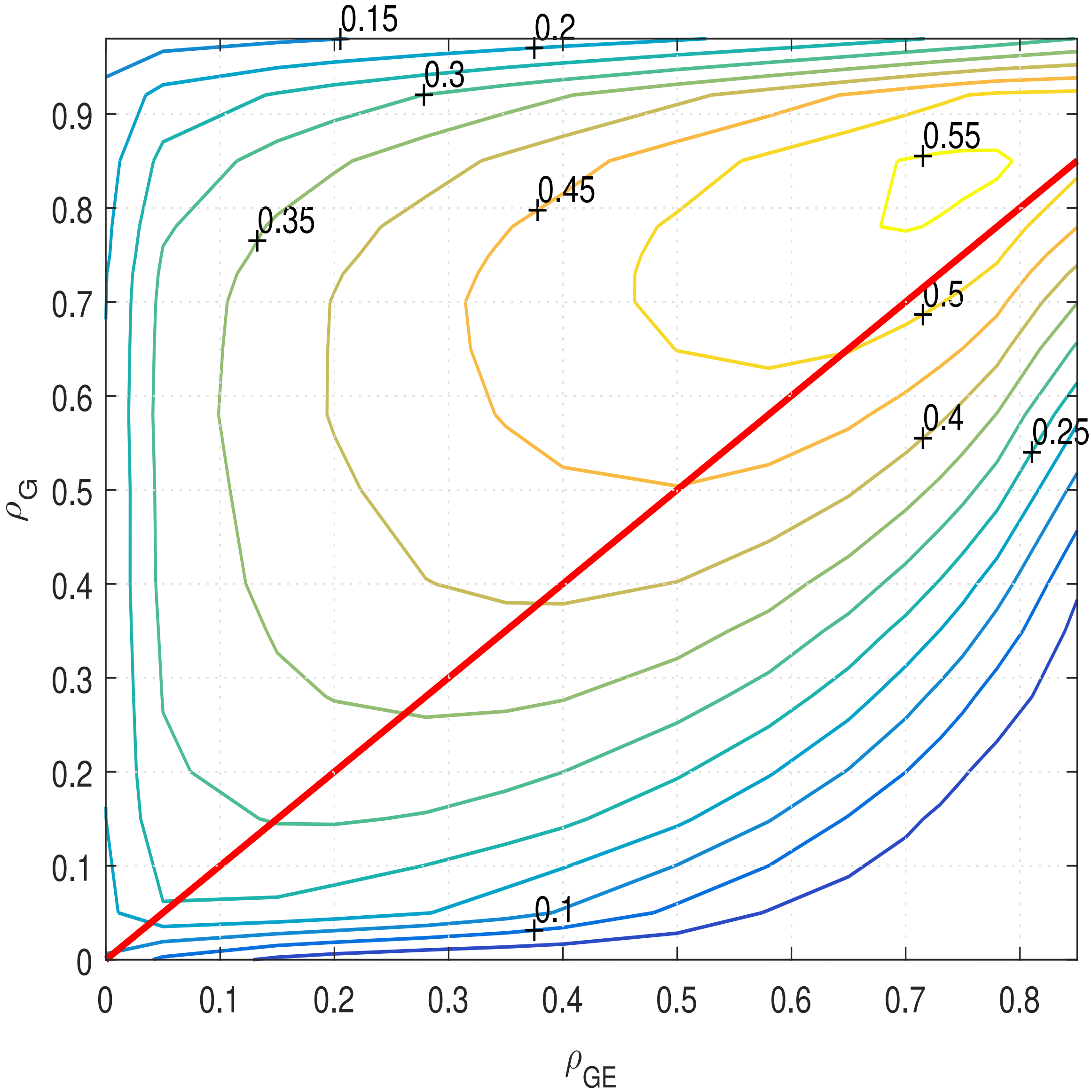


Figure 9: Mobility of non-optimizing ghost: μ_{MFP}

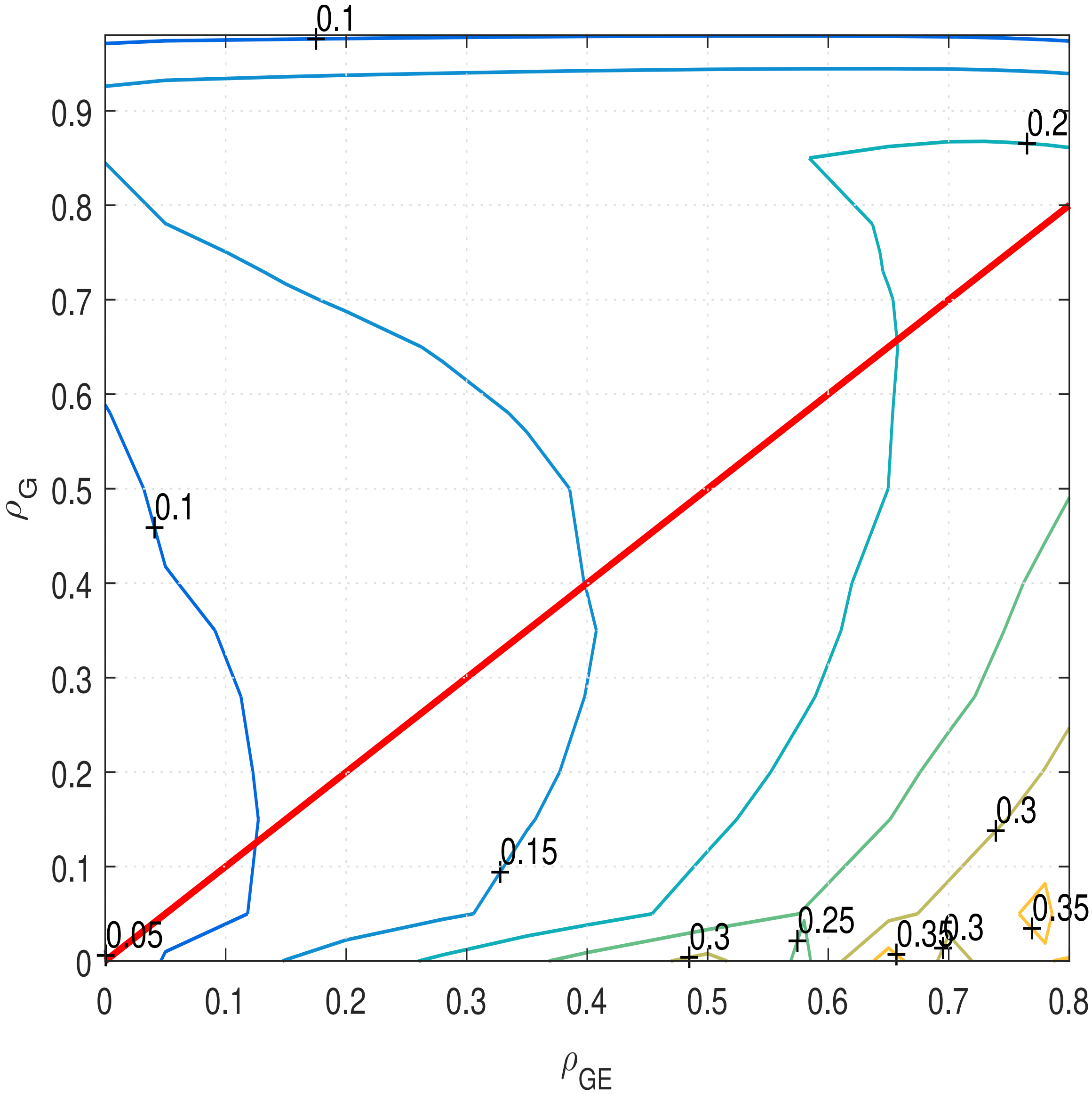


Figure 10: Mobility of non-optimizing ghost: μ_{2E}

