The First-Order Approach to Principal-Agent Problems
Author(s): William P. Rogerson
Source: *Econometrica*, Vol. 53, No. 6 (Nov., 1985), pp. 1357-1367
Published by: The Econometric Society
Stable URL: https://www.jstor.org/stable/1913212
Accessed: 26-05-2020 15:30 UTC

Econometrica, Vol. 53, No. 6 (November, 1985)

THE FIRST-ORDER APPROACH TO PRINCIPAL-AGENT PROBLEMS

BY WILLIAM P. ROGERSON[1]

The first-order approach to principal-agent problems involves relaxing the constraint that the agent choose an action which is utility maximizing to require instead only that the agent choose an action at which his utility is at a stationary point. Although more mathematically tractable, this approach is generally invalid. This paper identifies sufficient conditions—the monotone likelihood ratio condition and convexity of the distribution function condition—for the first-order approach to be valid. The Pareto-optimal wage contract is shown to be nondecreasing in output under these same conditions.

1. INTRODUCTION

MIRRLEES [5] WAS THE FIRST to point out that the standard method for analyzing the principal-agent problem is not generally correct. This method, the so-called first-order approach, involves weakening the constraint that the agent choose a utility-maximizing action to require instead only that the agent choose an action at which his utility is at a stationary point. The resulting problem is more mathematically tractable. However, as Mirrlees [5] has shown, necessary conditions for a contract to solve the first-order program are not generally even necessary conditions for the valid program. Therefore qualitative propositions about the nature of the Pareto-optimal contract derived from the first-order approach are not in general valid. This has motivated researchers to try to identify classes of cases where the first-order approach is valid.

In a subsequent paper, Mirrlees [7] considered the case where the probability functions determining output as a stochastic function of effort satisfy two properties called the monotone likelihood ratio condition (MLRC) and the convexity of the distribution function condition (CDFC).[2] He offered a proof that these two conditions imply that the first-order approach is valid, and furthermore, that the optimal wage schedule increases in output. However, the proof contains an error.[3] This paper offers a correct and much simpler proof of Mirrlees' proposition.

Grossman and Hart [1] have proven that when the MLRC and CDFC hold and the principal is risk-neutral that the Pareto-optimal contract is nondecreasing in output by directly analyzing the valid program instead of using the first-order approach. However, their proof does not generalize to the case where the principal is risk-averse.[4] Holmstrom [3] has identified another class of problems where the first-order approach is valid. This is the case where the distribution function over

[1] I would like to thank Kathleen Hagerty, Bengt Holmstrom, David Kreps, Rick Lambert, Steve Mathews, James Mirrlees, and Dilip Mookherjee for helpful comments and discussions.

[2] These are defined in Section 4.

[3] The proof relies on using comparative statics results concerning the agent's effort level as a function of the contract he is offered. For this to be a valid procedure, the agent's optimal actions must be unique. However, as will be seen, proving that the agent's effort choice is unique is the heart of the difficulty in showing that the first-order approach is valid.

[4] Grossman and Hart's proof depends on their Proposition 6, which is no longer true when the principal is risk-averse.

outcomes is a convex combination of two fixed distribution functions and the agent's effort determines the weights of the convex combination.

2. THE MODEL

The agent chooses an action from a real interval, A:

$$A = [\underline{a}, \bar{a}]. \tag{2.1}$$

The outcome can be one of N alternatives. Let

$$X = \{x_1, \ldots, x_N\} \tag{2.2}$$

denote the outcome space where the outcomes are dollar returns to the principal ordered from smallest to largest.

Let $p_j(a)$ denote the probability of outcome j occurring given that action a is taken. The following assumptions will be made about these functions.

ASSUMPTION (A.1): *$p_j(a) > 0$ for every $j \in \{1, \ldots, N\}$ and every $a \in A$.*

ASSUMPTION (A.2): *$p_j(a)$ is twice continuously differentiable for every $j \in \{1, \ldots, N\}$.*

Let $F_j(a)$ denote the corresponding distribution function:

$$F_j(a) = \sum_{i=1}^{j} p_i(a). \tag{2.3}$$

The principal is risk averse or risk neutral with utility function over income $u(y)$. The following assumptions are made.

ASSUMPTION (A.3): *u is defined over the real interval $(y_u, \infty]$ where y_u may be equal to $-\infty$.*

ASSUMPTION (A.4): $\text{Lim}_{y \to y_u} u(y) = -\infty$.

