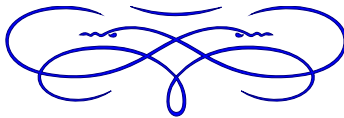


Calculs - Sommations



Exercice 1. Calculer

$$a) \sum_{1 \leq i \leq j \leq n} \frac{i}{j} \quad b) \sum_{1 \leq i \leq j \leq n} ij \quad c) \sum_{1 \leq i \leq j \leq n} i + j$$

Exercice 2. Calculer :

$$a) \sum_{1 \leq i \leq j \leq n} a^{i+j} \quad b) \sum_{1 \leq i \leq n} \sum_{1 \leq j \leq n} \sup(i, j)$$

Exercice 3. Calculer :

$$\sum_{i=0}^n \left[x^i \binom{n}{i} \sum_{k=i}^n \binom{n-i}{n-k} \right]$$

Exercice 4. Calculer :

$$a) \sum_{k=1}^{n^2} E(\sqrt{k}) \quad b) \sum_{k=1}^{n^3} E(\sqrt[3]{k})$$

Exercice 5. Calculer :

$$\sum_{i=0}^{2n} E\left(\frac{i}{2}\right) \binom{2n}{i} p^i (1-p)^{2n-i}$$

Exercice 6. Calculer :

$$\sum_{k=0}^{E\left(\frac{n}{3}\right)} \binom{n}{3k}$$

Indication : On pourra introduire $j = e^{\frac{2i\pi}{3}}$.

Exercice 7. Calculer :

$$S = \sum_{k=0}^n k^2 \binom{2n}{2k} \quad n > 1$$

Indication : On pourra introduire $S' = \frac{1}{4} \sum_{k=0}^n (2k+1)^2 \binom{2n}{2k+1}$.