Optimal Taxation with Heterogeneous Rates of Return

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Summary

- Study optimal *nonlinear* taxation of capital income when agents earn heterogeneous returns
- Assets: entrepreneurial technology with idiosyncratic return, and risk-free bonds in zero net supply ("investing" and "saving")
- Assume individual investment outputs imperfect substitutes
  - Eliminates undesirable "solution" in which only the most productive invest
- Static model: differential asset taxation
  - Optimal distortions on both types of assets hump-shaped
  - Tax on investing uniformly higher than saving
  - Distortions driven by *aggregate* DRS, informational frictions
- Dynamic model: preliminary solution
  - Exploit homogeneity to simplify planning problem
  - Wedge on investing history dependent, wedge on saving not

Motivation

- Thick tails in wealth distribution driven by capital income risk: Benhabib *et al.* (2011), Benhabib *et al.* (2019)
  - Multiplicative effect, rather than additive
- How should capital income, wealth be taxed?
  - Can tax capital income to redistribute from wealthy to poor
  - Efficiency: excessive capital taxation discourages investment and lowers output
- Mirrlees (1971): framework to consider redistribution/efficiency tradeoff in the presence of informational frictions

Literature Review

- Positive capital income taxes with informational frictions: Golosov *et al.* (2003), Kocherlakota (2005), Albanesi and Sleet (2006)
  - Common rate of return, labor market considerations
- Optimal taxation of entrepreneurs: Albanesi (2006), Scheuer (2014)
  - Entrepreneurial returns dependent on effort
  - Support for differential rates
- Guvenen *et al.* (2019): welfare improvement with *wealth* taxes in a similar model
- Heterogeneous returns: Shourideh (2014), Gerritsen *et al.* (2020)

Static Model: Households and Productivity

Households

- Continuum of households who differ in type θ, which determines *productivity* and *variety*
- Assume $\theta \in \Theta = \left[\underline{\theta}, \bar{\theta}\right]$, with CDF $F(\theta)$, privately known by household
- Household can borrow/lend b at common rate R
- Household of type θ can also invest k capital and produce θk of their variety of intermediate good
  - Sells to final good producer at price $p(\theta)$
  - Price-taker, so individual technologies are CRS

Static Model: Aggregate Production and Prices

- Final good producer combines intermediate goods to produce final good using CES aggregation technology:

$$Y = \left(\int_{\Theta} \left[\theta k\left(\theta\right) \right]^{\frac{\varepsilon-1}{\varepsilon}} dF\left(\theta\right) \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- $\varepsilon > 1$ elasticity of substitution
  - Ensures that at optimal solution, all types will invest
- From FG producer problem, prices are

$$p(\theta) = \left(\frac{Y}{\theta k\left(\theta\right)} \right)^{\frac{1}{\varepsilon}}$$

Static Model: Government

- Government levies taxes on income from investing $\theta kp(\theta)$ and saving Rb according to tax function T
- Taking T and p as given, household solves

$$\max_{k,b}\; u\left(w-k-b\right)+\beta u\left(\theta kp\left(\theta\right)+Rb-T\left(\theta kp,Rb\right)\right)$$

- Benevolent government chooses T to maximize social welfare:

$$\max\int_{\Theta} U\left(\theta\right)dF\left(\theta\right) \tag{1}$$

subject to revenue requirements and household optimality

Static Model: Mechanism Design Problem

- Following from Mirrlees (1971), can recast government's problem in terms of mechanism design
  - Revelation Principle: direct mechanism
  - Household reports θ and receives allocations
- Objective same as in (1)
- Feasibility constraints:

$$w \geq \int_{\Theta} \left[c_0 \left(\theta \right) + k \left(\theta \right) \right] dF \left(\theta \right)$$

$$\left(\int_{\Theta} \left[\theta k \left(\theta \right) \right]^{\frac{\varepsilon - 1}{\varepsilon}} dF \left(\theta \right) \right)^{\frac{\varepsilon}{\varepsilon - 1}} \geq \int_{\Theta} c_1 \left(\theta \right) dF \left(\theta \right)$$

