# Valuation of Contingent Convertibles with Derivatives


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# Valuation of Contingent Convertibles with Derivatives

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This thesis is dedicated to my parents for their love and support.
Thank you!

# Abstract

Financial crises have led to higher regulatory standards on the capital adequacy of banks. Banks are required to hold more capital with loss absorbance capacities on their balance sheet. In conjunction with this development, contingent convertible bonds (CoCos) have become an attractive instrument for banks to seek new capital. The defining characteristic of CoCos is the automatic conversion into common equity or the principal write-down when a certain ratio meets a predetermined trigger. Loss-absorbing capital is created, which instantly improves the capital structure of the distressed bank. However, the pricing of these hybrid instruments remains opaque. In this context, the thesis scrutinizes the valuation of CoCos with equity conversion mechanisms. The paper examines three dominant approaches: the structural approach in accordance to Pennacchi (2010), the credit derivative approach and the equity derivative approach both under De Spiegeleer and Schoutens (2012). Additionally, the application covers sensitivity analyses to understand the dynamics of the different methodologies further. Based on a case study of HSBC's perpetual subordinated contingent convertible securities (ISIN US404280AT69) the viability of the approaches is evaluated by analyzing their price tracking accuracy. Subsequently, the comprehensive software provides a basis for further applications of the pricing approaches.

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# Chapter 1

# Introduction

## 1.1 Background

Investors are overly restrictive in providing liquidity to financial institutions during periods of financial distress. In the past, governments were often in the situation to inject liquidity to financial markets to avoid disruptive insolvencies as no other market participant was inclined to do so. Government bailouts, however, externalize the cost of bankruptcy to taxpayers while distorting risk-taking incentives of banking professionals. Contingent convertibles (CoCos) aim to internalize these costs in the capital structure of systemically important financial institutions. CoCos are hybrid financial instruments that absorb losses under their specifications in case a pre-determined threshold fails to remain above a minimum trigger level. Then, debt automatically morphs to equity which instantly improves a bank's capitalization. Due to their loss-absorption capacity, CoCos are eligible to be categorized as regulatory capital under Basel III. (Avdjiev et al., 2013)

After the global financial crisis of 2008, regulators around the world have worked on two different objectives. On the one hand, they attempt to lower spillover effects on the economy due to bankruptcies of financial institutions. On the contrary, they aim to reduce the individual default probabilities of banks. One might achieve the latter objective by ensuring that banks have enough loss-absorbing capital on their balance sheet even in tough times. (De Spiegeleer and Schoutens, 2011) In this context, the Basel Committee on Banking Supervision (BCBS) specified that debt instruments are permitted as regulatory capital if they absorb losses to such an extent that taxpayers do not have to bear the costs. (Basel Committee on Banking Supervision, 2010b) Subsequently, this opened the door for CoCos and ever since, the regulatory treatment has been a major driver of past issues.

Besides the reforms of policy makers to raise the quality of regulatory capital, recent studies on CoCos highlight some advantages. Albul et al. (2015) show that CoCos lessen financial distress, whether caused by idiosyncratic or systemic shocks. They indicate lower default probabilities of banks and a smaller likelihood of costly bailouts by the public sector. Hilscher and Raviv (2014) support these findings. Additionally, they argue that an appropriate specification of a CoCo's building parts can eliminate the incentives of shareholders to asset-substitution. The problem of asset-substitution arises when managers undertake excessively risky investment decisions to maximize shareholder value at the expense of debtholders. (Bannier, 2011) Moreover, research evinces that CoCos increase the issuer's firm value as they reduce the cost of capital. (Albul et al., 2015; Von Furstenberg et al., 2011; Barucci and Del Viva, 2012) To sum up, one should consider CoCos in the liability structure of banks from an academic standpoint.

In addition to the positive perception of policy makers and academia, CoCos are well accepted by the financial industry. Banks value that the hybrid instrument enables them to refinance themselves while simultaneously satisfying the regulatory capital requirements at lower costs than with equity. (European Parliament, 2016) Between 2009 and 2015, financial institutions around the world issued CoCos worth USD 446.96 bn in 519 different issues. (Avdjiev et al., 2015) Albeit the amount of CoCos issued is relatively small compared to the market size of other financial products; they were brought into focus in early 2016. At this time, CoCos contributed to increased market volatility of some European banks. One should not whitewash the relevance of this episode concerning the potential systemic implications as it is likely that discussions on regulatory changes will emerge. Moreover, the question arises about the perception of CoCos by investors and the robustness of their pricing models. (European Parliament, 2016)

In this context, an investigation of different valuation concepts for CoCos is highly attractive for both investors as well as supervisory authorities. The paper contributes to a better understanding of relevant concepts. Additionally, the valuation approaches will be applied to a CoCo which was affected by the turbulences in early 2016.

## 1.2 Previous Studies

Various valuation approaches for CoCos have been developed over time covering different aspects of their nature. The variety of approaches is due to the hybrid character of CoCos which also makes them a highly interesting object of study. Wilkens and Bethke (2014) propose three groups to organize the broad universe: structural approaches, equity derivative approaches and credit derivative approaches. Additionally, Turfus and Shubert (2015) suggest a fourth category: hybrid equity-credit derivative approaches. A comprehensive compilation of relevant studies for each category can be found in table 1.1. Subsequently, the main idea of each type will be explained.

| **Structural Approaches** | **Equity Derivative Approaches** | **Credit Derivative Approaches** |
|---|---|---|
| Pennacchi (2010) | De Spiegeleer and Schoutens (2012) | De Spiegeleer and Schoutens (2012) |
| Albul et al. (2010) | Henriques and Doctor (2011) as cited by Erismann (2015) | Serjantov (2011) as cited by Wilkens and Bethke (2014) |
| Madan and Schoutens (2011) | Alvemar and Ericson (2012) | Alvemar and Ericson (2012) |
| Glasserman and Nouri (2012) | Corcuera et al. (2013) | Erismann (2015) |
| Alvemar and Ericson (2012) | Corcuera et al. (2014) | |
| Buergi (2013) | Teneberg (2015) | |
| Hilscher and Raviv (2014) | Erismann (2015) | |
| Pennacchi and Tchistyi (2015) | | |
| Cheridito and Xu (2015) | **Hybrid Equity-Credit Derivative Approaches** | |
| Erismann (2015) | Turfus and Shubert (2015) | |
| Sundaresan and Wang (2015) | | |

Table 1.1: Literature overview of valuation approaches for CoCos (Wilkens and Bethke, 2014; Erismann, 2015) with the examined methods.

**Structural approaches** try to capture all parameters that influence the issuer's ability to pay its liabilities. They are normally built upon a stochastic model which focusses on the variation in asset values relative to debt. (Duffie and Singleton, 2003) By contrast, **equity derivative approaches** emphasize the dependence of a CoCo's state on the share price and use equity derivatives to replicate their payoff. This model type follows the train of thought that the share price is the best proxy to track the solvency of the issuer. **Credit derivative approaches** assume an exogenously specified process for the migration of conversion probabilities. They apply the idea of reduced-form approaches to model the equity conversion intensity process of CoCos in line with a credit default intensity process. The rationale behind this approach is that CoCos are credit-risky instruments as their conversion depends on the issuer's solvency. (Wilkens and Bethke, 2014) **Hybrid equity-credit derivative approaches** capture the advantages of the latter two concepts. They model the share price and the conversion intensity as correlated stochastic processes. (Turfus and Shubert, 2015) A detailed literature review of the various pricing approaches is outlined in the following sections.

## Structural Approaches

Structural approaches offer a natural pricing framework for CoCos. They consider a bank's balance sheet structure as the most important value driver. Numerous structural approaches have been proposed in academia. All share common characteristics but vary in their application. (Wilkens and Bethke, 2014) For instance, they are often used to draw policy recommendations. A selection can be found in the following.

The study of Albul et al. (2010) is the first paper to provide analytic propositions to price CoCos by adapting the structural model of Leland (1994). The authors develop implications for the design of CoCos with the objective of maximizing the benefit for the issuer. Interestingly, their analysis is at first not limited to financial institutions. In fact, they argue that CoCos might generally be advantageous for corporates to optimize their capital structure. The authors further recommend the specific use of CoCos as tool for bank regulation. In this context, studies like Madan and Schoutens (2011), Hilscher and Raviv (2014) and Sundaresan and Wang (2015) analyze beneficial structures of CoCos.

In the scientific literature, the work of Pennacchi (2010) is often used as a reference article for structural approaches as he attempts to model the stochastic evolution of a bank's balance sheet to price CoCos (for further details see chapter 3.3). The author is able determine the value of CoCos by applying a jump-diffusion process to account for discontinuous asset returns. Capital ratios with mean-reverting tendency and a stochastic term-structure model shall improve the pricing accuracy. Based on the derived framework the author is able to capture several risk factors that may influence a CoCo's price. However, this is also the main shortcoming of the approach because the author does not address the parametrization of input factors in practice, which is also indicated by the work of Erismann (2015).

Madan and Schoutens (2011) implement a structural model utilizing conic finance theory. Classical Mertonian models (Merton, 1974) assume that assets are risky but liabilities are not. For instance, Alvemar and Ericson (2012) apply such a pure model pursuant to Merton (1974) to price CoCos. In contrast, the model of Madan and Schoutens (2011) assumes that liabilities are risky and correlated to the asset dynamics. The authors abandon the one-price-market idea and assume that bid-ask spreads exist. In addition, they argue that the Core Tier 1 ratio is potentially not optimal as trigger if one considers the presence of risky liabilities. As alternative they propose accounting triggers based on capital shortfall.

## Equity Derivative Approaches

Equity derivative approaches are an important category of valuation approaches which consider the share price as the best proxy. Most important approaches will be summarized in the following.

De Spiegeleer and Schoutens (2012) replicate the payout profile of a CoCo with a portfolio consisting of a straight bond, a knock-in forward and a set of binary down-and-in calls (for further details see chapter 3.3). Under the assumption that a CoCo will not convert to equity, one can assume that a CoCo is equivalent to a straight bond. Though, the knock-in forward simulates the conversion of a straight bond when a predetermined strike price is met. A CoCo investor would receive the shares at maturity if he or she is long a knock-in forward. However, this is a simplification which is reasonable under the assumption that dividend payments are cancelled in times of distress. Additionally, the foregone coupon payments of a straight bond at conversion are modeled with a short position in binary down-and-in calls. One of the main findings is that the assumed Black-Scholes setting does not sufficiently capture tail risks but which are inherent in CoCos.

Other approaches enhance the model dynamics by accounting for jumps and heavy tails. Erismann (2015) and Teneberg (2015) amend the model of De Spiegeleer and Schoutens (2012) by allowing for discontinuous returns. The calculations with regard to the binary down-and-in calls and the knock-in forward position accommodates a jump-diffusion process. Corcuera et al. (2013) also consider an equity derivative approach that reduces the valuation to a set of barrier options in which the trigger event is determined by the underlying hitting a certain barrier. They use smile conform models, more precisely, an exponential Lévy process incorporating jumps and heavy tails.

