

# Valuation of Contingent Convertibles with Derivatives



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# Abstract

Financial crises have led to higher regulatory standards on the capital adequacy of banks. Banks are required to hold more capital with loss absorbance capacities on their balance sheet. In conjunction with this development, contingent convertible bonds (CoCos) have become an attractive instrument for banks to seek new capital. The defining characteristic of CoCos is the automatic conversion into common equity when a predetermined trigger is met. Loss absorbing capital is created, which instantly improves the capital structure of the distressed bank. However, the pricing of these hybrid instruments remains opaque. In this context, the thesis scrutinizes the valuation of CoCos. Three dominant approaches are examined: the structural approach in accordance to Pennacchi (2010), the credit derivatives approach and the equity derivatives approach both pursuant to De Spiegeleer and Schoutens (2011). The application covers sensitivity analysis to further understand the dynamics of the different methodologies. Based on a case study the viability of those approaches is evaluated. Software is provided for further replication.

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# Chapter 1

## Introduction and Motivation

### 1.1 Introduction

| Face Value in USD bn | Issuer                      |
|----------------------|-----------------------------|
| 3.75                 | HSBC (GB)                   |
| 3.63                 | UBS (CH)                    |
| 3.15                 | Royal Bank of Scotland (GB) |
| 3.00                 | Barclays (GB)               |
| 2.70                 | UBS (CH)                    |
| 2.50                 | Credit Suisse (CH)          |
| 2.50                 | UBS (CH)                    |
| 2.45                 | HSBC (GB)                   |
| 2.25                 | ING (NL)                    |
| 2.06                 | Banco Santander (ESP)       |

Table 1.1: Largest CoCo issues in Europe from 2010 to 2016 (Dietegen, 2016)



## 1.2 Literature Overview

| Structural Approach         | Equity Derivative Approach         | Credit Derivative Approach         |
|-----------------------------|------------------------------------|------------------------------------|
| Pennacchi (2010)            | De Spiegeleer and Schoutens (2011) | De Spiegeleer and Schoutens (2011) |
| Glasserman and Nouri (2012) | Henriques and Doctor (2011)        |                                    |
| Madan and Schoutens (2011)  |                                    |                                    |
| Albul et al. (2010)         |                                    |                                    |
| Sundaresan and Wang (2015)  |                                    |                                    |
| Hilscher and Raviv (2014)   |                                    |                                    |
| Buergi (2013)               |                                    |                                    |

Table 1.2: Literature overview of valuation approaches for CoCos (Erismann, 2015)

### 1.2.1 Structural Approaches

### 1.2.2 Equity Derivative Approaches

### 1.2.3 Credit Derivative Approaches

## 1.3 Motivation

- 

## 1.4 Methodology

-

# Chapter 2

## Structure of CoCos

This chapter explains the nature of a new member in the family of hybrid securities; so-called contingent convertibles (CoCos). In the following, the general structure of these new instruments will be explained (section 2.1) including characteristic design features among others their trigger (section 2.2) and loss-absorption mechanism (section 2.3).

### 2.1 Definition

CoCos are hybrid financial instruments with automatic conversion mechanisms that morph debt into equity when the financial soundness of the issuer is at stake. In such a case, a predefined trigger event occurs. For instance, a regulatory capital ratio falls below a predefined threshold. That said, a write-down of the notional of the CoCo is also a viable loss-absorption mechanism. (De Spiegeleer and Schoutens, 2011) Figure 2.1 provides a good overview of the previously mentioned design characteristics. All anatomic aspects will be further explained in the subsequent sections.

CoCos are particularly interesting from the perspective of a regulatory authority because they might mitigate externalization costs of insolvencies and frictions due to spill-over effects. In times of distress, stakeholders might question the financial viability of the respective financial institute. However, a distressed bank does not have to approach new investors to issue new capital in extremely tough times as everything happens automatically. (De Spiegeleer and Schoutens, 2011)

In 2009, the Lloyds Banking Group was the first financial institute to issue this new financial instrument. They offered hybrid debt holders to swap their holdings into CoCos. (De Spiegeleer and Schoutens, 2011) In 2010, the Basel Committee on

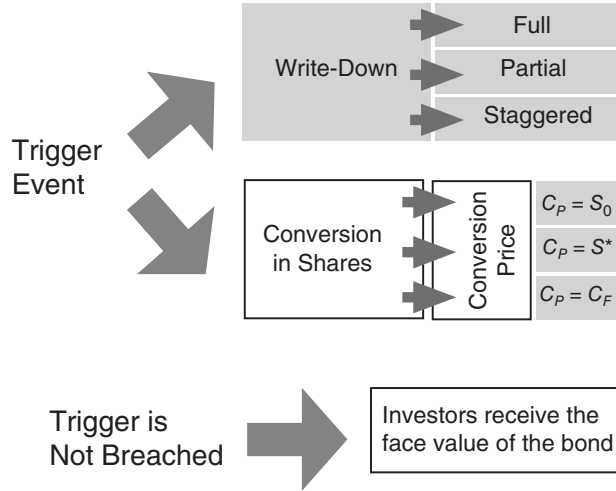


Figure 2.1: Anatomy of CoCos (De Spiegeleer et al., 2014)

Banking Supervision (BCBS) provided further impetus to the use of this instrument when it disclosed its proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. The general argumentation of the BCBS is that regulatory capital instruments have to be capable of absorbing financial losses in gone-concern phases.(on Banking Supervision, 2010).

## 2.2 Trigger Mechanism

The trigger is a key design element of a CoCo. In the following section, four different trigger mechanisms will be evaluated based on general design criteria as summarized by Erismann (2015). The concept of an accounting trigger will be illustrated in section 2.2.1 followed by the market trigger in section 2.2.2. Additionally, the regulatory trigger will be detailed in section 2.2.3 and the multi-variate trigger will be explained in section 2.2.4. Hereinafter, the trigger types will be evaluated based on the following criteria:

- **Clarity:** The trigger mechanism should be universally applicable irrespective of the jurisdiction in which the CoCo is traded.
- **Fixedness:** The definition of a CoCo's trigger should remain the same until maturity.
- **Frequency:** Data to which the trigger is linked should be updated at frequent intervals.

- **Objectiveness:** The trigger mechanism should be based on observable and well-known facts. There should be no room for subjectivity.
- **Publicity:** Data should be open to the general view of all market participants. This needs to be possible without concealment.
- **Transparency:** The trigger mechanism should have the property that investors can see it publicly, thereby reducing the chance of manipulation.

### 2.2.1 Accounting Trigger

CoCos with accounting trigger have a loss absorption mechanism which is inherently connected to the financial soundness of a bank's balance sheet. Accounting triggers are built upon capital ratios which compare a bank's regulatory capital with its assets. Conversion occurs if a pre-determined metric falls below a certain threshold. Hence, capital ratios are an objective indicator for a bank's solvency as they are defined uniformly for all financial institutions by regulatory authorities. (De Spiegeleer et al., 2014) Pazarbasioglu et al. (2011) note that accounting triggers are easy to price, intuitive and simple to implement. Issuing banks and regulators seem to have perceived the benefits of accounting triggers partly because a significant portion of CoCos use the common equity tier (CET1) ratio as reference metric. Examples of CoCos will be described shortly in section 3.

That said, objections against accounting triggers follow the line of thought that they just become active long after the need for loss absorbing capital arose. One might argue that accounting triggers assess the viability of financial institutions from a perspective that is far-removed from reality. (De Spiegeleer and Schoutens, 2011) Moreover, as accounting concept, book values are prone to manipulation and managerial dishonesty especially in times of distress. (McDonald, 2013)

Empirical findings bring up a further aspect. Haldane (2011) points out that major financial institutions, which either went bankrupt, were bailed out or were taken over under distress during the global financial crisis, reported similar CET 1 ratios right before the collapse of Lehman Brothers compared to their peers which coped relatively well with the collapse of the financial system. In this context, Haldane (2011)

highlights that market-based solvency measures<sup>1</sup> performed creditably as they showed clear signals of impending distress a year ahead of the bankruptcy of Lehman Brothers. Empirical evidence of Valukas (2010) further supports these findings. This leads to the conclusion that CoCos with accounting triggers might not reinforce distressed banks at the right time but instead produce more false positives, which means that CoCos of non-distressed banks trigger prematurely. Inefficiencies like higher funding costs could be the consequence. (Pazarbasioglu et al., 2011)

### 2.2.2 Market Trigger

A market trigger uses directly observable indicators like the issuing company's share price or credit default swap (CDS) spreads while assuming sufficiently efficient markets. The major advantage of those measures is that one can observe and verify them in real-time. (Haldane, 2011) Market triggers are widely discussed in academia and seen as preferable trigger mechanism. Calomiris and Herring (2013) pronounce themselves for using share prices. Besides, Haldane (2011) and Pazarbasioglu et al. (2011) contend to apply market-based capital ratios as trigger indicator. Their line of argumentation is based on some of the best-known examples of corporate defaults, which were indicated well before by a serious and continuous deterioration of a company's market capitalization.

This also applies to the aforementioned example of Lehman Brothers. If the bank had been obliged to issue CoCos with a trigger directly linked to its share price the decline in equity value would have triggered the conversion in due time. (Calomiris and Herring, 2013) Hence, the bankruptcy could have been prevented with a CoCo. However, practitioners argue that it comes at a risk to use market data as reference because markets are prone to manipulation. Moreover, self-fulfilling prophecies of share price dilution could lead to downward spirals that ultimately lead to CoCo conversions. (Pazarbasioglu et al., 2011) The severity of the aforementioned argumentation can be attenuated by using moving averages, which are less susceptible to manipulation.

### 2.2.3 Regulatory or Non-Viability Trigger

Regulatory or non-viability trigger is a conversion mechanism by which a CoCo is converted into equity at the discretion of the responsible supervisory authority. The

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<sup>1</sup>Haldane (2011) mentions three metrics: (1) market-based capital ratio (ratio of market capitalization to total assets), (2) market-based leverage ratio (ratio of market capitalization to total debt) and (3) Tobin's Q (ratio of market capitalization to book value of equity).

rationale behind this approach is that regulators want to limit the impact of any development that could pose a danger to the going-concern of a systemically important bank. Erismann (2015) Moreover, this kind of trigger would eliminate the periodicity problem of accounting data and the risk of market manipulation.

Though, it is very difficult for market participants to estimate the conversion probability of a CoCo with regulatory trigger. The valuation of such a hybrid instrument becomes opaque for market participants with limited information. (Alvemar and Ericson, 2012) One can also argue that a CoCo's marketability is weakened because of the greater uncertainty which could ultimately lead to higher funding costs. (De Spiegeleer et al., 2014)

#### **2.2.4 Multi-Variate Trigger**

The multi-variate trigger uses a combination of the aforementioned trigger mechanisms. It extends the accounting trigger of a CoCo by a universal regulatory trigger which covers severe states of the world adversely impacting the stability of the financial system. In this context, the Squam Lake Working Group (2009) supports the implementation of this dual trigger mechanism as it combines the best of two worlds. On the one hand, the bank-specific trigger serves as disciplining mechanism for a bank's management. It reduces the political pressure from the regulator who has to decide whether the systemic trigger is met. Second, if the conversion of a CoCo is only linked to a systemic trigger, even well capitalized banks would be forced to convert debt into equity during a systemic crisis. This would disincentivize financially sound banks to preserve their status quo.

#### **2.2.5 Evaluation of Trigger Types**

Generally, one can evaluate the aforementioned trigger types based on the design criteria as stated in the beginning.

|               | Accounting | Market | Regulatory | Multi-Variate |
|---------------|------------|--------|------------|---------------|
| Clarity       | medium     | high   | low        | medium        |
| Fixedness     | high       | high   | low        | high          |
| Frequency     | medium     | high   | low        | high          |
| Objectiveness | medium     | high   | low        | medium        |
| Publicity     | high       | high   | high       | high          |
| Transparency  | medium     | high   | low        | medium        |

Table 2.1: Evaluation of different trigger mechanisms based on design criteria. The status high means that the trigger type meets the requirements, whereas low implies that it does not satisfy the conditions. (Erismann, 2015)

We see in table 2.1 that the market trigger is a preferable mechanism as it fulfills all important design criteria. Furthermore, one has to take into account that Haldane (2011) described that market trigger were historically the preferable mechanism as it served as early-warning indicator. In section 3 different CoCos will be highlighted with their design elements also covering their trigger mechanism. This is a good way to understand whether banks and regulators followed the aforementioned rationale.

## 2.3 Loss-Absorption Mechanism

As mentioned before, banks can decide whether they want to issue CoCos with equity conversion or with a haircut on the notional.