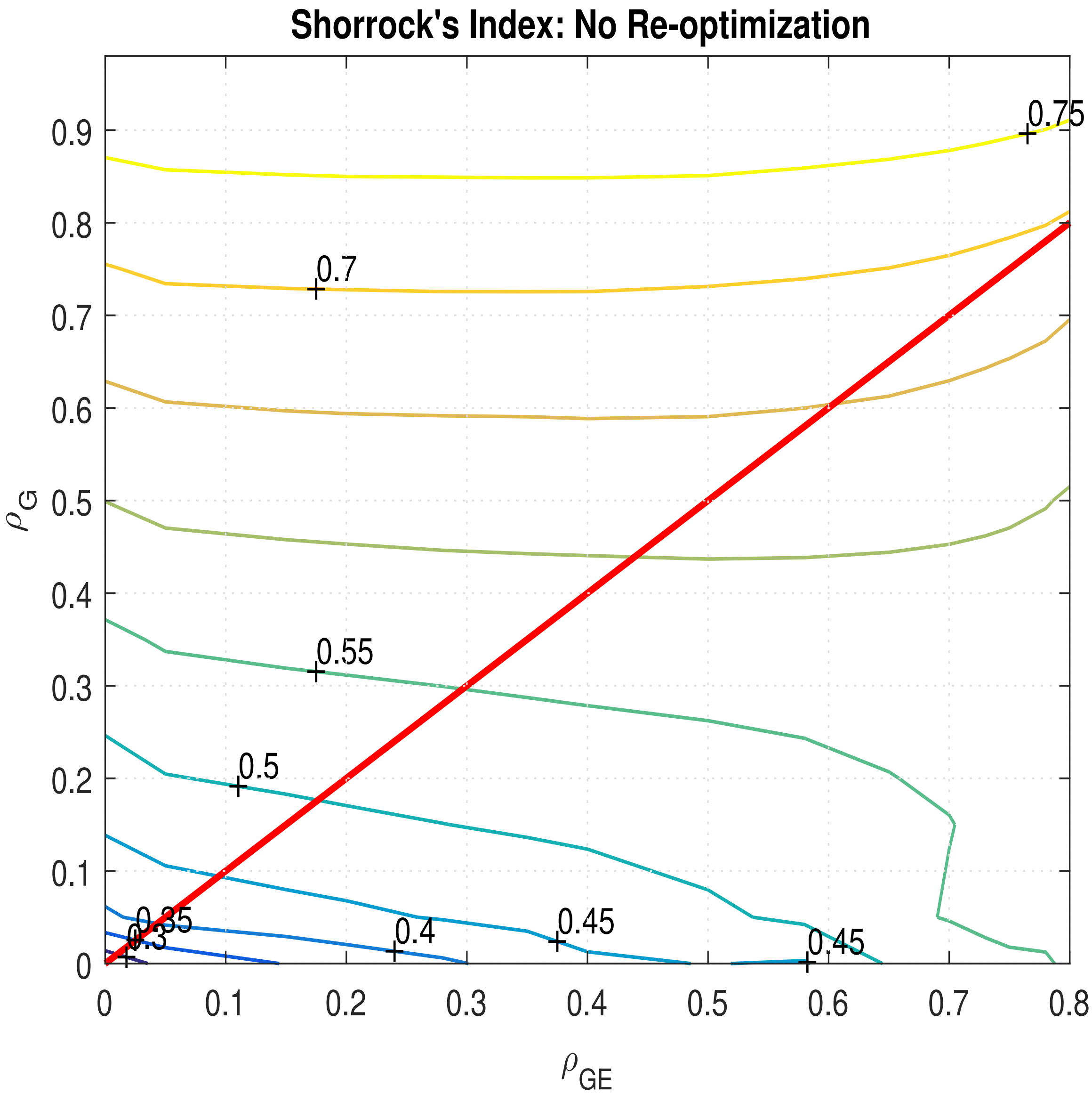


Figure 11: Mobility of non-optimizing ghost: μ_Sm

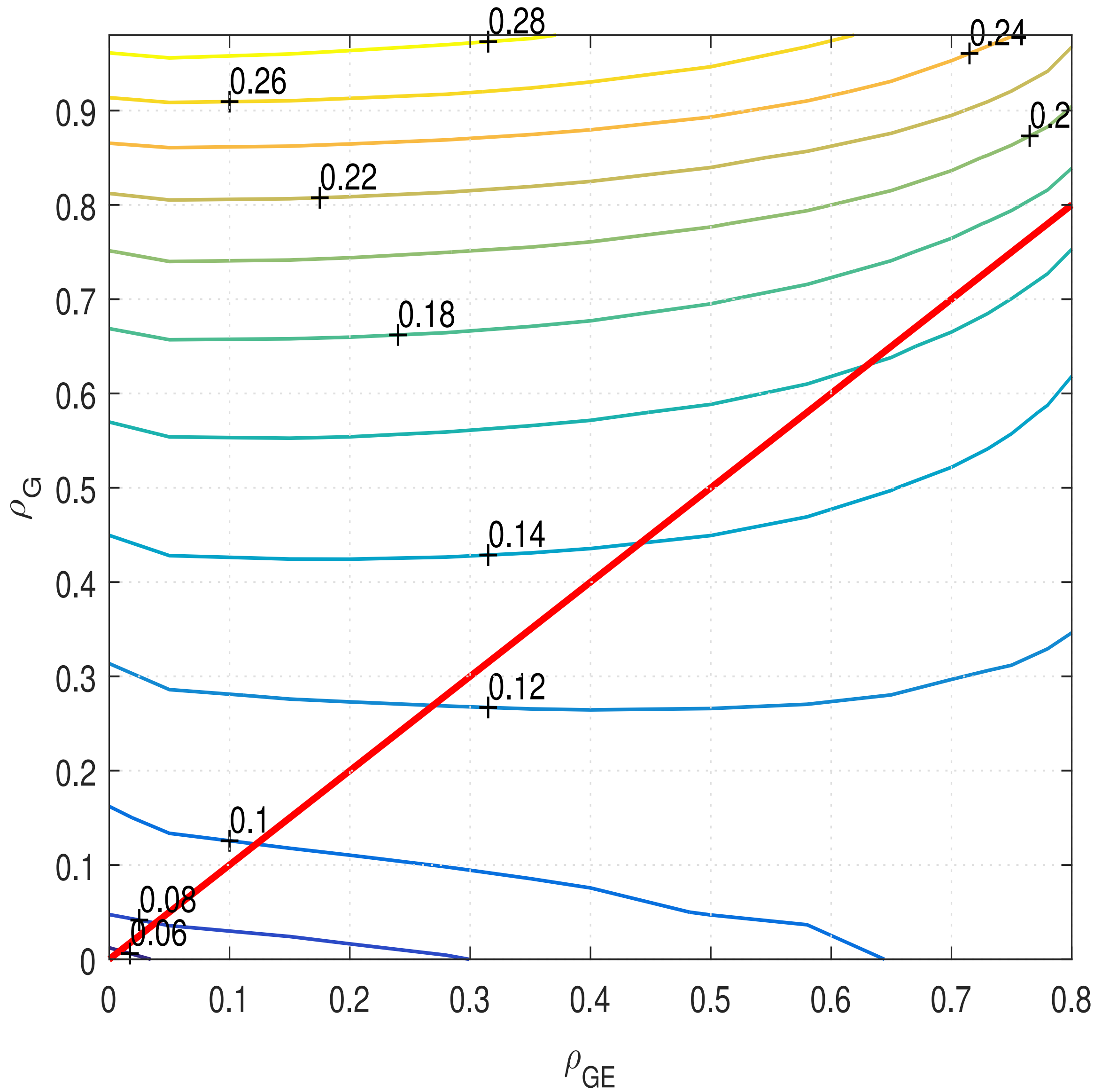


Figure 12: Mobility of non-optimizing ghost: μ_B

