Discrete Mathematics Homework II
Set Theory, Functions, Algorithms, and Integer Representations

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§2.1 Sets

1. List the members of these sets.

a) $\{x|x$ **is a real number such that** $x^2 = 1\}$

Will show the list of members as a set.
$\{1, -1\}$

b) $\{x|x$ **is a positive integer less than** $12\}$

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c) $\{x|x$ **is the square of an integer and** $x < 100\}$

$\{1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d) $\{x|x$ **is an integer such that** $x^2 = 2\}$

No members. $\{\}$ or $\emptyset$

3. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of airline flights from New York to New Delhi,
the set of nonstop airline flights from New York to New Delhi

If all airline flights from New York to New Delhi are non-stop, both sets would be equal meaning both would be subsets of each other. However, in the reasonable case that not all flights are nonstop, the second set is a subset of the first and first is not subset of second.

b) the set of people who speak English, the set of people who speak Chinese

Neither set is subset of the other.

c) the set of flying squirrels, the set of living creatures that can fly

First is subset of second. Second is not subset of first.

5. Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$

Equal

b) $\{\{1\}\}, \{1, \{1\}\}$

Not equal

c) $\emptyset, \{\emptyset\}$

Not equal, first is empty set, second is singleton set containing an empty set.

7. For each of the following sets, determine whether 2 is an element of that set.

a) $\{x \in \mathbb{R} | x$ **is an integer greater than** $1\}$

Yes, 2 is an element of this set.

b) $\{x \in \mathbb{R} | x$ **is the square of an integer**$\}$

No, this set does not contain 2.

c) $\{2, \{2\}\}$

Yes.

d) $\{\{2\}, \{\{2\}\}\}$

No.

e) $\{\{2\}, \{2, \{2\}\}\}$

No.

f) $\{\{\{2\}\}\}$

No.

9. Determine whether each of these statements is true or false.

a) $0 \in \emptyset$

False

b) $\emptyset \in \{0\}$

False

c) $\{0\} \subset \emptyset$

False

d) $\emptyset \subset \{0\}$

False

e) $\{0\} \in \{0\}$

True

f) $\{0\} \subset \{0\}$

False

g) $\{\emptyset\} \subseteq \{\emptyset\}$

True

11. Determine whether each of these statements is true or false.

a) $x \in \{x\}$

True

b) $\{x\} \subseteq \{x\}$

True

c) $\{x\} \in \{x\}$

False

d) $\{x\} \in \{\{x\}\}$

True

e) $\emptyset \subseteq \{x\}$

False

f) $\emptyset \in \{x\}$

False

17. Suppose that A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$. Show that $A \subseteq C$.

Say set A = {1, 2} and set B = {1, 2, 3}, where A $\subseteq$ B.
Then, set C = {1, 2, 3, 4, 5, 6}, such that B $\subseteq$ C.
$\therefore$ since {1, 2} $\subseteq$ {1, 2, 3, 4, 5, 6}, we see A $\subseteq$ C.

19. What is the cardinality of each of these sets?

a) $\{a\}$

1

b) $\{\{a\}\}$

1

c) $\{a, \{a\}\}$

2

d) $\{a, \{a\}, \{a, \{a\}\}\}$

3

21. Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$

Itself, $\{a\}$

b) $\{a, b\}$

$\{a, \{a, b\}\}$

c) $\{\emptyset, \{\emptyset\}\}$

$\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

25. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

...

32. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$

$A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$
$A \times B \times C = \{((a, x), 0), ((a, x), 1), ((a, y), 0), ((a, y), 1), ((b, x), 0), ((b, x), 1)\}$
...

41. Translate each of these quantifications into English and determine its truth value.

a) $\forall x \in \mathbb{R}(x^2 \neq -1)$

For all x in domain of real numbers, x squared is not equal to negative one. True

b) $\exists x \in \mathbb{Z}(x^2 = 2)$

There exists in domain of integers x, such that x squared is two. False

c) $\forall x \in \mathbb{Z}(x^2 > 0)$

For all x in domain of integers, x squared is greater than zero. True

d) $\exists x \in \mathbb{R}(x^2 = x)$

There exists in domain of real numbers x, such that x squared equals x. True, 1 satisfies.

