I

Lending and the Smoothing of Uninsurable Income

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An economy is studied in which there is a continuum of traders who are identical ex ante. These traders, who are infinite lived, maximize expected discounted utility. They have uncertainty in their individual endowments, but there is no aggregate uncertainty. No trader is able to observe the endowment or consumption of any other trader. The optimal allocation that is incentive compatible in this environment is characterized. It is shown that the wealth of a representative trader in this environment is a random walk and that consumption at a given date is a fixed fraction of the trader's wealth at that date. The allocation can be supported by the constrained exchange of infinite-lived bonds between the traders and an intermediary. The profit-maximization problem of such an intermediary is dual to the optimization problem of the coalition of traders.

1. Introduction

During the course of its lifetime, a typical household experiences considerable fluctuation in its level of consumption.[1] Some of this fluctuation is attributable to changes in aggregate economic conditions, but a substantial part is idiosyncratic. That is, much of the fluctuation of consumption over time for each *individual* household corresponds to random variation *among* households at each particular time.[2] In a large economy, this variation among households could be eliminated (in per capita terms, at least) by pooling the consumption of households that were comparable ex ante. Such pooling would smooth the consumption of each individual household not only across random events, but also over time. Most economic agents presumably would like the variability of their consumption to be reduced in both of these respects, if this reduction could be accomplished by a means such as pooling that would not reduce their expected level of consumption.[3]

In a sense, then, the actual pattern of consumption by households is in-

efficient. A Pareto-superior allocation with pooled consumption would be feasible in the sense that it would require no more goods and services to be provided than are actually produced. There is a more realistic sense of efficiency, though, in which the potential existence of such a Pareto-superior allocation does not necessarily show that the actual allocation is inefficient. This alternative sense of economic efficiency is formulated by adopting a notion of feasibility that recognizes that there are constraints on how commodities can be distributed as well as on how they can be produced. In particular, incentive compatibility is a constraint on distribution in an economy where agents have incomplete information. The main question to be studied in this paper is whether idiosyncratic fluctuations such as are actually observed in consumption by households can be consistent with efficiency in this more realistic sense.

The question can be sharpened in two ways. First, one can ask whether a specific stochastic process of consumption can be represented as an efficient allocation arising in some model economy. In particular, this question arises with respect to the permanent income hypothesis of Friedman (1957). That is, can households be represented as having wealth that follows a random walk, and as consuming a constant fraction of their wealth at each date?[4]

Second, one can ask about the connection between how consumption fluctuates and how the reallocation of income is supported. The hypothetical Pareto-improving reallocation mentioned above would be achieved by providing full insurance to households against contemporaneous random variations in their income. Some such insurance actually is provided (e.g., unemployment compensation), but households depend largely on credit markets rather than on insurance markets to smooth their income over time. Income is reallocated among households not primarily in the form of insurance premiums and indemnities, but in the form of loans financed by savings. Thus one can ask whether there is a model economy in which the allocations that would be supported by competitive credit markets can be distinguished from those that would be supported by insurance markets, and in which the allocations supported by credit markets would be efficient.

I will suggest an affirmative answer to both of these questions, an answer that will be supported by the analysis of a simple model of an economy populated by infinite-lived agents who receive endowments in the form of income that fluctuates randomly and independently, who attempt to maximize their expected discounted utility, who observe their own income as it is received, and who cannot communicate this information to others.[5] These assumptions about agents' endowments are intended to capture some of the economically significant features of more realistic but more complicated models, such as those that posit unobservable insurance risks or unobserved

levels of effort in labor supply. This model is described in Section 2 of the paper.

In this model economy, there are two constraints on the feasibility of allocations. First, an allocation certainly would not be feasible if its implementation required information in excess of what was available in the whole economy. The statistical independence of agents' endowments implies that even if they could pool their information, they could not discover anything about their future incomes. Thus, an agent's expected utility cannot be improved by having his net trade at any date depend on any information beyond the history of his own previous and current income. Rather than considering arbitrary net trades, then, we can restrict our attention to those net trades that are consistent with this informational restriction. These informationally constrained net trades, which can be thought of as being implemented by decentralized contracts, are described in Section 3.

Second, feasible allocations are constrained by the need to implement contracts on the basis of agents' unverifiable and unfalsifiable reports of their past and current incomes, rather than on the basis of actual observations. That is, incentive compatibility is a constraint on the feasibility of net trades in this environment.[6] This constraint is formulated in Section 4, and the problem of efficient allocation is stated there.

The characterization of the optimal contract is outlined in Section 5 and is accomplished in sections 6–10. This contract can be described by imputing to each agent a credit balance that fluctuates over time. The credit balance is the expected discounted present value of the agent's future net trade. The agent will elect to increase his credit balance at dates when he receives a unit of endowment and to reduce his balance at dates when he receives nothing. An agent who increases his balance must forgo some current consumption relative to an agent with an identical history who reduces his balance, but he will have a better prospect of future consumption. That is, agents make a choice at each date between current consumption and saving.

In Section 11, the time-series behavior of agents' wealth and consumption is compared with the rough empirical generalizations mentioned above. Given the parametric form of the model (including the specification of each agent's endowment as an i.i.d. stochastic process), an agent's wealth is a random walk. There is a fixed constant that approximately determines each agent's consumption at each date as a proportion of his wealth at that date.

Section 12 deals with the relationship between efficiency and competition in the model economy. This discussion is based on a duality theorem for welfare maximization and cost minimization that is proved in Section 7. Because the solutions of these two optimization problems coincide, the unique symmetric, efficient allocation of the model economy is perfectly compet-

itive in a sense suggested by Ostroy (1980). Specifically, the contract that supports this optimal allocation can be interpreted in terms of the constrained trading of infinite-lived, fixed-rate bonds between each agent and a competitive intermediary. The form of this endogenous credit constraint is significantly different from the kind of rationing that has typically been imposed as an exogenous specification in previous models of credit allocation.

2. The Economy

This section describes the theoretical model that is used to examine formally the questions that have been posed in the Introduction. The model specifies the endowment, tastes, and technology of an economy composed of many households. These households are infinite lived, and they consume a single, perishable, composite commodity at each date. Each household is exposed to endowment risk: that is, the amount of the commodity received as endowment by a particular household is not known beforehand. The subjective opinion (shared by all households) about this quantity (about the amount of commodity to be received by the particular household i on the particular date t) is described by representing the quantity as a random variable defined on a sample space K having probability measure k. (E_k will denote statistical expectation with respect to this measure.) In other words, the theory treats households as Bayesian agents, and the elements of K are sample points or "possible states of the world." For simplicity, it is assumed that each household will receive either no endowment or else one unit of endowment at each date. These random endowment quantities are identically and independently distributed, both across households and across dates. The mean of their distribution (i.e., the probability that one unit of the good is received in endowment), which will be denoted by p, is strictly between 0 and 1. Consequently the variance of the distribution is strictly less than 1.

