# Valuation of Contingent Convertibles with Derivatives



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#### Abstract

Financial crises have led to higher regulatory standards on the capital adequacy of banks. Banks are required to hold more capital with loss absorbance capacities on their balance sheet. In conjunction with this development, contingent convertible bonds (CoCos) have become an attractive instrument for banks to seek new capital. The defining characteristic of CoCos is the automatic conversion into common equity or the principal write-down when a predetermined trigger is met. Loss absorbing capital is created, which instantly improves the capital structure of the distressed bank. However, the pricing of these hybrid instruments remains opaque. In this context, the thesis scrutinizes the valuation of Three dominant approaches are examined: the structural approach in accordance to Pennacchi (2010), the credit derivatives approach and the equity derivatives approach both pursuant to De Spiegeleer and Schoutens (2011b). The application covers sensitivity analysis to further understand the dynamics of the different methodologies. Based on a case study the viability of those approaches is evaluated. Software is provided for further replication.

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## Chapter 1

## Introduction

## 1.1 Background

Investors are overly restrictive in providing liquidity to financial institutions during periods of financial distress. In the past, governments were often in the situation to inject liquidity to financial markets in order to avoid disruptive insolvencies as no other market participant was inclined to do so. Government bailouts, however, externalize the cost of bankruptcy to taxpayers while distorting risk-taking incentives of banking professionals. Contingent convertibles (CoCos) aim to internalize these costs in the capital structure of systemically important financial institutions. CoCos are hybrid financial instruments that absorb losses pursuant to their specifications in case a pre-determined threshold fails to remain above a minimum trigger level. Then, debt automatically morphs to equity which instantly improves a bank's capitalization. Due to their loss-absorption capacity, CoCos are eligible to be categorized as regulatory capital. (Avdjiev et al., 2013)

After the global financial crisis of 2008, regulators around the world have been working on two different objectives. On the one hand, they attempted to lower spillover effects on the economy due to bankruptcies of financial institutions. On the other hand, they tried to reduce the individual default probabilities of banks. The latter objective might be attained by ensuring that banks have enough loss absorbing capital on their balance sheet even in though times. (De Spiegeleer and Schoutens, 2011a) In this context, the Basel Committee on Banking Supervision (BCBS) specified that debt instruments are permitted as regulatory capital if losses are absorbed to such an extent that tax payers do not have to bear the costs. (Basel Committee on Banking Supervision, 2010b) Subsequently, this opened the door for CoCos and ever since, the regulatory treatment has been a major driver of past issues.

Besides the reforms of policy makers to raise the quality of regulatory capital, recent studies on CoCos highlight a number of advantages. Albul et al. (2015) show that CoCos lessen financial distress, whether caused by idiosyncratic or systemic shocks. They indicate lower default probabilities of banks and a smaller likelihood of costly bailouts by the public sector. Hilscher and Raviv (2014) support these findings. Additionally, they argue that an appropriate specification of a CoCo's building parts can eliminate the incentives of shareholders to asset-substitution. The problem of asset-substitution arises when managers undertake excessively risky investment decisions to maximize shareholder value at the expense of debtholders. (Bannier, 2011) Research evinces that CoCos increase the firm value of their issuer as they reduce the cost of capital. (Albul et al., 2015; Von Furstenberg et al., 2011; Barucci and Del Viva, 2012) From an academic standpoint, CoCos should be considered in the liability structure of banks.

In addition to the positive perception of policy makers and academia, CoCos are well accepted by the financial industry. Banks value that the hybrid instrument enables them to refinance themselves while simultaneously satisfying the regulatory capital requirements at lower costs than with equity. (European Parliament, 2016) Between 2009 and 2015, financial institutions around the world issued CoCos worth USD 446.96 bn in 519 different issues. (Avdjiev et al., 2015) Albeit the amount of CoCos issued is relatively small compared to the market size of other financial products, they were brought into focus in early 2016. At this time, CoCos contributed to increased market volatility in view of some European banks (i.a. Deutsche Bank and Credit Suisse). The relevance of this episode concerning the potential systemic implications should not be whitewashed as it is likely that discussions on regulatory changes will emerge. Having regard to this circumstances, the question arises about the perception of CoCos by investors and the robustness of their pricing models. (European Parliament, 2016)

In this context, an investigation and analysis of different valuation concepts for CoCos is highly interesting for both investors as well as supervisory authorities. The paper contributes to a better understanding of relevant concepts. Additionally, the valuation approaches will be applied to a CoCo which was affected by the turbulences in early 2016.

#### 1.2 Previous Studies

• pricing models for CoCo bonds proposed in literature can be grouped broadly into three main approaches

structural approach
equity derivative approach
credit derivative approach

- the diversity of pricing models is due to the hybrid nature of CoCo bonds
- several studies have been conducted on these different model types
- a comprehensive overview of relevant papers can be found in table 1.1

Structural Approach	Equity Derivative Approach	Credit Derivative Approach
Pennacchi (2010)	De Spiegeleer and Schoutens (2011b)	De Spiegeleer and Schoutens (2011b)
Albul et al. (2010)	Henriques and Doctor (2011)	Serjantov (2011) as cited by Wilkens and Bethke (2014)
Madan and Schoutens (2011)	Alvemar and Ericson (2012)	Alvemar and Ericson (2012)
Glasserman and Nouri (2012)	Corcuera et al. (2014)	Erismann (2015)
Alvemar and Ericson (2012)	Erismann (2015)	
Buergi (2013)		
Hilscher and Raviv (2014)		
Pennacchi and Tchistyi (2015)		
Turfus and Shubert (2015)		
Cheridito and Xu (2015)		
Erismann (2015)		
Sundaresan and Wang (2015)		

Table 1.1: Literature overview of valuation approaches for CoCos (Wilkens and Bethke, 2014; Erismann, 2015)

- structural approaches explicitly capture the typical trigger event and the purpose of CoCos as deleveraged tool
- attempts to capture the trigger event by modeling the CET1
- equity derivative approaches mainly reflect the dependence on share price as an indicator of both the financial health of a company and the value transfer at conversion
- model the equity price, taking this as a proxy for the financial health of the company and hence of its Common Equity Tier 1 ratio

- credit derivative approaches encapsulate the fact that CoCo bonds are creditrisky debt, paying coupons until maturity or until trigger or default
- models an equity conversion intensity process analogous to a credit default intensity process
- neither pure equity nor credit derivative models, however, lend themselves easily to reflecting the capital ratio trigger
- the various pricing approaches are outlined in the following sections

### Structural Approaches

- because most of the CoCos issued to date have expressed their trigger thresholds in terms of a predetermined capital ratio, structural models provide a natural pricing framework that considers the institution's balance sheet structure as the main price driver
- these models are economically fundamental because they are based on modeling the institution's assets and liabilities with the difference representing the institutions capital
- structural models describe processes for the institution's assets and liabilities and impose contingent capital conversion into equity when the critical capital-to-assets threshold has been reached
- within this framework, bankruptcy occurs when the value of the institution's assets drops below the value of its liabilities
- several structural models have been proposed in the literature
- all share common features but differ, for example in their applications
- Albul et al. (2010) focused on optimal capital structure and asset-substitution incentives
- Glasserman and Nouri (2012) considered CoCos that continuously convert sufficient nominal to maintain a capital ratio above the critical level
- Pennacchi (2010)
- Madan and Schoutens (2011)

- Alvemar and Ericson (2012)
- Buergi (2013)
- Hilscher and Raviv (2014)
- Pennacchi and Tchistyi (2015)
- Turfus and Shubert (2015)
- Cheridito and Xu (2015)
- Erismann (2015)
- Sundaresan and Wang (2015)

#### Equity Derivative Approaches

- main price driver is typically the share price of the company
- De Spiegeleer and Schoutens (2011b) replicated a CoCo via a straight bond, a knock-in forward, and a strip of binary down-and-in options
- the knock-in forward represents the trigger event, when the CoCo bondholder exchanges the bond for shares at a predetermined strike price
- because the shares are received at the trigger date and thus before the original maturity, the representation as a forward is a simplification; this assumption is usually justified because the company will likely not pay any dividends for some time after the conversion
- the binary down-and-in options reflect the cancellation of the coupon payments after the conversion has taken place
- equity derivative approach offers the advantage of a closed-form solution for the CoCo price and its sensitivities, as well as a straightforward parameterization similar to that of equity options
- enhancements proposed in the literature Corcuera et al. (2014), which require numerically more involved computation or Monte Carlo simulation, use smile confirm equity derivative formulas to represent the underlying risky asset dynamics, with an exponential Levy process reflecting jumps and heavy tails

- this approach is complemented by the modeling of a separate trigger process for the conversion in combination with a downward jump for the share price at conversion De Spiegeleer and Schoutens (2011b)
- De Spiegeleer and Schoutens (2011b)
- Henriques and Doctor (2011)
- Alvemar and Ericson (2012)
- Erismann (2015)

#### Credit Derivative Approaches

- main feature of a CoCo is conversion in the case of financial distress of the company
- the bond price is thus closely related to the company's financial health and default probability, and intensity-based credit modeling lends itself a pricing approach
- among others De Spiegeleer and Schoutens (2011b) and Serjantov (2011) suggested a pricing framework based on credit derivatives
- De Spiegeleer and Schoutens (2011b) formulated a general rule to derive the additional yield a CoCo must offer to compensate the holder for a premature repayment, which might amount to less than the bond's nominal
- a fundamental problem with this approach is the usual non-negligible stream of future coupon payments that the holder of the CoCo forfeits at conversion
- this effect cannot easily be incorporated explicitly, for example, into the recovery rate at conversion, without making it time dependent
- Serjantov (2011) offered a slightly different approach, explicitly taking the coupon payments into account
- to obtain the CoCo price, the repayment of the nominal at maturity and the coupon payments over the lifetime are weighted with cumulative survival probabilities representing the absence of trigger and default events until the respective payment and discounted to the pricing date

- the recovery rates for the default case and for he conversion case without default are distinct in this model and can thus differ
- the joint probability of both events happening within the same time interval is defined by a Gaussian copula in the simplest case, where the correlation between trigger and default events is a model input
- the credit derivatives model gives a quasi-closed-form solution for the CoCo price
- Alvemar and Ericson (2012)
- Erismann (2015)

### 1.3 Research Methodology

The objective of the thesis consists of an examination of three dominant pricing approaches for CoCos. All of them are widely discussed in academic literature as they are often used as basis for further model advancements. The utilized approaches are namely the structural approach of Pennacchi (2010), the equity derivative approach and the credit derivative approach both pursuant to De Spiegeleer and Schoutens (2011b). Hereinafter, chapter 2 provides an overview of the anatomy of CoCos. Characteristic building parts of the financial product will be discussed in detail in order to create an improved understanding of the mechanisms which drive the valuation of this hybrid instrument. In chapter 3, examples of past CoCos issues are highlighted covering the most important variations of the aforementioned design features. On this basis, chapter 4 studies the theoretical concepts behind each of the three valuation approaches. In addition, pricing examples provide an understanding of the data requirements of each model. In chapter 5, sensitivity analysis determine how different values of certain pricing parameters impact the valuation of CoCos. Chapter 6 comprises an empirical analysis on the price tracking accuracy of the aforementioned valuation approaches. Finally, a conclusion is reached in chapter 7.

## Chapter 2

## Structure of CoCos

This chapter explains the nature of a new member in the family of hybrid securities; so-called contingent convertibles (CoCos). In the following, the general structure of these new instruments will be explained (section 2.1) including characteristic design features among others their trigger (section 2.2) and loss-absorption mechanism (section 2.3).

#### 2.1 Definition

CoCos are hybrid capital instruments with automatic conversion mechanisms that morph debt into equity when the financial soundness of the issuer is at stake. In such a case, a predefined trigger event occurs. For instance, a regulatory capital ratio falls below a predefined threshold. That said, a write-down of the notional of the CoCo is also a viable loss-absorption mechanism. (De Spiegeleer and Schoutens, 2011b) Figure 2.1 provides a good overview of the previously mentioned design characteristics. All anatomic aspects will be further explained in the subsequent sections.

