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Dynamic Optimal Taxation with Private Information

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We study dynamic optimal taxation in a class of economies with private information. Optimal allocations in these environments are complicated and history-dependent. Yet, we show that they can be implemented as competitive equilibria in market economies supplemented with *simple* tax systems. The market structure in these economies is similar to that in Bewley (1986); agents supply labour and trade risk-free claims to future consumption, subject to a budget constraint and a debt limit. Optimal taxes are conditioned only on two observable characteristics—an agent's accumulated stock of claims, or wealth, and her current labour income. We show that optimal taxes are generally non-linear and non-separable in these variables and relate the structure of marginal wealth and income taxation to the properties of agent preferences.

1. INTRODUCTION

This paper studies optimal taxation in a class of dynamic economies with private information. We consider an environment in which agents' preferences are defined over consumption and labour, and each agent receives a privately observed sequence of i.i.d. preference shocks. Incentive-compatibility constraints stemming from private information imply that socially optimal, or *constrained–efficient*, allocations in this environment are complicated and history-dependent. Yet, we show that they can be implemented as competitive equilibria in market economies supplemented with *simple* tax systems. The market structure in these economies is identical to that in Bewley (1986), Huggett (1993) or Aiyagari (1994); agents can trade current consumption for claims to future consumption, subject to a budget constraint and a borrowing limit. These claims have a non-contingent pre-tax return. Crucially, taxes are conditioned upon only two observable characteristics of an agent: current wealth, given by the agent's accumulated stock of claims, and current labour income. They do not depend on any other aspect of an agent's past history.

Most models of dynamic optimal taxation follow the Ramsey approach, in which the set of fiscal instruments available to the government is *exogenously specified*.[1] Linear labour and capital income taxes are typically included in this set, while lump-sum taxes are ruled out. The exclusion of the latter is justified by appealing to incentive or administrative constraints, but these are not explicitly modelled. These exogenous restrictions on fiscal instruments represent frictions that the government seeks to ameliorate through its optimal choice of tax rates.

The approach we adopt in this paper builds on the optimal taxation literature initiated by Mirrlees (1971). Mirrlees assumes that agents receive privately observed shocks to their

1. Chari and Kehoe (1999) provide an excellent overview of this literature.

productivity. The incentive-compatibility constraints that stem from this private information then impose *endogenous* restrictions on optimal tax policies. Mirrlees characterizes those tax functions that induce agents to select constrained–efficient allocations. Such allocations exhibit a pattern of wedges between the social and an individual agent's shadow price of labour. Optimal marginal income tax rates simply "fill in" these wedges so that the constrained–efficient allocation satisfies the first order conditions of agents in the market economy with taxes.

The Mirrlees model is static, as have been most of its successors. Consequently, the properties of optimal taxes in dynamic economies with private information remain largely unexplored.[2] On the other hand, the dynamic contracting literature has extensively studied the properties of constrained–efficient allocations in such settings.[3] This literature has limited attention to implementation via direct mechanisms. Under such mechanisms, private agents report their privately observed shocks and allocations are made contingent on histories of reports. As Green (1987) and others have shown, when shocks are i.i.d., the constrained–efficient allocation can be implemented by a mechanism that is recursive in promised utilities.

Although direct mechanisms can be interpreted as tax systems, they seem divorced from the actual combination of markets and taxes that are used in practice to allocate resources, at least within modern economies.[4] This motivates our analysis of *fiscal implementations*. These are arrangements of markets and taxes that implement dynamic constrained–efficient allocations as competitive equilibria, in the spirit of Mirrlees. We focus on a class of fiscal implementations in which equilibrium allocations are recursive in an agent's wealth and taxes are conditioned only on current wealth and current labour earnings. In the same way that promised utility encodes an agent's history under a recursive direct mechanism, wealth encodes an agent's history in our fiscal implementations. The government is able to infer from an agent's wealth the continuation allocation to which she is entitled. Since the tax system is designed to induce the agent to choose this allocation, it is essential that taxes depend on wealth. In this way, the informational role of wealth crucially influences how it is taxed.

The existence of an optimal mechanism recursive in promised utilities does not imply the existence of a corresponding fiscal implementation. Under a direct mechanism, by adopting different reporting strategies an agent can obtain different allocations. In a market economy with taxes, an agent chooses from the set of budget-feasible allocations. Since the constrained–efficient allocation is incentive-compatible, it can be implemented if the set of budget-feasible allocations in the market economy equals the set of allocations available to an agent under the direct mechanism. However, in our simple fiscal implemetations, the tax system is conditioned only on an agent's current wealth and labour earnings. Consequently, an agent in the market economy might choose a labour supply that is consistent with constrained–efficient behaviour given a particular history of shocks, but then allocate her after-tax resources between consumption and savings in a way that matches constrained–efficient behaviour *given a different history of shocks*. The tax system cannot verify consistency of the previous period's labour earnings with this period's wealth, nor can it ensure that an agent's savings are consistent with her labour earnings. Surprisingly, when agents' preferences are separable in consumption and labour and when idiosyncratic shocks are i.i.d., we show that it is possible to design a tax system, conditioned on current wealth and labour earnings *only*, that induces agents to choose the constrained–efficient allocation.

As in the static Mirrlees model, dynamic constrained–efficient allocations exhibit a pattern of wedges. In particular, when preferences are additively separable in consumption and labour,

2. da Costa and Werning (2001) and Kocherlakota (2004*a*) also apply the Mirrlees approach in a dynamic setting. Golosov and Tsyvinski (2003*a*) apply a similar strategy to the analysis of disability insurance.

3. See, for example, Green (1987), Phelan and Townsend (1991), Atkeson and Lucas (1992, 1995) or Phelan (1994).

4. They may more closely resemble the arrangements used in simple village economies, see Ligon (1998).

they admit an intertemporal wedge between the social and an individual agent's shadow price of claims.[5] This wedge provides a rationale for asset taxation that is often absent from complete information Ramsey models. While implementation requires that the constrained–efficient allocation satisfies the agent's Euler equation in the market economy, there are many patterns of marginal asset taxes consistent with this condition. A natural first guess is that the optimal marginal asset tax at date $t+1$ simply matches the intertemporal wedge by equating the private and social shadow price of claims at date t. In general, however, such marginal asset taxes fail to ensure that the constrained–efficient allocation satisfies the agent's *second order necessary conditions* in the market economy. Underlying this failure is a potential complementarity between savings and labour supply. We provide an example in which this complementarity underpins a profitable joint deviation from the constrained–efficient allocation. In this joint deviation agents save too much in period t and work too little in period $t+1$. To remove the deviation marginal asset taxes at $t+1$ must covary negatively with the agent's labour income at this date. In this example, the optimal expected marginal asset tax equals zero and the intertemporal wedge is entirely generated by the negative covariance between marginal asset taxes and labour income. This covariance discourages saving by making claims a poor hedge against labour income risk.

More generally, we obtain recursive fiscal implementations that use a combination of a positive expected marginal asset tax and a negative covariance between marginal asset taxes and labour income to generate the intertemporal wedge. We show that the extent to which the tax systems implied by these implementations incorporate either of these two features is linked to the magnitude of wealth effects on labour supply in the planner's problem.

We explore the steady state properties of our optimal tax system in numerical examples. We find that marginal taxes are strongly sensitive to wealth in a neighbourhood of the borrowing limit. Marginal income taxes are decreasing in wealth and marginal asset taxes are decreasing in labour income. The intertemporal wedge is less than 1% over most of the wealth range, but is much larger close to the borrowing limit. The expected marginal asset tax falls steadily with wealth, while the absolute value of the covariance between the marginal asset tax and labour income falls sharply as wealth increases away from the limit.

Our paper is closely related to Kocherlakota (2004*a*). He also derives a tax system that implements constrained–optimal allocations in an environment similar to ours. His analysis allows for persistent idiosyncratic shocks. Kocherlakota's tax system is not recursive and does not exploit the information conveyed by an agent's asset position. Instead, it conditions taxes on an agent's entire history of labour earnings. Thus, his tax system, while more general, is also much more complex than ours. Interestingly, Kocherlakota's optimal tax system *always* implies a zero expected marginal asset tax. As we discuss in the body of the paper, results on marginal asset taxes are sensitive to the way in which the tax system uses information on the agents' past history.

The remainder of the paper proceeds as follows. In Section 2, we state the planner's problem and provide a recursive formulation for it that is closely related to that in Atkeson and Lucas (1992). In Section 3, we prove our main implementation result. Section 4 describes the optimal pattern of wedges that characterize constrained–efficient allocations and discusses the implications for taxes in a dynamic setting through a series of revealing examples. We present a numerical analysis of the optimal tax system in the steady state of an infinite period economy in Section 5. In this section, we also compare our optimal tax system with the findings of the static optimal non-linear income taxation literature. Section 6 concludes.

5. This result was first derived by Diamond and Mirrlees (1978) and Rogerson (1985). Golosov, Kocherlakota and Tsyvinski (2003) extend it to a very general setting.

2. THE PLANNER'S PROBLEM IN A FINITE PERIOD ECONOMY

In this section, we describe the planner's problem in a finite period economy. The economy is inhabited by a continuum of agents. These agents have preferences over stochastic sequences $\{c_t, y_t\}_{t=0}^{T}$ of consumption $c_t \in \mathbb{R}_+$ and labour $y_t \in \mathcal{Y} \equiv [0, \overline{y}]$ of the form

$$W_0(\{c_t, y_t\}_{t=0}^{T}) = E\left[\sum_{t=0}^{T} \beta^t [u(c_t) + \theta_t v(y_t)]\right]. \tag{1}$$

We assume that $u : \mathbb{R}_+ \to \mathcal{U} \subset \mathbb{R}$ and $v : \mathcal{Y} \to \mathcal{V} \subset \mathbb{R}$ are continuously differentiable, strictly concave and, respectively, strictly increasing and strictly decreasing functions. The variable $\theta_t \in \Theta \subset \mathbb{R}_+$ denotes an idiosyncratic preference shock. We assume that Θ is a compact set and that the preference shocks are distributed independently over time and across agents with probability distribution π.[6] We define a t-period history to be $\theta^t = (\theta_0, \ldots, \theta_t) \in \Theta^{t+1}$ and denote the corresponding probability distribution by π^t. We assume that the idiosyncratic shocks are *privately observed* by agents. The term $\theta_t v(y_t)$ denotes the disutility from labour at time t. Preference shocks alter the disutility of labour and the marginal rate of substitution between consumption and labour. They may, for example, be interpreted as short-lived shocks to health. We also assume that the production technology converts one unit of labour into one unit of output. The preference shock formulation that we adopt can easily be mapped into one in which agents receive privately observed productivity shocks that perturb their individual marginal rates of transformation of labour into output.

Each agent is identified with an initial lifetime utility promise w_0. Let Ψ_0 denote the distribution over such promises and let $\mathcal{W}_0 \subset \text{Range}(W_0)$ denote its support. It is convenient to state the planner's problem in terms of utility, rather than resource, variables. Given $\mathcal{W}_0$, define a *utility allocation* to be a sequence of functions $z = \{u_t, v_t\}_{t=0}^{T}$ with $u_t : \mathcal{W}_0 \times \Theta^{t+1} \to \mathcal{U}$ and $v_t : \mathcal{W}_0 \times \Theta^{t+1} \to \mathcal{V}$. Here u_t and v_t give the utility obtained by an agent from consumption and labour at date t as a function of that agent's utility promise and shock history. An individual utility allocation for an agent with initial promise w_0 will be denoted $z(w_0)$.[7] Let $C : \mathcal{U} \to \mathbb{R}_+$ denote the inverse of u and $Y : \mathcal{V} \to \mathcal{Y}$ the inverse of v. An individual utility allocation can be mapped into a consumption–labour allocation using the functions C and Y. Denote an agent's continuation utility from the individual utility allocation $z(w_0)$ after history θ^s by

$$U_{s+1}(z(w_0), \theta^s) = E\left[\sum_{t=s+1}^{T} \beta^{t-s-1}[u_t(w_0, \theta^t) + \theta_t v_t(w_0, \theta^t)] | \theta^s\right] \quad s = 0, \ldots, T-1$$

with $U_{T+1}\left(z(w_0), \theta^{T+1}\right) = 0$. Let $\overline{U}_s^{\text{nat}} \equiv \sup_{\varphi \in \mathcal{U}, \varsigma \in \mathcal{V}} \frac{1-\beta^{T+1-s}}{1-\beta}[\varphi + E\theta\varsigma]$ denote an upper bound for U_s.

We now introduce three restrictions on the set of utility allocations available to a planner. First, utility allocations must ensure that agents' utility promises are kept. Formally, they must satisfy the *promise-keeping condition*, for all $w_0 \in \mathcal{W}_0$,

$$w_0 = U_0(z(w_0)). \tag{2}$$

6. We also interpret $\pi(\theta)$ as the fraction of agents receiving the shock θ. In doing so we rely on the argument of Judd (1985).

7. The strict concavity of the problem ensures that it is optimal for a planner to treat all agents with the same utility promise identically. Hence, there is no loss of generality in assuming that agents with the same promise receive the same allocation.

Since agents privately observe their shock histories, we also require allocations to be incentive-compatible. Define a *reporting strategy* δ to be a sequence of functions $\{\delta_t\}_{t=0}^{T}$ with $\delta_t : \Theta^t \to \Theta$. We interpret δ_t as mapping an agent's history of shocks into a report concerning her current shock. Let $z(w_0;\delta)$ denote the composition of the individual utility allocation $z(w_0)$ and the reporting strategy δ. This is also an individual utility allocation. Let $\delta^* = \{\delta_t^*\}_{t=0}^{T}$ denote the *truthful reporting strategy*, where for all t, θ^t, $\delta_t^*(\theta^{t-1},\theta_t) = \theta_t$. We invoke the Revelation Principle and, without loss of generality, require that utility allocations induce agents to be truthful. Thus, we restrict attention to utility allocations z that satisfy the *incentive-compatibility condition*, for all $w_0 \in \mathcal{W}_0$,

$$\forall\delta,\ \ U_0(z(w_0;\delta^*)) \geq U_0(z(w_0;\delta)). \tag{3}$$

We say that a utility allocation $z = \{u_t, v_t\}_{t=0}^{T}$ is *temporarily incentive-compatible* if for all w_0

$$\begin{aligned}\forall t,\theta^{t-1},\theta,\theta',\quad & u_t(w_0,\theta^{t-1},\theta) + \theta v_t(w_0,\theta^{t-1},\theta) + \beta U_{t+1}(z(w_0),\theta^{t-1},\theta)\\ &\geq u_t(w_0,\theta^{t-1},\theta') + \theta v_t(w_0,\theta^{t-1},\theta') + \beta U_{t+1}(z(w_0),\theta^{t-1},\theta').\end{aligned} \tag{4}$$

The latter constraints imply that after each history of shocks, an agent is better off truthfully reporting her shock, rather than lying and being truthful thereafter. Equation (3) clearly implies (4) and, in a finite period setting, the reverse implication is also true.