If there were n households, then the variance of per capita endowment would be less than $1/n$. In the present theory, it is supposed that the population is so large that this variance is negligible. In fact, the theory concerns an idealized economy in which, although the endowment of each individual household is uncertain, the per capita endowment is p units with certainty.[7] To reconcile this idealization mathematically with the statistical independence of the endowments of households, the population of households is represented as a set H with a nonatomic measure h. (For mathematical convenience, h is normalized so that $h(H) = 1$. E_h denotes the integral over the population with respect to h.) If households were assumed to receive their endowment only at a single date, then this desired mathematical consistency would be assured by a theorem of Judd (1985). However, because the endowment of a household in this model is a stochastic process rather than a

single random variable, the present theory actually requires that endowment distributions of households at a given date be independent conditional on endowments already received before that date and that conditional population averages be equal to the corresponding conditional expectations. An example from Feldman and Gilles (1985) suggests that the construction used by Judd cannot simply be iterated to guarantee the consistency of these conditional statements. Nevertheless, the assumptions of the model are in fact consistent with set theory, including the axiom of choice. (The proof of consistency will be presented in a forthcoming paper.) A precise mathematical statement of the assumptions is now presented, following which the tastes of the households and technology of the economy will be specified.

The Endowment

The endowment is a function $Y:H \times K \times N \rightarrow \{0,1\}$. $Y(i,\theta,t)$ represents the amount of the consumption good that household i receives in endowment at date t, if the true state of the world is θ.

For any household, the set of sample points at which its endowment takes prescribed values at finitely many specified dates is an event (i.e., is a measurable subset of K). Formally, for any finite set F, let $\#F$ denote the number of elements of F. If i is a household, and if F and G are disjoint, finite subsets of N, then define the event $K_{i,F,G}$ to be the set of sample points θ such that $Y(i,\theta,t) = 1$ for $t \in F$ and $Y(i,\theta,t) = 0$ for $t \in G$. Then the fact that the endowment of household i is a Bernoulli process is expressed by

$$k(K_{i,F,G}) = p^{\#F}(1 - p)^{\#G}. \tag{1}$$

Also, the fact that the endowment of a household is independent of finite initial histories of the endowments of any finite set of other households is expressed by the statement that, if $i_1, \ldots, i_n$ are distinct households and if for $j \leq n$ the sets F_j and G_j are finite disjoint subsets of N, then

$$k\left(\bigcap_{j \leq n} K_{i_j,F_j,G_j}\right) = \prod_{j \leq n} k(K_{i_j,F_j,G_j}). \tag{2}$$

Equation (2) will not be used explicitly in the paper. However, it is assumed that each household will revise its expectations concerning its future endowment by conditioning its prior beliefs on its own past history alone and ignoring information that might be gained from observing the equilibrium behavior of other households. Equation (2) implies that, if each household has the capacity to observe only finitely many others, then conditioning on such observations would not cause any change in beliefs. Similarly, the efficiency concept to be formulated here supposes that the net trade offered to a household at any date will depend only on the past history of the house-

hold itself. The statistical independence expressed by (2) assures that these restrictions are consistent with the efficient use of information by the coalition of all households.

Measurable subsets of K are interpreted as events about which Bayesian households have subjective beliefs. Mathematically, though, the population H of households with the normalized measure h is also a probability space, and the expectation $E_h[Y(i,\theta,t)]$ is the per capita endowment received at date t in state of nature θ. It has already been mentioned that this quantity is assumed to be p with certainty. In fact, the stronger assumption is made that, in every state of nature θ, the function $Y(i,\theta,t)$ from $H \times N$ to $\{0,1\}$ is mathematically a Bernoulli process on H. The economic content of this assumption, combined with assumption (2), is that each household understands perfectly the pattern of endowment distribution that will occur in the economy and regards itself as being indistinguishable from other households in its endowment prospects, but that each household is behind a "veil of ignorance" regarding the relationship of its own endowment to the economy-wide pattern.

To state formally the assumption that Y is a Bernoulli process on H for every θ, subsets $H_{\theta,F,G}$ of the population are defined that are analogous to the events $K_{i,F,G}$. If θ is a state of nature (i.e., if $\theta \in K$), and if F and G are disjoint, finite subsets of N, then define the subset $H_{\theta,F,G}$ to be the set of households i such that $Y(i,\theta,t) = 1$ for $t \in F$ and $Y(i,\theta,t) = 0$ for $t \in G$. Then, for every $H_{\theta,F,G}$,

$$h(H_{\theta,F,G}) = p^{\#F}(1 - p)^{\#G}. \tag{3}$$

Consumption, Production, and Preferences

The households in H can collectively transform and reallocate their endowment Y. Their ability to do so, and their preferences among the allocations that might result, are now described.

An allocation is any function $Z: H \times K \times N \to R$ (R denotes the real numbers) such that the mean of the subjective probability distribution of every household concerning its income at each date, and the aggregate endowment in every state of nature at each date, are well defined and finite. That is, Z satisfies two measurability conditions:

For every $i \in H$ and $t \in N$, $Z(i,\theta,t)$ is integrable w.r.t. k,   (4)

and

For every $\theta \in K$ and $t \in N$, $Z(i,\theta,t)$ is integrable w.r.t. h.   (5)

A commodity bundle is a function $C: K \times N \to R$ such that, for every t, $C(\theta,t)$ is integrable as a function of θ. If Z is an allocation, then $Z(i,\theta,t)$ is

a commodity bundle for almost every i. An allocation is feasible if the discounted present value of aggregate consumption is, almost surely, equal to that of the endowment. That is, Z is feasible if

$$\sum_{i \in N} [\beta^t E_h Z(i,\theta,t)] = (1 - \beta)^{-1} p \text{ for almost all states } \theta. \tag{6}$$

Equation (6) describes a constant-returns technology that allows β units of consumption at time t to be transformed into one unit of consumption at time $t + 1$, or vice versa. It is assumed that $0 < \beta < 1$. This technology is available to the coalition of all traders, but no individual trader is able alone to transform consumption available at one date into consumption at another date. This technology has been introduced primarily to simplify the analysis by characterizing technical feasibility in terms of equation (6) alone, rather than being required to introduce a separate constraint at each date.