ASSUMPTION (A.5): *u is strictly increasing, concave, and twice continuously differentiable.*

The agent is risk averse with utility function over income and actions given by

$$v(y) - a. \tag{2.4}$$

Therefore a can be thought of as effort which the agent prefers to avoid.[5] The following is assumed.

ASSUMPTION (A.6): *v is defined over the real interval (y_v, ∞) where y_v may be $-\infty$.*

[5] The assumption that the agent's utility is linear in effort is made without loss of generality. Units of effort can always be chosen so that this is true.

ASSUMPTION (A.7): $\operatorname{Lim}_{y \to y_v} v(y) = -\infty$.

ASSUMPTION (A.8): *v is strictly increasing, strictly concave, and twice continuously differentiable.*

A contract between the principal and agent consists of an agreement over what wage will be paid to the agent conditional on the observed outcome. Let

$$(2.5) \qquad w = (w_1, \ldots, w_N)$$

denote a contract, where w_i is the wage paid to the agent if outcome x_i occurs. Let $U(w, a)$ and $V(w, a)$ denote the expected utility to, respectively, the principal and agent of the contract w when the agent chooses action a. These are determined as follows:

$$(2.6) \qquad U(w, a) = \sum_{j=1}^{N} p_j(a) u(x_j - w_j),$$

$$(2.7) \qquad V(w, a) = \left(\sum_{j=1}^{N} p_j(a) v(w_j) \right) - a.$$

Let U_a and U_{aa} denote the first and second derivatives of U with respect to a. Define V_a and V_{aa} similarly. Finally let V^* be the minimum expected utility level that the agent must be guaranteed.

A contract is Pareto optimal if no contract exists which gives the principal higher expected utility and gives the agent at least V^*.

DEFINITION: A contract, w, is said to be Pareto optimal if it solves the following program:

$$(2.8) \qquad \operatorname*{Max}_{w,a} U(w, a)$$

subject to

$$(2.9) \qquad V(w, a) \geq V^*,$$

$$(2.10) \qquad a \in \operatorname*{argmax}_{\hat{a} \in A} V(w, \hat{a}).$$

DEFINITION: Program (2.8)–(2.10) will be called the Pareto-optimization program.

In subsequent sections programs will be defined which are called the relaxed and doubly-relaxed Pareto-optimization programs. To clearly distinguish program (2.8)–(2.10) from these, it will sometimes be referred to as the unrelaxed Pareto-optimization program.

3. THE FIRST-ORDER APPROACH

Analysis of the Pareto-optimization program is difficult because the incentive compatibility constraint,

$$(3.1) \qquad a \in \underset{\hat{a} \in A}{\operatorname{argmax}}\ V(w, \hat{a}),$$

is actually a continuum of constraints of the form

$$(3.2) \qquad V(w, a) \geq V(w, \hat{a})$$

for every $\hat{a} \in A$. If (3.1) could be replaced with the requirement that the effort level be a stationary point for the agent,

$$(3.3) \qquad V_a(w, a) = 0,$$

then a solution could be calculated using the Kuhn–Tucker Theorem. This is called the first-order approach.

Substitution of (3.3) for (3.1) amounts to enlarging the constraint set over which U is maximized. Now all stationary points for the agent are included instead of merely the global maxima. Since the constraint set has been relaxed, the resulting program will be referred to as the relaxed program.

DEFINITION: The program created by substituting (3.3) for (2.10) is called the relaxed Pareto-optimization program.

Mirrlees [**5**] was the first to point out that the solutions to the relaxed and unrelaxed programs will not always be the same. In particular, necessary conditions for a contract to solve the relaxed program derived by using the Kuhn–Tucker Theorem may not even be necessary conditions for a contract to solve the unrelaxed program!

The problem with the first-order approach can be easily illustrated by a graphical example from Mirrlees [**5**]. Consider the program

$$(3.4) \qquad \underset{x,y}{\operatorname{Max}}\, f(x, y)$$

subject to

$$(3.5) \qquad y \in \underset{\hat{y} \in R}{\operatorname{argmax}}\ g(x, \hat{y}),$$

where f and g are smooth real-valued functions defined over R^2. The relaxed program corresponding to this is:

$$(3.6) \qquad \underset{x,y}{\operatorname{Max}}\, f(x, y)$$

subject to

$$(3.7) \qquad g_y(x, y) = 0.$$

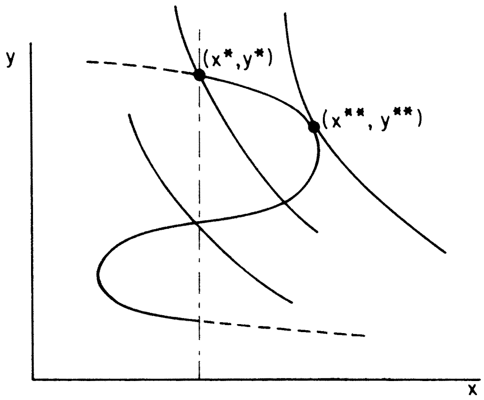

FIGURE 1.