Static Model: Mechanism Design Problem

- Additional constraints: incentive compatibility

$$U(\theta) \geq \ln\left(c_0\left(\hat{\theta}\right) + k\left(\hat{\theta}\right) - \frac{\hat{\theta} k\left(\hat{\theta}\right)}{\theta}\right) + \beta \ln c_1\left(\hat{\theta}\right), \quad \forall \theta, \hat{\theta} \in \Theta$$

  where

$$U(\theta) = \ln c_0(\theta) + \beta \ln c_1(\theta)$$

- Simplifying assumption: while the planner (government) cannot observe θ, the market for intermediate goods *can*
- If type θ claims to be of type $\hat{\theta}$, they still receive price $p(\theta)$

Static Model: Optimal Distortions

- From household problem, wedges are

$$\tau_k(\theta) = 1 - \frac{u'(c_0)}{\beta u'(c_1)\,\theta p(\theta)}$$
$$\tau_b(\theta) = 1 - \frac{u'(c_0)}{\beta R u'(c_1)}$$

- First and second partial derivatives of tax function T
- Optimal *distortions*, not necessarily optimal taxes

Static Model: Optimal Distortions

- From optimality conditions in social planner's problem, can state wedges as

$$\tau_k = \left(1 + \frac{k}{c_0}\right)\left(1 - \frac{R}{\theta p\left(\theta\right)}\right)$$
$$\tau_b = \frac{k}{c_0}\left(\frac{\theta p\left(\theta\right)}{R} - 1\right)$$

- Two key determinants: $\frac{k}{c_0}$, and $\frac{R}{\theta p(\theta)}$
- Both wedges nonnegative, and positive on interior of Θ
- $\frac{k}{c_0}$ pulls wedges upward, effect of $\frac{R}{\theta p(\theta)}$ depends on θ

Static Model: Marginal Product

Lemma

$R \leq \theta p(\theta)$, with equality if and only if $\theta \in \{\underline{\theta}, \overline{\theta}\}$.

- $\theta p(\theta)$ is the *societal* marginal product of capital at the optimum for type θ, which takes into account effect of k on p
- Using formula for prices,

$$\theta p(\theta) = \theta^{1-\frac{1}{\varepsilon}} \left(\frac{Y}{k\left(\theta\right)} \right)^{\frac{1}{\varepsilon}}$$

$\implies$ diminishing returns in aggregate

- Full information: planner wishes to equate $\theta p(\theta)$ with R
- Informational frictions prevent this, $\frac{\theta p(\theta)}{R}$ hump-shaped instead
- For high values of θ, this term pulls wedges towards zero

Static Model: Positive Wedge on Investing

- Consequence of Lemma: optimal distortions are positive on the interior of Θ
- Intuition for $\tau_k > 0$: planner distorts investment choice to prevent over-supply of variety θ
- Counterfactual: if households monopolists, investment wedge is lower and often negative; planner corrects for under-supply of variety

Static Model: Differential Asset Taxation

Proposition

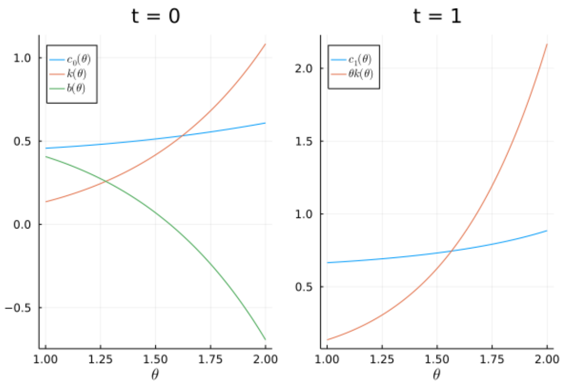
$\tau_k(\theta) \geq \tau_b(\theta)$, *with equality if and only if* $\theta \in \{\underline{\theta}, \overline{\theta}\}$.

- Optimality of positive τ_b stems directly from positive τ_k
  - Endowment is perfectly fungible between the two vehicles
- Larger τ_k: investing income is a signal of private information, savings income is not

Static Model: Numerical Example

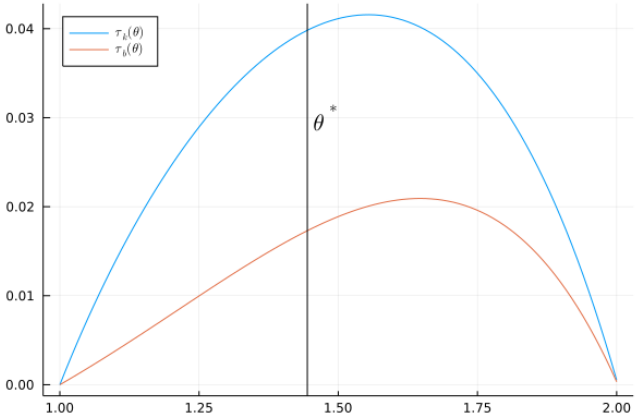
- $\theta \sim U[1, 2]$
- $\beta = 0.95$, $\varepsilon = 4$, $w = 1$
- Optimal allocations:

