

# Bayesian statistics project

## Frank-Wolfe Bayesian Quadrature: Probabilistic Integration with Theoretical Guarantees

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### References

- [1] François-Xavier Briol, Chris J. Oates, Mark Girolami, and Michael A. Osborne. Frank-wolfe bayesian quadrature: Probabilistic integration with theoretical guarantees.

### Notations

We start with some notations we will use along this report.

- $p(\cdot)$  denotes the function  $p : x \mapsto p(x)$ .

### Introduction

The goal of this article [1] is to compute efficiently the integrals of the form  $\int_{\mathcal{X}} f(x)p(x)dx$  where  $\mathcal{X} \subseteq \mathbb{R}^d$  is a measurable space,  $d \geq 1$  integer representing the dimension of the problem,  $p$  a probability density with respect to the Lebesgue measure on  $\mathcal{X}$  and  $f : \mathcal{X} \rightarrow \mathbb{R}$  is a *test-function*.

We will use the common approximation

$$\int_{\mathcal{X}} f(x)p(x)dx \approx \sum_{i=1}^n w_i f(x_i) \quad (1)$$

but of course the real challenge lies in the choice of sequences  $\{x_i\}$  and  $\{w_i\}$  :

- **Monte Carlo:**  $w_i = \frac{1}{n}$  and  $x_i$  realization of multivariate random variable  $X_i \stackrel{iid}{\sim} X$  where  $X$  has  $p(\cdot)$  as probability distribution.
- **Kernel herding:**
- **Quasi-Monte Carlo:**

In the **Frank-Wolfe Bayesian Quadrature**, we have

- 🔗  $\{w_i\}$  which appear naturally in the Bayesian Quadrature by taking the expectation of a posterior distribution (described in section 1)
- 🔗  $\{x_i\}$  which are chosen to minimize the posterior variance of our estimator (described in section 2)

The main interest of the method developed in [1] is the super fast *exponential* convergence to the true value compared to the other methods mentioned above.

## 1 Bayesian Quadrature

## 2 Frank-Wolfe algorithm