Finally, we require that allocations maintain the continuation utilities of agents above an exogenous lower bound, $\underline{U}_{t+1} < \overline{U}_{t+1}^{\text{nat}}$ at each date. Thus, utility allocations must satisfy, for all $w_0 \in \mathcal{W}_0$,

$$\forall t \in \{0,\ldots,T-1\},\ \ \theta^t,\ \ U_{t+1}(z(w_0)|\theta^t) \geq \underline{U}_{t+1}. \tag{5}$$

Define $\mathcal{W}_t = [\underline{U}_{t+1},\infty) \cap \text{Range}(U_t)$ to be the set of possible expected period t pay-offs for an agent, and let $\{G_t\}_{t=0}^{T}$ denote a sequence of exogenous planner (or government) consumption levels. The planner's cost objective is given by

$$\begin{aligned}D(z;\{G_t\}_{t=0}^{T},\Psi_0) = &\max_{t\in\{0,\ldots,T\}}\\ &\times\left\{\int_{w_0\in\mathcal{W}_0}\int_{\theta^t\in\Theta^{t+1}}\left[C(u_t(w_0,\theta^t)) - Y(v_t(w_0,\theta^t))\right]d\pi^t d\Psi_0 + G_t\right\}.\end{aligned} \tag{6}$$

The planner minimizes this cost objective subject to the promise-keeping, incentive-compatibility and utility bound constraints:

$$\begin{aligned}\mathcal{C}(\{\underline{U}_{t+1}\}_{t=0}^{T-1},\{G_t\}_{t=0}^{T},\Psi_0) &= \inf_z D(z;\{G_t\}_{t=0}^{T},\Psi_0)\\ \text{subject to:}\quad &\forall w_0,\ (2),\ (4)\ \text{and}\ (5).\end{aligned} \tag{7}$$

If $z^* = \{u_t^*, v_t^*\}_{t=0}^{T}$ attains the infimum in (7) then we will call z^* a *constrained–efficient allocation* (at $(\{\underline{U}_{t+1}\}_{t=0}^{T-1},\{G_t\}_{t=0}^{T},\Psi_0)$). We say that a triple $(\{\underline{U}_{t+1}\}_{t=0}^{T-1},\{G_t\}_{t=0}^{T},\Psi_0)$ is *consistent with resource clearing* if there exists a constrained–efficient allocation at $(\{\underline{U}_{t+1}\}_{t=0}^{T-1}$, $\{G_t\}_{t=0}^{T},\Psi_0)$ that satisfies, for $t \in \{0,\ldots T\}$,

$$\int_{w_0\in\mathcal{W}_0}\int_{\theta^t\in\Theta^{t+1}}\left[C(u_t(w_0,\theta^t)) - Y(v_t(w_0,\theta^t))\right]d\pi^t d\Psi_0 + G_t = 0.$$

We now describe a related economy and invoke a result of Atkeson and Lucas (1992) to establish that equilibrium allocations in this economy are constrained–efficient. Fix a triple

$(\{\underline{U}_{t+1}\}_{t=0}^{T-1}, \{G_t\}_{t=0}^{T}, \Psi_0)$ and, hence, $\{\mathcal{W}_t\}_{t=0}^{T}$. Define a *utility allocation rule* to be a collection of functions $\zeta = \{\{\varphi_t, \varsigma_t, \omega_{t+1}\}_{t=0}^{T-1}, \varphi_T, \varsigma_T\}$ with $\varphi_t : \mathcal{W}_t \times \Theta \to \mathcal{U}$, $\varsigma_t : \mathcal{W}_t \times \Theta \to \mathcal{V}$ and $\omega_{t+1} : \mathcal{W}_t \times \Theta \to \mathcal{W}_{t+1}$. $\varphi_t(w_t, \theta_t)$, $\varsigma_t(w_t, \theta_t)$ and $\omega_{t+1}(w_t, \theta_t)$ represent, respectively, the utility from current consumption, labour supply, and the period $t+1$ utility promise assigned to an agent with current utility promise w_t and shock θ_t. A utility allocation rule recursively induces a utility allocation as follows. Given ζ then for all $w_0 \in \mathcal{W}_0$, t and θ^t, let $u_t(w_0, \theta^t) = \varphi_t(w_t(w_0, \theta^{t-1}), \theta_t)$ and $v_t(w_0, \theta^t) = \varsigma_t(w_t(w_0, \theta^{t-1}), \theta_t)$, where for $t > 0$, $w_t(w_0, \theta^{t-1}) = \omega_t(w_{t-1}(w_0, \theta^{t-2}), \theta_{t-1})$ and $w_0(w_0, \theta^{-1}) = w_0$. Thus, the utility allocation rule uses utility promises to summarize past information. We denote by $z(\zeta)$ the utility allocation induced by ζ. Similarly, let $z(\zeta, w_0)$ denote the individual utility allocation induced by ζ from w_0.

Let $\{q_t\}_{t=0}^{T-1} \in \mathbb{R}_+^T$ denote a sequence of intertemporal prices and suppose a population of *component planners* each matched with a single agent. Assume that the representative component planner chooses a utility allocation rule to solve the following sequence of recursive problems. For $t \in \{0, \dots, T-1\}$,

$$B_t(w_t) = \inf_{\substack{\varphi:\Theta\to\mathcal{U}, \varsigma:\Theta\to\mathcal{V} \\ \omega:\Theta\to\mathcal{W}_{t+1}}} \int_\Theta [C(\varphi(\theta)) - Y(\varsigma(\theta)) + q_t B_{t+1}(\omega(\theta))] d\pi \tag{8}$$

subject to the temporary incentive-compatibility constraint

$$\forall \theta, \theta', \qquad \varphi(\theta) + \theta\varsigma(\theta) + \beta\omega(\theta) \geq \varphi(\theta') + \theta\varsigma(\theta') + \beta\omega(\theta'), \tag{9}$$

and the promise-keeping constraint

$$w_t = \int_\Theta [\varphi(\theta) + \theta\varsigma(\theta) + \beta\omega(\theta)] d\pi. \tag{10}$$

In the terminal period T, the component planner solves

$$B_T(w_T) = \inf_{\substack{\varphi:\Theta\to\mathcal{U} \\ \varsigma:\Theta\to\mathcal{V}}} \int_\Theta [C(\varphi(\theta)) - Y(\varsigma(\theta))] d\pi \tag{11}$$

subject to the temporary incentive-compatibility constraint

$$\forall \theta, \theta', \qquad \varphi(\theta) + \theta\varsigma(\theta) \geq \varphi(\theta') + \theta\varsigma(\theta'), \tag{12}$$

and the promise-keeping constraint

$$w_T = \int_\Theta [\varphi(\theta) + \theta\varsigma(\theta)] d\pi. \tag{13}$$

Denote the utility allocation rule that solves problems (8) and (11) by ζ^*. We assume that such a ζ^* exists and that each B_t is continuous. We provide sufficient conditions for each in Albanesi and Sleet (AS) (2004).

The optimal promise functions $\{\omega_{t+1}^*\}_{t=0}^{T-1}$ and the distribution Ψ_0 induce a sequence of cross sectional utility promise distributions Ψ_{t+1} according to

$$\forall S \in \mathbf{B}(\mathcal{W}_{t+1}): \quad \Psi_{t+1}(S) = \int 1_{\left\{\omega_{t+1}^*(w,\theta) \in S\right\}} d\pi \, d\Psi_t,$$

where $\mathbf{B}(\mathcal{W}_{t+1})$ denotes the Borel subsets of $\mathcal{W}_{t+1}$. Additionally, ζ^* and $\{\Psi_t\}_{t=0}^T$ imply a sequence of aggregate resource costs for $t = 0, \ldots, T$:

$$\int_{\mathcal{W}_t} \int_{\Theta} \left[C(\varphi_t^*(w,\theta)) - Y(\varsigma_t^*(w,\theta))\right] d\pi\, d\Psi_t + G_t.$$

Define a *component planner economy*, denoted $\mathcal{E}^{CP}(\{\underline{U}_{t+1}\}_{t=0}^{T-1}, \{G_t\}_{t=0}^T, \Psi_0)$, to be a continuum of component planners, an initial cross sectional distribution of utility promises Ψ_0, a sequence of continuation utility bounds $\{\underline{U}_{t+1}\}_{t=0}^{T-1}$, a sequence of markets for one period ahead claims to consumption between periods 0 and $T-1$, and a sequence of government consumptions $\{G_t\}_{t=0}^T$. We define an equilibrium of this economy as follows.

Definition 1. A sequence of intertemporal prices $\{q_t\}_{t=0}^{T-1}$, cost functions $\{B_t\}_{t=0}^T$, with $B_t : \mathcal{W}_t \to \mathbb{R}$, and cross sectional distributions of utility promises $\{\Psi_t\}_{t=1}^T$ and a utility allocation rule $\zeta^* = \{\{\varphi_t^*, \varsigma_t^*, \omega_{t+1}^*\}_{t=0}^{T-1}, \varphi_T^*, \varsigma_T^*\}$ are an **equilibrium** of $\mathcal{E}^{CP}(\{\underline{U}_{t+1}\}_{t=0}^{T-1}, \{G_t\}_{t=0}^T, \Psi_0)$ if:

(1) $\{B_t\}_{t=0}^{T-1}$ satisfy (8) and B_T satisfies (11);
(2) $\{\varphi_t^*, \varsigma_t^*, \omega_{t+1}^*\}$ attain the infima in the problems (8). $\{\varphi_T^*, \varsigma_T^*\}$ attain the infima in the problems (11);
(3) $\forall t, S \in \mathbf{B}(\mathcal{W}_{t+1})$, $\Psi_{t+1}(S) = \int 1_{\left\{\omega_{t+1}^*(w,\theta) \in S\right\}} d\pi\, d\Psi_t$;
(4) $\forall t$, $G_t + \int \left[C(\varphi_t^*(w,\theta)) - Y(\varsigma_t^*(w,\theta))\right] d\pi\, d\Psi_t = 0$.

The following lemma links such equilibria to constrained–efficient allocations and motivates our interest in them. Its proof is similar to that of Theorem 1, Atkeson and Lucas (1992) and is omitted.

Lemma 1. *Let ζ^* be an equilibrium utility allocation rule for the economy $\mathcal{E}^{CP}(\{\underline{U}_{t+1}\}_{t=0}^{T-1}, \{G_t\}_{t=0}^T, \Psi_0)$, then $z(\zeta^*)$ is constrained–efficient at $(\{\underline{U}_{t+1}\}_{t=0}^{T-1}, \{G_t\}_{t=0}^T, \Psi_0)$. Additionally, $(\{\underline{U}_{t+1}\}_{t=0}^{T-1}, \{G_t\}_{t=0}^T, \Psi_0)$ is consistent with resource clearing.*

We call such a ζ^* a *constrained-efficient utility allocation rule*. Given such a rule and the associated cost functions $\{B_t\}_{t=0}^T$ and using the fact that each B_t is strictly increasing (see AS, 2004) and, hence, invertible, we define $\mathcal{Y}_t^* : B_t(\mathcal{W}_t) \rightrightarrows \mathcal{Y}$ by

$$\mathcal{Y}_t^*(B_t(w_t)) \equiv \{y : y = Y(\varsigma_t^*(w_t,\theta)) \text{ some } \theta \in \Theta\} \subseteq \mathcal{Y}. \tag{14}$$

Thus, $\mathcal{Y}_t^*(b_t)$ is the set of labour supplies available to an agent with utility promise $B_t^{-1}(b_t)$ at t under ζ^*.

3. IMPLEMENTATION

We now show that a component planner equilibrium, and, hence, the associated constrained–efficient allocation, can be obtained as part of a competitive equilibrium in a market economy with taxes and borrowing constraints. In the market economy agents are endowed with an initial stock of non-contingent claims b_0. They enter each period t with claims b_t, they work y_t, pay taxes and, in periods $t \leq T-1$, they allocate their after-tax income between consumption c_t and purchases of claims b_{t+1}. In the terminal period T, they simply consume all after-tax income. All market trades undertaken by an agent are publicly observable. A government is exogenously assigned the spending levels $\{G_t\}_{t=0}^T$ and administers a tax system $\{T_t\}_{t=0}^T$. The tax system conditions an agent's tax payment in each period *only* on her current labour income y_t and her current claims b_t and not on any other aspect of her past history.

Formally, a *market economy with taxes and borrowing limits*, denoted $\mathcal{E}^{ME}(\{\underline{b}_{t+1}\}_{t=0}^{T-1}, \Lambda_0, \{G_t\}_{t=0}^{T}, \{T_t\}_{t=0}^{T})$, is a sequence of markets for one period ahead claims to consumption that open at each date $t \leq T-1$, a sequence of borrowing limits $\{\underline{b}_{t+1}\}_{t=0}^{T-1}$, an initial cross sectional distribution of claim holdings Λ_0, a sequence of government spending levels $\{G_t\}_{t=0}^{T}$, and a sequence of tax functions $\{T_t\}_{t=0}^{T}$, with $T_t : \mathcal{B}_t \times \mathcal{Y} \to \mathbb{R}$, where for $t > 0$, $\mathcal{B}_t \equiv \left[\underline{b}_t, \infty\right)$ and $\mathcal{B}_0$ denotes the support of Λ_0. We define a market allocation rule to be a sequence of functions $\hat{a} = \{\{\hat{c}_t, \hat{y}_t, \hat{b}_{t+1}\}_{t=0}^{T-1}, \hat{c}_T, \hat{y}_T\}$, with $\hat{c}_t : \mathcal{B}_t \times \Theta \to \mathbb{R}_+$, $\hat{y}_t : \mathcal{B}_t \times \Theta \to \mathcal{Y}$, and $\hat{b}_{t+1} : \mathcal{B}_t \times \Theta \to \mathcal{B}_{t+1}$. The functions $\hat{c}_t(b_t, \theta_t)$, $\hat{y}_t(b_t, \theta_t)$ and $\hat{b}_{t+1}(b_t, \theta_t)$ represent, respectively, consumption, labour supply, and savings at time t of an agent with current wealth b_t and shock θ_t. A competitive equilibrium of the market economy $\mathcal{E}^{ME}(\{\underline{b}_{t+1}\}_{t=0}^{T-1}, \Lambda_0, \{G_t\}_{t=0}^{T}, \{T_t\}_{t=0}^{T})$ is defined as follows.

Definition 2. A sequence of claims prices $\{\hat{q}_t\}_{t=0}^{T-1} \in \mathbb{R}_+^T$, value functions $\{V_t\}_{t=0}^{T}$, with $V_t : \mathcal{B}_t \to \mathbb{R}$, and cross sectional distributions of claim holdings $\{\Lambda_{t+1}\}_{t=0}^{T-1}$ and a market allocation rule $\hat{a} = \{\{\hat{c}_t, \hat{y}_t, \hat{b}_{t+1}\}_{t=0}^{T-1}, \hat{c}_T, \hat{y}_T\}$ is a **competitive equilibrium** of $\mathcal{E}^{ME}(\{\underline{b}_{t+1}\}_{t=0}^{T-1}, \Lambda_0, \{G_t\}_{t=0}^{T}, \{T_t\}_{t=0}^{T})$ if:

(1) for $t \in \{0, \ldots, T-1\}$, V_t and V_{t+1} satisfy

$$V_t(b) = \sup_{\substack{c:\Theta\to\mathbb{R}_+, y:\Theta\to\mathcal{Y},\\ b':\Theta\to\mathcal{B}_{t+1}}} \int [u(c(\theta)) + \theta v(y(\theta)) + \beta V_{t+1}(b'(\theta))] d\pi \tag{15}$$

subject to, for each θ, $b = c(\theta) - y(\theta) + T_t(b, y(\theta)) + \hat{q}_t b'(\theta)$; V_T satisfies

$$V_T(b) = \sup_{c:\Theta\to\mathbb{R}_+, y:\Theta\to\mathcal{Y}} \int [u(c(\theta)) + \theta v(y(\theta))] d\pi \tag{16}$$

subject to, for each θ, $b = c(\theta) - y(\theta) + T_T(b, y(\theta))$;

(2) $\{\hat{c}_t, \hat{y}_t, \hat{b}_{t+1}\}$ attain the suprema in the problems (15). $\{\hat{c}_T, \hat{y}_T\}$ attain the suprema in (16);

(3) $\forall S \in \mathbf{B}(\mathcal{B}_{t+1})$, $\Lambda_{t+1}(S) = \int 1_{\left\{\hat{b}_{t+1}(b,\theta)\in S\right\}} d\pi\, d\Lambda_t$;

(4) $\forall t$, $G_t + \int \left[\hat{c}_t(b, \theta) - \hat{y}_t(b, \theta)\right] d\pi\, d\Lambda_t = 0$.

