Discrete Mathematics Homework III
Mathematical Induction and Combinations

Nicholas Christiny

October 9, 2017

§5.1

3. Let $P(n)$ be the statement that $1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6$ for the positive integer n.

a) What is the statement $P(1)$?

Base case.

b) Show that $P(1)$ is true, completing the basis step of the proof.

c) What is the inductive hypothesis?

d) What do you need to prove in the inductive step?

e) Complete the inductive step, identifying where you use the inductive hypothesis.

f) Explain why these steps show that this formula is true whenever n is a positive integer.

5. Prove that $1^2 + 3^2 + 5^2 + ... + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$ whenever n is a nonnegative integer.

7. Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + ... + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$ whenever n is a nonnegative integer.

21. Prove that $2^n > n^2$ if n is an integer greater than 4.

31. Prove that 2 divides $n^2 + n$ whenever n is a positive integer.

39. Prove that if $A_1, A_2, ..., A_n$ and $B_1, B_2, ..., B_n$ are sets such that $A_j \subseteq B_j$ for $j = 1, 2, ..., n$, then

$$\bigcap_{j=1}^{n} A_j \subseteq \bigcap_{j=1}^{n} B_j.$$

43. Prove that if $A_1, A_2, ..., A_n$ are subsets of a universal set U , then

$$\bigcup_{k=1}^{n} A_k = \bigcap_{k=1}^{n} \overline{A_k}.$$