Figure: Allocations in the Static Model



Static Model: Numerical Wedges

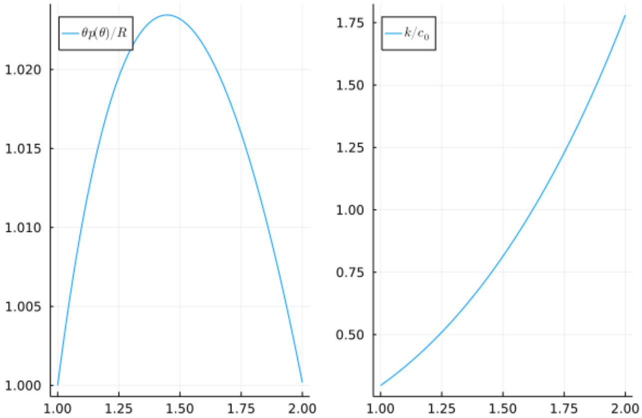
Figure: Wedges in the Static Model



Note: θ^* denotes the value of θ where the ratio $\frac{\theta p(\theta)}{R}$ attains its maximum and begins to decline. This ratio is the force that ultimately pulls the wedges down.

Static Model: Determinants of Wedges

Figure: Determinants of Wedges in the Static Model



Dynamic Model: IID Case

Households

- Time is discrete, each period plays out as follows:
  1. Agent realizes investing income
  $$y_t\left(\theta^{t-1}\right) = p_t\left(\theta^{t-1}\right)\theta_{t-1}k_t\left(\theta^{t-1}\right)$$
  2. Agent draws new type θ_t from $F(\theta)$, draws IID across time. Then, agent makes choices c_t, k_t, b_t
- Let $\theta^t = \{\theta_0, \theta_1, ..., \theta_t\}$ denote the history of shocks through time t

Dynamic Model: IID Case

Production

- Aggregate producer follows two-step process for Y_t:
  1. Combines intermediate capital goods to create single capital good $K_{t,f}$:

$$K_{t,f} = \left(\int \left[\theta_{t-1} k_t \left(\theta^{t-1} \right) \right]^{\frac{\varepsilon - 1}{\varepsilon}} d\mu_{t-1} \left(\theta^{t-1} \right) \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$

  2. Combines $K_{t,f}$ with labor L_t, to produce Y_t according to Cobb-Douglas technology, $Y_t = K_{t,f}^{\alpha} L_t^{1-\alpha}$
- Assumption: $L_t = L = 1 \; \forall t$, so final good output is

$$Y_t = K_{t,f}^{\alpha}$$

which ensures that a steady state in aggregates (Y, K_f) will exist

Dynamic Model: Planning Problem

- Let $\mu_t\left(\theta^t\right)$ denote the measure of period-t histories induced by the stochastic process for θ_t.
- The planner chooses allocations $\left\{c_t\left(\theta^t\right), k_{t+1}\left(\theta^t\right)\right\}_{t=0}^{\infty}$ to solve

$$\max \sum_{t=0}^{\infty} \beta^t \int u\left(c_t\left(\theta^t\right)\right) d\mu_t\left(\theta^t\right) \tag{2}$$

- Feasibility assumes entrepreneurs allocated capital share αY:

$$\begin{aligned}
\int \left[c_t\left(\theta^t\right) + k_{t+1}\left(\theta^t\right)\right] d\mu_t\left(\theta^t\right) & \\
&= \alpha\left(\int \left[\theta_{t-1} k_t\left(\theta^{t-1}\right)\right]^{\frac{\varepsilon-1}{\varepsilon}} d\mu_{t-1}\left(\theta^{t-1}\right)\right)^{\frac{\alpha\varepsilon}{\varepsilon-1}} \\
&= \int \theta_{t-1} p_t\left(\theta^{t-1}\right) k_t\left(\theta^{t-1}\right) d\mu_{t-1}\left(\theta^t\right)
\end{aligned}$$