## Credit Derivative Approaches

The price of a CoCo is directly linked to the issuer's solvency and default probability. Intensity-based credit modeling allows to develop comprehensive pricing approaches. In this connection, one should mention the work of De Spiegeleer and Schoutens (2012), Serjantov (2011) and Erismann (2015).

De Spiegeleer and Schoutens (2012) tackle the pricing problem with a credit-derivative approach (for further details see chapter 3.1). Their main contribution lies in the derivation of a closed-form solution of a CoCo's credit spread. In their model the spread follows a function of an exogenously defined trigger probability. The spread compensates for the risk that the CoCo converts to equity implying a loss for each investor. Their approach is an elegant way of bridging the gap between the prediction of conversion and the pricing of conversion risk. Though, the largest shortcoming of the model is that it fails to capture losses from cancelled coupons of triggered CoCos.

Erismann (2015) expands the model of De Spiegeleer and Schoutens (2012) by assuming that returns follow a jump-diffusion process. The approach models the exposure to return outliers of both signs and amplitudes. Finally, the author demonstrates that his approach is superior to De Spiegeleer and Schoutens (2012) considering price tracking accuracy. However, the case study on a HSBC CoCo reveals that the conclusions of Erismann (2015) and Wilkens and Bethke (2014) might not be as reliable as initially thought.

Serjantov (2011) as cited by Wilkens and Bethke (2014) develops a closed form solution to price CoCos. All cashflows are weighted with cumulative survival probabilities. In addition, the approach distinguishes between the conversion ratio without default and the recovery rate at default. The joint probability of both events happening in the same time interval is described with a Gaussian copula. Furthermore, this approach overcomes the shortcoming of the credit derivative approach of De Spiegeleer and Schoutens (2012) as it explicitly captures coupon payments.

## Hybrid Equity-Credit Derivative Approaches

Turfus and Shubert (2015) present a new pricing approach for CoCos. Their starting point is a stochastic model which captures interest rates, share prices and a conversion intensity process. The evolution of the first two is assumed to be determined by diffusive processes. By contrast, the share price is supposed to be governed by a jump-diffusion process which factors into a downward jump when the trigger level is touched. Both the share price and the conversion intensity process are modeled as correlated stochastic processes. For this very reason, the hybrid equity-credit derivative approach may be regarded as an important step forward because two direct benefits arise. On the one hand, the share price at conversion is modeled instead of being an input parameter and on the other hand, both equity and credit risk sensitivity can be estimated individually.

## 1.3 Research Methodology

The objective of the thesis consists of an examination of three dominant pricing approaches for CoCos similar to the proceedings of Alvemar and Ericson (2012), Erismann (2015) and Wilkens and Bethke (2014). All of the three valuation models are widely discussed in academic literature as they are often used as basis for further model advancements. The utilized approaches are namely the structural approach of Pennacchi (2010), the equity derivative approach and the credit derivative approach both pursuant to De Spiegeleer and Schoutens (2012). Hereinafter, chapter 2 provides an overview of the anatomy of CoCos. Characteristic building parts of the financial product will be discussed in detail in order to create an improved understanding of the mechanisms which drive the valuation of this hybrid instrument. Examples of past CoCos issues are highlighted covering the most important variations of the aforementioned design features. On this basis, chapter 3 studies the theoretical concepts behind each of the three valuation approaches. In addition, pricing examples provide an understanding of the data requirements of each model. In chapter 4, sensitivity analyses determine how different values of certain pricing parameters impact the valuation of CoCos. Chapter 5 comprises an empirical analysis on the price tracking accuracy of the aforementioned valuation approaches. Finally, a conclusion is reached in chapter 6.

# Chapter 2

# CoCo Structure

This chapter explains the nature of CoCos, a relatively new member of the family of hybrid securities. In the following, the general structure will be explained including characteristic design features among others their trigger and loss-absorption mechanism.

## 2.1 Definition

The name CoCo is used to define a hybrid capital instrument with an automatic conversion which morphs debt into equity when the financial soundness of the issuer is at stake. A write-down of the principal is also a viable loss-absorption mechanism to recapitalize the distressed bank. The loss-absorption mechanism is activated if a predefined trigger level is breached. (De Spiegeleer and Schoutens, 2012; Zähres, 2011). In this regard, figure 2.1 provides an overview of major design characteristics. Exactly these anatomic aspects will be explained in the subsequent sections.

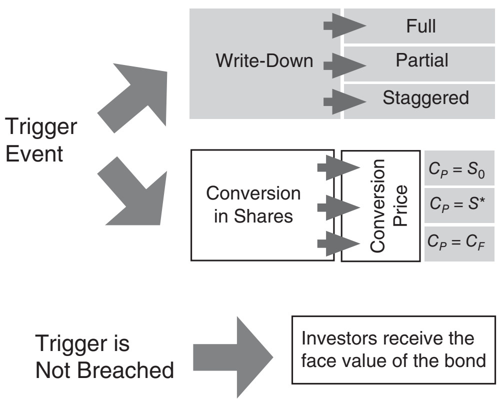


Figure 2.1: Anatomy of CoCos (De Spiegeleer et al., 2014)

CoCos are particularly interesting from the perspective of a regulatory authority because they might mitigate externalization costs of insolvencies and frictions due to spill-over effects. In times of distress, stakeholders might question the financial viability of the respective financial institute. Yet, the major advantage of CoCos is that a distressed bank does not have to approach new investors to issue new capital in extremely though times as everything happens automatically. (De Spiegeleer and Schoutens, 2012)

In 2009, the Lloyds Banking Group was the first financial institute which issued this new financial instrument. In an exchange offer, they asked hybrid debt holders to swap their holdings into CoCos. (De Spiegeleer and Schoutens, 2012) In 2010, the Basel Committee on Banking Supervision (BCBS) provided further impetus to the use of this instrument when it disclosed its proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. CoCos fit into their line of argumentation that regulatory capital instruments have to be capable of absorbing financial losses in gone-concern phases. (Basel Committee on Banking Supervision, 2010a)

## 2.2 Trigger

The trigger is a key design element of a CoCo which initializes the loss-absorption mechanism. The following sections will describe four different trigger mechanisms: accounting triggers, market triggers, regulatory triggers and multi-variate triggers.

### 2.2.1 Accounting Trigger

CoCos with accounting trigger have a loss absorption mechanism which relies on the financial soundness of a bank's balance sheet. Accounting triggers use capital ratios which compare a bank's regulatory capital with its assets. Capital ratios are an objective indicator for a bank's solvency as they are defined uniformly for all financial institutions by regulatory authorities. (De Spiegeleer et al., 2014) In addition, Pazarbasioglu et al. (2011) note that accounting triggers are easy to price, intuitive and straightforward to implement. Having said that, one might argue that accounting triggers assess the viability of financial institutions from a perspective that is far-removed from reality. A major objection against accounting triggers follows the line of thought that they just become active long after the need for loss-absorbing capital arose because companies publish accounting data infrequently. (De Spiegeleer and Schoutens, 2012) As accounting concept, book values are prone to manipulation and managerial dishonesty especially in times of distress. (McDonald, 2013)

Empirical findings bring up an another aspect. Haldane (2011) points out that major financial institutions, which either went bankrupt, were bailed out or were taken over under distress during the global financial crisis, reported similar CET 1 ratios right before the collapse of Lehman Brothers compared to their peers which coped relatively well with the crisis. In this context, Haldane (2011) highlights that market-based solvency measures performed well as they showed clear signals of impending distress a year ahead of the bankruptcy of Lehman Brothers. Empirical evidence of the United States Bankruptcy Court (2010) further supports these findings. One can conclude that CoCos with accounting triggers might not reinforce distressed banks at the right time but instead produce false positives, which means that CoCos of non-distressed banks trigger prematurely. Inefficiencies like higher funding costs could be the consequence. (Pazarbasioglu et al., 2011)

### 2.2.2 Market Trigger

A market trigger uses directly observable indicators like the issuing company's share price or credit default swap (CDS) spreads while assuming sufficiently efficient markets. The major advantage of those measures is that one can observe and verify them in real-time. (Haldane, 2011) Market triggers are widely discussed in academia and seen as a preferable trigger mechanism. Calomiris and Herring (2013) pronounce themselves for using share prices. Besides, Haldane (2011), Pazarbasioglu et al. (2011) and (Calomiris and Herring, 2013) contend to apply market-based capital ratios as trigger indicator. Their line of argumentation follows some of the best-known examples of corporate defaults, which have been indicated well before by a severe and continuous deterioration of a company's market capitalization.

In contrast, Sundaresan and Wang (2015) argue based on a structural approach that CoCos with market trigger do not lead to a unique share price equilibrium unless conversion results in a value transfer between shareholders and CoCo investors. Having said that, the design of dilutive conversion ratios to punish bank managers for taking excessive risks creates multiple equilibria which in turn makes CoCos susceptible to market manipulation. The authors conclude that regulation with the right intention might cause instability in the market, and the market itself may limit the impact of regulation. However, Hilscher and Raviv (2014) demonstrate that an appropriate design of CoCos can mitigate the risk of asset-substitution by exactly offsetting costs and benefits of shareholders when increasing the probability of conversion. Furthermore, Pennacchi and Tchistyi (2015) weaken the argumentation of Sundaresan and Wang (2015) as they demonstrate that a unique share price equilibrium exists for CoCos with perpetual maturity independent of their trigger type. The fact that 57.1% of CoCos, which banks issued between 2009 and 2015 do not have a set maturity, emphasizes the relevance of their findings. (European Parliament, 2016)

### 2.2.3 Regulatory or Non-Viability Trigger

The regulatory or non-viability trigger is a conversion mechanism by which a CoCo converts into equity at the discretion of the responsible supervisory authority. The rationale behind this approach is that regulators want to limit the impact of any development that could pose a danger to the going-concern of a systemically important bank. (Erismann, 2015) Moreover, this kind of trigger would eliminate the periodicity problem of accounting data and the risk of market manipulation.

Though, it is tough for market participants to estimate the conversion probability of a CoCo with a regulatory trigger. The valuation of such a hybrid instrument becomes opaque for market participants with limited information. (Alvemar and Ericson, 2012) One can also argue that a CoCo's marketability weakens because of the greater uncertainty which could ultimately lead to higher funding costs. (De Spiegeleer et al., 2014)

### 2.2.4 Multi-Variate Trigger

The multi-variate trigger combines an accounting trigger with a systemic trigger which covers severe states of the world. For instance, the Squam Lake Working Group (2009) argues that the implementation of such a dual trigger is preferable as it connects the best of two worlds. The bank-specific trigger serves as a direct disciplining mechanism for a bank's management. In parallel, it reduces the political pressure from the regulator who has to decide whether the systemic trigger is met. Moreover, if the conversion of a CoCo is only linked to a systemic trigger, even well-capitalized banks would be forced to convert debt into equity during a systemic crisis. But this would disincentivize financially sound banks to preserve their status quo.

