Discrete Mathematics Homework II
Set Theory, Functions, Algorithms, and Integer Representations

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§2.1 Sets

1. List the members of these sets.

a) $\{x | x$ **is a real number such that** $x^2 = 1\}$

Will show the list of members as a set.
$\{1, -1\}$

b) $\{x | x$ **is a positive integer less than** $12\}$

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c) $\{x | x$ **is the square of an integer and** $x < 100\}$

$\{1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d) $\{x | x$ **is an integer such that** $x^2 = 2\}$

No members. $\{\}$ or $\emptyset$

3. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of airline flights from New York to New Delhi,
the set of nonstop airline flights from New York to New Delhi

If all airline flights from New York to New Delhi are non-stop, both sets would be equal meaning both would be subsets of each other. However, in the reasonable case that not all flights are nonstop, the second set is a subset of the first and first is not subset of second.

b) the set of people who speak English, the set of people who speak Chinese

Neither set is subset of the other.

c) the set of flying squirrels, the set of living creatures that can fly

First is subset of second. Second is not subset of first.

5. Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$

Equal

b) $\{\{1\}\}, \{1, \{1\}\}$

Not equal

c) $\emptyset, \{\emptyset\}$

Not equal, first is empty set, second is singleton set containing an empty set.

7. For each of the following sets, determine whether 2 is an element of that set.

a) $\{x \in \mathbb{R} | x$ **is an integer greater than** $1\}$

Yes, 2 is an element of this set.

b) $\{x \in \mathbb{R} | x$ **is the square of an integer**$\}$

No, this set does not contain 2.

c) $\{2, \{2\}\}$

Yes.

d) $\{\{2\}, \{\{2\}\}\}$

No.

e) $\{\{2\}, \{2, \{2\}\}\}$

No.

f) $\{\{\{2\}\}\}$

No.

9. Determine whether each of these statements is true or false.

a) $0 \in \emptyset$

False

b) $\emptyset \in \{0\}$

False

c) $\{0\} \subset \emptyset$

False

d) $\emptyset \subset \{0\}$

False

e) $\{0\} \in \{0\}$

True

f) $\{0\} \subset \{0\}$

False

g) $\{\emptyset\} \subseteq \{\emptyset\}$

True

11. Determine whether each of these statements is true or false.

a) $x \in \{x\}$

True

b) $\{x\} \subseteq \{x\}$

True

c) $\{x\} \in \{x\}$

False

d) $\{x\} \in \{\{x\}\}$

True

e) $\emptyset \subseteq \{x\}$

False

f) $\emptyset \in \{x\}$

False

17. Suppose that A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$. Show that $A \subseteq C$.

Say set A = {1, 2} and set B = {1, 2, 3}, where A ⊆ B.
Then, set C = {1, 2, 3, 4, 5, 6}, such that B ⊆ C.
∴ since {1, 2} ⊆ {1, 2, 3, 4, 5, 6}, we see A ⊆ C.

19. What is the cardinality of each of these sets?

a) $\{a\}$

1

b) $\{\{a\}\}$

1

c) $\{a, \{a\}\}$

2

d) $\{a, \{a\}, \{a, \{a\}\}\}$

3

21. Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$

Itself, $\{a\}$

b) $\{a, b\}$

$\{a, \{a, b\}\}$

c) $\{\emptyset, \{\emptyset\}\}$

$\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

25. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

...

32. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$

$A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$
$A \times B \times C = \{((a, x), 0), ((a, x), 1), ((a, y), 0), ((a, y), 1), ((b, x), 0), ((b, x), 1)\}$
...

41. Translate each of these quantifications into English and determine its truth value.

a) $\forall x \in \mathbb{R}(x^2 \neq -1)$

For all x in domain of real numbers, x squared is not equal to negative one. True

b) $\exists x \in \mathbb{Z}(x^2 = 2)$

There exists in domain of integers x, such that x squared is two. False

c) $\forall x \in \mathbb{Z}(x^2 > 0)$

For all x in domain of integers, x squared is greater than zero. True

d) $\exists x \in \mathbb{R}(x^2 = x)$

There exists in domain of real numbers x, such that x squared equals x. True, 1 satisfies.