

Discrete Mathematics Homework II

Set Theory, Functions, Algorithms, and Integer Representations

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§2.1 Sets

1. List the members of these sets.

a) $\{x|x \text{ is a real number such that } x^2 = 1\}$

Will show the list of members as a set.

$\{1, -1\}$

b) $\{x|x \text{ is a positive integer less than } 12\}$

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c) $\{x|x \text{ is the square of an integer and } x < 100\}$

$\{1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d) $\{x|x \text{ is an integer such that } x^2 = 2\}$

No members. $\{\}$ or \emptyset

3. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

**a) the set of airline flights from New York to New Delhi,
the set of nonstop airline flights from New York to New Delhi**

If all airline flights from New York to New Delhi are non-stop, both sets would be equal meaning both would be subsets of each other. However, in the reasonable case that not all flights are nonstop, the second set is a subset of the first and first is not subset of second.

b) the set of people who speak English, the set of people who speak Chinese

Neither set is subset of the other.

c) the set of flying squirrels, the set of living creatures that can fly

First is subset of second. Second is not subset of first.

5. Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5\}, \{5, 3, 1\}$

Equal

b) $\{\{1\}\}, \{1, \{1\}\}$

Not equal

c) $\emptyset, \{\emptyset\}$

Not equal, first is empty set, second is singleton set containing an empty set.

7. For each of the following sets, determine whether 2 is an element of that set.

a) $\{x \in \mathbb{R} \mid x \text{ is an integer greater than } 1\}$

Yes, 2 is an element of this set.

b) $\{x \in \mathbb{R} \mid x \text{ is the square of an integer}\}$

No, this set does not contain 2.

c) $\{2, \{2\}\}$

Yes.

d) $\{\{2\}, \{\{2\}\}\}$

No.

e) $\{\{2\}, \{2, \{2\}\}\}$

No.

f) $\{\{\{2\}\}\}$

No.

9. Determine whether each of these statements is true or false.

a) $0 \in \emptyset$

False

b) $\emptyset \in \{0\}$

False

c) $\{0\} \subset \emptyset$

False

d) $\emptyset \subset \{0\}$

False

e) $\{0\} \in \{0\}$

True

f) $\{0\} \subset \{0\}$

False

g) $\{\emptyset\} \subseteq \{\emptyset\}$

True

11. Determine whether each of these statements is true or false.

a) $x \in \{x\}$

True

b) $\{x\} \subseteq \{x\}$

True

c) $\{x\} \in \{x\}$

False

d) $\{x\} \in \{\{x\}\}$

True

e) $\emptyset \subseteq \{x\}$

False

f) $\emptyset \in \{x\}$

False

17. Suppose that A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$. Show that $A \subseteq C$.

Say set $A = \{1, 2\}$ and set $B = \{1, 2, 3\}$, where $A \subseteq B$.

Then, set $C = \{1, 2, 3, 4, 5, 6\}$, such that $B \subseteq C$.

\therefore since $\{1, 2\} \subseteq \{1, 2, 3, 4, 5, 6\}$, we see $A \subseteq C$.

19. What is the cardinality of each of these sets?

a) $\{a\}$

1

b) $\{\{a\}\}$

1

c) $\{a, \{a\}\}$

2

d) $\{a, \{a\}, \{a, \{a\}\}\}$

3

21. Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$

Itself, $\{a\}$

b) $\{a, b\}$

$\{a, \{a, b\}\}$

c) $\{\emptyset, \{\emptyset\}\}$

$\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

25. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

...

32. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$

$A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

$A \times B \times C = \{((a, x), 0), ((a, x), 1), ((a, y), 0), ((a, y), 1), ((b, x), 0), ((b, x), 1)\}$

...

41. Translate each of these quantifications into English and determine its truth value.

a) $\forall x \in \mathbb{R}(x^2 \neq -1)$

For all x in domain of real numbers, x squared is not equal to negative one. True

b) $\exists x \in \mathbb{Z}(x^2 = 2)$

There exists in domain of integers x , such that x squared is two. False

c) $\forall x \in \mathbb{Z}(x^2 > 0)$

For all x in domain of integers, x squared is greater than zero. True

d) $\exists x \in \mathbb{R}(x^2 = x)$

There exists in domain of real numbers x, such that x squared equals x. True, 1 satisfies.