## 2.3 Loss-Absorption

As mentioned earlier, banks can decide whether to issue CoCos with an equity conversion- or a write-down mechanism. Conversion into equity means that a particular portion of a CoCo's notional will be converted into equity if a certain trigger event occurs. It is also possible to specify that the notional of a bond suffers a haircut if the issuer decides to use a write-down mechanism. The paper explains both types in detail.

| | **Lloyds** | **Credit Suisse** | **Barclays** | **Rabobank** | **ZKB** |
|---|---|---|---|---|---|
| **Full Name** | Enhanced Capital Notes | Tier 2 Buffer Capital Notes | Contingent Capital Notes | Senior Contingent Notes | Subordinated Tier 1 Notes |
| **ISIN** | XS0459088281 | XS0595225318 | US06740L8C27 | XS0496281618 | CH0143808332 |
| **Issue Date** | Dec 1, 2009 | Feb 24, 2011 | Nov 21, 2012 | Mar 19, 2010 | Jan 31, 2012 |
| **Maturity** | May 12, 2020 | Feb 24, 2041 | Nov 21, 2022 | Mar 19, 2020 | Perpetual |
| **Nominal** | GBP 7.5 bn | USD 2 bn | USD 3 bn | EUR 1.25 bn | CHF 590 mn |
| **Callability** | n/a | Callable from Aug 24, 2016 | n/a | n/a | Callable from Jun 20, 2017 |
| **Coupon** | 7.5884% | 7.875% | 7.625% | 6.875% | 3.5% |
| **Write-down** | n/a | n/a | Full by (100% of notional) | Partial (75% of notional) | Staggered (multiples of 25% of notional) |
| **Conversion price** | Fixed at GBP 0.59 (= $S_0$) | Floored at lowest of USD 20 and 30 day-VWAP | n/a | n/a | n/a |
| **Trigger** | Core Tier 1 capital ratio | CET 1 ratio | CET 1 ratio | Equity capital ratio (Member certificates to risk weighted assets) | CET 1 ratio |
| **Trigger Level** | 5% | 7% | 7% | 7% | 7% |

Table 2.1: CoCo examples with different loss-absorption mechanisms (Lloyds, 2009; Credit Suisse, 2011; Barclays, 2010; Rabobank, 2010; Zurich Cantonal Bank, 2013; De Spiegeleer and Schoutens, 2012)

Also, for each of the loss-absorption mechanism selected CoCos are characterized to gain a better sense of how banks implement these hybrid products in real-life. Broadly discussed CoCos of well-known financial institutions are picked out, among other things Lloyds, Credit Suisse, Barclays, Rabobank and Zurich Cantonal Bank. An overview of CoCos which have different characteristics concerning their loss-absorbency can be found in table 2.1. The first two apply similar equity conversion mechanisms whereas the latter three use different write-downs.

### 2.3.1 Conversion into Equity

The following sections clarify three of the most common structures which rely on the specification of the conversion price of a CoCo.

In the beginning, a few variables are introduced to study the equity conversion mechanism. First, the number of shares which a CoCo holder receives at conversion is given by the conversion rate $C_r$. Second, the conversion fraction $\alpha$ and the notional $N$ determines the conversion amount $\alpha N$. All of these parameters are defined ex-ante. One can now describe the relationship between the implied conversion price $C_p$ and the introduced parameters as follows: (De Spiegeleer et al., 2014)

$$C_p = \frac{\alpha N}{C_r} \tag{2.1}$$

The recovery rate $R_{CoCo}$ is derived from the conversion price $C_p$ and the share price $S^*$ at conversion. It becomes immediately evident from equation 2.2 that a CoCo investor is better off if $C_p$ is low since the recovery rate $R_{CoCo}$ becomes higher as more equity is created.

$$R_{CoCo} = \frac{S^*}{C_p} \tag{2.2}$$

If the CoCo converts into equity one can directly determine the occurring damage. The financial loss $L_{CoCo}$ of a CoCo investor can be expressed by the subsequent equation:

$$Loss_{CoCo} = N - C_r S^* = N - (1 - R_{CoCo}) = N\left(1 - \frac{S^*}{C_p}\right) \tag{2.3}$$

A CoCo may or may not trigger throughout its life. Nevertheless, one can describe the final payoff $V^{CoCo}$ of an investor at maturity $T$ as follows:

$$V_T^{CoCo} = \begin{cases} N & \text{if not triggered} \\ (1-\alpha)N + \frac{\alpha N}{C_p} S^* & \text{if triggered} \end{cases} \tag{2.4}$$

**Floating conversion price $C_p = S^*$**

Ex-ante an issuer can set a floating conversion price where $C_p$ is equal to $S^*$. In this regard, $S^*$ is the share price at conversion. Intuitively, the value of the stock price at precisely the trigger time is relatively small because the purpose of a CoCo is to help an undercapitalized bank in difficult times. If the issuer decides to specify a floating conversion price, the recovery rate of a CoCo holder will be 100%. However, current shareholders would carry the load of conversion. The main shortcoming of this approach is that regulators would not categorize this instrument to be adequate as a regulatory capital instrument. The dilution is potentially unbounded, and it is effectively not loss absorbing. (De Spiegeleer et al., 2014)

**Fixed conversion price** $C_p = S_0$

Fixed conversion means that the conversion price $C_p$ corresponds to the share price at the time of issue $S_0$. Hence, the specification fixes the number of shares upon conversion at the issue date. The conversion amount is known beforehand. Unlike the floating conversion price, there is a predetermined limit on the number of shares converted. (De Spiegeleer et al., 2014) In 2009, Lloyds issued Enhanced Capital Notes. At that time, the company was the first bank to refinance itself with CoCos. The CoCos convert into equity at a fixed conversion price which has been set to equal the share price at issue. The Enhanced Capital Notes trigger if the bank's Core Tier 1 capital (Basel II) fails to remain above the threshold of 5%. (Lloyds, 2009)

**Floored conversion price** $C_p = \max(S^*, S_F)$

One can also specify a floored conversion price where $C_p$ is equal to $\max(S^*, S_F)$. Hence, the conversion price $C_p$ is either equal to the floored share price $S_F$ or to the share price at conversion $S^*$. This approach represents a compromise between the floating and fixed conversion price as mentioned earlier. (De Spiegeleer et al., 2014) An example for the floored conversion price are the Tier 2 Buffer Capital Notes of Credit Suisse. The trigger event is specified to convert the Tier 2 Buffer Capital Notes into ordinary shares if the reported risk-based capital ratio is below 7%. The conversion price is floored at the average daily share price of the last 30 days or USD 20. It may also be that the CoCo converts if the FINMA determines that Credit Suisse has to be bailed out. (Credit Suisse, 2011)

### 2.3.2 Write-Down

The implementation of a loss-absorption mechanism with equity conversion mechanism entails several disadvantages which might reduce a CoCo's marketability. Portfolio managers with a mandate for bonds might face hard times to broaden their investment universe because CoCos with equity conversion mechanism are likely to convert into shares. Also, the write-down mechanism is preferable as investors know beforehand the potential loss. Shareholders could fear that the conversion dilutes their voting rights. Hence, both investors and shareholder have clear incentives to force banks to issue CoCos with write-down mechanism. (De Spiegeleer et al., 2014)

### Full write-down

The first way is to specify a full write-down of a CoCo's face value in case a certain capital ratio drops below a predetermined level. (De Spiegeleer et al., 2014) In 2012, Barclays launched the first high-trigger total-loss CoCo. The Contingent Capital Notes depreciate to a value of zero should the CET 1 ratio fail to remain above a minimum of 7%. Such a structure assumes already that the bank has a solid buffer before the trigger is met. (Barclays, 2010)

### Partial write-down

Another approach is that only a certain portion of the notional is wiped out. CoCos with write-off features are particularly suitable for cooperative banks that are deterred by their legal form to issue shares. (De Spiegeleer et al., 2014) In 2010, Rabobank was the first cooperative bank to issue Senior Contingent Notes with a loss-absorption mechanism that imposes a significant haircut on its principal. One-quarter of the notional is reimbursed at the trigger event if the equity capital ratio falls below 7%. The distinct haircut explains its high financing costs. (Rabobank, 2010)

### Staggered write-down

The third option consists of a staggered write-down, which means that a CoCo inherits a flexible write-down mechanism. Losses materialize up to the point to enhance a certain capital ratio to a fixed threshold. One might think of a gradual process. (De Spiegeleer et al., 2014) Zurich Cantonal Bank is wholly owned by the Canton of Zurich and hence, it is not listed. The bank has been in a similar situation to improve its regulatory capital as Rabobank. In this situation, the bank issued Subordinated Tier 1 Notes that are exemplary for CoCos with a staggered write-down mechanism. What has been new is that an investor faces a dilution of his or her holdings up to the point where the write-down lifts the regulatory capital up to its minimum level. Haircuts only materialize in multiples of 25%. (Zurich Cantonal Bank, 2013)

### Temporary write-down

Theoretically, it is possible that the write-down mechanism is only temporarily active. So, the haircut of the notional reverts upon restoration of the financial health of the issuing bank. However, regulators seek a permanent improvement in capitalization on conversion which makes it less likely that a temporary mechanism is embedded. (Avdjiev et al., 2013) For example, the Italian bank Intesa Sanpaolo issued Additional

Tier 1 Notes with an interim write-down mechanism. The bond is temporarily written off if the CET 1 ratio fails to remain above a level of 5.125%. (Intesa Sanpaolo, 2011)

## 2.4 Regulator's View

The purpose of the section is to reveal that the freedom in structuring is restricted. Financial institutions have applied all of the trigger types and loss-absorption mechanisms in reality, but regulators have clear preferences for the specification of important building parts like the trigger and the loss-absorption mechanism. In this context, the following sections describe the regulatory capital requirements on CoCos.

Under the terms of the Basel III framework, CoCos are eligible to be categorized as either Additional Tier 1 (AT1) or Tier 2 (T2) capital. Instruments that qualify for either of the categories can be found in table 2.2. Though, Basel III makes two principle demands. The first condition requires AT1 and T2 CoCos to incorporate a regulatory trigger. The second requirement consists of a full and permanent loss-absorption for AT1 CoCos. (Avdjiev et al., 2013)

| **Regulatory Capital** | | | |
|---|---|---|---|
| **Tier 2 (T2)** | | | |
| ↪ Non-CoCo subordinated debt | | | |
| ↪ Low-trigger CoCos | | | |
| **Additional Tier 1 (AT1)** | | | |
| ↪ Preferred shares | | | $\geq$ 8.0% RWA |
| ↪ High-trigger CoCos | | $\geq$ 6.0% RWA | |
| **Core Equity Tier 1 (CET1)** | | | |
| ↪ Common shares | $\geq$ 4.5% RWA | | |
| ↪ Retained earnings | | | |

Table 2.2: Position of CoCos in Basel III capital requirements (Avdjiev et al., 2013)

As mentioned, regulators demand the implementation of regulatory triggers. However, the trade-off between issuance costs and considerations regarding the categorization as either AT1 or T2 capital drives the exact specification of the trigger threshold. The lower the trigger is, the lower is also the loss-absorption capacity of a CoCo. Hence, low-trigger CoCos are cheaper to issue but only eligible as T2 capital. By contrast, high-trigger CoCos are required to have a minimum trigger which triggers a conversion if the CET1 capital falls below a threshold of 5.125% of RWA. (Avdjiev et al., 2013)

Regulatory requirements also influence the specification of the loss-absorption mechanism and maturity. One essential prerequisite for a CoCo to qualify as AT1 capital is that they have to incorporate a full write-down or complete conversion to equity mechanism. (Bleich, 2014) In addition, the loss-absorption must be permanent. Regulatory considerations are also important for the maturity decision. All AT1 CoCos should have a perpetual maturity. If this is not the case, the CoCo will only qualify as T2 capital. (Avdjiev et al., 2013)

# Chapter 3

# Valuation Concepts

## 3.1 Credit Derivative Approach

In financial markets, the reduced-form approach is widely used to price credit risk. It was originally introduced by Jarrow and Turnbull (1995) respectively Duffie and Singleton (1999). The credit derivative approach applies the reduced-form approach to CoCos. In this context, the derivation of a pricing formula for CoCos follows mainly De Spiegeleer and Schoutens (2012).