CoCos are particularly interesting from the perspective of a regulatory authority because they might mitigate externalization costs of insolvencies and frictions due to spill-over effects. In times of distress, stakeholders might question the financial viability of the respective financial institute. However, a distressed bank does not have to approach new investors to issue new capital in extremely though times as everything happens automatically. (De Spiegeleer and Schoutens, 2011b)

In 2009, the Lloyds Banking Group was the first financial institute to issue this new financial instrument. They offered hybrid debt holders to swap their holdings into CoCos.(De Spiegeleer and Schoutens, 2011b) In 2010, the Basel Committee on

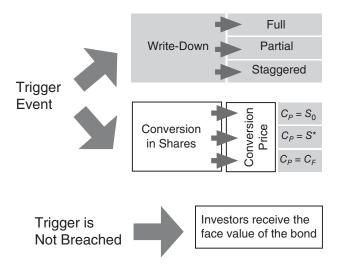


Figure 2.1: Anatomy of CoCos (De Spiegeleer et al., 2014)

Banking Supervision (BCBS) provided further impetus to the use of this instrument when it disclosed its proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. The general argumentation of the BCBS is that regulatory capital instruments have to be capable of absorbing financial losses in gone-concern phases. (Basel Committee on Banking Supervision, 2010a).

## 2.2 Trigger Mechanism

The trigger is a key design element of a CoCo. In the following section, four different trigger mechanisms will be evaluated based on general design criteria as summarized by Erismann (2015). The concept of an accounting trigger will be illustrated in section 2.2.1 followed by the market trigger in section 2.2.2. Additionally, the regulatory trigger will be detailed in section 2.2.3 and the multi-variate trigger will be explained in section 2.2.4. Hereinafter, the trigger types will be evaluated based on the following criteria:

- Clarity: The trigger mechanism should be universally applicable irrespective of the jurisdiction in which the CoCo is traded.
- Fixedness: The definition of a CoCo's trigger should remain the same until maturity.
- Frequency: Data to which the trigger is linked should be updated at frequent intervals.

- Objectiveness: The trigger mechanism should be based on observable and well-known facts. Their should be no room for subjectivity.
- Publicity: Data should be open to the general view of all market participants.
   This needs to be possible without concealment.
- **Transparency**: The trigger mechanism should have the property that investors can see it publicly, thereby reducing the chance of manipulation.

#### 2.2.1 Accounting Trigger

CoCos with accounting trigger have a loss absorption mechanism which is inherently connected to the financial soundness of a bank's balance sheet. Accounting triggers are built upon capital ratios which compare a bank's regulatory capital with its assets. Conversion occurs if a pre-determined metric falls below a certain threshold. Hence, capital ratios are an objective indicator for a bank's solvency as they are defined uniformly for all financial institutions by regulatory authorities. (De Spiegeleer et al., 2014) Pazarbasioglu et al. (2011) note that accounting triggers are easy to price, intuitive and simple to implement. Issuing banks and regulators seem to have perceived the benefits of accounting triggers partly because a significant portion of CoCos use the common equity tier (CET1) ratio as reference metric. Examples of CoCos will be described shortly in section 3.

That said, objections against accounting triggers follow the line of thought that they just become active long after the need for loss absorbing capital arose. One might argue that accounting triggers assess the viability of financial institutions from a perspective that is far-removed from reality. (De Spiegeleer and Schoutens, 2011b) Moreover, as accounting concept, book values are prone to manipulation and managerial dishonesty especially in times of distress. (McDonald, 2013)

Empirical findings bring up a further aspect. Haldane (2011) points out that major financial institutions, which either went bankrupt, were bailed out or were taken over under distress during the global financial crisis, reported similar CET 1 ratios right before the collapse of Lehman Brothers compared to their peers which coped relatively well with the collapse of the financial system. In this context, Haldane

(2011) highlights that market-based solvency measures<sup>1</sup> performed creditably as they showed clear signals of impending distress a year ahead of the bankruptcy of Lehman Brothers. Empirical evidence of United States Bankruptcy Court (2010) further supports these findings. This leads to the conclusion that CoCos with accounting triggers might not reinforce distressed banks at the right time but instead produce more false positives, which means that CoCos of non-distressed banks trigger prematurely. Inefficiencies like higher funding costs could be the consequence. (Pazarbasioglu et al., 2011)

#### 2.2.2 Market Trigger

A market trigger uses directly observable indicators like the issuing company's share price or credit default swap (CDS) spreads while assuming sufficiently efficient markets. The major advantage of those measures is that one can observe and verify them in real-time. (Haldane, 2011) Market triggers are widely discussed in academia and seen as preferable trigger mechanism. Calomiris and Herring (2013) pronounce themselves for using share prices. Besides, Haldane (2011) and Pazarbasioglu et al. (2011) contend to apply market-based capital ratios as trigger indicator. Their line of argumentation is based on some of the best-known examples of corporate defaults, which were indicated well before by a serious and continuous deterioration of a company's market capitalization.

This also applies to the aforementioned example of Lehman Brothers. If the bank had been obliged to issue CoCos with a trigger directly linked to its share price the decline in equity value would have triggered the conversion in due time. (Calomiris and Herring, 2013) Hence, the bankruptcy could have been prevented with a CoCo. However, practitioners argue that it comes at a risk to use market data as reference because markets are prone to manipulation. Moreover, self-fulfilling prophecies of share price dilution could lead to downward spirals that ultimately lead to CoCo conversions. (Pazarbasioglu et al., 2011) The severity of the aforementioned argumentation can be attentuated by using moving averages, which are less susceptible to manipulation.

<sup>&</sup>lt;sup>1</sup>Haldane (2011) mentions three metrices: (1) market-based capital ratio (ratio of market capitalization to total assets), (2) market-based leverage ratio (ratio of market capitalization to total debt) and (3) Tobin's Q (ratio of market capitalization to book value of equity).

#### 2.2.3 Regulatory or Non-Viability Trigger

Regulatory or non-viability trigger is a conversion mechanism by which a CoCo is converted into equity at the discretion of the responsible supervisory authority. The rationale behind this approach is that regulators want to limit the impact of any development that could pose a danger to the going-concern of a systemically important bank. Erismann (2015) Moreover, this kind of trigger would eliminate the periodicity problem of accounting data and the risk of market manipulation.

Though, it is very difficult for market participants to estimate the conversion probability of a CoCo with regulatory trigger. The valuation of such a hybrid instrument becomes opaque for market participants with limited information. (Alvemar and Ericson, 2012) One can also argue that a CoCo's marketability is weakened because of the greater uncertainty which could ultimately lead to higher funding costs. (De Spiegeleer et al., 2014)

#### 2.2.4 Multi-Variate Trigger

The multi-variate trigger uses a combination of the aforementioned trigger mechanisms. It extends the accounting trigger of a CoCo by a universal regulatory trigger which covers severe states of the world adversely impacting the stability of the financial system. In this context, the Squam Lake Working Group (2009) supports the implementation of this dual trigger mechanism as it combines the best of two worlds. On the one hand, the bank-specific trigger serves as disciplining mechanism for a bank's management. It reduces the political pressure from the regulator who has to decide whether the systemic trigger is met. Second, if the conversion of a CoCo is only linked to a systemic trigger, even well capitalized banks would be forced to convert debt into equity during a systemic crisis. This would disincentivize financially sound banks to preserve their status quo.

### 2.2.5 Evaluation of Trigger Types

Generally, one can evaluate the aforementioned trigger types based on the design criteria as stated in the beginning.

	Accounting	Market	Regulatory	Multi-Variate
Clarity	medium	high	low	medium
Fixedness	high	high	low	high
Frequency	medium	high	low	high
Objectiveness	medium	high	low	medium
Publicity	high	high	high	high
Transparency	medium	high	low	medium

Table 2.1: Evaluation of different trigger mechanisms based on design criteria. The status high means that the trigger type meets the requirements, whereas low implies that it does not satisfy the conditions. (Erismann, 2015)

We see in table 2.1 that the market trigger is a preferable mechanism as it fulfills all important design criteria. Furthermore, one has to take into account that Haldane (2011) described that market trigger were historically the preferable mechanism as it served as early-warning indicator. In section 3 different CoCos will be highlighted with their design elements also covering their trigger mechanism. This is a good way to understand whether banks and regulators followed the aforementioned rationale.

## 2.3 Loss-Absorption Mechanism

As mentioned before, banks can decide whether they want to issue CoCos with equity conversion or with a haircut on the notional.

#### 2.3.1 Conversion into Shares

The number of shares which a CoCo holder receives at conversion is given by the conversion rate  $C_r$ . This figure is specified ex-ante and the conversion into shares gives an investor in case a certain trigger threshold is breached. Besides, the conversion amount  $\alpha N$  is determined by the conversion fraction  $\alpha$  and the notional N. One can now derive the implied conversion price  $C_p$  with the following formula. (De Spiegeleer et al., 2014)

$$C_p = \frac{\alpha N}{C_r} \tag{2.1}$$

The recovery rate  $R_{CoCo}$  can be calculated based on the share price  $S^*$  at the trigger event and the conversion price  $C_p$ . One might recognize that a CoCo holder is better

off if  $C_p$  is low because the recovery rate  $R_{CoCo}$  will be higher as more equity will be created.

$$R_{CoCo} = \frac{S^*}{C_p} \tag{2.2}$$

If the CoCo is converted into shares one can directly derive the loss  $L_{CoCo}$  a CoCo holder has to bear.

$$Loss_{CoCo} = N - C_r S^* = N - (1 - R_{CoCo}) = N \left( 1 - \frac{S^*}{C_p} \right)$$
 (2.3)

Generally, the final payoff  $V^{CoCo}$  which an investor receives at maturity T is given by:

$$V_T^{CoCo} = \begin{cases} N & \text{if not triggered} \\ (1 - \alpha)N + \frac{\alpha N}{C_p} S^* & \text{if triggered} \end{cases}$$
 (2.4)

#### Floating conversion price

l At issuance one can set a floating conversion price where  $C_p$  is equal to  $S^*$ .  $S^*$  is the share price which is observed at the trigger event. Intuitively, the value of the share price at trigger time t is low as the purpose of the CoCo is to help undercapitalized banks in tough times. If the issuer chooses to set the conversion price at this specific level the recovery rate of a CoCo holder is 100% whereas the current shareholders carry the load of conversion. Regulators would not categorize this instrument to be adequate as regulatory capital instrument as the dilution is potentially unbounded and it is effectively not loss absorbing. (De Spiegeleer et al., 2014)

#### Fixed conversion price

Fixed conversion means that the conversion price  $C_p$  is equal to the share price of the issue date  $S_0$ . On the one hand, this means that the amount of shares upon conversion is fixed at the date of issue and one the other hand, the amount is known beforehand. This is in contrast to the floating conversion price where there was no limit on the amount of shares converted. (De Spiegeleer et al., 2014)

#### Floored conversion price

One can also specify a floored conversion price where  $C_p$  is equal to  $\max(S^*, S_F)$ . Hence, the conversion price  $C_p$  is either equal to the floored share price  $S_F$  or to the share price  $S^*$ , which is observed at the triggering event. It is a compromise between the floating and fixed conversion price as stated before. (De Spiegeleer et al., 2014)

#### 2.3.2 Write-Down

The implementation of a loss-absorption mechanism with equity conversion entails several disadvantages which might reduce the marketability of a CoCo. For instance, portfolio managers with a mandate to invest in corporate bonds may face hard times to broaden their investment universe because CoCos with equity conversion mechanism may be converted into shares. Furthermore, the write-down feature is preferable as investors know beforehand the potential loss. Shareholders might be concerned that their voting rights are diluted when a conversion occurs. Both investors and shareholder have clear incentives to influence a bank to issued CoCos with write-down mechanism.

#### Full write-down

The first way is to specify a full write-down of a CoCo's face value in case a certain capital ratio drops below a predetermined level.

#### Partial write-down

Another approach is that only a certain portion of the notional is wiped out.

#### Staggered write-down

The third option consists of a staggered write-down. This means that a CoCo inherits a flexible write-down mechanism. Losses materialize up to the point to enhance a certain capital ratio to a fixed threshold. One might think of a gradual process in which a haircut is imposed in multiples of 10%.

## Chapter 3

## CoCo Examples

For each of the aforementioned loss-absorption mechanisms selected CoCos are characterized hereinafter to gain a better sense of how these hybrid products are implemented in real-life. To that end, broadly discussed CoCos of well-known financial institutions are picked out, i.a. Lloyds, Credit Suisse, Barclays, Rabobank and Zurich Cantonal Bank.