Given an initial wealth b_0, an equilibrium market allocation rule induces a utility allocation $\hat{z}(\hat{a}, b_0)$ from b_0. We formally define an implementation as follows.

Definition 3. Let z^* be a constrained–efficient allocation at $(\{\underline{U}_{t+1}\}_{t=0}^{T-1}, \Psi_0, \{G_t\}_{t=0}^{T})$. We say that z^* is **implemented** by a competitive equilibrium in a market economy with taxes and borrowing limits $\mathcal{E}^{ME}(\{\underline{b}_{t+1}\}_{t=0}^{T-1}, \Lambda_0, \{G_t\}_{t=0}^{T}, \{T_t\}_{t=0}^{T})$ if:

(1) there exists a measurable function $f : \mathcal{W}_0 \to \mathbb{R}$ such that for each $S \in \mathbf{B}(\mathbb{R})$, $\Lambda_0(S) = \Psi_0(f^{-1}(S))$;

(2) $\mathcal{E}^{ME}(\{\underline{b}_{t+1}\}_{t=0}^{T-1}, \Lambda_0, \{G_t\}_{t=0}^{T}, \{T_t\}_{t=0}^{T})$ has a competitive equilibrium $\xi^{ME} = \{\{q_t\}_{t=0}^{T-1}, \hat{a}, \{V_t\}_{t=0}^{T}, \{\Lambda_{t+1}\}_{t=0}^{T-1}\}$ such that for each $w_0 \in \mathcal{W}_0$, $\hat{z}(\hat{a}, f(w_0)) = z^*(w_0)$.

If z^* can be implemented by a competitive equilibrium ξ^{ME} in a market economy $\mathcal{E}^{ME}(\{\underline{b}_{t+1}\}_{t=0}^{T-1}, \Lambda_0, \{G_t\}_{t=0}^{T}, \{T_t\}_{t=0}^{T})$, then $(\mathcal{E}^{ME}(\{\underline{b}_{t+1}\}_{t=0}^{T-1}, \Lambda_0, \{G_t\}_{t=0}^{T}, \{T_t\}_{t=0}^{T}), \xi^{ME})$ is said to be a **fiscal implementation** of z^*.

The first condition in the definition describes how the initial wealth distribution is set in the market economy. It implies that initial claim holdings will reveal the agent's initial utility promise to the government. The second condition is the central one. It requires that an agent with initial claim holdings of $f(w_0)$ in the market economy chooses the constrained–efficient individual utility allocation $z^*(w_0)$.

3.1. *Implementation in a two period economy*

Although our main fiscal implementation result applies to economies of arbitrary finite length, the key insights are most easily seen in a two period setting, and we will initially focus on this case. Subsequently, we extend our results to time horizons $T > 1$. Our approach is constructive. Given a component planner economy equilibrium, we propose an initial distribution of claims and a candidate equilibrium claims price for the market economy. We then derive a tax function and debt limits under which agents will be able to *afford* the constrained–efficient allocation from the component planner economy. The challenge lies in showing that the agents in the market economy do in fact choose this allocation.

Formally, let $\xi^{CP} = \{q, \zeta^*, \{B_t\}_{t=0}^1, \Psi_1\}$ denote the equilibrium of a two period component planner economy $\mathcal{E}^{CP}(\underline{U}, \{G_t\}_{t=0}^1, \Psi_0)$. We set the candidate equilibrium price in the market economy to be q, as in ξ^{CP}, and set $f = B_0$. We then structure the debt limits and tax system so that in period 0 an agent with wealth $B_0(w_0)$, $w_0 \in \mathcal{W}_0$ can afford to purchase each of the triples $\{C(\varphi_0^*(w_0,\theta)), Y(\varsigma_0^*(w_0,\theta)), B_1(\omega_1^*(w_0,\theta))\}$, $\theta \in \Theta$, while, in period 1, an agent with wealth $B_1(w_1)$, $w_1 \in \mathcal{W}_1$ can afford each pair $\{C(\varphi_1^*(w_1,\theta)),\ Y(\varsigma_1^*(w_1,\theta))\}$, $\theta \in \Theta$.

Under this arrangement, an agent with utility promise w_0 in the component planner economy is endowed with an initial quantity of claims equal to $B_0(w_0)$, the cost to a component planner of delivering w_0. The agent can then afford the constrained–efficient allocation if she saves an amount equal to the component planner's continuation cost. This identification of an agent's wealth with a component planner's costs is natural since the latter give the expected discounted net transfers to an agent under the constrained–efficient allocation. Moreover, if for each $B_0(w_0)$, $w_0 \in \mathcal{W}_0$ and $\theta \in \Theta$, an agent can be induced to save $B_1(\omega_1^*(w_0,\theta))$, then a government attempting to implement the constrained–efficient allocation can use an agent's savings to infer the continuation allocation to which she is entitled. Since the tax function will be designed to induce agents to choose this allocation, it will be essential that taxes depend on wealth and this informational role of wealth will crucially influence how it is taxed.

In period 1, we set the tax functions so that for each $B_1(w_1)$ and $Y(\varsigma_1^*(w_1,\theta))$, $w_1 \in \mathcal{W}_1$ and $\theta \in \Theta$,

$$T_1(B_1(w_1), Y(\varsigma_1^*(w_1,\theta))) = B_1(w_1) + Y(\varsigma_1^*(w_1,\theta)) - C(\varphi_1^*(w_1,\theta)). \tag{17}$$

It then follows that an agent with savings $B_1(w_1)$, $w_1 \in \mathcal{W}_1$ in the market economy can afford each of the period 1 allocations available to an agent with utility promise w_1 under ζ^*. Equation (17) only defines taxes for $\{b, y\} \in \text{Graph } \mathcal{Y}_1^*$. To prevent agents from choosing savings in period 0 outside of $B_1(\mathcal{W}_1) = [B_1(\underline{U}), \infty)$, we simply impose the borrowing limit $\underline{b} = B_1(\underline{U})$ and set $\mathcal{B}_1 = B_1(\mathcal{W}_1)$. We then extend T_1 onto the whole of $\mathcal{B}_1 \times \mathcal{Y}$, by choosing, for each $b = B_1(w_1)$, $w_1 \in \mathcal{W}_1$ and $y \in \mathcal{Y}/\mathcal{Y}_1^*(b)$, $T_1(b, y)$ so that

$$\forall \theta, \quad \varphi_1^*(w_1,\theta) + \theta\varsigma_1^*(w_1,\theta) \geq u(b + y - T_1(b, y)) + \theta v(y). \tag{18}$$

This ensures that no agent with wealth b would choose the labour supply y. The constrained–efficient allocation rule does not prescribe how this should be done, and we have some flexibility in selecting $T_1(b,\cdot)$ over such labour supplies. The procedure in period 0 is analogous, we set the tax function on Graph $\mathcal{Y}_0^*$ so that the agent can afford the relevant constrained–efficient allocations. Specifically, for each $B_0(w_0)$ and $Y(\varsigma_0^*(w_0,\theta))$, $w_0 \in \mathcal{W}_0$ and $\theta \in \Theta$, we set

$$T_0(B_0(w_0), Y(\varsigma_0^*(w_0,\theta))) = B_0(w_0) + Y(\varsigma_0^*(w_0,\theta)) - C(\varphi_0^*(w_0,\theta)) - q B_1(\omega_1^*(w_0,\theta)). \tag{19}$$

As in (18), we set taxes so that $y \in \mathcal{Y}/\mathcal{Y}_0^*(B_0(w_0))$ will not be chosen by an agent with initial wealth $B_0(w_0)$.

We have now defined taxes for all savings and labour supply levels that can occur in the market economy. Do the resulting tax functions succeed in implementing $z(\zeta^*)$? To understand why they might fail, it is useful to compare the sets of allocations available to an agent in the component planner and market economies. Let $\mathcal{Z}^{CP}(w_0)$ denote those utility allocations that an agent with initial promise w_0 can attain by adopting different reporting strategies under ζ^*. Let $\mathcal{Z}^{ME}(B_0(w_0))$ be the set of budget-feasible utility allocations available to an agent with initial wealth $B_0(w_0)$ in the market economy. If for each $w_0 \in \mathcal{W}_0$, $\mathcal{Z}^{CP}(w_0) = \mathcal{Z}^{ME}(B_0(w_0))$ then we would be able to rely on the incentive-compatibility of the constrained–efficient allocation to ensure that $z(\zeta^*)$ is implemented. It follows directly from the affordability of ζ^* in the market economy that $\mathcal{Z}^{CP}(w_0) \subseteq \mathcal{Z}^{ME}(B_0(w_0))$. But, the reverse set inclusion does not hold so that $\mathcal{Z}^{CP}(w_0) \subset \mathcal{Z}^{ME}(B_0(w_0))$.

Since the tax system is able to detect and deter agents from choosing wealth and labour supply pairs outside of Graph $\mathcal{Y}_t^*$, $t = 0, 1$, we can restrict attention in the market economy to those allocations that remain within these sets. However, even with this restriction, there remain utility allocations available to the agent in the market economy that are unavailable to her under ζ^*, and one or more of these may be preferred to the constrained–efficient allocation. These allocations involve an agent selecting a labour supply $y = Y(\varsigma_0^*(w_0, \theta))$ that is constrained–efficient given the history (w_0, θ) and a savings level $B_1(\omega_1^*(w_0', \theta'))$ that is constrained–efficient given some alternative history (w_0', θ'). Since taxes are conditioned on current wealth and labour earnings only, the tax system cannot in period 1 "look back" to the previous period's labour earnings and verify consistency with this period's wealth. Nor in period 0, can it prevent an agent from choosing a savings level that is inconsistent, from the point of view of ζ^*, with that period's labour supply. Despite this, the simple tax system we construct ensures that all allocations in $\mathcal{Z}^{ME}(B_0(w_0))/\mathcal{Z}^{CP}(w_0)$ are inferior to the constrained–efficient one.

Proposition 1 formally establishes the existence of a fiscal implementation for constrained–efficient allocations. The argument, given in the Appendix, relies on our assumptions that an agent's preferences are separable in consumption and labour and shocks are i.i.d. We provide intuition for the proof and discuss the role of our assumptions below.

Proposition 1. *Let $\xi^{CP} = \{q, \zeta^*, \{B_t\}_{t=0}^1, \Psi_1\}$ be an* ***equilibrium*** *of the component planner economy $\mathcal{E}^{CP}(\underline{U}, \{G_t\}_{t=0}^1, \Psi_0)$. Then, the associated constrained–efficient allocation can be* ***implemented*** *by a competitive equilibrium in a market economy with taxes and borrowing limits.*

The main step of the proof involves splitting the period 0 problem of an agent in the market economy and of a planner in the component planner economy into two stages. In the market economy, an agent in the first stage of period 0 selects a labour supply y_0 and an after-tax quantity of resources x. In the second stage, she allocates these resources between current consumption and savings. In period 1 she selects a labour supply and a consumption level. In the component planner economy, the planner first assigns utilities from labour supply ς_0 and interim utility promises d contingent on an agent's initial utility promise and shock report. In the second stage of period 0, she allocates d between utility from consumption φ_0 and a continuation utility promise ω. In period 1 she chooses utilities from labour ς_1 and consumption φ_1 as functions of ω and the agent's reported shock. Let $X^*(d)$ denote the resource cost to the planner of delivering the interim utility promise d to the agent and let $d^*(w_0, \theta)$ denote the optimal interim utility assigned to an agent with initial utility promise w_0 and shock report θ by the component planner. Figures A.1 and A.2 in the Appendix display timelines for both economies.

The proof proceeds by backward induction. We construct the period 1 tax system so that the choice set of an agent with wealth $B_1(w_1)$ in the market economy includes $\{Y(\varsigma_1^*(w_1, \theta)), C(\varphi_1^*(w_1, \theta))\}_{\theta\in\Theta}$. The incentive-compatibility of $\{Y(\varsigma_1^*(w_1, \theta)), C(\varphi_1^*(w_1, \theta))\}$ ensures that it will be chosen when the agent receives the shock θ. We then establish that in the second stage of period 0, an agent with after-tax resources $X^*(d)$ in the market economy chooses the same intertemporal allocation as a component planner with interim utility d. This is the key step of the proof. Finally, we construct the tax function in period 0 so that the choice set of an agent with wealth $B_0(w_0)$ includes $\{Y(\varsigma_0^*(w_0,\theta)), X^*(d^*(w_0,\theta))\}_{\theta\in\Theta}$. As in period 1, the incentive-compatibility of the constrained–efficient labour and resource pairs ensures that they will be chosen.

Our approach relies on the fact that continuation constrained–efficient allocations are measurable with respect to the agent's continuation utility and, hence, the planner's continuation cost. This is not true if shocks are persistent.[8] In addition, the proof of Proposition 1 hinges on a decomposition of period 0 into two stages whose only link is the interim utility promise d or quantity of resources $X^*(d)$ that is passed from the first to the second stage. The decomposition fails if $X^*(d)$ is not a sufficient state variable for the stage 2 intertemporal allocation problem in the market economy. This occurs when, given $X^*(d)$, the agent's intertemporal marginal rate of substitution in the second stage of period 0 depends upon her labour supply or her preference shock in the first stage. The first case occurs when preferences are non-separable in labour and consumption, the second when the preference shock θ is persistent. We briefly discuss the persistent shock case in the concluding remarks.

Remark 1. In our recursive fiscal implementation an agent's constrained–efficient continuation allocation is measurable with respect to her current wealth. Alternatively, we could imagine an implementation such that an agent's continuation allocation is measurable with respect to some other variable s_t where s_t depends on the past market choices of agents. Kocherlakota (2004*a*) proposes such an implementation. In this $s_t = y^{t-1} = \{y_0, \dots, y_{t-1}\}$, the past labour history of the agent, and wealth pays no role in conveying information to the government about the allocation to which the agent is entitled. Thus Kocherlakota considers using tax functions of the form $T_t(y^{t-1}, b_t, y_t)$. He imposes sufficient assumptions on the underlying constrained–efficient allocation to ensure that the tax system can detect deviations to utility allocations outside of $\mathcal{Z}^{CP}(w_0)$.[9] The tax system is then constructed so that the constrained–efficient allocation is affordable and all such deviations deliver an allocation to the agent that is inferior to the constrained–efficient one. Specifically, Kocherlakota restricts the functional form of the tax system to be $T_t^0(y^t) + T_t^1(y^t)b_t$ and chooses the functions $\{T_t^0, T_t^1\}$ to achieve these objectives. As described previously the tax system constructed in the proof of Proposition 1 can only directly detect a subset of deviations to allocations outside of $\mathcal{Z}^{CP}(w_0)$. If an agent deviates to a savings level b'_{t+1} that, given the agent's labour history, is inconsistent with implementation of the constrained–efficient allocation then, under this tax system, she will be induced to choose the continuation allocation of an agent whose constrained–efficient savings level is b'_{t+1}. Proposition 1 shows that this is sufficient for implementation if shocks are i.i.d. and preferences separable between consumption and labour.

