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Optimal Unemployment Insurance and Employment History

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In existing unemployment insurance programmes, it is standard to condition eligibility on the previous employment record of unemployed workers. The purpose of this article is to study conditions under which the efficient contract exhibits these properties. In order to do so, we characterize the optimal unemployment insurance contract in asymmetric information environments in which workers experience multiple unemployment spells. We show that if quits cannot be distinguished from layoffs, it is optimal to condition the benefits paid to unemployed workers on their employment history, in particular, the coverage should increase with the length of previous employment spells.

1. INTRODUCTION

In most existing unemployment insurance (UI) programs, eligibility depends on previous employment history. For instance, in the United States, a minimum of six months of employment is needed to qualify for benefits, and coverage ratios increase with the length of previous jobs. In addition, these restrictions have a significant impact on equilibrium outcomes. Evidence from Canada[1] shows that job termination rates double after a year, coincidental with the minimum number of periods required to qualify for unemployment benefits. This aspect has been neglected in theoretical work on optimal design which has focused on the simplified case of a single unemployment spell.[2] This article extends that research by considering the problem of optimal unemployment insurance design in a model of multiple unemployment spells.

We model unemployment insurance design as a repeated moral hazard problem. In our model, the search effort of unemployed workers cannot be monitored by the enforcement agency; as a consequence, the insurance mechanism must trade off incentives for job search with unemployment duration risk. The unemployment insurance programme specifies contingent transfers from the enforcement agency to the worker as a function of current and past

1. See Christofides and McKenna (1996) and Baker and Rea (1998).
2. See Baily (1978), Shavel and Weiss (1979) and Hopenhayn and Nicolini (1997). Exceptions that study on-the-job moral hazard problems are Wang and Williamson (1996, 2002) and Zhao (2000).

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employment/unemployment records. The optimal programme is the one that minimizes the budget given an *ex ante* expected utility for the worker.

As a first exploration we assume, in Section 2, that termination rates are exogenous. We show that previous results for the case of a single unemployment spell have their analogues in our multiple spells case: transfers to unemployed workers decrease with the length of their unemployment spell, while re-employment taxes increase with the length of that spell. Furthermore, these transfer schedules also decrease with previous unemployment spells. The intuition for these results is simple: since agents are risk averse and value consumption smoothing, optimal incentives are provided by using permanent and not temporary reductions in consumption. Hence, the longer a worker is unemployed, the lower is his permanent consumption level.

As job termination is exogenous, workers are completely insured against job loss. This property has two interesting implications. First, as optimal transfers to the unemployed do not depend on the length of previous employment spells, this environment does not provide a rationale to the employment history restrictions that motivates our analysis. Second, if an unemployed worker becomes employed and immediately loses this job, his replacement ratio is increased; thus, finding a job is a way to upgrade unemployment benefits. This suggests the possibility of a loophole in the optimal contract that can lead to opportunistic behaviour. Indeed, anecdotal evidence in countries with generous unemployment insurance programmes suggests that the UI programme may induce inefficient quits.

Opportunistic behaviour can only arise if the principal cannot distinguish quits from layoffs.[3] The article studies, in the remaining section, the design problem when quits and layoffs cannot be perfectly monitored.

We explore two ways in which opportunistic workers may take advantage of the UI contract in this modified scenario. First, we consider the case where the disutility of working and the generosity of unemployment insurance induce voluntary quits from socially efficient jobs. If the "no quit" constraint is not taken into account in the UI design, unemployed workers will look for jobs and, once employed, may quit just to upgrade their benefits. We show that if this constraint binds, it is optimal to condition the transfers on the agent's employment history. In particular, we show that unemployment benefits rise with tenure, and the tax paid while employed decreases with employment length. Second, we consider the opposite case, in which the UI programme may create inefficient matches. This situation can arise in environments with job heterogeneity where unemployed workers may take socially inefficient jobs just to upgrade future unemployment benefits. The UI programme increases the private−but not the social−value of jobs, and therefore it may create an adverse selection problem. We show that in the presence of such adverse selection problem, the optimal contract shares most of the qualitative properties of the previous case: future unemployment benefits rise and re-employment taxes decrease with employment tenure. It is interesting thus, that even though unobservable quits potentially create two very different incentive problems, both can be handled in a qualitatively similar way: increasing coverage with work experience.[4]

The main contribution of the article is to consider incentives and UI design in the presence of repeated unemployment spells. Papers by Wang and Williamson (1996, 2002) and Zhao (2000) also derive optimal contracts that exhibit employment dependence. These papers differ

3. One interpretation of the evidence provided by Christofides and McKenna (1996) and Baker and Rea (1998) is that there are many workers in Canada that quit right after a 12-month tenure to claim UI benefits.

4. It should be noted, however, that the quantitative impact of the two incentive problems on the optimal contract may well be different.

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from ours in that they introduce a moral hazard problem for employed workers.[5] Even though the incentive problem we study–opportunistic quits–is of very different nature, their results are closely related to the first of our two scenarios, when the no-quit constraint binds. Wang and Williamson account for general equilibrium effects–and allow for private savings. However, given the complexity of their environment, only simulation results are provided. While our environment abstracts from some of those features, we provide very general qualitative properties of the optimal contract. The paper by Zhao (which is contemporaneous to our article) also provides a general characterization in a simple environment with unobservable job effort.

The article proceeds as follows: in Section 2 we describe the model with repeated unemployment spells, in an environment where quits and layoffs can be distinguished by the principal. We relax this assumption in Section 3 and study the optimal design when moral hazards and adverse selection incentive problems are both present.

2. THE MODEL

We study the unemployment insurance problem within the framework of standard repeated moral hazard models, where the principal (enforcement agency) cannot monitor unemployed workers' search effort.[6] The preferences of the agent are given by

$$E \sum_{t=0}^{\infty} \beta^t \left[u \left(c_t \right) - a_t \right] \tag{1}$$

where c_t and a_t are consumption and effort at time t, $\beta < 1$ is the discount factor, and E is the expectations operator. The function u is strictly increasing, strictly concave and unbounded above.

For convenience, we assume that effort while at work is zero. As long as the principal can monitor quits, this assumption is just a normalization of the utility function and is therefore without loss of generality. In the next section, where we analyse the case in which the principal cannot distinguish quits from layoffs, this assumption is relaxed.

We assume that positive effort is required to generate job offers while workers are unemployed. In particular, we assume that a_t can take only two values, 0 or 1. When effort is zero, the probability of finding a job is also zero; while if it is one, the probability of finding a job is some positive number strictly smaller than one called p.[7] We assume that the choice of effort cannot be monitored by the principal. Finally, we also assume that all jobs are identical, offering a constant wage w over time and terminating at a constant and exogenous rate λ.

We follow the literature in repeated moral hazard and assume that the principal can directly control the consumption of the agent or, equivalently, monitor its wealth.[8] An unemployment insurance contract specifies, for each period, a net transfer to the agent and, if the agent is unemployed, a recommended action as a function of the realized history. The transfer while

5. Wang and Williamson assume that on-the-job effort affects the probability to keep the job, while in Zhao it also affects the probability distribution of output.

6. For early examples of dynamic mortal hazard models, see Phelan and Townsend (1991), Rogerson (1985) and Spear and Srivastava (1987).

7. In Hopenhayn and Nicolini (1997) and in a preliminary version of this article, we allowed for a continuum of effort levels. It significantly complicates the analysis without providing additional insights.