	Lloyds	Credit Suisse	Barclays	Rabobank	Zurich Cantonal Bank
Full Name	Enhanced Capital Notes	Tier 2 Buffer Capital Notes	Contingent Capital Notes	Senior Contingent Notes	Subordinated Tier 1 Notes
ISIN	XS0459088281	XS0595225318	US06740L8C27	XS0496281618	CH0143808332
Issue Date	Dec 1, 2009	Feb 24, 2011	Nov 21, 2012	Mar 19, 2010	Jan 31, 2012
Maturity	May 12, $2020$	Feb 24, 2041	Nov 21, 2022	Mar 19, 2020	Perpetual
Nominal	GBP 7.5 bn	USD 2 bn	USD 3 bn	EUR 1.25 bn	CHF 590 mn
Callability	n/a	Callable from Aug 24, 2016	n/a	n/a	Callable from Jun 20, 2017
Coupon	7.5884%	7.875%	7.625%	6.875%	3.5%
Write-down	n/a	n/a	Full by (100% of notional)	Partial (75% of notional)	Staggered (multiples of 25% of notional)
Conversion price	Fixed at GBP $0.59 = S_0$	Floored at lowest of USD 20 and 30 day-VWAP	n/a	n/a	n/a
Trigger	Core Tier 1 capital ratio	CET 1 ratio	CET 1 ratio	Equity capital ra- tio (Member cer- tificates to risk weighted assets)	CET 1 ratio
Trigger Level	5%	7%	7%	7%	7%

Table 3.1: CoCo examples with different loss-absorption mechanisms (Lloyds, 2009; Credit Suisse, 2011; Barclays, 2010; Rabobank, 2010; Zurich Cantonal Bank, 2013)

An overview of CoCos which have different characteristics with respect to their lossabsorbency can be found in table 3.1. The first two apply similar equity conversion mechanisms whereas the latter three use different write-downs.

## 3.1 Lloyds – Enhanced Capital Notes

In 2009, Lloyds issued Enhanced Capital Notes (ECN) in the amount of GBP 9 bn in an debt exchange offer. At that time, the company was the first bank to refinance itself with CoCos. The ECN inherit an accounting-based trigger which is disclosed each quarter. These instruments convert into equity should the bank's Core Tier 1 capital (Basel II) fail to remain above the threshold of 5%. If this is the case, the conversion mechanism morphs the entire face value to a fixed amount of ordinary shares. But today, banks in the UK move to Basel III. Yet, this leads to stricter definitions of risk-weighting compared to Basel II. Thus, the initial threshold of 5% Core Tier 1 capital ratio is not sufficient as it translates to a significantly lower CET 1 ratio under Basel III. Regulators believe that the ECN might not buttress the balance sheet in times of stress. Hence, the supervisory authorities did not consider them in their latest stress tests. Subsequently, Lloyds has started to call in the CoCos. (Lloyds, 2009)

### 3.2 Credit Suisse – Tier 2 Buffer Capital Notes

The rationale of Credit Suisse's issue of its Tier 2 Buffer Capital Notes (T2BCN) in 2011 follows the recommendation of the Swiss Commission of Experts. Their goal is to establish measures which further solidify the capital structure of the Swiss banking sector. In this context, CoCos play an important role for Credit Suisse in strengthening its capital base to 19% until 2019. In order to achieve the given objectives, USD 2 bn in T2BCN have been raised with a coupon of 7.875%. The trigger event is specified to convert the T2BCN into ordinary shares if the reported risk-based capital ratio is below 7%. The conversion price is floored at the average daily share price of the last 30 days or USD 20. It may also be that the CoCo is converted if the FINMA determines that Credit Suisse needs to be bailed out. (Credit Suisse, 2011)

## 3.3 Barclays – Contingent Capital Notes

In 2012, Barclays launched the first high-trigger total-loss CoCo. The Contingent Capital Notes depreciate to a value of zero should the CET 1 ratio fail to remain above a minimum of 7%. Accordingly, the bond pays a coupon of 7.625%. Such a structure assumes already that the bank has a solid buffer before the trigger is actually met. (Barclays, 2010)

### 3.4 Rabobank – Senior Contingent Notes

CoCos with write-off features are particularly suitable for cooperative banks that are deterred by their legal form to issue shares. In 2010, Rabobank was the first cooperative bank to issue EUR 1.25 bn in Senior Contingent Notes with a loss-absorption mechanism that imposes a considerable haircut on its principal. One quarter of the notional is reimbursed at the trigger event if the equity capital ratio falls below 7%. The distinct haircut explains the high financing costs of the instrument. (Rabobank, 2010)

## 3.5 Zurich Cantonal Bank– Subordinated Tier 1 Notes

Zurich Cantonal Bank is wholly owned by the Canton of Zurich and hence, it is not listed. The bank has been in a similar situation to improve its regulatory capital as Rabobank. In this situation, the bank issued Subordinated Tier 1 Notes that are exemplary for CoCos with a staggered write-down mechanism. What has been new is that an investor faces a dilution of his or her holdings up to the point where the write-down lifts the regulatory capital up to its minimum level. Haircuts materialize only in multiples of 25%. Zurich Cantonal Bank (2013)

## Chapter 4

# Theory of Pricing

## 4.1 Credit Derivative Approach

In financial markets the reduced-form approach is widely used in order to price credit risk. It was originally introduced by Jarrow and Turnbull (1995) respectively Duffie and Singleton (1999). The credit derivative approach applies the reduced-form approach to CoCos. In this context, the derivation of a pricing formula for CoCos follows mainly De Spiegeleer and Schoutens (2011b).

### 4.1.1 Reduced-Form Approach and Credit Triangle

The reduced-form approach is an elegant way of bridging the gap between the prediction of default and the pricing of default risk of a straight bond. In the following, we investigate the link between estimated default intensities and credit spreads under the reduced-form approach. (Lando, 2009)

Let  $\tau$  denote the random time of default of some company. It is assumed that the distribution of  $\tau$  has a continuous density function f, so that the distribution function F and the curve of survival probabilities q are related as follows:

$$P(\tau \le t) = F(t) = 1 - q(t) = \int_0^t f(s)ds$$
, with  $t \ge 0$  (4.1)

The hazard rate respectively the default intensity  $\lambda$  is defined as follows:

$$\lambda(t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} P(\tau \le t + \Delta | \tau > t) = \frac{F'(t)}{q(t)} = \frac{F'(t)}{1 - F(t)} = -\frac{d}{dt} \log q(t)$$
 (4.2)

Intuitively, the hazard rate is the default rate per year as of today. Using 4.2 we can derive a formula for the survival probability:

$$q(t) = \exp\left(-\int_0^t \lambda(s)ds\right) \tag{4.3}$$

For our application of the reduced-form approach we assume that the hazard rate  $\lambda(t)$  is a deterministic function of time. In reality  $\lambda(t)$  is not deterministic but itself stochastic. That fits in with the fact that credit spreads are not static but stochastically varying over time. (Schmidt, 2015) However, we further consider the hazard rate to be constant in order to simplify the problem. Hence, a constant hazard rate  $\lambda(t) = \lambda$  implies an exponential distribution of the default time:

$$F(t) = 1 - q(t) = 1 - \exp(-\lambda t) \tag{4.4}$$

Under the idealized assumption of a flat zero interest rate curve, a flat spread curve and continuous spread payments, the default intensity  $\lambda$  can be calculated directly from the credit spread s and the recovery rate R by the rule of thumb formula (Schmidt, 2015), which is also known as credit triangle:

$$\lambda = \frac{s}{1 - R} \tag{4.5}$$

Finally, this relationship makes it possible to determine the default probability F from the credit spread s and vice versa.

#### 4.1.2 Adaption to CoCos

In accordance with the aforementioned reduced-form approach, De Spiegeleer and Schoutens (2011b) assume that the probability  $F^*$ , which measures the likelihood that a CoCo triggers within the next T-t years, follows similar mechanics as the default probability of a straight bond does. Under the credit derivative approach the probability  $F^*$  can be expressed as follows:

$$F^* = 1 - \exp\left[-\lambda_{Trigger}(T - t)\right] \tag{4.6}$$

Additionally, the credit derivative approach models  $F^*$  with the first exit time equation used in barrier option pricing under a Black-Scholes setting. (Su and Rieger, 2009) Hence, the probability  $F^*$  that the trigger level  $S^*$  is touched within the next T-t years is given by the following equation with the continuous dividend yield q, the continuous interest rate r, the drift  $\mu$ , the volatility  $\sigma$  and the current share price S of the issuing company:

$$F^* = \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) - \mu(T-t)}{\sigma\sqrt{(T-t)}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}}\Phi\left(\frac{\log\left(\frac{S^*}{S}\right) + \mu(T-t)}{\sigma\sqrt{(T-t)}}\right)$$
(4.7)

In this regard, a CoCo's credit spread  $s_{CoCo}$  can be approximated by the credit triangle, where  $R_{CoCo}$  denotes the recovery rate of a CoCo and  $L_{CoCo}$  is the loss rate:

$$s_{CoCo} = (1 - R_{CoCo}) \lambda_{Trigger} = L_{CoCo} \lambda_{Trigger}$$
(4.8)

In the trigger event, the face value N converts into  $C_r$  shares worth  $S^*$ . The loss of a long position in a CoCo is therefore determined by the conversion price  $C_p$ :

$$Loss_{CoCo} = N - C_r S^* = N \left( 1 - R_{CoCo} \right) = N \left( 1 - \frac{S^*}{C_p} \right)$$
 (4.9)

By combining 4.6, 4.8 and 4.9 we see that the credit spread  $s_{CoCo}$  of a CoCo with maturity T at time t is driven by its major design elements, the trigger level  $S^*$  and the conversion price  $C_p$ :

$$s_{CoCo_t} = -\frac{\log(1 - F^*)}{(T - t)} \left( 1 - \frac{S^*}{C_p} \right) \tag{4.10}$$

Subsequently, a pricing formula for CoCos under the credit derivative approach can be derived. The present value  $V^{cd}$  at time t is given by:

$$V_t^{cd} = \sum_{i=1}^{T} c_i \exp\left[-(r + s_{CoCo_t})(t_i - t)\right] + N \exp\left[-(r + s_{CoCo_t})(T - t)\right]$$
(4.11)

In summary, the credit derivative approach provides us with a concise method to price CoCos. However, one has to bear in mind its largest shortcoming. Losses from cancelled coupons of triggered CoCos are not taken into account in the valuation. Hence, the credit derivative approach naturally overestimates the price of CoCos, but it equips investors with a simple rule of thumb formula.

### 4.1.3 Parameter Classification and Adjustment

	Description	Usage	Source
$\overline{T}$	CoCo maturity	Static input	Term sheet
N	CoCo nominal	Static input	Term sheet
c	CoCo coupon rate	Static input	Term sheet
$S_0$	Initial share price of the issuer	Dynamic input	Market data
$S^*$	Trigger share price	Static input	Term sheet
$C_p$	CoCo nominal conversion price	Static input	Term sheet
r	Risk-free interest rate	Static input	Market data
q	Dividend yield	Static input	Market data
$\sigma$	Implied volatility	Static input	Market data

Table 4.1: Parameter classification of the credit derivative approach (Wilkens and Bethke, 2014)

#### 4.1.4 Model Application

	Value	Comment
T	10yrs	Maturity
N	100	Nominal
c	6.00	Annual coupon rate
$S_0$	100	Initial share price of the bank
$S^*$	35	Trigger share price
$C_p$	65	Nominal conversion price
r	1.00%	Risk-free interest rate
q	2.00%	Dividend yield
$\sigma$	30.00%	Implied volatility

Table 4.2: Parameter specification of credit derivative approach application (Alvemar and Ericson, 2012)

## 4.2 Equity Derivative Approach

In order to assess the value of a CoCo, investors can use a method which depends on equity derivatives. (De Spiegeleer and Schoutens, 2011b; De Spiegeleer et al., 2014) The so-called equity derivative approach attempts to compensate for the main drawback of the credit derivative approach, since it takes into account that coupon payments might be cancelled if the trigger of a CoCo was touched.

Subsequently, the valuation can be divided into two steps: In the first step, the value of a CoCo is determined without coupon payments. Such a CoCo is called Zero-Coupon CoCo. In the second step, one has to incorporate the coupon payments in the pricing formula while keeping in mind that they might be knocked out. The closed-form solution is derived under a Black-Scholes setting.