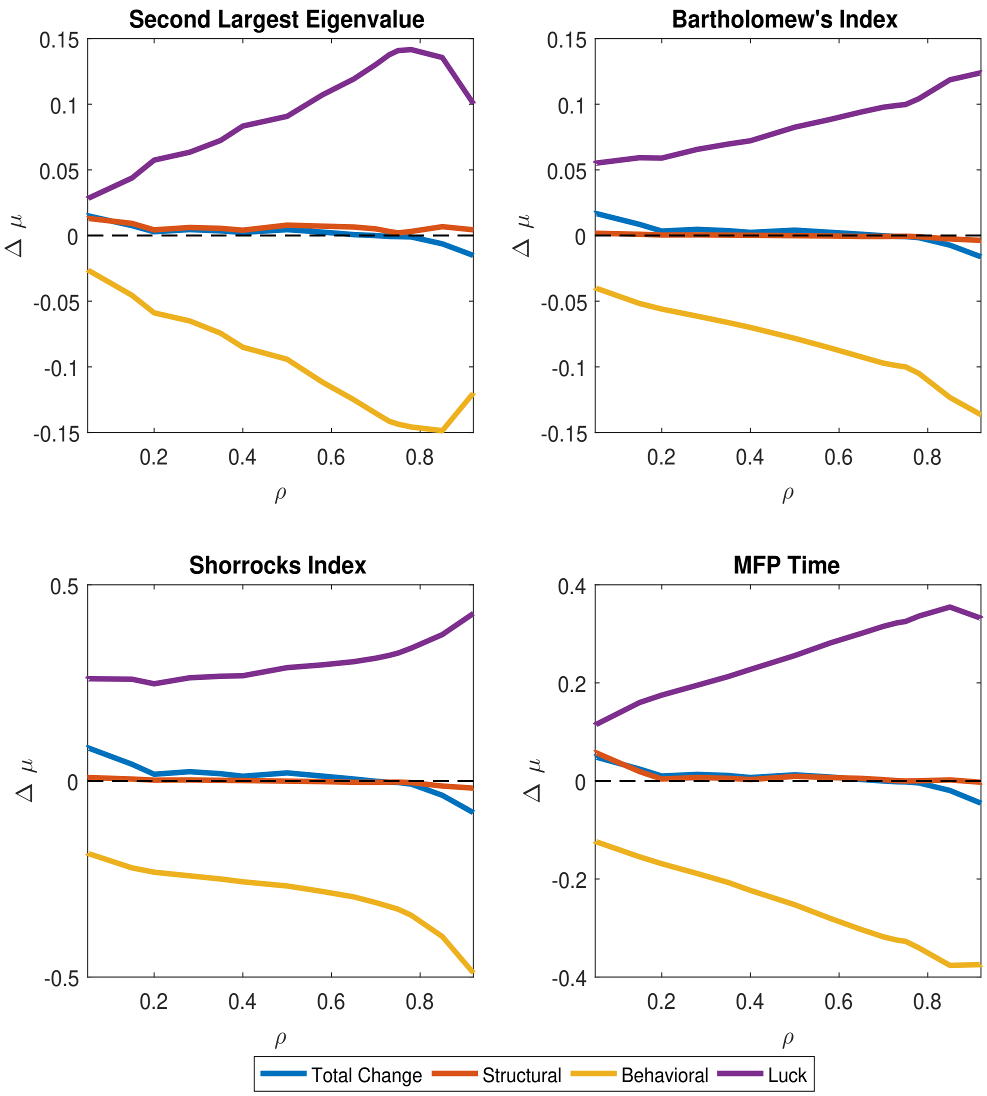


Figure 13: Decomposition of change in mobility as ρ increases

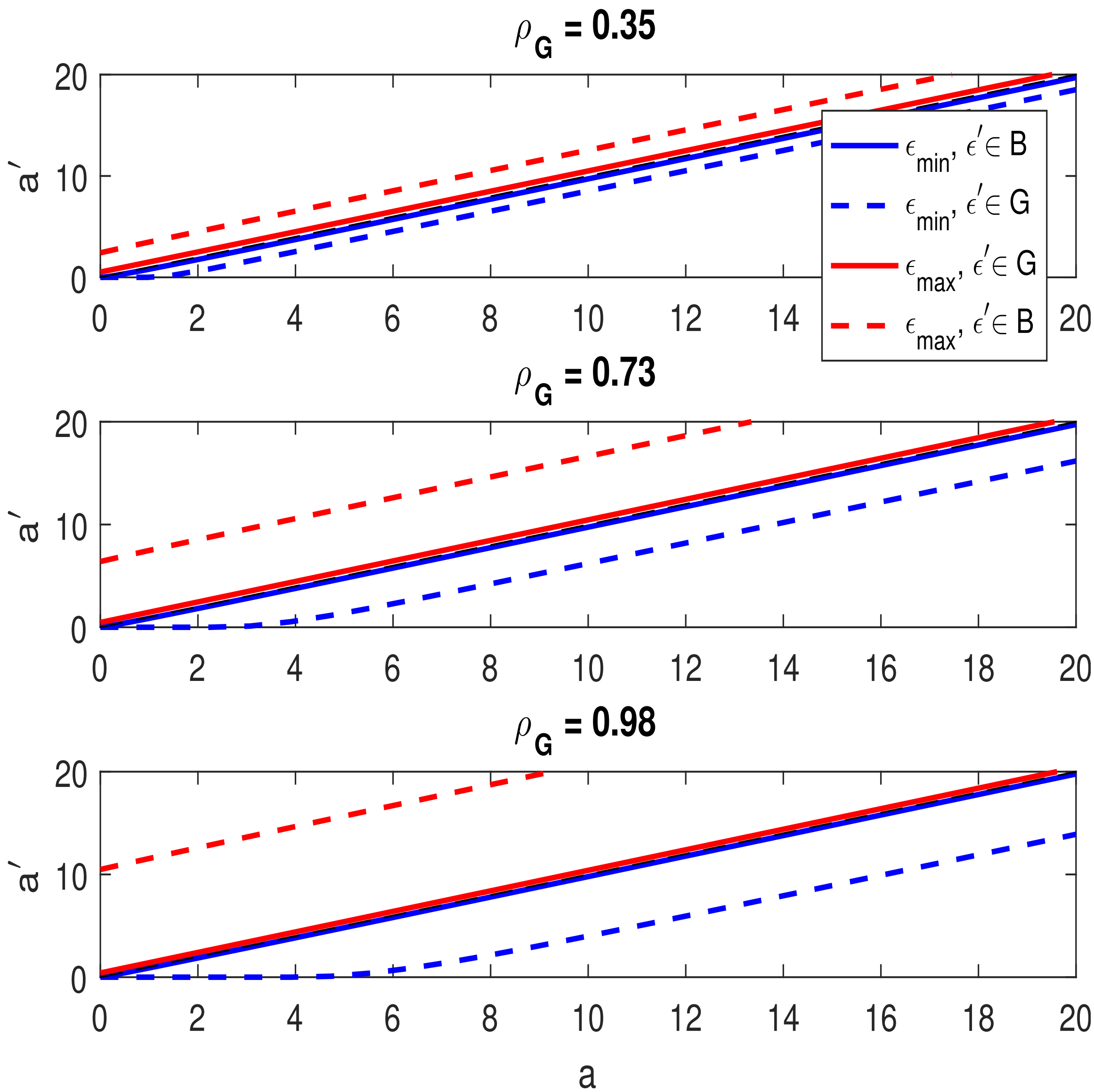
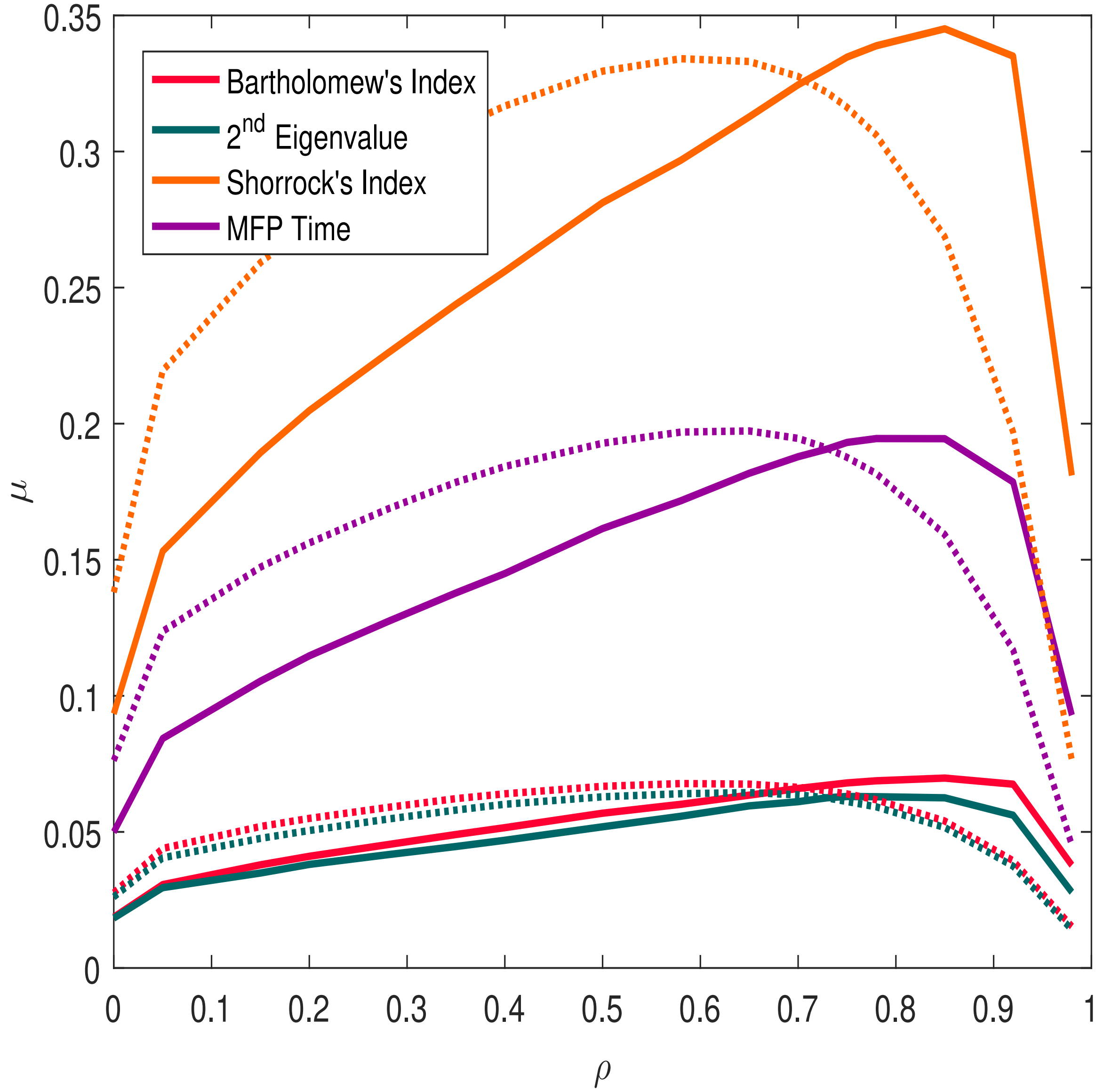


