CHAPTER 1. NONPARAMETRIC REGRESSION AND CHANGE-POINT PROBLEMS

1.1. INTRODUCTION

In this chapter we describe the basic mathematical tools used for two statistical problems: that of nonparametric regression and of change-point estimation. The techniques of this chapter apply in several ways for the construction and analysis of image estimators. These applications will first appear in Chapter 4. The purpose of this chapter is to give a simple introduction to nonparametric regression and to change-point estimation in a self-sufficient form. We do not propose an overview of all estimation techniques available for these problems. For nonparametric regression we study only an important class of locally-polynomial estimators which contains the popular kernel estimator as a special case, and the class of piecewise-polynomial estimators. For the change-point problem we consider the maximum likelihood estimator. The results that we prove in this chapter are related mainly to the convergence rates of the estimators.

1.2. THE NONPARAMETRIC REGRESSION PROBLEM

Let X,Y be random variables and let $(X_1,Y_1),\ldots,(X_n,Y_n)$ be n independent pairs of random variables having the same distribution as (X,Y). The problem of nonparametric regression with random design consists in estimation of the function

$$f(x) = E(Y|X=x)$$

based on observations $(X_1,Y_1),\ldots,(X_n,Y_n)$. Here $E(Y|X=x)$ is the conditional expectation of Y for fixed $X=x$. Random variables X_i are called design points. The design is defined as a collection $\mathcal{X}=(X_1,\ldots,X_n)$. Note that

$$(1.1) \qquad Y_i = f(X_i) + \xi_i,\ i = 1,\ldots,n,$$

where ξ_i are independent random variables such that $E(\xi_i|X_i) = 0$.