#### 4.2.1 Step One - Zero-Coupon CoCo

To determine the present value of a Zero-Coupon CoCo  $V^{zcoco}$  at maturity T we can use equation 2.4. The underlying assumption of the equity derivative approach is that the triggering of a CoCo respectively of a Zero-Coupon CoCo is equivalent to the share price falling below the level  $S^*$ .

$$V_T^{zcoco} = \begin{cases} N & \text{if not triggered} \\ (1-\alpha)N + \frac{\alpha N}{C_p}S^* & \text{if triggered} \end{cases}$$

$$= N \mathbb{1}_{\{\tau > T\}} + \left[ (1-\alpha)N + \frac{\alpha N}{C_p}S^* \right] \mathbb{1}_{\{\tau \le T\}}$$

$$= N + \left[ \frac{\alpha N}{C_p}S^* - \alpha N \right] \mathbb{1}_{\{\tau \le T\}}$$

$$= N + \left[ C_rS^* - \alpha N \right] \mathbb{1}_{\{\tau \le T\}}$$

$$= N + C_r \left[ S^* - \frac{\alpha N}{C_r} \right] \mathbb{1}_{\{\tau \le T\}}$$

$$= N + C_r \left[ S^* - C_p \right] \mathbb{1}_{\{\tau < T\}}$$

$$= (4.12)$$

It may be inferred that the financial payoff of equation 4.12 consists of two components (Erismann, 2015): (1) the face value N of a zero bond and (2) a long position in  $C_r$  shares generating a payoff only if the CoCo materializes at time  $\tau$ . This component can be approximated with a knock-in forward. The intuition behind equation 4.12 is that if the share price falls below a certain level  $S^*$ , an investor will use the face value N to exercise the knock-in forward. That said, the investor is committed to buy the amount of  $C_r$  shares for the price of  $C_p$  at maturity T.

Before maturity the present value of a Zero-Coupon CoCo  $V^{zcoco}$  can be determined by adding up the present value of a zero bond  $V^{zb}$  and the present value of a knock-in

forward  $V_t^{kifwd}$ . Hereinafter, the components will be explained briefly.

$$V_t^{zcoco} = V_t^{zb} + V_t^{kifwd} (4.13)$$

with

$$V_t^{zb} = N \exp\left[-r(T-t)\right] \tag{4.14}$$

Moreover, the long position in shares at time t can be approximated with the respective closed-form solution of a knock-in forward. (Hull, 2006)

$$V_t^{kifwd} = C_r \left[ S_t \exp\left[-q\left(T - t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda} \Phi\left(y_1\right) - K \exp\left[-r\left(T - t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda - 2} \Phi\left(y_1 - \sigma\sqrt{T - t}\right) - K \exp\left[-r\left(T - t\right)\right] \Phi\left(-x_1 - \sigma\sqrt{T - t}\right) + S_t \exp\left[-q\left(T - t\right)\right] \Phi\left(-x_1\right) \right]$$

$$(4.15)$$

with

$$C_r = \frac{\alpha N}{C_p}$$

$$K = C_p$$

$$\lambda = \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}$$

$$x_1 = \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{T - t}} + \lambda\sigma\sqrt{T - t}$$

$$y_1 = \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{T - t}} + \lambda\sigma\sqrt{T - t}$$

It is important, however, to recognize that a subtle difference exists between the actual economic payoff of equation 4.12 and its replication with a knock-in forward, since the knock-in forward replicates an economic ownership of shares at maturity T. Though, the triggering of a CoCo forces investors to accept the conversion immediately. This could lead to an economic ownership of shares at trigger time  $\tau$  and, thus, prior to T. Therefore, one could argue that receiving a knock-in forward in the trigger

event disregards the dividends which a shareholder would receive in particular when a CoCo triggers early. De Spiegeleer and Schoutens (2011b) argue that dividends can be neglected because distressed banks are likely to behave with great restraint when it comes to dividend payments.

#### 4.2.2 Step Two - Adding Coupons

As mentioned earlier, the first step excludes coupon payments from the valuation. Yet, in this step we want to include them in order to replicate the exact payout profile of a CoCo. Therefore, we replace the zero bond in equation 4.13 with a straight bond with regular coupon payments c. Besides, a third component has to be added which takes into account the foregone coupon payments if the trigger is touched. This can be modeled with a short position in k binary down-and-in calls with maturity  $t_i$ . Those binary down-and-in calls are knocked in if the trigger  $S^*$  is met and hence, offset all future coupon payments.

The price of a straight bond can be determined with:

$$V_t^{sb} = \sum_{i=1}^{T} c_i \exp\left[-r(t_i - t)\right] + N \exp\left[-r(T - t)\right]$$
 (4.16)

To price the down-and-in calls one might use the formula of Rubinstein and Reiner (1991):

$$V_t^{bdic} = \alpha \sum_{i=1}^k c_i \exp\left(-rt_i\right) \left[\Phi\left(-x_{1i} + \sigma\sqrt{t_i}\right) + \left(\frac{S^*}{S_t}\right)^{2\lambda - 2} \Phi\left(y_{1i} - \sigma\sqrt{t_i}\right)\right]$$

$$(4.17)$$

with

$$x_{1i} = \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i}$$
$$y_{1i} = \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i}$$
$$\lambda = \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}$$

To sum up, the theoretical price of a CoCo  $V^{ed}$  at time t pursuant the equity derivative approach consists of three components: (1) a straight bond  $V^{sb}$ , (2) a knock-inforward  $V^{kifwd}$  and (3) a set of binary down-and-in calls  $V^{bdic}$ :

$$V_t^{ed} = V_t^{sb} + V_t^{kifwd} - V_t^{bdic} (4.18)$$

## 4.2.3 Parameter Classification and Adjustment

	Description	Usage	Source
T	CoCo maturity	Static input	Term sheet
N	CoCo nominal	Static input	Term sheet
c	CoCo coupon rate	Static input	Term sheet
$\alpha$	CoCo nominal conversion factor	Static input	Term sheet
$S_0$	Initial share price of the issuer	Dynamic input	Market data
$S^*$	Trigger share price	Static input	Term sheet
$C_p$	CoCo nominal conversion price	Static input	Term sheet
r	Risk-free interest rate	Static input	Market data
q	Dividend yield	Static input	Market data
$\sigma$	Implied volatility	Static input	Market data

Table 4.3: Parameter classification of the equity derivative approach (Wilkens and Bethke, 2014)

#### 4.2.4 Model Application

	Value	Comment
$\overline{T}$	10yrs	Maturity
N	100	Nominal
c	6.00	Annual coupon rate
$\alpha$	1	Nominal conversion factor
$S_0$	100	Initial share price of the bank
$S^*$	35	Trigger share price
$C_p$	65	Nominal conversion price
r	1.00%	Risk-free interest rate
q	2.00%	Dividend yield
$\sigma$	30.00%	Implied volatility

Table 4.4: Parameter specification of equity derivative approach application (Alvemar and Ericson, 2012)

## 4.3 Structural Approach

A third alternative to price CoCos is the structural approach of Pennacchi (2010). The idea has its roots in the seminal work of Merton (1974), which aims to explain a company's default based on the relationship of its assets and liabilities under a standard Black-Scholes setting. Pennacchi (2010)'s approach expands the idea by modeling the stochastic evolution of a bank's balance sheet respectively of its components. In the following, the assets' rate of return process will be explained. Thereafter, we will outline the assumptions of the model regarding the various liabilities a bank issues to refinance itself including deposits, equity and coupon bonds in the form of CoCos. Lastly, a pricing formula will be illustrated.

## 4.3.1 Structural Banking Model

#### Bank Assets and Asset-To-Deposit Ratio

Pennacchi (2010) assumes that a bank holds a portfolio of loans, equities and offbalance sheet positions as assets whose returns follow a jump-diffusion process. The change of this portfolio  $A_t$  is determined by the rate of return and the cash inrespectively outflows. In this context, the symbol \* is used to point out the change in value of the portfolio which can be quantified by the rate of return, excluding net cashflows. The aforementioned instantaneous rate of return is denoted as  $dA_t^*/A_t^*$  and follows a stochastic process as stated below under the risk-neutral probability measure  $\mathbb{Q}$ :

$$\frac{dA_t^*}{A_t^*} = (r_t - \lambda_t k_t) dt + \sigma dz + (Y_{q_{t^-}} - 1) dq_t$$
(4.19)

It should be noted that  $r_t$  stands for the risk-free interest rate as defined by the Cox et al. (1985) term-structure model which will be discussed shortly. dz is a Brownian motion, whereby  $\sigma$  denotes the volatility of returns of the aforementioned asset portfolio.  $q_t$  is a Poisson counting process which increases by one whenever a Poissondistributed event respectively a jump occurs. Hence, the variable  $dq_t$  is one whenever such a jump takes place and zero otherwise. The risk-neutral probability that a jump happens is equal to  $\lambda_t dt$  where  $\lambda_t$  stands for the intensity of the jump process. Variable  $Y_{q_t}$  is a i.i.d. random variable drawn from  $\ln(Y_{q_t}) \sim \Phi(\mu_y, \sigma_y^2)$  at time t where  $\mu_y$  stands for the mean jump size and  $\sigma_y$  denotes the standard deviation of jumps. In case the random variable  $Y_{q_{t-}}$  is greater than one, an upward shift in the bank's asset value can be observed. If the value is smaller than one a downward jump takes place. Given that the risk-neutral expected proportional jump  $k_t$ is defined as  $k_t = E_t^{\mathbb{Q}}[Y_{q_{t^-}} - 1]$ , one can determine  $k_t$  with the following formula:  $k_t = \exp(\mu_y + \frac{1}{2}\sigma_y^2) - 1$ . Thus, the risk-neutral expected change in  $A^*$  from the jump element  $(Y_{q_{t-}} - 1)dq_t$  equals  $\lambda_t k_t dt$  in dt. To sum up, the value development of a bank's asset portfolio  $A_t^*$  follows largely a continuous process. But disruptive jumps may occur as illustrated below in the graph 4.1.

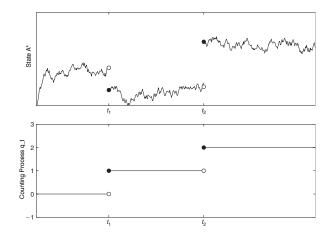


Figure 4.1: The first graph shows two jumps in the state variable  $A^*$  at discrete time points. Additionally, the corresponding Poisson counting process  $q_t$  is highlighted in the second graph. (Aït-Sahalia and Hansen, 2009)

The risk-neutral process of bank assets  $A_t$  including the net cashflows is equal to the assets' rate of return less interest payments  $r_t$  respectively premium payments  $h_t$  to deposit holders proportionally to their deposits  $D_t$ . Furthermore, one has to subtract the coupon payments  $c_t$  to CoCo investors proportionally to the face value B.

$$dA_t = [(r_t - \lambda k) A_t - (r_t + h_t) D_t - c_t B] dt + \sigma A_t dz + (Y_{q_{t-}} - 1) A_t dq \qquad (4.20)$$

By substituting variable  $x_t$  with  $A_t/D_t$  and anticipating the deposit growth process  $g(x_t - \hat{x})$  as pointed out by equation 4.31, the risk neutral process of the asset-to-deposit ratio equals:

$$\frac{dx_t}{x_t} = \frac{dA_t}{A_t} - \frac{dD_t}{D_t} 
= \left[ (r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g(x_t - \hat{x}) \right] dt + \sigma dz + (Y_{q_{t-}} - 1) dq_t$$
(4.21)

with

$$b_t = \frac{B}{D_t} \tag{4.22}$$

Lastly, an application of Itô's lemma for jump-diffusion processes leads to the following formula for the asset-to-deposit ratio process:

$$d\ln(x_t) = \left[ (r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g(x_t - \hat{x}) - \frac{1}{2}\sigma^2 \right] dt$$

$$+ \sigma dz + \ln Y_{q,-} dq_t$$
(4.23)

#### Default-Free Term Structure

Pennacchi (2010) applies the term-structure specifications of Cox et al. (1985) to model the risk-neutral process of the instantaneous risk-free interest rate  $dr_t$  which is defined as follows:

$$dr_t = \kappa \left(\bar{r} - r_t\right) dt + \sigma_r \sqrt{r_t} d\zeta \tag{4.24}$$

Note that  $\kappa$  is the speed of convergence,  $\bar{r}$  is the long-run equilibrium interest rate,  $r_t$  is the continuous short-term interest rate,  $\sigma_r$  is the instantaneous volatility and  $d\zeta$  is a Brownian motion.

