姓名：吳添毅　學號：R10944011　系所：網媒所　HW1

1.

Yes, sum of sinusoids of equal frequency is still a sinusoid of the same frequency.

By "The angle sum rule" we know,

$$A\sin(wt+\phi) = A\sin(wt)\cos(\phi) + A\cos(wt)\sin(\phi) = A'\sin(wt) + A''\cos(wt) \quad \text{——— (1)}$$

where $\begin{cases} A' = A\cos(\phi) \\ A'' = A\sin(\phi) \end{cases}$ ——— (2)

For $\begin{cases} A' = A\cos(\phi) \\ A'' = A\sin(\phi) \end{cases} \Rightarrow \begin{cases} \cos\phi = \dfrac{A'}{A} \\ \sin\phi = \dfrac{A''}{A} \end{cases} \Rightarrow \begin{cases} \sin^2\phi + \cos^2\phi = \dfrac{A'^2 + A''^2}{A^2} = 1 \\ \dfrac{\sin\phi}{\cos\phi} = \dfrac{A''}{A'} = \tan\phi \end{cases} \Rightarrow \begin{cases} A = \sqrt{A'^2 + A''^2} \\ \phi = \arctan\left(\dfrac{A''}{A'}\right) \end{cases}$ ——— (3)

Sum of sinusoids can be divided in 3 cases:

Case 1: sum of sin and cos: $A\sin(wt+\alpha) + B\cos(wt+\beta)$

use equation (1) $\Rightarrow A'\sin(wt) + A''\cos(wt) + B'\cos(wt) - B''\sin(wt)$ where $\begin{cases} A' = A\cos\alpha \\ A'' = A\sin\alpha \\ B' = B\cos\beta \\ B'' = B\sin\beta \end{cases}$

$$= (A'-B'')\sin(wt) + (A''+B')\cos(wt)$$

$$= C\sin(wt+\phi) \quad \text{where} \begin{cases} C = \sqrt{(A'-B'')^2 + (A''+B')^2} \\ \phi = \arctan\left(\dfrac{A''+B'}{A'-B''}\right) \end{cases}$$

Because A', A'', B', B'' are constants, C and ϕ are also constants

Hense, $A\sin(wt+\alpha) + B\cos(wt+\beta) = C\sin(wt+\phi)$ is sinusoid of same frequency

Case 2: sum of 2 sines: $A\sin(wt+\alpha) + B\sin(wt+\beta)$

use equation (1) $\Rightarrow A'\sin(wt) + A''\cos(wt) + B'\sin(wt) + B''\cos(wt)$ where $\begin{cases} A' = A\cos\alpha \\ A'' = A\sin\alpha \\ B' = B\cos\beta \\ B'' = B\sin\beta \end{cases}$

$$= (A'+B')\sin(wt) + (A''+B'')\cos(wt)$$

$$= C\sin(wt+\phi) \quad \text{where} \begin{cases} C = \sqrt{(A'+B')^2 + (A''+B'')^2} \\ \phi = \arctan\left(\dfrac{A''+B''}{A'+B'}\right) \end{cases}$$

Because, A', A'', B', B'' are constants, C and ϕ are also constants.

Hense, $A\sin(wt+\alpha) + B\sin(wt+\beta) = C\sin(wt+\phi)$ is sinusoid of same frequency.

Case 3: sum of 2 cosines: $A\cos(wt+\alpha) + B\cos(wt+\beta)$

use equation (1) $\Rightarrow A'\cos(wt) - A''\sin(wt) + B'\cos(wt) - B''\sin(wt)$ $\begin{cases} A' : A\cos\alpha \\ A'' = A\sin\alpha \\ B' : B\cos\beta \\ B'' : B\sin\beta \end{cases}$

$$= (-A''-B'')\sin(wt) + (A'+B')\cos(wt)$$

$$= C\sin(wt+\phi) \quad \text{where} \begin{cases} C = \sqrt{(-A''-B'')^2 + (A'+B')^2} \\ \phi = \arctan\left(\dfrac{A'+B'}{-A''-B''}\right) \end{cases}$$

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Because A', A'', B', B'' are constants, C and ϕ are also constants.

Hense, $A\cos(\omega t+\alpha) + B(\omega t+\beta) = C\sin(\omega t+\phi)$ is sinusoid of same frequency.

Hense, all cases in sum of sinusoids we have proved that it's still sinusoid of same frequency. #

2.

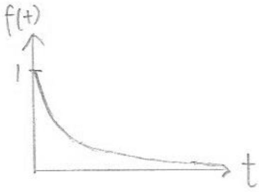
DC component can be obtained by $a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$

$T_0 = 0.04$, $\int_0^{T_0} x(t)\,dt = \frac{0.04\times 1}{2} = 0.02$ $\therefore a_0 = \frac{0.02}{0.04} = 0.5$

$\therefore$ DC component is 0.5. #

3.

Figure for $f(x) = e^{-at}$



$u(t) = \begin{cases} 1 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$

$$F\{f(t)\}(w) = F(w) = \int_{-\infty}^{\infty} e^{-at} u(t) \cdot e^{-jwt}\,dt = \int_0^{\infty} e^{-at} \cdot e^{-jwt}\,dt$$

$$= \int_0^{\infty} e^{-(a+jw)t}\,dt$$

$$= \frac{1}{-(a+jw)} e^{-(a+jw)t}\Big|_0^{\infty} = 0 - \frac{1}{-(a+jw)} = \frac{1}{a+jw}$$ #

4.

Gaussian Function : $g(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-t^2}{2\sigma^2}}$

Fourier Transform of $g(t)$:

$$G(w) = F\{g(t)\} = \int_{-\infty}^{\infty} g(t) \cdot e^{-jwt} dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[t^2 + jwt \cdot 2\sigma^2]} dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[t^2 + j2wt\sigma^2 + (jw\sigma^2)^2 - (jw\sigma^2)^2]} dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[t + jw\sigma^2]^2} \cdot e^{\frac{(jw\sigma^2)^2}{2\sigma^2}} dt$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{\frac{(jw\sigma^2)^2}{2\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}[t + jw\sigma^2]^2} dt \quad \begin{pmatrix} u = t + jw\sigma^2 \\ du = dt \end{pmatrix}$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{\frac{(jw\sigma^2)^2}{2\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2\sigma^2}} du$$

(Gaussian Function' property : Total area under Gaussian Function integrals is 1.)

$$= \underbrace{\frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{\frac{-u^2}{2\sigma^2}} du}_{= 1} \cdot e^{\frac{(jw\sigma^2)^2}{2\sigma^2}} = e^{\frac{(jw\sigma^2)^2}{2\sigma^2}}$$

$$= e^{-\frac{1}{2}(w\sigma)^2}$$ is also gaussian function #

5.

$x(t) = v(t)\cos(2\pi f_c t) \qquad f_c = 700\text{Hz}, \quad v(t) = 5 + 4\cos(40\pi t)$

$$= (5 + 4\cos(40\pi t))\cos(1400\pi t)$$

$$= 5\cos(1400\pi t) + 4\cos(1400\pi t)\cos(40\pi t) \quad \left(\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]\right)$$

$$= 5\cos(1400\pi t) + 2\cos(1440\pi t) + 2\cos(1360\pi t)$$

$$F\{x(t)\} = F(w) = 5\pi\delta(w - 1400\pi) + 5\pi\delta(w + 1400\pi) + 2\pi\delta(w - 1440\pi) + 2\pi\delta(w + 1440\pi)$$
$$+ 2\pi\delta(w - 1360\pi) + 2\pi\delta(w + 1360\pi)$$ #