8. According to Engen and Gruber (2001), the median 25 to 64-year-old worker has gross financial assets equivalent to less than 3 weeks of income, and the average unemployment spell for those becoming unemployed is approximately 13·1 weeks. Gruber (1997) shows that consumption falls by more than 20% for workers that enter unemployment with no claims to insurance.

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unemployed, c^u, can be interpreted as the unemployment benefit, so the ratio c^u/w corresponds to the replacement rate. Note that, as in the recent literature, we do not restrict the analysis to contracts where the replacement rate is fixed over time.[9]

We associate to each contract an expected discounted utility to the agent V and a cost to the principal C. This cost is measured by the expected discounted value of the net transfers required to provide the agent with utility V. These values assume that the agent responds to the contract rationally maximizing (1) by choosing the search effort. Given a level of lifetime utility for the agent at time zero, the optimal contract minimizes the cost of granting that utility to the agent in an incentive compatible way.

Incentive problems arise when the principal is interested in implementing positive search effort. In the following analysis we restrict to this case. The cost of implementing positive search effort increases with the promised utility to the agent. As a consequence, for sufficiently high initial utility levels, implementing high effort may not be optimal. In the Appendix we solve the general problem and characterize the set of utilities such that the high effort is indeed optimal. The analysis in this section corresponds to utility levels within this set.[10]

2.1. *Recursive contract*

Consider first the situation of an unemployed worker. In each period, the contract specifies current consumption c and effort a, together with continuation values, contingent on the employment status of the agent at the end of the period. Let V^e correspond to next period entitlement if the worker is employed, and V^u the corresponding expected discounted utility if he is not. Since V is the expected discounted utility offered by the contract to the worker at the beginning of the current period, and we consider the case where $a = 1$, then

$$V = u(c) - 1 + \beta\left(pV^e + (1-p)V^u\right). \tag{2}$$

Given that the principal cannot observe effort, and it is optimal to set $a = 1$, the contract must satisfy the following incentive compatibility constraint:

$$\beta p\left(V^e - V^u\right) \geq 1. \tag{3}$$

Note that in this recursive formulation, a contract delivers a current transfer and promises future utility to the agent. This promised utility will be attained using an implicit sequence of future transfers. Thus, associated to each possible value for V, there is an associated cost C for the principal, given by the minimum budget required to offer the agent utility V. Thus, $C(V)$ denotes the minimal budget necessary to provide a lifetime expected utility V to an unemployed worker.

Consider now an employed agent who is entitled to a certain utility level. This utility level will be attained by current consumption while at work, c, and promised utilities, contingent on whether the job continues or not. Associated to this utility level there is a cost to the principal–that can be negative–denoted by W. Let $W(V)$ be the minimum possible budget required to provide utility V to an employed agent. We assume, as it is standard in the literature, that the principal discounts the future at the same rate as the agent. Then, the optimal problem

9. For a detailed discussion of the optimal time path of the replacement rate and a quantitative evaluation in a single unemployment spell environment, see Hopenhayn and Nicolini (1997).

10. In Appendix A.1 we also show that for higher utility levels the solution is either trivial–because there is no incentive problem–or it is a lottery between the trivial solution and the one we solve for in this section.

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when the worker is employed is given by

$$W(V) = \min_{c, V^e, V^u} c - w + \beta[(1-\lambda) W\left(V^e\right) + \lambda C\left(V^u\right)]. \tag{4}$$

$$\text{subject to} \quad : \quad u(c) + \beta\left[(1-\lambda)V^e + \lambda V^u\right] = V. \tag{5}$$

With probability λ employment terminates, and the worker is granted continuation utility V^u at a cost to the principal $C(V^u)$; with probability $(1-\lambda)$ employment continues, and the worker is granted continuation utility V^e at a cost to the principal $W(V^e)$. The optimal choice of c, V^e and V^u minimize the cost of granting the value V to the employed worker.

Given the wage w, the difference $w - c$ can be interpreted as a tax the worker pays once employed. Note that we do not restrict to contracts where consumption is constant, so tax rates will generally depend on the individual history of each agent; in particular, we allow for tax rates that depend on the number and length of previous unemployment spells.[11]

Consider now the dynamic programming equation when the worker is unemployed:

$$C(V) = \min_{c, V^e, V^u} c + \beta[pW\left(V^e\right) + (1-p)C\left(V^u\right)]. \tag{6}$$

$$\text{subject to} \quad : \quad u(c) - 1 + \beta\left[pV^e + (1-p)V^u\right] = V. \tag{7}$$

$$\text{and} \quad : \quad \beta p\left(V^e - V^u\right) \geq 1. \tag{8}$$

In the current period, the unemployed worker receives a transfer c. With probability p the worker is employed next period and is granted continuation utility V^e at a cost to the principal $W(V^e)$; with probability $(1-p)$, the worker remains unemployed and is granted continuation utility V^u at a cost to the principal $C(V^u)$. The optimal choices of c, V^e and V^u minimize the cost of granting this initial value V to the unemployed worker subject to the incentive compatibility constraint.

2.2. *Characterization of the solution*

It is straightforward to verify that both functions $C(V)$ and $W(V)$ are increasing and strictly convex, as the corresponding return functions are linear, and the function u in the constraints is strictly concave.[12] Consequently, these functions are almost everywhere differentiable.

From the first-order and envelope conditions of the problem when the agent is employed, we obtain:

$$W'(V) = \frac{1}{u'(c^e)} = W'\left(V^e\right) = C'\left(V^u\right). \tag{9}$$

By the strict convexity of W, it follows that $V = V^e$ so promised utility remains constant while employed. This has two important implications. First, consumption of an employed worker is constant. If $c^e < w$, the worker will be taxed, otherwise he will get a subsidy. Second, promised utility in case of unemployment, V^u, is also constant while employment lasts, and therefore independent of the length of the employment spell. Hence there is no

11. For a detailed analysis of the optimal time path for this tax and a quantitative evaluation in a single unemployment spell environment, see Hopenhayn and Nicolini (1997).

12. For a standard proof, replace the control variable c by its utility index $u = u(c)$, which is a monotone transformation. With this transformation, all constraints are linear. Since the inverse function $c(u)$ is convex, so is the return function. Convexity of the value functions follows immediately from standard dynamic programming arguments.

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employment dependence in the optimal unemployment insurance plan. This follows quite naturally from the fact that employment termination is exogenous. As there is no information problem while the agent is working, it is optimal for the risk-neutral principal to completely insure the agent against the job termination shock. In this environment, the employment history-related restrictions that we mentioned in the first paragraph of the article are inefficient, since the length of the employment spell provides no valuable information to the principal.

However, it turns out that the characterization of the optimal contract when workers are unemployed–the $C(V)$ problem–suggests that the result may not be robust to natural generalizations of the environment, like allowing positive effort on the job or allowing for job heterogeneity. In what follows, we characterize the optimal contract problem when the agent is unemployed and explain why we believe it is interesting to study those generalizations.

If we let δ be the multiplier of the incentive constraint, the first order and envelope conditions of this problem are

$$C'(V) = \frac{1}{u'(c^u)}. \tag{10}$$

$$C'(V^u) = C'(V) - \delta\frac{p}{1-p}. \tag{11}$$

$$W'(V^e) = C'(V) + \delta. \tag{12}$$

It is straightforward to show that the incentive constraint always binds.[13] The next proposition characterizes the properties of the optimal contract.

