Take-home test 1

Semester 2, 2021

Dear Tung Vu

This is your test paper for Take-home test 1, for MTH20012 Series and Transforms. Please take the time to read the following instructions.

- There are 100 marks in total.
- This assessment item is worth 20% of your overall grade.
- Fully justify all of your working.
- Be clear! Full marks are only awarded if your setting out is clear.
- Note: by "closed form expression" I mean a simple expression, like a number, or a simple function – e.g. a closed form expression for the series $1 + x + x^2 + x^3 + \ldots$ is $1/(1 - x)$ (valid for $|x| < 1$).
- This test paper is unique to you. Make sure that you answer the questions on *your* test paper. (Questions on other test papers are similar, but will have different constants.)
- **Your submission must be 100% your own work.** You are encouraged to consult with Nathan, your tutor, MASH, and your fellow students about the material you have studied in MTH20012 Series and Transforms. But, you must not seek improper help with your questions, or share your work with other students or anyone else. **It is academic misconduct to improperly share the test questions on "tutoring" (contract cheating) websites.** The consequence of improperly sharing any question will in most cases be to receive a score of zero for the entire assessment, with this action recorded permanently on your student file. Please see the Unit Outline for more information about what constitutes academic misconduct. Also see https://www.swinburne.edu.au/current-students/manage-course/exams-results-assessment/plagiarism-academic-integrity/.
- Please upload Part A, Part B, and Part C as separate pdf files to Canvas at the appropriate submission points.
- **The submission deadline is Friday, 10 September, 11:59pm. This deadline applies to Parts A, B, and C.**
- Late submissions will be penalised at a rate of 10% per day, up to a maximum of 5 calendar days. After 5 days no more submissions will be accepted, and a score of 0 will be recorded.
- If there are special circumstances which mean you cannot submit your test prior to the deadline, for example a medical or family emergency, please get in touch with Nathan as early as possible.

I wish you the best of luck in completing the test!

Kind regards,

Nathan

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Part A

1. **[4 marks]** Determine the following limit, fully justifying your answer.

$$\lim_{n\to\infty} \frac{-10 - 2n + 14n^2 + 7n^3}{14 + 5n - 2n^3}$$

2. **[4 marks]** Determine the following limit, fully justifying your answer.

$$\lim_{n\to\infty} \frac{3(3^n) + 2n^n}{n^2 - 3(n!)}$$

3. **[10 marks]** Determine the following limit, fully justifying your answer.

$$\lim_{n\to\infty} \frac{4}{\sqrt{6n} - \sqrt{6n + 7\sqrt{n}}}$$

4. **[8 marks]** Determine if the following series converges, fully justifying your answer. If it converges find its sum.

$$\sum_{n=0}^{\infty} \frac{(-2)^{n-3} 3^{2n+2}}{5^{2n-4}}$$

5. **[4 marks]** Expand out the following series, and then provide a numerical value for the sum that is correct to 6 decimal places.

$$\sum_{k=2}^{4} \frac{(-2)^k}{2k + 11}$$

Part B

6. **[6 marks]** Verify that the integral test can be applied to the following series, and then use the integral test to determine if the series converges.

$$\sum_{n=3}^{\infty} \frac{2}{(n+4)^2}$$

7. **[10 marks]** Verify that the comparison test can be applied to the following series, then use the comparison test to establish whether the series converges.

$$\sum_{n=1}^{\infty} \frac{4}{9n-4}$$

8. **[6 marks]** Use the root test to determine whether the following series is absolutely convergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 5^n n^{n-3}}{7^{n+2}}$$

9. **[8 marks]** Determine a closed form expression for the sum of the following series, fully justifying your answer.

$$\sum_{n=2}^{\infty} \frac{72}{(4n+3)(4n+7)}$$

Part C

10. **[12 marks]** Consider the following series:

$$\sum_{n=1}^{\infty} \frac{x^n}{5n\,\pi^{2n+2}},$$

with $x \in \mathbb{R}$. Determine for which values of x the series is convergent, and for which values of x the series is absolutely convergent.

11. **[8 marks]** Determine the disc of convergence of the power series:

$$\sum_{n=0}^{\infty} \frac{(2n+4)!}{(n+2)!\,(n+5)!} z^n,$$

with $z \in \mathbb{C}$.

12. **[20 marks total]**

(a) **[10 marks]** Determine the disc of convergence of the power series:

$$f(z) = \sum_{n=0}^{\infty} \frac{2^{n+1} z^n}{3^{n+5}},$$

with $z \in \mathbb{C}$, and derive a closed form expression for the function $f(z)$.

(b) **[10 marks]** Hence or otherwise obtain a closed form expression for the power series:

$$g(z) = \sum_{n=0}^{\infty} \frac{(n+2)2^{n+1} z^n}{3^{n+5}},$$

fully justifying your answer.