Remark 2. An immediate implication of the tax system that we construct in the proof of Proposition 1 is that, for each $B_0(w_0)$, $w_0 \in \mathcal{W}_0$,

$$E[T_0(B(w_0), Y(\varsigma_0^*(w_0,\theta_0)))|w_0] = E[B(w_0) + Y(\varsigma_0^*(w_0,\theta_0)) - X_0^*(d_0^*(w_0,\theta_0))|w_0] = 0. \quad (20)$$

Similarly $E[T_1(B(w_1), Y(\varsigma_1^*(w_1,\theta_1)))|w_1] = 0$. Thus, the tax system is solely redistributive and raises no revenue to finance the government spending levels $\{G_t\}_{t=0}^1$. In our implementation, such

8. See, for example, Fernandes and Phelan (2000).

9. Specifically, he assumes that the constrained efficient consumption allocation is measurable with respect to the history of past (constrained efficient) labour supplies.

spending is financed via an appropriate setting of the initial distribution of claims. In particular, to extract net resources from the population of agents, the government must hold claims against them at date 0.

A straightforward iteration of the argument underlying the proof of Proposition 1 allows us to extend this result to economies of arbitrary finite length. Formally, we have:

Proposition 2. *Let ξ^{CP} be an **equilibrium** of the component planner economy $\mathcal{E}^{CP}(\{\underline{U}_{t+1}\}_{t=0}^{T-1}, \{G_t\}_{t=0}^{T}, \Psi_0)$. Then, the associated constrained–efficient allocation can be **implemented** by a competitive equilibrium in a market economy with taxes and borrowing limits.*

In AS (2004) we provide sufficient conditions for a fiscal implementation to exist in economies of infinite horizon as well.

4. PROPERTIES OF OPTIMAL TAX FUNCTIONS

To derive the properties of the optimal tax function, we proceed in two steps. First, we describe the implications of constrained–efficient allocations for the pattern of wedges between individual and social shadow prices. Since these have been obtained elsewhere we relegate their derivation to AS (2004). We then analyse the implications of these wedges for optimal tax functions.

Constrained–efficient allocations imply a wedge between an agent's shadow price and the social shadow price of labour. We call this the *effort wedge*. Formally, the component planner's first order conditions imply

$$-\theta v'(Y(\varsigma_t^*(w_t,\theta)))/u'(C(\varphi_t^*(w_t,\theta))) \leq 1, \tag{21}$$

with strict inequality when the incentive constraints bind. The L.H.S. of this inequality gives the agent's shadow price. The linear technology ensures that the social shadow price is 1.

The implications of the effort wedge for optimal non-linear income taxation are well known from the static public finance literature (*e.g.* Mirrlees, 1971). If a consumption–labour allocation $(c_t^*(w_t,\theta),\ y_t^*(w_t,\theta)) = (C(\varphi_t^*(w_t,\theta)),\ Y(\varsigma_t^*(w_t,\theta)))$ is to be implemented at date t with the tax function T_t, it must be such that

$$\begin{aligned}(c_t^*(w_t,\theta), y_t^*(w_t,\theta)) &\in \arg\sup_{c,y} u(c)+\theta v(y) \\ \text{s.t. } c+T_t(B_t(w_t),y) &= y+B_t(w_t)-q_{t+1}B_{t+1}(\omega_{t+1}^*(w_t,\theta)).\end{aligned} \tag{22}$$

In particular, if $T_t(B_t(w_t),\cdot)$ is differentiable at $y_t^*(w_t,\theta)$, then

$$\frac{\partial T_t(B_t(w_t), y_t^*(w_t,\theta))}{\partial y} = 1+\frac{\theta v'(y_t^*(w_t,\theta))}{u'(c_t^*(w_t,\theta))} \tag{23}$$

and the marginal income tax at $(B_t(w_t), y_t^*(w_t,\theta))$ equals the effort wedge.

In dynamic models with additively separable preferences there is an additional wedge. If the lower bound on continuation utilities does not bind, then the component planner's intertemporal Euler equation implies the following *inverted Euler equation*:

$$\frac{1}{u'(c_t^*(w_t,\theta))} = \frac{q_t}{\beta}E_{\theta'}\left[\frac{1}{u'(c_{t+1}^*(\omega_{t+1}^*(w_t,\theta),\theta'))}\right], \tag{24}$$

where $c_{t+j}^*(w_{t+j},\theta) = C(\varphi_{t+j}^*(w_{t+j},\theta))$. From Jensen's inequality, we then have

$$q_t \leq \beta E_t\left[\frac{u'(c_{t+1}^*(\omega_{t+1}^*(w_t,\theta),\theta'))}{u'(c_t^*(w_t,\theta))}\right], \tag{25}$$

with strict inequality if the incentive-compatibility constraint binds. Thus, a wedge occurs between the social shadow price of claims q_t and the individual agent's private shadow price $\beta E_{\theta'}[u'(c^*_{t+1}(\omega^*_{t+1}(w_t,\theta),\theta'))]/\, u'(c^*_t(w_t,\theta))$. We refer to this as the *intertemporal wedge*.

The intertemporal wedge was first derived by Diamond and Mirrlees (1978) and Rogerson (1985). Golosov *et al.* (2003) establish that it is present in a very large class of private information economies. This wedge stems from the adverse effect of savings on incentives. Higher saving at t reduces the correlation between an agent's consumption and her labour supply at $t+1$ and, hence, exacerbates the incentive problem at $t+1$. The intertemporal wedge adjusts for this additional marginal social cost of saving. Just as the effort wedge gives rise to positive marginal labour income taxes, the intertemporal wedge provides a rationale for asset taxation. Specifically, implementation of a constrained–efficient consumption and savings allocation $\{c^*_t(w_t,\theta),$ $B_{t+1}(\omega^*_{t+1}(w_t,\theta)),\ c^*_{t+1}(\omega^*_{t+1}(w_t,\theta),\cdot)\}$ requires

$$\begin{aligned}&\{c^*_t(w_t,\theta), B_{t+1}(\omega^*_{t+1}(w_t,\theta)), c^*_{t+1}(\omega^*_{t+1}(w_t,\theta),\cdot)\} \in \arg\max\nolimits_{c_t,b_{t+1},c_{t+1}} u(c_t)+\beta E_t u(c_{t+1})\\ &\text{s.t.} \qquad c_t + T_t(B_t(w_t), y^*_t(w_t,\theta)) = y^*_t(w_t,\theta) + B_t(w_t) - q_t b_{t+1},\end{aligned}$$

and

$$\begin{aligned}\forall\theta',\quad c_{t+1}(\theta') + T_{t+1}(b_{t+1}, y^*_{t+1}(\omega^*_{t+1}(w_t,\theta),\theta')) &= y^*_{t+1}(\omega^*_{t+1}(w_t,\theta),\theta')\\ &\quad + b_{t+1} - q_{t+1}B_{t+2}(\omega^*_{t+2}(\omega^*_{t+1}(w_t,\theta),\theta')).\end{aligned}$$

Hence, assuming that the function T_{t+1} is differentiable in its first argument, T_{t+1} must be consistent with the agent's Euler equation holding at this allocation:

$$\begin{aligned}q_t = \beta E_t\Bigg[&\left(1-\frac{\partial T_{t+1}(B_{t+1}(\omega^*_{t+1}(w_t,\theta)), y^*_{t+1}(\omega^*_{t+1}(w_t,\theta),\theta'))}{\partial b}\right)\\ &\times\frac{u'(c^*_{t+1}(\omega^*_{t+1}(w_t,\theta),\theta'))}{u'(c^*_t(w_t,\theta))}\Bigg].\end{aligned} \tag{26}$$

Condition (26) places a linear restriction upon marginal asset taxes, but it does not, in general, uniquely determine them. In contrast (23) uniquely pins down the marginal labour income tax. A positive intertemporal wedge implies that

$$0 < E_t\left[\frac{\partial T_{t+1}}{\partial b}\frac{u'(c^*_{t+1})}{u'(c^*_t)}\right] = E_t\left[\frac{\partial T_{t+1}}{\partial b}\right]E_t\left[\frac{u'(c^*_{t+1})}{u'(c^*_t)}\right] + \mathrm{Cov}_t\left[\frac{\partial T_{t+1}}{\partial b},\frac{u'(c^*_{t+1})}{u'(c^*_t)}\right]. \tag{27}$$

This decomposition illustrates two ways in which asset taxation can generate an intertemporal wedge. The first is to set a positive expected marginal asset tax $E_t\left[\frac{\partial T_{t+1}}{\partial b}\right] > 0$ so that the expected return on savings is reduced. The second is to set marginal asset taxes so that the after-tax return on savings covaries positively with consumption, generating $\mathrm{Cov}_t\left[\frac{\partial T_{t+1}}{\partial b},\frac{u'(c^*_{t+1})}{u'(c^*_t)}\right] > 0$. This discourages savings in period t by making claims a less effective hedge against period $t+1$ consumption risk. Clearly, a positive covariance requires that $\partial T_{t+1}/\partial b$ depends on y_{t+1} and, hence, that the tax function is additively non-separable in wealth and labour income. By an argument in AS (2004), $c^*_{t+1}(w_{t+1},\cdot)$ and $y^*_{t+1}(w_{t+1},\cdot)$ are monotone decreasing in θ. Consequently, if the cross partial $\partial^2 T_{t+1}/\partial b\partial y$ exists on $\mathcal{Y}^*_{t+1}(B_{t+1}(\omega^*_{t+1}(w_t,\theta)))$ then $\mathrm{Cov}_t\left[\frac{\partial T_{t+1}}{\partial b},\frac{u'(c^*_{t+1})}{u'(c^*_t)}\right] > 0$ implies that $\partial^2 T_{t+1}(B_{t+1}(\omega^*_{t+1}(w_t,\theta)),y)/\partial b\partial y < 0$ for at least some $y \in \mathcal{Y}^*_{t+1}(B_{t+1}(\omega^*_{t+1}(w_t,\theta)))$. Overall and crucially, (27) shows that *a positive intertemporal wedge does not necessarily translate into a positive expected marginal asset tax.*

To explore the respective roles of the expected and the stochastic components of marginal asset taxation in generating the intertemporal wedge, we present two examples in order of complexity of the physical environment. In the first example it is *essential* that marginal asset taxes generate a positive covariance between after-tax returns on assets and labour earnings. Absent this covariance, an agent could increase her lifetime utility with a deviation in which she saves more in period t and reduces her labour supply in period $t+1$, relative to the constrained–efficient allocation. In this example, *the optimal expected marginal asset tax equals zero*. The second example illustrates the link between the dependence of the constrained–efficient labour allocation in period $t+1$ on wealth and a positive expected marginal asset tax.

4.1. *Example 1: Zero expected marginal asset tax*

4.1.1. The component planner's problem. In this example we consider a modified two period component planner economy in which agents neither receive preference shocks nor work in period 0. In period 1, shocks are drawn from $\Theta = \{\underline{\theta}, \overline{\theta}\}$, $\underline{\theta} < \overline{\theta}$, while labour supply choices are made from the discrete set $\mathcal{Y} = \{\underline{y}, \overline{y}\}$, $\underline{y} < \overline{y}$. Define $\Delta y = \overline{y} - \underline{y}$ and $\Delta v = v(\underline{y}) - v(\overline{y})$. We assume that the initial utility promise distribution is degenerate so that $\Psi_0(w_0) = 1$ and $\mathcal{W}_0 = \{w_0\}$. We also assume that there is no lower bound on the continuation utility of agents so that $\mathcal{W}_1 =$ Range (U_1). Let q be an intertemporal price. We choose a pair of, possibly negative, government spending levels, $\{G_t\}_{t=0}^{1}$ to ensure that an equilibrium of the component planner economy with price q exists. In this equilibrium, component planners solve problems of the form

$$\begin{aligned} &\text{Period 0} \quad B_0(w_0) = \inf_{\varphi \in \mathcal{U}, \omega \in \mathcal{W}_1} C(\varphi) + q B_1(\omega) \\ &\text{s.t.} \qquad\qquad w_0 = \varphi + \beta \omega \end{aligned} \tag{28}$$

and

$$\begin{aligned} &\text{Period 1} \quad B_1(w_1) = \inf_{\varphi: \Theta \to \mathcal{U}, \varsigma: \Theta \to \{v(\underline{y}), v(\overline{y})\}} \sum_{\theta \in \Theta} \{C(\varphi(\theta)) - Y(\varsigma(\theta))\} \pi(\theta) \\ &\text{s.t.} \qquad\qquad w_1 = \sum_{\theta \in \Theta} \{\varphi(\theta) + \theta \varsigma(\theta)\} \pi(\theta) \\ &\qquad\qquad\qquad \forall \theta, \theta', \quad \varphi(\theta) + \theta \varsigma(\theta) \geq \varphi(\theta') + \theta \varsigma(\theta'). \end{aligned} \tag{29}$$

Let $\zeta^* = \{\varphi_0^*, \varphi_1^*, \varsigma_1^*, \omega^*\}$ denote a solution to (28) and (29). The following simple lemma summarizes some properties of ζ^* under an assumption on C. We make this assumption throughout the example.

Lemma 2. *Assume that* $\inf_{u \in \mathcal{U}} C(u + \underline{\theta} \Delta v) - C(u) < \Delta y < \sup_{u \in \mathcal{U}} C(u) - C(u - \underline{\theta} \Delta v)$.

(1) *There exists a* $\underline{w}_1$ *such that for* $w_1 < \underline{w}_1$, $\varsigma_1^*(w_1, \overline{\theta}) = v(\overline{y})$ *and for* $w_1 > \underline{w}_1$, $\varsigma_1^*(w_1, \overline{\theta}) = v(\underline{y})$. *Similarly, there exists a* $\overline{w}_1 \geq \underline{w}_1$, *such that for* $w_1 < \overline{w}_1$, $\varsigma_1^*(w_1, \underline{\theta}) = v(\overline{y})$ *and for* $w_1 > \overline{w}_1$, $\varsigma_1^*(w_1, \underline{\theta}) = v(\underline{y})$.

(2) *There exists a* $\underline{w}_0$ *such that for* $w_0 \leq \underline{w}_0$, $\omega^*(w_0) \leq \underline{w}_1$ *and for* $w_0 > \underline{w}_0$, $\omega^*(w_0) > \underline{w}_1$. *Similarly, there exists a* $\overline{w}_0 \geq \underline{w}_0$, *such that for* $w_0 < \overline{w}_0$, $\omega^*(w_0) < \underline{w}_1$ *and for* $w_0 \geq \overline{w}_0$, $\omega^*(w_0) \geq \overline{w}_1$.

Proof. See AS (2004). ‖

It follows that if $w_0 \in (\underline{w}_0, \overline{w}_0)$ then $\varsigma_1^*(\omega^*(w_0), \underline{\theta}) = v(\overline{y})$, $\varsigma_1^*(\omega^*(w_0), \overline{\theta}) = v(\underline{y})$ and the period 1 incentive constraints bind:

$$\varphi_1^*(\omega^*(w_0), \underline{\theta}) - \varphi_1^*(\omega^*(w_0), \overline{\theta}) = \underline{\theta} \left[\varsigma_1^*(\omega^*(w_0), \overline{\theta}) - \varsigma_1^*(\omega^*(w_0), \underline{\theta}) \right] > 0. \tag{30}$$

We assume that this is the case in the remainder. We refer to the effect of changes in w_1 on the continuation labour allocation $\{\varsigma_1^*(w_1, \underline{\theta}), \varsigma_1^*(w_1, \overline{\theta})\}$ as a wealth effect. In this example, the discreteness of the labour supply and shock sets imply that small changes in w_1 have no wealth effects when $w_1 \in (\underline{w}_1, \overline{w}_1)$.