Proposition 1. *The unemployment benefit decreases and the re-employment tax increases with the length of the unemployment spell.*

Proof. Note that the above first-order conditions imply

$$C'(V) > C'\left(V^u\right)$$

and by the convexity of $C(\cdot)$ this implies that $V^u < V$, so the continuation utility decreases while the worker is unemployed. Moreover, the envelope condition

$$C'(V) = \frac{1}{u'(c^u)}$$

implies that the benefit is increasing with V. Thus, over a single unemployment spell, the benefit decreases with the length of the spell. On the other hand, as the incentive compatibility constraint binds, the optimal continuation values V^u and V^e move in the same direction. Consequently, V^e also decreases with the length of the unemployment spell. From the envelope condition of the problem while the worker is employed,

$$W'(V) = \frac{1}{u'(c^e)}.$$

As the wage is constant, this means that the tax increases with the duration of the unemployment spell. ‖

13. Assume not, then V^e and V^u are invariant over time and $\frac{1}{u'(c^e)} = W'(V^e) = C'(V^u) = \frac{1}{u'(c^e)}$ for all periods. Thus, the sequence of consumption is independent of the employment state and unemployed workers have no incentive to choose $a = 1$.

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This proposition is a straightforward extension of Hopenhayn and Nicolini (1997) to the case of multiple unemployment spells: the use of higher re-employment taxes as part of the incentive structure follows from the efficiency of permanent income punishments when workers have a preference for consumption smoothing.

Corollary 1. *All net future contingent transfers increase with the value of V.*

Proof. This follows immediately from the last proposition. ‖

The Corollary highlights the persistent effect of rewards and punishments. In particular, it implies that the replacement ratio and the re-employment tax both depend in a non-trivial way on all previous unemployment spells and on their duration.

It should be noted, however, that there is no simple statistic to compare different unemployment/employment histories. If two workers a and b experience the same number of unemployment spells but each spell is longer for worker a, he will face higher taxes or lower replacement ratios. But besides this very strong ordering of unemployment histories, little else can be said. For example, consider two workers a and b that start employed with identical V (and thus the same consumption). Now, suppose b remains employed for the next two periods whereas a loses the job after the first period and is re-employed after one period of unemployment. From equations (9–12) it follows that the final continuation utility of a will exceed that of b, so his consumption will be higher (tax will be lower).[14] This occurs loosely because, although there is a reward for finding a job, there is no punishment for losing it. The previous discussion highlights that while finding a job is a way to "upgrade" the level of unemployment benefits, losing the job does not "downgrade" the coverage. This asymmetry may create adverse selection problems in the decision of both creating and destroying job matches, as discussed in the next section.

In summary, in providing incentives for search effort, the optimal contract generates a loophole, which could be exploited by workers to upgrade their unemployment benefits. In what follows, we discuss this issue.

2.3. *A loophole in the optimal contract*

The empirical literature on job duration provides evidence that hazard rates for job termination tend to rise sharply at the time workers become eligible for unemployment insurance, suggesting the existence of opportunistic quits.[15] In this section we identify a loophole in the optimal insurance programme derived above that leaves room for such opportunistic behaviour.

Proposition 2. *Consider an unemployed worker with promised future utility equal to V. If the worker finds a job and is fired the following period, the optimal contract will offer him a utility level $\overline{V^u} > V$.*

Proof. Letting V^e denote the promised utility when the worker finds the job, from the above first-order conditions for the problems defining C and W, it follows that

$$C'(V) < W'(V^e) = C'(\overline{V^u}).$$

By the convexity of the function C, it follows that $\overline{V^u} > V$. ‖

14. This fine point was indicated by a referee.

15. As we mentioned before, in Canada job termination rates double at the 12 month period, which is the minimum period for eligibility (see Christofides and McKenna (1996) and Baker and Rea (1998)).

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This proposition has two critical implications. First, an immediate implication of the incentive constraint is that $V^e > V^u$. However, this does not imply that, once employed, the worker is better off keeping the job rather than losing it. In particular, since $\overline{V^u} > V^u$ it could be the case that $\overline{V^u} > V^e$. This would give rise to opportunistic quit behaviour. Second, taking a job, no matter how short-lived it is, upgrades unemployment benefits. This could give incentives for workers to *fake* employment, *e.g.* taking a bad job and then quitting, just to upgrade their unemployment insurance benefits. In either case, workers could abuse the UI system only if the principal is not able to monitor perfectly the causes of job termination. We analyse these cases in the following section.

3. QUITS AND LAYOFFS

In this section we assume that the principal cannot distinguish quits from layoffs. This is important, since UI is designed to protect workers against exogenous shocks. As quits may be considered, at least to some extent, endogenous, only fired workers should qualify for benefits. To cope with this problem, many unemployment insurance programmes require involuntary separations to re-qualify for benefits. Admittedly, however, this distinction between involuntary and voluntary separations is hard to establish in practice,[16] leaving room for opportunistic behaviour. Therefore, it is natural to extend the model and study the optimal contract when the principal cannot distinguish quits form layoffs.

Two possible forms of opportunistic behaviour may arise. First, if workers face a very generous unemployment insurance, they may find optimal to quit, collect benefits and avoid the disutility of working. This case is considered in Section 3.1 and is more closely related to the work of Wang and Williamson (1996, 2002) and Zhao (2000). The incentive problem is different though, since in our case the unobservable variable is not effort while working but the decision to quit.[17]

Second, when job offers are heterogeneous and the principal cannot monitor the quality of jobs, workers may take bad jobs and soon quit from them just to upgrade their unemployment benefits. This case is considered in Section 3.2. As we show below, when any of these two cases occurs, it is optimal to condition benefits on employment duration. In particular, we show that in order to prevent either form of opportunistic behaviour, replacement ratios rise with employment duration and the tax paid by an employed worker decreases with tenure.

3.1. *Quitting from good jobs*

So far we assumed that effort at work does not affect the utility of an employed worker. For the analysis in the previous sections, this was just a normalization with no effect on any qualitative results. However, when quits and layoffs cannot be distinguished, workers could decide to quit opportunistically. As the following proposition shows, the contract derived above is not immune to such behaviour.

16. This is indeed the interpretation that Baker and Rea (1998) provide (see p. 82).

17. Incentive problems while at work of the kind they analyse may be resolved within the firm–worker relationship. In this case, voluntary quits of the kind analysed here may be less of an issue. This would be the case, for instance, in a simple efficiency wage model, since the wage contract is chosen such that the worker never has incentives to quit. We thank a referee for this comment.

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Proposition 3. *Suppose $e \geq 0$ is the disutility of effort for an employed worker. Let V^e and V^u denote the utility value in the optimal contract, for a currently employed worker with initial utility V^e,who remains employed or loses the job, respectively.*

1. *If $e = 0$, then $V^e \geq V^u$.*
2. *If $e > \frac{1}{p}$, then $V^u > V^e$.*

Proof. See Appendix. ‖

This proposition implies that when the disutility of effort at work is sufficiently high, once employed, workers would certainly prefer to quit after one period[18] and claim unemployment benefits, rather than continuing to be employed. To prevent this behaviour, the optimal contract must guarantee that for an employed worker

$$V^e \geq V^u, \tag{13}$$

where V^e and V^u correspond to the utility levels of keeping the job and losing it, respectively.

As in the previous model, given that utility is linear in effort but concave in consumption, there is a high enough value for utility such that it is inefficient for the agent to work. Thus, depending on the starting value of the state, V^e, it may not be optimal to impose this constraint. The results that we characterize in what follows only hold if it is indeed optimal to have the agent working next period.

