Discrete Mathematics Homework III
Mathematical Induction and Combinations

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§5.1

3. Let $P(n)$ be the statement that $1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6$ for the positive integer n.

a) What is the statement $P(1)$?

Basis step.

b) Show that $P(1)$ is true, completing the basis step of the proof.

$P(1) = 1(1+1)(2\cdot 1+1)/6 = 2\cdot(3/6) = 1$. $P(1)$ is a true statement.

c) What is the inductive hypothesis?

The inductive hypothesis is that if basis step applies for $P(n)$ then $P(n+1)$ is true as well.

d) What do you need to prove in the inductive step?

...

e) Complete the inductive step, identifying where you use the inductive hypothesis.

...

f) Explain why these steps show that this formula is true whenever n is a positive integer.

...

5. Prove that $1^2 + 3^2 + 5^2 + ... + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$ whenever n is a nonnegative integer.

...

7. Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + ... + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$ whenever n is a nonnegative integer.

...

21. Prove that $2^n > n^2$ if n is an integer greater than 4.

...

31. Prove that 2 divides $n^2 + n$ whenever n is a positive integer.

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39. Prove that if $A_1, A_2, ..., A_n$ and $B_1, B_2, ..., B_n$ are sets such that $A_j \subseteq B_j$ for $j = 1, 2, ..., n$, then

$$\bigcap_{j=1}^{n} A_j \subseteq \bigcap_{j=1}^{n} B_j.$$

43. Prove that if $A_1, A_2, ..., A_n$ are subsets of a universal set U , then

$$\bigcup_{k=1}^{n} A_k = \bigcap_{k=1}^{n} \overline{A_k}.$$

...

§6.1

1. There are 18 mathematics majors and 325 computer science majors at a college.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

...

b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

...

5. Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?

...

9. How many different three-letter initials are there that begin with an A?

...

27. A committee is formed consisting of one representative from each of the 50 states in the United States, where the representative from a state is either the governor or one of the two senators from that state. How many ways are there to form this committee?

...

29. How many license plates can be made using either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters?

...

31. How many license plates can be made using either two or three uppercase English letters followed by either two or three digits?

...

47. In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if

a) the bride must be next to the groom?

...

b) the bride is not next to the groom?

...

c) the bride is positioned somewhere to the left of the groom?

...

§6.2

1. Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.

...

3. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

a) How many socks must he take out to be sure that he has at least two socks of the same color?

...

b) How many socks must he take out to be sure that he has at least two black socks?

...

7. Let n be a positive integer. Show that in any set of n consecutive integers there is exactly one divisible by n.

...

9. What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

...

§6.3

1. List all the permutations of $\{a, b, c\}$.

$\{a, b, c\}$, $\{a, c, b\}$, $\{c, b, a\}$, $\{b, a, c\}$, $\{b, c, a\}$, $\{c, a, b\}$