Let $z^* = z(\zeta^*, w_0)$ denote the constrained–efficient utility allocation induced by ζ^* from w_0. Also, let $\alpha^* = \{c_0^*, c_1^*, y_1^*, B_1^*\}$ denote the associated constrained–efficient resource allocation and period 1 planner cost, where $c_0^* = C(\varphi_0^*(w_0))$, $B_1^* = B_1(\omega^*(w_0))$, $y_1^*(\theta) = Y(\varsigma_1^*(\omega^*(w_0), \theta))$ and $c_1^*(\theta) = C(\varphi_1^*(\omega^*(w_0), \theta))$. By assumption, $\mathcal{Y}_1^*(B_1^*) = \mathcal{Y} = \{\underline{y}, \overline{y}\}$. The first order conditions for φ_0^*, φ_1^*, ω^* and the period 1 envelope condition imply the inverted Euler equation:

$$\frac{1}{u'(c_0^*)} = \frac{q}{\beta}\left[\frac{\pi(\overline{\theta})}{u'(c_1^*(\overline{\theta}))} + \frac{\pi(\underline{\theta})}{u'(c_1^*(\underline{\theta}))}\right]. \tag{31}$$

4.1.2. A condition for implementation. Notice that in the planning problem above, *all* agents have identical histories at the beginning of period 1 and all receive the same continuation constrained–efficient allocation in that period. Consequently, in a fiscal implementation, the government does not need to use an agent's wealth or any other variable to infer the particular continuation allocation to which the agent is entitled. As in Section 3, we assume that all agents receive an initial endowment of claims equal to $B_0(w_0)$. We also assume that there are no taxes in period 0 of the market economy.[10] The solution to the component planner's problem (28) and the agent's period 0 budget constraint in the market economy then imply that all agents must save B_1^* if z^* is to be implemented. To induce an agent to make this choice, the implementation needs to ensure that the agent is worse off if she deviates to a wealth level $B_1(w_1) \neq B_1^*$. One way to do this is to follow the procedure in the proof of Proposition 1 and construct a tax system that induces the agent to choose the continuation allocation implied by ζ^* at w_1 if she deviates to $B_1(w_1) \neq B_1^*$. However, there are many other tax systems that can induce agents to choose B_1^* and, hence, z^*. In this section, we show that all such tax systems differentiable in wealth satisfy a striking condition; they imply a zero expected marginal asset tax at B_1^*.

Define a candidate set of tax functions $\mathcal{T}$ equal to all functions $T : \mathbb{R} \times \mathcal{Y} \to \mathbb{R}$, differentiable in their first argument and satisfying

$$T(B_1^*, y_1^*(\theta)) = B_1^* + y_1^*(\theta) - c_1^*(\theta), \tag{32}$$

and

$$qu'(c_0^*) = \beta E\left[\left(1 - \frac{\partial T}{\partial b}(B_1^*, y_1^*)\right) u'(c_1^*)\right]. \tag{33}$$

Incentive-compatibility of the constrained–efficient allocation then immediately guarantees that *if* the agent saves B_1^*, she will choose the constrained–efficient labour supply:

$$\overline{y} \in \arg\max_{y \in \{\underline{y}, \overline{y}\}} u(B_1^* - T(B_1^*, y) + y) + \underline{\theta} v(y), \tag{34}$$

$$\underline{y} \in \arg\max_{y \in \{\underline{y}, \overline{y}\}} u(B_1^* - T(B_1^*, y) + y) + \overline{\theta} v(y). \tag{35}$$

Thus, all tax systems in $\mathcal{T}$ are consistent with the agent's first order conditions holding at the constrained–efficient allocation. Coupled with the initial wealth endowment $B_0(w_0)$ all of these tax systems render the constrained–efficient allocation affordable for the agent. A very natural candidate tax system is one that is additively separable in labour income and savings: $\hat{T}(b, y) \equiv \hat{T}_0(y) + \hat{T}_1 b$, with $\hat{T}_1$ chosen to satisfy

$$qu'(c_0^*) = \beta(1 - \hat{T}_1)E\left[u'(c_1^*)\right], \tag{36}$$

10. These assumptions provide a convenient normalization of the *level* of taxes and savings. The characterization of marginal asset tax rates that we give below does not depend in any essential way on this normalization.

and $\hat{T}_0$ set so that $\hat{T}_0(y_1^*(\theta)) = (1-\hat{T}_1)B_1^* + y_1^*(\theta) - c_1^*(\theta)$. As we show in the following lemma, this tax system fails to implement the desired allocation.

Lemma 3. *z^* cannot be implemented in a market economy with the tax function $\hat{T}_0(y) + \hat{T}_1 b$.*

Proof. Assume instead that z^* can be implemented with the tax function $\hat{T}(b, y) = \hat{T}_0(y) + \hat{T}_1 b$. The binding incentive-compatibility constraint (30) in the component planner's problem implies that an agent can obtain a lifetime expected utility equal to that from z^* by choosing an alternative allocation in which she saves B_1^*, and selects $\underline{y}$ and $c_1^*(\overline{\theta})$ in all states in period 1. The binding incentive-compatibility constraint also implies $c_1^*(\overline{\theta}) < c_1^*(\underline{\theta})$. It follows that

$$qu'(c_0^*) = \beta(1-\hat{T}_1)E[u'(c_1^*)] < \beta(1-\hat{T}_1)u'(c_1^*(\overline{\theta})).$$

Thus, z^* is dominated by an allocation in which the agent saves $B_1^* + \varepsilon$ (for $\varepsilon > 0$ and small) and chooses $\underline{y}$ regardless of her shock. This contradicts the optimality of z^* for the agent in the market economy. ||

Why does this tax function fail? Although it satisfies the first order conditions of the agent at the constrained–efficient allocation, it permits a complementarity between saving and effort. To see this, define F by

$$F(b_1, y(\underline{\theta}), y(\overline{\theta}); T) = u(B_0(w_0) - qb_1) + \beta E[u(b_1 - T(b_1, y) + y) + \theta v(y)].$$

F gives the agent's pay-off in the market economy in terms of her savings and labour allocation. The discrete approximation to the cross partial of F in $(b_1, y(\underline{\theta}))$ at the constrained–efficient allocation is

$$\begin{aligned}\frac{\Delta}{\Delta y}\left[\frac{\partial F}{\partial b}\right](\hat{T}) &= \frac{1}{\underline{y}-\overline{y}}\left\{\frac{\partial F}{\partial b}(B_1^*, \underline{y}, \underline{y}; \hat{T}) - \frac{\partial F}{\partial b}(B_1^*, \overline{y}, \underline{y}; \hat{T})\right\}\\ &= \frac{1}{\underline{y}-\overline{y}}\beta(1-\hat{T}_1)\{u'(c_1^*(\overline{\theta})) - u'(c_1^*(\underline{\theta}))\}\pi(\underline{\theta}) < 0.\end{aligned}$$

This complementarity then underpins a profitable joint deviation for the agent in which she saves too much in period 0 and works too little in period 1.[11]

$\hat{T}$ is the only additively separable tax function that renders the constrained–efficient allocation affordable and that satisfies the first order conditions of the agent in the market economy at this allocation. Since it fails to implement the constrained–efficient allocation, it follows that no optimal tax function can be additively separable and, in particular, that marginal asset taxes must depend on labour income. Lemma 4 obtains a sharper characterization of marginal asset taxes. In this lemma, we show that optimal marginal asset taxes must be chosen so that they equate the marginal value of an extra unit of savings *across all states*:

$$qu'(c_0^*) = \beta\left(1-\frac{\partial T}{\partial b}(B_1^*, \underline{y})\right)u'(c_1^*(\overline{\theta})),\ qu'(c_0^*) = \beta\left(1-\frac{\partial T}{\partial b}(B_1^*, \overline{y})\right)u'(c_1^*(\underline{\theta})). \tag{37}$$

11. This finding is related to Golosov and Tsyvinski (2003*a*) who consider the design of optimal disability insurance. They show that disability benefits must be made contingent on an age-dependent asset level.

We refer to equations such as (37) as *state-by-state Euler equations*. They imply that regardless of her choice of labour in period 1, the agent does not find it profitable to make a small adjustment to her savings. They also imply that the complementarity term is set to zero:

$$\frac{\Delta}{\Delta y}\left[\frac{\partial F}{\partial b}\right](T) = \frac{1}{\underline{y}-\overline{y}}\beta\left\{\left(1-\frac{\partial T}{\partial b}(B_1^*,\underline{y})\right)u'(c_1^*(\overline{\theta}))-\left(1-\frac{\partial T}{\partial b}(B_1^*,\overline{y})\right)u'(c_1^*(\underline{\theta}))\right\}\pi(\underline{\theta})$$
$$= 0.$$

Finally, as we show, they imply marginal asset taxes that have a negative covariance with consumption and that are zero on average.

Lemma 4. *Any tax function T differentiable in b that implements z^* must satisfy*

$$qu'(c_0^*) = \beta\left(1-\frac{\partial T}{\partial b}(B_1^*,\underline{y})\right)u'(c_1^*(\overline{\theta})),\ qu'(c_0^*) = \beta\left(1-\frac{\partial T}{\partial b}(B_1^*,\overline{y})\right)u'(c_1^*(\underline{\theta})).$$

Moreover, all such tax functions satisfy:

(1) $\frac{\partial T}{\partial b}(B_1^*,\underline{y}) > \frac{\partial T}{\partial b}(B_1^*,\overline{y})$;
(2) $\frac{\partial T}{\partial b}(B_1^*,\underline{y})\pi(\overline{\theta}) + \frac{\partial T}{\partial b}(B_1^*,\overline{y})\pi(\underline{\theta}) = 0.$

Proof. See Appendix. ||

The tax system constructed in the proof of Proposition 1 satisfies the conditions of Lemma 4. To check this directly, recall that for all pairs in Graph $\mathcal{Y}_1^*$ this tax system satisfies

$$T(B_1(w_1), Y(\varsigma_1^*(w_1,\theta))) = B_1(w_1) + Y(\varsigma_1^*(w_1,\theta)) - C(\varphi_1^*(w_1,\theta)). \tag{38}$$

For $w_1 \in (\underline{w}_1, \overline{w}_1)$, there are no local wealth effects on labour supply, *i.e.* $\partial\varsigma_1^*(w_1,\theta)/\partial w_1 = 0$. It then follows from the component planner's period 1 incentive and promise-keeping constraints that $\varphi_1^*(w_1,\theta) = w_1 + K(\theta)$, where $K(\theta)$ is independent of w_1. Thus, for each $(\theta, w_1) \in \Theta \times (\underline{w}_1, \overline{w}_1)$, $\partial\varphi_1^*(w_1,\theta)/\partial w_1 = 1$. Totally differentiating (38) evaluated at $(\theta, w_1) \in \Theta \times (\underline{w}_1, \overline{w}_1)$, with respect to w_1 then gives[12]

$$\frac{\partial T}{\partial b}(B_1(w_1), Y(\varsigma_1^*(w_1,\theta))) = 1 - C'(\varphi_1^*(w_1,\underline{\theta}))\frac{1}{B_1'(w_1)}. \tag{39}$$

Also, for an initial $w_0 \in (\underline{w}_0, \overline{w}_0)$, the component planner's period 0 first order condition implies

$$C'(\varphi_1^*(\omega^*(w_0),\theta))\frac{1}{B_1'(\omega^*(w_0))} = \frac{q}{\beta}\frac{u'(c_0^*)}{u'(c_1^*(\theta))}. \tag{40}$$

Combining (39) and (40) gives the state-by-state Euler equations (37). It then follows from the argument underlying the last part of Lemma 4 that the expected marginal asset tax $E_\theta[\frac{\partial T(B_1^*, y_1^*(\theta))}{\partial b}]$ is zero.

This example can be generalized by allowing the initial distribution of utility promises Ψ_0 to be non-degenerate. If we retain the assumption that $\mathcal{W}_0 \subset (\underline{w}_0, \overline{w}_0)$, then, in the component

12. We show that B_1 is differentiable on $(\underline{w}_1, \overline{w}_1)$ in AS (2004).

planning problem, agents with different initial utility promises receive the same constrained–efficient labour allocation, but different constrained–efficient consumption allocations in period 1. The implementation must now be such that the tax function keeps track of sufficient information about an agent's past to permit it to induce the appropriate consumption allocation for the agent in period 1. As Proposition 1 shows, an agent's wealth is sufficient information and the tax function constructed in the proof of that proposition succeeds in implementing this constrained–efficient allocation. It may be shown, once again, that this tax function exhibits zero expected marginal asset taxes across the range of savings chosen by agents in period 0.

4.2. *Example 2: Positive expected marginal asset taxes*

4.2.1. Component planner's problem. We now alter Example 1 by setting Θ equal to $[\underline{\theta},\overline{\theta}]$ and by assuming that π admits a strictly positive density ρ on this interval. Thus, in period 1, the component planner solves

$$B_1(w_1)=\inf_{\varphi:\Theta\to\mathcal{U},\varsigma:\Theta\to\{\upsilon(\underline{y}),\upsilon(\overline{y})\}}\int_{\underline{\theta}}^{\overline{\theta}}\{C(\varphi(\theta))-Y(\varsigma(\theta))\}\rho(\theta)d\theta$$

$$\text{s.t.}\qquad w_1=\int_{\underline{\theta}}^{\overline{\theta}}\{\varphi(\theta)+\theta\varsigma(\theta)\}\rho(\theta)d\theta,$$

$$\forall\theta,\theta',\quad \varphi(\theta)+\theta\varsigma(\theta)\ge\varphi(\theta')+\theta\varsigma(\theta').$$

We keep the period 0 component planner problem the same as in Example 1. In particular, we assume that $\Psi_0(\{w_0\})=1$ for some w_0.

The incentive-compatibility constraints for the period 1 component planner problem imply that any feasible ς is decreasing and that if $\varsigma(\theta)=\varsigma(\theta')$, then $\varphi(\theta)=\varphi(\theta')$, $\forall\,\theta,\theta'\in\Theta$. Thus, this problem can be rewritten as follows:

$$B_1(w_1)=\inf_{\substack{\{\underline{\varphi},\overline{\varphi}\in\mathcal{U},\\ \hat{\theta}\in[\underline{\theta},\overline{\theta}]\}}}[\{C(\overline{\varphi})-Y(\overline{\upsilon})\}\Pi(\hat{\theta})+\{C(\underline{\varphi})-Y(\underline{\upsilon})\}(1-\Pi(\hat{\theta}))]$$

$$\text{s.t.}\qquad w_1=\left\{\overline{\varphi}\Pi[\hat{\theta}]+\overline{\upsilon}\int_{\underline{\theta}}^{\hat{\theta}}\theta\rho(\theta)d\theta\right\}+\left\{\underline{\varphi}(1-\Pi[\hat{\theta}])+\underline{\upsilon}\int_{\hat{\theta}}^{\overline{\theta}}\theta\rho(\theta)d\theta\right\},$$

$$\overline{\varphi}=\underline{\varphi}+\hat{\theta}\Delta\upsilon,$$

where $\Pi[\hat{\theta}]=\int_{\underline{\theta}}^{\hat{\theta}}\rho(\theta)d\theta$, $\underline{\upsilon}=\upsilon(\underline{y})$, $\overline{\upsilon}=\upsilon(\overline{y})$ and $\Delta\upsilon=\underline{\upsilon}-\overline{\upsilon}>0$. In this problem, the component planner chooses a cut-off value for shocks $\hat{\theta}$ and a utility allocation of the form: $\{\varphi(\theta),\varsigma(\theta)\}=\{\overline{\varphi},\overline{\upsilon}\}$, $\theta<\hat{\theta}$ and $\{\varphi(\theta),\varsigma(\theta)\}=\{\underline{\varphi},\underline{\upsilon}\}$, $\theta\ge\hat{\theta}$. Let $\{\underline{\varphi}_1^*,\overline{\varphi}_1^*,\hat{\theta}_1^*\}$ denote optimal choices of $\underline{\varphi}$, $\overline{\varphi}$ and $\hat{\theta}$ as functions of the component planner's period 1 utility promise w_1. We assume that $\hat{\theta}_1^*(\omega^*(w_0))\in(\underline{\theta},\overline{\theta})$. We now refer to the effect of w_1 on $\hat{\theta}_1^*$ as a wealth effect. It summarizes the consequences of different w_1 values for the constrained–efficient labour allocation. The following lemma gives a sufficient condition for the wealth effect to be negative. We assume that this condition holds for the remainder of the example. It is satisfied whenever u is CARA or CRRA with coefficient of relative risk aversion greater than or equal to $1/2$.