§2.2

3. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

a) $A \cup B$

$\{0, 1, 2, 3, 4, 5, 6\}$

b) $A \cap B$

$\{3\}$

c) $A - B$

$\{1, 2, 4, 5\}$

d) $B - A$

$\{0, 6\}$

19. Show that if A and B are sets, then

a) $A - B = A \cap \overline{B}$

Venn Diagram:

b) $(A \cap B) \cup (A \cap \overline{B}) = A$

Venn Diagram:

25. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

a) $A \cap B \cap C$

$\{0, 2, 4, 6\} \cap \{4, 5, 6, 7, 8, 9, 10\} = \{4, 6\}$

b) $A \cup B \cup C$

$\{0, 1, 2, 3, 4, 5, 6, 8, 10\} \cup \{4, 5, 6, 7, 8, 9, 10\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c) $(A \cup B) \cap C$

$\{0, 1, 2, 3, 4, 5, 6, 8, 10\} \cap \{4, 5, 6, 7, 8, 9, 10\} = \{4, 5, 6, 8, 10\}$

d) $(A \cap B) \cup C$

$\{0, 2, 4, 6\} \cup \{4, 5, 6, 7, 8, 9, 10\} = \{0, 2, 4, 5, 6, 7, 8, 9, 10\}$

27. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a) $A \cap (B - C)$

Venn Diagram:

b) $(A \cap B) \cup (A \cap C)$

Venn Diagram:

c) $(A \cap B) \cup (A \cap \overline{C})$

Venn Diagram:

32. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$

$\{2, 5\}$

37. Show that if A is a subset of a universal set U , then

a) $A \oplus A = \emptyset$

Venn Diagram:

b) $A \oplus \emptyset = A$

Venn Diagram:

c) $A \oplus U = \overline{A}$

Venn Diagram:

d) $A \oplus \overline{A} = U$

Venn Diagram:

§2.3

1. Why is f not a function from $\mathbb{R}$ to $\mathbb{R}$ if

a) $f(x) = 1/x$**?**

Because f(zero) is dividing by zero.

b) $f(x) = \sqrt{x}$**?**

Because any negative value such as x = -1, gives $\sqrt{-1}$, an imaginary number part of complex set $\mathbb{C}$.

c) $f(x) = \pm\sqrt{(x^2+1)}$**?**

There are two possible values for every f(x) - not a function by definition.

9. Find these values.

a) $\lceil \frac{3}{4} \rceil$

1

b) $\lfloor \frac{7}{8} \rfloor$

0

c) $\lceil -\frac{3}{4} \rceil$

0

d) $\lfloor -\frac{7}{8} \rfloor$

-1

e) $\lceil 3 \rceil$

3

f) $\lfloor -1 \rfloor$

-1

g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$

2

h) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

1

10. Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.

a) $f(a) = b, f(b) = a, f(c) = c, f(d) = d$

One-to-one

b) $f(a) = b, f(b) = b, f(c) = d, f(d) = c$

Not one-to-one

c) $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

Not one-to-one

11. Which functions in Exercise 10 are onto?

Only a. is onto function.

23. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

a) $f(x) = 2x + 1$

One-to-one and onto, Yes bijection.

b) $f(x) = x^2 + 1$

Not one-to-one, so no bijection.

c) $f(x) = x^3$

Not one-to-one, not bijective.

d) $f(x) = (x^2 + 1)/(x^2 + 2)$

...

32. Let $f(x) = 2x$ where the domain is the set of real numbers. What is

a) $f(\mathbb{Z})$**?**

b) $f(\mathbb{N})$**?**

c) $f(\mathbb{R})$**?**

...

33. Suppose that g is a function from A to B and f is a function from B to C.

a) Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.

b) Show that if both f and g are onto functions, then $f \circ g$ is also onto.

45. Let f be a function from A to B. Let S be a subset of B. Show that $f^{-}1(S) = \overline{f^{-}1(S)}$**.**

...

§3.1

1. List all the steps used by Algorithm 1 to find the maximum of the list 1, 8, 12, 9, 11, 2, 14, 5, 10, 4.

Make first element max, max equals 1.
For all subsequent elements, if that element is higher than max, replace max.
Step 2, 8 replaces 1 as max.
Step 3, 12 becomes new max.
Step 4, nothing happens since $9 < 12$
Same for steps 5 and 6 but at step 7, $14 > 12$ so max becomes 14.
Iterate and check rest of elements and
return max when all elements have been checked.

3. Devise an algorithm that finds the sum of all the integers in a list.

Procedure: Sum(L: list of n integers)
total := o
for i:= 1 to n
    total = Li + total
return total

4. Describe an algorithm that takes as input a list of n integers and produces as output the largest difference obtained by subtracting an integer in the list from the one following it.

```
Procedure: BigDiff(L: list of n integers)
biggestDiff := o
for i:= 2 to (n-1)
     if biggestDiff less than abs(L_{i-1} - L_i)
              Replace biggestDiff with value
return biggestDiff
```

6. Describe an algorithm that takes as input a list of n integers and finds the number of negative integers in the list.

```
Procedure: NumberOfNegatives(L: list of n integers)
count := o
for i:= 1 to n
              if L_i less than 0
              count = count + 1
return count
```