If Z is an allocation, then $Z - Y$ is a net trade. Net trades belong to the same linear space of functions as do allocations. However, when one of these functions is considered as a net trade, it is called feasible if it is the difference between a feasible allocation and the endowment. Equivalently, net trade B is feasible if $\Sigma_{i \in N} [\beta^t E_h B(i,\theta,t)] = 0$ for almost all states θ. Note that, if B is a net trade, then $B(i,\theta,t)$ represents the amount that trader i borrows at time t in state θ.

All traders share the same preferences concerning their own consumption. These preferences are derived from maximization of the expected discounted value of a temporary-utility function of consumption. If consumption (at a time and in a state) is x, then the temporary utility of consumption is given by the CARA function $W : R \rightarrow R$ defined by

$$W(x) = -e^{-rx}, \tag{7}$$

where $r > 0$ is the coefficient of absolute risk aversion. The utility of a commodity bundle is the discounted expected value of W. Formally, let Y_i denote the commodity bundle that trader i receives in the allocation Y. That is, for all t and θ, $Y_i(\theta,t) = Y(i,\theta,t)$. Then the discounted expected utility to i of receiving a consumption bundle C in net trade is the utility $U^i(C)$ defined by

$$U^i(C) = E_k \left[\sum_{i \in N} \beta^t W(Y_i(\theta,t) + C(\theta,t)) \right]. \tag{8}$$

Until this point, the model described here has been essentially a version of the model of competitive exchange under uncertainty that was originally introduced by Arrow (1964). In that model, traders are assumed to have common information about the state of nature. In particular, it is assumed

that the disposable income of every trader is publicly observable at the time of its receipt. Under this assumption of public information, it is feasible and efficient for all the traders to pool their income and redistribute it evenly among themselves. By agreeing to do so, they fully insure themselves against the idiosyncratic risks that they face. By (1) and (3), and the strict concavity of W, this full-insurance allocation is the unique core allocation under complete information.

In contrast, in addition to the assumption that traders do not observe their disposable income until the time of its receipt, it is also assumed here that each trader's transitory income is directly observable by him alone. Thus the insurance arrangement just described is unworkable, because under it traders would have incentive to deny that they had received disposable income when in fact they had received it. In this environment of incomplete information, the traders need to design an allocation that, besides being feasible in the materials-balance sense, can be implemented on the basis of traders' disposable-income reports that are potentially subject to misrepresentation. The characterization of the efficient symmetric allocation that is subject to these constraints, and that of the contract that implements it, are the tasks of this paper.

3. Net Trades Specifiable by Contracts

Since traders are anonymous and their consumption bundles in the endowment are i.i.d. random processes, it is natural to consider allocations that assign each trader a net trade that is a function of that person's own endowment, that allow borrowing in each state and at each date to depend only on disposable income received in that state and by that date, and that treat all traders symmetrically. Such an allocation is completely specified by the consumption bundle (or the net consumption bundle) that it assigns to a representative trader. Consumption bundles of this sort, or contracts, are now defined. In this section, the unobservability of income before its receipt is taken account of while the privacy of this observation is temporarily ignored.

Define the space S of deterministic income streams by $S = \{0,1\}^N$. A contract is a function $\Gamma: S \times N \to R$ that is consistent with the temporal structure of the trader's observation of his own income. $\Gamma(\sigma,t)$ is what the trader borrows at t if his income stream to date has been $\langle\sigma_0,\sigma_1,\ldots,\sigma_t\rangle$. [For convenience, denote this finite sequence (which is of length $t + 1$) by $\langle\sigma\rangle_{t+1}$. Let $\langle\sigma\rangle_0$ denote the empty sequence.] Then the temporal-structure requirement on Γ is that, for all income streams σ and τ and for all dates t,

$$\langle\sigma\rangle_{t+1} = \langle\tau\rangle_{t+1} \Rightarrow \Gamma(\sigma,t) = \Gamma(\tau,t). \tag{9}$$

A contract Γ defines a net trade Γ^* by

$$\Gamma^*(i,\phi,t) = \Gamma(\langle Y(i,\theta,0),Y(i,\theta,1),\ldots\rangle,t) \tag{10}$$

for all i, θ, and t.

Actually, the feasibility of Γ^* and the expected discounted utility that it will yield each trader can be determined by considering Γ alone. To do this, consider the probability measure on income streams that is induced by the consumption bundle that a trader i receives in the endowment, according to (1). Give the product topology to $S = \{0,1\}^N$ and define s to be the unique Borel probability measure that assigns probability p to $\{\sigma|\sigma_t = 1\}$ for every t and that makes these events independent. The operator E_s denotes the integral with respect to s. Now, Γ^* is a feasible net trade if

$$\sum_{t \in N} [\beta^t E_s \Gamma(\sigma,t)] = 0, \tag{11}$$

because the integral of Γ over S is equivalent to the population average of Γ^* over H $a.s.$ by (3). The discounted expected utility U^i of the net-trade consumption bundle assigned to trader i by Γ^* is constant for almost all i by (1), and this constant $U(\Gamma)$ is defined by

$$U(\Gamma) = E_s\left[\sum_{t \in N} \beta^t W(\sigma_t + \Gamma(\sigma,t))\right]. \tag{12}$$

4. Incentive Compatibility and the Coalition's Optimization Problem

In the formulas above, the consumption bundle received by a trader in the endowment has been treated as an argument of the contract Γ. Actually, though, a trader's borrowing is supposed to be a function of his report about his income, rather than of the income itself. For the formulas correctly to describe the equilibrium, then, the trader must decide to report truthfully at each date. Contract Γ is incentive compatible if truth is always the trader's utility-maximizing report.

Formally, define a reporting plan to be a measurable function $\pi:S \to S$ that satisfies the same temporal-structure requirement as does a contract. That is,

$$\langle \sigma \rangle_{t+1} = \langle \tau \rangle_{t+1} \Rightarrow (\pi(\sigma))_t = (\pi(\tau))_t. \tag{13}$$

If trader i follows reporting plan π, then at date t he will receive net trade $\Gamma(\pi(\sigma),t)$ if his true endowment is σ. Define the contract $\Gamma \circ \pi$ by $\Gamma \circ \pi(\sigma,t)$

$= \Gamma(\pi(\sigma),t)$. Then Γ is incentive compatible if truth is the best reporting plan; that is, if P is the set of reporting plans, then

$$U(\Gamma) = \max_{\pi \in P} U(\Gamma \circ \pi) > -\infty. \tag{14}$$

Now the optimal credit contract can be defined. It is the contract Γ that maximizes $U(\Gamma)$ subject to the feasibility constraint (11) and the incentive compatibility constraint (14).