### 2.3.1 Conversion into Shares

The number of shares which a CoCo holder receives at conversion is given by the conversion rate  $C_r$ . This figure is specified ex-ante and the conversion into shares gives an investor in case a certain trigger threshold is breached. Besides, the conversion amount  $\alpha N$  is determined by the conversion fraction  $\alpha$  and the notional  $N$ . One can now derive the implied conversion price  $C_p$  with the following formula. (De Spiegeleer et al., 2014)

$$C_p = \frac{\alpha N}{C_r} \quad (2.1)$$

The recovery rate  $R_{CoCo}$  can be calculated based on the share price  $S^*$  at the trigger event and the conversion price  $C_p$ . One might recognize that a CoCo holder is better

off if  $C_p$  is low because the recovery rate  $R_{CoCo}$  will be higher as more equity will be created.

$$R_{CoCo} = \frac{S^*}{C_p} \quad (2.2)$$

If the CoCo is converted into shares one can directly derive the loss  $L_{CoCo}$  a CoCo holder has to bear.

$$Loss_{CoCo} = N - C_r S^* = N - (1 - R_{CoCo}) = N \left(1 - \frac{S^*}{C_p}\right) \quad (2.3)$$

Generally, the final payoff  $V^{CoCo}$  which an investor receives at maturity  $T$  is given by:

$$V_T^{CoCo} = \begin{cases} N & \text{if not triggered} \\ (1 - \alpha)N + \frac{\alpha N}{C_p} S^* & \text{if triggered} \end{cases} \quad (2.4)$$

### Floating conversion price

At issuance one can set a floating conversion price where  $C_p$  is equal to  $S^*$ .  $S^*$  is the share price which is observed at the trigger event. Intuitively, the value of the share price at trigger time  $t$  is low as the purpose of the CoCo is to help undercapitalized banks in tough times. If the issuer chooses to set the conversion price at this specific level the recovery rate of a CoCo holder is 100% whereas the current shareholders carry the load of conversion. Regulators would not categorize this instrument to be adequate as regulatory capital instrument as the dilution is potentially unbounded and it is effectively not loss absorbing. (De Spiegeleer et al., 2014)

### Fixed conversion price

Fixed conversion means that the conversion price  $C_p$  is equal to the share price of the issue date  $S_0$ . On the one hand, this means that the amount of shares upon conversion is fixed at the date of issue and on the other hand, the amount is known beforehand. This is in contrast to the floating conversion price where there was no limit on the amount of shares converted. (De Spiegeleer et al., 2014)

### Floored conversion price

One can also specify a floored conversion price where  $C_p$  is equal to  $\max(S^*, S_F)$ . Hence, the conversion price  $C_p$  is either equal to the floored share price  $S_F$  or to the share price  $S^*$ , which is observed at the triggering event. It is a compromise between the floating and fixed conversion price as stated before. (De Spiegeleer et al., 2014)



### **2.3.2 Write-Down**

The implementation of a loss-absorption mechanism with equity conversion entails several disadvantages which might reduce the marketability of a CoCo. For instance, portfolio managers with a mandate to invest in corporate bonds may face hard times to broaden their investment universe because CoCos with equity conversion mechanism may be converted into shares. Furthermore, the write-down feature is preferable as investors know beforehand the potential loss. Shareholders might be concerned that their voting rights are diluted when a conversion occurs. Both investors and shareholder have clear incentives to influence a bank to issued CoCos with write-down mechanism.

#### **Full write-down**

The first way is to specify a full write-down of a CoCo's face value in case a certain capital ratio drops below a predetermined level.

#### **Partial write-down**

Another approach is that only a certain portion of the notional is wiped out.

#### **Staggered write-down**

The third option consists of a staggered write-down. This means that a CoCo inherits a flexible write-down mechanism. Losses materialize up to the point to enhance a certain capital ratio to a fixed threshold. One might think of a gradual process in which a haircut is imposed in multiples of 10%.

# Chapter 3

## CoCo Examples

For each of the aforementioned loss-absorption mechanisms selected CoCos are characterized hereinafter to gain a better sense of how these hybrid products are implemented in real-life. To that end, broadly discussed CoCos of well-known financial institutions are picked out, i.a. Lloyds, Credit Suisse, Barclays, Rabobank and Zurich Cantonal Bank.

|                  | Lloyds                       | Credit Suisse                               | Barclays                   | Rabobank   | Zurich Cantonal Bank                     |
|------------------|------------------------------|---|----------------------------|--|--|
| Full Name        | Enhanced Capital Notes       | Tier 2 Buffer Capital Notes                 | Contingent Capital Notes   | Senior Contingent Notes  | Subordinated Tier 1 Notes                |
| ISIN             | XS0459088281                 | XS0595225318                                | US06740L8C27               | XS0496281618   | CH0143808332                             |
| Issue Date       | Dec 1, 2009                  | Feb 24, 2011                                | Nov 21, 2012               | Mar 19, 2010   | Jan 31, 2012                             |
| Maturity         | May 12, 2020                 | Feb 24, 2041                                | Nov 21, 2022               | Mar 19, 2020   | Perpetual                                |
| Nominal          | GBP 7.5 bn                   | USD 2 bn                                    | USD 3 bn                   | EUR 1.25 bn  | CHF 590 mn                               |
| Callability      | n/a                          | Callable from Aug 24, 2016                  | n/a                        | n/a  | Callable from Jun 20, 2017               |
| Coupon           | 7.5884%                      | 7.875%                                      | 7.625%                     | 6.875%   | 3.5%                                     |
| Write-down       | n/a                          | n/a   | Full by (100% of notional) | Partial (75% of notional)  | Staggered (multiples of 25% of notional) |
| Conversion price | Fixed at GBP 0.59 (= $S_0$ ) | Floored at lowest of USD 20 and 30 day-VWAP | n/a                        | n/a  | n/a                                      |
| Trigger          | Core Tier 1 capital ratio    | CET 1 ratio                                 | CET 1 ratio                | Equity capital ratio (Member certificates to risk weighted assets) | CET 1 ratio                              |
| Trigger Level    | 5%                           | 7%  | 7%                         | 7%   | 7%                                       |

Table 3.1: CoCo examples with different loss-absorption mechanisms (Lloyds, 2009; Credit Suisse, 2011; Barclays, 2010; Rabobank, 2010; Zurich Cantonal Bank, 2013)

An overview of CoCos which have different characteristics with respect to their loss-absorbency can be found in table 3.1. The first two apply similar equity conversion mechanisms whereas the latter three use different write-downs.

### **3.1 Lloyds – Enhanced Capital Notes**

In 2009, Lloyds issued Enhanced Capital Notes (ECN) in the amount of GBP 9 bn in an debt exchange offer. At that time, the company was the first bank to refinance itself with CoCos. The ECN inherit an accounting-based trigger which is disclosed each quarter. These instruments convert into equity should the bank's Core Tier 1 capital (Basel II) fail to remain above the threshold of 5%. If this is the case, the conversion mechanism morphs the entire face value to a fixed amount of ordinary shares. But today, banks in the UK move to Basel III. Yet, this leads to stricter definitions of risk-weighting compared to Basel II. Thus, the initial threshold of 5% Core Tier 1 capital ratio is not sufficient as it translates to a significantly lower CET 1 ratio under Basel III. Regulators believe that the ECN might not buttress the balance sheet in times of stress. Hence, the supervisory authorities did not consider them in their latest stress tests. Subsequently, Lloyds has started to call in the CoCos. (Lloyds, 2009)

### **3.2 Credit Suisse – Tier 2 Buffer Capital Notes**

The rationale of Credit Suisse's issue of its Tier 2 Buffer Capital Notes (T2BCN) in 2011 follows the recommendation of the Swiss Commission of Experts. Their goal is to establish measures which further solidify the capital structure of the Swiss banking sector. In this context, CoCos play an important role for Credit Suisse in strengthening its capital base to 19% until 2019. In order to achieve the given objectives, USD 2 bn in T2BCN have been raised with a coupon of 7.875%. The trigger event is specified to convert the T2BCN into ordinary shares if the reported risk-based capital ratio is below 7%. The conversion price is floored at the average daily share price of the last 30 days or USD 20. It may also be that the CoCo is converted if the FINMA determines that Credit Suisse needs to be bailed out. (Credit Suisse, 2011)

### **3.3 Barclays – Contingent Capital Notes**

In 2012, Barclays launched the first high-trigger total-loss CoCo. The Contingent Capital Notes depreciate to a value of zero should the CET 1 ratio fail to remain above a minimum of 7%. Accordingly, the bond pays a coupon of 7.625%. Such a structure assumes already that the bank has a solid buffer before the trigger is actually met. (Barclays, 2010)

### **3.4 Rabobank – Senior Contingent Notes**

CoCos with write-off features are particularly suitable for cooperative banks that are deterred by their legal form to issue shares. In 2010, Rabobank was the first cooperative bank to issue EUR 1.25 bn in Senior Contingent Notes with a loss-absorption mechanism that imposes a considerable haircut on its principal. One quarter of the notional is reimbursed at the trigger event if the equity capital ratio falls below 7%. The distinct haircut explains the high financing costs of the instrument. (Rabobank, 2010)

### **3.5 Zurich Cantonal Bank– Subordinated Tier 1 Notes**

Zurich Cantonal Bank is wholly owned by the Canton of Zurich and hence, it is not listed. The bank has been in a similar situation to improve its regulatory capital as Rabobank. In this situation, the bank issued Subordinated Tier 1 Notes that are exemplary for CoCos with a staggered write-down mechanism. What has been new is that an investor faces a dilution of his or her holdings up to the point where the write-down lifts the regulatory capital up to its minimum level. Haircuts materialize only in multiples of 25%. Zurich Cantonal Bank (2013)

# Chapter 4

## Theory of Pricing

### 4.1 Credit Derivative Approach

In financial markets the reduced-form approach is widely used in order to price credit risk. It was originally introduced by Jarrow and Turnbull (1995) respectively Duffie and Singleton (1999). The credit derivative approach applies the reduced-form approach to CoCos. In this context, the derivation of a pricing formula for CoCos follows mainly De Spiegeleer and Schoutens (2011).

#### 4.1.1 Reduced-Form Approach and Credit Triangle

The reduced-form approach is an elegant way of bridging the gap between the prediction of default and the pricing of default risk of a straight bond. In the following, we investigate the link between estimated default intensities and credit spreads under the reduced-form approach. (Lando, 2009)

Let  $\tau$  denote the random time of default of some company. It is assumed that the distribution of  $\tau$  has a continuous density function  $f$ , so that the distribution function  $F$  and the curve of survival probabilities  $q$  are related as follows:

$$P(\tau \leq t) = F(t) = 1 - q(t) = \int_0^t f(s)ds, \text{ with } t \geq 0 \quad (4.1)$$

The hazard rate respectively the default intensity  $\lambda$  is defined as follows:

$$\lambda(t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} P(\tau \leq t + \Delta | \tau > t) = \frac{F'(t)}{q(t)} = \frac{F'(t)}{1 - F(t)} = -\frac{d}{dt} \log q(t) \quad (4.2)$$

Intuitively, the hazard rate is the default rate per year as of today. Using 4.2 we can derive a formula for the survival probability:

$$q(t) = \exp \left( - \int_0^t \lambda(s)ds \right) \quad (4.3)$$

For our application of the reduced-form approach we assume that the hazard rate  $\lambda(t)$  is a deterministic function of time. In reality  $\lambda(t)$  is not deterministic but itself stochastic. That fits in with the fact that credit spreads are not static but stochastically varying over time. (Schmidt, 2015) However, we further consider the hazard rate to be constant in order to simplify the problem. Hence, a constant hazard rate  $\lambda(t) = \lambda$  implies an exponential distribution of the default time:

$$F(t) = 1 - q(t) = 1 - \exp(-\lambda t) \quad (4.4)$$

Under the idealized assumption of a flat zero interest rate curve, a flat spread curve and continuous spread payments, the default intensity  $\lambda$  can be calculated directly from the credit spread  $s$  and the recovery rate  $R$  by the rule of thumb formula (Schmidt, 2015), which is also known as credit triangle:

$$\lambda = \frac{s}{1 - R} \quad (4.5)$$

Finally, this relationship makes it possible to determine the default probability  $F$  from the credit spread  $s$  and vice versa.

#### 4.1.2 Adaption to CoCos

In accordance with the aforementioned reduced-form approach, De Spiegeleer and Schoutens (2011) assume that the probability  $F^*$ , which measures the likelihood that a CoCo triggers within the next  $T - t$  years, follows similar mechanics as the default probability of a straight bond does. Under the credit derivative approach the probability  $F^*$  can be expressed as follows:

$$F^* = 1 - \exp[-\lambda_{Trigger}(T - t)] \quad (4.6)$$

Additionally, the credit derivative approach models  $F^*$  with the first exit time equation used in barrier option pricing under a Black-Scholes setting. (Su and Rieger, 2009) Hence, the probability  $F^*$  that the trigger level  $S^*$  is touched within the next  $T - t$  years is given by the following equation with the continuous dividend yield  $q$ , the continuous interest rate  $r$ , the drift  $\mu$ , the volatility  $\sigma$  and the current share price  $S$  of the issuing company:

$$F^* = \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) - \mu(T - t)}{\sigma\sqrt{(T - t)}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) + \mu(T - t)}{\sigma\sqrt{(T - t)}}\right) \quad (4.7)$$

In this regard, a CoCo's credit spread  $s_{CoCo}$  can be approximated by the credit triangle, where  $R_{CoCo}$  denotes the recovery rate of a CoCo and  $L_{CoCo}$  is the loss rate:

$$s_{CoCo} = (1 - R_{CoCo}) \lambda_{Trigger} = L_{CoCo} \lambda_{Trigger} \quad (4.8)$$

In the trigger event, the face value  $N$  converts into  $C_r$  shares worth  $S^*$ . The loss of a long position in a CoCo is therefore determined by the conversion price  $C_p$ :

$$Loss_{CoCo} = N - C_r S^* = N (1 - R_{CoCo}) = N \left(1 - \frac{S^*}{C_p}\right) \quad (4.9)$$

By combining 4.6, 4.8 and 4.9 we see that the credit spread  $s_{CoCo}$  of a CoCo with maturity  $T$  at time  $t$  is driven by its major design elements, the trigger level  $S^*$  and the conversion price  $C_p$ :

$$s_{CoCo_t} = -\frac{\log(1 - F^*)}{(T - t)} \left(1 - \frac{S^*}{C_p}\right) \quad (4.10)$$

Subsequently, a pricing formula for CoCos under the credit derivative approach can be derived. The present value  $V^{cd}$  at time  $t$  is given by:

$$V_t^{cd} = \sum_{i=1}^T c_i \exp[-(r + s_{CoCo_t})(t_i - t)] + N \exp[-(r + s_{CoCo_t})(T - t)] \quad (4.11)$$

In summary, the credit derivative approach provides us with a concise method to price CoCos. However, one has to bear in mind its largest shortcoming. Losses from cancelled coupons of triggered CoCos are not taken into account in the valuation. Hence, the credit derivative approach naturally overestimates the price of CoCos, but it equips investors with a simple rule of thumb formula.

