BAYESIAN STATISTICS PROJECT

Frank-Wolfe Bayesian Quadrature: Probabilistic Integration with Theoretical Guarantees

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Notations

We start with some notations we will use along this report.

• $p(\cdot)$ denotes the function $p: x \mapsto p(x)$.

Introduction

The goal of this article [1] is to compute efficiently the integrals of the form $\int_{\mathcal{X}} f(x)p(x)\mathrm{d}x$ where $\mathcal{X}\subseteq\mathbb{R}^d$ is a measurable space, $d\geq 1$ integer representing the dimension of the problem, p a probability density with respect to the Lebesgue measure on \mathcal{X} and $f:\mathcal{X}\to\mathbb{R}$ is a *test*-function.

We will use the common approximation

$$\int_{\mathscr{X}} f(x)p(x)\mathrm{d}x \approx \sum_{i=1}^{n} w_{i}f(x_{i})$$
 (1)

but of course the real challenge lies in the choice of sequences $\{x_i\}$ and $\{w_i\}$:

- Monte Carlo: $w_i = \frac{1}{n}$ and x_i realization of multivariate random variable $X_i \stackrel{iid}{\sim} X$ where X has $p(\cdot)$ as probability distribution.
- Kernel herding:
- Quasi-Monte Carlo:

In the Frank-Wolfe Bayesian Quadrature, we have

- $\{w_i\}$ which appear naturally in the Bayesian Quadrature by taking the expectation of a posterior distribution (described in section 2)
- $\{x_i\}$ which are chosen to minimize the posterior variance of our estimator (described in section 3)

The main interest of the method developed in [1] is the super fast *exponential* convergence to the true value compared to the other methods mentioned above.

- 1 Theoretical framework
- 2 Bayesian Quadrature
- 3 Frank-Wolfe algorithm

References

[1] François-Xavier Briol, Chris J. Oates, Mark Girolami, and Michael A. Osborne. Frank-wolfe bayesian quadrature: Probabilistic integration with theoretical guarantees.