Discrete Mathematics Homework IV
Relations

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§9.1

1. List the ordered pairs in the relation R from $A = 0, 1, 2, 3, 4$ to $B = 0, 1, 2, 3$, where $(a, b)R$ if and only if

a) $a = b$**.**

$$R = \{(0,0),(1,1),(2,2),(3,3)\}$$

b) $a + b = 4$**.**

$$R = \{(1,3),(2,2),(3,1),(4,0)\}$$

c) $a > b$**.**

$$R = \{(1,0),(2,0),(2,1),(3,0),(3,1),(3,2),(4,0),(4,1),(4,2),(4,3)\}$$

d) $a|b$**.**

$$R = \{(1,0),(1,1),(1,2),(1,3),(2,0),(2,2),(3,0),(3,3),(4,0)\}$$

3. For each of these relations on the set $\{1,2,3,4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

Reflexive, no. Symmetric, no. Anti-symmetric, no, more than one edge connects 2-points. Transitive? Yes.

b) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$

Reflexive? Yes Symmetric? Yes Anti-symmetric? No. Transitive? Yes.

c) $\{(2,4),(4,2)\}$

Reflexive? No. Symmetric? Yes. Anti-symmetric? No. Transitive? No.

d) $\{(1,2),(2,3),(3,4)\}$

Reflexive? No. Symmetric? No. Anti-symmetric? Yes. Transitive? No.

e) $\{(1,1),(2,2),(3,3),(4,4)\}$

Reflexive? Yes. Symmetric? Yes. Anti-symmetric? Yes. Transitive? Yes.

f) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

Reflexive? No.Symmetric? No. Anti-symmetric? No. Transitive? No. None.

6. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x,y)R$ if and only if

a)$x+y=0$**.**

Reflexive, yes.Symmetric, yes. Anti-symmetric, no. Transitive, yes.

c) $x-y$ **is a rational number.**

...

e) $x \cdot y \geq 0$**.**

Reflexive, no. Symmetric, yes. Anti-symmetric, no. Transitive, yes.

h) $x = 1$ **or** $y = 1$**.**

Reflexive, no. Symmetric, no. Anti-symmetric, yes. Transitive, yes?

7. Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

b.) $xy \geq 1$

Reflexive? No. Symmetric, Yes. Anti-symmetric, No. Transitive, Yes.

c) $x = y + 1$ **or** $x = y - 1$**.**

Reflexive, No. Symmetric, Yes. Anti-symmetric, No. Transitive, No.

d) $x \equiv y(mod 7)$**.**

Reflexive, No. Symmetric, No. Anti-symmetric, Yes. Transitive, No.

g) $x = y^2$**.**

Reflexive, No. Symmetric, No. Anti-symmetric, Yes. Transitive, No.

§9.5

1. Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

Reflexive? Yes. Symmetric, Yes. Transitive, Yes. Yes, it is an equivalence relation.

b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$

Reflexive, No. Not transitive. Not an equivalence relation.

c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$

Reflexive, yes. Symmetric, yes. Transitive, Yes. It is an equivalence relation.

d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Reflexive, Yes. Symmetric, yes. Transitive, No. Not an equivalence relation.

e) $\{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,2),(3,3)\}$

Reflexive, yes. Symmetric, no. Transitive, No. Not an equivalence relation.

7. Show that the relation of logical equivalence on the set of all compound propositions is an equivalence relation. What are the equivalence classes of F and T?

Logically equivalent propositions have the same truth tables. p logically equivalent to q is reflexive, because p is logically equivalent to p; it is also symmetric, since statement p is logically equivalent to q is the same as q is logically equivalent to p; and while p is logically equivalent to q, if q is logically equivalent to r, then p is logically equivalent to r as well, which means transitive property holds. Therefore, it is an equivalence relation.
The equivalence class of T is the set of all propositions that are tautologies.
The equivalence class of F is the set of all propositions that are contradictions.

15. Let R be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $a+d=b+c$. Show that R is an equivalence relation.

Reflexive? $((a,b),(b,a)) \in R$, because $a+b=b+a$
Symmetric? $a+b=b+c$ so $c+b=d+a$
Transitive? $a+d=b+c$, and $c+e=d+f$, so $a+d+c+e=b+c+d+f$, so $a+e=b+f$

In Exercises 21–23 determine whether the relation with the directed graph shown is an equivalence relation.