Figure 14: Portfolio decisions

Figure 15: Mobility across ρ; Incomplete markets vs. partial insurance



Solid - 2 assets; Dashed - 1 asset

Figure 16: *

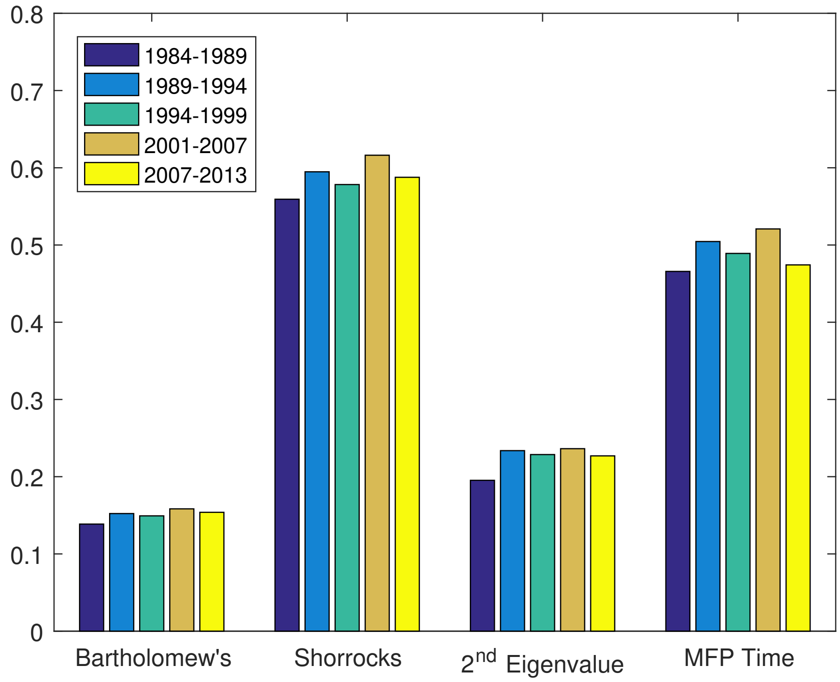


Figure 17: Short-Horizon Mobility

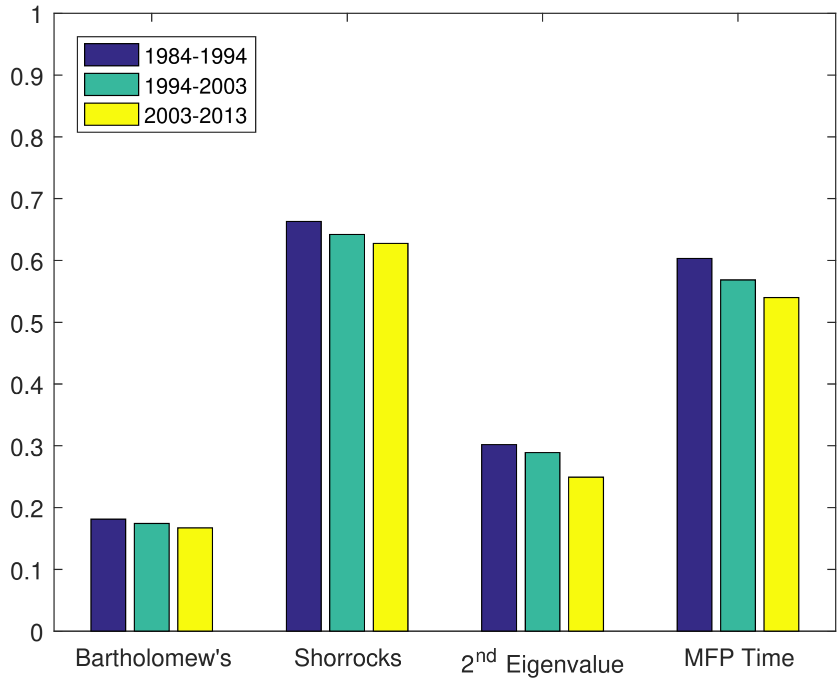


Figure 18: Medium-Horizon Mobility

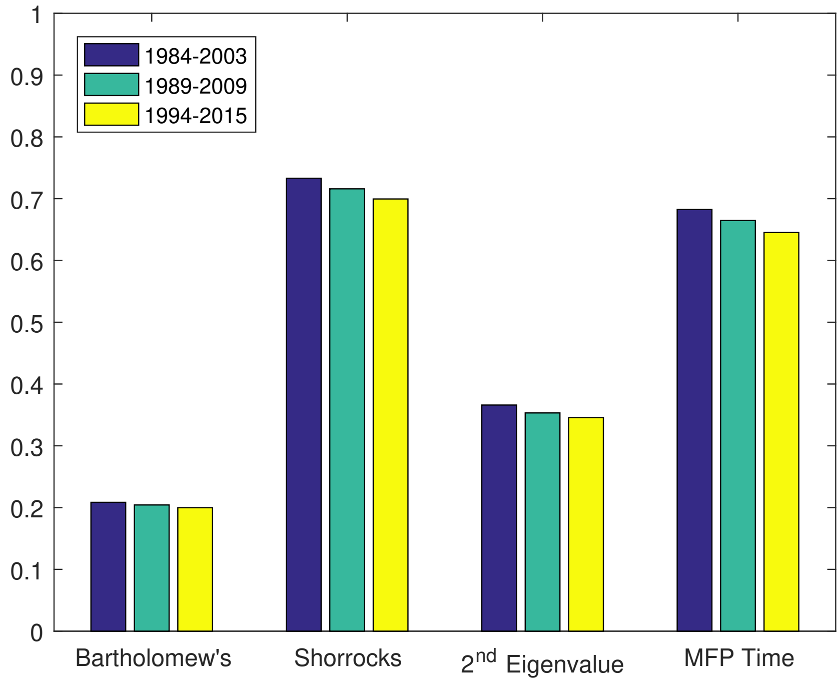


Figure 19: Long-Horizon Mobility

Figure 20: Bootstrapped Mobility Measures, Short Horizon

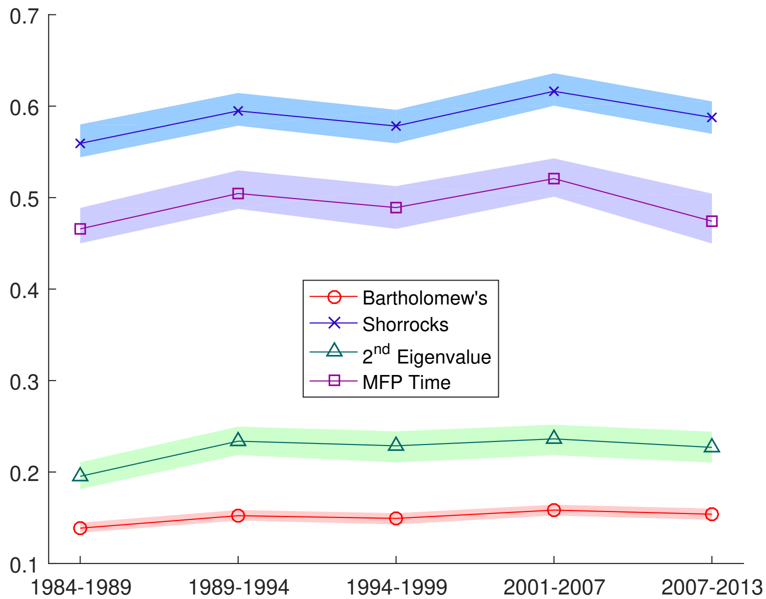