Adding constraint (13) to the optimization problem defined by equation (4) and the promise-keeping constraint (5), we obtain a programme that is almost identical to the one considered for an unemployed worker: if binding, the incentives needed to deter job quits are quite similar in nature to those needed to induce job search. Not surprisingly, the first-order conditions for the two problems are almost identical. Consequently, if the no-quit constraint (13) binds,

$$C'\left(V^u\right) < W'(V) < W'\left(V^e\right), \tag{14}$$

and if it does not bind

$$C'\left(V^u\right) = W'(V) = W'\left(V^e\right). \tag{15}$$

Together with the envelope conditions, the first equation implies that consumption falls when the worker loses the job (incomplete replacement) and it rises if the worker remains employed.

Let $\left\{V_t^e\right\}$ and $\left\{V_t^u\right\}$ denote the optimal continuation utilities derived from this programme for a worker that starts employment at $t = 0$ with an entitlement of utility V_0^e. The following proposition shows that if the no-quit constraint binds in any period, then it must bind in all, as long as it is efficient to have the agent working.

Proposition 4. *Suppose there exists an elapsed duration time t such that the constraint $V_t^e \geq V_t^u$ binds and that it is optimal to keep the worker employed. Then, the sequences V_t^e and V_t^u are strictly increasing along the employment spell.*

Proof. See Appendix. ‖

18. The sufficient condition $e > 1/p$ is far from necessary. On the other hand, note that no matter how large e is, the incentive constraint of the unemployed problem ensures that workers that find a job are better off taking that job.

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Corollary 2. *If the value of e is high enough so that the no-quit constraint binds at the beginning of an employment spell, consumption will be strictly increasing (taxes decreasing) with tenure during this spell and so will be the unemployment benefit.*

Proof. Follows immediately from the previous proposition and the envelope conditions. ‖

It follows from these results that the utility given by these two sequences is strictly increasing in the employment tenure. In particular, this implies that taxes decrease with tenure and replacement ratios increase with the length of the previous employment spell.

The intuition behind these results is the same as the one behind the falling replacement rates for an unemployed worker. The contract must make employment an attractive state and thus reward duration. Optimal rewards are permanent income rewards. Thus, consumption in both future employment and unemployment states must increase with tenure on the job. Finally note that when the incentive constraint binds, using equation (14) and the envelope conditions,

$$\frac{1}{u'\left(c_{t+1}^u\right)} = C'\left(V_{t+1}^u\right) < W'\left(V_t^e\right) = \frac{1}{u'\left(c_t^e\right)}$$

so consumption decreases when a worker becomes unemployed, *i.e.* the replacement rate (after taxes) is less than one as occurs in most unemployment insurance programmes.

3.1.1. The optimal sequence of benefits.

Let us now turn to the problem of the principal when the worker is unemployed. Formally, the problem for an unemployed worker−given by equations (6), (7) and (8)−remains unchanged, except for the definition of the function W, which is still increasing and convex. As far as the incentive constraint for search binds, the qualitative properties remain unchanged.

Proposition 5. *The search incentive constraint binds.*

Proof. We may distinguish two cases: (1) the *no quit* incentive constraint does not bind; (2) the *no quit* incentive constraint binds. For the first case, the standard proof sketched in footnote 11 applies. Consider now the second case. Letting V^e denote the initial value when the worker takes the job and $V_1^e = V_1^u$ the continuation values after the first period. Assume towards a contradiction that the search incentive constraint in the prior period did not bind. This implies (i) consumption c in the first period of employment is exactly the same as in the previous period, (ii) the continuation utility if unemployed equals initial utility, $V^u = V$, and (iii) $C'(V) = W'(V^e) > C'\left(V_1^u\right)$ (follows from 14) so $V_1^u < V^u$. It follows that

$$\begin{aligned} V^u &= V \geq u(c) + \beta V^u > u(c) + \beta V_1^u \\ &= u(c) + \beta\left[(1-\lambda)V_1^e + \lambda V_1^u\right] \\ &= V^e - e \end{aligned}$$

implying that $V^u > V^e$, which obviously violates the search incentive constraint. ‖

As a consequence, with the exception of the employment dependence properties indicated above, all other qualitative characteristics of the unemployment insurance programme (*i.e.* that benefits decrease and taxes increase with the length of the current and previous unemployment spells) remain unchanged.

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3.2. *Accepting bad jobs*

In this section we explore another source of incentive problems, namely, the possibility that a worker may take an undesirable match and then quit just to upgrade his/her unemployment benefit. In order to evaluate this possibility, we extend the model allowing for heterogenous job matches, the quality of which cannot be perfectly monitored by the principal. In contrast to current unemployment insurance generosity, which will typically lead workers to be excessively picky, future generosity may indeed have the opposite effect.[19]

We consider an environment where, besides the job opportunities described so far, we allow for a second type of jobs. These new jobs are socially inefficient in the sense that, in the absence of unemployment insurance, no worker would ever accept them. However, with UI, the presence of such job opportunities introduces new constraints into the optimal contract problem. In what follows, we solve this problem, taking explicitly into account these additional constraints. We show how employment dependence arises as an optimal response to prevent workers from exploiting these opportunities.

We make the following assumptions regarding these second type, or *bad* jobs.

1. *Bad jobs* are socially inefficient because they provide a lower flow of utility. These jobs pay the same wage w as the good jobs, but generate disutility d per period.
2. Bad jobs arrive every period with probability one, independently of the effort level.
3. The termination rate of bad jobs is the same as good jobs (λ).
4. The principal can monitor job transitions between bad jobs.

A natural alternative to assumption 1 would be to define the bad job as one with a lower wage w_0. In the absence of unemployment insurance, the two problems are identical, letting $d = u(w) - u\,(w_0)$. However, this is not true for an unemployment insurance programme that involves transfers during the employment state, since the marginal valuation of such transfers will differ depending on the wage received. Our separability assumption avoids this complication, and thus simplifies considerably the analysis.[20] In addition, for the analysis that follows, it is key that the principal cannot observe the quality of the job. We find it more natural to assume heterogeneity in utility, which is unobservable. Assumption 2 is made for simplicity and it captures the idea that a bad job is more readily available than good jobs. Assumption 3 is also made for simplicity, and it is without loss of generality: in the working paper version we show that the results also hold for any arbitrary termination rate of bad jobs although the analysis becomes substantially more complicated. Assumption 4 makes assumption 3 operational: if the principal cannot monitor job transitions, effective termination rates are zero, because when one job terminates the worker can immediately take another one. Note that assumption 4 is also without loss of generality as it becomes irrelevant when the termination rate of bad jobs is zero.

As in the previous section, we assume that quits and layoffs cannot be distinguished by the principal.

The bad job provides a costly way to send a signal of employment that the principal cannot distinguish from the good job. We assume d is high enough such that in the absence of unemployment insurance no unemployed worker would take the bad job. However, an unemployment insurance contract with no employment dependence like the one considered

19. Mortensen (1977) considers this latter effect in a search model with multiple spells.

20. We conjecture that similar properties of the optimal unemployment insurance programme would emerge without this separability assumption. However, either strong conditions must be imposed in preferences to ensure that the problem is convex or randomizations might be optimal.