Constraint (3.7) will be a smooth one-dimensional manifold in R^2. This is the s-shaped curve in Figure 1.

Not all points in the manifold necessarily satisfy (3.5), however. Some of the stationary points may be local minima, saddle points, or local but not global maxima. Let the part of the manifold which is a dashed line denote the points which satisfy (3.5). Three of the contour surfaces for f are drawn in the figure. As drawn, it is clear that (x^*, y^*) solves the unrelaxed program, while (x^{**}, y^{**}) solves the relaxed program. Furthermore, (x^*, y^*) does not satisfy the necessary conditions for the relaxed program derived from the Kuhn-Tucker Theorem.

4. THE SUFFICIENT CONDITIONS

This section presents and describes the conditions which will be shown to be sufficient for the first-order approach to be valid. The first is the monotone likelihood-ratio condition (MLRC).

DEFINITION: The functions $\{p_j(a)\}_{j=1}^N$ are said to satisfy the MLRC if $\hat{a} \leq \hat{\hat{a}}$ implies that $p_i(\hat{a})/p_i(\hat{\hat{a}})$ is nonincreasing in i.

Milgrom [4] has shown that the MLRC is equivalent to the following statistical property. Suppose that a statistician begins with a prior over the agent's action choice, observes the level of output, and then updates his prior to calculate a posterior on the agent's action choice. Let $G(a|x)$ denote the posterior probability distribution given outcome x is observed. Then the MLRC is equivalent to

$$(4.1) \qquad x \leq \hat{x} \Rightarrow G(a|\hat{x}) \leq G(a|x) \quad \text{for every} \quad a \in A.$$

That is, observation of a higher level of output allows the statistical inference that the agent worked harder in the sense of stochastic dominance. Milgrom [4] has also shown that in a differentiable world, the MLRC is equivalent to $p_i'(a)/p_i(a)$ being nondecreasing in i for every a.

The MLRC also implies that increases in effort cause output to increase in the sense of stochastic dominance. This will be called the stochastic dominance condition (SDC).

DEFINITION: The functions $\{p_j(a)\}_{j=1}^N$ are said to satisfy the SDC if $F_j'(a)$ is nonpositive for every $j \in \{1, \ldots, N\}$ and $a \in A$.

LEMMA 1:[6] *The MLRC implies the SDC.*

PROOF: See Whitt [9]. *Q.E.D.*

The second property that the probability functions will be assumed to possess is the convexity of the distribution function condition (CDFC).

DEFINITION: The functions $\{p_j(a)\}_{j=1}^N$ satisfy the CDFC if $F_j''(a)$ is nonnegative for every $j \in \{1, \ldots, N\}$ and $a \in A$.

By the MLRC, $F_j(a)$ decreases in a, i.e., the probability of an outcome less than or equal to x_j decreases as the agent works harder. The CDFC requires that the function decrease at a decreasing rate, i.e. the CDFC is a form of stochastic diminishing returns to scale. Note, however, that if output is determined by a stochastic production function with diminishing returns to scale in each state of nature, the implied distribution function over output will not, in general, exhibit the CDFC.

An example of a family of densities satisfying the MLRC and CDFC is as follows:[7]

$$F_i(a) = \left(\frac{x_i}{x_N}\right)^{a-\underline{a}}. \tag{4.2}$$

5. PROOF OF THE VALIDITY OF THE FIRST-ORDER APPROACH

It will be useful to introduce a further relaxation of the Pareto-optimization program by weakening the incentive compatibility constraint even further. This will be called the doubly-relaxed program.

DEFINITION: The following program will be called the doubly-relaxed Pareto-optimization program:

$$\operatorname*{Max}_{w,a} U(w, a) \tag{5.1}$$

[6] I would like to thank David Kreps for originally suggesting a proof of this which appeared in an earlier version of this paper. An anonymous referee subsequently identified a reference (Whitt [9]) which contains a different proof.