A zero bond can be priced using the Cox et al. (1985) specifications under the noarbitrage assumption. This implies that the price of a risk-free zero bond at time t that pays the amount of  $\in 1$  in  $\tau = T - t$  is given by:

$$P(r_t, \tau) = A(\tau) \exp[-B(\tau) r_t]$$
(4.25)

with

$$A(\tau) = \left\{ \frac{2\theta \exp\left[ (\theta + \kappa) \frac{\tau}{2} \right]}{(\theta + \kappa) \left[ \exp\left( \theta \tau \right) - 1 \right] + 2\theta} \right\}^{2\kappa \bar{r}/\sigma_r^2}$$

$$B(\tau) = \frac{2 \left[ \exp(\theta \tau) - 1 \right]}{(\theta + \kappa) \left[ \exp(\theta \tau) - 1 \right] + 2\theta}$$

$$\theta = \sqrt{\kappa^2 + 2\sigma_r^2}$$

The cost of replication of a risk-free coupon bond that pays a continuous coupon of  $c_r dt$  is equal to a set of zero bonds which can be priced with equation 4.25. Therefore, the fair coupon rate  $c_r$  of such a coupon bond at time t, which is issued at par, equals:

$$c_{r} = \frac{1 - A(\tau) \exp\left[-B(\tau) r_{t}\right]}{\int_{0}^{\tau} A(s) \exp\left[-B(s) r_{t}\right] ds}$$

$$\approx \frac{1 - A(\tau) \exp\left[-B(\tau) r_{t}\right]}{\sum_{i=1}^{i=n} A(\Delta t \times i) \exp\left[-B(\Delta t \times i) r_{t}\right] \Delta t}$$
(4.26)

with

$$n = \frac{\tau}{\Delta t} \tag{4.27}$$

#### Deposits and Insurance Premium

Bank deposits are not riskless because depositors may suffer losses if a bank's asset value  $A_t$  is worth less than the deposits  $D_t$ . That said, one can assume that a bank is closed by the deposit insurer when the asset-to-deposit ratio  $x_t$  is less or equal to one. A bank might become distressed due to continuous downward movements in its asset value. Then, the bank will be shut down with  $A_{t_b} = D_t$  and subsequently, depositors will not face any loss. However, depositors may experience severe losses when a downward jump in asset value happens at a discrete point in time,  $\hat{t}$ . It may be that the downward jump in asset value exceeds the bank's capital. If such a jump occurs the instantaneous proportional loss to deposits will equal  $(D_t - Y_{q_t} - A_{\hat{t}}) / D_t$ .

The fair deposit insurance premium  $h_t$  for deposit holders can be derived with equation 4.28. The equation illustrates that  $h_t$  is closely related to the asset-to-deposit ratio  $x_t$ :

$$h_t = \lambda \left[ \Phi(-d_1) - x_{t^-} \exp\left(\mu_y + \frac{1}{2}\sigma_y^2\right) \Phi(-d_2) \right]$$
 (4.28)

with

$$d_1 = \frac{\ln(x_{t^-}) + \mu_y}{\sigma_y} \tag{4.29}$$

$$d_2 = d_1 + \sigma_y \tag{4.30}$$

The model assumes that a bank pays continuously a total interest and deposit premium of  $(r_t + h_t) D_t dt$  to each depositor. Hence, one can recognize that the deposits of the bank change only because of comparatively higher deposit inflows than outflows. Empirical research of Adrian and Shin (2010) suggests that banks have a target capital ratio and that deposit growth is positively related to the bank's current asset-to deposit ratio:

$$\frac{dD_t}{D_t} = g\left(x_t - \hat{x}\right)dt\tag{4.31}$$

 $\hat{x} > 1$  is a bank's target asset-to-deposit ratio with g being a positive constant. Whenever the actual asset-to-deposit ratio is higher than its target,  $x_t > \hat{x}$ , a bank will shrink its balance sheet. Thus, the deposit growth rate  $g(x_t - \hat{x})$  in the time interval dt, leads to a mean-reverting tendency for the bank's asset-to-deposit ratio  $x_t$ .

### **Equity and Conversion Threshold**

As stated originally, the conversion of a CoCo at time  $t_c$  occurs when the asset-to-deposit ratio  $x_{t_c}$  meets the trigger level  $\bar{x}_{t_c}$ . The conversion threshold can also be expressed relative to the original equity-to-deposits ratio  $\bar{e}$ . This is favourable because the equity value is directly observable in the market whereas the asset value is not. The relationship between the equity threshold  $\bar{e}$  and the asset-to-deposit threshold  $\bar{x}_{t_c}$  can be summarized as follows:

$$\bar{e} = \frac{E_{t_c}}{D_{t_c}} = \frac{A_{t_c} - D_{t_c} - pB}{D_{t_c}} = \bar{x}_{t_c} - 1 - pb_{t_c}$$
(4.32)

Hence, it is possible to specify exactly the conversion trigger of a CoCo bond. This will be important for the valuation part.

### CoCos

The valuation of a CoCo can be accomplished with a Monte Carlo simulation of both the asset and the deposit process. Along the asset-to-deposit ratio process, the CoCo pays coupons and the nominal at maturity unless the CoCo has not been triggered. If the trigger event occurs the conversion amount is paid out. (Wilkens and Bethke, 2014) The price of the CoCo  $V^{st}$  is equal to the risk-neutral expectation of the aforementioned cashflows as derived by Pennacchi (2010):

$$V_0^{st} = E_0^{\mathbb{Q}} \left[ \int_0^T \exp\left(-\int_0^t r_s ds\right) v\left(t\right) dt \right]$$
(4.33)

Please note that v(t) stands for a CoCo's coupon payment at date t which equals  $c_t B$  as long as the CoCo has not been triggered. If the CoCo does not convert until maturity T, a final payout of B will be performed. However, if the CoCo triggers early at time  $t_c$ , there is the one-time cashflow of pB. Parameter p determines the maximum conversion amount of new equity per par value of contingent capital. Thereafter, v(t) is zero.

### 4.3.2 Parameter Classification and Adjustment

	Description	Usage	Source
T	CoCo maturity Static input Ter		Term sheet
B	CoCo nominal	Static input	Term sheet
c	c CoCo coupon rate Static input Term		Term sheet
p	CoCo nominal conversion factor Static input Term shee		Term sheet
$x_0$	Initial asset-to-deposit ratio Dynamic input Balance shee		Balance sheet
$\hat{x}$	Target asset-to-deposit ratio Static input Assumption		Assumption
g	Mean-reversion speed Static input Assump		Assumption
$\sigma$	σ Annual asset return volatility Static input Assur		Assumption
$\lambda$	$\lambda$ Jump intensity in asset return process Static input A		Assumption
$\mu_y$	Mean jump size in asset return process	Static input	Assumption
$\sigma_y$	Jump volatility in asset return process	Static input	Assumption
$r_0$	Dynamic input Market		Market data
$\bar{r}$	Long-term risk-free interest rate	Static input	Market data
$\sigma_r$	$r_r$ Interest rate volatility Static input Ma		Market data
$\kappa$	$\kappa$ Speed of convergence Static input		Assumption
$\rho$	Correlation between Brownian motion for as-	Static input	Market data
$\bar{e}$	set returns and interest rate process  Conversion threshold of the market value of original shareholders' equity to deposit value	Static input	Balance sheet
$b_0$	Ratio of the contingent capital's nominal to Dynamic input Balance sh the initial value of deposits		Balance sheet

Table 4.5: Parameter classification of the structural approach (Wilkens and Bethke, 2014)

### 4.3.3 Model Application

	Value	Comment
T	10yrs	Maturity
B	100.00%	Nominal
c	6.00%	Annual coupon rate
p	1	Nominal conversion factor
$x_0$	1.15	Initial asset-to-deposit ratio
$\hat{x}$	1.1	Target asset-to-deposit ratio
g	0.5	Mean-reversion speed
$\sigma$	2.00%	Annual asset return volatility
$\lambda$	1	Jump intensity in asset return process
$\mu_y$	-1.00%	Mean jump size in asset return process
$\sigma_y$	2.00%	Jump volatility in asset return process
$r_0$	1.00%	Risk-free interest rate
$\bar{r}$	6.90%	Long-term risk-free interest rate
$\sigma_r$	7.00%	Interest rate volatility
$\kappa$	11.40%	Speed of convergence
$\rho$	-20.00%	Correlation between Brownian motion for asset returns
		and interest rate process
$\bar{e}$	2.00%	Conversion threshold of the market value of original
		shareholders' equity to deposit value
$b_0$	4.00%	Ratio of contingent capital's nominal to the initial de- posit value
		poste varue

Table 4.6: Parameter specification of structural approach application (Alvemar and Ericson, 2012; Pennacchi, 2010)

# Chapter 5

# Dynamics and Sensitivity Analysis

### 5.1 Credit Derivative Approach

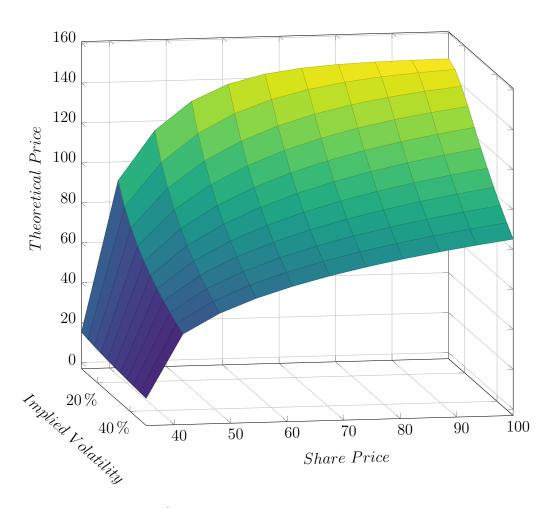


Figure 5.1: CoCo price  $V^{cd}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011b) as function of share price S and implied volatility  $\sigma$ 

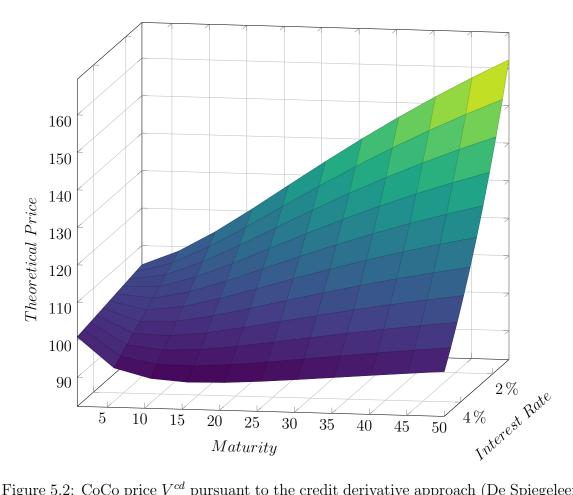


Figure 5.2: CoCo price  $V^{cd}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011b) as function of maturity T and risk-free interest rate r

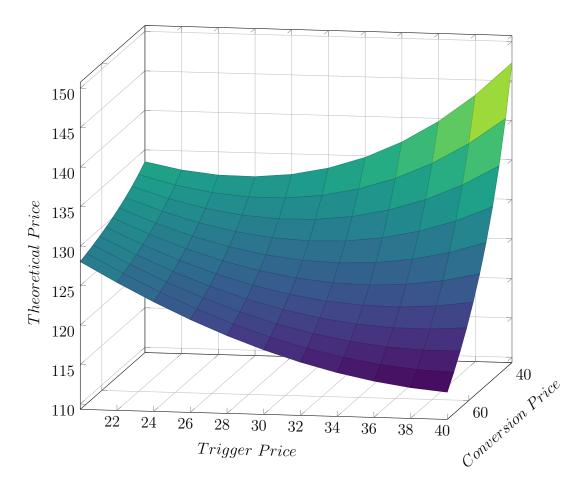


Figure 5.3: CoCo price  $V^{cd}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011b) as function of trigger price  $S^*$  and conversion price  $C_p$ 

### 5.2 Equity Derivative Approach

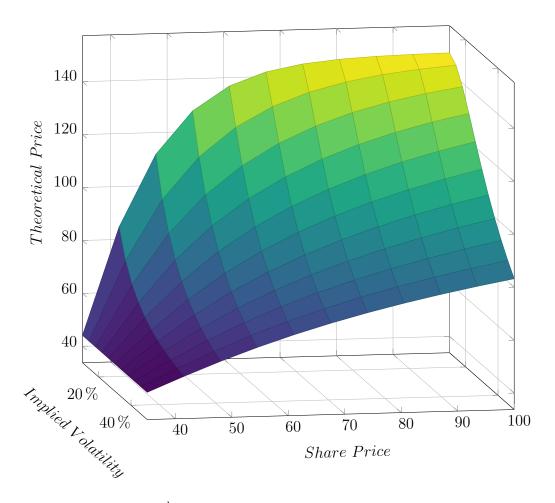


Figure 5.4: CoCo price  $V^{ed}$  pursuant to the equity derivative approach (De Spiegeleer and Schoutens, 2011b) as function of share price S and volatility  $\sigma$ 