### 3.1.1 Reduced-Form Approach and Credit Triangle

The reduced-form approach is an elegant way of bridging the gap between the prediction of default and the pricing of default risk of a straight bond. In the following, we investigate the link between estimated default intensities and credit spreads under the reduced-form approach. (Lando, 2009)

Let $\tau$ denote the random time of default of some company. It is assumed that the distribution of $\tau$ has a continuous density function $f$, so that the distribution function $F$ and the curve of survival probabilities $q$ are related as follows:

$$P(\tau \leq t) = F(t) = 1 - q(t) = \int_0^t f(s)ds, \text{ with } t \geq 0 \tag{3.1}$$

The hazard rate respectively the default intensity $\lambda$ is defined as follows:

$$\lambda(t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} P(\tau \leq t + \Delta | \tau > t) = \frac{F'(t)}{q(t)} = \frac{F'(t)}{1 - F(t)} = -\frac{d}{dt} \log q(t) \tag{3.2}$$

Intuitively, the hazard rate is the default rate per year as of today. Using 3.2 we can derive a formula for the survival probability:

$$q(t) = \exp\left(-\int_0^t \lambda(s)ds\right) \tag{3.3}$$

For our application of the reduced-form approach we assume that the hazard rate $\lambda(t)$ is a deterministic function of time. In reality $\lambda(t)$ is not deterministic but itself stochastic. That fits in with the fact that credit spreads are not static but stochastically varying over time. (Schmidt, 2015) However, we further consider the hazard rate to be constant in order to simplify the problem. Hence, a constant hazard rate $\lambda(t) = \lambda$ implies an exponential distribution of the default time:

$$F(t) = 1 - q(t) = 1 - \exp(-\lambda t) \tag{3.4}$$

Under the idealized assumption of a flat zero interest rate curve, a flat spread curve and continuous spread payments, the default intensity $\lambda$ can be calculated directly from the credit spread $s$ and the recovery rate $R$ by the rule of thumb formula (Schmidt, 2015), which is also known as credit triangle:

$$\lambda = \frac{s}{1 - R} \tag{3.5}$$

Finally, this relationship makes it possible to determine the default probability $F$ from the credit spread $s$ and vice versa.

### 3.1.2 Adaption to CoCos

In accordance with the aforementioned reduced-form approach, De Spiegeleer and Schoutens (2012) assume that the probability $F^*$, which measures the likelihood that a CoCo triggers within the next $T - t$ years, follows similar mechanics as the default probability of a straight bond does. Under the credit derivative approach the probability $F^*$ can be expressed as follows:

$$F^* = 1 - \exp\left[-\lambda_{Trigger}(T - t)\right] \tag{3.6}$$

Additionally, the credit derivative approach models $F^*$ with the first exit time equation used in barrier option pricing under a Black-Scholes setting. (Su and Rieger, 2009) Hence, the probability $F^*$ that the trigger level $S^*$ is touched within the next $T - t$ years is given by the following equation. The continuous dividend yield $q$, the continuous interest rate $r$, the drift $\mu$, the volatility $\sigma$ and the current share price $S$ of the issuing company are relevant variables:

$$F^* = \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) - \mu(T-t)}{\sigma\sqrt{(T-t)}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) + \mu(T-t)}{\sigma\sqrt{(T-t)}}\right) \tag{3.7}$$

In this regard, a CoCo's credit spread $s_{CoCo}$ can be approximated by the credit triangle, where $R_{CoCo}$ denotes the recovery rate of a CoCo and $L_{CoCo}$ is the loss rate:

$$s_{CoCo} = (1 - R_{CoCo})\,\lambda_{Trigger} = L_{CoCo}\lambda_{Trigger} \tag{3.8}$$

In the trigger event, the face value $N$ converts into $C_r$ shares worth $S^*$. The conversion price $C_p$ therefore determines the loss of a long position in a CoCo:

$$Loss_{CoCo} = N - C_r S^* = N\,(1 - R_{CoCo}) = N\left(1 - \frac{S^*}{C_p}\right) \tag{3.9}$$

By combining 3.6, 3.8 and 3.9 we see that the credit spread $s_{CoCo}$ of a CoCo with maturity $T$ at time $t$ is driven by its major design elements, the trigger level $S^*$ and the conversion price $C_p$:

$$s_{CoCo_t} = -\frac{\log(1 - F^*)}{(T - t)}\left(1 - \frac{S^*}{C_p}\right) \tag{3.10}$$

Subsequently, a pricing formula for CoCos under the credit derivative approach can be derived. The present value $V^{cd}$ at time t is given by:

$$V_t^{cd} = \sum_{i=1}^{T} c_i \exp\left[-(r + s_{CoCo_t})(t_i - t)\right] + N \exp\left[-(r + s_{CoCo_t})(T - t)\right] \tag{3.11}$$

In summary, the credit derivative approach provides us with a concise method to price CoCos. However, one has to bear in mind the largest shortcoming. The valuation does not take into account the losses from canceled coupons of triggered CoCos. Hence, the credit derivative approach naturally overestimates the price of CoCos, but it equips investors with a simple rule of thumb formula.

### 3.1.3 Parameter Classification and Adjustment

The credit derivative approach requires several model inputs which can be found in table 3.1. Generally, one can separate three different types. Static inputs comprise parameters that are specified ex-ante and are assumed to be constant. Dynamic inputs are updated regularly. Fitting parameters are used to fit the model prices to market prices. (Wilkens and Bethke, 2014)

| | Description | Usage | Source |
|---|---|---|---|
| $T$ | Maturity | Static input | Term sheet |
| $N$ | Notional | Static input | Term sheet |
| $c$ | Coupon rate | Static input | Term sheet |
| $S_t$ | Share price | Dynamic input | Market data |
| $S^*$ | Trigger share price | Fitting parameter | – |
| $C_p$ | Conversion price | Static input | Term sheet |
| $r$ | Risk-free interest rate | Dynamic input | Market data |
| $q$ | Dividend yield | Static input | Market data |
| $\sigma_E$ | Share price volatility | Dynamic input | Market data |

Table 3.1: Parameter classification of the credit derivative approach (Wilkens and Bethke, 2014)

All static inputs can be found in the term sheets of the respective CoCo. But beyond the model depends upon further dynamic inputs among others the share price $S$, the risk-free interest rate $r$ and the share price volatility $\sigma$. The daily share price $S$ is directly observed in the market. The risk-free interest rate $r$ is derived from sovereign bonds with the same maturity and currency. Furthermore, the input parameter $q$ relies on the three year average dividend yield of the issuing company. The share price volatility $\sigma$ is derived based on a yearly average volatility on a reference stock market index of the last five years similar to Alvemar and Ericson (2012). In addition, the only degree of freedom in the credit derivative approach is the fitting parameter $S^*$ respectively the trigger share price. Because this input variable is neither a static parameter nor a market parameter, $S^*$ is adjusted by minimizing the root mean squared deviation of realized and estimated CoCo prices. (Erismann, 2011)

### 3.1.4 Model Application

In the following a fictive CoCo is priced with the credit derivative approach pursuantly the specifications as stated in table 3.2. After installing both R, a programming language for statistical computing, and Rstudio, an open-source integrated development environment, a reader can easily price the aforementioned CoCo example with the source code of chapter A.1. The CoCo specifications are also used to analyze the price sensitivity in regard to certain input parameters. The results of the sensitivity analysis for the credit derivative approach can be found in section B.1. In addition, this example data set is also used to apply the equity derivative approach and the structural approach.

| | **Value** | **Comment** |
|---|---|---|
| $T$ | 10yrs | Maturity |
| $N$ | 100% | Nominal |
| $c$ | 6.00% | Coupon rate |
| $S_0$ | 120 | Initial share price |
| $S^*$ | 60 | Trigger share price |
| $C_p$ | 75 | Conversion price |
| $r$ | 3.00% | Risk-free interest rate |
| $q$ | 0.00% | Dividend yield |
| $\sigma_E$ | 30.00% | Share price volatility |

Table 3.2: Specification of input variables for a generic CoCo under the credit derivative approach (Alvemar and Ericson, 2012)

The spread of the CoCo $s_{CoCo}$, which compensates an investor for the risk of equity conversion, equals 1.46%. Moreover, the price $V^{cd}$ of the fictive CoCo under the credit derivative approach equates to 111.31.

## 3.2 Equity Derivative Approach

To assess the value of a CoCo, investors can use a method which depends on equity derivatives. (De Spiegeleer and Schoutens, 2012; De Spiegeleer et al., 2014) The so-called equity derivative approach attempts to compensate for the main drawback of the credit derivative approach since it takes into account that coupon payments might be canceled if the trigger of a CoCo was touched.

Subsequently, the valuation can be divided into two steps: In the first step, the value of a CoCo is determined without coupon payments. Such a CoCo is called Zero-Coupon CoCo. In the second phase, one has to incorporate the coupon payments in the pricing formula while keeping in mind that they might be knocked out. The closed-form solution is derived under a Black-Scholes setting.

### 3.2.1 Step One - Zero-Coupon CoCo

To determine the present value of a Zero-Coupon CoCo $V^{zcoco}$ at maturity $T$ we can use equation 2.4. The underlying assumption of the equity derivative approach is that the triggering of a CoCo respectively of a Zero-Coupon CoCo is equivalent to the share price falling below the share price level $S^*$.

$$\begin{aligned}
V_T^{zcoco} &= \begin{cases} N & \text{if not triggered} \\ (1-\alpha)N + \frac{\alpha N}{C_p} S^* & \text{if triggered} \end{cases} \\
&= N \mathbb{1}_{\{\tau > T\}} + \left[ (1-\alpha) N + \frac{\alpha N}{C_p} S^* \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + \left[ \frac{\alpha N}{C_p} S^* - \alpha N \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + \left[ C_r S^* - \alpha N \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + C_r \left[ S^* - \frac{\alpha N}{C_r} \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + C_r \left[ S^* - C_p \right] \mathbb{1}_{\{\tau \leq T\}}
\end{aligned} \tag{3.12}$$

It may be inferred that the financial payoff of equation 3.12 consists of two components (Erismann, 2015): (1) the face value $N$ of a zero bond and (2) a long position in $C_r$ shares generating a payoff only if the CoCo materializes at time $\tau$. This component can be approximated with a knock-in forward. The intuition behind equation 3.12 is that if the share price falls below a certain level $S^*$, an investor will use the face value $N$ to exercise the knock-in forward. That said, the investor is committed to buy the amount of $C_r$ shares for the price of $C_p$ at maturity $T$.

