

1.

Yes, sum of sinusoids of equal frequency is still a sinusoid of the same frequency.

By "The angle sum rule" we know,

$$A \sin(\omega t + \phi) = A \sin(\omega t) \cos(\phi) + A \cos(\omega t) \sin(\phi) = A' \sin(\omega t) + A'' \cos(\omega t) \quad (1)$$

$$\text{where } \begin{cases} A' = A \cos(\phi) \\ A'' = A \sin(\phi) \end{cases} \quad (2)$$

$$\text{For } \begin{cases} A' = A \cos(\phi) \\ A'' = A \sin(\phi) \end{cases} \Rightarrow \begin{cases} \cos \phi = \frac{A'}{A} \\ \sin \phi = \frac{A''}{A} \end{cases} \Rightarrow \begin{cases} \sin^2 \phi + \cos^2 \phi = \frac{A'^2 + A''^2}{A^2} = 1 \\ \frac{\sin \phi}{\cos \phi} = \frac{A''}{A'} = \tan \phi \end{cases} \Rightarrow \begin{cases} A = \sqrt{A'^2 + A''^2} \\ \phi = \arctan\left(\frac{A''}{A'}\right) \end{cases} \quad (3)$$

Sum of sinusoids can be divided in 3 cases:

Case 1: Sum of sin and cos: $A \sin(\omega t + \alpha) + B \cos(\omega t + \beta)$

$$\text{use equation (1)} \Rightarrow A' \sin(\omega t) + A'' \cos(\omega t) + B' \cos(\omega t) - B'' \sin(\omega t) \quad \text{where } \begin{cases} A' = A \cos \alpha \\ A'' = A \sin \alpha \\ B' = B \cos \beta \\ B'' = B \sin \beta \end{cases}$$

$$= (A' - B'') \sin(\omega t) + (A'' + B') \cos(\omega t)$$

$$= C \sin(\omega t + \phi) \quad \text{where } \begin{cases} C = \sqrt{(A' - B'')^2 + (A'' + B')^2} \\ \phi = \arctan\left(\frac{A'' + B'}{A' - B''}\right) \end{cases}$$

Because A', A'', B', B'' are constants, C and ϕ are also constants.

Hence, $A \sin(\omega t + \alpha) + B \cos(\omega t + \beta) = C \sin(\omega t + \phi)$ is sinusoid of same frequency.

Case 2: sum of 2 sines: $A \sin(\omega t + \alpha) + B \sin(\omega t + \beta)$

$$\text{use equation (1)} \Rightarrow A' \sin(\omega t) + A'' \cos(\omega t) + B' \sin(\omega t) + B'' \cos(\omega t) \quad \text{where } \begin{cases} A' = A \cos \alpha \\ A'' = A \sin \alpha \\ B' = B \cos \beta \\ B'' = B \sin \beta \end{cases}$$

$$= (A' + B') \sin(\omega t) + (A'' + B'') \cos(\omega t)$$

$$= C \sin(\omega t + \phi) \quad \text{where } \begin{cases} C = \sqrt{(A' + B')^2 + (A'' + B'')^2} \\ \phi = \arctan\left(\frac{A'' + B''}{A' + B'}\right) \end{cases}$$

Because, A', A'', B', B'' are constants, C and ϕ are also constants.

Hence, $A \sin(\omega t + \alpha) + B \sin(\omega t + \beta) = C \sin(\omega t + \phi)$ is sinusoid of same frequency.

Case 3: sum of 2 cosines: $A \cos(\omega t + \alpha) + B \cos(\omega t + \beta)$

$$\text{use equation (1)} \Rightarrow A' \cos(\omega t) - A'' \sin(\omega t) + B' \cos(\omega t) - B'' \sin(\omega t) \quad \begin{cases} A' = A \cos \alpha \\ A'' = A \sin \alpha \\ B' = B \cos \beta \\ B'' = B \sin \beta \end{cases}$$

$$= (A' - B'') \cos(\omega t) - (A'' + B') \sin(\omega t)$$

$$= C \sin(\omega t + \phi) \quad \text{where } \begin{cases} C = \sqrt{(A' - B'')^2 + (A'' + B')^2} \\ \phi = \arctan\left(\frac{A'' + B'}{A' - B''}\right) \end{cases}$$

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Because A', A'', B', B'' are constants, C and ϕ are also constants.