Lemma 5. *If $C'''>0$, then* $\frac{\partial\hat{\theta}_1^*}{\partial w_1}\le 0$.

Proof. See AS (2004). ||

Let $\underline{c}_1^* = C(\underline{\varphi}_1^*(\omega^*(w_0)))$, $\overline{c}_1^* = C(\overline{\varphi}_1^*(\omega^*(w_0)))$. To economize on notation we also use $\hat{\theta}_1^*$ without an argument to denote $\hat{\theta}_1^*(\omega^*(w_0))$ below.

4.2.2. Implementation. Without loss of generality, we assume that agents in the market economy also work $\overline{y}$ when their shock is below a critical $\hat{\theta}$ and $\underline{y}$ when it is above this level. We restrict attention to implementations that rely on a period 1 tax function that depends on wealth and labour income and that is differentiable in wealth. An agent's budget constraint may then be used to re-express her preferences as a function of a savings level b_1 and a threshold shock $\hat{\theta}$:

$$\begin{aligned} F(b_1,\hat{\theta}) &= u(B_0(w_0)-qb_1)+\beta u(b_1+\overline{y}-T(b_1,\overline{y}))\Pi(\hat{\theta}) \\ &\quad +\beta\{u(b_1+\underline{y}-T(b_1,\underline{y}))(1-\Pi(\hat{\theta}))-\beta\Delta v\int_{\underline{\theta}}^{\hat{\theta}}\theta\rho(\theta)d\theta+\beta v(\underline{y}). \end{aligned} \tag{41}$$

Successful implementation requires a tax function that induces agents to choose $b_1 = B_1^*$ and $\hat{\theta} = \hat{\theta}_1^*$. This tax function must be such that the agent's first order conditions hold at $(B_1^*, \hat{\theta}_1^*)$:

$$\begin{aligned} \frac{\partial F(B_1^*,\hat{\theta}_1^*)}{\partial b_1} &= -qu'(c_0^*)+\beta\left\{\left(1-\frac{\partial T}{\partial b}(B_1^*,\overline{y})\right)u'(\overline{c}_1^*)\Pi(\hat{\theta}_1^*)\right. \\ &\quad \left.+\left(1-\frac{\partial T}{\partial b}(B_1^*,\underline{y})\right)u'(\underline{c}_1^*)(1-\Pi(\hat{\theta}_1^*))\right\}=0, \end{aligned}$$

and

$$\frac{\partial F(B_1^*,\hat{\theta}_1^*)}{\partial\hat{\theta}} = [\{u(\overline{c}_1^*)+\hat{\theta}_1^* v(\overline{y})\}-\{u(\underline{c}_1^*)+\hat{\theta}_1^* v(\underline{y})\}]\rho(\hat{\theta}_1^*)=0.$$

The second derivatives of F evaluated at $(B_1^*, \hat{\theta}_1^*)$ are

$$\begin{aligned} \frac{\partial^2 F(B_1^*,\hat{\theta}_1^*)}{\partial b_1^2} &= q^2u''(c_0^*)+\beta\left\{\left(1-\frac{\partial T}{\partial b}(B_1^*,\overline{y})\right)^2 u''(\overline{c}_1^*)\Pi(\hat{\theta}_1^*)\right. \\ &\quad \left.+\left(1-\frac{\partial T}{\partial b}(B_1^*,\underline{y})\right)^2 u''(\underline{c}_1^*)(1-\Pi(\hat{\theta}_1^*))\right\}<0, \end{aligned} \tag{42}$$

and

$$\frac{\partial^2 F(B_1^*,\hat{\theta}_1^*)}{\partial\hat{\theta}^2} = -\beta\Delta v\rho(\hat{\theta}_1^*)<0. \tag{43}$$

The cross partial of the objective is

$$\frac{\partial^2 F(B_1^*,\hat{\theta}_1^*)}{\partial b_1\partial\hat{\theta}} = \beta\left\{\left(1-\frac{\partial T}{\partial b}(B_1^*,\overline{y})\right)u'(\overline{c}_1^*)-\left(1-\frac{\partial T}{\partial b}(B_1^*,\underline{y})\right)u'(\underline{c}_1^*)\right\}\rho(\hat{\theta}_1^*). \tag{44}$$

This last expression can be interpreted as implying that locally around their constrained–efficient values, there is a complementarity between saving b_1 and the threshold shock $\hat{\theta}$ in the market economy. This complementarity is only eliminated when $\left(1-\frac{\partial T}{\partial b}(B_1^*,\overline{y})\right)u'(\overline{c}_1^*) = \left(1-\frac{\partial T}{\partial b}(B_1^*,\underline{y})\right)u'(\underline{c}_1^*) = \frac{q}{\beta}u'(c_0^*)$. In other words it is only eliminated when marginal asset taxes satisfy the state-by-state Euler equations. The determinant of the Hessian for the agent's market economy problem evaluated at the constrained–efficient allocation is

$$|H| = -\left[q^2 u''(c_0^*) + \beta\left\{\left(1-\frac{\partial T}{\partial b}(B_1^*,\overline{y})\right)^2 u''(\overline{c}_1^*)\Pi(\hat{\theta}_1^*)\right.\right.$$
$$\left.\left.+\left(1-\frac{\partial T}{\partial b}(B_1^*,\underline{y})\right)^2 u''(\underline{c}_1^*)(1-\Pi(\hat{\theta}_1^*))\right\}\right]\{\beta\Delta v\rho(\hat{\theta}_1^*)\}$$
$$-\left[\beta\left\{\left(1-\frac{\partial T}{\partial b}(B_1^*,\overline{y})\right)u'(\overline{c}_1^*)-\left(1-\frac{\partial T}{\partial b}(B_1^*,\underline{y})\right)u'(\underline{c}_1^*)\right\}\rho(\hat{\theta}_1^*)\right]^2.$$

$|H| \geq 0$ is a second order necessary condition for the constrained–efficient allocation to be optimal in the market economy. This condition requires that the complementarity term in the final line not be too large.[13] In contrast to Example 1, it does not require that the complementarity term be zero.

One tax function that succeeds in implementing the constrained–efficient allocation, and that must therefore satisfy the above first and second order necessary conditions is that constructed in Proposition 1. Recall once again that this is constructed using the entire utility allocation rule, with for each w_1,

$$T(B_1(w_1),\underline{y}) = B_1(w_1)+\underline{y}-C(\underline{\varphi}_1^*(w_1)), \tag{45}$$
$$T(B_1(w_1),\overline{y}) = B_1(w_1)+\overline{y}-C(\overline{\varphi}_1^*(w_1)). \tag{46}$$

Totally differentiating these equations with respect to w_1 and using the fact that $\overline{\varphi}_1^*(w_1) = \underline{\varphi}_1^*(w_1)+\hat{\theta}_1^*(w_1)\Delta v$ and that, under our assumption, $\partial\hat{\theta}_1^*/\partial w_1 < 0$,

$$\begin{aligned}\left(1-\frac{\partial T}{\partial b_1}(B_1(w_1),\overline{y})\right)u'(C(\overline{\varphi}_1^*(w_1))) &= \frac{1}{B_1'(w_1)}\left\{\frac{\partial\underline{\varphi}_1^*}{\partial w_1}(w_1)+\frac{\partial\hat{\theta}_1^*}{\partial w_1}(w_1)\Delta v\right\}\\ &< \frac{1}{B_1'(w_1)}\frac{\partial\underline{\varphi}_1^*}{\partial w_1}(w_1)\\ &= \left(1-\frac{\partial T}{\partial b_1}(B_1(w_1),\underline{y})\right)u'(C(\underline{\varphi}_1^*(w_1))).\end{aligned} \tag{47}$$

Thus, the state-by-state Euler equations no longer hold. Moreover, in this case, the size of the expected marginal asset tax rate is linked to the size of wealth effects on the efficient labour allocation. Under the assumption that these wealth effects are negative, the expected marginal asset tax is positive. To see why, first recall that the tax function constructed in Proposition 1 associates different savings levels to different utility promises and, hence, to different efficient continuation allocations. Thus, under the assumption that the wealth effect is negative, $\frac{\partial\hat{\theta}_1^*}{\partial w_1} < 0$, agents with different savings levels are induced to choose different labour allocations in period 1. Let $\widetilde{\theta}_1^*(b)$ denote the agent's choice of shock threshold in the market economy as a function of her period 1 wealth. The reproduction of negative wealth effects in the market economy requires that $\frac{\partial\widetilde{\theta}_1^*}{\partial b}(B_1(w_1)) = -\frac{\partial^2 F(B_1(w_1),\widetilde{\theta}_1^*(B_1(w_1)))}{\partial b_1\partial\hat{\theta}}\Big/\frac{\partial^2 F(B_1(w_1),\widetilde{\theta}_1^*(B_1(w_1)))}{\partial\hat{\theta}^2} < 0$ or, equivalently, that $\frac{\partial^2 F(B_1(w_1),\widetilde{\theta}_1^*(B_1(w_1)))}{\partial b_1\partial\hat{\theta}} < 0$. Combining this inequality, equation (44) and the agent's Euler

13. Kocherlakota (2004*b*) analyses a model of unemployment insurance with hidden search effort and hidden saving. He identifies a complementarity between these variables similar to that between b_1 and $\hat{\theta}$. He shows how this complementarity can preclude an application of the first order approach to such contracting problems.

equation gives

$$\left(1-\frac{\partial T}{\partial b_1}(B_1^*,\overline{y})\right)u'(\overline{c}_1^*)=\frac{q}{\beta}u'(c_0^*)+k_1$$
$$\left(1-\frac{\partial T}{\partial b_1}(B_1^*,\underline{y})\right)u'(\underline{c}_1^*)=\frac{q}{\beta}u'(c_0^*)+k_2$$

where $k_1\Pi(\hat{\theta}_1^*)+k_2(1-\Pi(\hat{\theta}_1^*))=0$ and $k_2>k_1$ from (47). But then, using the component planner's first order conditions and the fact that $\overline{c}_1^*>\underline{c}_1^*$:

$$E\left(\frac{\partial T}{\partial b_1}(B_1^*,y)\right)=1-\left\{\frac{q}{\beta}u'(c_0^*)+k_1\right\}\frac{\Pi(\hat{\theta}_1^*)}{u'(\overline{c}_1^*)}-\left\{\frac{q}{\beta}u'(c_0^*)+k_2\right\}\frac{1-\Pi(\hat{\theta}_1^*)}{u'(\underline{c}_1^*)}$$
$$=-\left[k_1\frac{\Pi(\hat{\theta}_1^*)}{u'(\overline{c}_1^*)}+k_2\frac{1-\Pi(\hat{\theta}_1^*)}{u'(\underline{c}_1^*)}\right]>0.$$

4.3. *Utility bounds and borrowing limits*

The previous examples focus on the implications of the incentive-compatibility constraint for the tax system. The lower bound on the continuation utilities in the component planning problems also has implications both for taxes and the structure of asset markets. The argument in the proof of Proposition 1 implies that this lower bound can be implemented with a lower bound on an agent's claim holdings. In the implementation of Section 3, an agent with utility promise w_t and shock θ is induced to save $B_{t+1}(\omega_t^*(w_t,\theta))$, the corresponding continuation cost of a component planner. If the component planner is restricted to making utility promises in excess of $\underline{U}_{t+1}$, then this implementation requires agents to hold claims in excess of $\underline{b}_{t+1}=B_{t+1}(\underline{U}_{t+1})$. This borrowing limit will bind on those agents with low after-tax resources, $x_t=b_t+y_t-T_t(b_t,y_t)$, in period t.

A binding borrowing limit has implications for both the agents' intertemporal Euler equations and the intertemporal wedge. Letting $\hat{\eta}_{t+1}$ denote the multiplier on the borrowing limit the intertemporal wedge can be decomposed as

$$\underbrace{\frac{\beta E_t u'(c_{t+1})-q_t u'(c_t)}{\beta E_t u'(c_{t+1})}}_{\text{Intertemporal wedge}}=\underbrace{\frac{E_t\left[\frac{\partial T_{t+1}}{\partial b}(b_{t+1},y_{t+1})\,u'(c_{t+1})\right]}{E_t u'(c_{t+1})}}_{\text{Tax component}}-\underbrace{\frac{\hat{\eta}_{t+1}}{E_t u'(c_{t+1})}}_{\text{Limit component}}. \tag{48}$$

The first component is induced by the tax system, the second by the multiplier on the borrowing limit. Clearly, the second component is only present when the borrowing limit binds. However, the lower utility bound in the planner's problem also has implications for the optimal tax system. Specifically, the bound restricts the planner's ability to use continuation utilities to provide incentives for truthful revelation. Thus, the planner must rely more heavily on variations in current consumption to provide incentives. Close to the lower bound, the constrained–efficient allocation will then exhibit greater consumption variability and larger effort wedges. These characteristics translate into greater curvature of optimal tax functions at wealth levels close to the borrowing limit.

5. NUMERICAL ANALYSIS

To shed further light on the properties of the optimal tax system, we turn to numerical examples. We set parameters according to recent calibrations of Bewley economies with endogenous labour supply. However, our examples are intended to be illustrative rather than a fully calibrated quantitative exercise. For reasons of space, we only report one example in detail below. However, we indicate those properties that are robust across other examples that we have computed.

5.1. *Calibration and numerical procedure*

We adopt the utility function

$$U(c,y;\theta)=\kappa\frac{c^{1-\sigma}}{1-\sigma}+(1-\kappa)\frac{(\overline{y}-\theta y)^{1-\gamma}}{1-\gamma}. \tag{49}$$

Here, θ may be interpreted as a cost of effort shock.[14] This preference specification is common in macroeconomics.[15]

The numerical parameters for this economy are $\left\{\kappa,\sigma,\gamma,\overline{y},\beta,\underline{U},\Theta,\pi,\{G_t\}_{t=0}^{\infty}\right\}$. For our benchmark case, we follow Heathcote, Storesletten and Violante (2003) in setting the preference parameters $(1-\kappa)/\kappa$ to 1·184, $\overline{y}$ to 1, σ to 1·461, γ to 2·54. This parameterization implies that their model matches the fraction of time devoted to labour and the wage-hours correlation for the U.S. It implies a Frisch elasticity of labour supply of 0·3. In addition, we set β to 0·90. In the benchmark case, we assume that $1/\theta$ is distributed uniformly on the interval $[0{\cdot}2, 1{\cdot}2]$. We set $\underline{U}$ to $-3{\cdot}48$, which translates into a borrowing limit of $-2{\cdot}14$. This value of $\underline{U}$ lies between the lifetime utility that an agent would attain if she were at her "natural" debt limit[16] in a Bewley economy without taxes, which is clearly $-\infty$, and the lifetime utility under autarky without taxes and markets, equal to $-2{\cdot}74$. Government consumption is constant over time and equal to 0·1 in each period, which amounts to approximately 30% of steady state aggregate output.

We numerically solve for the steady state of a component planner economy.[17] In the steady state, the price of claims is constant at q, the component planner's cost function B and optimal policy functions, $\{\varphi^*,\varsigma^*,\omega^*\}$ are time invariant, and the cross sectional distribution of utility promises, Ψ, is a fixed point of the Markov operator implied by ω^*. Our algorithm solves the recursive component planner problem using numerical dynamic programming techniques at each intertemporal price. We use the policy functions from this problem to obtain an approximation to the limiting distribution over utility promises. We iterate on the intertemporal price until this distribution is consistent with resource feasibility. Finally, we construct the time invariant tax function $T(b,y)$ on Graph $\mathcal{Y}^*$, where $\mathcal{Y}^*(b)=\{y:y=Y(\varsigma^*(B^{-1}(b),\theta)),\ \theta\in\Theta\}$, from the solution to the component planning problem using the procedure of the proof of Proposition 1.

