Identitati si inegalitati

1. Identitati:

Fie $a, b, c \in \mathbf{R}$ si $m, n \in \mathbf{N}$

a) $(a \pm b)^2 = a^2 \pm 2ab + b^2$,

b) $(a \pm b)^3 = a^3 \pm 3a^2 b + 3ab^2 \pm b^3$,

c) $a^2 - b^2 = (a - b)(a + b)$,

d) $a^3 \mp b^3 = (a \mp b)(a^2 \pm ab + b^2)$,

e) $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1})$

f) $a^{2m+1} + b^{2m+1} = (a + b)(a^{2m} - a^{2m-1}b + a^{2m-2}b - \ldots - ab^{2m-1} + b^{2m})$

g) $a^{2m} - b^{2m} = (a^m - b^m)(a^m + b^m)$

h) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

2. Inegalitati:

a) daca $0 < a \le b$ atunci $a^p \le b^p$ $(p \in \mathbf{R}^+)$ si $\sqrt[q]{a} \le \sqrt[q]{b}$ $(q \in \mathbf{R})$

b) $\left| |a| - |b| \right| \le |a + b| \le |a| + |b|$, $a, b \in \mathbf{R}$

c) $|a + b + \ldots + l| \le |a| + |b| + \ldots + |l|$ $\forall a, b, \ldots, l \in \mathbf{R}$

d) daca $a < b$ si $a, b, m, n > 0$ atunci $a < \dfrac{ma + nb}{m + n} < b$

e) $a^2 + b^2 + c^2 \ge ab + bc + ca$, $\forall a, b, c \in \mathbf{R}$

f) $a^3 + b^3 + c^3 \ge 3abc$, $\forall a, b, c \in \mathbf{R}$

g) $\dfrac{a_1 + a_2 + \ldots + a_n}{n} \ge \sqrt[n]{a_1 a_2 \ldots a_n}$, $a_1, a_2, \ldots, a_n \in \mathbf{R}$

h) $(1 + a)(1 + b) \ldots (1 + t) > 1 + (a + b + \ldots + t)$, $\forall a, b, \ldots, t \ge 0$

i) $1 - a^n < n(1 - a)$, $a > 0$,

j) $n^{n+1} > (n + 1)^n$, $3 < n \in \mathbf{N}$,

k) $\sqrt{n^n} < n! < \left(\dfrac{n+1}{2} \right)^n$, $n \in \mathbf{N}^*$,

l) $(a_1^2 + a_2^2 + \ldots + a_n^2)(b_1^2 + b_2^2 + \ldots + b_n^2) \ge (a_1 b_1 + a_2 b_2 + \ldots + a_n b_n)^2$, $a_1, b_1, \ldots, a_n, b_n \in \mathbf{R}$,

m) daca $0 < a < 1$ si $x < y$ atunci $a^x > a^y$,

n) daca $a > 0$ si $x < y$ atunci $a^x < a^y$,

o) daca $0 < a < 1$ si $x < y$ atunci $\log_a x > \log_a y$,

p) daca $a > 0$ si $x < y$ atunci $\log_a x < \log_a y$.