Before maturity the present value of a Zero-Coupon CoCo $V_t^{zcoco}$ can be determined by adding up the present value of a zero bond $V_t^{zb}$ and the present value of a knock-in forward $V_t^{kifwd}$. Hereinafter, the components will be explained briefly.

$$V_t^{zcoco} = V_t^{zb} + V_t^{kifwd} \tag{3.13}$$

with

$$V_t^{zb} = N \exp\left[-r(T-t)\right] \tag{3.14}$$

Moreover, the long position in shares at time $t$ can be approximated with the respective closed-form solution of a knock-in forward. (Hull, 2006)

$$\begin{aligned} V_t^{kifwd} = C_r \Bigg[ & S_t \exp\left[-q\left(T-t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda} \Phi\left(y_1\right) \\ & - K \exp\left[-r\left(T-t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \Phi\left(y_1 - \sigma\sqrt{T-t}\right) \\ & - K \exp\left[-r\left(T-t\right)\right] \Phi\left(-x_1 - \sigma\sqrt{T-t}\right) \\ & + S_t \exp\left[-q\left(T-t\right)\right] \Phi\left(-x_1\right) \Bigg] \end{aligned} \tag{3.15}$$

with

$$\begin{aligned} C_r &= \frac{\alpha N}{C_p} \\ K &= C_p \\ \lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2} \\ x_1 &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \\ y_1 &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \end{aligned}$$

It is important, however, to recognize that a subtle difference exists between the actual economic payoff of equation 3.12 and its replication with a knock-in forward, since the knock-in forward replicates an economic ownership of shares at maturity $T$. Though, the triggering of a CoCo forces investors to accept the conversion immediately. This could lead to an economic ownership of shares at trigger time $\tau$ and, thus, prior to $T$. Therefore, one could argue that receiving a knock-in forward in the trigger event disregards the dividends which a shareholder would receive in particular when a CoCo triggers early. De Spiegeleer and Schoutens (2012) argue that dividends can be neglected because distressed banks are likely to behave with great restraint when it comes to dividend payments.

### 3.2.2 Step Two - Adding Coupons

As mentioned earlier, the first step excludes coupon payments from the valuation. Yet, in this step we want to include them in order to replicate the exact payout profile of a CoCo. Therefore, we replace the zero bond in equation 3.13 with a straight bond with regular coupon payments $c$. Besides, a third component has to be added which takes into account the foregone coupon payments if the trigger is touched. This can be modeled with a short position in $k$ binary down-and-in calls with maturity $t_i$. Those binary down-and-in calls are knocked in if the trigger $S^*$ is met and hence, offset all future coupon payments.

The price of a straight bond can be determined with:

$$V_t^{sb} = \sum_{i=1}^{T} c_i \exp\left[-r(t_i - t)\right] + N \exp\left[-r(T - t)\right] \tag{3.16}$$

Furthermore, the formula of Rubinstein and Reiner (1991) can be used to price the down-and-in calls:

$$\begin{aligned} V_t^{bdic} = \alpha \sum_{i=1}^{k} c_i \exp\left(-r t_i\right) \Bigg[ & \Phi\left(-x_{1i} + \sigma\sqrt{t_i}\right) \\ & + \left(\frac{S^*}{S_t}\right)^{2\lambda - 2} \Phi\left(y_{1i} - \sigma\sqrt{t_i}\right) \Bigg] \end{aligned} \tag{3.17}$$

with

$$\begin{aligned} x_{1i} &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ y_{1i} &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ \lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2} \end{aligned}$$

To sum up, the theoretical price of a CoCo $V^{ed}$ at time $t$ pursuant the equity derivative approach consists of three components: (1) a straight bond $V^{sb}$, (2) a knock-in-forward $V^{kifwd}$ and (3) a set of binary down-and-in calls $V^{bdic}$:

$$V_t^{ed} = V_t^{sb} + V_t^{kifwd} - V_t^{bdic} \tag{3.18}$$

### 3.2.3 Parameter Classification and Adjustment

The equity derivative approach requires the same model inputs as the credit derivative approach. All parameters are outlined in table 3.3. The input variables are adjusted in the same way as already described in section 3.1.3.

| | **Description** | **Usage** | **Source** |
|---|---|---|---|
| $T$ | Maturity | Static input | Term sheet |
| $N$ | Notional | Static input | Term sheet |
| $c$ | Coupon rate | Static input | Term sheet |
| $\alpha$ | Conversion factor | Static input | Term sheet |
| $S_t$ | Share price | Dynamic input | Market data |
| $S^*$ | Share price | Fitting parameter | – |
| $C_p$ | Conversion price | Static input | Term sheet |
| $r$ | Risk-free interest rate | Dynamic input | Market data |
| $q$ | Dividend yield | Static input | Market data |
| $\sigma_E$ | Share price volatility | Dynamic input | Market data |

Table 3.3: Parameter classification of the equity derivative approach (Wilkens and Bethke, 2014)

### 3.2.4 Model Application

A fictive CoCo is priced based on the values shown in table 3.4. The source code for the equity derivative approach can be found in chapter A.2.

| | **Value** | **Comment** |
|---|---|---|
| $T$ | 10yrs | Maturity |
| $N$ | 100% | Nominal |
| $c$ | 6.00% | Coupon rate |
| $\alpha$ | 1 | Conversion factor |
| $S_0$ | 120 | Initial share price |
| $S^*$ | 60 | Trigger share price |
| $C_p$ | 75 | Nominal conversion price |
| $r$ | 3.00% | Risk-free interest rate |
| $q$ | 0.00% | Dividend yield |
| $\sigma_E$ | 30.00% | Share price volatility |

Table 3.4: Specification of input variables for a generic CoCo

The price of the CoCo can be separated into the individual components as presented in diagram 3.1. In that sense, the value of the risk-free coupon bond makes up a major portion of a CoCo's value under the equity derivative approach. The value of the straight bond $V^{sb}$ is equal to 125.14. The short position in the set of binary down-and-in calls $V^{bdic}$ is equivalent to -15.88. Furthermore, the value of the knock-in forward corresponds to -1.80.

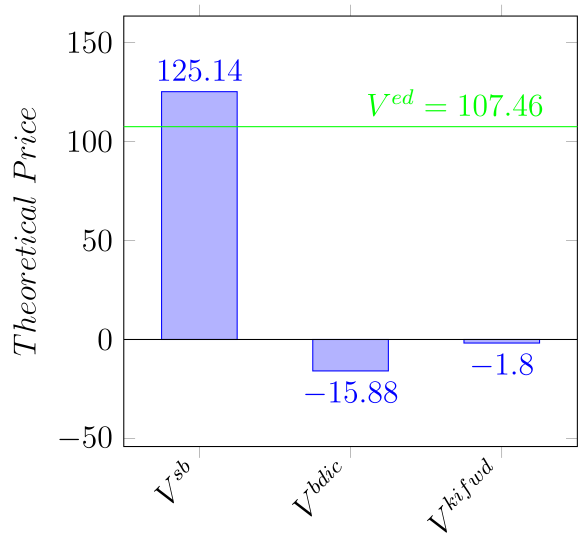


Figure 3.1: CoCo price $V^{ed}$ and separation of its components under the equity derivative approach

Thus, the price of the fictive CoCo $V^{ed}$ under the equity derivative approach is equal to 107.46.

## 3.3 Structural Approach

A third alternative to price CoCos is the structural approach of Pennacchi (2010). The idea has its roots in the seminal work of Merton (1974), which aims to explain a company's default based on the relationship of its assets and liabilities under a standard Black-Scholes setting. Pennacchi (2010)'s approach expands the idea by modeling the stochastic evolution of a bank's balance sheet respectively of its components. In the following, the assets' rate of return process will be explained. Thereafter, we will outline the assumptions of the model regarding the various liabilities a bank issues to refinance itself including deposits, equity and coupon bonds in the form of CoCos. Lastly, a pricing formula will be illustrated.

### 3.3.1 Structural Banking Model

**Bank Assets and Asset-To-Deposit Ratio**

Pennacchi (2010) assumes that a bank holds a portfolio of loans, equities and off-balance sheet positions as assets whose returns follow a jump-diffusion process. The change of this portfolio $A_t$ is determined by the rate of return and the cash in- respectively outflows. In this context, the symbol $*$ is used to point out the change in value of the portfolio which can be quantified by the rate of return, excluding net cashflows. The aforementioned instantaneous rate of return is denoted as $dA_t^*/A_t^*$ and follows a stochastic process as stated below under the risk-neutral probability measure:

$$\frac{dA_t^*}{A_t^*} = (r_t - \lambda_t k_t)\, dt + \sigma dz + \left(Y_{q_{t-}} - 1\right) dq_t \tag{3.19}$$

It should be noted that $r_t$ stands for the risk-free interest rate as defined by the Cox et al. (1985) term-structure model which will be discussed shortly. $dz$ is a Brownian motion, whereby $\sigma$ denotes the volatility of returns of the aforementioned asset portfolio. $q_t$ is a Poisson counting process which increases by one whenever a Poisson-distributed event respectively a jump occurs. Hence, the variable $dq_t$ is one whenever such a jump takes place and zero otherwise. The risk-neutral probability that a jump happens is equal to $\lambda_t dt$ where $\lambda_t$ stands for the intensity of the jump process. Variable $Y_{q_{t-}}$ is a i.i.d. random variable drawn from $\ln(Y_{q_{t-}}) \sim \Phi\left(\mu_y, \sigma_y^2\right)$ at time $t$ where $\mu_y$ stands for the mean jump size and $\sigma_y$ denotes the standard deviation of jumps. In case the random variable $Y_{q_{t-}}$ is greater than one, an upward shift in the bank's asset value can be observed. If the value is smaller than one a downward jump takes place. Given that the risk-neutral expected proportional jump $k_t$ is defined as $k_t = E_t^{\mathbb{Q}}\left[Y_{q_{t-}} - 1\right]$, one can determine $k_t$ with the following formula: $k_t = \exp(\mu_y + \frac{1}{2}\sigma_y^2) - 1$. Thus, the risk-neutral expected change in $A^*$ from the jump element $(Y_{q_{t-}} - 1)dq_t$ equals $\lambda_t k_t dt$ in $dt$. To sum up, the value development of a bank's asset portfolio $A_t^*$ follows largely a continuous process. But disruptive jumps may occur as illustrated in the graph 3.2.

