Optimal Taxation with Heterogeneous Rates of Return
Two-Type Case
June 9, 2020

1 Two-Type Case

1.1 Immobile Capital

Here, I consider a model in which households are one of two types $\theta \in \{\theta_L, \theta_h\}$ with $\Pr(\theta = \theta_H) = \pi$. The social planner solves the usual problem:

$$\max \sum_{i \in \{L,H\}} \left[u(w - k(\theta_i)) + \beta u(c(\theta))\right] \pi_i \tag{1}$$

s.t.

$$\sum_{i \in \{L,H\}} \left[\theta_i k(\theta_i) - c(\theta_i)\right] \pi_i \geq E \tag{2}$$

$$u(w - k(\theta_H)) + \beta u(c(\theta_H)) \geq u\left(w - \frac{\theta_L}{\theta_H} k(\theta_L)\right) + \beta u(c(\theta_L)) \tag{3}$$

$$u(w - k(\theta_L)) + \beta u(c(\theta_L)) \geq u\left(w - \frac{\theta_H}{\theta_L} k(\theta_H)\right) + \beta u(c(\theta_H)) \tag{4}$$

I use the following parametrization:

$$\beta = 0.95 \qquad w = 1.2$$
$$E = 0 \qquad \pi = 0.3$$

In general, the planner requires that households of type θ_L invest in the first period. This level of investment can be low, particularly if θ_L is far below 1 and θ_H is large in relation to θ_L. However, it does seem that for most specifications, the planner finds it optimal to have $k(\theta_L) > 0$, likely due in part to satisfy the incentive constraint for the high type (3). Figure 1 shows the optimal levels of investment for the high and low types, for two different values of θ_L, and on the x-axis the ratio of θ_H to θ_L.

1.2 Mobile Capital

Here I consider the same model as in section 1.1, but allow for the borrowing and lending of risk-free bonds b between the households in the first period. Thus, the planner's problem becomes

$$\max \sum_{i \in \{L,H\}} \left[u(w - k(\theta_i) - b(\theta_i)) + \beta u(c(\theta))\right] \pi_i \tag{5}$$

s.t.

$$\sum_{i \in \{L,H\}} \left[\theta_i k(\theta_i) - c(\theta_i)\right] \pi_i \geq E \tag{6}$$

$$u(w - k(\theta_H) - b(\theta_H)) + \beta u(c(\theta_H)) \geq u\left(w - \frac{\theta_L}{\theta_H} k(\theta_L) - b(\theta_L)\right) + \beta u(c(\theta_L)) \tag{7}$$

$$u(w - k(\theta_L) - b(\theta_L)) + \beta u(c(\theta_L)) \geq u\left(w - \frac{\theta_H}{\theta_L} k(\theta_H) - b(\theta_H)\right) + \beta u(c(\theta_H)) \tag{8}$$

