

# Trading wind energy based on probabilistic forecasts of wind generation and market quantities

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# What is the article about ?

## Context

- Portfolio of wind energy (can be solar)
- Liberalized electricity markets

## Goals

- Propose an operational trading strategy (*based on the quantile of wind power production*)
- Assess its performance

## Inputs of the model

### Forecasts of

- Wind power production
- Spot market prices
- Imbalance prices (*regulating market prices*)

*Remark : Not a forecasting problem !*

# What are the main assumptions ?

## Assumptions

- Price-taker
- No practical limitations
- Don't care about the risk : only the long run matters , ie we may face severe losses on the short run
- No curtailment
- PTU (*Program Time Units*) are independent : no market dynamic.
- Imbalance volumes are never rewarded

## Notations

- $k$  : a specific PTU
- $\tilde{W}_k$  : amount of energy contracted in the spot market
- $W_k$  : stochastic production of wind energy
- $\rho_k$  : revenue
- $\rho_k^{(S)}$  : revenue from spot
- $\rho_k^{(\uparrow/\downarrow)}$  : revenue from balancing
- $\pi_k^{(S)}$  : spot market price
- $\pi_k^{(\downarrow)}$  : down-regulation price
- $\pi_k^{(\uparrow)}$  : up-regulation price

## Relations that hold

- $\rho_k = \rho_k^{(S)} + \rho_k^{(\uparrow/\downarrow)}$
- $\rho_k^{(S)} = \pi_k^{(S)} \tilde{W}_k$
- $\rho_k^{(\uparrow/\downarrow)} = \begin{cases} \pi_k^{(\downarrow)} (W_k - \tilde{W}_k), & W_k \geq \tilde{W}_k \\ \pi_k^{(\uparrow)} (W_k - \tilde{W}_k), & W_k < \tilde{W}_k \end{cases}$
- $\pi_k^{(\downarrow)} \leq \pi_k^{(S)} \leq \pi_k^{(\uparrow)}$

# Reformulating the revenue

## Reformulating the revenue

- $\rho_k = \pi_k^{(S)} W_k + C_k^{(\uparrow/\downarrow)}$
- $C_k^{(\uparrow/\downarrow)} = \begin{cases} \psi_k^{(\downarrow)} (W_k - \tilde{W}_k), & W_k \geq \tilde{W}_k \\ \psi_k^{(\uparrow)} (W_k - \tilde{W}_k), & W_k < \tilde{W}_k \end{cases}$
- $\psi_k^{(\downarrow)} = \pi_k^{(\downarrow)} - \pi_k^{(S)}$
- $\psi_k^{(\uparrow)} = \pi_k^{(\uparrow)} - \pi_k^{(S)}$

## Idea

- revenue =  $\left( \text{term ind. from } \tilde{W}_k \right) + (\Delta \text{price} \cdot \Delta \text{imb. volumes})$

# Maximizing the revenue

## Expected Utility Maximization (EUM)

- We want to find  $\tilde{W}_k = \arg \max_{\tilde{W}_k} \mathbb{E} \{ \rho_k \}$
- ... which becomes  $\tilde{W}_k = \arg \max_{\tilde{W}_k} \mathbb{E} \left\{ C_k^{(\uparrow/\downarrow)} \right\}$

## Reformulating $C_k^{(\uparrow/\downarrow)}$

- $$\mathbb{E} \left\{ C_k^{(\downarrow/\uparrow)} \right\} = \underbrace{\int_0^{+\infty} \int_0^{\tilde{W}_k} \psi_k^{(\uparrow)} \left( W_k - \tilde{W}_k \right) dP_{W_k} dP_{\psi_k^{(\uparrow)}}}_{\tilde{W}_k \geq W_k: \text{ short position}}$$
$$+ \underbrace{\int_{-\infty}^0 \int_{\tilde{W}_k}^{W^{(max)}} \psi_k^{(\downarrow)} \left( W_k - \tilde{W}_k \right) dP_{W_k} dP_{\psi_k^{(1)}}}_{\tilde{W}_k < W_k: \text{ long position}}$$

# A stochastic optimization problem

## A stochastic optimization problem

- Idea: getting rid of  $\psi_k^{(\downarrow)}$  and  $\psi_k^{(\uparrow)}$
- $\mathbb{E} \left\{ C_k^{(\downarrow/\uparrow)} \right\} = \widehat{\psi}_k^{(\uparrow)} \int_0^{W_k} (W_k - \widetilde{W}_k) dP_{W_k}$   
 $+ \widehat{\psi}_k^{(\downarrow)} \int_{\widetilde{W}_k}^{W_k^{(max)}} (W_k - \widetilde{W}_k) dP_{W_k}$
- ... where  $\widehat{\psi}_k^{(\uparrow)} = \int_0^{+\infty} \psi_k^{(\uparrow)} dP_{\psi_k^{(\uparrow)}}$
- ... and  $\widehat{\psi}_k^{(\downarrow)} = \int_{-\infty}^0 \psi_k^{(\downarrow)} dP_{\psi_k^{(\downarrow)}}$

## Remark

- Are  $\left\{ \psi_{k,t}^{(\downarrow)} \right\}_t$  and  $\left\{ \psi_{k,t}^{(\uparrow)} \right\}_t$  stationary ?