5. Characterizing the Optimal Contract: An Overview

Sections 6–10 are devoted to characterizing the optimal contract and establishing its optimality. An overview of this fairly lengthy argument may be helpful before launching into the details of its proof.

First, the proof requires some understanding of incentive compatibility. One definition of incentive compatibility is that truth-telling is an optimal solution of the infinite-horizon stochastic programming problem defined by a contract. The variational and transversality conditions for the optimality of truth-telling are studied in Section 6. The variational condition, to be called temporary incentive compatibility (t.i.c.), will play an important role throughout the proof.

The credit balance of a trader is introduced in Section 7. The feasibility constraint (11) can be interpreted as stating that each trader must initially have a credit balance of zero. Therefore, the coalition's optimization problem is an instance of finding the incentive-compatible contract that maximizes expected discounted utility subject to a constraint on the trader's initial credit balance. The dual to this problem is finding the incentive-compatible contract that minimizes the initial credit balance required for the trader to attain a specified level of expected discounted utility. It will be shown that a solution to the dual problem is also a solution to the primal problem.

At any point, the trader's credit balance is the expected discounted value of the future borrowings to which that trader is entitled. Thus, when the dual problem is viewed as a stochastic, dynamic, cost-minimization problem for the coalition, the credit-balance process can be used to define a cost function that satisfies an inequality resembling the functional equation of dynamic programming. This is shown in Section 8. The operator defining the functional inequality is called the t.i.c. operator.

Temporary incentive compatibility suggests a finite-dimensional version of the dual problem. This temporary dual problem is used in Section 9 to characterize the minimum fixed point of the t.i.c. operator that could possibly be the cost function of a feasible contract. A contract that actually does

have this as its cost function is constructed in Section 10. By duality, this contract is also the solution to the primal utility-maximization problem of the coalition.

6. Temporary Incentive Compatibility

Suppose that, beginning at date 1, the coalition could observe the income of each trader, but that income at date 0 were private information. That is, $\Gamma(\sigma,t)$ would depend on the trader's report of σ_0 but on the actual values of subsequent disposable income. If telling the truth at date 0 would be optimal for the trader, then Γ will be called temporarily incentive compatible (t.i.c.) at $\varnothing$ (the trader's history prior to date 0). More generally, given any finite history $\langle\sigma\rangle_t$, Γ will be called t.i.c. if, had he previously reported $\langle\sigma\rangle_t$ as his income and were he to be constrained to report his income truthfully after date t, the trader would voluntarily give a truthful report at date t no matter what income he actually received then.

Formally, define (σ,t) (the node at $\langle\sigma\rangle_t$) by

$$(\sigma,t) = \{\langle\tau^0,\tau^1\rangle \in S^2 | \langle\sigma\rangle_t = \langle\tau^0\rangle_t = \langle\tau^1\rangle_t, \text{ and } \tau_t^k = k \text{ for } k = 0,1\}. \quad (15)$$

Also, for every finite history $\langle\sigma\rangle_t$, define $V(\Gamma,\sigma,t)$ to be the trader's expected utility from consumption determined by Γ beginning at date t, discounted to date t. Formally, letting $E_s[|\langle\ \rangle_t]$ denote conditional expectation with respect to history before date t, define

$$V(\Gamma,\sigma,t) = E_s\left[\sum_{n\in N} \beta^n W(\sigma_{t+n} + \Gamma(t + n))|\langle\ \rangle_t\right](\sigma). \quad (16)$$

[Note that τ in (16) is the variable of integration for E_s.] Finally, define Γ to be t.i.c. at (σ,t) if, for all $\langle\tau^0,\tau^1\rangle \in (\sigma,t)$,

$$W(k + \Gamma(\tau^k,t)) + \beta V(\Gamma,\tau^k,t + 1)$$
$$\geq W(k + \Gamma(\tau^{1-k},t)) + \beta V(\Gamma,\tau^{1-k},t + 1). \quad (17)$$

The characterization of the optimal contract requires three results about temporary incentive compatibility. First, if Γ is t.i.c. at the initial node $(\sigma,0)$ and is incentive compatible after date 0, then Γ is incentive compatible. Second, if Γ is t.i.c. at every node and if the present discounted value of sanctions imposed under Γ in the distant future approaches zero uniformly, then Γ is incentive compatible. Third, if Γ is t.i.c. at (σ,t), then in event (σ,t) the trader will borrow more at date t if he has not received income at that date than if he has. These three results are now stated formally and proved.

Define a contract to be (σ,t)-incentive compatible $[(\sigma,t)\text{-i.c.}]$ if, having received income stream $\langle\sigma\rangle_t$ before date t and having reported this truthfully, the trader would thereafter have no incentive to lie. Formally, Γ is (σ,t)-i.c. if

$$V(\Gamma,\sigma,t) = \max\{V(\Gamma \circ \pi,\sigma,t)|\langle\pi(\sigma)\rangle_t = \langle\sigma\rangle_t\}. \qquad (18)$$

Note that incentive compatibility as defined in (14) is $(\sigma,0)$-incentive compatibility.

LEMMA 1. *Γ is (σ,t)-i.c. if and only if it is t.i.c. at (σ,t) and is both $(\tau^0,t + 1)$-i.c. and $(\tau^1,t + 1)$-i.c., where $\langle\tau^0,\tau^1\rangle \in (\sigma,t)$.*

PROOF. This is an instance of Bellman's (1957) principle of optimality. Q.E.D.

Typically, optimal policies in infinite-horizon dynamic problems can be characterized in terms of a variational condition like (17) if a transversality condition, stating that events in the remote future will have negligible consequences for present decisions, is satisfied. Equation (19) below is the transversality condition for the trader's problem of choosing the optimal reporting rule in response to a contract.

LEMMA 2. *If Γ satisfies*

$$\lim_{t\to\infty}[\inf_{\sigma\in S} \beta^t V(\Gamma,\sigma,t)] = 0, \qquad (19)$$

then Γ is incentive compatible if and only if it is t.i.c. at every node.