The risk-neutral process of bank assets $A_t$ including the net cashflows is equal to the assets' rate of return less interest payments $r_t$ respectively premium payments $h_t$ to deposit holders proportionally to their deposits $D_t$. Furthermore, one has to subtract the coupon payments $c_t$ to CoCo investors proportionally to the face value $B$.

$$dA_t = \left[(r_t - \lambda k)\, A_t - (r_t + h_t)\, D_t - c_t B\right] dt + \sigma A_t dz + \left(Y_{q_{t-}} - 1\right) A_t dq \tag{3.20}$$

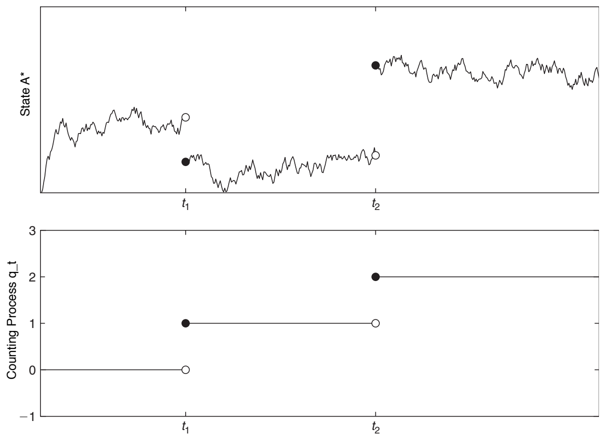


Figure 3.2: The first graph shows two jumps in the state variable $A^*$ at discrete time points. Additionally, the corresponding Poisson counting process $q_t$ is highlighted in the second graph. (Aït-Sahalia and Hansen, 2009)

By substituting variable $x_t$ with $A_t/D_t$ and anticipating the deposit growth process $g\left(x_t - \hat{x}\right)$ to behave as pointed out by equation 3.31, the risk neutral process of the asset-to-deposit ratio equals:

$$\begin{aligned}\frac{dx_t}{x_t} &= \frac{dA_t}{A_t} - \frac{dD_t}{D_t} \\ &= \left[\left(r_t - \lambda k\right) - \frac{r_t + h_t + c_t b_t}{x_t} - g\left(x_t - \hat{x}\right)\right] dt + \sigma dz + \left(Y_{q_{t-}} - 1\right) dq_t\end{aligned} \tag{3.21}$$

with

$$b_t = \frac{B}{D_t} \tag{3.22}$$

Lastly, an application of Itô's lemma for jump-diffusion processes leads to the following formula for the asset-to-deposit ratio process:

$$\begin{aligned}d\ln\left(x_t\right) = &\left[\left(r_t - \lambda k\right) - \frac{r_t + h_t + c_t b_t}{x_t} - g\left(x_t - \hat{x}\right) - \frac{1}{2}\sigma^2\right] dt \\ &+ \sigma dz + \ln Y_{q_{t-}} dq_t\end{aligned} \tag{3.23}$$

### Default-Free Term Structure

Pennacchi (2010) applies the term-structure specifications of Cox et al. (1985) to model the risk-neutral process of the instantaneous risk-free interest rate $dr_t$ which is defined as follows:

$$dr_t = \kappa\left(\bar{r} - r_t\right) dt + \sigma_r \sqrt{r_t} d\zeta \tag{3.24}$$

Note that $\kappa$ is the speed of convergence, $\bar{r}$ is the long-run equilibrium interest rate, $r_t$ is the continuous short-term interest rate, $\sigma_r$ is the instantaneous volatility and $d\zeta$ is a Brownian motion.

A zero bond can be priced using the Cox et al. (1985) specifications under the no-arbitrage assumption. This implies that the price of a risk-free zero bond at time t that pays the amount of 1 currency unit in $\tau = T - t$ is given by:

$$P\left(r_t, \tau\right) = A\left(\tau\right) \exp\left[-B\left(\tau\right) r_t\right] \tag{3.25}$$

with

$$A\left(\tau\right) = \left\{ \frac{2\theta \exp\left[\left(\theta + \kappa\right) \dfrac{\tau}{2}\right]}{\left(\theta + \kappa\right)\left[\exp\left(\theta\tau\right) - 1\right] + 2\theta} \right\}^{2\kappa\bar{r}/\sigma_r^2}$$

$$B\left(\tau\right) = \frac{2\left[\exp\left(\theta\tau\right) - 1\right]}{\left(\theta + \kappa\right)\left[\exp\left(\theta\tau\right) - 1\right] + 2\theta}$$

$$\theta = \sqrt{\kappa^2 + 2\sigma_r^2}$$

The cost of replication of a risk-free coupon bond that pays a continuous coupon of $c_r dt$ is equal to a set of zero bonds which can be priced with equation 3.25. Therefore, the fair coupon rate $c_r$ of such a coupon bond at time $t$, which is issued at par, equals:

$$\begin{aligned} c_r &= \frac{1 - A\left(\tau\right) \exp\left[-B\left(\tau\right) r_t\right]}{\int_0^\tau A\left(s\right) \exp\left[-B\left(s\right) r_t\right] ds} \\ &\approx \frac{1 - A\left(\tau\right) \exp\left[-B\left(\tau\right) r_t\right]}{\sum_{i=1}^{i=n} A\left(\Delta t \times i\right) \exp\left[-B\left(\Delta t \times i\right) r_t\right] \Delta t} \end{aligned} \tag{3.26}$$

with

$$n = \frac{\tau}{\Delta t} \tag{3.27}$$

### Deposits and Insurance Premium

Bank deposits are not riskless because depositors may suffer losses if a bank's asset value $A_t$ is worth less than the deposits $D_t$. That said, one can assume that a bank is closed by the deposit insurer when the asset-to-deposit ratio $x_t$ is less or equal to one. A bank might become distressed due to continuous downward movements in its asset value. Then, the bank will be shut down with $A_{t_b} = D_t$ and subsequently, depositors will not face any loss. However, depositors may experience severe losses when a downward jump in asset value happens at a discrete point in time, $\hat{t}$. It may be that the downward jump in asset value exceeds the bank's capital. If such a jump occurs the instantaneous proportional loss to deposits will equal $\left(D_t - Y_{q_{t^-}} A_{\hat{t^-}}\right)/D_t$.

The fair deposit insurance premium $h_t$ for deposit holders can be derived with equation 3.28. The equation illustrates that $h_t$ is closely related to the asset-to-deposit ratio $x_t$:

$$h_t = \lambda \left[ \Phi\left(-d_1\right) - x_{t^-} \exp\left(\mu_y + \frac{1}{2}\sigma_y^2\right) \Phi\left(-d_2\right) \right] \tag{3.28}$$

with

$$d_1 = \frac{\ln\left(x_{t^-}\right) + \mu_y}{\sigma_y} \tag{3.29}$$

$$d_2 = d_1 + \sigma_y \tag{3.30}$$

The model assumes that a bank pays continuously a total interest and deposit premium of $\left(r_t + h_t\right) D_t dt$ to each depositor. Hence, one might recognize that the deposits change only because of comparatively higher deposit inflows than outflows. Empirical research of Adrian and Shin (2010) suggests that banks have a target capital ratio and that deposit growth is positively related to the current asset-to deposit ratio:

$$\frac{dD_t}{D_t} = g\left(x_t - \hat{x}\right) dt \tag{3.31}$$

$\hat{x} > 1$ is a bank's target asset-to-deposit ratio with $g$ being a positive constant. Whenever the actual asset-to-deposit ratio is higher than its target, $x_t > \hat{x}$, a bank will shrink its balance sheet. Thus, the deposit growth rate $g\left(x_t - \hat{x}\right)$ in the time interval $dt$, leads to a mean-reverting tendency for the asset-to-deposit ratio $x_t$.

### Equity and Conversion Threshold

As stated originally, the conversion of a CoCo at time $t_c$ occurs when the asset-to-deposit ratio $x_{t_c}$ meets the trigger level $\bar{x}_{t_c}$. The conversion threshold can also be expressed relative to the original equity-to-deposits ratio $\bar{e}$. This is favourable because the equity value is directly observable in the market whereas the asset value is not. The relationship between the equity threshold $\bar{e}$ and the asset-to-deposit threshold $\bar{x}_{t_c}$ can be summarized as follows:

$$\bar{e} = \frac{E_{t_c}}{D_{t_c}} = \frac{A_{t_c} - D_{t_c} - pB}{D_{t_c}} = \bar{x}_{t_c} - 1 - pb_{t_c} \tag{3.32}$$

Hence, it is possible to specify exactly the conversion trigger of a CoCo bond, which will be substantial for the valuation part.

**CoCos**

The valuation of a CoCo can be accomplished with a Monte-Carlo simulation of both the asset and the deposit process. Along the asset-to-deposit ratio process, the CoCo pays coupons and the nominal at maturity unless the CoCo has not been triggered. If the trigger event occurs the conversion amount is paid out. (Wilkens and Bethke, 2014) The price of the CoCo $V^{st}$ is equal to the risk-neutral expectation of the aforementioned cashflows as derived by Pennacchi (2010):

$$V_0^{st} = E_0^{\mathbb{Q}} \left[ \int_0^T \exp\left( - \int_0^t r_s ds \right) v(t)\, dt \right] \tag{3.33}$$

Please note $v(t)$ stands for a coupon payment at date $t$ which equals $c_t B$ as long as the CoCo has not been triggered. If the CoCo does not convert until maturity $T$, a final payout of $B$ will be performed. However, if the CoCo triggers early at time $t_c$, there is the one-time cashflow of $pB$. Parameter $p$ determines the maximum conversion amount of new equity per par value of contingent capital. Thereafter, $v(t)$ is zero.

### 3.3.2 Parameter Classification and Adjustment

The biggest challenge of the structural approach is the accurate estimation of its input parameters. A reliable estimation is not straightforward as most of the variables are not directly observable in the market. (De Spiegeleer et al., 2014) A complete overview of all input variables is presented in table 3.5.

