Optimal Taxation with Heterogeneous Rates of Return

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Summary

- Study optimal *nonlinear* taxation in an economy where agents are heterogeneous in the (stochastic) returns to their investment
- Static model
  - Risky investment is susbidized according to a nonlinear function of return
  - Wedge on risk-free savings of productive entrepreneurs is positive, increasing in return
- Dynamic model
  - IID case: optimal wedges are independent of history
- Implementation
  - In progress: how to implement constrained-efficient allocations with taxes and transfers?

Motivation

- US distribution of wealth exhibits thick (Pareto) tails
  - Benhabib *et al.* (2011): the heavy tail is generated by capital, rather than labor, income risk
  - Tail populated by entrepreneurs, investors, business owners
  - Need to understand optimal taxation of those whose income derives from investment, rather than labor effort
- Capital income, wealth taxation have been topics of debate
  - Saez and Zucman (2019): argue for wealth taxation
  - This introduces obvious disincentives to invest
  - Mirrlees: framework in which to study the optimal tradeoff between *redistribution* and *efficiency*

Related Literature

- Static Mirrleesian taxation: Mirrlees (1971), Diamond (1998), Saez (2001)
  - Government can set arbitrarily nonlinear taxes, subject to informational frictions
- Dynamic extensions: Golosov *et al.* (2003), Kocherlakota (2005), Albanesi and Sleet (2006)
  - Possibility of a positive optimal wedge on savings
  - Intertemporal return constant across the population
- Heterogeneous returns: Shourideh (2014), Phelan (2019b), Phelan (2019a)

Static Model: Agents, Assets, and Production

- Continuum of agents, $i \in [0,1]$
- Two periods, $t = 0, 1$
- Agents have privately-known type $\theta \in \Theta = \left[\underline{\theta}, \overline{\theta}\right]$
    - Distributed according to differentiable CDF $F(\theta)$
- Two assets
    - Risk-free savings bond: zero net supply, return R
    - Risky entrepreneurial endeavor: stochastic return θ
- Initial endowment w
- Output: an agent of type θ who invests k into risky technology produces

$$y = \begin{cases} \theta k & \text{with probability } \alpha \\ 0 & \text{with probability } 1 - \alpha \end{cases} \tag{1}$$

Static Model: Taxes

Government

- Observes incomes y and Rb, but not θ or k
- Levies taxes according to tax function $T(y, Rb)$
- Mirrleesian problem: must set T to maximize total utility, subject to the constraint that T cannot induce individuals to invest a "suboptimal" amount

Static Model: Taxes

- Agents maximize expected utility, solving

$$\mathcal{U}(\theta) = \max_{k \geq 0, b \geq 0} \log c_0 + \beta \mathbb{E}[\log c_1]$$

$$\text{s.t.}$$

$$c_0 \leq w - k - b$$

$$c_1 \leq \begin{cases} \theta k + Rb - T(y, Rb) & y > 0 \\ Rb - T(y, Rb) & y = 0 \end{cases}$$

  given type θ, initial endowment w, and taxes T
- Government's objective, then, is to maximize

$$\int_{\underline{\theta}}^{\bar{\theta}} \mathcal{U}(\theta) f(\theta) d\theta$$

  by choice of T, subject to budget and incentive constraints

Static Model: Mechanism Design Problem

- Mirrlees (1971): optimal taxation problem can be recast as one of *mechanism design*
  - Revelation principle: can focus on direct mechanism
- Planner collects reports of type θ from agents, gives allocations

$$\{c_0(\theta), k(\theta), b(\theta), c_1(\theta, y), c_1(\theta, 0)\}_{\theta \in \Theta}$$

  - $c_1(\theta, y)$ and $c_1(\theta, 0)$ give consumption at $t = 1$ following a successful and unsuccessful investment, respectively

Static Model: Mechanism Design Problem

Planner's problem:

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{U}(\theta)\, dF(\theta)$$

subject to

$$\log c_0(\theta) + \beta[\alpha \log c_1(\theta, y) + (1-\alpha)\log c_1(\theta, 0)] = \mathcal{U}(\theta)$$

$$\int_{\underline{\theta}}^{\overline{\theta}} [c_0(\theta) + k(\theta)]\, dF(\theta) \le w$$

$$\int_{\underline{\theta}}^{\overline{\theta}} [\alpha c_1(\theta, y) + (1-\alpha)c_1(\theta, 0)]\, dF(\theta) \le \int_{\underline{\theta}}^{\overline{\theta}} \alpha\theta k(\theta)\, dF(\theta)$$

Static Model: Incentive Constraints

- Additional constraints in planner's problem: incentive compatibility
  - Resulting from informational frictions: planner cannot observe θ or k
- Agents can "lie" in two ways

Static Model: Incentive Constraints

- First incentive constraint: $\forall \theta \in \Theta$,

$$\theta \in \arg\max_{\hat{\theta}} u\left(c_0\left(\hat{\theta}\right) + k\left(\hat{\theta}\right) - \frac{\hat{\theta}}{\theta} k\left(\hat{\theta}\right)\right) + \beta\left[\alpha u\left(c_1\left(\hat{\theta}, y\right)\right) + (1-\alpha) u\left(c_1\left(\hat{\theta}, 0\right)\right)\right] \quad (2)$$