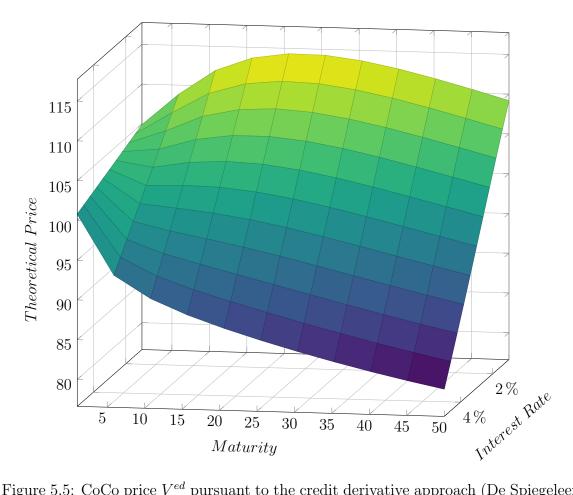


Figure 5.5: CoCo price  $V^{ed}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011b) as function of maturity T and risk-free interest rate r

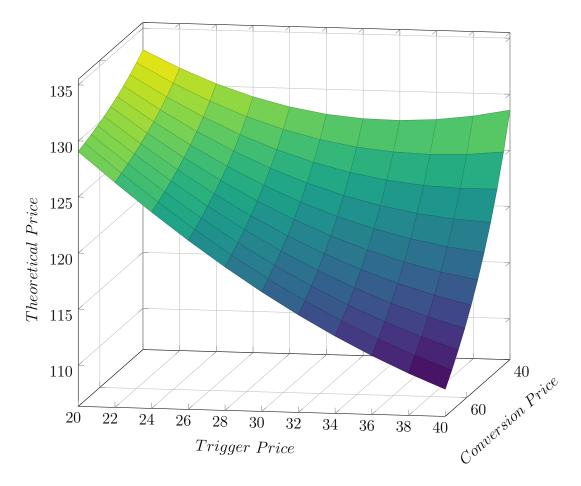


Figure 5.6: CoCo price  $V^{ed}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011b) as function of trigger price  $S^*$  and conversion price  $C_p$ 

### 5.3 Structural Approach

1

<sup>&</sup>lt;sup>1</sup>The Monte-Carlo simulation runs in the Amazon Elastic Compute Cloud (EC2) as the service provides a re-sizable compute capacity which is key to quickly scale the computing requirements. If one wants to replicate the simulations it is recommended to follow the instructions of Shekel (2015) to set up a Rstudio server on Amazon EC2.

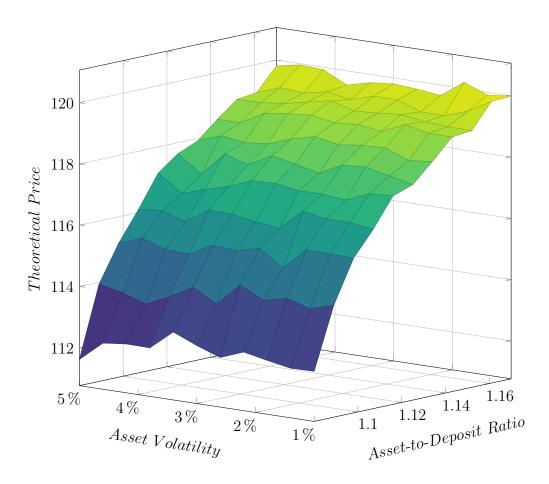


Figure 5.7: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of initial asset-to-deposit ratio  $x_0$  and asset volatility  $\sigma$ 

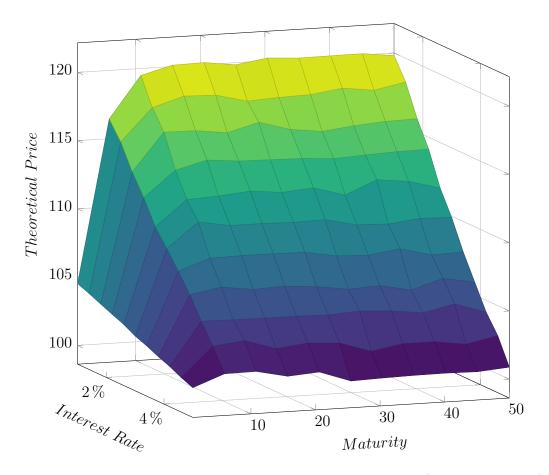


Figure 5.8: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of maturity T and interest rate r

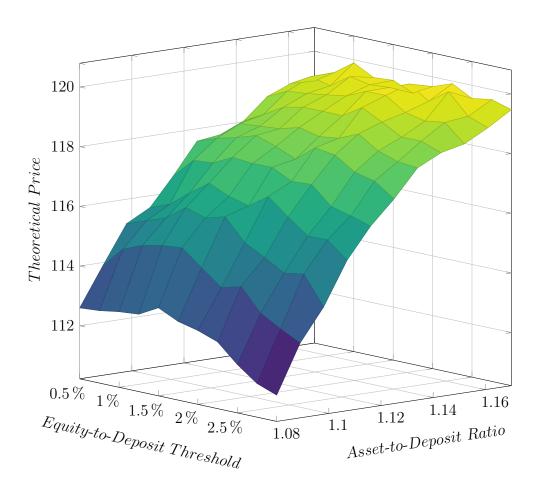


Figure 5.9: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of initial asset-to-deposit ratio  $x_0$  and equity-to-deposit threshold  $\bar{e}$ 

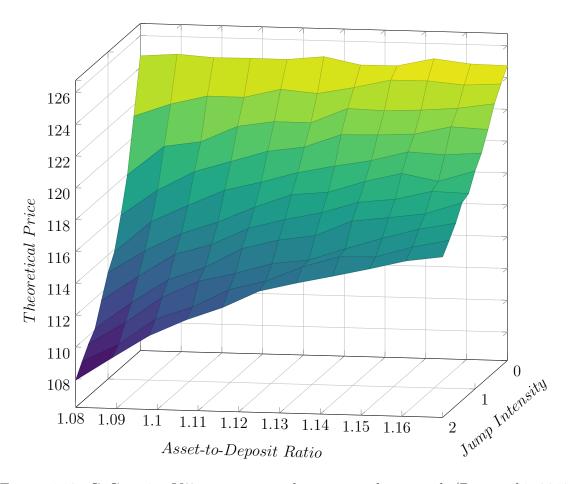


Figure 5.10: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of initial asset-to-deposit ratio  $x_0$  and jump intensity  $\lambda$ 

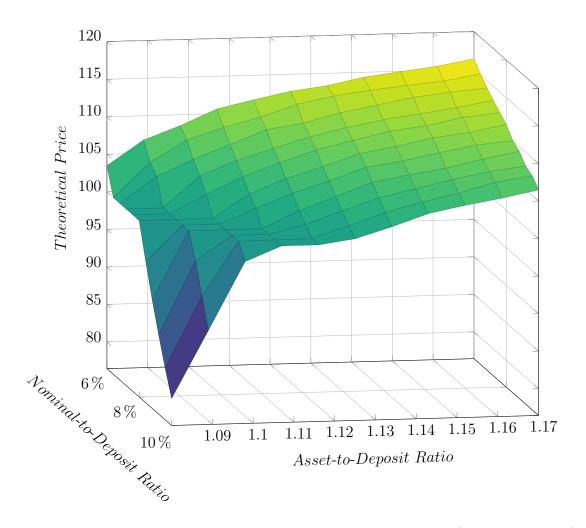


Figure 5.11: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of initial asset-to-deposit ratio  $x_0$  and initial ratio of contingent capital's nominal to the initial value of deposits

# Chapter 6

# Empirical Analysis and Model Comparison

- 6.1 Data Description
- 6.1.1 Deutsche Bank
- 6.2 Model Parametrization
- 6.3 Model Comparison
- 6.3.1 Qualitative Analysis
- 6.3.2 Quantitative Analysis

# Chapter 7

# Conclusion

# Appendix A<br/>Sample Title

### Appendix B

# Code - Credit Derivative Approach

The following source code is an implementation of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
# Price of Contingent Convertible Bond
2 price_coco_cd <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma){
    spread_coco <- calc_spread_coco(t, T, S_t, S_star, C_p, r, q, sigma)
4
    V_t_{-coco} \leftarrow N * exp(-(r + spread_{-coco}) * (T - t))
     for (t in 1:T)
       V_t_{-coco} \leftarrow V_t_{-coco} + c_i * exp(-(r + spread_{-coco}) * t)
9
10
    return (V_t_coco)
11 }
12
13 # Calculation of Trigger Probability
14 calc_p_star <- function(t, T, S_t, S_star, r, q, sigma){
    p_star \leftarrow pnorm((log(S_star / S_t) - calc_mu(r, q, sigma) * (T - t)))
        (\operatorname{sigma} * \operatorname{sqrt}(T - t)) + (S_{\operatorname{star}} / S_{\operatorname{t}})^{2} * \operatorname{calc_mu}(r, q, \operatorname{sigma}) /
       sigma^2) * pnorm((log(S_star / S_t) + calc_mu(r, q, sigma) * (T - t
      )) / (sigma * sqrt(T - t)))
    return (p_star)
16
17 }
19 # Calculation of Drift of Underlying
calc_mu \leftarrow function(r, q, sigma)
    mu <\!\!- r - q - sigma^2 / 2
    return (mu)
22
23 }
25 # Spread of CoCo Bond
26 calc_spread_coco <- function(t, T, S_t, S_star, C_p, r, q, sigma){
    spread\_coco <- log(1 - calc\_p\_star(t, T, S\_t, S\_star, r, q, sigma))
      / (T - t) * (1 - S_star / C_p)
     return (spread_coco)
31 # Pricing Example
```

price\_coco\_cd(t <- 0, T <- 5, S\_t <- 40, S\_star <- 20, C\_p <- 25, c\_i <- 7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3)

# Appendix C

# Code - Equity Derivative Approach

The following source code is an implementation of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
# Price of Contingent Convertible Bond
2 price_coco_ed <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma,
      alpha){
    V_t_{ed} \leftarrow price_cb(t, T, c_i, r, N) - price_dibi(t, T, S_t, S_star, c_i)
      i, r, q, sigma, alpha) + price_difwd(t, T, S_t, S_star, C_p, r, N, q
      , sigma, alpha)
4
    return (V_t_ed)
5
6 }
8 # Price of Corporate Bond
9 price_cb \leftarrow function(t, T, c_i, r, N){
    V_t_c = V_t - cb < N * exp(-r * (T - t))
11
    for (t in 1:T) {
12
    V_{-}t_{-}cb \leftarrow V_{-}t_{-}cb + c_{-}i * exp(-r * t)
13
15
    return (V_t_cb)
16
17 }
19 # Price of Binary Option
price_dibi <- function(t, T, S_t, S_star, c_i, r, q, sigma, alpha){
    V_t_dibi < 0
23
    i <- t
    k <- T
24
25
    for (i in 1:k) {
26
    V_t_dibi \leftarrow V_t_dibi + c_i * exp(-r * i) * (pnorm(-calc_x_1_i(S_t, S_t)) + (pnorm(-calc_x_1_i(S_t, S_t)))
      _star , sigma , r , q , i ) + sigma * sqrt(i)) + (S_star / S_t)^(2 * calc
      _{lambda(r, q, sigma) - 2)} * pnorm ( calc_y_1_i(S_t, S_star, sigma, r)
      , q, i) - sigma * sqrt(i)))
28
```

```
V_t_dibi <- alpha * V_t_dibi
30
31
               return (V_t_dibi)
32
33
34
35 # Price of Down-And-In Forward
price_difwd <- function(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha){
               V_{-}t_{-}difwd <- \ calc_{-}conversion_{-}rate\left(C_{-}p\,,\ N,\ alpha\right)\ *\ (S_{-}t\ *\ exp\left(-\ q\ *\ (T_{-}t_{-})\right)
                        - t)) * (S_star / S_t) ^ (2 * calc_lambda(r, q, sigma)) * pnorm(
                     calc_y_1(t, T, S_t, S_{star}, r, q, sigma)) - C_p * exp(-r * (T - t))
                        * (S_star / S_t)^2 = (S_star /
                    1(t, T, S_t, S_{star}, r, q, sigma) - sigma * sqrt(T - t)) - C_p * exp
                    (-r * (T - t)) * pnorm(-calc_x_1(t, T, S_t, S_star, r, q, sigma) +
                        sigma * sqrt(T - t)) + S_t * exp(-q * (T - t)) * pnorm(- calc_x_1(T - t)) * pnorm(-calc_x_1(T 
                    t, T, S<sub>-</sub>t, S<sub>-</sub>star, r, q, sigma)))
38
                return (V_t_difwd)
39
40 }
41
42 # Calculation of Conversion Rate
43 calc_conversion_rate <- function(C_p, N, alpha){
               C_r \leftarrow alpha * N / C_p
                return (C<sub>-</sub>r)
46
47 }
48
49 # Calculation of additional Parameters
50 \operatorname{calc}_{-x_{-}1_{-}i} \leftarrow \operatorname{function}(S_{-}t, S_{-}\operatorname{star}, \operatorname{sigma}, r, q, t_{-}i)
               sigma) * sigma * sqrt(t_i)
                return(x_1_i)
53
54
       calc_y_1_i \leftarrow function(S_t, S_star, sigma, r, q, t_i)
56
               57
                    sigma) * sigma * sqrt(t_i)
59
                return(y_1_i)
60 }
61
62 calc_lambda <- function(r, q, sigma){
               lambda \leftarrow (r - q + sigma^2 / 2) / sigma^2
63
64
                return (lambda)
65
66
67
       calc_x_1 \leftarrow function(t, T, S_t, S_star, r, q, sigma)
68
69
               x_1 \leftarrow log(S_t / S_star) / (sigma * sqrt(T - t)) + calc_lambda(r, q, t)
                    sigma) * sigma * sqrt(T - t)
70
               return(x_1)
71
72 }
73
```

```
74 calc_y_1 <- function(t, T, S_t, S_star, r, q, sigma){
75    y_1 <- log(S_star / S_t) / (sigma * sqrt(T - t)) + calc_lambda(r, q, sigma) * sigma * sqrt(T - t)

76    return(y_1)

78 }

79    # Pricing Example
81 price_coco_ed(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <- 7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3, alpha <- 1)</pre>
```