In general, one can distinguish three parameter types. The first group comprises parameters which are directly observable in the market respectively in a CoCo's term sheet. The second category encompasses variables that are linked to a bank's balance sheet or strategy and are thus semi-observable. Finally, the third group covers parameters which are in fact not observable and are determined based on expert judgement or calibration to market data. (Wilkens and Bethke, 2014) Hereinafter, major input variables and their adjustment will be described.

| | Description | Usage | Source |
|---|---|---|---|
| $T$ | Maturity | Static input | Term sheet |
| $B$ | Notional | Static input | Term sheet |
| $c$ | Coupon rate | Static input | Term sheet |
| $p$ | Conversion factor | Static input | |
| $\hookrightarrow S^*$ | Trigger share price | Fitting param. | – |
| $\hookrightarrow C_p$ | Conversion price | Static input | Term sheet |
| $x_t$ | Asset-to-deposit ratio | Dynamic input | |
| $\hookrightarrow S_t$ | Share price | Dynamic input | Market data |
| $\hookrightarrow n_t$ | Number of shares | Dynamic input | Market data |
| $\hookrightarrow D_t$ | Deposit value | Dynamic input | Balance sheet |
| $\hat{x}$ | Target asset-to-deposit ratio | Static input | |
| $\hookrightarrow A_{\text{Target}}$ | Target asset value | Static input | Term sheet |
| $\hookrightarrow D_{\text{Target}}$ | Target deposit value | Static input | Term sheet |
| $g$ | Mean-reversion speed | Static input | Expert judgm. |
| $\sigma_A$ | Annual asset return volatility | Dynamic input | |
| $\hookrightarrow D_t$ | Deposit value | Dynamic input | Balance sheet |
| $\hookrightarrow S_t$ | Share price | Dynamic input | Market data |
| $\hookrightarrow n_t$ | Number of shares | Dynamic input | Market data |
| $\hookrightarrow \sigma_E$ | Historic share price volatility | Static input | Market data |
| $\hookrightarrow r_t$ | Risk-free interest rate | Dynamic input | Market data |
| $\lambda$ | Jump intensity in asset return process | Static input | Expert judgm. |
| $\mu_y$ | Mean jump size in asset return process | Static input | |
| $\sigma_y$ | Jump volatility in asset return process | Static input | |
| $\hookrightarrow S_{\text{past}}$ | Historic share price data | Static input | Market data |
| $r_t$ | Risk-free interest rate | Dynamic input | Market data |
| $\bar{r}$ | Long-term risk-free interest rate | Static input | |
| $\sigma_r$ | Interest rate volatility | Static input | |
| $\kappa$ | Speed of convergence | Static input | |
| $\hookrightarrow r_{\text{past}}$ | Set of historic risk-free interest rate data | Static input | Market data |
| $\rho$ | Correlation between Brownian motion for asset returns and interest rate process | Static input | |
| $\hookrightarrow S_{\text{past}}$ | Historic share price data | Static input | Market data |
| $\hookrightarrow r_{\text{past}}$ | Historic risk-free interest rate data | Static input | Market data |
| $\bar{e}$ | Conversion threshold of the market value of original shareholders' equity to deposit value | Static input | |
| $\hookrightarrow S^*$ | Trigger share price | Fitting param. | – |
| $\hookrightarrow n_0$ | Initial number of shares | Static input | Term sheet |
| $\hookrightarrow D_0$ | Initial deposit value | Static input | Term sheet |
| $b_0$ | Ratio of the contingent capital's nominal to the initial value of deposits | Dynamic input | |
| $\hookrightarrow D_t$ | Initial deposit value | Dynamic input | Balance sheet |
| $\hookrightarrow B$ | Contingent capital nominal | Static input | Term sheet |

Table 3.5: Parameter classification of the structural approach

### Bank Assets and Asset-To-Deposit Ratio

The structural approach models the development of the asset-to-deposit ratio $x_t$ over time. On this account, the paper attempts to obtain $x_t$ with equity and deposit estimates. The equity component is equivalent to the daily observable market capitalization $S_t n_t$. Moreover, deposit values are inferred from a bank's quarterly published balance sheet data while assuming that all liabilities are deposits. The deposit level $D_t$ is interpolated between the disclosure of financial statements. (Wilkens and Bethke, 2014) Hence one can determine the asset-to-deposit ratio with the following equation: $x_t = (S_t n_t + D_t)/D_t$. Furthermore, the target asset-to-deposit ratio $\hat{x}$ is driven by the strategy of the issuing bank and can be described as follows: $\hat{x} = A_{\text{Target}}/D_{\text{Target}}$.

The asset volatility $\sigma_A$ is also an important input variable of the asset process but is not observable on a daily basis. Therefore, the paper evaluates the asset volatility based on its relation to the share price volatility $\sigma_E$ as described by Merton (1974). The source code can be found in section C.1.2. Moreover, the structural model assumes that asset returns follow a jump-diffusion process. This implies that the probability of extreme asset returns is larger than predicted by normally distributed asset returns. To estimate the parameters that govern the distribution of the jump size, the paper assumes that historical share price returns are a reliable proxy for asset jumps. The mean jump size $\mu_y$ and the jump volatility $\sigma_y$ are estimated based on assumptions on the jump intensity $\lambda$, which is in turn used to specify the number of exceedances over a threshold. (Longin and Solnik, 2001) The implementation is presented in section C.1.3.

### Default-Free Term Structure

For the Cox et al. (1985) model pricing parameters are estimated with the approach of Remillard (2013b). The approach calibrates the long-term risk-free interest rate $\bar{r}$, the interest rate volatility $\sigma_r$ and the speed of convergence $\kappa$. To do so, maximum likelihood techniques for dependent observations are applied. The approach takes a series of daily sovereign bond yields as input which have maturities of one, three, five and ten years. The implementation of this approach can be found in section C.1.1. Additionally, the correlation between the geometric Brownian motion for asset returns and the interest rate process $\rho$ is approximated with the five year daily correlation between the price of a sovereign bond of the same tenure respectively denomination and a related stock market index.

#### Deposits and Insurance Premium

A reasonable mean-reversion speed $g$ for deposits is assumed to be equal to 0.5. The assumption follows the assessment of Pennacchi (2010). He argues that this estimate for $g$ gives a plausible deposit's half time of around 3 years. The deposit's half-time in turn describes the time it takes for the deposit value to move half the distance towards its target value.

#### Equity and Conversion Threshold

Similar to the credit and equity derivative approach the conversion factor $p$ depends on the trigger share price $S^*$ and the conversion price $C_p$: $p = S^*/C_p$. Moreover, the conversion threshold $\bar{e}$ of a CoCo also relies on the trigger share price $S^*$ as outlined by $\bar{e} = E_{\text{Trigger}}/D_0 = nS^*/D_0$.

### 3.3.3 Model Application

The parameters shown in table 3.6 serve to price a generic CoCo pursuant to the structural approach. The pricing results can be found in figure 3.3.

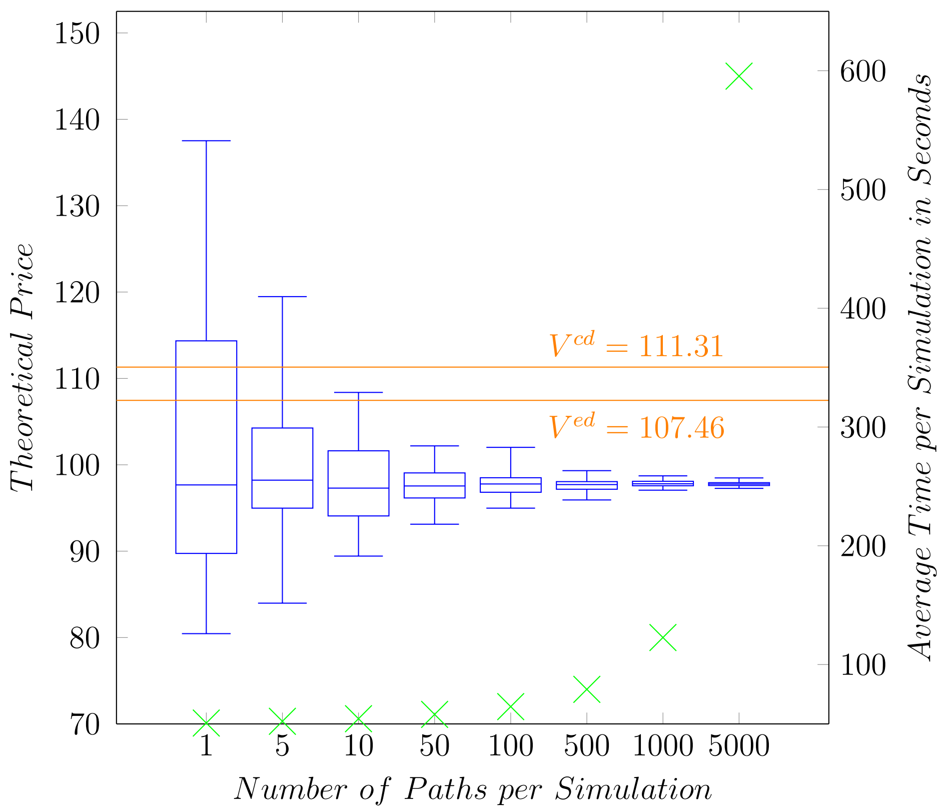


Figure 3.3: Comparison of pricing results for a generic CoCo

Running the Monte-Carlo simulation with different number of paths per simulation leads to a broad range of prices. As shown by the blue whisker plots one can directly see the minimum, the 25%-percentile, the median, the 75%-percentile and the maximum for each path number. One hundred simulations have been conducted for each boxplot. It becomes apparent that the price stability is tightly connected to the average time per simulation as shown by the green crosses. For 5000 paths per simulation one can derive a median price of $V^{sa}$ of 97.77. The derived value is significantly lower than those of the credit derivative approach $V^{cd}$ and the equity derivative approach $V^{ed}$ which are highlighted in yellow. This might be the case due to the fact that the structural approach accounts for discontinuous returns.

| | Value | Comment |
|---|---|---|
| $T$ | 10yrs | Maturity |
| $B$ | 100.00% | Notional |
| $c$ | 6.00% | Coupon rate |
| $p$ | 0.8 | Conversion factor |
| $\hookrightarrow S^*$ | 60 | Trigger share price |
| $\hookrightarrow C_p$ | 75 | Conversion price |
| $x_0$ | 1.1364 | Initial asset-to-deposit ratio |
| $\hookrightarrow S_0$ | 120 | Initial share price |
| $\hookrightarrow n$ | 1 | Number of shares |
| $\hookrightarrow D_0$ | 880 | Initial deposit value |
| $\hat{x}$ | 1.12 | Target asset-to-deposit ratio |
| $\hookrightarrow A_{\text{Target}}$ | 1000 | Target asset value |
| $\hookrightarrow D_{\text{Target}}$ | 892.86 | Target deposit value |
| $g$ | 10 | Mean-reversion speed |
| $\sigma_A$ | 3.63% | Asset volatility |
| $\hookrightarrow D_0$ | 880 | Initial deposit value |
| $\hookrightarrow n$ | 1 | Number of shares |
| $\hookrightarrow S_0$ | 120 | Initial share price |
| $\hookrightarrow \sigma_E$ | 30% | Historic share price volatility |
| $\hookrightarrow r_0$ | 3.00% | Initial risk-free interest rate |
| $\lambda$ | 2 | Jump intensity in asset return process |
| $\mu_y$ | 0 | Mean jump size in asset return process |
| $\sigma_y$ | 2.00% | Jump volatility in asset return process |
| $\hookrightarrow S_{\text{past}}$ | | Historic share price data |
| $r_0$ | 3.00% | Risk-free interest rate |
| $\bar{r}$ | 6.00% | Long-term risk-free interest rate |
| $\sigma_r$ | 5.00% | Interest rate volatility |
| $\kappa$ | 4.00% | Speed of convergence |
| $\hookrightarrow r_{\text{past}}$ | | Set of historic risk-free interest rate data |
| $\rho$ | 50% | Correlation between Brownian motion for asset returns and interest rate process |
| $\hookrightarrow S_{\text{past}}$ | | Historic share price data |
| $\hookrightarrow r_{\text{past}}$ | | Historic risk-free interest rate data |
| $\bar{e}$ | 6.81% | Conversion threshold of the market value of shareholders' equity to original deposit value |
| $\hookrightarrow S^*$ | 60 | Trigger share price |
| $\hookrightarrow n$ | 1 | Number of shares |
| $\hookrightarrow D_0$ | 880 | Initial deposit value |
| $b_0$ | 3.41% | Ratio of contingent capital's nominal to initial deposit value |
| $\hookrightarrow D_0$ | 880 | Initial deposit value |
| $\hookrightarrow CC$ | 30 | Contingent capital value |

Table 3.6: Specification of input variables for a generic CoCo under the structural approach

# Chapter 4

# Sensitivity Analyses

In the following sections, the price sensitivity of all three valuation approaches with respect to certain input variables will be examined. Sensitivity analyses are especially useful to quantify the impact a variable has on the valuation if it varies from what was initially assumed. The paper outlines a set of scenarios. All analyses use the already introduced fictive CoCo to ensure that the results are comparable.