- Truth-telling, $\theta = \hat{\theta}$ is dominant strategy
- First-order approach, following Jewitt (1988):

$$\mathcal{U}'(\theta) = \frac{k(\theta)}{\theta} u'\left(c_0(\theta)\right) \quad (3)$$

Static Model: Incentive Constraints

- Second incentive constraint: $\forall \theta, \hat{\theta} \in \Theta$,

$$\mathcal{U}(\theta) \geq \max_{\hat{\theta}} u\left(c_0(\hat{\theta}) + k(\hat{\theta})\right) + \beta u\left(c_1(\hat{\theta}, 0)\right) \tag{4}$$

- From (3), $\mathcal{U}$ is weakly increasing in θ, so it is sufficient to impose

$$\mathcal{U}(\underline{\theta}) \geq u\left(c_0\left(\theta\right) + k\left(\theta\right)\right) + \beta u\left(c_1\left(\theta, 0\right)\right) \tag{5}$$

for all $\theta \in \Theta$, and treat $\mathcal{U}\left(\underline{\theta}\right)$ as a parameter

Static Model: Wedges

- The optimal wedges are defined as follows:

$$\tau_k(\theta) = 1 - \frac{u'(c_0(\theta))}{\alpha\beta\theta u'(c_1(\theta, y))} \tag{6}$$

$$\tau_b(\theta) = 1 - \frac{u'(c_0)}{\beta R\left[\alpha u'(c_1(\theta, y)) + (1-\alpha)u'(c_1(\theta, 0))\right]} \tag{7}$$

- Related to partial derivatives of the tax function:

$$\begin{aligned}\tau_k(\theta) &= T_1(y, Rb)\\ \tau_b(\theta) &= T_2(y, Rb)\end{aligned}$$

- Kocherlakota (2005): these are not exactly equivalent to taxes, but nevertheless give measure of how decisions are *distorted*

Static Model: Results

Proposition 1: Risk and Incentives

$k(\theta) > 0 \implies \alpha\theta > R$. Furthermore,

$$c_1(\theta, y) = c_1(\theta, 0) + \frac{\beta\phi(\theta)}{\lambda_1(1-\alpha)} \tag{8}$$

where λ_1 is the multiplier on feasibility in the second period, and $\phi(\theta)$ is the multiplier on the second incentive constraint.

- Government plays role in selecting between entrepreneurs and "lenders"
- The function (8) describes how incentives are provided: more investment $\implies$ larger difference in $c_1(\theta, y)$ and $c_1(\theta, 0)$

Static Model: Optimal Wedges

Proposition 2: Optimal Wedges

If $k(\theta) > 0$, $\tau_k(\theta) \leq 0$, with equality if $\theta \in \{\underline{\theta}, \bar{\theta}\}$, and $\tau_b(\theta) > 0$. If $k(\theta) = 0$, meanwhile, $\tau_b(\theta) = 0$.

- Entrepreneurs: face a possibly non-monotonic pattern of subsidies to risky investment, positive wedge on risk-free saving
- Borrowers ($k = 0$): face no distortions

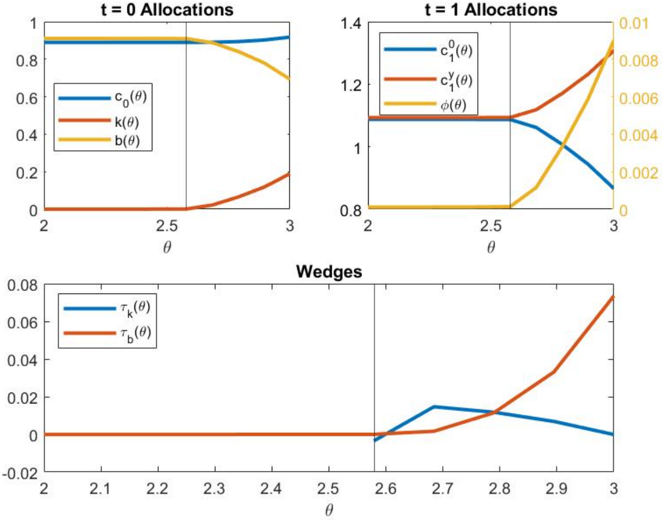
Static Model: Numerical Example

Partial solution to cost-minimization problem, parameterized as follows:

$$\alpha = 0.5 \qquad \left\{\underline{\theta}, \bar{\theta}\right\} = \{1.0, 3.0\}$$
$$\beta = 0.95 \qquad w = 1.8$$
$$R = 1.29 \qquad \mathcal{U}^* = -0.03$$

Static Model: Numerical Example

Figure: Allocations in the Planner's Problem



Dynamic Model: Setup and Timing

- Static model cannot differentiate between capital income and wealth
- We consider a dynamic extension of the model in order to study wealth accumulation
- Time is again discrete, $t = 0, 1, ...$
- Agents draw new type θ_t in each period
  - We assume that draws are *i.i.d.*
  - This way, promised utility is a sufficient statistic for keeping track of history