Figure 21: Bootstrapped Mobility Measures, Medium Horizon

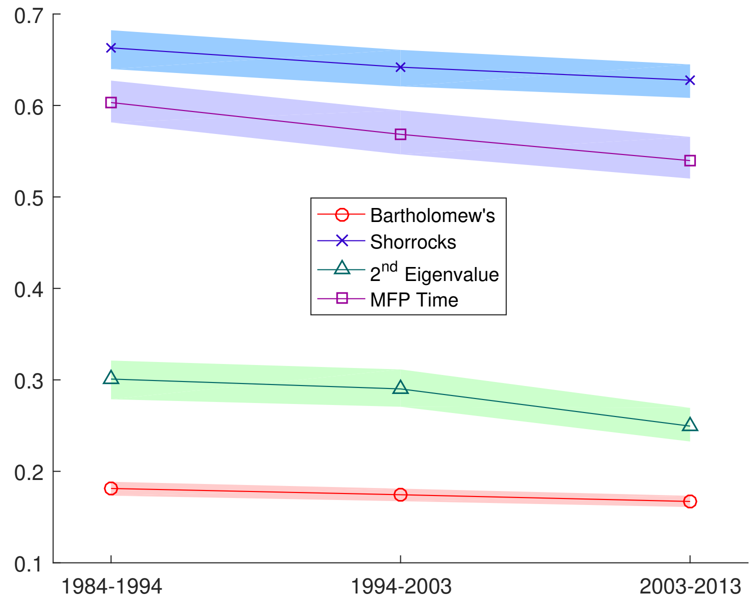


Figure 22: Bootstrapped Mobility Measures, Long Horizon

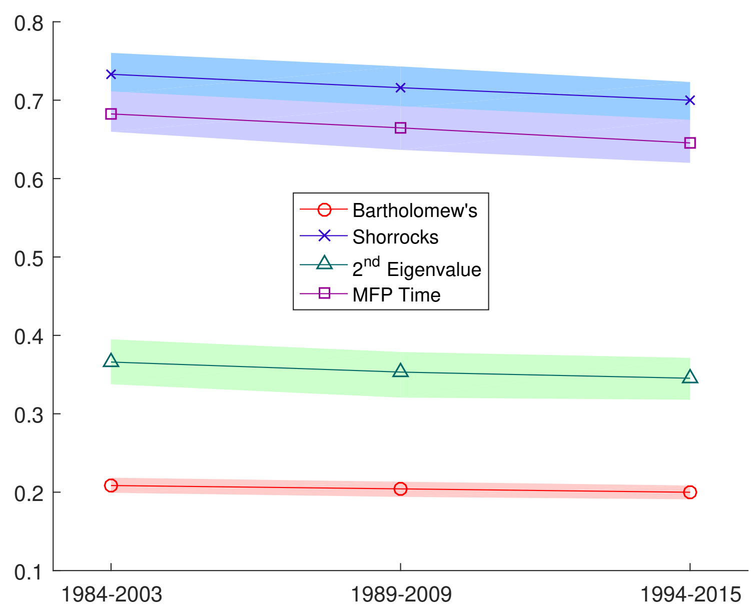


Figure 23: Decomposition: Short Horizon

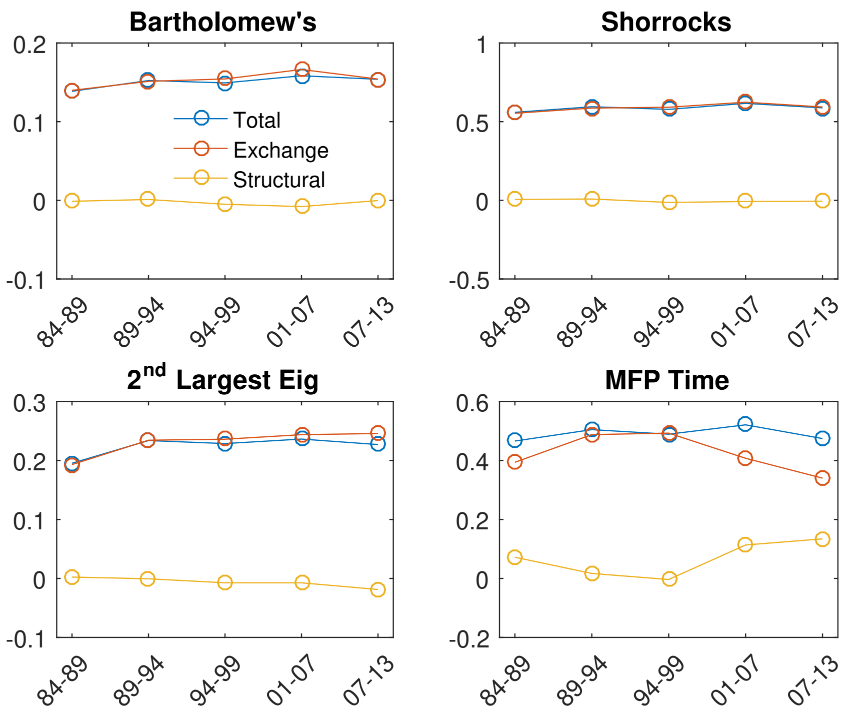


Figure 24: Decomposition: Medium Horizon

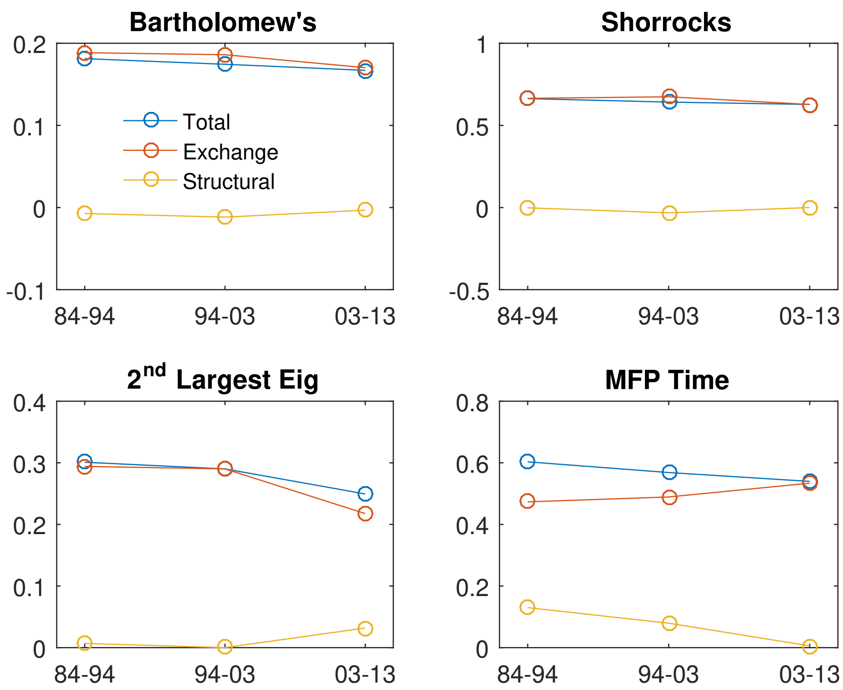


Figure 25: Decomposition: Long Horizon

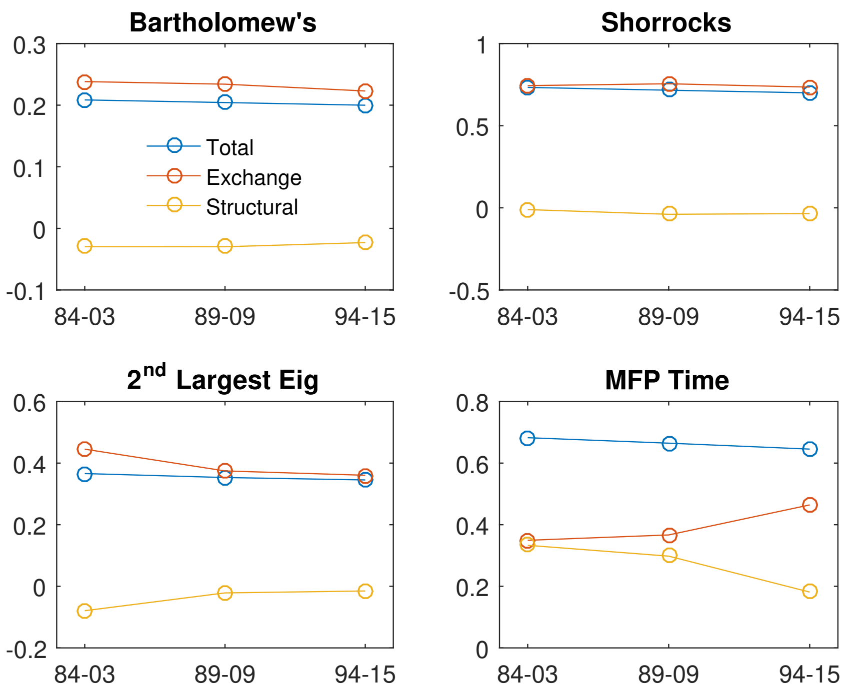


Table 5: Movement Through Distribution (OLS)