PROOF. Temporary incentive compatibility at every node is an immediate consequence of Lemma 1 if Γ is incentive compatible. To prove the converse, a contradiction will be obtained from the assumption that Γ is t.i.c. at every node but that $U(\Gamma \circ \pi) > U(\Gamma)$. First it will be shown that, if any π satisfies this inequality, then there is a π that satisfies it and that involves truthful revelation except at finitely many nodes. The application of t.i.c. at these exceptional nodes will lead to the contradiction.

Define reporting plan π^n by specifying $(\pi^n(\sigma))_t$ to be equal to $(\pi(\sigma))_t$ if $t < n$ and to σ_t if $t \geq n$. By (12) and (16),

$$U(\Gamma \circ \pi^n) = E_s\left[\sum_{t<n} \beta^t W(\sigma_t + \Gamma \circ \pi(\sigma,t))\right] + \beta^n E_s V(\Gamma,\pi(\sigma),n). \qquad (20)$$

Considering the right-hand side of (20) as $n \to \infty$, the monotone convergence theorem applies to the first expectation and the bounded convergence theorem applies to the second. Taking these limits yields

$$\lim_{n\to\infty} U(\Gamma \circ \pi^n) = U(\Gamma \circ \pi). \qquad (21)$$

In π^n, there is misrepresentation of income in at most 2^n nodes. Thus, by (21), it can be assumed without loss of generality that the number of nodes where π involves misrepresentation is already finite, and in fact that π has been chosen so that this number is as small as possible.

That π possesses this minimality property will now be contradicted. Choose σ and t so that t is as large as possible subject to $(\pi(\sigma))_t = 1 - \sigma_t$. Then define π' to be equal to π except that $(\pi'(\sigma))_t = \sigma_t$. Because π is t.i.c. at (σ,t), $U(\Gamma \circ \pi') \geq U(\Gamma \circ \pi)$. This is a contradiction, because π' involves misrepresentation at one fewer node than π does.   Q.E.D.

Temporary incentive compatibility has an intuitively appealing consequence—that traders with identical past histories will currently borrow amounts that are inversely related to their current levels of disposable income.

LEMMA 3. *If $\langle \tau^0, \tau^1 \rangle \in (\sigma,t)$ and Γ is t.i.c. at (σ,t), then $\Gamma(\tau^0,t) \geq \Gamma(\tau^1,t)$ and $V(\Gamma,\tau^1,t + 1) \geq V(\Gamma,\tau^0,t + 1)$.*

PROOF. Because $W(1 + x) = e^{-r}W(x)$, (17) is equivalent to

$$(-(e^{-r}))^k[W(\Gamma(\tau^1,t)) - W(\Gamma(\tau^0,t))]$$

$$\leq (-1)^k\beta[V(\Gamma,\tau^0,t + 1) - V(\Gamma,\tau^1,t + 1)]. \quad (22)$$

Adding the instances of (22) for $k = 0,1$ together yields

$$(1 - e^{-r})[W(\Gamma(\tau^1,t)) - W(\Gamma(\tau^0,t)] \leq 0. \quad (23)$$

This implies that $\Gamma(\tau^0,t) \geq \Gamma(\tau^1,t)$, because W is increasing. Then $V(\Gamma,\tau^1,t + 1) \geq V(\Gamma,\tau^0,t + 1)$, or (17) would fail for $k = 1$.   Q.E.D.

7. A Duality Theorem for Credit Balances

The trader's credit balance at a node is the discounted expected value at that node of the future payments specified by the contract, conditional on the past history. Formally, define the stochastic process B of credit balances by

$$B(\Gamma,\sigma,t) = E_s\left[\sum_{n \in N} \beta^n\Gamma(\tau,t + n)|\langle \ \rangle_t\right](\sigma). \quad (24)$$

Note that, besides specifying the balance of the representative trader at the beginning of the contract, $B(\Gamma,\sigma,0)$ also is almost surely the aggregate discounted cost of fulfilling the contract for all traders. Thus the initial balance defines a constraint in the coalition's primal problem of writing an efficient contract, and it serves as the objective function in the dual to that problem. It is now shown that a solution to the dual problem is a solution to the primal problem as well.

LEMMA 4. *If $U(\Gamma^*) = u^*$ and $B(\Gamma^*,\sigma,0) = b^*$, and if Γ^* solves the dual problem*

P_δ: *minimize $B(\Gamma,\sigma,0)$ subject to Γ incentive compatible and*

$$U(\Gamma) \geq u^*,$$

then Γ^* also solves

P_π: *maximize $U(\Gamma)$ subject to Γ incentive compatible and*

$$B(\Gamma,\sigma,0) \leq b^*.$$

PROOF. In the proof, reference will be made to the amount that a trader with disposable income k needs to borrow to achieve temporary utility w. Denote this amount by $g(k,w)$. Formally, g is defined by

$$W(k + g(k,w)) = w. \tag{25}$$

Solving (25) yields

$$g(k,w) = -r^{-1}\ln(-w) - k. \tag{26}$$

Suppose that Γ^* does not solve P_π. It must be shown that it does not solve P_δ either. That is, either Γ^* is not incentive compatible or else there exists an incentive-compatible contract Γ' such that $U(\Gamma') > u^*$ and $B(\Gamma',\sigma,0) \leq b^*$.

Since Γ^* solves P_δ, thought, it must be incentive compatible. Consider the other possibility, then. By Lemma 1, Γ' is t.i.c. at $(\sigma,0)$ and it is $(\tau^k,1)$-i.c. for $k = 0, 1$, where $\langle\tau^0,\tau^1\rangle \in (\sigma,0)$. For each $n \in N$, define Γ^n to be the contract that yields a temporary utility level n^{-1} less than Γ' at date 0 to a trader who receives a unit of disposable income, regardless of what the trader reports, and that is identical to Γ' thereafter. That is,

$$\Gamma^n(\sigma,0) = g(1,W(1 + \Gamma'(\sigma,0)) - n^{-1}) \tag{27}$$

and for $t > 0$,

$$\Gamma^n(\sigma,t) = \Gamma'(\sigma,t). \tag{28}$$

For every n, Γ^n has the property that $B(\Gamma^n,\sigma,0) < b^*$, and $\lim_{n\to\infty} U(\Gamma^n) = U(\Gamma')$. Consider n sufficiently large so that $U(\Gamma^n) > u^*$. If Γ^n is incentive compatible, then Γ^* does not solve P_δ. Because Γ^n is $(\tau^k,1)$-i.c. for $k = 0$, 1, since it is identical to Γ' after date 0, it is sufficient to prove that it is t.i.c. at $(\sigma,0)$. It satisfies (17) for $k = 1$ by construction.