Dynamic Model: Setup and Timing

Each period plays out as follows:

1. Agents realize their capital income y_t according to

$$y_t = \begin{cases} \theta_{t-1} k_t & \text{with probability } \alpha \\ 0 & \text{with probability } 1 - \alpha \end{cases} \tag{9}$$

2. Agents draw new type θ_t from $F(\theta)$
3. Agents make consumption and savings choices $c_t(\theta^t, y^t), k_{t+1}(\theta^t, y^t)$, and $b_{t+1}(\theta^t, y^t)$
   - Superscript notation denotes history, e.g. $y^t = \{y_0, y_1, ..., y_t\}$

Dynamic Model: Dual Planner's Problem

- $\mu_t(\theta^t, y^t)$: measure of period-t histories induced by the stochastic process governing θ_t and the random nature in y_t
- Promised utility:

$$w_{t+1}\left(\theta^t, y^{t+1}\right) = \sum_{s=t+1}^{\infty} \beta^{s-t-1} \int u\left(c_s\left(\theta^s, y^s\right)\right) d\mu_s\left(\theta^s, y^s | \theta^t, y^{t+1}\right)$$

  allocated to agent with history (θ^t, y^t), conditional on realization of y_{t+1}
- We consider the dual planner's problem: minimize the time-0 discounted cost of delivering total utility w

Dynamic Model: Dual Planner's Problem

Recursive formulation:

$$C(w) = \min_{c,k',w'_y,w'_0,\mathcal{U}}$$
$$\int \left\{ c(\theta) - \frac{\alpha\theta k(\theta)}{R} + R^{-1}\left[\alpha C\left(w'_y(\theta)\right) + (1-\alpha) C\left(w'_0(\theta)\right)\right] \right\} dF(\theta) \tag{10}$$

subject to

$$\int \mathcal{U}(\theta)\, dF(\theta) \geq w$$
$$\mathcal{U}(\theta) = u(c) + \beta\left[\alpha w'_y(\theta) + (1-\alpha) w'_0(\theta)\right]$$
$$\mathcal{U}'(\theta) = u'(c)\frac{k}{\theta}$$
$$\mathcal{U}(\underline{\theta}) \geq u(c+k) + \beta w'_0$$

Dynamic Model: Closed-form solution

Proposition 3: Solution to the Dynamic Model

If $u(c) = \log c$, then (10) admits the following closed-form solution:

$$\begin{aligned} C(w) &= A \exp((1-\beta)w) & w'(\theta, 0, w) &= w'(\theta, 0) + w \\ k(\theta, w) &= k(\theta) \exp((1-\beta)w) & w'(\theta, y, w) &= w'(\theta, y) + w \\ c(\theta, w) &= c(\theta) \exp((1-\beta)w) & \mathcal{U}(\theta, w) &= \mathcal{U}(\theta) + w \end{aligned}$$

for some $A, c(\theta), k(\theta), w'(\theta, y), w'(\theta, 0)$, and $\mathcal{U}(\theta)$.

- A similar result holds if utility is CRRA with $\sigma \neq 1$.

Dynamic Model: Wedges

- Proposition 3 shows that wedges in the planner's cost minimization problem are *history-independent*.
- Instead, the wedges only depend on the pair θ_t, θ_{t+1}
- For example, the wedge on risky investment is given by

$$\begin{aligned}\tau_{t,k}\left(\theta^t, y^t\right) &= 1 - \frac{u'\left(c_t\left(\theta^t, y^t\right)\right)}{\alpha\beta\theta_t u'\left(c_{t+1}\left(\theta^{t+1}, \{y^t, y\}\right)\right)} \\ &= 1 - \frac{1/c\left(\theta_t\right)}{\alpha\beta\theta_t\left\{1/c\left(\theta_{t+1}\right)\exp\left[\left(1-\beta\right)w^y_{t+1}\left(\theta_t\right)\right]\right\}}\end{aligned}$$

Implementation

- Thus far, we have focused on optimal *wedges* in the mechanism design problem
- The question remains: how to implement these wedges with taxes and transfers in a competitive market economy?

Implementation: Wealth Subsidy

- The lack of history-dependence in the solution to the dynamic planner's problem makes implementation somewhat difficult
- At the optimum, we need two individuals with different histories but the same current and one period-ahead types to face identical distortions
- If we want to implement a progressive tax on capital income, then to accomplish this result, we would need to pair this tax with a wealth *subsidy*
  - Progressive tax would need to be "undone" to preserve history independence

Conclusion

- We have studied optimal taxation of capital income in an environment with informational frictions
- Static model: positive wedge on risk-free savings, non-monotonic marginal subsidies on capital income for entrepreneurs
  - No distortions for workers, or “lenders”
- Dynamic model: with i.i.d. productivity shocks, optimal wedges are history-independent

Next Steps

- Full solution to static model
- Characterization of history-independent wedges
- Dynamic model: persistence in θ
  - Less tractable, but perhaps more intuitive
- Implementation: construct a tax-and-transfer system that generates the optimal allocations as a competitive equilibrium

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