21.

Reflexive? Yes. Symmetric? Yes. Transitive? No. Not an equivalence relation.

23.

Reflexive? Yes. Symmetric? Yes. Transitive? No. Not an equivalence relation.

35. What is the congruence class $[n]5$ (that is, the equivalence class of n with respect to congruence modulo 5) when n is

a) 2?

$[a]_m = \{\ldots, a-2m, a-m, a, a+m, a+2m, \ldots\}$
$= \{\ldots, -8, -3, 2, 7, 12, \ldots\}$

b) 3?

$= \{\ldots, -7, -2, 3, 8, 13, \ldots\}$

c) 6?

$= \{\ldots, -4, 1, 6, 11, 16, \ldots\}$

d) −3?

$= \{\ldots, -13, -8, -3, 2, 7, \ldots\}$

39.

a) What is the equivalence class of $(1, 2)$ with respect to the equivalence relation in Exercise 15?

...

b) Give an interpretation of the equivalence classes for the equivalence relation R in Exercise 15. [Hint: Look at the difference a − b corresponding to (a, b).]

...

§9.6

1. Which of these relations on $\{0, 1, 2, 3\}$ are partial orderings? Determine the properties of a partial ordering that the others lack.

a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

Reflexive, yes. Anti-symmetric, yes. Transitive, yes proven earlier. Partial ordering confirmed.

b) $\{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$

Reflexive, yes. Anti-symmetric, no. Not transitive. Not a partial ordering.

c) $\{(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)\}$

Reflexive, yes. Anti-symmetric, yes. Transitive, yes. Partial ordering.

d) $\{(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

Reflexive, yes. Anti-symmetric, yes. Transitive, yes. Partial ordering.

e) $\{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,2),(3,3)\}$

Reflexive, yes. Anti-symmetric, no. Transitive, no. Not a partial ordering.

5. Which of these are posets?

a) $(\mathbb{Z},=)$

Reflexive? $x = x$, yes. Anti-symmetric? $x = y$,$y = x$, yes. Transitive? $x = y$,$y = z$,$x = z$, yes. Therefore, yes this is a poset.

b) $(\mathbb{Z},\neq)$

Reflexive, $x \neq x$ No. Anti-symmetric, $x \neq y, y \neq x$, can't really tell anything definite, not anti-symmetric. Same reason not transitive. Not a poset.

c) $(Z,\geq)$

Reflexive? $x \geq x$ Yes. Anti-symmetric? $x \geq y$,$y \geq x$, yes. Transitive? $x \geq y, y \geq z, x \geq z$, yes. Yes, this is a poset.

d) $(Z,\nmid)$

Reflexive? No, since $x \nmid x$ is false. Right away, not a poset.

23. Draw the Hasse diagram for divisibility on the set

a) $\{1,2,3,4,5,6,7,8\}$**.**

b) $\{1,2,3,5,7,11,13\}$**.**

c) $\{1,2,3,6,12,24,36,48\}$**.**

d) $\{1,2,4,8,16,32,64\}$**.**

In Exercises 25–27 list all ordered pairs in the partial ordering with the accompanying Hasse diagram.

25.

Must be reflexive, anti-symmetric, and transitive to be a poset. Hasse diagrams show do not show any loops or directed edges for transitive and anti-symmetric pairs. In this problem, we can work backwards to the directed graph to obtain the ordered pairs of relation.
Therefore, $(a,a),(b,b),(c,c),(d,d),(a,b),(a,c),(a,d),(b,c),(b,d)$

27.

$(a, a), (b, b), (c, c), (d, d), (e, e), (f, f), (g, g), (a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g),$
$(c, d), (c, e), (c, f), (c, g), (g, d), (g, e), (g, f)$

32. Answer these questions for the partial order represented by this Hasse diagram.

a) Find the maximal elements.

m, l

b) Find the minimal elements.

a, b, c

c) Is there a greatest element?

No.

d) Is there a least element?

No.

e) Find all upper bounds of $\{a, b, c\}$.

l, g

f) Find the least upper bound of $\{a, b, c\}$, if it exists.

f, d

g) Find all lower bounds of $\{f, g, h\}$.

... uncertain

h) Find the greatest lower bound of $\{f, g, h\}$, if it exists.

... not sure

43. Determine whether the posets with these Hasse diagrams are lattices.

a)

Yes.

b)

No.

c)

Yes.