### Appendix D

### Code - Structural Approach

The following source code is an implementation of the Structural Approach (Pennacchi, 2010) with support from Quantum (2011) in translating the source code of Pennacchi (2010) from GAUSS to R.

```
# Price of Contingent Convertible Bond
price_coco_sa <- function(T , nsimulations , rho , kappa , r_bar, r0,
      sigma\_r\;,\;\; mu\_Y,\;\; sigma\_Y,\;\; lambda\;,\;\; g\;,\;\; x\_hat\;,\;\; b0\;,\;\; p\;,\;\; e\_bar\;,\;\; sigma\_x\;,\;\; x0\;,
       B, coupon){
3
    ndays \leftarrow T * 250
4
    dt <- T / ndays
5
    # Get Brownian motions
    result <- sim_corrProcess(T, nsimulations, rho, ndays, dt)
    dz_1 \leftarrow result dz_1
    dz_2corr <- result$dz_2corr
11
    # Simulate Cox et al. (1985) term-structure process
12
    r <- sim_interestrate(kappa, r_bar, r0, sigma_r, dz_2corr, ndays,
13
      nsimulations, dt)
14
    # Simulate price of contingent convertible bond with a Monte-Carlo
15
      simulation
    V_{-t-sa} \leftarrow get_{-price}(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y,
16
      sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_x, x0, B, coupon) *
     return (V_t_sa)
18
19
20
21 # Create correlated Brownian motions for asset and interest rate process
22 \text{ sim\_corrProcess} \leftarrow \text{function}(T, \text{ nsimulations}, \text{ rho}, \text{ ndays}, \text{ dt}) 
23
    # Compute the Choleski factorization of a real symmetric positive-
24
      definite square matrix.
    chol_RHO \leftarrow t(chol(matrix(c(1, rho, rho, 1), nrow = 2)))
```

```
# Random generation for the normal distribution with mean equal to 0
      and standard deviation equal to 1
    dz_1 <- matrix(1, ndays, nsimulations)
    dz_2 <- matrix(1, ndays, nsimulations)
    for (j in 1: nsimulations)
30
31
      dz_1[, j] < rnorm(ndays) * sqrt(dt)
      dz_2[, j] \leftarrow rnorm(ndays) * sqrt(dt)
33
34
35
    # Create correlated Brownian motions using Cholesky-decomposition for
36
      the Cox et al. (1985) term-structure process
    dz_2corr <- matrix(1, ndays, nsimulations)
37
    for (j in 1: nsimulations)
38
      for (i in 1:ndays)
40
41
        dz_2 corr[i, j] \leftarrow dz_1[i, j] * chol_RHO[2, 1] + dz_2[i, j] * chol_RHO[2, 1]
42
     RHO[2, 2]
43
44
45
    return(list("dz_1" = dz_1, "dz_2corr" = dz_2corr))
46
47 }
48
49 # Simulate Cox et al. (1985) term-structure process
50 sim_interestrate <- function(kappa, r_bar, r0, sigma_r, dz_2corr, ndays,
       nsimulations, dt) {
    r <- matrix(r0, ndays + 1, nsimulations)
51
52
    for (j in 1: nsimulations)
53
54
      for (i in 1:ndays)
55
56
        r[i + 1, j] \leftarrow r[i, j] + kappa * (r_bar - r[i, j]) * dt + sigma_r
57
      * sqrt(r[i, j]) * dz_2corr[i, j]
59
60
    return(r)
61
62 }
64 get_price <- function(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y,
     sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_x, x0, B, coupon){
    # Define parametres
66
    phi <- matrix(rbinom( ndays %*% nsimulations, 1, dt * lambda), ndays,
67
      nsimulations)
68
69
    ln_Y <- matrix (rnorm (ndays %*% nsimulations, mu_Y, sigma_Y), ndays,
      nsimulations)
70
    # Ratio of contingent capital's nominal to the value of deposits
    b <- matrix (b0, ndays + 1, nsimulations)
```

```
73
      h <- matrix (1, ndays, nsimulations)
74
75
      # Paramter for jump diffusion process
76
      k < -\exp(mu_Y + 0.5 * sigma_Y^2) - 1
78
      # Target asset-to-deposit ratio
      x_bar0 < -1 + e_bar + p * b0
80
      x_bar < matrix(x_bar0, ndays + 1, nsimulations)
81
82
      # Asset-to-deposit ratio
83
      x \leftarrow matrix(x0, ndays + 1, nsimulations)
84
      \ln x0 \leftarrow \text{matrix}(\log(x0), \text{ndays} + 1, \text{nsimulations})
85
      ln_x <- ln_x0
86
      trigger_dummy <- matrix(1, ndays + 1, nsimulations)
88
89
      # Simulate asset-to-deposit ratio and trigger events
90
      for (j in 1: nsimulations)
91
92
         for (i in 1:ndays)
93
94
           d_1 < (\ln x[i, j] + mu_Y) / sigma_Y
           d_2 \leftarrow d_1 + sigma_Y
96
97
           h[i, j] \leftarrow lambda * (pnorm(-d_1) - exp(ln_x[i, j]) * exp(mu_Y +
98
        0.5 * sigma_Y^2 * pnorm(-d_2)
99
           100
101
           ln_{\,-}x\,[\,i \ + \ 1\,,\ j\,] \ < -\ ln_{\,-}x\,[\,i \ ,\ j\,] \ + \ (\ (\,r\,[\,i\,\,,\ j\,] \ - \ lambda \ * \ k\,) \ - \ (\,r\,[\,i\,\,,\ j\,]
102
        \begin{array}{l} +\; h\,[\,i\;,\;\;j\,]\; +\; coupon\; *\; b\,[\,i\;,\;\;j\,])\; /\; exp\,(\,\ln_{\,-}\!x\,[\,i\;,\;\;j\,])\; -\; g\; *\; (\,exp\,(\,\ln_{\,-}\!x\,[\,i\;,\;\;j\,])\; -\; x_-hat\,)\; -\; 0.5\; *\; sigma_-x\,\,^2)\; *\; dt\; +\; sigma_-x\; *\; sqrt\,(\,dt\,)\; *\; dz_-1\,[\,i\;,\;\;j\,] \\ \end{array}
       j + \ln_{-}Y[i,j] * phi[i,j]
103
           x[i + 1, j] \leftarrow \exp(\ln_{-}x[i + 1, j])
104
           x_bar[i + 1, j] < 1 + e_bar + p * b[i + 1, j]
107
           if(x[i + 1, j] >= x_bar[i + 1, j] \&\& trigger_dummy[i, j] > 0.5)
108
109
              trigger_dummy[i + 1, j] <- 1
110
           } else
111
112
              trigger_dummy[i + 1, j] \leftarrow 0
114
        }
115
116
117
118
      cashflows <- matrix(c(rep(coupon * dt, ndays - 1), B), ndays,
        nsimulations) * trigger_dummy[1:ndays, ]
119
      # Determine cashflows for each simulation
      for (j in 1: nsimulations) {
```

```
for (i in 2:ndays) {
122
           if(cashflows[i, j] = 0 \&\& p * b[sum(trigger_dummy[, j]) + 1, j]
123
       <= x[sum(trigger_dummy[ , j]) + 1, j] - 1){
             cashflows[i, j] \leftarrow p * B
124
             break
125
           else if (\cosh flows[i, j] = 0 \&\& 0 < x[sum(trigger_dummy[, j]) +
       1\,,\;\;j\;]\;-\;1\;\;\&\&\;\;x\left[\,\mathrm{sum}\,(\;\mathrm{trig}\,\mathrm{g}\,\mathrm{e}\,\mathrm{r}\,\,\text{-dummy}\left[\  \  ,\;\;j\;\right]\,)\;\,+\;1\,,\;\;j\;\right]\;-\;1\;<\;p\;*\;\;b\left[\,\mathrm{sum}\,(\,
       trigger_dummy[ , j]) + 1, j]) \{
             cashflows[i, j] \leftarrow (x[sum(trigger\_dummy[, j]) + 1, j] - 1) * B
128
       / b[sum(trigger\_dummy[ , j]) + 1, j]
             break
129
130
           else {
             cashflows [i, j] <- cashflows [i, j]
133
134
135
      list\_discounted\_cashflows \leftarrow rep(0, nsimulations)
136
137
     # Discount cashflows for each simulation
138
     for (j in 1: nsimulations)
139
        disc_cashflows <- 0
141
        int_r < 0
142
143
        for (i in 1:ndays)
144
145
          int_r \leftarrow int_r + r[i, j] * dt
146
           disc_cashflows <- disc_cashflows + exp(- int_r) * cashflows[i, j]
        list_discounted_cashflows[j] <- disc_cashflows
149
     }
150
151
     # Calculate arithmetic average over all simulations as present value
152
       of contingent convertibles bond
     V_t_sa <- mean(list_discounted_cashflows)
153
154
155
     return (V_t_sa)
156
157
158 # Pricing Example
price_coco_sa(T <- 10, nsimulations <- 10000, rho <- - 0.2, kappa <-
       0.114, r_bar <- 0.069, r0 <- 0.01, sigma_r <- 0.07, mu_Y <- -0.01,
       sigma_Y \leftarrow 0.02, lambda \leftarrow 1, g \leftarrow 0.5, x_hat \leftarrow 1.1, b0 \leftarrow 0.04, p
       <-1, e_bar <-0.02, sigma_x <-0.02, x0 <-1.15, B <-1, coupon <-
       0.06)
```

# Appendix E

# Code - Sensitivity Analysis

### E.1 Credit Derivative Approach

The following source code is an implementation of the sensitivity analysis of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
source('CreditDerivativeApproach.R')
3 # CoCo price V^cd as function of share price S and volatility sigma
4 createData_CD_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
    data \leftarrow matrix(1, 121, 3)
    counter <- 1
    for (S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))
      for (sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_
     \max - \operatorname{sigma} - \min (10)
10
        data [counter, 1] <- S_increment
11
        12
     increment, S_star <- 35, C_p <- 65, c_i <- 6, r <- 0.01, N <- 100, q
      <- 0.02, sigma <- sigma_increment)
        data[counter, 3] <- sigma_increment
        counter <- counter + 1
15
16
    write.table(data, file = "createData_CD_S_sigma.txt", row.names =
17
     FALSE, quote=FALSE)
18 }
19
20 # CoCo price V^cd as function of maturity T and risk-free interest rate
  createData_CD_T_r <- function(T_min, T_max, r_min, r_max){
21
    data \leftarrow matrix(1, 121, 3)
22
    counter <- 1
    for(T_{increment} in seq(from=T_{min}, to=T_{max}, by=((T_{max}-T_{min})/10)))
24
25
      for (r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
26
```