§2.2

3. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

a) $A \cup B$

$$\{0, 1, 2, 3, 4, 5, 6\}$$

b) $A \cap B$

$$\{3\}$$

c) $A - B$

$$\{1, 2, 4, 5\}$$

d) $B - A$

$$\{0, 6\}$$

19. Show that if A and B are sets, then

a) $A - B = A \cap \overline{B}$

Venn Diagram:

b) $(A \cap B) \cup (A \cap \overline{B}) = A$

Venn Diagram:

25. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

a) $A \cap B \cap C$

$$\{0, 2, 4, 6\} \cap \{4, 5, 6, 7, 8, 9, 10\} = \{4, 6\}$$

b) $A \cup B \cup C$

$$\{0, 1, 2, 3, 4, 5, 6, 8, 10\} \cup \{4, 5, 6, 7, 8, 9, 10\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

c) $(A \cup B) \cap C$

$$\{0, 1, 2, 3, 4, 5, 6, 8, 10\} \cap \{4, 5, 6, 7, 8, 9, 10\} = \{4, 5, 6, 8, 10\}$$

d) $(A \cap B) \cup C$

$$\{0, 2, 4, 6\} \cup \{4, 5, 6, 7, 8, 9, 10\} = \{0, 2, 4, 5, 6, 7, 8, 9, 10\}$$

27. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a) $A \cap (B - C)$

Venn Diagram:

b) $(A \cap B) \cup (A \cap C)$

Venn Diagram:

c) $(A \cap B) \cup (A \cap \overline{C})$

Venn Diagram:

**32. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$
 $\{2, 5\}$**

37. Show that if A is a subset of a universal set U , then

a) $A \oplus A = \emptyset$

Venn Diagram:

b) $A \oplus \emptyset = A$

Venn Diagram:

c) $A \oplus U = \overline{A}$

Venn Diagram:

d) $A \oplus \overline{A} = U$

Venn Diagram:

§2.3

1. Why is f not a function from \mathbb{R} to \mathbb{R} if

a) $f(x) = 1/x$?

Because $f(\text{zero})$ is dividing by zero.

b) $f(x) = \sqrt{x}$?

Because any negative value such as $x = -1$, gives $\sqrt{-1}$, an imaginary number part of complex set \mathbb{C} .

c) $f(x) = \pm\sqrt{(x^2 + 1)}$?

There are two possible values for every $f(x)$ - not a function by definition.

9. Find these values.

a) $\lceil \frac{3}{4} \rceil$

1

b) $\lfloor \frac{7}{8} \rfloor$

0

c) $\lceil -\frac{3}{4} \rceil$

0

d) $\lfloor -\frac{7}{8} \rfloor$

-1

e) $\lceil 3 \rceil$

3

f) $\lfloor -1 \rfloor$

-1

g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$

2

h) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

1

10. Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.

a) $f(a) = b, f(b) = a, f(c) = c, f(d) = d$

One-to-one

b) $f(a) = b, f(b) = b, f(c) = d, f(d) = c$

Not one-to-one

c) $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

Not one-to-one

11. Which functions in Exercise 10 are onto?

Only a. is onto function.

23. Determine whether each of these functions is a bijection from \mathbb{R} to \mathbb{R} .

a) $f(x) = 2x + 1$

One-to-one and onto, Yes bijection.

b) $f(x) = x^2 + 1$

Not one-to-one, so no bijection.

c) $f(x) = x^3$

Not one-to-one, not bijective.

d) $f(x) = (x^2 + 1)/(x^2 + 2)$

...

32. Let $f(x) = 2x$ where the domain is the set of real numbers. What is

a) $f(\mathbb{Z})$?

b) $f(\mathbb{N})$?

c) $f(\mathbb{R})$?

...

33. Suppose that g is a function from A to B and f is a function from B to C .

a) Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.

b) Show that if both f and g are onto functions, then $f \circ g$ is also onto.

45. Let f be a function from A to B . Let S be a subset of B . Show that $f^{-1}(S) = \overline{f^{-1}(S)}$.

...

§3.1

1. List all the steps used by Algorithm 1 to find the maximum of the list 1, 8, 12, 9, 11, 2, 14, 5, 10, 4.

Make first element max, max equals 1.

For all subsequent elements, if that element is higher than max, replace max.

Step 2, 8 replaces 1 as max.

Step 3, 12 becomes new max.

Step 4, nothing happens since $9 < 12$

Same for steps 5 and 6 but at step 7, $14 > 12$ so max becomes 14.

Iterate and check rest of elements and

return max when all elements have been checked.