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before may increase the private value of these jobs. We also assume, as it is standard in the search literature that, while employed, the worker cannot look for another job.[21]

Instantaneous utility derived from the bad job is

$$u(w) - d,$$

where d is such that

$$d > \alpha[u(w) - (u(0) - 1)]. \tag{16}$$

This inequality is imposed to make sure that, in the absence of insurance, the worker is better off searching for the good job than taking the bad one. Indeed, if we let V^s denote the value that an unemployed worker has–in the absence of any insurance–if it is optimal to search, it is easy to check that:

$$(1-\beta)V^s = \alpha(u(0) - 1) + (1-\alpha)u(w), \tag{17}$$

where

$$\alpha = \frac{1-\beta(1-\lambda)}{1-\beta(1-(\lambda+p))}.$$

Equation (17) expresses the flow value of utility of this unemployed worker as a weighted sum of the flow when unemployed $(u(0) - 1)$ and the flow when employed $(u(w))$, where the weights α and $1-\alpha$ are derived from the rates of exit from unemployment p and from employment λ, together with the discount factor.

On the other hand, the value of taking the bad job and quitting after one period is given by:

$$V^b = u(w) - d + \beta V^s.$$

The worker will not take the bad job, if and only if $V^b \leq V^s$, *i.e.*

$$u(w) - d < (1-\beta)V^s$$

which after substituting for V^s delivers condition (16).

3.2.1. The self-selection constraint. Consider a given unemployment insurance contract, specifying net transfers to the worker τ_t as a function of the state (employed, unemployed) and the employment history h. Suppose that this history is such that the worker is unemployed at time zero. Let V indicate the value that the contract gives to the worker at this node. As in the previous section, let V^u denote the value if the worker remains unemployed, V^e the value if the worker becomes employed and c the current consumption (equal to the current period transfer τ). The continuation value V^e can be described by the two sequences: $\{c_t\}_{t=0}^{\infty}$, $\left\{V_t^u\right\}_{t=1}^{\infty}$, which denote, respectively, the consumption in the first, second, third, etc. periods of employment

21. This assumption is important when considering the utility of an agent that takes a bad job and implies that, in that state, the agent cannot simultaneously search for a good job. If we allow for search on the job, it may even be socially optimal for unemployed agents to take the bad jobs–because they would not be bad!–while searching for the good job. Allowing for search on the job, would raise the bound on the cost d in equation (16).

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and the continuation values if the job is terminated in the first, second, third, etc. periods of employment. Taking into account the constant probability of termination λ, it follows that

$$V^e \leq \sum_{t=0}^{\infty} \beta^t (1-\lambda)^t \left[u(c_t) + \beta\lambda V_{t+1}^u\right]. \tag{18}$$

In turn, the value of taking the bad job and quitting after T periods is given by:

$$Q_T = \sum_{t=0}^{T-1} \beta^t \,(1-\lambda)^t \,[u(c_t) - d + \lambda\beta V_{t+1}^u] + (1-\lambda)^T \,\beta^T V_T^u.$$

In order for the worker not to take this alternative, it must be the case that for all $T \geq 0$, $V^u \geq Q_T$, *i.e.*

$$V^u \geq \sum_{t=0}^{T-1} \beta^t \,(1-\lambda)^t \,[u(c_t) - d + \lambda\beta V_{t+1}^u] + (1-\lambda)^T \,\beta^T V_T^u, \tag{19}$$

for all $T = 1, 2, 3...$

Notice that for a given value entitlement V^e, these constraints restrict the particular paths $\left\{c_t, V_{t+1}^u\right\}$ that can be used to support that value. The optimal choice of this path determines the nature of employment dependence in the unemployment insurance contract. This problem is analysed in the following section.

3.2.2. Employment dependence: the optimal choice of $\left\{c_t, V_{t+1}^u\right\}$. The optimal contract we solve now is more general than the one we solved in Section 2. There we characterized the transfers conditional on effort being high, which was the only non-trivial case. Now, the contract can first specify a recommendation to take the bad job and transfers between the principal and the agent. If the recommendation is not to take the bad job, then the contract specifies a recommended effort level and transfers. As before, for some values of the states (V^e, V^u), it may be optimal to recommend the bad job or zero effort and the solution is trivial. The contract only has incentive problems if the recommendation is not to take the bad job and the search effort is high. We proceed now to characterize the transfers in that case.

The optimal path $\left\{c_t, V_{t+1}^u\right\}$ is the solution to the following problem:

$$W(V^e, V^u) = \min_{c_t, V_{t+1}^u} \sum_{t=0}^{\infty} \beta^t (1-\lambda)^t \left[c_t - w + \beta\lambda C(V_{t+1}^u)\right] \tag{20}$$

subject to (18) and (19).

Before solving the problem, we must make sure that the choice set is well defined, since (18) imposes a lower bound on the sequence of controls, whereas (19) imposes upper bounds. Lemma 3 in the Appendix establishes that this set is non-empty.

We now characterize the solution for the optimal path. Letting γ and μ_t be the Lagrange multipliers of constraints (18) and (19) respectively, the first-order conditions[22] for the optimal (20) are

$$\frac{1}{u'(c_t)} = \gamma - \sum_{j=t+1}^{\infty} \mu_j, \tag{21}$$

22. The argument to prove that the value function is convex is the same as before, since the added constraints are linear in utilities.

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and

$$C'(V^u_{t+1}) = \gamma - \sum_{j=t+1}^{\infty} \mu_j - \frac{(1-\lambda)}{\lambda}\mu_{t+1}, \tag{22}$$

and the envelope conditions

$$\frac{\partial W(V^e, V^u)}{\partial V^e} = W_1(V^e, V^u) = \gamma, \tag{23}$$

$$\frac{\partial W(V^e, V^u)}{\partial V^u} = W_2\left(V^e, V^u\right) = -\sum_{j=1}^{\infty} \mu_j. \tag{24}$$

The interpretation of these partial derivatives is clear. First notice that if all multipliers μ_t are zero, the optimal path is constant, independent of the length of the employment spell. This would be the case if d is arbitrarily large and would be equivalent to the unconstrained problem discussed earlier. Second, the Kuhn–Tucker theorem implies that $\mu_t \geq 0$ for all t, so unless the self-selection constraint ceases to bind, both V^u_{t+1} and c_t will be below their unconstrained values.

The problem discussed above is the dynamic counterpart to standard static adverse selection problems. A classic example is the work of Spence (1973), where asymmetric costs of education among high- and low-skill workers allow separation of the former from the latter. In our model, staying at the bad job entails a flow cost in utility units equal to d, so the total cost of taking the bad job for T periods is equal to

$$d\frac{1-\beta^T}{1-\beta},$$

which is increasing with T. Recall that the adverse selection problem arises as unemployed workers may have incentives to take the bad job, since this entails an upgrade in the unemployment benefit. To prevent workers from doing this, the upgrade in benefits must be bounded above by the constraint that makes workers indifferent between that upgrade and the cost of staying at the bad job. As the cost of staying in the bad job is increasing with job tenure, so is the upgrade in benefits the contract can offer workers. Eventually, as T grows, the cost of staying for so long in the bad job may become so high that the limit on the upgrade may not be binding anymore. This suggests that after some period T, the cost of holding a bad job is high enough so that separating *good* from *bad* jobs creates no inefficiency. The same intuition suggests that the profiles of consumption c_t and V^u_{t+1} should increase up to that period. This is indeed the case, as indicated in the following proposition.

Proposition 6. *(i) There exists a period T, such that the sequences $\{c_t\}$ and $\left\{V^u_{t+1}\right\}$ are strictly increasing up to that period and constant thereafter. (ii) If the search IC constraint (3) binds, then $T < \infty$ and $\mu_t = 0$ for all $t \geq T$.*

Proof. See Appendix. ‖

In order for the worker to have the right incentives, the optimal contract must punish early termination of jobs. Thus, the tax is a decreasing function of job length and the benefit received in case of unemployment is an increasing function. Eventually, for long enough

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duration, the incentive problems disappear and both the tax and future promised utility in case of unemployment become constant over time while employment lasts.