5.2. *Numerical results*

The optimal tax function T for the benchmark parameterization is illustrated in Figure 1 on the set Graph $\mathcal{Y}^*$. Taxes are negative at low wealth and low labour income levels. Across the whole of its domain, the cross partial of the tax function, $\frac{\partial^2 T(b,y)}{\partial b\partial y}$, is negative. This is especially marked, and

14. θ can also be interpreted as the reciprocal of a productivity shock. Then, y should be interpreted as the agent's output.

15. These preferences retain the key property of additive separability in consumption and labour. They drop the inessential property of multiplicative separability in the shock and the utility from labour. Since they are not bounded, we assume that the tax functions we compute do not admit an infinite sequence of deviations that raise the agents' pay-off above their constrained efficient one.

16. The natural debt limit is the maximal borrowing that an agent can service. Given the bound on the agent's per period output, this limit is finite, but it translates into a utility bound of $-\infty$.

17. We do not have a proof of the existence of a steady state in our environment. The numerical policy functions we compute indicate that the Markov process for utility promises possesses an ergodic distribution. As in Atkeson and Lucas (1995), $\underline{U}$ is essential to ensure this.

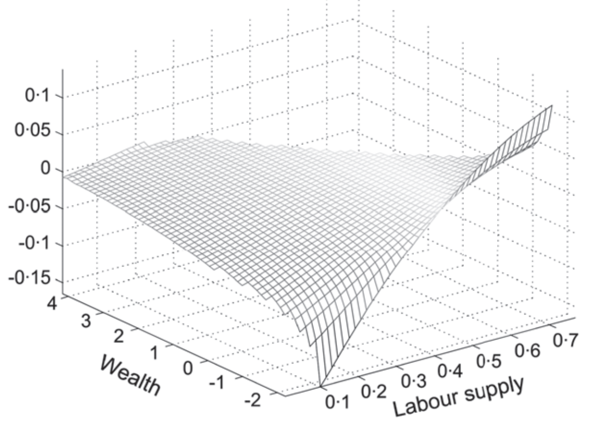

FIGURE 1
The tax function, $T(b, y)$

the curvature of the tax function especially large, close to the borrowing limit. These properties conform with the discussion in section 4.3 and imply that the marginal taxes of low wealth agents are very sensitive to wealth.

Figure 2 shows $\frac{\partial T(b,y)}{\partial y}$, the marginal labour income tax, as a function of y. Each curve corresponds to a different wealth level b. The negative cross partial $\frac{\partial^2 T(b,y)}{\partial b \partial y}$ implies that marginal labour income taxes are decreasing in *wealth*. These features are common across many other examples that we have computed. In contrast, we have found the dependence of the marginal labour income tax on *labour income* to be quite sensitive to the choice of utility function and shock distribution. In our benchmark parameterisation reported here, marginal labour income taxes have an inverted U shape as a function of income, holding wealth fixed. At the lowest and highest labour supplies at each wealth level, the marginal income tax is zero. At intermediate levels it is positive.[18]

This sensitivity of the dependence of marginal labour income taxes on labour income to parameters characterises much of the work on static non-linear income taxation. Consequently, there are few general results in that literature. Mirrlees (1971) obtained marginal income tax rates that are low and slightly declining in income, while Diamond (1998) and Saez (2001) find marginal income taxes that are high and sharply declining in income at low income levels.[19] Diamond and Saez's results have been interpreted as being consistent with the empirical phasing out of social benefits for low *income* agents as their income rises. In contrast, our numerical findings suggest that the transfers received by low *wealth* agents should be rapidly phased out as their labour incomes increase.

The implications of the intertemporal wedge for marginal asset taxes are illustrated in Figures 3 and 4. Figure 3 plots $\frac{\partial T}{\partial b}(\cdot, y)$ against b for different fixed labour income levels y. Now the negative cross partial $\frac{\partial^2 T(b,y)}{\partial b \partial y}$ implies that marginal asset taxes covary negatively with

18. The same pattern has been found in the static non-linear tax literature when similar assumptions on preferences and shocks are made. The zero marginal income taxes at the lowest and highest labour supplies stem from the fact that the incentive-compatibility constraint does not bind at these points. See the Supplement and Seade (1977) for further details.

19. The low value of marginal income taxes in Mirrlees (1971) stems from his choice of utility function: $\log c + \log(1 - l)$, which implies a high labour supply elasticity. The monotonically declining pattern of rates in income stems from his assumption of a log-normal distribution of shocks. Diamond (1998) and Saez (2001) assume lower labour supply elasticities and a (calibrated) Pareto shock distribution, and obtain higher marginal income taxes.

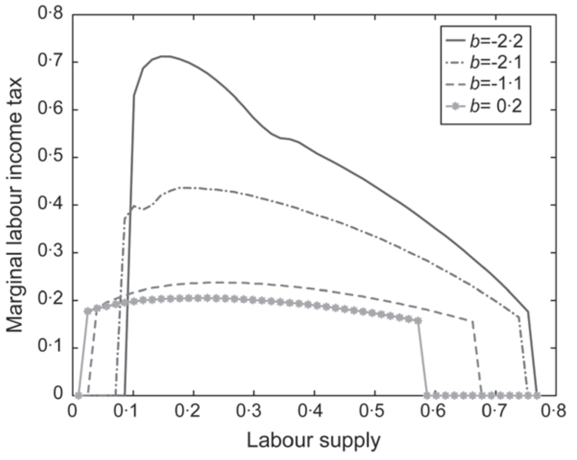

FIGURE 2

The marginal labour income tax, $\frac{\partial T(b,y)}{\partial y}$

income, being high at low income levels and low at high ones. As with marginal income taxes, variation in marginal asset taxes is greatest close to the borrowing limit. Figure 4 explicitly relates the tax function to the intertemporal wedge. Recall from (27) that the contribution of the tax function to the intertemporal wedge in (48) can be decomposed into an expected marginal tax and a covariance component, and is equal to the intertemporal wedge at wealth levels strictly greater than the borrowing limit. Figure 4 shows the contribution of the tax function to the wedge (solid line), as well as the expected marginal asset tax (dashed line), and the covariance (dash–dot) component. Over most of the wealth range the intertemporal wedge is less than 1% in value, but close to the borrowing limit it becomes much larger rising to about 16%. The expected marginal asset tax peaks at a little over 2% at the borrowing limit, and then falls steadily with wealth. The covariance component is also decreasing in wealth, but it is much larger close to the limit and falls off much more quickly as wealth increases. Consequently, the covariance component plays the major role in generating the intertemporal wedge only when the agent's wealth is small and the wedge is large.

We model taxes as a function of labour income and the *stock of wealth*. The marginal asset taxes drawn in Figure 3 can be converted into marginal asset *income* taxes by multiplying them by a factor of $1/(1-q)$. This conversion indicates that optimal marginal asset income taxes are quite large and very sensitive to labour income, especially close to the borrowing limit. The optimal expected marginal asset income tax peaks at over 20% and exceeds 10% over much of the wealth range.

6. CONCLUDING REMARKS

We study optimal taxation in a class of dynamic economies with private information. We show that constrained–efficient allocations in this environment can be implemented as competitive equilibria in market economies with taxes. The optimal tax system is simple and conditions only upon current wealth and current labour earnings. The incentive compatibility constraints shape the features of the resulting optimal tax system. We analytically derive implications for both income and asset taxation and further explore them in numerical examples.

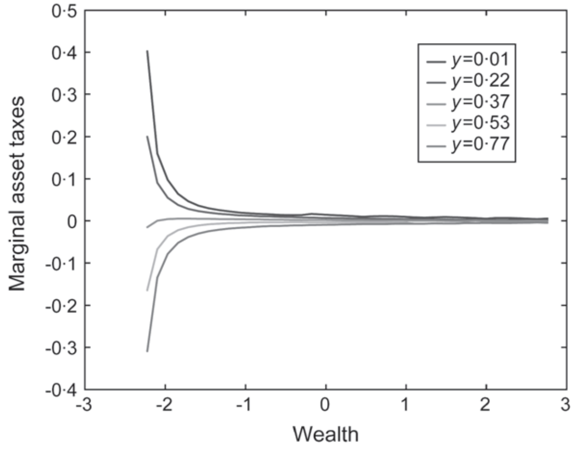

FIGURE 3

The marginal asset tax, $\frac{\partial T(b,y)}{\partial b}$

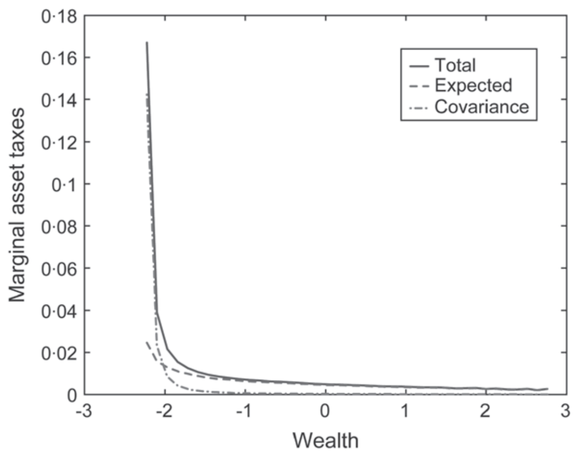

FIGURE 4

The tax contribution to the intertemporal wedge

Our implementation relies on a recursive formulation of the planner's problem. However, this is only valid for the case in which the agent's shocks are i.i.d. Consequently, our fiscal implementation will not work when shocks are persistent. In this case, wealth levels do not adequately describe past histories. Moreover, when shocks are persistent an agent's current shock influences her intertemporal marginal rate of substitution. Thus, the agent's intertemporal allocation of consumption and savings will depend upon this shock as well as her after-tax quantity of resources. The simple decomposition of the agent's and component planner's within period problem on which the proof of Proposition 1 relies will no longer hold.

Kocherlakota (2004*a*) provides an alternative fiscal implementation that works even with persistent shocks. In Kocherlakota's formulation, the government keeps track of an agent's entire history of labour supplies and condition taxes upon this history. The government does not use wealth to summarize aspects of an agent's past history.

It remains an open question as to whether there exists a fiscal implementation intermediate between ours and Kocherlakota's that would be valid for an economy with persistent shocks. Fernandes and Phelan (2000) and Doepke and Townsend (2002) have shown that there do exist recursive formulations of the planning problem for Markovian shocks. These rely on a vector of utility promises to keep track of histories. Similarly, a recursive fiscal implementation for an economy with persistent shocks could not rely on an agent's stock of non-contingent claims alone to keep track of past histories. It would be necessary to augment the state space. For example, it may be possible to use an agent's portfolio position in a richer asset market structure in conjunction with truncated labour histories to encode past shock histories. We leave this important extension to future work.

Our fiscal implementation embeds specific assumptions about the relative roles of markets and government policy. In particular, no private insurance contracts are allowed with the current market structure. In practice, government welfare programmes and private insurance contracts are complementary in providing incentives and determining the extent of risk-sharing supported in a competitive equilibrium. Exploring this complementarity could provide important insight in cross-country differences in government policies.[20]

APPENDIX: PROOFS

Proof of Proposition 1. We directly construct a two period market economy with taxes and borrowing limits. We assume a market for claims opens in period 0. We set $f = B_0$ and set Λ_0 to satisfy condition (1) in Definition 3. We set the government spending shocks to $\{G_t\}_{t=0}^{1}$, the debt limit to $\underline{b} = B_0(\underline{U})$ and $\mathcal{B}_1$ to $[\underline{b}, \infty)$. The (candidate) equilibrium price in the market economy is set to q. The proof will be complete if we can find taxes that ensure that for each w_0, an agent with initial wealth $B_0(w_0)$ in the market economy chooses the allocation $z(\zeta^*, w_0)$, where $\zeta^* = \{\{\varphi_t^*, \varsigma_t^*\}_{t=0}^{1}, \omega^*\}$.

The argument is in three steps that work back from period 1 to period 0. In the first step, a tax function is found such that an agent with a stock of claims $B_1(w_1)$ in period 1 will choose the same allocation as is awarded to an agent with a utility promise of w_1 in the component planner economy. Period 0 is divided into two stages in both the market and the component planner economy. In the second stage, the intertemporal allocation of a given quantity of resources between time 0 consumption and claims is obtained. In the first stage, the labour–resource allocation is determined. The next step of the proof shows that the agent's second stage problem in the market economy is the dual of the corresponding second stage component planner's problem. In the final step, a tax function is found such that an agent with an initial stock of claims $B_0(w_0)$ chooses the same labour and resource pair as would be awarded to an agent with utility promise w_0 in the component planner economy. We give the argument for $\Theta = [\underline{\theta}, \overline{\theta}]$ and for $\varsigma_t^*(w_0, \cdot)$ continuous each $w_0 \in \mathcal{W}_0$. These assumptions simplify the exposition; neither is essential.

Period 1: A component planner with assigned utility promise $w_1 \in \mathcal{W}_1$ solves the problem

$$B_1(w_1) = \inf_{\varphi:\Theta\to\mathcal{U}, \varsigma:\Theta\to\mathcal{V}} \int [C(\varphi(\theta)) - Y(\varsigma(\theta))]\, d\pi, \tag{A.1}$$

s.t. $w_1 = \int [\varphi(\theta) + \theta\varsigma(\theta)]\, d\pi$ and $\forall \theta, \theta' \in \Theta$, $\varphi(\theta) + \theta\varsigma(\theta) \geq \varphi(\theta') + \theta\varsigma(\theta')$. Denote the policy functions that attain the infima in the problems (A.1) by $\varsigma_1^* : \mathcal{W}_1 \times \Theta \to \mathcal{V}$ and $\varphi_1^* : \mathcal{W}_1 \times \Theta \to \mathcal{U}$. Let y_1^* and c_1^* denote the corresponding constrained–efficient resource allocation defined by $y_1^*(B_1(w_1), \theta) = Y(\varsigma_1^*(w_1, \theta))$ and $c_1^*(B_1(w_1), \theta) = C(\varphi_1^*(w_1, \theta))$.

Next consider the period 1 problem of an agent in the market economy confronting a tax function T_1. An agent with wealth $b_1 \in \mathcal{B}_1$ solves

$$V_1(b_1) = \sup_{c:\Theta\to\mathbb{R}_+, y:\Theta\to\mathcal{Y}} \int \left[u(c(\theta)) + \theta v(y(\theta))\right] d\pi \tag{A.2}$$

subject to the budget constraint, for each $\theta \in \Theta$, $b_1 = c(\theta) - y(\theta) + T_1(b_1, y(\theta))$.