To show that (17) holds for $k = 0$, its equivalent (22) will be established. Note that, by (25) and (27),

$$W(\Gamma^n(\tau^k,0)) = -e^{-r[-r^{-1}\ln(-(W(1+\Gamma'(\tau^k,0))-n^{-1}))-1]}$$

$$= e^r[W(1 + \Gamma'(\tau^k,0)) - n^{-1}]. \tag{29}$$

Hence

$$[W(\Gamma''(\tau^1,0)) - W(\Gamma''(\tau^0,0))] = e^r[W(\Gamma'(\tau^1,0)) - W(\Gamma'(\tau^0,0))]. \quad (30)$$

By (28) and (30), Lemma 3 implies that (22) must hold of Γ'' for $k = 0$ since it holds of Γ'. Q.E.D.

8. A Cost Function for Contracts

Lemma 4 suggests the following question: Given a level u of welfare, what is a lower bound on the initial credit balance required to guarantee u using an incentive compatible contract? A lower bound $f_\mu(u)$ is characterized in this and the following two sections, and then a contract that achieves this lower bound is constructed for every $u \in R_-$. In particular, the contract that achieves utility level $f_\mu^{-1}(0)$ must be the efficient, incentive-compatible contract by Lemma 4.

Given a contract Γ, say that a function $f : R_- \to R_\infty$ is a cost function for Γ if, for every node (σ,t),

$$f(V(\Gamma,\sigma,t)) = \inf\{B(\Gamma,\tau,n) | V(\Gamma,\tau,n) \geq V(\Gamma,\sigma,t)\} \quad (31)$$

and for all $u \in R_-$,

$$f(u) \geq f_0(u) = -r^{-1}(1 - \beta)^{-1}[\ln(1 - \beta) + \ln(-u)] - p. \quad (32)$$

LEMMA 5. *For every incentive-compatible contract Γ, there is at least one cost function f_Γ.*

PROOF. $f_0(x)$ is the expected discounted cost of providing utility level x to a trader if incentive compatibility is not required—that is, if the trader can be fully insured. Under the incentive-compatibility constraint, then, providing a utility level at least as great as u must have an expected discounted cost of at least $f_0(u)$. In particular, the infimum in (32) must be at least $f_0(V(\Gamma,\sigma,t))$. Therefore, the function f_Γ defined by $f_\Gamma(u) = \inf\{B(\Gamma,\tau,n) | V(\Gamma,\tau,n) \geq u\}$ (where the infimum over the empty set is infinite) satisfies both (31) and (32). Q.E.D.

It is standard to solve discounted infinite-horizon optimization problems by using "backward induction" to converge from an arbitrary candidate for the cost function to the cost function of the optimal decision rule. As usually formulated [e.g., by Denardo (1967)], this method relies heavily on the boundedness of the cost function. Since the cost function here is unbounded, this method has to be modified. However, the basic idea is the same. Given some cost function f, it is assumed that for any utility levels v_0 and $v_1 < 0$,

there is a contract Γ such that $V(\Gamma,\tau^k,1) = v_k$ and $B(\Gamma,\tau^k,1) = f(v_k)$ for $k = 0, 1$, where $\langle \tau^0,\tau^1 \rangle \in (\sigma,0)$. If $W(k + \Gamma(\tau^k,0)) = w_k$, then the expected discounted cost $B(\Gamma,\sigma,0)$ of Γ would be equal to

$$b(w,v) = (1 - p)[g(0,w_0) + \beta f(v_0)] + p[g(1,w_1) + \beta f(v_1)]. \quad (33)$$

The expected discounted utility $U(\Gamma)$ would be equal to

$$u = (1 - p)[w_0 + \beta v_0] + p[w_1 + \beta v_1]. \quad (34)$$

By (22), Γ would be incentive compatible if for $k = 0,1$,

$$(-(e^{-r}))^k[w_1 - w_0] \geq (-1)^k\beta[v_0 - v_1]. \quad (35)$$

Now, letting $\geq$ denote the pointwise-comparison partial ordering on functions, let $F = \{f | f \geq f_0\}$. For $f \in F$, define Tf by

$$[Tf](u) = \inf\{b(w,v) | w \text{ and } v \text{ satisfy (34) and (35) for } k = 0,1\}. \quad (36)$$

The following three facts about T can be established by standard arguments.

LEMMA 6. *(a)* $T:F \to F$. *(b)* $Tf_0 \geq f_0$. *(c) If Γ is an incentive-compatible contract and f_Γ is the cost function for Γ defined in the proof of Lemma 5, then $Tf_\Gamma \leq f_\Gamma$.*

9. The Minimum Fixed Point of T

Motivated by the functional form of f_0, cost functions of the form

$$f(u) = c - r^{-1}(1 - \beta)^{-1} \ln(-u) \quad (37)$$

will now be considered. Define $x_k = w_k/u$, $y_k = v_k/u$, $q_0 = (1 - p)$, and $q_1 = p$. Then (33) is equivalent to

$$b(w,v) = \sum_{k=0,1} q_k[-r^{-1} \ln(-ux_k) - k + \beta(c - r^{-1}(1 - \beta)^{-1} \ln(-uy_k))]$$

$$= -[r^{-1} - \beta(r^{-1}(1 - \beta)^{-1})] \ln(-u) - p$$

$$+ \beta c - r^{-1} \sum_{k=0,1} q_k[\ln(x_k) + \beta(1 - \beta)^{-1} \ln(y_k)]. \quad (38)$$

Noting that $r^{-1} + \beta(r^{-1}(1 - \beta)^{-1}) = r^{-1}(1 - \beta)^{-1}$, (38) can be written as

$$b(w,v) = j(c,x,y) - r^{-1}(1 - \beta)^{-1} \ln(-u), \quad (39)$$

where

$$j(c,x,y) = \beta c - p - r^{-1} \sum_{k=0,1} q_k[\ln(x_k) + \beta(1 - \beta)^{-1} \ln(y_k)]. \quad (40)$$

Moreover, the constraints (34) and (35) respectively are equivalent to

$$1 = \sum_{k=0,1} q_k[x_k + \beta y_k] \quad (41)$$

and

$$(-(e^{-r}))^k[x_1 - x_0] \geq (-1)^k \beta[y_0 - y_1]. \quad (42)$$

Thus, for f of form (37), Tf is defined by

$$[Tf](u) = \inf\{j(c,x,y) - r^{-1}(1 - \beta)^{-1} \ln(-u)$$

$$|x \text{ and } y \text{ satisfy (41) and (42) for } k = 0,1\}. \quad (43)$$

By (43), $[Tf](u)$ is the solution of a convex minimization problem. The next lemma states the properties of the solution.