As we mentioned above, employment duration plays the role of a signal analogous to education in Spence (1973), where high productivity workers must invest enough in education to separate themselves from low productivity workers. The larger is the cost of education for low productivity workers in Spence's model, the cheaper it is to separate the high productivity ones and less investment in education is needed. The intuition of this comparative static exercise in Spence is similar to the intuition of the last proposition. As the disutility flow accumulates over time, the cost of staying on the bad job is increasing with time. Therefore, as time goes by, it becomes less costly to separate the two job types and better coverage can be provided with employment length.

Replacement rates. Using (21) and (22) it follows that

$$\begin{aligned} C'(V^u_{t+1}) - \frac{1}{u'(c^e_t)} &= -\sum_{j=t+1}^{\infty} \mu_j - \frac{(1-\lambda)}{\lambda}\mu_{t+1} + \sum_{j=t+1}^{\infty} \mu_j \\ &= -\frac{(1-\lambda)}{\lambda}\mu_{t+1}. \end{aligned}$$

This term is strictly negative until the self-selection constraint stops binding. By the envelope theorem $C'\left(V^u_{t+1}\right) = 1/u'\left(c^u_t\right)$, this implies that $c^u_t < c^e_t$ so consumption falls when becoming unemployed. As in the case of opportunistic quits, the (after tax) replacement rates are less than one.

Note that there is no incentive problem if $T = 0$, which would occur if the flow disutility d is large relative to the upgrade in UI benefits. It can be established,[23] however, that there is a non-empty set of parameters for which $T > 0$ and it is optimal for the planner to prevent workers from taking the alternative–bad–jobs, as we imposed in the problem above. The argument relies on shrinking the time period–and appropriately rescaling probabilities–in such a way that it becomes essentially costless for the agent to take the bad job and quit an instant later.[24]

3.2.3. The optimal sequence of benefits.

Let us now turn to the problem of the principal when the worker is unemployed. The problem is the same as before, except for the definition of the function W. The first-order conditions (11) and (12) given in Section 2.2 must be replaced by:

$$\begin{aligned} C'(V^u) &= C'(V) - \delta\frac{p}{1-p} - \frac{p}{1-p}W_2\left(V^e, V^u\right) \\ &= C'(V) - \delta\frac{p}{1-p} + \frac{p}{1-p}\sum_{j=1}^{\infty}\mu_j \end{aligned} \tag{25}$$

and

$$W_1\left(V^e, V^u\right) = C'\left(V\right) + \delta. \tag{26}$$

23. Provided a continuity condition holds.

24. We thank the comment of one referee that made us think about this issue.

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Using (25), (26) and (24) it follows that

$$C'(V) = (1-p)\,C'\left(V^u\right) + pW_1\left(V^e\right) - p\sum_{j=1}^{\infty}\mu_j. \tag{27}$$

Letting c denote the current consumption level for an unemployed worker, c_u consumption in the following period if unemployed and c_e consumption if employed, equations (10) and (21) (for $t=0$) imply the familiar inverse Euler equation:

$$\frac{1}{u'(c)} = (1-p)\,\frac{1}{u'(c_{\mathrm{u}})} + p\frac{1}{u'(c_{\mathrm{e}})},$$

which holds for all previous cases too.

If $\delta = 0$, equation (25) implies that $C'(V^u) \geq C'(V)$, so $V^u \geq V$ and $V \geq \frac{u(c)}{1-\beta}$. Using equations (21), (23) and (26) it also follows that $C'(V) = \gamma \geq \frac{1}{u'(c_t)}$, which implies that consumption while employed $c_t \leq c$. Moreover, using (22) it follows that $C'\left(V^u_{t+1}\right) \leq \gamma$ and thus $V^u_t \leq V$. Consequently,

$$\begin{aligned}
V^e &= \sum_{t=0}^{\infty}\beta^t(1-\lambda)^t\left[u(c_t) + \beta\lambda V^u_{t+1}\right]\\
&\leq \sum_{t=0}^{\infty}\beta^t(1-\lambda)^t\left[u(c) + \beta\lambda V\right]\\
&\leq \sum_{t=0}^{\infty}\beta^t(1-\lambda)^t\left[(1-\beta)\,V + \beta\lambda V\right]\\
&= V \leq V^u
\end{aligned}$$

so the worker would choose not to search. Hence $\delta > 0$ and the search incentive constraint binds.

With the exception of the employment dependence properties indicated above, the other properties of the optimal contract remain the same as in the previous section: benefits decrease and taxes increase with the length of the current and previous unemployment spells. Although qualitative properties are alike, the opportunistic quits and adverse selection problems considered above are likely to have a quantitative impact on the optimal contract. We conjecture that, as the function W becomes steeper and the marginal cost of providing utility at employment increases, the initial replacement ratios are likely to be higher but decrease more rapidly. Furthermore, in a setup where there is a range of search effort choices, one may conjecture that the optimal choice of effort will decrease. A full quantitative evaluation of the impact of this adverse section problem is left for future research.

4. FINAL REMARKS

In this article we derive an optimal unemployment insurance programme as the solution to a repeated principal agent problem. This article extends previous work allowing for multiple employment/unemployment spells. We show that replacement rates decrease as a function of current and previous unemployment spells. We also show that taxes paid by employed workers also increase in all previous unemployment spells. When job termination is exogenous, the history of current and previous employment duration has no effect on replacement rates or taxes.

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In practice, most unemployment insurance schemes condition replacement ratios on the duration of previous employment spells. This article provides a theoretical foundation for such practice. When quits and layoffs cannot be perfectly distinguished by the enforcing agency, the above optimal contract can lead to opportunistic quit behaviour. Moreover, workers may also have an incentive to accept bad matches and soon quit from them, simply to upgrade their unemployment insurance benefits. Preventing such behaviour imposes further constraints on the design of the insurance contract. Once these additional constraints are introduced, the optimal programme calls for incomplete replacement ratios, which increase with the length of previous unemployment spells and taxes that decrease with tenure on the job.

Incentives are provided to make the employment state more attractive. Hence, the worker's expected future utility decreases with the length of unemployment spells and increases with the length of employment spells. This explains the duration dependence of unemployment benefits and taxes. Moreover, a change of state from unemployment to employment is rewarded while the opposite is penalized, accounting for the incomplete replacement rate policy. The additional sources of opportunistic quit behaviour examined in this article increase the cost of providing search incentives for unemployed workers. We conjecture that this is likely to result in higher initial replacement ratios for the unemployed but a steeper decline with the length of unemployment.

APPENDIX A. PROOFS

A.1. *Convexity of the optimal programme when $e \in \{0, 1\}$*

The optimal contract problem when the worker is unemployed has a discrete choice variable–the effort level. The general problem, as we will show, is not convex, so the unconstrained optimum may involve the use of lotteries.