### 4.1.3 Parameter Classification and Adjustment

|          | Description                       | Usage         | Source      |
|----------|-----------------------------------|---------------|-------------|
| $T$      | CoCo maturity                     | Static input  | Term sheet  |
| $N$      | CoCo nominal                      | Static input  | Term sheet  |
| $c$      | CoCo coupon rate                  | Static input  | Term sheet  |
| $S_0$    | Initial share price of the issuer | Dynamic input | Market data |
| $S^*$    | Trigger share price               | Static input  | Term sheet  |
| $C_p$    | CoCo nominal conversion price     | Static input  | Term sheet  |
| $r$      | Risk-free interest rate           | Static input  | Market data |
| $q$      | Dividend yield                    | Static input  | Market data |
| $\sigma$ | Implied volatility                | Static input  | Market data |

Table 4.1: Parameter classification of the credit derivative approach (Wilkins and Bethke, 2014)

### 4.1.4 Model Application

|          | Value  | Comment                         |
|----------|--------|---------------------------------|
| $T$      | 10yrs  | Maturity                        |
| $N$      | 100    | Nominal                         |
| $c$      | 6.00   | Annual coupon rate              |
| $S_0$    | 100    | Initial share price of the bank |
| $S^*$    | 35     | Trigger share price             |
| $C_p$    | 65     | Nominal conversion price        |
| $r$      | 1.00%  | Risk-free interest rate         |
| $q$      | 2.00%  | Dividend yield                  |
| $\sigma$ | 30.00% | Implied volatility              |

Table 4.2: Parameter specification of credit derivative approach application (Alvemar and Ericson, 2012)

## 4.2 Equity Derivative Approach

In order to assess the value of a CoCo, investors can use a method which depends on equity derivatives. (De Spiegeleer and Schoutens, 2011; De Spiegeleer et al., 2014) The so-called equity derivative approach attempts to compensate for the main drawback



of the credit derivative approach, since it takes into account that coupon payments might be cancelled if the trigger of a CoCo was touched.

Subsequently, the valuation can be divided into two steps: In the first step, the value of a CoCo is determined without coupon payments. Such a CoCo is called Zero-Coupon CoCo. In the second step, one has to incorporate the coupon payments in the pricing formula while keeping in mind that they might be knocked out. The closed-form solution is derived under a Black-Scholes setting.

### 4.2.1 Step One - Zero-Coupon CoCo

To determine the present value of a Zero-Coupon CoCo  $V^{zcoco}$  at maturity  $T$  we can use equation 2.4. The underlying assumption of the equity derivative approach is that the triggering of a CoCo respectively of a Zero-Coupon CoCo is equivalent to the share price falling below the level  $S^*$ .

$$\begin{aligned}
V_T^{zcoco} &= \begin{cases} N & \text{if not triggered} \\ (1 - \alpha)N + \frac{\alpha N}{C_p} S^* & \text{if triggered} \end{cases} \\
&= N \mathbb{1}_{\{\tau > T\}} + \left[ (1 - \alpha) N + \frac{\alpha N}{C_p} S^* \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + \left[ \frac{\alpha N}{C_p} S^* - \alpha N \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + [C_r S^* - \alpha N] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + C_r \left[ S^* - \frac{\alpha N}{C_r} \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + C_r [S^* - C_p] \mathbb{1}_{\{\tau \leq T\}} \tag{4.12}
\end{aligned}$$

It may be inferred that the financial payoff of equation 4.12 consists of two components (Erismann, 2015): (1) the face value  $N$  of a zero bond and (2) a long position in  $C_r$  shares generating a payoff only if the CoCo materializes at time  $\tau$ . This component can be approximated with a knock-in forward. The intuition behind equation 4.12 is that if the share price falls below a certain level  $S^*$ , an investor will use the face value  $N$  to exercise the knock-in forward. That said, the investor is committed to buy the amount of  $C_r$  shares for the price of  $C_p$  at maturity  $T$ .

Before maturity the present value of a Zero-Coupon CoCo  $V^{zcoco}$  can be determined by adding up the present value of a zero bond  $V^{zb}$  and the present value of a knock-in

forward  $V_t^{kifwd}$ . Hereinafter, the components will be explained briefly.

$$V_t^{zcoco} = V_t^{zb} + V_t^{kifwd} \quad (4.13)$$

with

$$V_t^{zb} = N \exp[-r(T-t)] \quad (4.14)$$

Moreover, the long position in shares at time  $t$  can be approximated with the respective closed-form solution of a knock-in forward. (Hull, 2006)

$$\begin{aligned} V_t^{kifwd} = C_r & \left[ S_t \exp[-q(T-t)] \left( \frac{S^*}{S_t} \right)^{2\lambda} \Phi(y_1) \right. \\ & - K \exp[-r(T-t)] \left( \frac{S^*}{S_t} \right)^{2\lambda-2} \Phi(y_1 - \sigma\sqrt{T-t}) \\ & - K \exp[-r(T-t)] \Phi(-x_1 - \sigma\sqrt{T-t}) \\ & \left. + S_t \exp[-q(T-t)] \Phi(-x_1) \right] \end{aligned} \quad (4.15)$$

with

$$\begin{aligned} C_r &= \frac{\alpha N}{C_p} \\ K &= C_p \\ \lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2} \\ x_1 &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \\ y_1 &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \end{aligned}$$

It is important, however, to recognize that a subtle difference exists between the actual economic payoff of equation 4.12 and its replication with a knock-in forward, since the knock-in forward replicates an economic ownership of shares at maturity  $T$ . Though, the triggering of a CoCo forces investors to accept the conversion immediately. This could lead to an economic ownership of shares at trigger time  $\tau$  and, thus, prior to  $T$ . Therefore, one could argue that receiving a knock-in forward in the trigger

event disregards the dividends which a shareholder would receive in particular when a CoCo triggers early. De Spiegeleer and Schoutens (2011) argue that dividends can be neglected because distressed banks are likely to behave with great restraint when it comes to dividend payments.

### 4.2.2 Step Two - Adding Coupons

As mentioned earlier, the first step excludes coupon payments from the valuation. Yet, in this step we want to include them in order to replicate the exact payout profile of a CoCo. Therefore, we replace the zero bond in equation 4.13 with a straight bond with regular coupon payments  $c$ . Besides, a third component has to be added which takes into account the foregone coupon payments if the trigger is touched. This can be modeled with a short position in  $k$  binary down-and-in calls with maturity  $t_i$ . Those binary down-and-in calls are knocked in if the trigger  $S^*$  is met and hence, offset all future coupon payments.

The price of a straight bond can be determined with:

$$V_t^{sb} = \sum_{i=1}^T c_i \exp[-r(t_i - t)] + N \exp[-r(T - t)] \quad (4.16)$$

To price the down-and-in calls one might use the formula of Rubinstein and Reiner (1991):

$$V_t^{bdic} = \alpha \sum_{i=1}^k c_i \exp(-rt_i) \left[ \Phi(-x_{1i} + \sigma\sqrt{t_i}) + \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \Phi(y_{1i} - \sigma\sqrt{t_i}) \right] \quad (4.17)$$

with

$$\begin{aligned} x_{1i} &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ y_{1i} &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ \lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2} \end{aligned}$$

To sum up, the theoretical price of a CoCo  $V^{ed}$  at time  $t$  pursuant the equity derivative approach consists of three components: (1) a straight bond  $V^{sb}$ , (2) a knock-in-forward  $V^{kifwd}$  and (3) a set of binary down-and-in calls  $V^{bdic}$ :

$$V_t^{ed} = V_t^{sb} + V_t^{kifwd} - V_t^{bdic} \quad (4.18)$$

### 4.2.3 Parameter Classification and Adjustment

|          | Description                       | Usage         | Source      |
|----------|-----------------------------------|---------------|-------------|
| $T$      | CoCo maturity                     | Static input  | Term sheet  |
| $N$      | CoCo nominal                      | Static input  | Term sheet  |
| $c$      | CoCo coupon rate                  | Static input  | Term sheet  |
| $\alpha$ | CoCo nominal conversion factor    | Static input  | Term sheet  |
| $S_0$    | Initial share price of the issuer | Dynamic input | Market data |
| $S^*$    | Trigger share price               | Static input  | Term sheet  |
| $C_p$    | CoCo nominal conversion price     | Static input  | Term sheet  |
| $r$      | Risk-free interest rate           | Static input  | Market data |
| $q$      | Dividend yield                    | Static input  | Market data |
| $\sigma$ | Implied volatility                | Static input  | Market data |

Table 4.3: Parameter classification of the equity derivative approach (Wilkins and Bethke, 2014)

#### 4.2.4 Model Application

|          | Value  | Comment                         |
|----------|--------|---------------------------------|
| $T$      | 10yrs  | Maturity                        |
| $N$      | 100    | Nominal                         |
| $c$      | 6.00   | Annual coupon rate              |
| $\alpha$ | 1      | Nominal conversion factor       |
| $S_0$    | 100    | Initial share price of the bank |
| $S^*$    | 35     | Trigger share price             |
| $C_p$    | 65     | Nominal conversion price        |
| $r$      | 1.00%  | Risk-free interest rate         |
| $q$      | 2.00%  | Dividend yield                  |
| $\sigma$ | 30.00% | Implied volatility              |

Table 4.4: Parameter specification of equity derivative approach application (Alvemar and Ericson, 2012)

### 4.3 Structural Approach

A third alternative to price CoCos is the structural approach of Pennacchi (2010). The idea has its roots in the seminal work of Merton (1974), which aims to explain a company's default based on the relationship of its assets and liabilities under a standard Black-Scholes setting. Pennacchi (2010)'s approach expands the idea by modeling the stochastic evolution of a bank's balance sheet respectively of its components. In the following, the assets' rate of return process will be explained. Thereafter, we will outline the assumptions of the model regarding the various liabilities a bank issues to refinance itself including deposits, equity and coupon bonds in the form of CoCos. Lastly, a pricing formula will be illustrated.

#### 4.3.1 Structural Banking Model

##### Bank Assets and Asset-To-Deposit Ratio

Pennacchi (2010) assumes that a bank holds a portfolio of loans, equities and off-balance sheet positions as assets whose returns follow a jump-diffusion process. The change of this portfolio  $A_t$  is determined by the rate of return and the cash in- respectively outflows. In this context, the symbol  $*$  is used to point out the change in value of the portfolio which can be quantified by the rate of return, excluding net

cashflows. The aforementioned instantaneous rate of return is denoted as  $dA_t^*/A_t^*$  and follows a stochastic process as stated below under the risk-neutral probability measure  $\mathbb{Q}$ :

$$\frac{dA_t^*}{A_t^*} = (r_t - \lambda_t k_t) dt + \sigma dz + (Y_{q_t-} - 1) dq_t \quad (4.19)$$

It should be noted that  $r_t$  stands for the risk-free interest rate as defined by the Cox et al. (1985) term-structure model which will be discussed shortly.  $dz$  is a Brownian motion, whereby  $\sigma$  denotes the volatility of returns of the aforementioned asset portfolio.  $q_t$  is a Poisson counting process which increases by one whenever a Poisson-distributed event respectively a jump occurs. Hence, the variable  $dq_t$  is one whenever such a jump takes place and zero otherwise. The risk-neutral probability that a jump happens is equal to  $\lambda_t dt$  where  $\lambda_t$  stands for the intensity of the jump process. Variable  $Y_{q_t-}$  is a i.i.d. random variable drawn from  $\ln(Y_{q_t-}) \sim \Phi(\mu_y, \sigma_y^2)$  at time  $t$  where  $\mu_y$  stands for the mean jump size and  $\sigma_y$  denotes the standard deviation of jumps. In case the random variable  $Y_{q_t-}$  is greater than one, an upward shift in the bank's asset value can be observed. If the value is smaller than one a downward jump takes place. Given that the risk-neutral expected proportional jump  $k_t$  is defined as  $k_t = E_t^{\mathbb{Q}}[Y_{q_t-} - 1]$ , one can determine  $k_t$  with the following formula:  $k_t = \exp(\mu_y + \frac{1}{2}\sigma_y^2) - 1$ . Thus, the risk-neutral expected change in  $A^*$  from the jump element  $(Y_{q_t-} - 1)dq_t$  equals  $\lambda_t k_t dt$  in  $dt$ . To sum up, the value development of a bank's asset portfolio  $A_t^*$  follows largely a continuous process. But disruptive jumps may occur as illustrated below in the graph 4.1.

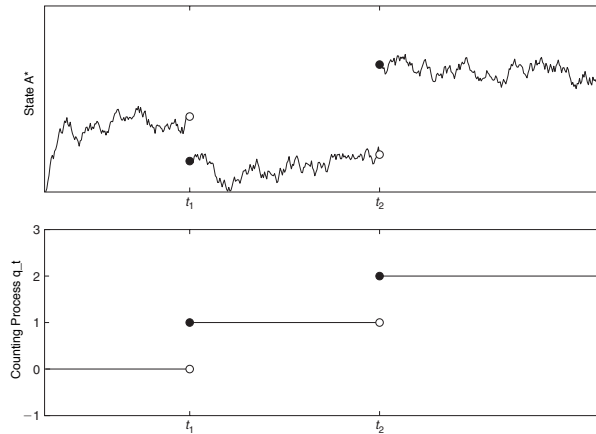


Figure 4.1: The first graph shows two jumps in the state variable  $A^*$  at discrete time points. Additionally, the corresponding Poisson counting process  $q_t$  is highlighted in the second graph. (Ait-Sahalia and Hansen, 2009)

The risk-neutral process of bank assets  $A_t$  including the net cashflows is equal to the assets' rate of return less interest payments  $r_t$  respectively premium payments  $h_t$  to deposit holders proportionally to their deposits  $D_t$ . Furthermore, one has to subtract the coupon payments  $c_t$  to CoCo investors proportionally to the face value  $B$ .

$$dA_t = [(r_t - \lambda k) A_t - (r_t + h_t) D_t - c_t B] dt + \sigma A_t dz + (Y_{q_t-} - 1) A_t dq \quad (4.20)$$

By substituting variable  $x_t$  with  $A_t/D_t$  and anticipating the deposit growth process  $g(x_t - \hat{x})$  as pointed out by equation 4.31, the risk neutral process of the asset-to-deposit ratio equals:

$$\begin{aligned} \frac{dx_t}{x_t} &= \frac{dA_t}{A_t} - \frac{dD_t}{D_t} \\ &= \left[ (r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g(x_t - \hat{x}) \right] dt + \sigma dz + (Y_{q_t-} - 1) dq_t \end{aligned} \quad (4.21)$$

with

$$b_t = \frac{B}{D_t} \quad (4.22)$$

Lastly, an application of Itô's lemma for jump-diffusion processes leads to the following formula for the asset-to-deposit ratio process:

$$\begin{aligned} d \ln(x_t) &= \left[ (r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g(x_t - \hat{x}) - \frac{1}{2} \sigma^2 \right] dt \\ &\quad + \sigma dz + \ln Y_{q_t-} dq_t \end{aligned} \quad (4.23)$$

## Default-Free Term Structure

Pennacchi (2010) applies the term-structure specifications of Cox et al. (1985) to model the risk-neutral process of the instantaneous risk-free interest rate  $dr_t$  which is defined as follows:

$$dr_t = \kappa (\bar{r} - r_t) dt + \sigma_r \sqrt{r_t} d\zeta \quad (4.24)$$

Note that  $\kappa$  is the speed of convergence,  $\bar{r}$  is the long-run equilibrium interest rate,  $r_t$  is the continuous short-term interest rate,  $\sigma_r$  is the instantaneous volatility and  $d\zeta$  is a Brownian motion.