[7] I would like to thank Steve Mathews for suggesting this example.

subject to

(5.2) $\quad V(w, a) \geq V^*,$

(5.3) $\quad V_a(w, a) \geq 0.$

Before proceeding with the formal proof, the idea underlying it will be outlined. The intuition for the proof is most clearly presented for the case where $A = R$ and the possibility of corner solutions for the agent's action choice can be ignored. Let B denote the set of all contract-action pairs, (w, a), which satisfy the unrelaxed constraint set. Let C denote all (w, a) which satisfy the relaxed constraint set. C contains B. To show that the first-order approach is valid it is therefore clearly sufficient to show that if (w^*, a^*) maximizes $U(w, a)$ over C, that (w^*, a^*) is an element of B. In particular it is sufficient to show the following statement:

If (w^*, a^*) maximizes $U(w, a)$ over C then

(i) $\quad V_a(w^*, a^*) = 0,$

(ii) $\quad V_{aa}(w^*, a) \leq 0 \quad \text{for every} \quad a \in A.$

By definition, condition (i) holds for every element of C. Therefore it suffices to show (ii). Unfortunately, to prove (ii), one must be able to prove that the Lagrange multiplier on the incentive constraint, (3.3), is nonnegative when the relaxed program is solved. (See Mirrlees [7].) However, the only known method of proving that the multiplier is nonnegative requires the *assumption* that the first-order approach is valid. (See Holmstrom [**2**].)

To avoid this circularity, an even more relaxed program is considered in this paper. Let D denote the doubly-relaxed constraint set. D contains C. Clearly the first-order approach would be valid if it could be shown that if (w^*, a^*) maximizes $U(w, a)$ over D, then (w^*, a^*) is an element of B. In particular it is sufficient to show that if (w^*, a^*) maximizes $U(w, a)$ over D then (i) and (ii), above, are satisfied. Because the incentive constraint is now an *inequality* constraint, it is immediate that the Lagrange multiplier associated with it is nonnegative. Existing arguments by Mirrlees [7] then show that (ii) holds. However, (i) is no longer satisfied, trivially; it may be the case that $V_a(w^*, a^*) > 0$. A strong economic intuition suggests, however, that (i) should be binding. If (i) is not binding, the agent wishes to work harder than he is allowed to by the principal—the landowner must hire guards to prevent his sharecroppers from sneaking back to the fields at night and doing a little extra work! It seems likely that in such a situation, less risk could be imposed on the agent (with a consequent improvement in risk bearing) until his incentive to work is reduced to the correct level. This will be shown to be the case.

The remainder of this section presents and formally proves the above results. Proposition 1 and its three corollaries contain the results of this section. After their statement, Proposition 1 is proven by a series of lemmas. Two existence assumptions are required for the results.

ASSUMPTION (A.9): *A solution to the doubly-relaxed program exists.*

ASSUMPTION (A.10): A solution to the unrelaxed program exists with $a > \underline{a}$.

Grossman and Hart [1] have shown that Assumptions (A.1)-(A.8) are sufficient to guarantee existence for the unrelaxed program if it is additionally assumed that the constraint set is nonempty, i.e. a contract exists under which both parties can survive. The same technique of proof can be used to prove existence of a solution to the doubly-relaxed program. It is additionally assumed here that a solution to the unrelaxed program exists which involves more than the lowest possible level of work effort.

PROPOSITION 1: *Suppose (A.1)-(A.10), the MLRC, and the CDFC hold, and that (w, a) is a solution to the doubly-relaxed program.*

(i) *Then (w, a) is also a solution to the unrelaxed program.*

(ii) *If $a < \bar{a}$, it is also a solution to the relaxed program.*

(iii) *The wage contract is monotone nondecreasing in output, i.e.,*

$$w_i \leq w_{i+1} \tag{5.4}$$

for every i.

(iv) *If $a < \bar{a}$ then the principal's expected utility is nondecreasing in effort, i.e.,*

$$U_a(w, a) \geq 0. \tag{5.5}$$

PROOF: See the lemmas below. *Q.E.D.*

COROLLARY 1: *Suppose that (A.1)-(A.10), the MLRC, and the CDFC hold. Then for $a > \underline{a}$, (w, a) solves the doubly-relaxed program if and only if it solves the unrelaxed program.*

PROOF: If (w, a) solves the doubly-relaxed program it solves the unrelaxed program by Proposition 1. To show the reverse, assume that (w, a) solves the unrelaxed program. Assume, for contradiction, that (w, a) does not solve the doubly-relaxed program. Let (w^*, a^*) be a solution to the doubly-relaxed program.