Dynamic Model: Incentive Constraints

- Promise utility allocated to an agent of history θ^t is

$$w_{t+1}\left(\theta^t\right) = \sum_{s=t+1}^{\infty} \beta^{s-t-1} \int u\left(c_s\left(\theta^s\right)\right) d\mu_s\left(\theta^s \middle| \theta^t\right)$$

- The *local* incentive constraints require

$$\frac{\partial U_t\left(\theta^t\right)}{\partial \theta_t} = u'\left(c_t\left(\theta^t\right)\right)\frac{k_{t+1}\left(\theta^t\right)}{\theta_t}$$

Dynamic Model: Solving the Planning Problem

- We focus on the dual (cost-minimization) problem of a *component* planner
  - Considers the problem of a single agent of history θ^{t-1}
  - Takes the path of prices $\{p_{s+1}(\theta^s)\}_{s\geq t}$ as given

Dynamic Model: Component Planner's Problem

$$\min_{\substack{c_\tau(\theta^\tau), k_{\tau+1}(\theta^\tau),\\ U_\tau(\theta^\tau), w_{\tau+1}(\theta^\tau)}} \sum_{\tau=t}^{\infty} \left(\prod_{s=t}^{\tau-1} R_s \right)^{-1} \left\{ \int \left[c_\tau\left(\theta^\tau\right) + k_{\tau+1}\left(\theta^\tau\right) \right] d\mu_\tau\left(\theta^\tau\right) - \int p_\tau\left(\theta^{\tau-1}\right) \theta_{\tau-1} k_\tau\left(\theta^{\tau-1}\right) d\mu_{\tau-1}\left(\theta^{\tau-1}\right) \right\}$$

subject to

$$w_t\left(\theta^{t-1}\right) = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \int u\left[c_\tau\left(\theta^\tau\right)\right] d\mu_\tau\left(\theta^\tau\right)$$

$$U_\tau\left(\theta^\tau\right) = u\left[c_\tau\left(\theta^\tau\right)\right] + \beta w_{\tau+1}\left(\theta^\tau\right)$$

$$\frac{\partial U_\tau\left(\theta^\tau\right)}{\theta_\tau} = \frac{k_{\tau+1}\left(\theta^\tau\right)}{\theta_\tau c_\tau\left(\theta^\tau\right)}$$

given $p_\tau\left(\theta^{\tau-1}\right)$ and R_s

Dynamic Model: Exploiting Homogeneity

- Although the aggregate value function (cost-minimization) is not homogeneous, the component value function *is*, as we assume that the CP takes prices as given
  - Similar to Angeletos (2007)
- This allows us to derive a recursive formulation
- Implies that the policy function for $k_{t+1}(\theta^t)$ can be written as

$$\begin{aligned} k_{t+1}\left(\theta^t\right) &= \overline{k}_{t+1}\left(\theta_t\right)\exp\left[\left(1-\beta\right)w_t\left(\theta^{t-1}\right)\right] \\ &= \overline{k}_{t+1}\left(\theta_t\right)\exp\left[\left(1-\beta\right)\left(w'\left(\theta_{t-1}\right)+\ldots+w'\left(\theta_0\right)+w_0\right)\right] \end{aligned}$$

for some functions $\overline{k}\left(\theta_t\right)$ and $w'\left(\theta_t\right)$, in the case of log utility

Dynamic Model: Exploiting Homogeneity

- This decomposition implies a similar process for prices:

$$p_{t+1}\left(\theta^{t}\right) = \overline{p}_{t}\left(\theta^{t-1}\right)\hat{p}_{t}\left(\theta_{t}\right)$$

- Can decompose price into $\overline{p}$, which encodes history, and $\hat{p}$, which only depends on θ_t
- "Common price" $\overline{p}$ evolves according to

$$\overline{p}_{t+1}\left(\theta^{t}\right) = \overline{p}_{t}\left(\theta^{t-1}\right)\tilde{p}\left(\theta_{t}\right)$$

  where

$$\tilde{p}\left(\theta_{t}\right) = \exp\left[-\frac{(1-\beta)}{\varepsilon}w'\left(\theta_{t}\right)\right]$$