A zero bond can be priced using the Cox et al. (1985) specifications under the no-arbitrage assumption. This implies that the price of a risk-free zero bond at time  $t$  that pays the amount of €1 in  $\tau = T - t$  is given by:

$$P(r_t, \tau) = A(\tau) \exp[-B(\tau) r_t] \quad (4.25)$$

with

$$A(\tau) = \left\{ \frac{2\theta \exp\left[(\theta + \kappa) \frac{\tau}{2}\right]}{(\theta + \kappa) [\exp(\theta\tau) - 1] + 2\theta} \right\}^{2\kappa\bar{r}/\sigma_r^2}$$

$$B(\tau) = \frac{2[\exp(\theta\tau) - 1]}{(\theta + \kappa) [\exp(\theta\tau) - 1] + 2\theta}$$

$$\theta = \sqrt{\kappa^2 + 2\sigma_r^2}$$

The cost of replication of a risk-free coupon bond that pays a continuous coupon of  $c_r dt$  is equal to a set of zero bonds which can be priced with equation 4.25. Therefore, the fair coupon rate  $c_r$  of such a coupon bond at time  $t$ , which is issued at par, equals:

$$\begin{aligned} c_r &= \frac{1 - A(\tau) \exp[-B(\tau) r_t]}{\int_0^\tau A(s) \exp[-B(s) r_t] ds} \\ &\approx \frac{1 - A(\tau) \exp[-B(\tau) r_t]}{\sum_{i=1}^{i=n} A(\Delta t \times i) \exp[-B(\Delta t \times i) r_t] \Delta t} \end{aligned} \quad (4.26)$$

with

$$n = \frac{\tau}{\Delta t} \quad (4.27)$$

## Deposits and Insurance Premium

Bank deposits are not riskless because depositors may suffer losses if a bank's asset value  $A_t$  is worth less than the deposits  $D_t$ . That said, one can assume that a bank is closed by the deposit insurer when the asset-to-deposit ratio  $x_t$  is less or equal to one. A bank might become distressed due to continuous downward movements in its asset value. Then, the bank will be shut down with  $A_{t_b} = D_t$  and subsequently, depositors will not face any loss. However, depositors may experience severe losses when a downward jump in asset value happens at a discrete point in time,  $\hat{t}$ . It may be that the downward jump in asset value exceeds the bank's capital. If such a jump occurs the instantaneous proportional loss to deposits will equal  $(D_t - Y_{q_t^-} A_{\hat{t}^-}) / D_t$ .

The fair deposit insurance premium  $h_t$  for deposit holders can be derived with equation 4.28. The equation illustrates that  $h_t$  is closely related to the asset-to-deposit ratio  $x_t$ :

$$h_t = \lambda \left[ \Phi(-d_1) - x_{t^-} \exp\left(\mu_y + \frac{1}{2}\sigma_y^2\right) \Phi(-d_2) \right] \quad (4.28)$$



with

$$d_1 = \frac{\ln(x_{t-}) + \mu_y}{\sigma_y} \quad (4.29)$$

$$d_2 = d_1 + \sigma_y \quad (4.30)$$

The model assumes that a bank pays continuously a total interest and deposit premium of  $(r_t + h_t) D_t dt$  to each depositor. Hence, one can recognize that the deposits of the bank change only because of comparatively higher deposit inflows than outflows. Empirical research of Adrian and Shin (2010) suggests that banks have a target capital ratio and that deposit growth is positively related to the bank's current asset-to-deposit ratio:

$$\frac{dD_t}{D_t} = g(x_t - \hat{x}) dt \quad (4.31)$$

$\hat{x} > 1$  is a bank's target asset-to-deposit ratio with  $g$  being a positive constant. Whenever the actual asset-to-deposit ratio is higher than its target,  $x_t > \hat{x}$ , a bank will shrink its balance sheet. Thus, the deposit growth rate  $g(x_t - \hat{x})$  in the time interval  $dt$ , leads to a mean-reverting tendency for the bank's asset-to-deposit ratio  $x_t$ .

### Equity and Conversion Threshold

As stated originally, the conversion of a CoCo at time  $t_c$  occurs when the asset-to-deposit ratio  $x_{t_c}$  meets the trigger level  $\bar{x}_{t_c}$ . The conversion threshold can also be expressed relative to the original equity-to-deposits ratio  $\bar{e}$ . This is favourable because the equity value is directly observable in the market whereas the asset value is not. The relationship between the equity threshold  $\bar{e}$  and the asset-to-deposit threshold  $\bar{x}_{t_c}$  can be summarized as follows:

$$\bar{e} = \frac{E_{t_c}}{D_{t_c}} = \frac{A_{t_c} - D_{t_c} - pB}{D_{t_c}} = \bar{x}_{t_c} - 1 - pb_{t_c} \quad (4.32)$$

Hence, it is possible to specify exactly the conversion trigger of a CoCo bond. This will be important for the valuation part.

### CoCos

The valuation of a CoCo can be accomplished with a Monte Carlo simulation of both the asset and the deposit process. Along the asset-to-deposit ratio process, the CoCo pays coupons and the nominal at maturity unless the CoCo has not been

triggered. If the trigger event occurs the conversion amount is paid out. (Wilkins and Bethke, 2014) The price of the CoCo  $V^{st}$  is equal to the risk-neutral expectation of the aforementioned cashflows as derived by Pennacchi (2010):

$$V_0^{st} = E_0^{\mathbb{Q}} \left[ \int_0^T \exp \left( - \int_0^t r_s ds \right) v(t) dt \right] \quad (4.33)$$

Please note that  $v(t)$  stands for a CoCo's coupon payment at date  $t$  which equals  $c_t B$  as long as the CoCo has not been triggered. If the CoCo does not convert until maturity  $T$ , a final payout of  $B$  will be performed. However, if the CoCo triggers early at time  $t_c$ , there is the one-time cashflow of  $pB$ . Parameter  $p$  determines the maximum conversion amount of new equity per par value of contingent capital. Thereafter,  $v(t)$  is zero.

### 4.3.2 Parameter Classification and Adjustment

|            | Description  | Usage         | Source        |
|------------|--|---------------|---------------|
| $T$        | CoCo maturity  | Static input  | Term sheet    |
| $B$        | CoCo nominal   | Static input  | Term sheet    |
| $c$        | CoCo coupon rate   | Static input  | Term sheet    |
| $p$        | CoCo nominal conversion factor   | Static input  | Term sheet    |
| $x_0$      | Initial asset-to-deposit ratio   | Dynamic input | Balance sheet |
| $\hat{x}$  | Target asset-to-deposit ratio  | Static input  | Assumption    |
| $g$        | Mean-reversion speed   | Static input  | Assumption    |
| $\sigma$   | Annual asset return volatility   | Static input  | Assumption    |
| $\lambda$  | Jump intensity in asset return process   | Static input  | Assumption    |
| $\mu_y$    | Mean jump size in asset return process   | Static input  | Assumption    |
| $\sigma_y$ | Jump volatility in asset return process  | Static input  | Assumption    |
| $r_0$      | Initial risk-free interest rate  | Dynamic input | Market data   |
| $\bar{r}$  | Long-term risk-free interest rate  | Static input  | Market data   |
| $\sigma_r$ | Interest rate volatility   | Static input  | Market data   |
| $\kappa$   | Speed of convergence   | Static input  | Assumption    |
| $\rho$     | Correlation between Brownian motion for asset returns and interest rate process            | Static input  | Market data   |
| $\bar{e}$  | Conversion threshold of the market value of original shareholders' equity to deposit value | Static input  | Balance sheet |
| $b_0$      | Ratio of the contingent capital's nominal to the initial value of deposits                 | Dynamic input | Balance sheet |

Table 4.5: Parameter classification of the structural approach (Wilkins and Bethke, 2014)

### 4.3.3 Model Application

|            | Value   | Comment  |
|------------|---------|--|
| $T$        | 10yrs   | Maturity   |
| $B$        | 100.00% | Nominal  |
| $c$        | 6.00%   | Annual coupon rate   |
| $p$        | 1       | Nominal conversion factor  |
| $x_0$      | 1.15    | Initial asset-to-deposit ratio   |
| $\hat{x}$  | 1.1     | Target asset-to-deposit ratio  |
| $g$        | 0.5     | Mean-reversion speed   |
| $\sigma$   | 2.00%   | Annual asset return volatility   |
| $\lambda$  | 1       | Jump intensity in asset return process   |
| $\mu_y$    | -1.00%  | Mean jump size in asset return process   |
| $\sigma_y$ | 2.00%   | Jump volatility in asset return process  |
| $r_0$      | 1.00%   | Risk-free interest rate  |
| $\bar{r}$  | 6.90%   | Long-term risk-free interest rate  |
| $\sigma_r$ | 7.00%   | Interest rate volatility   |
| $\kappa$   | 11.40%  | Speed of convergence   |
| $\rho$     | -20.00% | Correlation between Brownian motion for asset returns and interest rate process            |
| $\bar{e}$  | 2.00%   | Conversion threshold of the market value of original shareholders' equity to deposit value |
| $b_0$      | 4.00%   | Ratio of contingent capital's nominal to the initial deposit value                         |

Table 4.6: Parameter specification of structural approach application (Alvemar and Ericson, 2012; Pennacchi, 2010)

# Chapter 5

## Dynamics and Sensitivity Analysis

### 5.1 Credit Derivative Approach

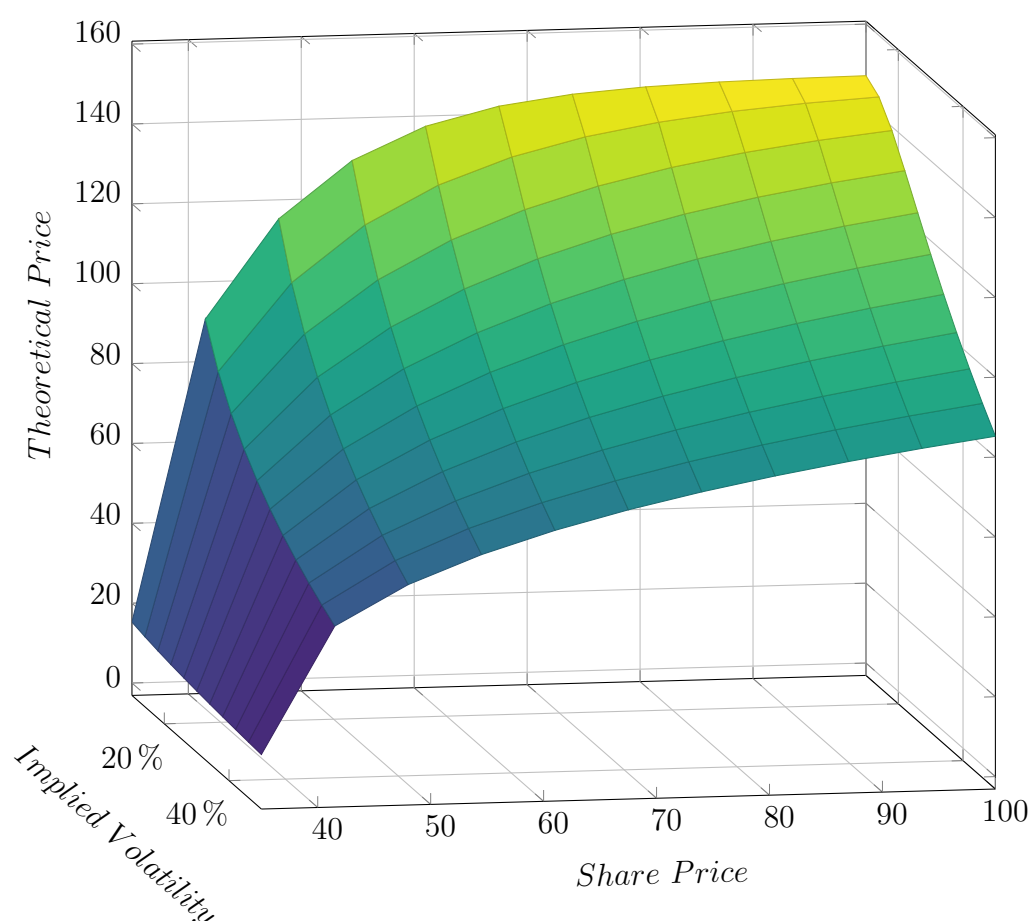


Figure 5.1: CoCo price  $V^{cd}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011) as function of share price  $S$  and implied volatility  $\sigma$