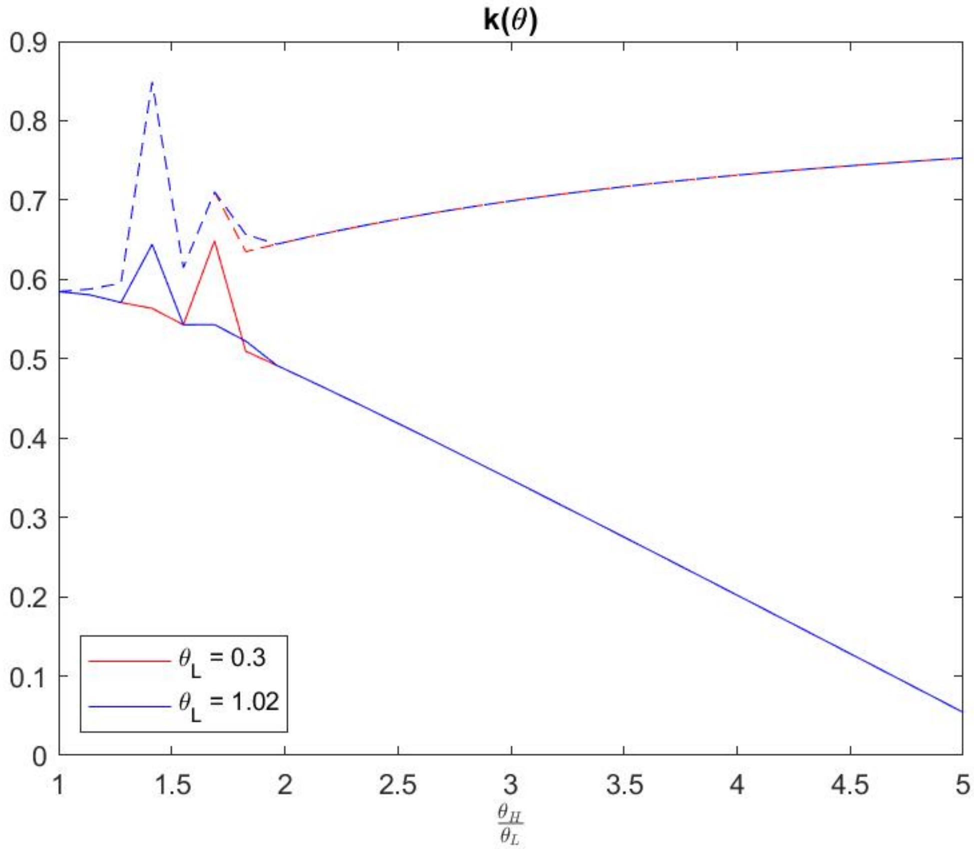

Figure 1: Optimal Investment. The solid lines show $k(\theta_L)$, and the dashed lines $k(\theta_H)$.

Aside from the allowance for borrowing and lending, the model is the same as in section 1.1. In this case, if θ_H is sufficiently large relative to θ_L, then $k(\theta_L) = 0$. The optimal borrowing and lending allocations call for θ_H types to be net borrowers in the first period, allowing them to invest more, and for θ_L types to be net lenders. Figure 2 show the optimal investment k and lending b for each type, as in Figure 1.

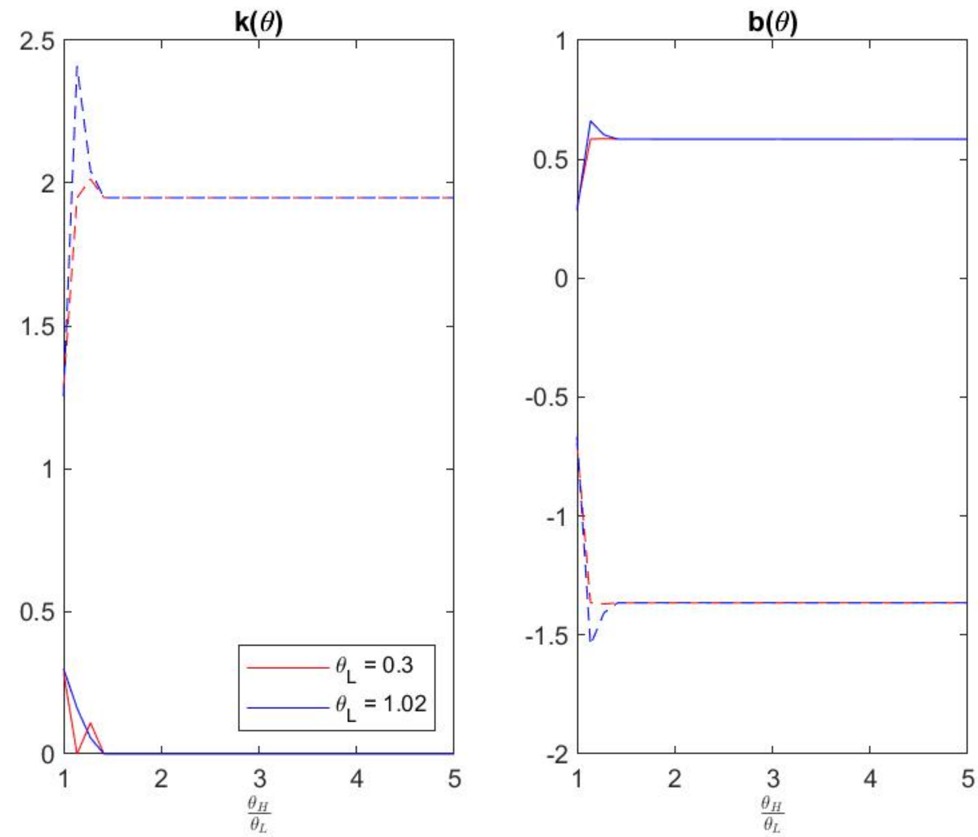

Figure 2: Optimal Investment and Lending. The dashed lines show the allocations for θ_H, and the solid lines θ_L.