Dynamic Model: Recursive Formulation

Assuming constant R, the recursive problem is:

$$C(w,\overline{p}) = \min_{\substack{c(\theta),k'(\theta),\\ w'(\theta),U(\theta)}} \int \left\{ c\left(\theta\right) + k'\left(\theta\right) + R^{-1}\left[C\left(w'\left(\theta\right), \overline{p}\cdot\tilde{p}\left(\theta\right)\right) - \right.\right.$$

$$\left.\left. \overline{p}\cdot\hat{p}\left(\theta\right)\theta k'\left(\theta\right)\right]\right\} dF\left(\theta\right)$$

subject to

$$w \leq \int U(\theta) dF(\theta)$$

$$U(\theta) = u\left(c(\theta)\right) + \beta w'(\theta)$$

$$U'(\theta) = u'\left(c(\theta)\right)\frac{k'(\theta)}{\theta}$$

Dynamic Model: Exploiting Homogeneity Once More

Proposition

Suppose $u(c) = \ln c$. Then, the component planner's problem has the following solution:

$$C\left(w,\overline{p}\right) = A\left(\overline{p}\right)e^{(1-\beta)w} \qquad w'\left(\theta,w,\overline{p}\right) = w'\left(\theta,\overline{p}\right) + w$$

$$c\left(\theta,w,\overline{p}\right) = c\left(\theta,\overline{p}\right)e^{(1-\beta)w} \qquad U\left(\theta,w,\overline{p}\right) = U\left(\theta,\overline{p}\right) + w$$

$$k'\left(\theta,w,\overline{p}\right) = k'\left(\theta,\overline{p}\right)e^{(1-\beta)w}$$

for some functions $A\left(\overline{p}\right), c(\theta,\overline{p}), k'(\theta,\overline{p}), w'(\theta,\overline{p}), U(\theta,\overline{p})$.

- Implication: we can solve the above problem for $w = 0$ to obtain "baseline" functions $A\left(\overline{p}\right), c(\theta,\overline{p}), k'(\theta,\overline{p}), w'(\theta,\overline{p}), U(\theta,\overline{p})$.

Dynamic Model: Full Solution

- The "baseline" functions solve the following recursion:

$$A\left(\overline{p}\right) = \min_{\substack{c(\theta),k'(\theta),\\ w'(\theta),U(\theta)}}$$

$$\int \left[c\left(\theta\right) + k'\left(\theta\right) + R^{-1}\left\{A\left(\overline{p}\cdot\tilde{p}\left(\theta\right)\right)\exp\left(\left(1-\beta\right)w'(\theta)\right) - \overline{p}\cdot\hat{p}\left(\theta\right)\theta k'\left(\theta\right)\right\}\right] dF\left(\theta\right)$$

subject to

$$0 = \int U(\theta,\overline{p})dF(\theta)$$

$$U(\theta,\overline{p}) = \ln\left(c(\theta,\overline{p})\right) + \beta w'(\theta,\overline{p})$$

$$U'(\theta,\overline{p}) = \frac{k'(\theta,\overline{p})}{\theta c(\theta,\overline{p})}$$

Dynamic Model: Optimal Wedges

- We can characterize the optimal wedges on saving and investing, respectively, as follows:

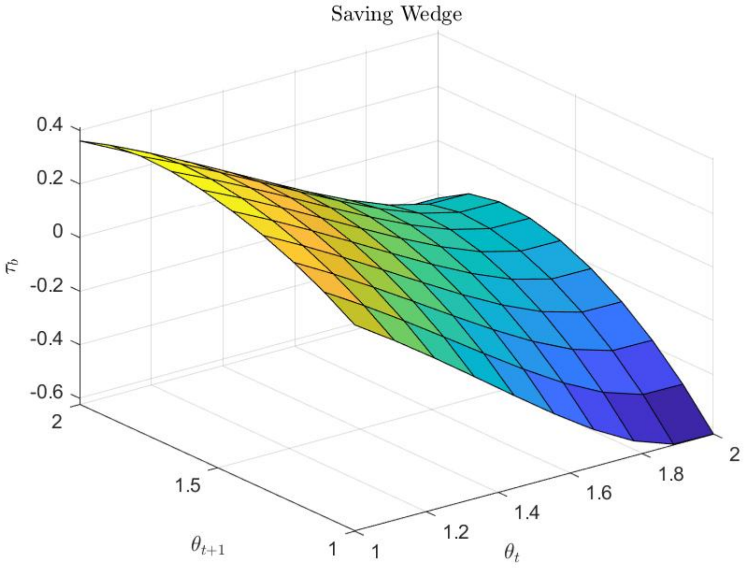
$$\tau_{t+1,b}\left(\theta^{t+1}\right) = 1 - \frac{c_{t+1}\left(\theta_{t+1}\right)\exp\left[\left(1-\beta\right)w_{t+1}\left(\theta_{t+1}\right)\right]}{\beta R c_t\left(\theta_t\right)}$$

$$\tau_{t+1,k}\left(\theta^{t+1}\right) = 1 - \frac{c_{t+1}\left(\theta_{t+1}\right)\exp\left[\left(1-\beta\right)w_{t+1}\left(\theta_{t+1}\right)\right]}{\beta c_t\left(\theta_t\right)\theta_t p_{t+1}\left(\theta^t\right)}$$