Let

$$\begin{aligned} C_0(V) &= \min_{c, V^u} c + \beta C\left(V^u\right) \\ \text{subject to} &: \quad u(c) + \beta V^u = V \end{aligned}$$

and

$$\begin{aligned} C_1(V) &= \min_{c, V^u, V^e} c + \beta \left\{ pW\left(V^e\right) + (1-p)\, C\left(V^u\right) \right\} \qquad \text{(A1)} \\ \text{subject to} &: \quad u(c) - 1 + \beta \left\{ pV^e + (1-p)\, V^u \right\} = V \\ & \quad\;\; p\left(V^e - V^u\right) \geq 1 \end{aligned}$$

$$\begin{aligned} C(V) &= \min_{q, V_0, V_1} (1-q)\, C_0(V_0) + qC_1(V_1) \qquad \text{(A2)} \\ \text{subject to} &: \quad (1-q)\, V_0 + qV_1 = V \end{aligned}$$

where C_0 and C_1 are the cost of recommending the low and high effort level today respectively, whereas C is the cost of a lottery between the high effort level and the low effort level with the corresponding promised utilities. We show in the next proposition that the problem so defined is convex, so no further randomizations can improve upon the optimal solution.

Lemma 1. *The functions C_0, C_1 and C are convex.*

Proof. If C is convex, it is immediate to verify that the problem defined by equation (A1) defines convex functions C_0 and C_1. We now show that if these two functions are convex, then equation (A2) defines a convex function C. Let (q, V_0, V_1) be solutions for V and q', V_0', V_1' be solutions for V'. Let

$$V^\lambda = \lambda V + (1-\lambda)\, V',$$

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and for this starting value take as choice variables

$$\begin{aligned} q^{\lambda} &= \lambda q + (1-\lambda) q' \\ V_0^{\lambda} &= \frac{\lambda (1-q) V_0 + (1-\lambda)\left(1-q'\right) V_0'}{1-q^{\lambda}} \\ V_1^{\lambda} &= \frac{\lambda q V_1 + (1-\lambda) q' V_1'}{q^{\lambda}}. \end{aligned}$$

Simple algebra shows that this is a feasible solution for V^{λ}. Therefore, by definition

$$C(V^{\lambda}) \leq \left(1-q^{\lambda}\right) C_0\left(V_0^{\lambda}\right) + q^{\lambda} C_1\left(V_1^{\lambda}\right).$$

Convexity of C_0 and C_1 implies that

$$C_0\left(V_0^{\lambda}\right) \leq \frac{\lambda (1-q) C_0(V_0) + (1-\lambda)(1-q') C_0\left(V_0'\right)}{(1-q^{\lambda})}$$

and

$$C_1\left(V_1^{\lambda}\right) \leq \frac{\lambda q C_1(V_1) + (1-\lambda) q' C_1\left(V_1'\right)}{q^{\lambda}}.$$

It therefore follows that

$$\left(1-q^{\lambda}\right) C_0\left(V_0^{\lambda}\right) + q^{\lambda} C_1\left(V_1^{\lambda}\right) \leq \lambda C(V) + (1-\lambda) C\left(V'\right),$$

which completes the proof. ‖

We do now show that the derivative of the C_1 function is larger than the derivative of the C_0 function, such that they display the single crossing property.

Lemma 2. *For any* V, $C_0'(V) < C_1'(V)$.

Proof. From the first-order and envelope conditions of the C_0 and C_1 problem we obtain

$$C_0'(V) = \frac{1}{u'(c_0)} = C'(V_0^u) \tag{A3}$$

$$u(c_0) + \beta V_0^u = V \tag{A4}$$

$$C_1'(V) = \frac{1}{u'(c_1)} > C'(V_1^u) \tag{A5}$$

$$u(c_1) + \beta V_1^u = V. \tag{A6}$$

Now, assume that $c_0 < c_1$. The result follows from the concavity of u. Now, assume that $c_0 \geq c_1$. Then, (A4) and (A6) and the convexity of C imply that

$$C'(V_0^u) \leq C'(V_1^u).$$

The result follows from (A3) and (A5). ‖

We can now characterize the function $C(V)$. First, if either $C_0(V)$ or $C_1(V)$ is below the other for all the domain, then $C(V)$ coincides with that curve. If they cross, by the single crossing property, they cross only once. As we already showed, $C_1(V)$ has a larger slope so it will intersect $C_0(V)$ from below. Therefore, as can be seen from the first order condition, $C(V)$ will coincide with C_1 for low values of V, it will be linear for intermediate values of V,[25] and will coincide with C_0 for high values of V. The analysis in the article is relevant for low values of V.

25. This means that randomizations are optimal.

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A.2. *Proof of Proposition 3*

Consider an employed worker. Let V^u denote the utility of quitting. Under the optimal contract described in Section 2.2, this is the utility obtained by quitting any period. Consider the strategy: "stay working this period but quit the next". The utility obtained equals

$$u\left(c^e\right) - e + \beta V^u.$$

In turn, the strategy of quitting immediately gives utility:

$$V^u = u\left(c^u\right) - 1 + \beta\left[pV^e + (1-p)\tilde{V}^u\right]$$

where $\tilde{V}^u$ is the utility if still unemployed next period, given a utility of V^u this period. Once the worker is unemployed, the incentive constraint binds. This expression can be written as

$$V^u = u\left(c^u\right) + \beta\tilde{V}^u.$$

Under the above optimal contract, $c^e = c^u$, so the net gain of delaying one period the quit decision is:

$$-e + \beta\left(V_u - \tilde{V}_u\right).$$

If $e = 0$, as $V_u \geq \tilde{V}_u$, it follows that the agent has no loss by delaying the quitting decision. Applying this principle recursively, the worker is better off by not quitting.

To prove part 2, suppose by way of contradiction that $e > 1/p$ but the value of employment $V^e \geq V^u$. Let $\tilde{V}^e$ denote the utility that an unemployed worker with initial value V^u is entitled to if he finds a job. We first show that $\tilde{V}^e \geq V^u$. Let $\tilde{c}^e$ be the first period consumption of this employed worker. As proved in Section 2.2 consumption rises if an unemployed worker finds a job, so $\tilde{c}^e \geq c^u = c^e$. Since consumption is monotonic in value, this implies that $\tilde{V}^e \geq V^e$ which by the contradiction assumptions is no less than V^u. The incentive constraint for the unemployed worker implies that $\beta\left(\tilde{V}^e - \tilde{V}^u\right) = 1/p$, which together with the previous result implies that $\beta\left(V^u - \tilde{V}^u\right) \leq 1/p < e$, so the strategy of quitting immediately dominates the strategy of waiting one more period. Applying this result recursively proves that $V^e < V^u$. ‖

A.3. *Proof of Proposition 4*

First note that if this constraint does not bind at time T, then from equation (15) it follows that $V_T^e = V_{T+1}^e$ and by iterative application $V_t^e = V_T^e$ for all $t \geq T$. Moreover, as V_t^e remains constant so does V_t^u and thus $V_t^e \geq V_t^u$ for all $t \geq T$ and the constraint does not bind for any future period. Next we show that if the constraint binds in some period T, then it must bind in all subsequent periods. The combination of these two results proves the first part of the proposition. Suppose, by way of contradiction, that (13) binds at t, *i.e.* $V_{t+1}^u = V_{t+1}^e$, but it does not bind at $t+1$. Using equations (14) and (15) it follows that:

$$C'\left(V_{t+1}^u\right) < W'\left(V_t^e\right) < W'\left(V_{t+1}^e\right)$$

and

$$C'\left(V_{t+2}^u\right) = W'\left(V_{t+1}^e\right) = W'\left(V_{t+2}^e\right).$$

By strict convexity of functions C and W, and since $V_{t+1}^u = V_{t+1}^e$ it follows that

$$V_{t+2}^u > V_{t+1}^u = V_{t+1}^e = V_{t+2}^e,$$

which violates the no-quit constraint.

