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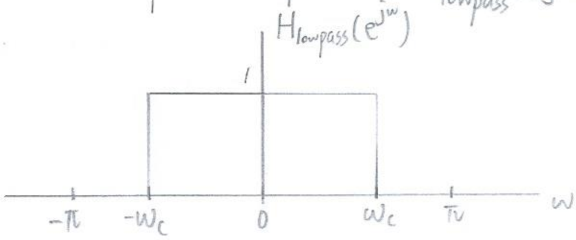
1、

Ideal low pass filter 的 impulse response $h_{lowpass}[n] = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwn}\,dw = \frac{\sin w_c n}{\pi n}$

Frequency response :



則 $H_{hp}(e^{jw}) = 1 - (H_{lowpass}(e^{jw}))$ 做 Inverse Fourier Transform

$$h_{highpass}[n] = F^{-1}\{1 - H_{lowpass}(e^{jw})\}$$

$$= F^{-1}\{1\} - F^{-1}\{H_{lowpass}(e^{jw})\} = \delta[n] - \frac{\sin w_c n}{\pi n} \quad \#$$

2、

(a) Let $w = e^{-j\frac{2\pi}{N}} \Rightarrow w^N = 1$ $\quad \therefore \sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}} = \sum_{n=0}^{N-1} w^n$

$$= 1 + w + w^2 + \cdots + w^{N-1}$$

$$= \frac{w^N - 1}{w - 1} = 0 \qquad \text{Q.E.D.} \ \#$$

(b) $$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k n}{N}} = \overset{1}{\cancel{x[0]}} + \overset{1}{\cancel{x[1]}} e^{-j\frac{2\pi k}{N}} + \overset{1}{\cancel{x[2]}} e^{-j\frac{2\pi\cdot 2k}{N}} + \cdots + \overset{1}{\cancel{x[N-1]}} e^{-j\frac{2\pi(N-1)k}{N}}$$

where $k = 0, 1, 2, \cdots, N-1$

$$X[0] = \underbrace{1 + 1 + \cdots + 1}_{N\text{個}} = N$$

$$X[1] = 1 + e^{-j\frac{2\pi}{N}} + e^{-j\frac{2\pi\cdot 2}{N}} + \cdots + e^{-j\frac{2\pi(N-1)}{N}} = \sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}} = 0$$

$$X[2] = 1 + e^{-j\frac{2\pi\cdot 2}{N}} + e^{-j\frac{2\pi\cdot 4}{N}} + \cdots + e^{-j\frac{2\pi\cdot 2(N-1)}{N}} = \sum_{n=0}^{N-1} e^{-j\frac{2\pi\cdot 2n}{N}} = 0$$

$\vdots$

$$X[N-1] = 1 + e^{-j\frac{2\pi(N-1)}{N}} + e^{-j\frac{2\pi\cdot 2(N-1)}{N}} + \cdots + e^{-j\frac{2\pi(N-1)(N-1)}{N}} = \sum_{n=0}^{N-1} e^{-j\frac{2\pi(N-1)n}{N}} = 0$$

$$\therefore X[k] = \begin{cases} N & , k = 0 \\ 0 & , k = 1, 2, \cdots, N-1 \end{cases}$$

$\#$

3.

(a) $x[n] = [2, 0, 1, 0]$, $y[n] = [1, -1, 0, 0]$

$y[-n] = [1, 0, 0, -1]$

$y[-n+1] = [-1, 1, 0, 0]$

$y[-n+2] = [0, -1, 1, 0]$

$y[-n+3] = [0, 0, -1, 1]$

$h[n] = x[n] \otimes y[n]$

$h[0] = 2\times 1 + 0 + 0 + 0 = 2$

$h[1] = 2\times -1 + 0 + 0 + 0 = -2$

$h[2] = 0 + 0 + 1\times 1 + 0 = 1$

$h[3] = 0 + 0 + 1\times -1 + 0 = -1$

$\therefore x[n] \otimes y[n] = [2, -2, 1, -1]$ #

(b)

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{4}} & e^{-j\frac{4\pi}{4}} & e^{-j\frac{6\pi}{4}} \\ 1 & e^{-j\frac{4\pi}{4}} & e^{-j\frac{8\pi}{4}} & e^{-j\frac{12\pi}{4}} \\ 1 & e^{-j\frac{6\pi}{4}} & e^{-j\frac{12\pi}{4}} & e^{-j\frac{18\pi}{4}} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-0.5j} & -1 & e^{0.5j} \\ 1 & -1 & 1 & -1 \\ 1 & e^{0.5j} & -1 & e^{-0.5j} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \end{bmatrix}$$ #

$$Y[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-0.5j} & -1 & e^{0.5j} \\ 1 & -1 & 1 & -1 \\ 1 & e^{0.5j} & -1 & e^{-0.5j} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - e^{-0.5j} \\ 2 \\ 1 - e^{0.5j} \end{bmatrix} = \begin{bmatrix} 0 \\ 1+i \\ 2 \\ 1-i \end{bmatrix}$$ #

(c)

$$Z[k] = X[k]\,Y[k] = \begin{bmatrix} 0 \\ 1+i \\ 6 \\ 1-i \end{bmatrix}$$ #

(d)

$$z[n] = \frac{1}{4} \sum_{k=0}^{3} Z[k]\, W_N^{-kn} \quad , \; n = 0, 1, 2, 3$$

$$z[n] = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 0 \\ 1+i \\ 6 \\ 1-i \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ -8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \\ -1 \end{bmatrix}$$ #

4.

已知 $g[n] = v[2n]$, $h[n] = v[2n+1]$ where $0 \le n < N$

$$
\begin{aligned}
V[k] &= \sum_{n=0}^{2N-1} v[n]\, W_{2N}^{nk} \\
&= \sum_{n=0}^{N-1} v[2n]\, W_{2N}^{2nk} + \sum_{n=0}^{N-1} v[2n+1]\, W_{2N}^{(2n+1)k} \\
&= \sum_{n=0}^{N-1} v[2n]\, W_{2N}^{2nk} + W_{2N}^{k} \sum_{n=0}^{N-1} v[2n+1]\, W_{2N}^{2nk} \\
&= \sum_{n=0}^{N-1} g[n]\, W_{N}^{nk} + W_{2N}^{k} \sum_{n=0}^{N-1} h[n]\, W_{N}^{nk} \quad \text{where } 0 \le k \le 2N-1 \\
&= G[k \bmod N] + W_{2N}^{k} H[k \bmod N]
\end{aligned}
$$

$\therefore f[k] = W_{2N}^{k}$ #