Since $a > \underline{a}$, it must be true that $V_a(w, a) \geq 0$, and therefore (w, a) is an element of the doubly-relaxed constraint set. Since it does not solve the doubly-relaxed program, $U(w^*, a^*) > U(w, a)$. However, by Proposition 1, (w^*, a^*) is in the unrelaxed constraint set. Therefore (w, a) cannot solve the unrelaxed program. *Q.E.D.*

COROLLARY 2: *Suppose that (A.1)-(A.10), the MLRC, and the CDFC hold and that a solution to the doubly-relaxed program exists with $a < \bar{a}$. Then for $a \in (\underline{a}, \bar{a})$, (w, a) solves the doubly-relaxed program if and only if it solves the relaxed program if and only if it solves the unrelaxed program.*

PROOF: The proof is similar to that of Corollary 1. *Q.E.D.*

COROLLARY 3: *Suppose that (A.1)–(A.10), the MLRC, and CDFC hold and that (w, a) is a solution to the unrelaxed program with $a > \underline{a}$.*

(i) *Wages are monotone nondecreasing in output (i.e., (5.4) holds).*

(ii) *If $a < \bar{a}$, the principal's expected utility is nondecreasing in effort (i.e., (5.5) holds).*

PROOF: This follows from Proposition 1(iii) and (iv) and Corollary 1. *Q.E.D.*

The proof of Proposition 1 now follows in a series of Lemmas. Assumptions (A.1)–(A.10), the MLRC, and the CDFC will be assumed to hold throughout.

LEMMA 2: *If (w, a) solves the doubly-relaxed program, there exist nonnegative real numbers λ and δ such that*

$$\frac{u'(x_i - w_i)}{v'(w_i)} = \lambda + \delta \frac{p_i'(a)}{p_i(a)} \tag{5.6}$$

for every $i \in \{1, \dots, N\}$ and

$$U_a + \lambda V_a + \delta V_{aa} \begin{cases} \leqslant 0, \\ = 0, \\ \geqslant 0, \end{cases} \quad \text{for} \quad a \begin{cases} = \underline{a}, \\ \in (\underline{a}, \bar{a}), \\ = \bar{a}. \end{cases} \tag{5.7}$$

PROOF: These are the Kuhn–Tucker necessary conditions. *Q.E.D.*

The reason for relaxing the relaxed program even further is to guarantee that the multiplier, δ, is nonnegative.[8] The work of Mirrlees [**6**] has shown that if δ is nonnegative then the MLRC and (5.6) imply that wages are monotone nondecreasing in output. Mirrlees [7] has also shown that the CDFC then implies that the agent's expected utility is concave in action. For completeness these results are presented and proven as Lemmas 3 and 4.

LEMMA 3: *If (w, a) solves the doubly-relaxed program, then wages are monotone nondecreasing in output.*

[8] Rick Lambert has pointed out to me that when the principal is risk neutral, δ can be proven to be positive without recourse to the doubly-relaxed program. The corresponding condition to (5.6) for the relaxed program then is

$$\frac{1}{v'(w_i)} = \lambda + \delta \frac{p_i'(a)}{p_i(a)}.$$

If δ is nonpositive, wages are nonincreasing in output. By the SDC the agent therefore chooses $a = \underline{a}$. Therefore for $a > \underline{a}$, δ must be positive.

PROOF: This follows immediately from (5.6) because the MLRC implies that $p_i'(a)/p_i(a)$ is nondecreasing in i. *Q.E.D.*

LEMMA 4: *If (w, a) solves the doubly-relaxed program, then the agent's expected utility at w is concave in action.*

PROOF: The agent's expected utility can be rewritten as

$$V = \sum_{i=1}^{N} \Delta_i \left[\sum_{j=i}^{N} p_j(a) \right] - a \tag{5.8}$$

where

$$\Delta_i = \begin{cases} v(w_i) - v(w_{i-1}), & i > 1, \\ v(w_1), & i = 1. \end{cases} \tag{5.9}$$

Therefore

$$V_{aa} = \sum_{i=2}^{N} \Delta_i \left[\sum_{j=i}^{N} p_j''(a) \right]. \tag{5.10}$$

Because w_i is nondecreasing, each of the Δ_i terms is nonnegative. By the CDFC each of the terms in square brackets in nonpositive. Therefore $V_{aa} \leqslant 0$. *Q.E.D.*

Lemma 5 now provides the key step in the proof of Proposition 1. It shows that the further relaxation of the relaxed program does not change the solution.