That V_t^e (and thus V_t^u) strictly increases when the no-quit constraint binds follows immediately from equation (14). ‖

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A.4. *Proofs for Section 3.2*

Lemma 3. *For all V^u and $V^e = V^u + \frac{1}{p\beta}$ there exist paths $\{c_t, V^u_{t+1}\}$ satisfying constraints (3), (18) and (19). Moreover, the paths can be chosen so that constraint (3) binds.*

Proof. Pick any sequence $\{c_t, V^u_t\}$ such that constraint (19) is satisfied with equality for all T, so the worker is indifferent between quitting or not any period. It follows that

$$V^u = \sum_{t=0}^{\infty} \beta^t (1-\lambda)^t \left[u(c_t) - d + \beta\lambda V^u_{t+1}\right]. \tag{A7}$$

Let $\hat{V}^e$ denote the value of taking a good job that is associated to the same path $\{c_t, V^u_t\}$. Subtracting (A7) from (18), it follows that:

$$\hat{V}^e - V^u = \frac{d}{1-\beta(1-\lambda)}.$$

Using equation (16) it follows that

$$\hat{V}^e - V^u > \frac{u(w) - (u(0) - 1)}{1-\beta(1-(\lambda+p))}. \tag{A8}$$

To evaluate the right hand side of this equation, note that for search to be at all valuable, it must be the case that $(1-\beta)V^s > u(0)$, where the second term is the value of never searching. In conjunction with equation (17) this implies that

$$(u(w) - u(0)) > \frac{\alpha}{(1-\alpha)} = \frac{1-\beta(1-\lambda)}{\beta p}$$

which together with (A8) implies that $\hat{V}^e - V^u > \frac{1}{\beta p}$. This path satisfies the incentive constraint. Moreover, it follows that by decreasing some components of the sequence, an alternative path that satisfies constraint (19) and gives a value $V^e < \hat{V}^e$, and $V^e - V^u = \frac{1}{\beta p}$ can be found. ‖

The remainder of this section proves that the optimal plan has strictly increasing sequences $\{c_t, V^u_{t+1}\}$ up to some T and constant thereafter. The following lemmas give intermediate results that are used in the proof.

Lemma 4. *Let Q_T be as defined in Section 3.2.1. Then, $Q_{T+1} - Q_T$ has the same sign as $u(c_T) - d + \beta V^u_{T+1} - V^u_T$.*

Proof. Follows immediately by noting that:

$$\begin{aligned}
Q_{T+1} &= \sum_{t=0}^{T} \beta^t (1-\lambda)^t \left(u(c_t) - d + \beta\lambda V^u_{t+1}\right) + \beta^{T+1}(1-\lambda)^{T+1} V^u_{T+1} \\
&= Q_T - \beta^T (1-\lambda)^T V^u_T + \beta^T (1-\lambda)^T \left(u(c_T) - d + \beta V^u_{T+1}\right) \\
&= Q_T + \beta^T (1-\lambda)^T \left(u(c_T) - d + \beta V^u_{T+1} - V^u_T\right). \quad \|
\end{aligned}$$

Lemma 5. *$V^u_{t+2} - V^u_{t+1}$ has the same sign as $\mu_{t+1} - (1-\lambda)\mu_{t+2}$.*

Proof. From equation (22) it follows that

$$C'\left(V^u_{t+2}\right) - C'\left(V^u_{t+1}\right) = \sum_{j=t+1}^{\infty} \mu_j + \frac{(1-\lambda)}{\lambda}\mu_{t+1} - \sum_{j=t+2}^{\infty} \mu_j - \frac{(1-\lambda)}{\lambda}\mu_{t+2} = \frac{1}{\lambda}\left(\mu_{t+1} - (1-\lambda)\mu_{t+2}\right).$$

The claim follows from the convexity of C. ‖

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Lemma 6. *Suppose $V_{t-1}^u \geq V_{t+1}^u$ and $Q_t = V^u$. Then $V_{t+1}^u < V_t^u$ and $\mu_{t+2} > 0$.*

Proof. If $Q_t = V^u$, the incentive constraints imply that $Q_{t-1} \leq V^u = Q_t$, and $Q_t = V^u \geq Q_{t+1}$. These inequalities and Lemma 4 imply that

$$u(c_{t-1}) - d + \beta V_t^u \geq V_{t-1}^u \geq V_t^u \geq u(c_t) - d + \beta V_{t+1}^u > u(c_{t-1}) - d + \beta V_{t+1}^u,$$

(where $c_t > c_{t-1}$ follows from (21)) so $V_t^u > V_{t+1}^u$. Finally, note that Lemma 5 implies that if $\mu_{t+2} = 0$, then $V_{t+2}^u - V_{t+1}^u \geq 0$, contradicting the previous inequality. ‖

Lemma 7. *In the optimal plan the sequences $\left\{c_t, V_{t+1}^u\right\}$ are non-decreasing. Moreover, whenever $\mu_{t+s} > 0$, for some $s \geq 0$, then $c_{t+1} > c_t$, and $V_{t+1}^u > V_t^u$. In addition, if $\mu_{t+s} = 0$ for all $s \geq 0$, then $c_{t+s} = c_t$ and $V_{t+1+s}^u = V_{t+1}^u$ for all $s \geq 0$.*

Proof. The results for $\{c_t\}$ follow immediately from (21). To show the results regarding V_t^u, consider first the case $\mu_t > 0$ and suppose towards a contradiction that $V_t^u \geq V_{t+1}^u$. Applying repeatedly Lemma 6, $V_{t+s}^u > V_{t+s+1}^u$ and $\mu_{t+s} > 0$ for all s. But as $\sum \mu_s < \infty$, $C'\left(V_t^u\right) \to \gamma$ and so eventually it must increase. Now suppose instead that $\mu_t = 0$. By the first-order conditions (22) it follows that $V_{t+1}^u \geq V_t^u$ and that $V_{t+1}^u = V_t^u$ only when $\mu_{t+s} = 0$ for all $s \geq 0$. ‖

The previous Lemma proved part (i) of Proposition 6. We now prove part (ii).

In every period T where the self-selection constraint binds, $Q_T = V^u$, so

$$\begin{aligned}
V^u &= Q_T = \sum_{t=0}^{T-1} \beta^t (1-\lambda)^t \left[u(c_t) - d + \beta\lambda V_{t+1}^u\right] + \beta^T (1-\lambda)^T V_T^u \\
&= \frac{-d\left[1 - [\beta(1-\lambda)]^T\right]}{1-\beta(1-\lambda)} + \sum_{t=0}^{T-1} \beta^t (1-\lambda)^t \left[u(c_t) + \beta\lambda V_{t+1}^u\right] + \beta^T (1-\lambda)^T V_T^u \\
&\leq \frac{-d\left[1 - [\beta(1-\lambda)]^T\right]}{1-\beta(1-\lambda)} + V^e,
\end{aligned}$$

where the last inequality follows from the fact that it is never optimal to quit from the good job. This implies that

$$V^e - V^u \geq \frac{d\left[1 - [\beta(1-\lambda)]^T\right]}{1-\beta(1-\lambda)}.$$

Now, assume towards a contradiction that for any $T \in \mathbb{R}$, $\exists\, s > 0, \ni \mu_{T+s} > 0$. Then,

$$V^e - V^u \geq \frac{d}{1-\beta(1-\lambda)}.$$

But as in the proof of Lemma 3, this in turn implies that $V^e - V^u > \frac{1}{\beta p}$. As a consequence, the incentive compatibility constraint does not hold with equality, and V^e can be reduced, lowering the total cost. This completes the proof.

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