```
data [counter, 1] <- T_increment
         data[counter, 2] <- price_coco_cd(t <- 0, T <- T_increment, S_t <-
29
       100, S_{star} < 35, C_{p} < 65, C_{i} < 6, r < r_{increment}, N < 100,
       q < -0.02, sigma < -0.3)
         data [counter, 3] <- r_increment
         counter \leftarrow counter + 1
33
    write.table(data, file = "createData_CD_T_r.txt", row.names = FALSE,
34
      quote=FALSE)
35
36
37 # CoCo price V<sup>cd</sup> as function of trigger price S<sup>*</sup> and conversion price
  createData_CD_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_
     \max) {
    data \leftarrow matrix(1, 121, 3)
39
    counter <- 1
40
    for (S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S_
41
      star_max-S_star_min)/10))
42
      for (C_p_increment in seq (from=C_p_min, to=C_p_max, by=((C_p_max-C_p_
43
      \min(10)
44
         data [counter, 1] <- S_star_increment
45
         data [counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- 100, S_
46
      star \leftarrow S_star_increment, C_p \leftarrow C_p_increment, c_i \leftarrow 6, r \leftarrow 0.01,
      N < -100, q < -0.02, sigma < -0.3
         data [counter, 3] <- C_p_increment
         counter <- counter + 1
50
    write.table(data, file = "createData_CD_Sstar_Cp.txt", row.names =
51
     FALSE, quote=FALSE)
52
54 createData_CD_S_sigma(35.01, 100, 0.1, 0.5)
_{55} createData_CD_T_r(1, 50, 0.01, 0.05)
createData_CD_Sstar_Cp(20, 40, 40, 70)
```

### E.2 Equity Derivative Approach

The following source code is an implementation of the sensitivity analysis of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
source('EquityDerivativeApproach.R')

# CoCo price V^ed as function of share price S and volatility sigma
createData_ED_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
   data <- matrix(1, 121, 3)
   counter <- 1
   for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))</pre>
```

```
8
       for (sigma_increment in seq (from=sigma_min, to=sigma_max, by=((sigma_
      \max - \operatorname{sigma}_{-\min} (10)
         data [counter, 1] <- S_increment
11
         data[counter, 2] \leftarrow price\_coco\_ed(t \leftarrow 0, T \leftarrow 10, S_t \leftarrow S_t)
12
      increment, S_star <- 35, C_p <- 65, c_i <- 6, r <- 0.01, N <- 100, q
       <-0.02, sigma <- sigma_increment, alpha <-1)
         data[counter, 3] <- sigma_increment
13
         counter \leftarrow counter + 1
14
15
16
     write.table(data, file = "createData_ED_S_sigma.txt", row.names =
17
      FALSE, quote=FALSE)
18
19
20 # CoCo price V^ed as function of maturity T and risk-free interest rate
  createData_ED_T_r <- function(T_min, T_max, r_min, r_max){
     data \leftarrow matrix(1, 121, 3)
     counter <- 1
23
     for (T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
24
       for (r_increment in seq (from=r_min, to=r_max, by=((r_max-r_min)/10)))
26
27
         data [counter, 1] <- T_increment
28
         data [counter, 2] <- price_coco_ed(t <- 0, T <- T_increment, S_t <-
29
       100, S_star < 35, C_p < 65, c_i < 6, r < r_increment, N < 100,
       q \leftarrow 0.02, sigma \leftarrow 0.3, alpha \leftarrow 1
         data [counter, 3] <- r_increment
         counter <- counter + 1
31
32
    }
33
     write.table(data, file = "createData_ED_T_r.txt", row.names = FALSE,
      quote=FALSE)
35
36
37 # CoCo price V^ed as function of trigger price S^* and conversion price
38 createData_ED_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_
      \max) {
     data \leftarrow matrix(1, 121, 3)
39
     counter <- 1
40
     for (S_star_increment in seq (from=S_star_min, to=S_star_max, by=((S_
41
      star_max-S_star_min)/10))
42
       for (C_p_increment in seq (from=C_p_min, to=C_p_max, by=((C_p_max-C_p_
43
      \min(10)
44
45
         data [counter, 1] <- S_star_increment
         data[counter, 2] < price_coco_ed(t < 0, T < 10, S_t < 100, S_t
46
      star <- S<sub>-</sub>star<sub>-</sub>increment, C<sub>-</sub>p <- C<sub>-</sub>p<sub>-</sub>increment, c<sub>-</sub>i <- 6, r <- 0.01,
       N < -\ 100\,,\ q < -\ 0.02\,,\ sigma < -\ 0.3\,,\ alpha < -\ 1)
         data [counter, 3] <- C_p_increment
```

### E.3 Structural Approach

The following source code is an implementation of the sensitivity analysis of the Structural Approach (Pennacchi, 2010) written in R.

```
source ('Structural Approach .R')
 3 # CoCo price V^st as function of initial asset-to-deposit ratio x_0 and
                 volatility sigma
  4 createData_SA_x0_sigma <- function(x0_min, x0_max, sigma_min, sigma_max)
            data \leftarrow matrix(1, 121, 3)
  5
            counter <- 1
  6
             for(x0\_increment in seq(from=x0\_min, to=x0\_max, by=((x0\_max-x0\_min)/max))
                 10)))
  8
                   for (sigma_increment in seq (from=sigma_min, to=sigma_max, by=((sigma_
  9
                \max - \operatorname{sigma} - \min (10)
10
                         data[counter, 1] \leftarrow x0\_increment
11
                         data [counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 10000,
12
                {\rm rho} < -\ -\ 0.2 \,, \ {\rm kappa} < -\ 0.114 \,, \ {\rm r\_bar} < -\ 0.069 \,, \ {\rm r0} < -\ 0.01 \,, \ {\rm sigma\_r} < -\ -\ 0.01 \,, \ {\rm roma\_r} < -\ 0.01 \,, \ {\rm roma\_r} < -\ 0.01 \,, \ {\rm roma\_r} < -\ -\ 0.01 \,, \ {\rm roma
                   0.07\,,\ mu\_Y<-\ -0.01\,,\ sigma\_Y<-\ 0.02\,,\ lambda<-\ 1\,,\ g<-\ 0.5\,,\ x\_hat
                <-1.1, b0 < -0.04, p < -1, e_bar < -0.02, sigma_x < -0.02, x0 < -0.02
                1.15, B <- 1, coupon <- 0.06)
                         data [counter, 3] <- sigma_increment
13
                         counter <- counter + 1
14
15
16
             write.table(data, file = "createData_SA_x0_sigma_2.txt", row.names =
                FALSE, quote=FALSE)
18
19
20 # CoCo price V^sa as function of maturity T and risk-free interest rate
      createData_SA_T_r <- function(T_min, T_max, r_min, r_max){
            data \leftarrow matrix(1, 121, 3)
            counter <- 1
23
             for (T_{increment} in seq(from=T_{min}, to=T_{max}, by=((T_{max}-T_{min})/10)))
24
25
                   for (r_increment in seq (from=r_min, to=r_max, by=((r_max-r_min)/10)))
```

```
data[counter, 1] \leftarrow T_increment
28
         data [counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 10000,
      rho < -0.2, kappa < 0.114, r_bar < 0.069, r0 < -0.01, sigma_r < -0.069
       0.07, mu_Y <- -0.01, sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat
      <-1.1, b0 < -0.04, p < -1, e_bar < -0.02, sigma_x < -0.02, x0 < -0.02
      1.15, B <- 1, coupon <- 0.06)
         data[counter, 3] <- r_increment
30
         counter \leftarrow counter + 1
31
       }
32
33
     write.table(data, file = "createData_SA_T_r_2.txt", row.names = FALSE,
34
       quote=FALSE)
35
36
37 # CoCo price V^sa as function of initial asset-to-deposit ratio x_0 and
      equity-to-deposit threshold bar_e
38 createData_SA_x0_ebar <- function(x0_min, x0_max, ebar_min, ebar_max){
     data \leftarrow matrix(1, 121, 3)
     counter <- 1
40
     for (x0\_increment in seq(from=x0\_min, to=x0\_max, by=((x0\_max-x0\_min)/
41
42
       for (ebar_increment in seq (from=ebar_min, to=ebar_max, by=((ebar_max-
43
      ebar_min)/10)))
44
         data [counter, 1] <- x0_increment
45
         data[counter, 2] \leftarrow price\_coco\_sa(T \leftarrow 10, nsimulations \leftarrow 10000,
46
      rho < -0.2, kappa < 0.114, r_bar < 0.069, r0 < -0.01, sigma_r < -0.069
       0.07\,,\ \mathrm{mu\_Y} < -\ -0.01\,,\ \mathrm{sigma\_Y} < -\ 0.02\,,\ \mathrm{lambda} < -\ 1\,,\ \mathrm{g} < -\ 0.5\,,\ \mathrm{x\_hat}
      <-1.1\,,\ b0<-0.04\,,\ p<-1\,,\ e\_bar<-0.02\,,\ sigma\_x<-0.02\,,\ x0<-
      1.15, B <- 1, coupon <- 0.06)
         data [counter, 3] <- ebar_increment
47
         counter \leftarrow counter + 1
48
       }
49
50
     write.table(data, file = "createData_SA_x0_ebar_2.txt", row.names =
51
      FALSE, quote=FALSE)
52 }
53
_{54} # CoCo price V^st as function of initial asset-to-deposit ratio x_0 and
      jump intensity in asset return process lambda
55 createData_SA_x0_lambda <- function(x0_min, x0_max, lambda_min, lambda_
      \max) {
     data \leftarrow matrix(1, 121, 3)
     counter <- 1
     for(x0\_increment in seq(from=x0\_min, to=x0\_max, by=((x0\_max-x0\_min)/
58
      10)))
59
60
       for (lambda_increment in seq (from=lambda_min, to=lambda_max, by=((
      lambda_max-lambda_min)/10))
61
         data [counter, 1] <- x0_increment
```

```
data[counter, 2] \leftarrow price\_coco\_sa(T \leftarrow 10, nsimulations \leftarrow 10000,
      {\rm rho} < -\ -\ 0.2 \,, \ {\rm kappa} < -\ 0.114 \,, \ {\rm r\_bar} < -\ 0.069 \,, \ {\rm r0} < -\ 0.01 \,, \ {\rm sigma\_r} < -\ -
       0.07, mu_Y <- -0.01, sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat
      <-1.1, b0 < -0.04, p < -1, e_bar < -0.02, sigma_x < -0.02, x0 < -
      1.15, B <- 1, coupon <- 0.06)
         data [counter, 3] <- lambda_increment
         counter \leftarrow counter + 1
      }
66
67
    write.table(data, file = "createData_SA_x0_lambda_2.txt", row.names =
68
     FALSE, quote=FALSE)
69
70
71 # CoCo price V^st as function of initial asset-to-deposit ratio x_0 and
      initial ratio of contingent capital to deposits b0
_{72} createData_SA_x0_b0 <- function(x0_min, x0_max, b0_min, b0_max){
    data \leftarrow matrix(1, 121, 3)
73
    counter <- 1
74
    for (x0\_increment in seq(from=x0\_min, to=x0\_max, by=((x0\_max-x0\_min)/
76
       for (b0_increment in seq(from=b0_min, to=b0_max, by=((b0_max-b0_min)/
      10)))
      {
78
         data [counter, 1] <- x0_increment
79
         data [counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 10000,
80
      rho < -0.2, kappa < 0.114, r_bar < 0.069, r0 < -0.01, sigma_r < -0.069
       0.07, mu_Y <- -0.01, sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat
      <-1.1\,,\ b0<-\ 0.04\,,\ p<-\ 1,\ e\_bar<-\ 0.02\,,\ sigma\_x<-\ 0.02\,,\ x0<-
      1.15, B <- 1, coupon <- 0.06)
         data [counter, 3] <- b0_increment
81
         counter <- counter + 1
82
      }
83
84
    write.table(data, file = "createData_SA_x0_b0_2.txt", row.names =
85
      FALSE, quote=FALSE)
86
88 createData_SA_x0_sigma(1.08, 1.17, 0.01, 0.05)
s9 createData_SA_T_r(1, 50, 0.01, 0.05)
90 createData_SA_x0_ebar(1.08, 1.17, 0.005, 0.03)
gi createData_SA_x0_lambda(1.08, 1.17, 0, 2)
92 createData_SA_x0_b0(1.08, 1.17, 0.1, 0.06)
```

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