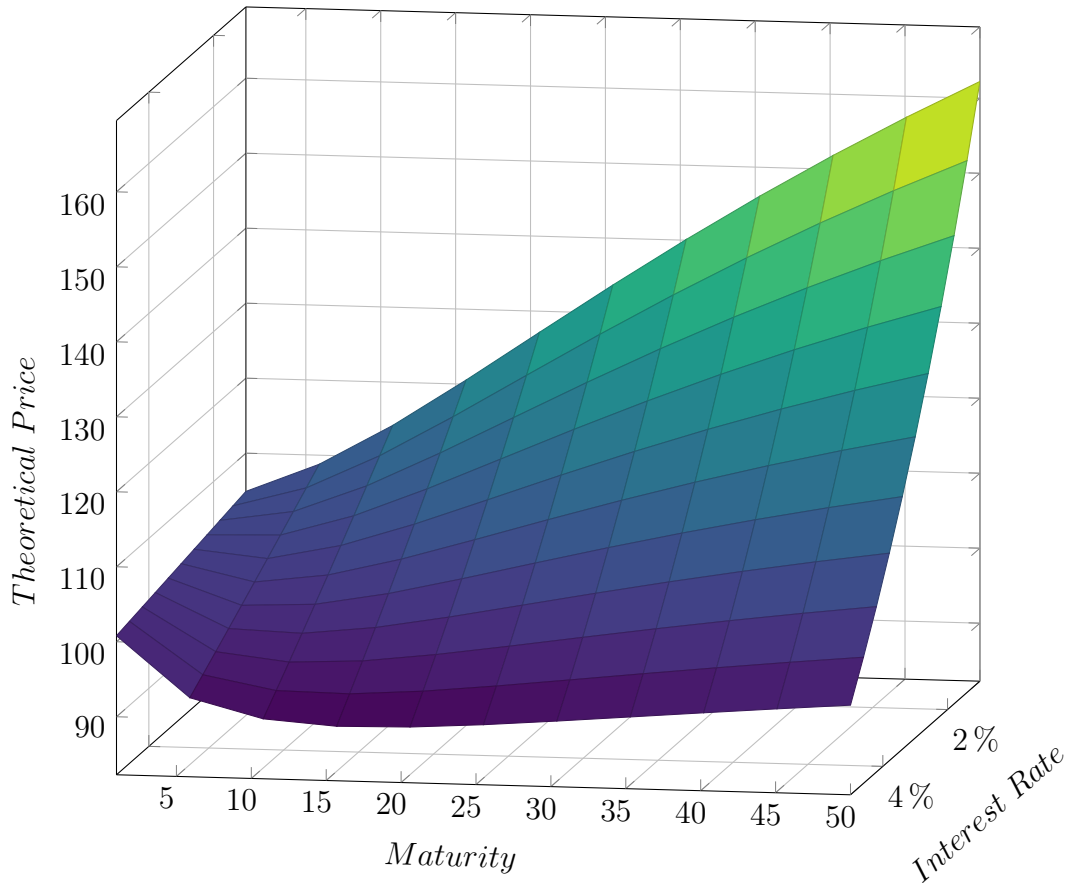


Figure 5.2: CoCo price  $V^{cd}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011) as function of maturity  $T$  and risk-free interest rate  $r$

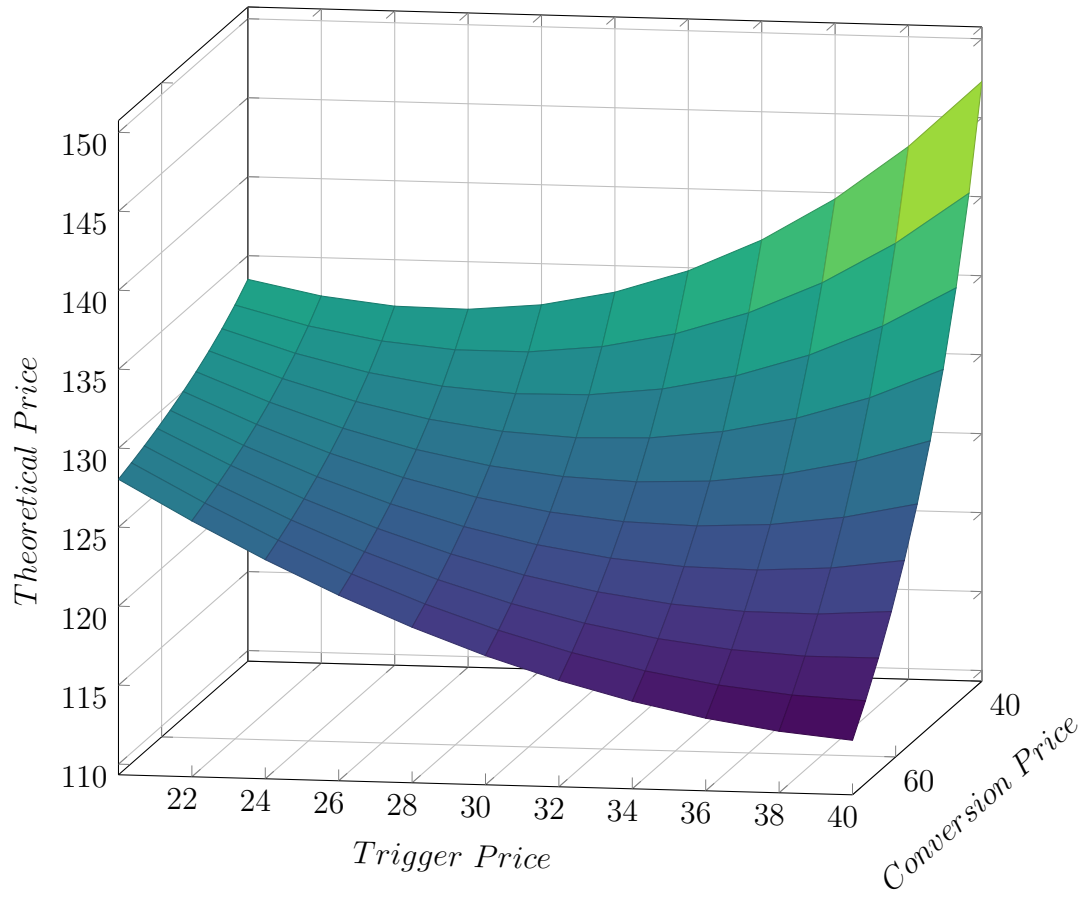


Figure 5.3: CoCo price  $V^{cd}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011) as function of trigger price  $S^*$  and conversion price  $C_p$

## 5.2 Equity Derivative Approach

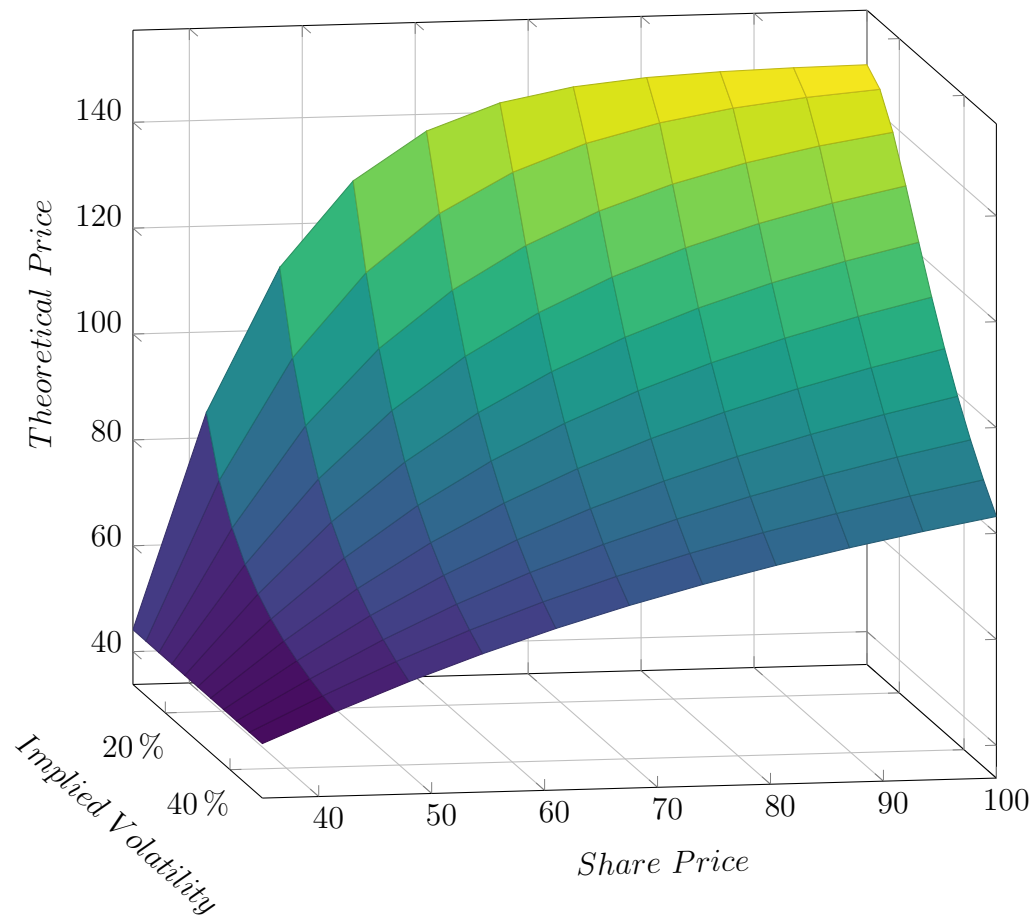


Figure 5.4: CoCo price  $V^{ed}$  pursuant to the equity derivative approach (De Spiegeleer and Schoutens, 2011) as function of share price  $S$  and volatility  $\sigma$



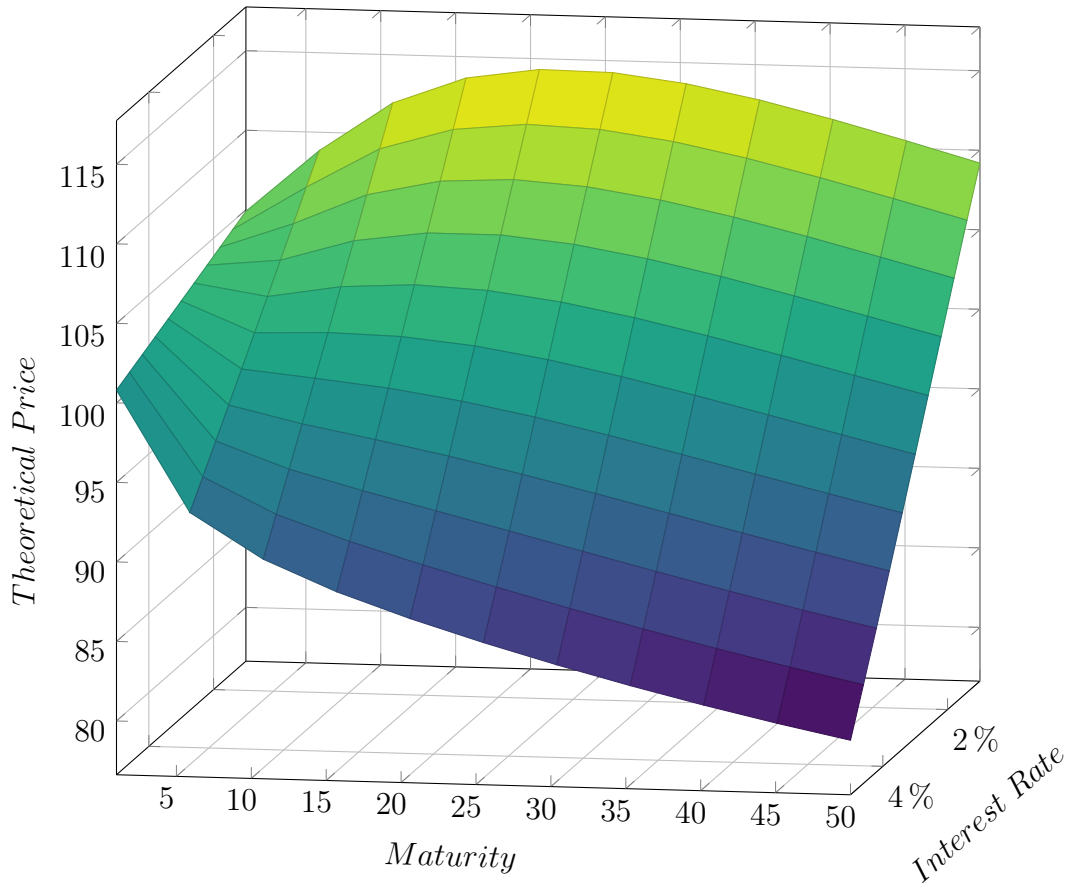


Figure 5.5: CoCo price  $V^{ed}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011) as function of maturity  $T$  and risk-free interest rate  $r$

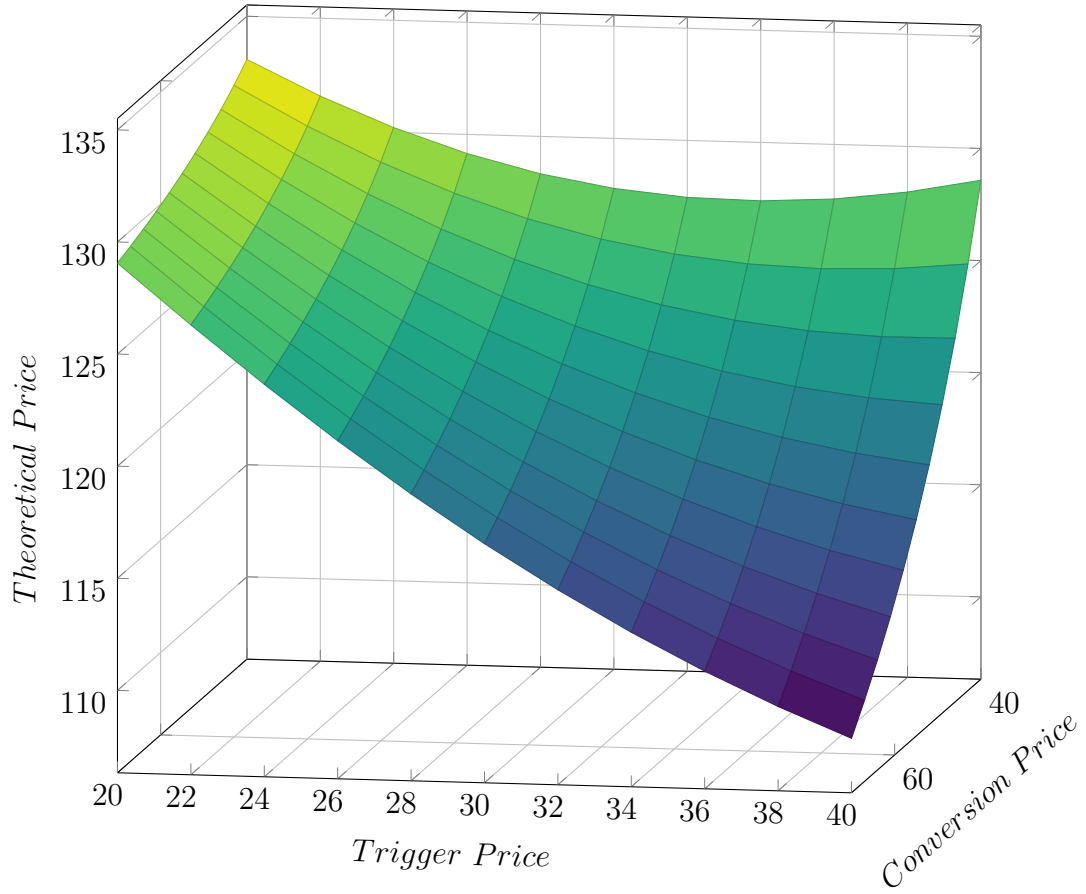


Figure 5.6: CoCo price  $V^{ed}$  pursuant to the credit derivative approach (De Spiegeleer and Schoutens, 2011) as function of trigger price  $S^*$  and conversion price  $C_p$

### 5.3 Structural Approach

1

---

<sup>1</sup>The Monte-Carlo simulation runs in the Amazon Elastic Compute Cloud (EC2) as the service provides a re-sizable compute capacity which is key to quickly scale the computing requirements. If one wants to replicate the simulations it is recommended to follow the instructions of Shekel (2015) to set up a Rstudio server on Amazon EC2.