3. Devise an algorithm that finds the sum of all the integers in a list.

Procedure: Sum(L: list of n integers)

total := 0

for i:= 1 to n

total = Li + total

return total

4. Describe an algorithm that takes as input a list of n integers and produces as output the largest difference obtained by subtracting an integer in the list from the one following it.

```
Procedure: BigDiff(L: list of n integers)
biggestDiff := 0
for i:= 2 to (n-1)
    if biggestDiff less than abs(Li-1 - Li)
        Replace biggestDiff with value
return biggestDiff
```

6. Describe an algorithm that takes as input a list of n integers and finds the number of negative integers in the list.

```
Procedure: NumberOfNegatives(L: list of n integers)
count := 0
for i:= 1 to n
    if Li less than 0
        count = count + 1
return count
```

9. A palindrome is a string that reads the same forward and backward. Describe an algorithm for determining whether a string of n characters is a palindrome.

```
Procedure: PalindromeDetect(text: string)

if text is ""
    return false
left:= 0
right := text.length - 1
while (left < right)
    if text[left] not equal to text[right]
        return false
    left:= left + 1
    right right - 1

return true
```

13. List all the steps used to search for 9 in the sequence 1, 3, 4, 5, 6, 8, 9, 11 using

a) a linear search.

Set a zero or 1-based index counter
check first value, 1. It is not target, 9.
increment index counter
Move on, check second term 3, not 9.
increment index counter
Third term 4, not 9 move on,
increment index counter
check fourth term 5, not 9, move to next,
increment index counter
check 6, not 9 move on to next,
increment index counter
check value of 8 not 9, move to next value
increment index counter
check value of 9 is same as target 9
return index counter for position of target

b) a binary search.

Set some initial counters to zero or 1 per preference
Divide ordered sequence in two, check value versus top index of lower half
For our sequence search, $5 < 9$, so we take top half of sequence and reassign indices, to keep track of original positions
We continue to bisect, 6,8 and 9,11
check target versus top of bottom bisection shows $8 < 9$, so we repeat bisection of top half.
this gives 8 and 9, checking bottom half versus target shows $8 < 9$,
the algorithm will continue to bisect leaving just 9
a final check shows 9's index position that can be traced back using the index counter,
position 7 for 1-based index

20. Describe an algorithm for finding both the largest and the smallest integers in a finite sequence of integers.

Procedure: BigSmall(L: list of n integers)

max := 0

min := 0 for i:= 1 to n

 if max less than L_n

 max := L_n

 if min greater than L_n

 min := L_n

return (max, min)

27. The ternary search algorithm

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7. Adapt the bubble sort algorithm so that it stops when no interchanges are required. Express this more efficient version of the algorithm in pseudocode.

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§4.2

1. Convert the decimal expansion of each of these integers to a binary expansion.

a) 231

(Please note work for these shown separately at end of HW)

$(1110\ 0111)_2$

b) 4532

$(1\ 0001\ 1011\ 0100)_2$

2. Convert the decimal expansion of each of these integers to a binary expansion.

a) 321

$(1\ 0000\ 0101)_2$

b) 1023

$(11\ 1111\ 1111)_2$

3. Convert the binary expansion of each of these integers to a decimal expansion.

a) $(1\ 1111)_2$

31

b) $(10\ 0000\ 0001)_2$

513

c) $(1\ 0101\ 0101)_2$

341

d) $(110\ 1001\ 0001\ 0000)_2$

26,897

5. Convert the octal expansion of each of these integers to a binary expansion.

b) $(1604)_8$

$(1110\ 0100)_2$

d) $(2417)_8$

$(1\ 0100\ 1111)_2$

6. Convert the binary expansion of each of these integers to an octal expansion.

b) $(1010\ 1010\ 1010)_2$

$(5252)_8$

d) $(101\ 0101\ 0101\ 0101)_2$

$(52525)_8$

7. Convert the hexadecimal expansion of each of these integers to a binary expansion

b) $(135AB)_{16}$

$(1\ 00110101\ 1010\ 1011)_2$

d) $(DEFACED)_{16}$

$(1101\ 1110\ 1111\ 1010\ 1100\ 1110\ 1101)_2$

11. Convert $(1011\ 0111\ 1011)_2$ from its binary expansion to its hexadecimal expansion.

$(B7B)_{16}$

12. Convert $(1\ 1000\ 0110\ 0011)_2$ from its binary expansion to its hexadecimal expansion.

$(1863)_{16}$