	Dependent variable:		
	$\Delta p_{i,t}$		
	Short	Medium	Long
Married During Interval	0.018***	0.061***	0.080***
	(0.004)	(0.007)	(0.011)
Divorced During Interval	−0.049***	−0.058***	−0.075***
	(0.006)	(0.007)	(0.013)
Non-white (Head)	0.006*	0.009*	0.016
	(0.003)	(0.005)	(0.012)
Mean Income (Thousands)	0.0001***	0.0001***	0.0002***
	(0.00001)	(0.00002)	(0.0001)
College Degree (Head)	0.022***	0.034***	0.058***
	(0.003)	(0.004)	(0.009)
Owned Real Estate	−0.002	−0.006	0.001
	(0.003)	(0.004)	(0.009)
Owned a Farm or Business	−0.004	0.0003	−0.014
	(0.003)	(0.004)	(0.009)
Owned Stocks	0.012***	0.014***	0.027***
	(0.003)	(0.004)	(0.010)
Received Inheritance	0.061***	0.044***	0.047***
	(0.004)	(0.005)	(0.010)
Observations	27,334	13,798	3,219
R^2	0.049	0.088	0.201
Adjusted R^2	0.048	0.087	0.198
Residual Std. Error	0.836	0.943	1.087
F Statistic	93.426***	101.387***	67.040***
Note:	*p<0.1; **p<0.05; ***p<0.01		
	All dollar values in constant 2016 USD		

Table 6: Upward Movements (Probit)

	Dependent variable:		
	$\Pr(\Delta q_{i,t} > 0 \mid X)$		
	Short	Medium	Long
Married During Interval	0.168***	0.398***	0.456***
	(0.007)	(0.009)	(0.013)
Divorced During Interval	−0.165***	−0.253***	−0.323***
	(0.010)	(0.010)	(0.016)
Non-white (Head)	0.042***	0.047***	−0.140***
	(0.005)	(0.007)	(0.016)
Mean Income (Thousands)	−0.0002***	−0.0001***	−0.0002**
	(0.00002)	(0.00003)	(0.0001)
College Degree (Head)	0.067***	0.146***	0.249***
	(0.004)	(0.006)	(0.012)
Owned Real Estate	0.012***	0.019***	0.011
	(0.004)	(0.006)	(0.012)
Owned a Farm or Business	0.0003	0.018***	−0.048***
	(0.005)	(0.006)	(0.012)
Owned Stocks	0.091***	0.110***	0.195***
	(0.004)	(0.006)	(0.013)
Received Inheritance	0.279***	0.175***	0.254***
	(0.007)	(0.008)	(0.012)
Observations	27,334	13,798	3,219

Note: *p<0.1; **p<0.05; ***p<0.01
All dollar values in constant 2016 USD

Table 7: Downward Movements (Probit)

	Dependent variable:		
	$\Pr(\Delta q_{i,t} < 0 \vert X)$		
	Short	Medium	Long
Married During Interval	-0.014^{**}	-0.112^{***}	-0.347^{***}
	(0.007)	(0.010)	(0.015)
Divorced During Interval	0.334^{***}	0.330^{***}	0.408^{***}
	(0.009)	(0.010)	(0.016)
Non-white (Head)	-0.089^{***}	-0.090^{***}	-0.180^{***}
	(0.005)	(0.007)	(0.015)
Mean Income (Thousands)	-0.002^{***}	-0.002^{***}	-0.003^{***}
	(0.00004)	(0.0001)	(0.0001)
College Degree (Head)	-0.122^{***}	-0.128^{***}	-0.237^{***}
	(0.005)	(0.006)	(0.012)
Owned Real Estate	0.022^{***}	0.061^{***}	0.092^{***}
	(0.004)	(0.006)	(0.011)
Owned a Farm or Business	0.039^{***}	0.042^{***}	0.157^{***}
	(0.005)	(0.006)	(0.012)
Owned Stocks	-0.094^{***}	-0.072^{***}	-0.016
	(0.004)	(0.006)	(0.012)
Received Inheritance	-0.387^{***}	-0.287^{***}	-0.228^{***}
	(0.008)	(0.008)	(0.012)
Observations	27,334	13,798	3,219
Note:	*p<0.1; **p<0.05; ***p<0.01		
	All dollar values in constant 2016 USD		

Table 8: Upward Jumps (Probit)

	Dependent variable:		
	$\Pr(\Delta q_{i,t} \geq 2 \vert X)$		
	Short	Medium	Long
Married During Interval	0.195***	0.435***	0.280***
	(0.010)	(0.011)	(0.016)
Divorced During Interval	−0.053***	−0.121***	−0.137***
	(0.014)	(0.014)	(0.019)
Non-white (Head)	0.129***	0.083***	−0.161***
	(0.007)	(0.010)	(0.022)
Mean Income (Thousands)	−0.00005	0.0001	−0.0001
	(0.00003)	(0.00004)	(0.0001)
College Degree (Head)	0.139***	0.189***	0.283***
	(0.007)	(0.008)	(0.015)
Owned Real Estate	0.107***	0.056***	0.088***
	(0.007)	(0.008)	(0.015)
Owned a Farm or Business	0.182***	0.235***	0.084***
	(0.007)	(0.008)	(0.015)
Owned Stocks	0.005	0.075***	0.268***
	(0.007)	(0.008)	(0.017)
Received Inheritance	0.354***	0.229***	0.137***
	(0.010)	(0.010)	(0.015)
Observations	27,334	13,798	3,219

Note: *p<0.1; **p<0.05; ***p<0.01
All dollar values in constant 2016 USD

Table 9: Downward Jumps (Probit)

	Dependent variable:		
	$\Pr(\Delta q_{i,t} \leq -2 \mid X)$		
	Short	Medium	Long
Married During Interval	0.011	-0.085^{***}	-0.302^{***}
	(0.010)	(0.015)	(0.021)
Divorced During Interval	0.299^{***}	0.412^{***}	0.470^{***}
	(0.013)	(0.013)	(0.021)
Non-white (Head)	0.074^{***}	-0.041^{***}	-0.153^{***}
	(0.007)	(0.010)	(0.021)
Mean Income (Thousands)	-0.002^{***}	-0.002^{***}	-0.002^{***}
	(0.0001)	(0.0001)	(0.0002)
College Degree (Head)	-0.090^{***}	-0.117^{***}	-0.226^{***}
	(0.007)	(0.009)	(0.017)
Owned Real Estate	0.033^{***}	0.141^{***}	-0.010
	(0.007)	(0.008)	(0.015)
Owned a Farm or Business	0.214^{***}	0.178^{***}	0.309^{***}
	(0.007)	(0.009)	(0.015)
Owned Stocks	-0.019^{***}	-0.089^{***}	-0.015
	(0.007)	(0.008)	(0.016)
Received Inheritance	-0.358^{***}	-0.252^{***}	-0.140^{***}
	(0.014)	(0.013)	(0.017)
Observations	27,334	13,798	3,219
Note:			*p<0.1; **p<0.05; ***p<0.01
			All dollar values in constant 2016 USD

Appendix: Directional Mobility and Demographics

10.1 A Measure of Directional Mobility

We find that splitting our panels along demographic groups–particularly, along educational attainment and race–reveals notable differences in mobility. All of our measures treat upward and downward mobility identically, so for the purposes of this section, we consider the following measure of directional mobility: for an $n \times n$ matrix Π, we define the measure $\mathbf{x} = (x_d, x_u)$, where

$$x_d = \frac{2}{n(n-1)} \sum_{i=2}^{n} \sum_{j=1}^{i-1} \Pi_{i,j} \, |i-j|$$

and

$$x_u = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Pi_{i,j} \, |i-j|$$

This measure is similar to the Bartholomew measure, but it is restricted to the entries below (above) the diagonal for downward (upward) mobility.