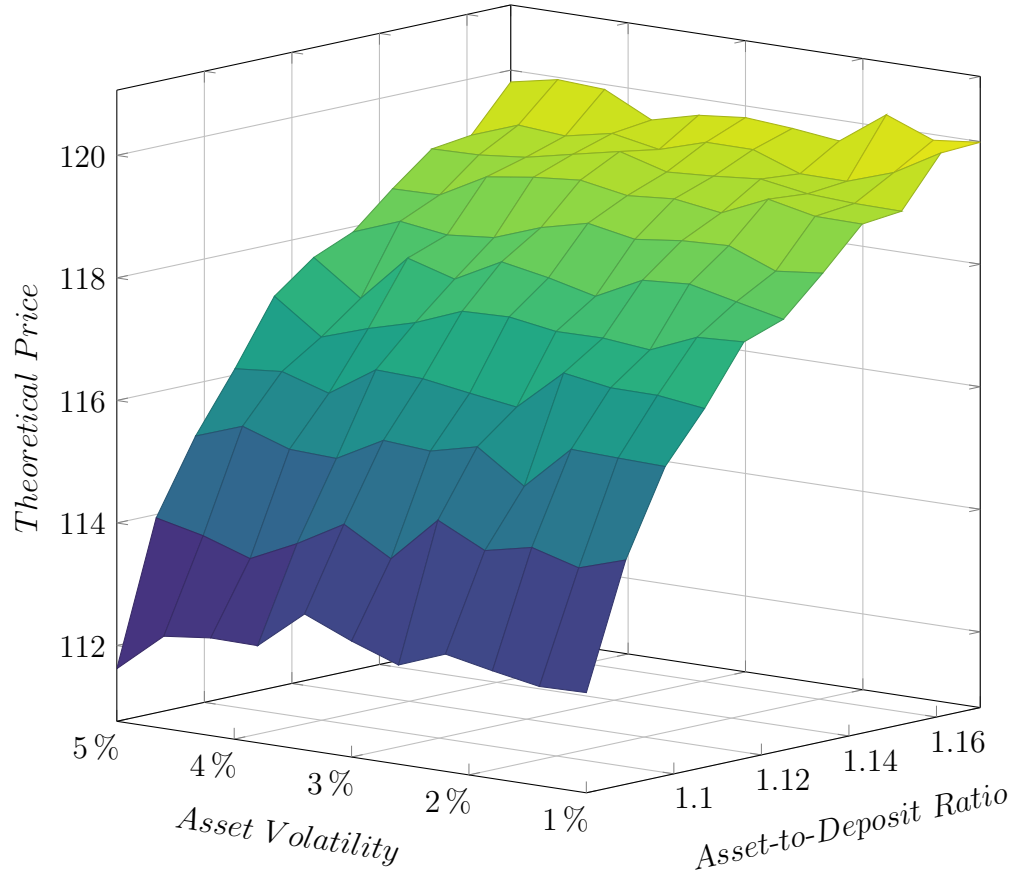


Figure 5.7: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of initial asset-to-deposit ratio  $x_0$  and asset volatility  $\sigma$

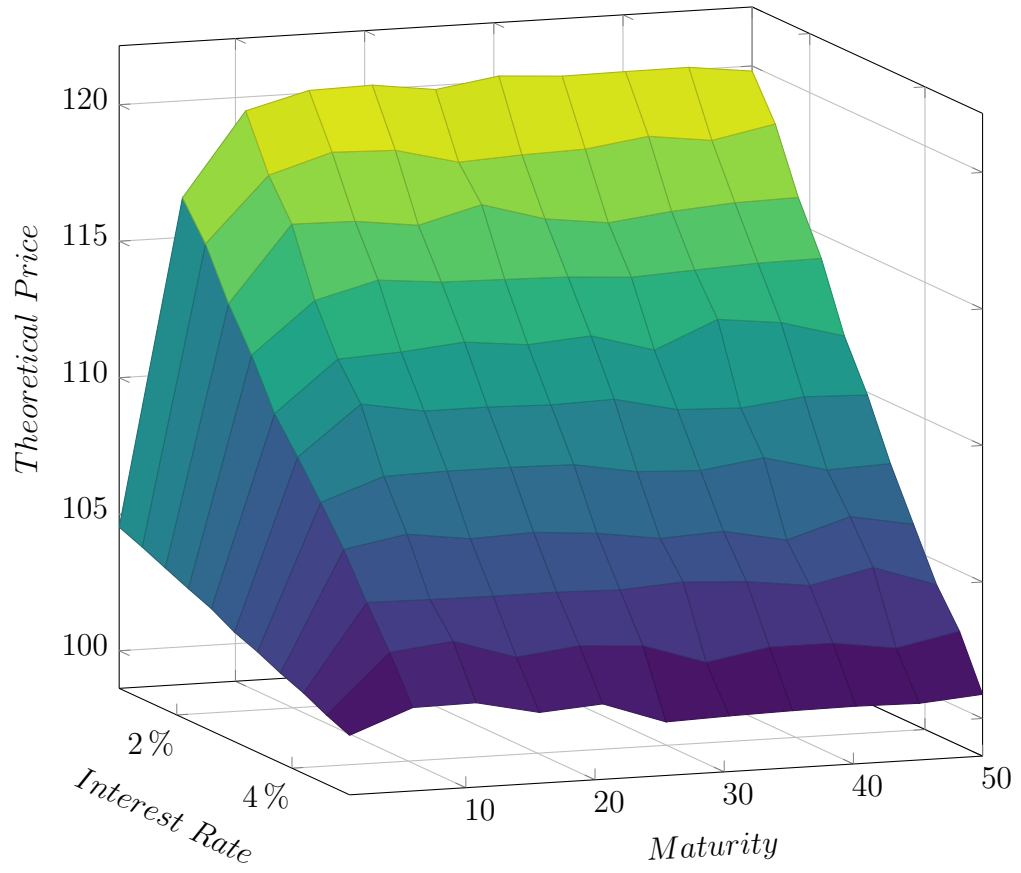


Figure 5.8: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of maturity  $T$  and interest rate  $r$

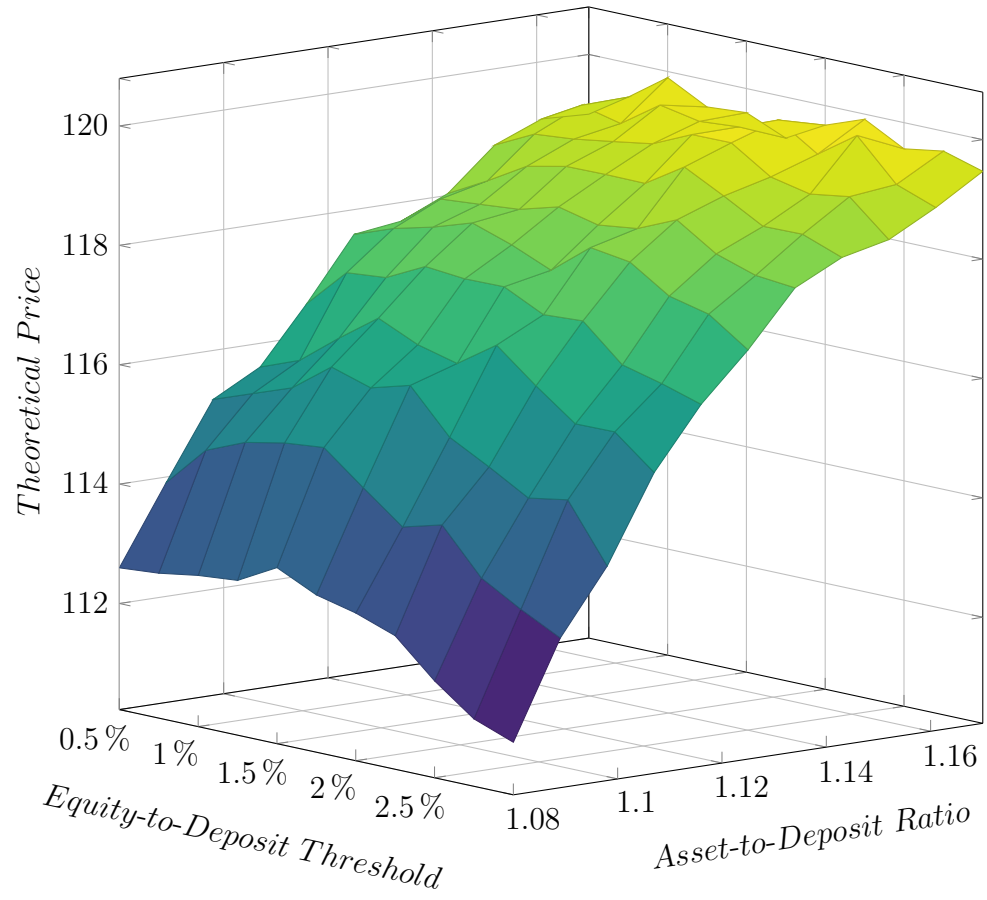


Figure 5.9: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of initial asset-to-deposit ratio  $x_0$  and equity-to-deposit threshold  $\bar{e}$

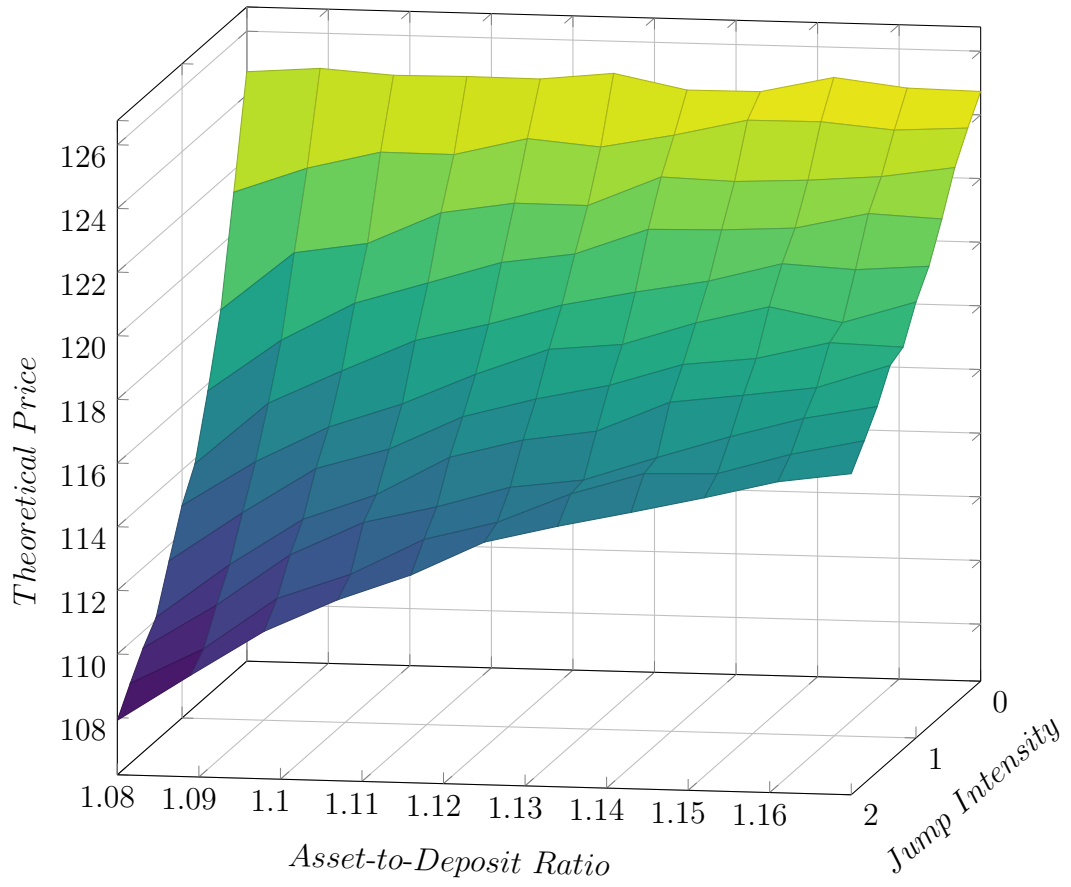


Figure 5.10: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of initial asset-to-deposit ratio  $x_0$  and jump intensity  $\lambda$

