Take-home test 2

Semester 2, 2021

Dear Andrew Martini

This is your test paper for Take-home test 2, for MTH20012 Series and Transforms. Please take the time to read the following instructions.

- There are 100 marks in total.
- This assessment item is worth 20% of your overall grade.
- Fully justify all of your working.
- Be clear! Full marks are only awarded if your setting out is clear.
- This test paper is unique to you. Make sure that you answer the questions on *your* test paper. (Questions on other test papers are similar, but will have different constants.)
- **Your submission must be 100% your own work.** You are encouraged to consult with Nathan, your tutor, MASH, and your fellow students about the material you have studied in MTH20012 Series and Transforms. But, you must not seek improper help with your questions, or share your work with other students or anyone else. **It is academic misconduct to improperly share the test questions on "tutoring" (contract cheating) websites.** The consequence of improperly sharing any question will in most cases be to receive a score of zero for the entire assessment, with this action recorded permanently on your student file. Please see the Unit Outline for more information about what constitutes academic misconduct. Also see https://www.swinburne.edu.au/current-students/manage-course/exams-results-assessment/plagiarism-academic-integrity/.
- Please upload Part A, Part B, and Part C as separate pdf files to Canvas at the appropriate submission points.
- **The submission deadline is Monday, 11 October, 11:59pm. This deadline applies to Parts A, B, and C.**
- Late submissions will be penalised at a rate of 10% per day, up to a maximum of 5 calendar days. After 5 days no more submissions will be accepted, and a score of 0 will be recorded.
- If there are special circumstances which mean you cannot submit your test prior to the deadline, for example a medical or family emergency, please get in touch with Nathan as early as possible.

I wish you the best of luck in completing the test!

Kind regards,

Nathan

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Part A

1. **[10 marks]** Sketch the graph of an even function $f(x)$ that is defined for all $x \in \mathbb{R}$ and which has the following properties.

   - $f(0) = -2$
   - A local maximum at $x = -4$
   - A local minimum at $x = 7$
   - $$\lim_{x \to -\infty} f(x) = \infty$$

   The graph must be clearly labelled, and there is no need to write a formula for $f(x)$.

2. **[10 marks]** Sketch the graph of an odd function $g(t)$ that is defined for $t \neq 0$ and which has the following properties.

   - $$\lim_{t \to 0^+} g(t) = \infty$$
   - A local maximum at $t = 2$
   - A local maximum at $t = -10$
   - $$\lim_{t \to -\infty} g(t) = \infty$$

   The graph must be clearly labelled, and there is no need to write a formula for $g(t)$.

3. **[5 marks]** Let $A_e(z)$ be an even function, $B_e(z)$ an even function, $C_o(z)$ an odd function, and $D_o(z)$ an odd function. Determine if the function

   $$B_e(A_e(z))C_o(D_o(z))$$

   is even, odd, or neither.

4. **[5 marks]** Let $P_e(s)$ be an even function, $Q_o(s)$ an odd function, $R_o(s)$ an odd function, and $S_o(s)$ an odd function. Determine whether the function

   $$P_e(Q_o(s)) + R_o(S_o(s))$$

   is odd, even, or neither.

Part B

5. **[35 marks total]** Consider the function $f(t)$ defined for $t \in (0, 8\pi]$

$$f(t) = 4\cos\left(\frac{t}{8}\right) \qquad 0 < t \leq 8\pi$$

Note: the same function is studied in Questions 5 and 6. This allows you to partially cross-check your answers **but** you *must* use the appropriate methods for each question, namely standard (trigonometric) Fourier series methods for Question 5, and complex Fourier series methods for Question 6.

Zero marks will be awarded for any answer without the appropriate working.

(a) **[3 marks]** Sketch the full-range periodic extension of $f(t)$, showing three periods.

(b) **[10 marks]** Determine the full-range Fourier series of $f(t)$.

(c) **[3 marks]** Sketch the even periodic extension of $f(t)$, showing three periods.

(d) **[5 marks]** Determine the half-range cosine expansion of $f(t)$.

(e) **[3 marks]** Explain the underlying reason for the interesting result you obtained for the half-range cosine expansion.

(f) **[3 marks]** Sketch the odd periodic extension of $f(t)$, showing three periods.

(g) **[5 marks]** Determine the half-range sine expansion of $f(t)$.

(h) **[3 marks]** Explain the underlying reason for any similarities you observe between the half-range sine expansion and the full-range expansion.

Part C

6. **[35 marks total]** Consider the function $f(t)$ defined for $t \in \mathbb{R}$

$$f(t) = \begin{cases} 4\cos\left(\frac{t}{8}\right) & 0 < t \leq 8\pi \\ f(t+8\pi) & \text{for all } t \end{cases}$$

Note: the same function is studied in Questions 5 and 6. This allows you to partially cross-check your answers **but** you *must* use the appropriate methods for each question, namely standard (trigonometric) Fourier series methods for Question 5, and complex Fourier series methods for Question 6.

Zero marks will be awarded for any answer without the appropriate working.

(a) **[18 marks]** Determine the complex Fourier series of $f(t)$.

(b) **[7 marks]** Using Parseval's theorem for complex Fourier series, derive an expression for

$$\int_0^{8\pi} \left(4\cos\left(\frac{t}{8}\right)\right)^2 dt = 64\pi$$

as an infinite series.

(c) **[10 marks]** From your expression for the complex Fourier series, determine the trigonometric Fourier series of $f(t)$.