LEMMA 7. *There are x^* and y^* that are the minimizing values that define $[Tf](u)$. These are constants that do not depend on c or on u. $0 < y_1^* < y_0^* < \beta^{-1}$.*

The function $j(c,x^*,y^*)$ is a contraction mapping in c. By (43), Lemma 7, and the fixed-point theorem for contraction mappings, there is a fixed point f_μ of T that is of form (37) and that is the limit of the increasing sequence of functions $\langle T^n f_0 \rangle_{n \in N}$. This and Lemma 6 imply:

LEMMA 8. *For every incentive-compatible contract Γ, $f_\mu \leq f_\Gamma$.*

10. The Optimal Contract

It is now clear how to define the optimal contract Γ. First, define $V(\Gamma,\sigma,t)$ recursively by

$$V(\Gamma,\sigma,0) = f_\mu^{-1}(0) \quad (44)$$

and

$$V(\Gamma,\sigma,t + 1) = y_{\sigma_t}^* V(\Gamma,\sigma,t). \quad (45)$$

Next, define $\Gamma(\sigma,t)$ by

$$\Gamma(\sigma,t) = g(\sigma_t,x_{\sigma_t}^* V(\Gamma,\sigma,t)). \quad (46)$$

Note that, by (46) and the bounds on y^* stated in Lemma 7, the hypothesis (19) of Lemma 2 holds. From (19) and (46), it follows that the stochastic process M defined by

$$M(\sigma,t) = \sum_{n<t} \beta^n W(\sigma_n + \Gamma(\sigma,n)) + \beta^t V(\Gamma,\sigma,t) \qquad (47)$$

is a bounded martingale. Applying Doob's martingale convergence theorem (Breiman 1968, Theorem 5.23) to M shows that (16) holds.[8] Then Lemma 2 implies:

LEMMA 9. Γ *is incentive compatible.*

A parallel argument shows that (24) holds, so that:

LEMMA 10. Γ *is feasible.*

Finally, by (44), $B(\Gamma,\sigma,0) = 0$. Therefore, Γ solves P_δ. By Lemma 4:

THEOREM. Γ *solves the coalition's optimization problem* P_π.

11. The Time-series Behavior of Consumption and Wealth

In this section, it is shown that the optimal contract Γ can be supported by the issuance of infinite-lived bonds. This representation of the contract provides a characterization of the stochastic processes of consumption and credit balances for the representative consumer.

To simplify notation, define $V_0 = V(\Gamma,\sigma,0)$ and define three stochastic processes X, Y, and Z on S by

$$X(\sigma,t) = x^*_{\sigma_t} \qquad (48)$$

$$Y(\sigma,t) = y^*_{\sigma_t} \qquad (49)$$

$$Z(\sigma,t) = -r^{-1} \ln(Y(\sigma,t)). \qquad (50)$$

By induction using (45), it is seen that

$$V(\Gamma,\sigma,t + 1) = \left[\prod_{u \le t} Y(\sigma,u) \right] V_0. \qquad (51)$$

By (44), the initial credit balance is zero. By (37), this means that

$$B(\sigma,0) = c - r^{-1}(1 - \beta)^{-1} \ln(-V_0) = 0. \qquad (52)$$

Again using (37), (51) and (52) imply that

$$B(\sigma,t + 1) = f_\mu(V(\Gamma,\sigma,t + 1)) = (1 - \beta)^{-1} \sum_{u \le t} Z(\sigma,u). \qquad (53)$$

Since Z is an i.i.d. sequence of random variables, (53) shows that the credit balance of the representative trader is a random walk. The trader's consumption at node (σ,t) is $\sigma_t + \Gamma(\sigma,t)$. Using (26), (45), and (51), this can be evaluated to be

$$\sigma_t + \Gamma(\sigma,t) = \sum_{u<t} Z(\sigma,u) - r^{-1} \ln(-X(\sigma, t)V_0). \tag{54}$$

Friedman (1957) has suggested that a household will attempt to consume the annuitized value of its "permanent income" at every date. That is, the preferred consumption of the household will be the amount of interest that its wealth earns at the market rate. Equations (53) and (54) show that the traders in the model economy fit this description closely. To see this, recall that β^{-1} is the marginal rate of transformation of income from one date to the next: in other words, $\beta^{-1} - 1$ is the interest rate in the economy. For β close to 1, Taylor's theorem shows that this interest rate is approximately equal to $1 - \beta$. According to the permanent income hypothesis, then, $(1 - \beta)B(\sigma,t)$ should be equal to $\sigma_t + \Gamma(\sigma,t)$. By (53) and (54), the difference between these quantities is $r^{-1} \ln(-X(\sigma,t)V_0)$. This difference is thus an i.i.d. sequence of random variables with finite variance. Asymptotically, the difference will be negligible relative to the credit balance, since the latter is a random walk, which has a variance that tends to infinity. Thus it is a good approximation to say that the representative trader consumes the annuitized value of his wealth at every date.

12. The Optimal Contract as an Outcome of Competition

Comparing (53) and (54) also leads to a characterization of the trader's credit balance and consumption in terms of holding and trading infinite-lived bonds. To discuss below some issues that are raised by this characterization, let us assume that traders are allowed to buy and sell bonds only through an intermediary. At every date u, the trader pays $\sigma_u + r^{-1} \ln(-X(\sigma,u)V_0)$ to the intermediary, and the trader receives $Z(\sigma,u)$ bonds in return. At every date t thereafter, each of the bonds pays one unit of consumption. Thus the discounted value at date t of the proceeds to be received at t and future dates from the trader's bond transaction at date u is $(1 - \beta)^{-1}Z(\sigma,u)$, and the sum of these values for all dates prior to t is $B(\sigma,t)$.