For the purposes of exposition, we will refer to the non-college and nonwhite groups as the "disadvantaged" group. We also note that, although our measure of directional mobility places greater weight on larger movements, the difference in upward and downward mobility across groups is not driven by a few large movements. Irrespective of the horizon or specific time period, for the disadvantaged group nearly every below (above) diagonal element of the transition matrix is greater (less) than the corresponding element in the transition matrix for the advantaged group.

10.2 Differences in Mobility Across Demographics

10.2.1 Education

First, we divide our sample into families wherein the head had a college degree at the start of the sample period, and those who did not. We then use the same method as before to construct mobility matrices that capture the wealth transitions of families in each subsample.

Evidence from the PSID data suggests that families whose heads have a college degree experience higher upward mobility. As an example, consider the following two matrices from

2003-2013:

$$\Pi_C^{03-13} = \begin{bmatrix} 0.435 & 0.261 & 0.179 & 0.103 & 0.022 \\ 0.245 & 0.328 & 0.250 & 0.130 & 0.047 \\ 0.114 & 0.140 & 0.327 & 0.301 & 0.118 \\ 0.041 & 0.031 & 0.168 & 0.430 & 0.330 \\ 0.008 & 0.003 & 0.036 & 0.180 & 0.774 \end{bmatrix} \Pi_{NC}^{03-13} = \begin{bmatrix} 0.592 & 0.294 & 0.092 & 0.019 & 0.003 \\ 0.273 & 0.428 & 0.227 & 0.059 & 0.014 \\ 0.154 & 0.240 & 0.408 & 0.168 & 0.030 \\ 0.064 & 0.101 & 0.229 & 0.453 & 0.153 \\ 0.042 & 0.039 & 0.064 & 0.293 & 0.563 \end{bmatrix}$$

These matrices show that families with a college-educated head experience higher upward mobility, lower persistence in the lower wealth quintiles, and higher persistence in the upper wealth quintiles. Families in this subsample have a good chance of getting to high levels of wealth, and when they do so, they tend to stay there. By contrast, families wherein the head did not have a college degree experienced higher downward mobility, higher persistence in lower quintiles, and lower persistence in upper quintiles. These patterns are even more pronounced at long horizon (see Table 10). These results suggest that the mobility matrices over longer time periods may be built by two distinct groups: college-educated families making larger contributions to the above-diagonal elements, and the families without a college-educated making larger contributions to the below-diagonal elements.

10.2.2 Race

Splitting our sample by race yields similar results. Below, we report the ten-year wealth transition matrices over the period 2003-2013 for households with white (Π_W) and nonwhite (Π_{NW}) families:

Here we see that families with a white head experience high levels of upward mobility, low persistence in lower quintiles, and high persistence in upper quintiles. Nonwhite families, by contrast, are more likely to make downward movements, and experience relatively higher persistence in low quintiles, and lower persistence in higher quintiles. We can see, for example, that a nonwhite family who started off in the first quintile had about a one in ten chance of reaching one of the top three quintiles, compared to the roughly one in five chance faced by a white family at making the same transition from the same starting point. Once again, this is a pattern that holds true at a long horizon (Table 11).