LEMMA 5: *Suppose (w, a) is a solution to the doubly-relaxed program and $a < \bar{a}$. Then $V_a(w, a) = 0$.*

PROOF: If $\delta > 0$, constraint (5.3) is binding and $V_a = 0$. Now suppose, instead, that $\delta = 0$. It will be shown that V_a still equals zero. Since $\delta = 0$, by (5.6), w is now a first-best risk-sharing contract for the principal and agent. Therefore $x_i - w_i$ is nondecreasing in i.

The principal's expected utility is

$$U = \sum_{i=1}^{N} p_i(a) u(w_i - x_i). \tag{5.11}$$

This can be rewritten as

$$U = \sum_{i=1}^{N} \Delta_i \left[\sum_{j=i}^{N} p_j(a) \right] \tag{5.12}$$

where

$$\Delta_i = \begin{cases} v(x_i - w_i) - v(x_{i-1} - w_{i-1}), & i > 1, \\ v(x_1 - w_1), & i = 1. \end{cases} \tag{5.13}$$

Therefore

$$(5.14) \qquad U_a = \sum_{i=2}^{N} \Delta_i \left[\sum_{j=1}^{N} p_j'(a) \right].$$

By the SDC (which is implied by the MLRC) each of the terms in square brackets is nonnegative. Since the principal's net income, $x_i - w_i$, is nondecreasing in i, each of the Δ_i terms is also nonnegative. Therefore U_a is nonnegative. Note as well that since $\delta = 0$, by (5.6) λ must be strictly positive. Since $\delta = 0$, $\lambda > 0$, and $U_a \geqslant 0$, it follows from (5.7) that $V_a \leqslant 0$. However, by (5.3) $V_a \geqslant 0$. Therefore $V_a = 0$. *Q.E.D.*

Proposition 1(i) now follows from Lemmas 4 and 5. If (w^*, a^*) solves the doubly-relaxed program, then V satisfies

$$(5.15) \qquad V_{aa}(w^*, a) \leqslant 0 \quad \text{for every} \quad a \in [\underline{a}, \bar{a}],$$

$$(5.16) \qquad V_a(w^*, a^*) = 0 \quad \text{if} \quad a^* \in [\underline{a}, \bar{a}),$$

and

$$(5.17) \qquad V_a(w^*, a^*) \geqslant 0 \quad \text{if} \quad a^* = \bar{a}.$$

These conditions imply that the agent's action choice is a global maximum and, consequently (w^*, a^*) is also an element of the unrelaxed constraint set. Since (w^*, a^*) maximizes the principal's expected utility over the doubly-relaxed constraint set, by (A.10) it clearly also maximizes it over the unrelaxed constraint set. Thus, (w^*, a^*) solves the unrelaxed program.

Propositions 1(ii) and 1(iii) follow immediately from, respectively, Lemmas 5 and 3. Proposition 1(iv) follows from (5.7) since $V_a = 0$ and $V_{aa} \leqslant 0$.

Stanford University

Manuscript received February, 1983; final revision received October, 1984.

REFERENCES

[1] GROSSMAN, SANFORD, AND OLIVER HART: "An Analysis of the Principal-Agent Problem," *Econometrica*, 51(1983), 7-45.

[2] HOLMSTROM, B.: "Moral Hazard and Observability," *Bell Journal of Economics*, 10(1979), 74-91.

[3] ———: "A Note on the First-Order Approach in Principal Agent Models," mimeo, 1984.

[4] MILGROM, PAUL: "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12(1981), 380-391.

[5] MIRRLEES, J.: "The Theory of Moral Hazard and Unobservable Behavior—Part I," mimeo, Nuffield College, Oxford, 1975.

[6] ———: "The Optimal Structure of Authority and Incentives Within an Organization," *Bell Journal of Economics*, 7(1976), 105-131.

[7] ———: "The Implications of Moral Hazard for Optimal Insurance," Seminar given at Conference held in honor of Karl Borch, Bergen, Norway, mimeo, 1979.

[8] SHAVELL, S.: "Risk Sharing and Incentives in the Principal and Agent Relationship," *Bell Journal of Economics*, 10(1979), 55-73.

[9] WHITT, WARD: "Uniform Conditional Stochastic Order," *Journal of Applied Probability*, (1980), 112-123.