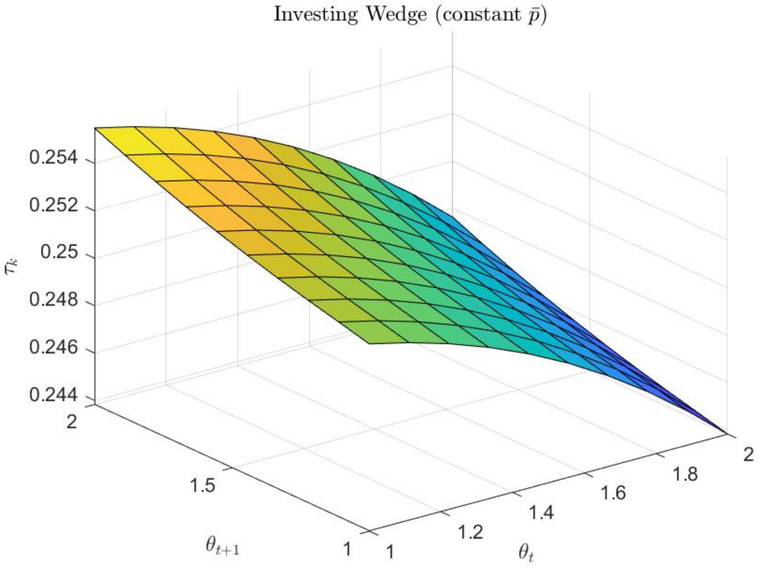
- Prices are given by

$$p_{t+1}\left(\theta^t\right) = \alpha K_f^{\alpha-1}\left(\frac{K_f}{\theta_t k_{t+1}\left(\theta_t\right)e^{(1-\beta)w_t\left(\theta^{t-1}\right)}}\right)^{\frac{1}{\varepsilon}}$$

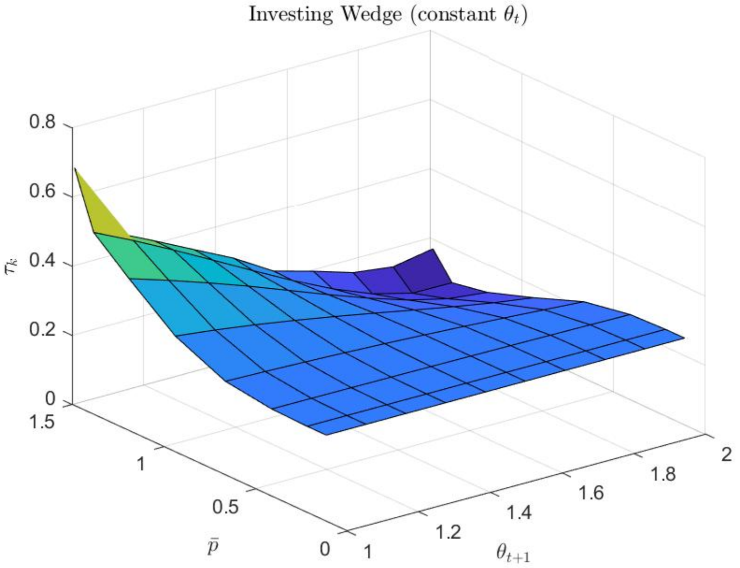
Dynamic Model: Savings Wedge



Dynamic Model: Investing Wedge



Dynamic Model: Investing Wedge



Conclusion

- Studied optimal nonlinear taxation of capital income with heterogeneous returns
- Static model: positive, hump-shaped wedges
  - Investing income subject to strictly larger distortions than savings income
- Dynamic model: derived recursive form, solved simplified planning problem
  - Investing wedge history-dependent and regressive in promise utility
  - Savings wedge independent of history prior to t

Remaining Work

- Static model: stochastic returns
- Infinite horizon: persistent shocks
- Implementation in decentralized economy with taxes, transfers, and private borrowing/lending contracts

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