## Solution

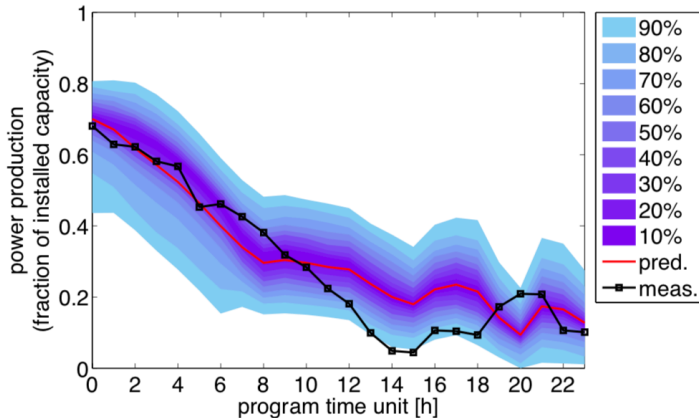
- $\tilde{W}_k = F_{W_k}^{-1} \left( \frac{|\hat{\psi}_k^{(\downarrow)}|}{|\hat{\psi}_k^{(\uparrow)}| + |\hat{\psi}_k^{(\downarrow)}|} \right)$
- $F_{W_k}$  cumulative distribution function of  $W_k$ .

## Remark

- A probabilistic forecast of  $W_k$  is needed

# Probabilistic forecast of $W_k$

Probabilistic forecast of production for a wind power portfolio in Eastern Denmark provided at 11 AM.



# How to compute unknown values ?

## Estimators of unknown values

- spot price:  $\hat{\pi}_k^{(S)} = \mathbb{E} \left\{ \pi_k^{(S)} \right\}$
- spread between spot and imbalance prices :
  - Down :  $\hat{\psi}_k^{(\downarrow)} = \mathbb{E} \left\{ \psi_k^{(\downarrow)} | \psi_k^{(\downarrow)} < 0 \right\}$
  - Up :  $\hat{\psi}_k^{(\uparrow)} = \mathbb{E} \left\{ \psi_k^{(\uparrow)} | \psi_k^{(\uparrow)} > 0 \right\}$

## Remark

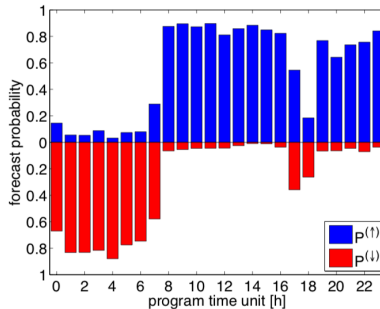
- $P_k^{(\downarrow)} = P \left\{ \psi_k^{(\downarrow)} < 0 \right\}$  and  $P_k^{(\uparrow)} = P \left\{ \psi_k^{(\uparrow)} > 0 \right\}$  need to be computed
- ... since  $\hat{\psi}_k^{(\downarrow)} \neq \hat{\psi}_k^{(\downarrow)} | \psi_k^{(\downarrow)} < 0$  but  $\hat{\psi}_k^{(\downarrow)} = \hat{\psi}_k^{(\downarrow)} | \psi_k^{(\downarrow)} < 0 \cdot P_k^{(\downarrow)}$
- ... same for  $\hat{\psi}_k^{(\uparrow)}$  ...

# How to compute unknown values ?

Estimators of  $P_k^{(\downarrow)}$  and  $P_k^{(\uparrow)}$

- $P_k^{(\downarrow)} = P\left\{\psi_k^{(\downarrow)} < 0\right\}$
- $P_k^{(\uparrow)} = P\left\{\psi_k^{(\uparrow)} > 0\right\}$

Example of forecast probabilities of up and down



## A successful strategy ?

- Simulation results show a PNL of 100K euros /MW from the 1st March 2008 to the 31st December 2008.
- not modeling its interaction with the system. An aggressive position can make the system switch ...
- Model improvements ?
  - Imposing new constraints through a *risk-aversion parameter*...
  - $\tilde{W}_k^{\nu, a_\nu} = \min \left\{ \max \left\{ \tilde{W}_k, \hat{W}_k \cdot (1 - a_\nu) \right\}, \hat{W}_k \cdot (1 + a_\nu) \right\}$
  - ... where  $a_\nu \in \{0.1, 0.2\}$