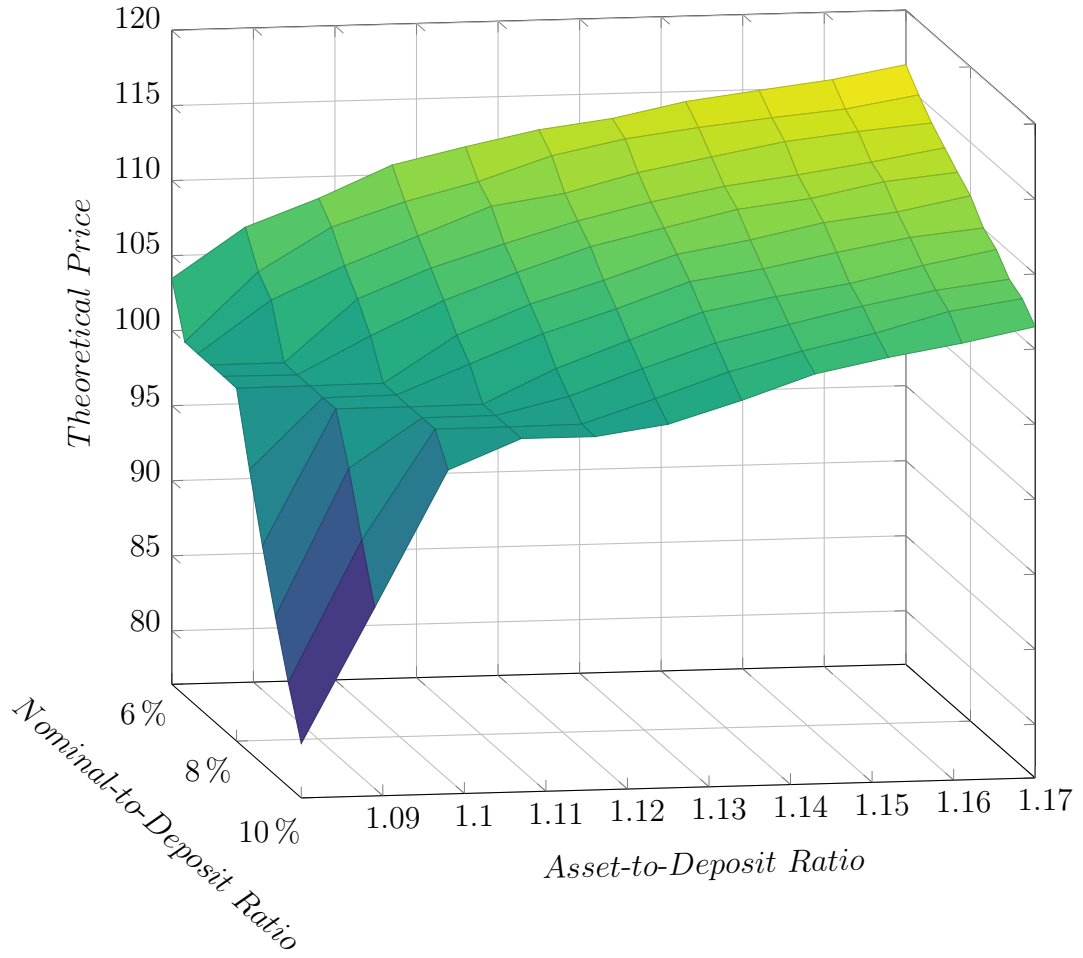


Figure 5.11: CoCo price  $V^{sa}$  pursuant to the structural approach (Pennacchi, 2010) as function of initial asset-to-deposit ratio  $x_0$  and initial ratio of contingent capital's nominal to the initial value of deposits

# Chapter 6

## Empirical Analysis and Model Comparison

### 6.1 Data Description

#### 6.1.1 Deutsche Bank

### 6.2 Model Parametrization

### 6.3 Model Comparison

#### 6.3.1 Qualitative Analysis

#### 6.3.2 Quantitative Analysis



## Chapter 7

## Conclusion

# Appendix A

## Sample Title

## Appendix B

### Code - Credit Derivative Approach

The following source code is an implementation of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2011) written in R.

```
1 # Price of Contingent Convertible Bond
2 price_coco_cd <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma){
3
4   spread_coco <- calc_spread_coco(t, T, S_t, S_star, C_p, r, q, sigma)
5   V_t_coco <- N * exp(-(r + spread_coco) * (T - t))
6
7   for (t in 1:T){
8     V_t_coco <- V_t_coco + c_i * exp(-(r + spread_coco) * t)
9   }
10  return(V_t_coco)
11 }
12
13 # Calculation of Trigger Probability
14 calc_p_star <- function(t, T, S_t, S_star, r, q, sigma){
15   p_star <- pnorm((log(S_star / S_t) - calc_mu(r, q, sigma) * (T - t)) /
16     (sigma * sqrt(T - t))) + (S_star / S_t)^(2 * calc_mu(r, q, sigma) /
17     sigma^2) * pnorm((log(S_star / S_t) + calc_mu(r, q, sigma) * (T - t)
18     )) / (sigma * sqrt(T - t)))
19   return(p_star)
20 }
21
22 # Calculation of Drift of Underlying
23 calc_mu <- function(r, q, sigma){
24   mu <- r - q - sigma^2 / 2
25   return(mu)
26 }
27
28 # Spread of CoCo Bond
29 calc_spread_coco <- function(t, T, S_t, S_star, C_p, r, q, sigma){
30   spread_coco <- - log(1 - calc_p_star(t, T, S_t, S_star, r, q, sigma))
31     / (T - t) * (1 - S_star / C_p)
32   return(spread_coco)
33 }
34
35 # Pricing Example
```

```
32 price_coco_cd(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <-  
    7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3)
```

# Appendix C

## Code - Equity Derivative Approach

The following source code is an implementation of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2011) written in R.

```
1 # Price of Contingent Convertible Bond
2 price_coco_ed <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma,
3   alpha){
4   V_t_ed <- price_cb(t, T, c_i, r, N) - price_dibi(t, T, S_t, S_star, c_i,
5     r, q, sigma, alpha) + price_difwd(t, T, S_t, S_star, C_p, r, N, q,
6     sigma, alpha)
7
8   return(V_t_ed)
9 }
10
11 # Price of Corporate Bond
12 price_cb <- function(t, T, c_i, r, N){
13   V_t_cb <- N * exp(-r * (T - t))
14
15   for (t in 1:T){
16     V_t_cb <- V_t_cb + c_i * exp(-r * t)
17   }
18
19   return(V_t_cb)
20 }
21
22 # Price of Binary Option
23 price_dibi <- function(t, T, S_t, S_star, c_i, r, q, sigma, alpha){
24   V_t_dibi <- 0
25
26   i <- t
27   k <- T
28
29   for (i in 1:k) {
30     V_t_dibi <- V_t_dibi + c_i * exp(- r * i) * (pnorm(- calc_x_1_i(S_t, S_star, sigma, r, q, i) + sigma * sqrt(i)) + (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm ( calc_y_1_i(S_t, S_star, sigma, r, q, i) - sigma * sqrt(i)))
31   }
32 }
```

```

30 V_t_dibi <- alpha * V_t_dibi
31
32 return(V_t_dibi)
33 }
34
35 # Price of Down-And-In Forward
36 price_difwd <- function(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha){
37   V_t_difwd <- calc_conversion_rate(C_p, N, alpha) * (S_t * exp(- q * (T
    - t)) * (S_star / S_t) ^ (2 * calc_lambda(r, q, sigma)) * pnorm(
    calc_y_1(t, T, S_t, S_star, r, q, sigma)) - C_p * exp(- r * (T - t))
    * (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm(calc_y_
    1(t, T, S_t, S_star, r, q, sigma) - sigma * sqrt(T - t)) - C_p * exp
    (- r * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma) +
    sigma * sqrt(T - t)) + S_t * exp(- q * (T - t)) * pnorm(- calc_x_1(
    t, T, S_t, S_star, r, q, sigma)))
38
39   return(V_t_difwd)
40 }
41
42 # Calculation of Conversion Rate
43 calc_conversion_rate <- function(C_p, N, alpha){
44   C_r <- alpha * N / C_p
45
46   return(C_r)
47 }
48
49 # Calculation of additional Parameters
50 calc_x_1_i <- function(S_t, S_star, sigma, r, q, t_i){
51   x_1_i <- log(S_t / S_star) / (sigma * sqrt(t_i)) + calc_lambda(r, q,
    sigma) * sigma * sqrt(t_i)
52
53   return(x_1_i)
54 }
55
56 calc_y_1_i <- function(S_t, S_star, sigma, r, q, t_i){
57   y_1_i <- log(S_star / S_t) / (sigma * sqrt(t_i)) + calc_lambda(r, q,
    sigma) * sigma * sqrt(t_i)
58
59   return(y_1_i)
60 }
61
62 calc_lambda <- function(r, q, sigma){
63   lambda <- (r - q + sigma^2 / 2) / sigma^2
64
65   return(lambda)
66 }
67
68 calc_x_1 <- function(t, T, S_t, S_star, r, q, sigma){
69   x_1 <- log(S_t / S_star) / (sigma * sqrt(T - t)) + calc_lambda(r, q,
    sigma) * sigma * sqrt(T - t)
70
71   return(x_1)
72 }
73

```

```

74 calc_y_1 <- function(t, T, S_t, S_star, r, q, sigma){
75   y_1 <- log(S_star / S_t) / (sigma * sqrt(T - t)) + calc_lambda(r, q,
      sigma) * sigma * sqrt(T - t)
76
77   return(y_1)
78 }
79
80 # Pricing Example
81 price_coco_ed(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <-
      7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3, alpha <- 1)

```

# Appendix D

## Code - Structural Approach

The following source code is an implementation of the Structural Approach (Pennacchi, 2010) with support from Quantnet (2011) in translating the source code of Pennacchi (2010) from GAUSS to R.

```
1 # Price of Contingent Convertible Bond
2 price_coco_sa <- function(T , nsimulations , rho , kappa , r_bar, r0,
   sigma_r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_x, x0,
   B, coupon){
3
4   ndays <- T * 250
5   dt <- T / ndays
6
7   # Get Brownian motions
8   result <- sim_corrProcess(T, nsimulations , rho, ndays, dt)
9   dz_1 <- result$dz_1
10  dz_2corr <- result$dz_2corr
11
12  # Simulate Cox et al. (1985) term-structure process
13  r <- sim_interstrate(kappa, r_bar, r0, sigma_r, dz_2corr, ndays,
   nsimulations, dt)
14
15  # Simulate price of contingent convertible bond with a Monte-Carlo
   simulation
16  V_t_sa <- get_price(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y,
   sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_x, x0, B, coupon) *
   100
17
18  return(V_t_sa)
19 }
20
21 # Create correlated Brownian motions for asset and interest rate process
22 sim_corrProcess <- function(T, nsimulations , rho, ndays, dt){
23
24   # Compute the Choleski factorization of a real symmetric positive-
   definite square matrix.
25   chol_RHO <- t(chol(matrix(c(1, rho, rho, 1), nrow = 2)))
26 }
```



```

27 # Random generation for the normal distribution with mean equal to 0
    and standard deviation equal to 1
28 dz_1 <- matrix(1, ndays, nsimulations)
29 dz_2 <- matrix(1, ndays, nsimulations)
30 for(j in 1:nsimulations)
31 {
32   dz_1[ , j] <- rnorm(ndays) * sqrt(dt)
33   dz_2[ , j] <- rnorm(ndays) * sqrt(dt)
34 }
35
36 # Create correlated Brownian motions using Cholesky-decomposition for
    the Cox et al. (1985) term-structure process
37 dz_2corr <- matrix(1, ndays, nsimulations)
38 for(j in 1:nsimulations)
39 {
40   for(i in 1:ndays)
41   {
42     dz_2corr[i, j] <- dz_1[i, j] * chol_RHO[2, 1] + dz_2[i, j] * chol_
        RHO[2, 2]
43   }
44 }
45
46 return(list("dz_1" = dz_1, "dz_2corr" = dz_2corr))
47 }
48
49 # Simulate Cox et al. (1985) term-structure process
50 sim_interestrates <- function(kappa, r_bar, r0, sigma_r, dz_2corr, ndays,
    nsimulations, dt){
51   r <- matrix(r0, ndays + 1, nsimulations)
52
53   for(j in 1:nsimulations)
54   {
55     for(i in 1:ndays)
56     {
57       r[i + 1, j] <- r[i, j] + kappa * (r_bar - r[i, j]) * dt + sigma_r
        * sqrt(r[i, j]) * dz_2corr[i, j]
58     }
59   }
60
61   return(r)
62 }
63
64 get_price <- function(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y,
    sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_x, x0, B, coupon){
65
66   # Define parametres
67   phi <- matrix(rbinom( ndays %%% nsimulations, 1, dt * lambda), ndays,
    nsimulations)
68
69   ln_Y <- matrix(rnorm(ndays %%% nsimulations, mu_Y, sigma_Y), ndays,
    nsimulations)
70
71   # Ratio of contingent capital's nominal to the value of deposits
72   b <- matrix(b0, ndays + 1, nsimulations)

```

```

73
74 h <- matrix(1, ndays, nsimulations)
75
76 # Paramter for jump diffusion process
77 k <- exp(mu_Y + 0.5 * sigma_Y^2) - 1
78
79 # Target asset-to-deposit ratio
80 x_bar0 <- 1 + e_bar + p * b0
81 x_bar <- matrix(x_bar0, ndays + 1, nsimulations)
82
83 # Asset-to-deposit ratio
84 x <- matrix(x0, ndays + 1, nsimulations)
85 ln_x0 <- matrix(log(x0), ndays + 1, nsimulations)
86 ln_x <- ln_x0
87
88 trigger_dummy <- matrix(1, ndays + 1, nsimulations)
89
90 # Simulate asset-to-deposit ratio and trigger events
91 for(j in 1:nsimulations)
92 {
93   for(i in 1:ndays)
94   {
95     d_1 <- (ln_x[i, j] + mu_Y) / sigma_Y
96     d_2 <- d_1 + sigma_Y
97
98     h[i, j] <- lambda * (pnorm(- d_1) - exp(ln_x[i, j]) * exp(mu_Y +
99       0.5 * sigma_Y^2) * pnorm(-d_2))
100
101     b[i + 1, j] <- b[i, j] * exp(- g * (exp(ln_x[i, j]) - x_hat) * dt)
102
103     ln_x[i + 1, j] <- ln_x[i, j] + ( (r[i, j] - lambda * k) - (r[i, j]
104       + h[i, j] + coupon * b[i, j]) / exp(ln_x[i, j]) - g * (exp(ln_x[i,
105       j]) - x_hat) - 0.5 * sigma_x^2) * dt + sigma_x * sqrt(dt) * dz_1[i,
106       j] + ln_Y[i, j] * phi[i, j]
107
108     x[i + 1, j] <- exp(ln_x[i + 1, j])
109
110     x_bar[i + 1, j] <- 1 + e_bar + p * b[i + 1, j]
111
112     if(x[i + 1, j] >= x_bar[i + 1, j] && trigger_dummy[i, j] > 0.5)
113     {
114       trigger_dummy[i + 1, j] <- 1
115     } else
116     {
117       trigger_dummy[i + 1, j] <- 0
118     }
119   }
120 }
121
122 cashflows <- matrix(c(rep(coupon * dt, ndays - 1), B), ndays,
123   nsimulations) * trigger_dummy[1:ndays, ]
124
125 # Determine cashflows for each simulation
126 for(j in 1:nsimulations){

