Parent-child pair makes up a dynasty Parent allocates lifetime income y between consumption c and investment in child's wealth z. Payoff to parent:

$$u(c) + w_0(z) + w_1(e)$$

where w_0 is the intrinsic parental utility from the child's wealth, and w_1 the milestone utility from passing a threshold a, with $e = \max\{z - a, 0\}$.

$\Psi(\lambda y, F^\lambda) = \lambda\Psi(y, F)$

$\min\{y, \min F\} \leq \Psi(y, F) \leq \max(y, \max F)$

The parent's problem is

$$\max_z u(y - k(z)) + w_0(z) + w_1(\max\{z - a, 0\})$$

$$w_0'(z_0) = \frac{u'(y - k(z_0))}{f'(k(z_0))}$$

and

$$w_0'(z_1) + w_1'(z_1 - a) = \frac{u'(y - k(z_1))}{f'(k(z_1))}$$

$$\left(y - \frac{z}{\rho}\right)^{1-\sigma} + \delta\left[z^{1-\sigma} + \pi\left(\max\{z - a, 0\}\right)^{1-\sigma}\right]$$

$$\left(1 - \frac{g}{\rho}\right)^{1-\sigma} + \delta\left[g^{1-\sigma} + \pi\left(\max\{g - r, 0\}\right)^{1-\sigma}\right]$$

$r \equiv z/y$

$$\left(1 - \frac{g(r)}{\rho}\right)^{-\sigma} = \delta\rho\left[g(r)^{-\sigma} + \pi\left(g(r) - r\right)^{-\sigma}\right]$$

$$\left(1 - \frac{g}{\rho}\right)^{-\sigma} = \delta\rho g^{-\sigma}$$

$\underline{g}$