There is a clear sense in which this long-term relationship between traders and the intermediary is competitive. To begin with, the intermediary earns zero profits. No competing intermediary could successfully bid traders away by offering an alternative long-term contract. By construction of the optimal contract Γ, any contract that would provide higher levels of utility to traders must require the intermediary offering it to provide a subsidy. In particular, then, the optimal contract defines a no-surplus allocation in the sense of Ostroy (1980).

It remains an open question whether the optimal allocation is supported by a system of Walrasian prices. In fact, it is difficult even to formulate

this question. The price system in question would be a continuous linear functional on some linear topological space of commodity bundles. In this paper, no topology on commodity bundles has been defined. A price system supporting the optimal allocation would have to be an incomplete-markets system, because the allocation in which every trader is fully insured (consuming p units of income at every date with certainty) is the only Walras equilibrium with complete markets. The events on which a trader's consumption is contingent in the optimal allocation are defined in terms of his own endowment, however, so there would have to be a vast array of contingent commodities—defined for sufficiently many events to make the endowment of every trader measurable.

Whether or not there exist supporting prices in principle, the interpretation of the optimal contract as a long-term relationship between traders and an intermediary suggests that the allocation would in fact be supported by a set of market institutions that might include rationing of credit by the intermediary in some circumstances. Rationing typically has been described as a constraint on the total amount of debt that a trader would be allowed to owe.[9] Since a trader's credit balance determined by the optimal contract is a random walk, that sort of constraint is not imposed. Rather, the *rate* at which a trader increases his debt is bounded. A trader may owe so much interest on his existing debt that it will exceed the amount of new debt that he issues. In this case, he will make a net payment to the intermediary even if he has received no income.

13. Conclusion

The problem of optimal incentive-compatible allocation has been studied in a simple model of an economy where traders have private information about their endowments. This optimal allocation has been characterized in sufficient detail to describe the stochastic process of a representative trader's consumption and wealth. This stochastic process has the properties described by the permanent income hypothesis. The contract that supports the optimal allocation is competitive in the sense that no other incentive-compatible contract could guarantee traders equally high utility unless it provided them with a subsidy.

Several questions have not been resolved, though, even in the context of this highly simplified model. One of these questions concerns whether the imposition of incentive compatibility as a constraint will affect the characterization of efficient production as well as of efficient allocation. Recall that in the model economy there is a technology that can transform consumption from one date to the next at exactly the same rate as the traders' rate of pure time preference. Since the aggregate endowment is the same at

every date, this technology would not be used if agents were fully insured. Since full insurance cannot be provided here, though, it is possible that the optimal incentive-compatible allocation requires a technological transformation of the aggregate endowment.

Since the stochastic process of traders' consumption in the optimal allocation resembles the consumption of actual households, one might also study whether the optimal allocation could be supported by market institutions that would resemble actual institutions of intermediated credit. When the optimal allocation has been interpreted in this paper in terms of intermediated credit, the intermediary has been described as offering a very limited choice to traders. There is a fixed quantity of bonds that a trader is allowed to buy at a single date and a fixed quantity that he is allowed to sell, and he must make exactly one of these two permissible transactions. It is plausible, though, that traders could be induced to make these choices from a constraint set that looks more like a budget set, or like some other contract that is observed in actual financial markets. In particular, it would be of interest to know whether the optimal allocation could be supported by the intermediary charging a spread between bid and ask prices on bonds, or whether there must be explicit quantity rationing of credit.

Finally, the model studied in this paper is severely restricted: there is no aggregate uncertainty, there is no private information about future endowments, and traders are always able to honor contracts that require them to make arbitrarily large payments. I hope that the analysis developed here will also be applicable to more realistic models that do not incorporate these stringent simplifying assumptions.

Notes

1. Much of the research reported in this paper was done while I was a visiting fellow at the Institute for Advanced Studies, Hebrew University of Jerusalem. I would like to thank Jacques Cremer, Christian Gilles, Arnold Kling, Robert E. Lucas, Jr., Julio Rotemberg, and Neil Wallace for their comments on earlier drafts.

2. This equivalence is implied by the ergodic theorem of probability theory.

3. On the surface, this statement may seem inconsistent with the choice of consumers to make occasional large expenditures on commodities such as medical care and consumer durables. The theories of human capital and of household production resolve this dilemma by treating these expenditures as investment rather than as consumption.

4. This statement is suggested by Friedman's (1957) permanent income hypothesis. Obviously, since actual households have a lower bound on their feasible consumption, the description is not literally true. The sense in which it is an approximation has been clarified by Yaari (1976), Schechtman (1976), Bewley (1977), Hall (1978), and others. Statistical tests of the random-walk hypothesis using data on consumption, beginning with Hall (1978), have had mixed results [cf. Shapiro (1984) and the work cited by him]. However, the characterization given here is intended only as a highly stylized contrast between the performance of actual economies and what would be expected in an environment with complete information.

Jovanovic (1983) and Scheinkman and Weiss (forthcoming) have shown that, in an environment with aggregate uncertainty as well as purely individual uncertainty, allocations that resemble those derived here on the individual level may also involve business-cycle phenomena on the aggregate level. The present paper may be viewed as providing an explanation of the restrictions on market structure that they assume.

5. Radner (1981) and Rubenstein and Yaari (1983) have studied incentive problems in environments where there are infinite-lived agents who maximize asymptotic-average utility and where there is imperfect information. Their results can be restated to apply also to the specific kind of incomplete information that is assumed here. Because the agents have no time preference, it is possible for agents in noncooperative equilibrium to achieve the best utility levels that would be possible if they were to act cooperatively under complete and perfect information. In contrast, when traders maximize discounted utility, they are unable to attain the same utility levels that they could if information were complete. The model studied here is essentially identical to that of Townsend (1982), except that Townsend imposed an exogenous finite bound on the length of contracts. There is some work in a similar spirit in monetary economics, including Gale (1982, chap. 6) and Bewley (1983).

6. See Harris and Townsend (1981).

7. Roy Radner has emphasized in discussion that full insurance is not possible in an economy with finitely many traders who have statistically independent endowments, because with positive probability all would simultaneously suffer a loss. However, traders who were ex ante identical would pool their endowments, and by doing so they would greatly decrease their individual exposure to risk.

8. A bounded martingale converges both almost surely and also in the function space L^1. That is, if $M(\sigma,t)$ is a bounded martingale, then it converges almost surely to a random variable $M^*(\sigma)$, and $M(\sigma,t) = E_s[M^*|\langle \ \rangle_t](\sigma)$.

9. For example, Friedman (1957) suggests that a household can borrow only to the extent of its nonhuman wealth.

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