Define $\mathcal{Y}_1^*(b)$ as in (14) and for $b \in \mathcal{B}_1$ and $y \in \mathcal{Y}_1^*(b)$ set $T_1(b_1, y)$ according to (17). Since $\Theta = [\underline{\theta}, \overline{\theta}]$ and $\varsigma_1^*(B_1^{-1}(b), \cdot)$ is continuous, $\mathcal{Y}_1^*(b)$ is an interval of the form $[\underline{y}_1(b), \overline{y}_1(b)]$ with $\underline{y}_1(b) = y_1^*(b, \overline{\theta})$ and $\overline{y}_1(b) = y_1^*(b, \underline{\theta})$. For $y > \overline{y}_1(b_1)$, set $T_1(b_1, y) > T_1(b_1, \overline{y}_1(b_1))$ and such that $u(b_1 + y - T_1(b_1, y)) + \underline{\theta} v(y) < u(b_1 + \overline{y}_1(b_1) -$

20. Golosov and Tsyvinski (2003*b*) consider this issue.

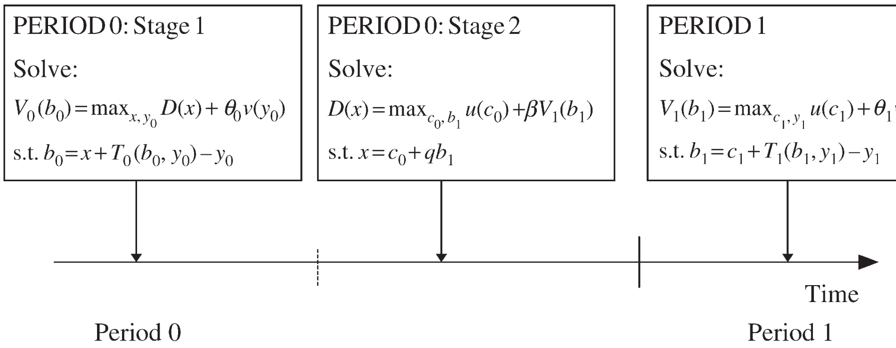

FIGURE A.1

Timeline for the market economy

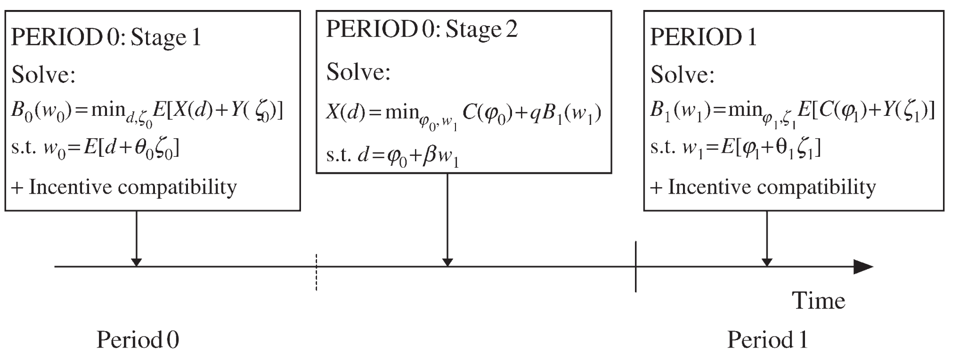

FIGURE A.2

Timeline for the component planner economy

$T_1(b_1,\overline{y}_1(b_1)))+\underline{\theta}v(\overline{y}_1(b_1))$. For example, extend $T_1(b_1,\cdot)$ linearly on $\mathcal{Y}/\mathcal{Y}_1^*(b_1)$ by setting $T_1(b_1,y)=T_1(b_1,\overline{y}_1(b_1))$ $+\ \overline{\tau}_1(y-\overline{y}_1(b_1))$, where $\overline{\tau}_1=[u'(c_1^*(b_1,\overline{y}_1(b_1)))-\underline{\theta}v'(\overline{y}_1(b_1))]/u'(c_1^*(b_1,\overline{y}_1(b_1)))$. Similarly, for $y<\underline{y}_1(b_1)$, set $T_1(b_1,y)<T_1(b_1,\underline{y}_1(b_1))$ and such that $u(b_1+y-T_1(b_1,y))+\overline{\theta}v(y)<u(b_1+\underline{y}_1(b_1)-T_1(b_1,\underline{y}_1(b_1)))+\overline{\theta}v(\underline{y}_1(b_1))$. For example, set $T_1(b_1,y)=T_1(b_1,\underline{y}_1(b_1))+\underline{\tau}_1(y-\underline{y}_1(b_1))$, where $\underline{\tau}_1=[u'(c_1^*(b_1,\underline{y}_1(b_1)))-\overline{\theta}v'(\underline{y}_1(b_1))]/u'(c_1^*(b_1,\underline{y}_1(b_1)))$.

Consider an agent in the market economy in period 1 with wealth $b_1=B_1(w_1)$ and shock θ. Under (17), if the agent chooses labour $y_1^*(b_1,\theta')\in\mathcal{Y}_1^*(b_1)$, she obtains consumption $c_1^*(b_1,\theta')$. By construction, this provides the utility pair $(\varphi_1^*(w_1,\theta'),\ \varsigma_1^*(w_1,\theta'))$. By choosing different labour levels in $\mathcal{Y}_1^*(b_1)$, the agent can obtain the entire set of period 1 report-contingent resource allocations available to an agent with utility promise w_1 in the component planner economy. Incentive-compatibility implies that amongst these, the agent obtains the highest pay-off from $(c_1^*(b_1,\theta),y_1^*(b_1,\theta))$. For $y\in\mathcal{Y}/\mathcal{Y}_1^*(b_1)$, consider first an agent choosing $y=\overline{y}_1(b_1)+\delta$, $\delta>0$. By construction, $u(b_1+y-T_1(b_1,y))+\theta v(y)$ $<u(b_1+\overline{y}_1(b_1)-T_1(b_1,\overline{y}_1(b_1)))+\theta v(\overline{y}_1(b_1))+(\theta-\underline{\theta})(v(y)-v(\overline{y}_1(b_1)))<u(b_1+\overline{y}_1(b_1)-T_1(b_1,\overline{y}_1(b_1)))+\theta v(\overline{y}_1(b_1))$. Thus, for all $y>\overline{y}_1(b_1)$, the agent is better off reducing her labour to $\overline{y}_1(b_1)$. By a similar argument the agent would never choose $y<\underline{y}_1(b_1)$. It follows that the agent will choose the allocation $(c_1^*(b_1,\theta),y_1^*(b_1,\theta))$ and, hence, the utility pair $(\varphi_1^*(w_1,\theta),\ \varsigma_1^*(w_1,\theta))$. Since b_1 and θ were arbitrary, it follows that for all $b_1'\in\mathcal{B}_1$ and $\theta'\in\Theta$, an agent will choose $(c_1^*(b_1',\theta'),y_1^*(b_1',\theta'))$ when confronted with the tax function T_1. The agent's value function in the market economy, V_1, defined in (A.2), then equals B_1^{-1} and, hence, is strictly increasing.

Period 0: We divide the agent's problem in the market economy into two stages. In the first, the agent chooses labour y_0 and resources x. In the second, she allocates x between current consumption c_0 and claims b_1. Similarly, the component planner's problem can be subdivided. In the first stage, the agent reports her shock and receives a utility from labour, ς_0, and an interim utility promise d. In the second stage, the planner allocates the interim promise between utility from current consumption φ_0 and a continuation utility promise w_1.

Period 0, *second stage*: Consider the second stage problem of a component planner with interim utility promise $d \in \mathcal{D} = \{\varphi + \beta w : \varphi \in \mathcal{U}, w \in \mathcal{W}_1\}$:

$$X(d) = \inf_{\varphi_0 \in \mathcal{U}, w_1 \in \mathcal{W}_1} C(\varphi_0) + qB_1(w_1) \tag{A.3}$$

subject to: $d = \varphi_0 + \beta w_1$. It is straightforward to verify that X is strictly increasing. Let $\widetilde{\varphi}_0^* : \mathcal{D} \to \mathcal{U}$ and $\widetilde{\omega}^* : \mathcal{D} \to \mathcal{W}_1$ denote policy functions that attain the infima in the problems (A.3). Define the corresponding resource allocation functions by $\widetilde{c}_0^*$ and $\widetilde{b}_1^*$, where $\widetilde{c}_0^*(X(d)) = C(\widetilde{\varphi}_0^*(d))$ and $\widetilde{b}_1^*(X(d)) = B_1(\widetilde{\omega}^*(d))$.

Next consider the agent's second stage problem in the market economy. The agent allocates $x \geq q\underline{b}$ units of resources across current consumption and savings to solve

$$D(x) = \sup_{c_0 \in \mathbb{R}_+, b_1 \geq \underline{b}} u(c_0) + \beta V_1(b_1)$$

subject to $x = c_0 + qb_1$. The allocation $(\widetilde{c}_0^*(x), \widetilde{b}_1^*(x))$ is optimal for the agent in this problem. To see this suppose that there was some alternative allocation (c', b') such that $x = c' + qb'$, $c' \in \mathbb{R}_+$, $b' \geq \underline{b}$ and $u(c') + \beta V_1(b') > d = X^{-1}(x)$. Then, since $d \in \mathcal{D}$, and u and V_1 are continuous and monotone, there exists an allocation (c^+, b^+) with $c^+ \leq c'$ and $b^+ \leq b'$ with at least one of these inequalities strict such that $u(c^+) + \beta V_1(b^+) = d$. But then $(u(c^+), V_1(b^+))$ attains the interim utility promise d and has a cost strictly less than x. This contradicts the optimality of $\widetilde{\varphi}_0^*$ and $\widetilde{\omega}^*$ at d for the component planner's problem.

Period 0, *first stage*: In this stage, a component planner with utility promise $w_0 \in \mathcal{W}_0$ solves

$$B_0(w_0) = \inf_{d:\Theta \to \mathcal{D}, \varsigma:\Theta \to \mathcal{V}} \int [X(d(\theta)) - Y(\varsigma(\theta))] d\pi, \tag{A.4}$$

s.t. $w_0 = \int [X(d(\theta)) + \theta\varsigma(\theta)] d\pi$ and $\forall \theta, \theta' \in \Theta$, $d(\theta) + \theta\varsigma(\theta) \geq d(\theta') + \theta\varsigma(\theta')$. Denote the policy functions that attain the infima in these problems by $d_0^* : \mathcal{W}_0 \times \Theta \to \mathcal{D}$ and $\varsigma_0^* : \mathcal{W}_0 \times \Theta \to \mathcal{V}$. Let $y_0^*(B_0(w_0), \theta) = Y(\varsigma_0^*(w_0, \theta))$ and $x_0^*(B_0(w_0), \theta) = X(d_0^*(w_0, \theta))$.

Next consider the first stage problem of an agent in the market economy with initial wealth $b_0 \in \mathcal{B}_0$, $\mathcal{B}_0 = B_0(\mathcal{W}_0)$ under a tax function T_0:

$$V_0(b_0) = \sup_{xc:\Theta \to \mathbb{R}_+, y:\Theta \to \mathcal{Y}} \int \left[D(x(\theta)) + \theta v(y(\theta))\right] d\pi \tag{A.5}$$

subject to the budget constraint, for each $\theta \in \Theta$, $b_0 = x(\theta) - y(\theta) + T_0(b, y(\theta))$.

Define $\mathcal{Y}_0^*(b)$ as in (14). For each $b_0 \in \mathcal{B}_0$ and θ, set $T_0(b_0, y_0^*(b_0, \theta)) = b_0 + y_0^*(b_0, \theta) - x_0^*(b_0, \theta)$. Since $\Theta = [\underline{\theta}, \overline{\theta}]$ and $\varsigma_0^*(B_0^{-1}(b), \cdot)$ is continuous, $\mathcal{Y}_0^*(b)$ is an interval of the form $[\underline{y}_0(b), \overline{y}_0(b)]$. For $y > \overline{y}_0(b_0)$, set $T_0(b_0, y) > T_0(b_0, \overline{y}_0(b_0))$ and such that $D(b_0 + y - T_0(b_0, y)) + \underline{\theta}v(y) < D(b_0 + \overline{y}_0(b_0) - T_0(b_0, \overline{y}_0(b_0))) + \underline{\theta}v(\overline{y}_0(b_0))$. Similarly, for $y < \underline{y}_0(b_0)$, set $T_0(b_0, y) < T_0(b_0, \underline{y}_0(b_0))$ and such that $D(b_0 + y - T_0(b_0, y)) + \overline{\theta}v(y) < D(b_0 + \underline{y}_0(b_0) - T_0(b_0, \underline{y}_0(b_0))) + \overline{\theta}v(\underline{y}_0(b_0))$. Then, the set of budget-feasible labour and resource combinations (x_0, y_0) for an agent with initial wealth $b_0 = B_0(w_0)$ and shock θ_0 in the market economy includes those available to an agent with initial promise w_0 in the component planner problem. By incentive-compatibility $(x_0^*(b_0, \theta), y_0^*(b_0, \theta))$ is optimal for the agent amongst these. In the market economy, the agent can also increase her labour above $\overline{y}_0(b_0)$ or reduce it below $\underline{y}_0(b_0)$. However, as in period 1, allocations obtained in this way are sub-optimal. Let $c_0^*(b_0, \theta) = \widetilde{c}_0^*(x_0^*(b_0, \theta))$ and $b^*(b_0, \theta) = \widetilde{b}_1^*(x_0^*(b_0, \theta))$.

Combining the previous arguments, it follows that it is optimal for an agent in the market economy, endowed with a stock of claims $b_0 = B_0(w_0)$ and confronting the price q and the tax system $\{T_0, T_1\}$, to make choices that induce the indvidual utility allocation $z(\zeta^*, w_0)$. ||

Proof of Lemma 4. Under our normalization, implementation requires that the agent saves B_1^*. It also requires that she obtains the consumption level $c_1^*(\underline{\theta})$ if she chooses $\overline{y}$ and $c_1^*(\overline{\theta})$ if she chooses $\underline{y}$. Define $\underline{T}_1$ by $qu'(c_0^*) = \beta(1 - \underline{T}_1)\, u'(c_1^*(\overline{\theta}))$ and $\overline{T}_1$ by $qu'(c_0^*) = \beta(1 - \overline{T}_1) u'(c_1^*(\underline{\theta}))$.

Now, suppose that the tax function $T(b, y)$ implements z^* in the market economy and that $\frac{\partial T}{\partial b}(B_1^*, \underline{y}) < \underline{T}_1$. Then, as in the proof of Lemma 3, the agent can save B_1^* and select $\underline{y}$ regardless of her shock. This is feasible and delivers the same pay-off, w_0, to the agent as z^*. However, since $\frac{\partial T}{\partial b}(B_1^*, \underline{y}) < \underline{T}_1$, $qu'(c_0^*) < \beta(1 - \frac{\partial T}{\partial b}(B_1^*, \underline{y}))u'(c_1^*(\overline{\theta}))$, so that the agent can do even better and obtain a pay-off above w_0 by saving slightly more than B_1^*, and selecting an effort of $\underline{y}$ regardless of her shock. It follows that if $T(b, y)$ implements z^* then $\frac{\partial T}{\partial b}(B_1^*, \underline{y}) \geq \underline{T}_1$. Similarly, if $\frac{\partial T}{\partial b}(B_1^*, \underline{y}) > \underline{T}_1$, the agent can improve on the planner's solution by saving slightly less than B_1^* and choosing $\underline{y}$ regardless of her shock. Thus, $\frac{\partial T}{\partial b}(B_1^*, \underline{y}) = \underline{T}_1$. It then follows from the definitions of $\underline{T}_1$ and $\overline{T}_1$ that z^* is consistent with the agent's Euler equation only if $\frac{\partial T}{\partial b}(B_1^*, \overline{y}) = \overline{T}_1$.

Condition 1 in the lemma follows from the definitions of $\underline{T}_1$ and $\overline{T}_1$ and the fact that $c_1^*(\overline{\theta}) < c_1^*(\underline{\theta})$. For Condition 2, combine the definitions of $\underline{T}_1$ and $\overline{T}_1$ with the component planner's intertemporal first order condition (31) to obtain

$$\underline{T}_1\pi(\overline{\theta}) + \overline{T}_1\pi(\underline{\theta}) = 1 - \frac{q}{\beta}u'(c_0^*)\left\{\frac{\pi(\overline{\theta})}{u'(c_1^*(\overline{\theta}))} + \frac{\pi(\underline{\theta})}{u'(c_1^*(\underline{\theta}))}\right\} = 0. \quad \|$$

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