Figure 26: Calibration: σ_z across ρ_z for fixed Gini

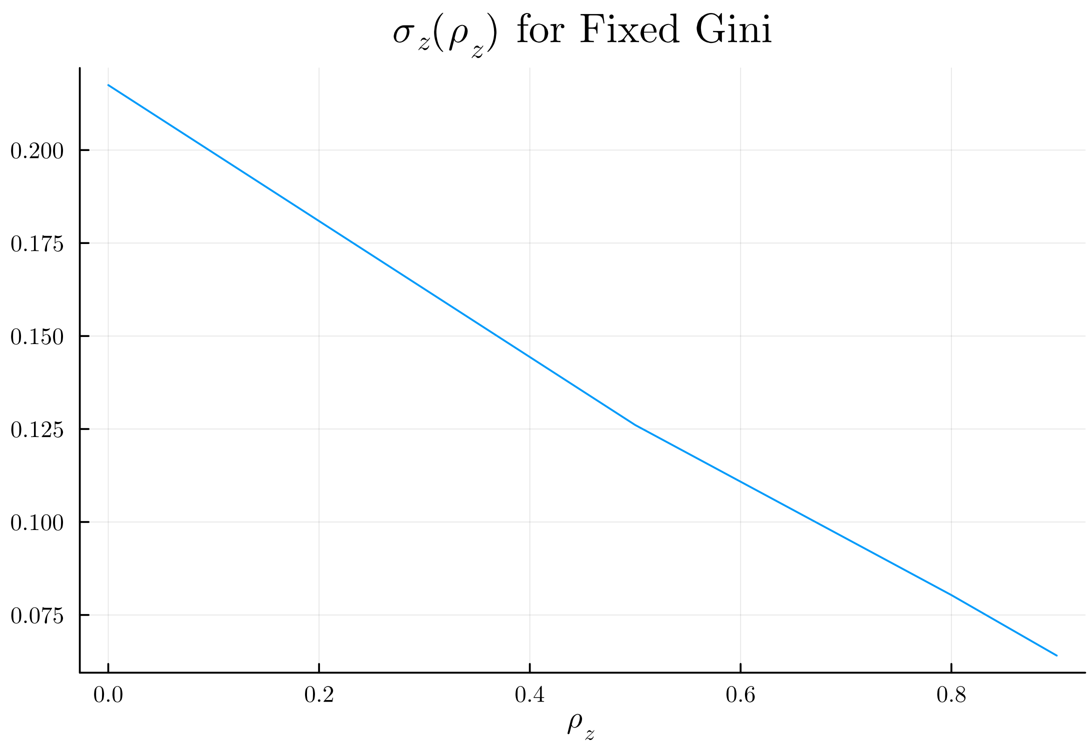


Figure 27: Mobility Measures Across Return Persistence ρ_z

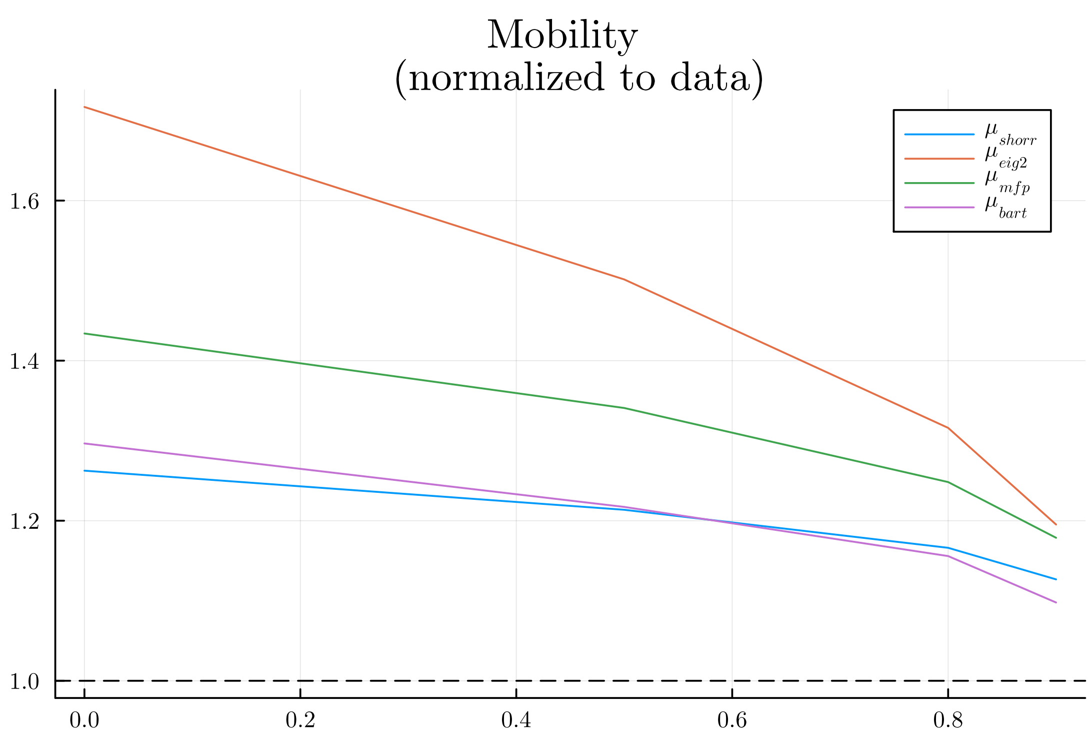


Figure 28: Top Wealth Shares Across Return Persistence ρ_z

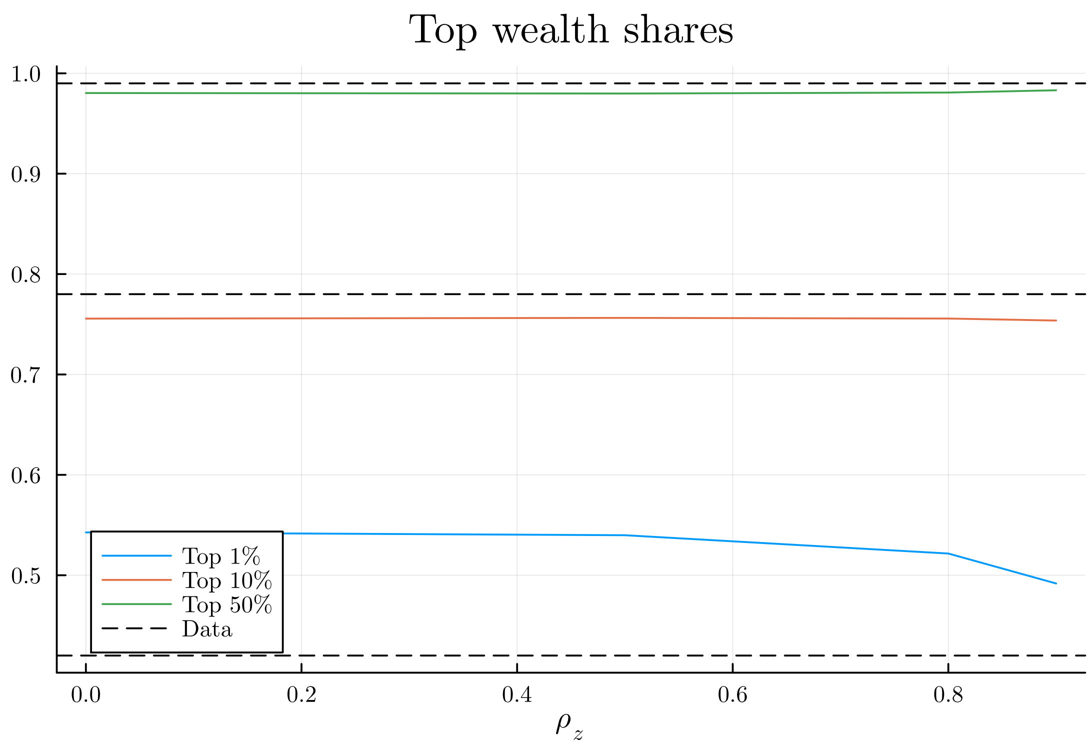
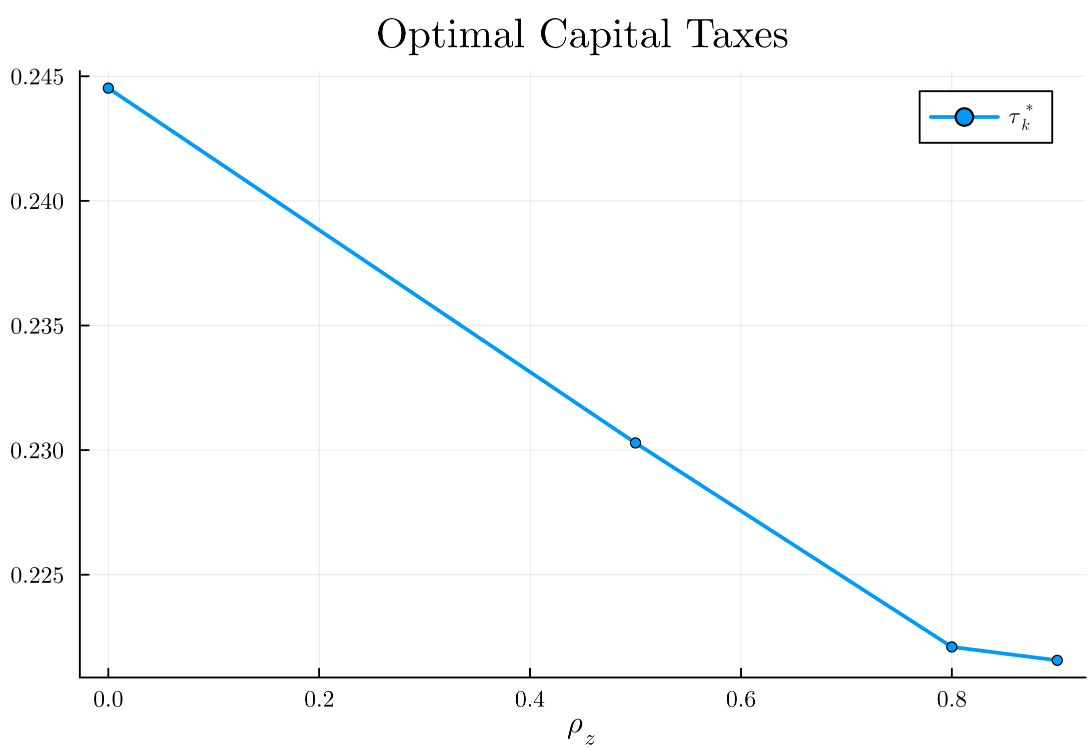


Figure 29: Optimal Taxes Across Return Persistence ρ_z



$$\Pi_W^{03-13} = \begin{bmatrix} 0.480 & 0.314 & 0.137 & 0.055 & 0.014 \\ 0.232 & 0.397 & 0.245 & 0.093 & 0.033 \\ 0.118 & 0.167 & 0.390 & 0.254 & 0.071 \\ 0.033 & 0.062 & 0.188 & 0.473 & 0.244 \\ 0.014 & 0.010 & 0.045 & 0.223 & 0.709 \end{bmatrix}$$

$$\Pi_{NW}^{03-13} = \begin{bmatrix} 0.624 & 0.274 & 0.084 & 0.016 & 0.001 \\ 0.299 & 0.424 & 0.218 & 0.051 & 0.008 \\ 0.183 & 0.284 & 0.382 & 0.126 & 0.025 \\ 0.136 & 0.124 & 0.271 & 0.345 & 0.124 \\ 0.090 & 0.090 & 0.077 & 0.282 & 0.462 \end{bmatrix}$$

Table 10: Twenty-Year College Breakdown

$$\Pi_C^{89-09} : \begin{bmatrix} 0.289 & 0.211 & 0.244 & 0.122 & 0.133 \\ 0.167 & 0.188 & 0.333 & 0.210 & 0.101 \\ 0.059 & 0.129 & 0.294 & 0.329 & 0.188 \\ 0.049 & 0.079 & 0.238 & 0.311 & 0.323 \\ 0.019 & 0.023 & 0.047 & 0.211 & 0.700 \end{bmatrix}$$

$$\Pi_{NC}^{89-09} : \begin{bmatrix} 0.600 & 0.286 & 0.081 & 0.022 & 0.010 \\ 0.298 & 0.422 & 0.155 & 0.088 & 0.038 \\ 0.144 & 0.312 & 0.293 & 0.166 & 0.086 \\ 0.099 & 0.131 & 0.314 & 0.325 & 0.131 \\ 0.011 & 0.082 & 0.142 & 0.290 & 0.475 \end{bmatrix}$$

Table 11: Twenty-Year Race Breakdown

$$\Pi_W^{89-09} : \begin{bmatrix} 0.402 & 0.303 & 0.176 & 0.077 & 0.042 \\ 0.204 & 0.327 & 0.229 & 0.163 & 0.076 \\ 0.096 & 0.192 & 0.295 & 0.268 & 0.149 \\ 0.065 & 0.093 & 0.290 & 0.334 & 0.218 \\ 0.013 & 0.045 & 0.092 & 0.247 & 0.603 \end{bmatrix}$$

$$\Pi_{NW}^{89-09} : \begin{bmatrix} 0.657 & 0.258 & 0.058 & 0.010 & 0.017 \\ 0.371 & 0.433 & 0.143 & 0.040 & 0.013 \\ 0.174 & 0.428 & 0.290 & 0.072 & 0.036 \\ 0.192 & 0.250 & 0.250 & 0.212 & 0.096 \\ 0.059 & 0.176 & 0.059 & 0.235 & 0.471 \end{bmatrix}$$