```

```

122   for(i in 2:ndays){
123     if(cashflows[i, j] == 0 && p * b[sum(trigger_dummy[, j]) + 1, j]
124     <= x[sum(trigger_dummy[, j]) + 1, j] - 1 ){
125       cashflows[i, j] <- p * B
126       break
127     }
128     else if(cashflows[i, j] == 0 && 0 < x[sum(trigger_dummy[, j]) +
129     1, j] - 1 && x[sum(trigger_dummy[, j]) + 1, j] - 1 < p * b[sum(
130     trigger_dummy[, j]) + 1, j]){
131       cashflows[i, j] <- (x[sum(trigger_dummy[, j]) + 1, j] - 1) * B
132       / b[sum(trigger_dummy[, j]) + 1, j]
133       break
134     }
135     else{
136       cashflows[i, j] <- cashflows[i, j]
137     }
138   }
139   list_discounted_cashflows <- rep(0, nsimulations)
140
141   # Discount cashflows for each simulation
142   for(j in 1:nsimulations)
143   {
144     disc_cashflows <- 0
145     int_r <- 0
146
147     for(i in 1:ndays)
148     {
149       int_r <- int_r + r[i, j] * dt
150       disc_cashflows <- disc_cashflows + exp(- int_r) * cashflows[i, j]
151     }
152     list_discounted_cashflows[j] <- disc_cashflows
153   }
154
155   # Calculate arithmetic average over all simulations as present value
156   # of contingent convertibles bond
157   V_t_sa <- mean(list_discounted_cashflows)
158
159   return(V_t_sa)
160 }
161
162 # Pricing Example
163 price_coco_sa(T <- 10, nsimulations <- 10000, rho <- - 0.2, kappa <-
164   0.114, r_bar <- 0.069, r0 <- 0.01, sigma_r <- 0.07, mu_Y <- -0.01,
165   sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat <- 1.1, b0 <- 0.04, p
166   <- 1, e_bar <- 0.02, sigma_x <- 0.02, x0 <- 1.15, B <- 1, coupon <-
167   0.06)

```

# Appendix E

## Code - Sensitivity Analysis

### E.1 Credit Derivative Approach

The following source code is an implementation of the sensitivity analysis of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2011) written in R.

```
1 source('CreditDerivativeApproach.R')
2
3 # CoCo price  $V^{\text{cd}}$  as function of share price  $S$  and volatility  $\sigma$ 
4 createData_CD_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
5   data <- matrix(1, 121, 3)
6   counter <- 1
7   for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))
8   {
9     for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
10    {
11      data[counter, 1] <- S_increment
12      data[counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- S_increment, S_star <- 35, C_p <- 65, c_i <- 6, r <- 0.01, N <- 100, q <- 0.02, sigma <- sigma_increment)
13      data[counter, 3] <- sigma_increment
14      counter <- counter + 1
15    }
16  }
17  write.table(data, file = "createData_CD_S_sigma.txt", row.names = FALSE, quote=FALSE)
18 }
19
20 # CoCo price  $V^{\text{cd}}$  as function of maturity  $T$  and risk-free interest rate  $r$ 
21 createData_CD_T_r <- function(T_min, T_max, r_min, r_max){
22   data <- matrix(1, 121, 3)
23   counter <- 1
24   for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
25   {
26     for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
27     {
```

```

28     data[counter, 1] <- T_increment
29     data[counter, 2] <- price_coco_cd(t <- 0, T <- T_increment, S_t <-
100, S_star <- 35, C_p <- 65, c_i <- 6, r <- r_increment, N <- 100,
q <- 0.02, sigma <- 0.3)
30     data[counter, 3] <- r_increment
31     counter <- counter + 1
32   }
33 }
34 write.table(data, file = "createData_CD-T-r.txt", row.names = FALSE,
quote=FALSE)
35 }
36
37 # CoCo price V^cd as function of trigger price S^* and conversion price
C_p
38 createData_CD_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p-
max){
39   data <- matrix(1, 121, 3)
40   counter <- 1
41   for(S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S-
star_max-S_star_min)/10)))
42   {
43     for(C_p_increment in seq(from=C_p_min, to=C_p_max, by=((C_p_max-C_p-
min)/10)))
44     {
45       data[counter, 1] <- S_star_increment
46       data[counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- 100, S-
star <- S_star_increment, C_p <- C_p_increment, c_i <- 6, r <- 0.01,
N <- 100, q <- 0.02, sigma <- 0.3)
47       data[counter, 3] <- C_p_increment
48       counter <- counter + 1
49     }
50   }
51   write.table(data, file = "createData_CD-Sstar-Cp.txt", row.names =
FALSE, quote=FALSE)
52 }
53
54 createData_CD-S-sigma(35.01, 100, 0.1, 0.5)
55 createData_CD-T-r(1, 50, 0.01, 0.05)
56 createData_CD-Sstar-Cp(20, 40, 40, 70)

```

## E.2 Equity Derivative Approach

The following source code is an implementation of the sensitivity analysis of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2011) written in R.

```

1 source('EquityDerivativeApproach.R')
2
3 # CoCo price V^ed as function of share price S and volatility sigma
4 createData_ED-S-sigma <- function(S_min, S_max, sigma_min, sigma_max){
5   data <- matrix(1, 121, 3)
6   counter <- 1
7   for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))

```

```

8 {
9   for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
10   {
11     data[counter, 1] <- S_increment
12     data[counter, 2] <- price_coco_ed(t <- 0, T <- 10, S_t <- S_increment, S_star <- 35, C_p <- 65, c_i <- 6, r <- 0.01, N <- 100, q <- 0.02, sigma <- sigma_increment, alpha <- 1)
13     data[counter, 3] <- sigma_increment
14     counter <- counter + 1
15   }
16 }
17 write.table(data, file = "createData_ED_S_sigma.txt", row.names = FALSE, quote=FALSE)
18 }
19
20 # CoCo price V^ed as function of maturity T and risk-free interest rate r
21 createData_ED_T_r <- function(T_min, T_max, r_min, r_max){
22   data <- matrix(1, 121, 3)
23   counter <- 1
24   for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
25   {
26     for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
27     {
28       data[counter, 1] <- T_increment
29       data[counter, 2] <- price_coco_ed(t <- 0, T <- T_increment, S_t <- 100, S_star <- 35, C_p <- 65, c_i <- 6, r <- r_increment, N <- 100, q <- 0.02, sigma <- 0.3, alpha <- 1)
30       data[counter, 3] <- r_increment
31       counter <- counter + 1
32     }
33   }
34   write.table(data, file = "createData_ED_T_r.txt", row.names = FALSE, quote=FALSE)
35 }
36
37 # CoCo price V^ed as function of trigger price S^* and conversion price C_p
38 createData_ED_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_max){
39   data <- matrix(1, 121, 3)
40   counter <- 1
41   for(S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S_star_max-S_star_min)/10)))
42   {
43     for(C_p_increment in seq(from=C_p_min, to=C_p_max, by=((C_p_max-C_p_min)/10)))
44     {
45       data[counter, 1] <- S_star_increment
46       data[counter, 2] <- price_coco_ed(t <- 0, T <- 10, S_t <- 100, S_star <- S_star_increment, C_p <- C_p_increment, c_i <- 6, r <- 0.01, N <- 100, q <- 0.02, sigma <- 0.3, alpha <- 1)
47       data[counter, 3] <- C_p_increment

```

```

48     counter <- counter + 1
49   }
50 }
51 write.table(data, file = "createData_ED_Sstar_Cp.txt", row.names =
  FALSE, quote=FALSE)
52 }
53
54 createData_ED_S_sigma(35.01, 100, 0.1, 0.5)
55 createData_ED_T_r(1, 50, 0.01, 0.05)
56 createData_ED_Sstar_Cp(20, 40, 40, 70)

```

## E.3 Structural Approach

The following source code is an implementation of the sensitivity analysis of the Structural Approach (Pennacchi, 2010) written in R.

```

1 source('StructuralApproach.R')
2
3 # CoCo price  $V^{st}$  as function of initial asset-to-deposit ratio  $x_0$  and
  volatility  $\sigma$ 
4 createData_SA_x0_sigma <- function(x0_min, x0_max, sigma_min, sigma_max)
  {
5   data <- matrix(1, 121, 3)
6   counter <- 1
7   for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/
  10)))
8   {
9     for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_
  max-sigma_min)/10)))
10    {
11      data[counter, 1] <- x0_increment
12      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 10000,
  rho <- - 0.2, kappa <- 0.114, r_bar <- 0.069, r0 <- 0.01, sigma_r <-
  0.07, mu_Y <- -0.01, sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat
  <- 1.1, b0 <- 0.04, p <- 1, e_bar <- 0.02, sigma_x <- 0.02, x0 <-
  1.15, B <- 1, coupon <- 0.06)
13      data[counter, 3] <- sigma_increment
14      counter <- counter + 1
15    }
16  }
17  write.table(data, file = "createData_SA_x0_sigma_2.txt", row.names =
  FALSE, quote=FALSE)
18 }
19
20 # CoCo price  $V^{sa}$  as function of maturity  $T$  and risk-free interest rate
   $r$ 
21 createData_SA_T_r <- function(T_min, T_max, r_min, r_max){
22   data <- matrix(1, 121, 3)
23   counter <- 1
24   for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
25   {
26     for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))

```

```

27     {
28       data[counter, 1] <- T_increment
29       data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 10000,
rho <- - 0.2, kappa <- 0.114, r_bar <- 0.069, r0 <- 0.01, sigma_r <-
0.07, mu_Y <- -0.01, sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat
<- 1.1, b0 <- 0.04, p <- 1, e_bar <- 0.02, sigma_x <- 0.02, x0 <-
1.15, B <- 1, coupon <- 0.06)
30       data[counter, 3] <- r_increment
31       counter <- counter + 1
32     }
33   }
34   write.table(data, file = "createData_SA_T_r_2.txt", row.names = FALSE,
quote=FALSE)
35 }
36
37 # CoCo price  $V^s_a$  as function of initial asset-to-deposit ratio  $x_0$  and
equity-to-deposit threshold  $\bar{e}$ 
38 createData_SA_x0_ebar <- function(x0_min, x0_max, ebar_min, ebar_max){
39   data <- matrix(1, 121, 3)
40   counter <- 1
41   for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/
10)))
42   {
43     for(ebar_increment in seq(from=ebar_min, to=ebar_max, by=((ebar_max-
ebar_min)/10)))
44     {
45       data[counter, 1] <- x0_increment
46       data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 10000,
rho <- - 0.2, kappa <- 0.114, r_bar <- 0.069, r0 <- 0.01, sigma_r <-
0.07, mu_Y <- -0.01, sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat
<- 1.1, b0 <- 0.04, p <- 1, e_bar <- 0.02, sigma_x <- 0.02, x0 <-
1.15, B <- 1, coupon <- 0.06)
47       data[counter, 3] <- ebar_increment
48       counter <- counter + 1
49     }
50   }
51   write.table(data, file = "createData_SA_x0_ebar_2.txt", row.names =
FALSE, quote=FALSE)
52 }
53
54 # CoCo price  $V^s_t$  as function of initial asset-to-deposit ratio  $x_0$  and
jump intensity in asset return process  $\lambda$ 
55 createData_SA_x0_lambda <- function(x0_min, x0_max, lambda_min, lambda_
max){
56   data <- matrix(1, 121, 3)
57   counter <- 1
58   for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/
10)))
59   {
60     for(lambda_increment in seq(from=lambda_min, to=lambda_max, by=((
lambda_max-lambda_min)/10)))
61     {
62       data[counter, 1] <- x0_increment

```



```

63     data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 10000,
rho <- - 0.2, kappa <- 0.114, r_bar <- 0.069, r0 <- 0.01, sigma_r <-
0.07, mu_Y <- -0.01, sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat
<- 1.1, b0 <- 0.04, p <- 1, e_bar <- 0.02, sigma_x <- 0.02, x0 <-
1.15, B <- 1, coupon <- 0.06)
64     data[counter, 3] <- lambda_increment
65     counter <- counter + 1
66   }
67 }
68 write.table(data, file = "createData_SA_x0_lambda_2.txt", row.names =
FALSE, quote=FALSE)
69 }
70
71 # CoCo price  $V^st$  as function of initial asset-to-deposit ratio  $x_0$  and
initial ratio of contingent capital to deposits  $b_0$ 
72 createData_SA_x0_b0 <- function(x0_min, x0_max, b0_min, b0_max){
73   data <- matrix(1, 121, 3)
74   counter <- 1
75   for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/
10)))
76   {
77     for(b0_increment in seq(from=b0_min, to=b0_max, by=((b0_max-b0_min)/
10)))
78     {
79       data[counter, 1] <- x0_increment
80       data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 10000,
rho <- - 0.2, kappa <- 0.114, r_bar <- 0.069, r0 <- 0.01, sigma_r <-
0.07, mu_Y <- -0.01, sigma_Y <- 0.02, lambda <- 1, g <- 0.5, x_hat
<- 1.1, b0 <- 0.04, p <- 1, e_bar <- 0.02, sigma_x <- 0.02, x0 <-
1.15, B <- 1, coupon <- 0.06)
81       data[counter, 3] <- b0_increment
82       counter <- counter + 1
83     }
84   }
85   write.table(data, file = "createData_SA_x0_b0_2.txt", row.names =
FALSE, quote=FALSE)
86 }
87
88 createData_SA_x0_sigma(1.08, 1.17, 0.01, 0.05)
89 createData_SA_T_r(1, 50, 0.01, 0.05)
90 createData_SA_x0_ebar(1.08, 1.17, 0.005, 0.03)
91 createData_SA_x0_lambda(1.08, 1.17, 0, 2)
92 createData_SA_x0_b0(1.08, 1.17, 0.1, 0.06)

```

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