Hence, $A\cos(\omega t + \alpha) + B(\omega t + \beta) = C\sin(\omega t + \phi)$ is sinusoid of same frequency.

Hence, all cases in sum of sinusoids we have proved that it's still sinusoid of same frequency. #

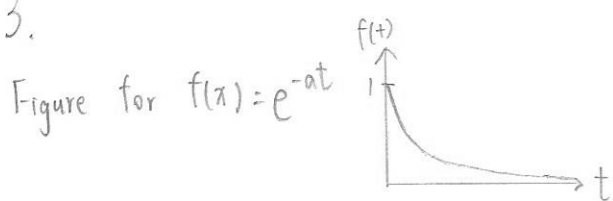
2.

DC component can be obtained by $a_0 = \frac{1}{T_0} \int_0^{T_0} x(t) dt$

$$T_0 = 0.04, \quad \int_0^{T_0} x(t) dt = \frac{0.04 \times 1}{2} = 0.02 \quad \therefore a_0 = \frac{0.02}{0.04} = 0.5$$

\therefore DC component is 0.5. #

3.



$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\begin{aligned} F\{f(t)\}(\omega) &= F(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t) \cdot e^{-j\omega t} dt = \int_0^{\infty} e^{-at} \cdot e^{-j\omega t} dt \\ &= \int_0^{\infty} e^{-(a+j\omega)t} dt \\ &= \frac{1}{-(a+j\omega)} e^{-(a+j\omega)t} \Big|_0^{\infty} = 0 - \frac{1}{-(a+j\omega)} = \frac{1}{a+j\omega} \# \end{aligned}$$

4.

Gaussian Function: $g(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}}$

Fourier Transform of $g(t)$:

$$G(\omega) = F\{g(t)\} = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[t^2 + j\omega t \cdot 2\sigma^2]} dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[t^2 + j2\omega t\sigma^2 + (j\omega\sigma^2)^2 - (j\omega\sigma^2)^2]} dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[t + j\omega\sigma^2]^2} \cdot e^{\frac{(j\omega\sigma^2)^2}{2\sigma^2}} dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{\frac{(j\omega\sigma^2)^2}{2\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[t + j\omega\sigma^2]^2} dt \quad \left(\begin{array}{l} u = t + j\omega\sigma^2 \\ du = dt \end{array} \right)$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{\frac{(j\omega\sigma^2)^2}{2\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2\sigma^2}} du \quad \left(\begin{array}{l} \text{Gaussian Function 'property':} \\ \text{Total area under Gaussian Function integrals} \\ \text{is 1.} \end{array} \right)$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2\sigma^2}} du \cdot e^{\frac{(j\omega\sigma^2)^2}{2\sigma^2}} = e^{\frac{(j\omega\sigma^2)^2}{2\sigma^2}}$$

$$= e^{-\frac{1}{2}(\omega\sigma)^2} \text{ is also gaussian function \#}$$

5.

$$x(t) = v(t) \cos(2\pi f_c t) \quad f_c = 700 \text{ Hz}, \quad v(t) = 5 + 4 \cos(40\pi t)$$

$$= (5 + 4\cos(40\pi t)) \cos(1400\pi t)$$

$$= 5 \cos(1400\pi t) + 4 \cos(1400\pi t) \cos(40\pi t) \quad \left(\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)] \right)$$

$$= 5 \cos(1400\pi t) + 2 \cos(1440\pi t) + 2 \cos(1360\pi t)$$

$$F\{x(t)\} = F(\omega) = 5\pi \delta(\omega - 1400\pi) + 5\pi \delta(\omega + 1400\pi) + 2\pi \delta(\omega - 1440\pi) + 2\pi \delta(\omega + 1440\pi)$$

$$+ 2\pi \delta(\omega - 1360\pi) + 2\pi \delta(\omega + 1360\pi) \quad \#$$