## 4.1 Credit Derivative Approach

For the credit derivative approach, we investigate a set of scenarios concerning the underlying share price $S$, the share price volatility $\sigma_E$, a CoCo's maturity $T$, the risk-free interest rate $r$, the trigger share price $S^*$ and the conversion price $C_p$. The same scenarios are also analyzed for the equity derivative approach because the input parameters are the same.

Varying share prices $S$ and share price volatilities $\sigma_E$ have a decisive impact on CoCo prices. The effect of both variables is presented in figure 4.1. The diagram shows that higher share price levels lead to higher CoCo prices. This can be justified by the fact that it becomes less likely that the share price $S$ falls below the trigger share price $S^*$. Hence, the probability that investors face losses due to an equity conversion is lower. The compensation for that risk respectively the conversion spread $s_{CoCo}$ decreases. With surging share price volatilities $\sigma_E$, CoCo investors demand higher yields to compensate for rising conversion probabilities. For that reason, the conversion spread $s_{CoCo}$ increases which in turn leads to lower CoCo prices. However, the influence of this effect diminishes with rising share prices $S$ as it becomes less likely that stock prices below the predefined trigger share price $S^*$.

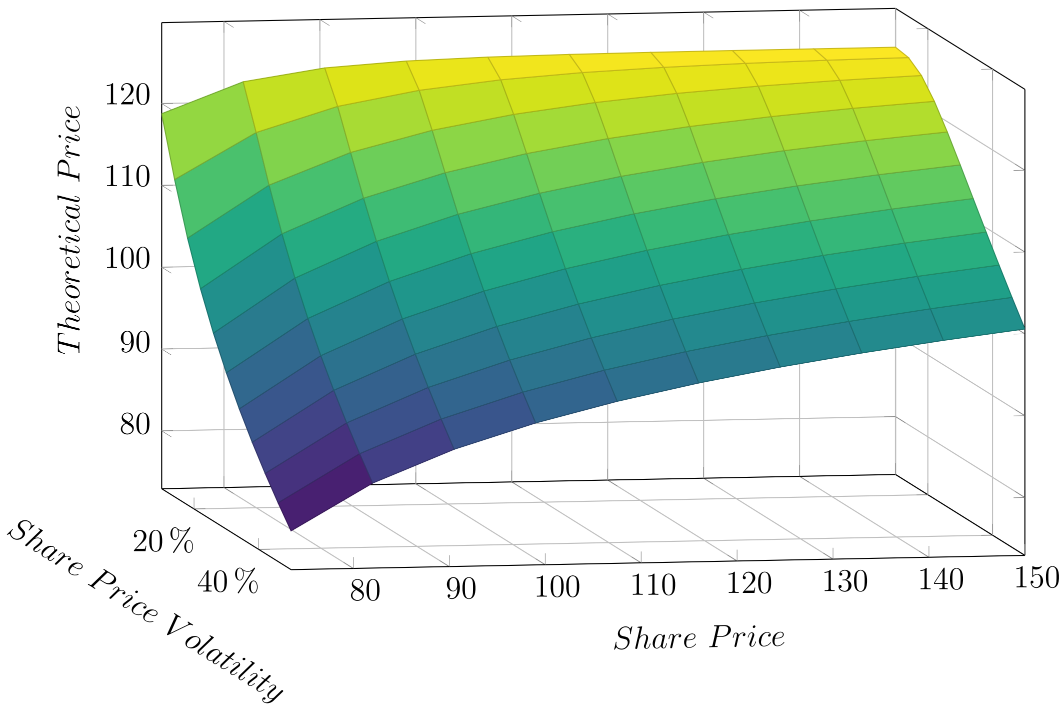


Figure 4.1: CoCo price pursuant to the credit derivative approach as function of share price $S$ and share price volatility $\sigma_E$

Furthermore, figure 4.2 shows the reaction of the CoCo price due to changes in maturity $T$ and the risk-free interest rate $r$. One can observe that higher risk-free interest rate levels lead to lower CoCo prices. By contrast, for rising maturities, one can observe higher CoCo prices, except for the combination of high-interest rates and a maturity of ten years. In this scenario, the risk-free interest rate effect outweighs the maturity effect. Also, when analyzing the development of the conversion spread $s_{CoCo}$ one can also see that the conversion spread is at its maximum with a maturity of ten years. The influence of parameter $T$ on the value of a CoCo increases significantly with lower interest rate levels. The highest CoCo price can be found for the combination of low-interest rates and high maturities. One can explain the observation by compounding effects and comparatively high discount factors especially for the notional.

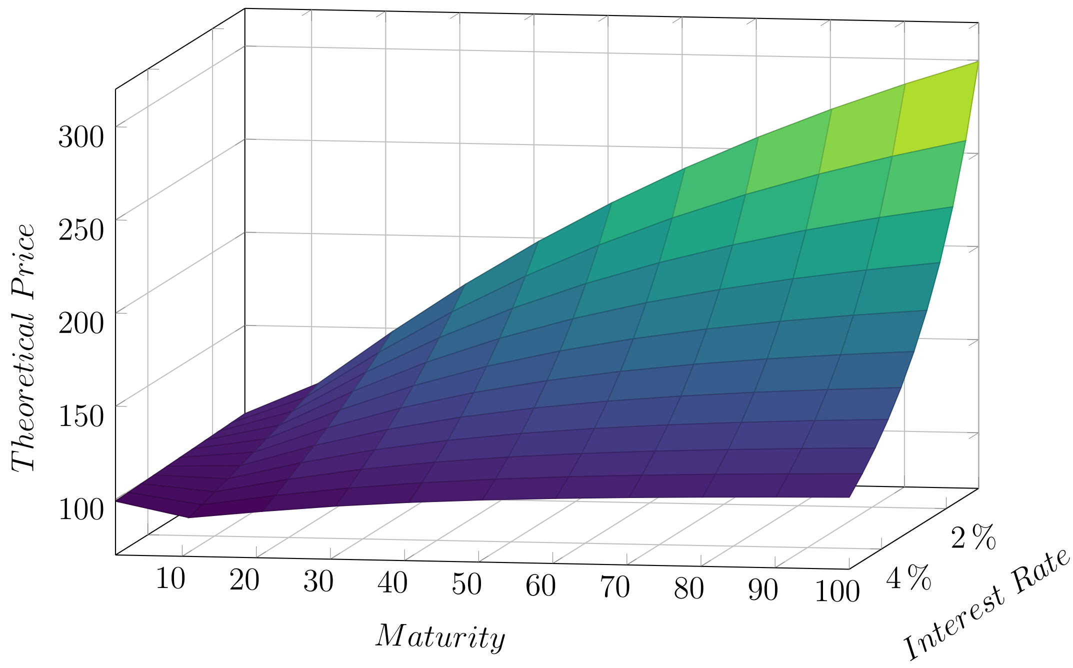


Figure 4.2: CoCo price pursuant to the credit derivative approach as function of maturity $T$ and risk-free interest rate $r$

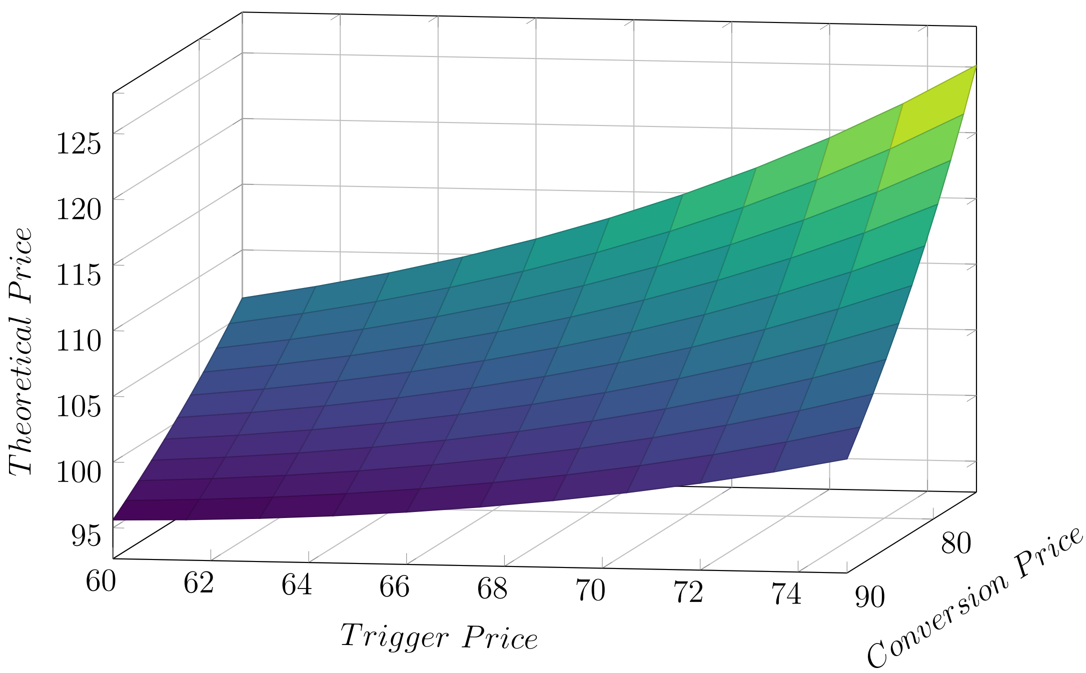


Figure 4.3: CoCo price pursuant to the credit derivative approach as function of trigger price $S^*$ and conversion price $C_p$

Figure 4.3 illustrates the sensitivity of the CoCo price with respect to the trigger price $S^*$ and the conversion price $C_p$. The graph reveals that lower conversion prices result in higher CoCo prices while keeping the trigger price constant. This is because the recovery rate $R_{CoCo}$ rises. The opposite effect is visible for low trigger prices while holding the conversion price constant. A low trigger price implies that the likelihood of conversion is lower due to a higher distance between the actual share price $S$ and the trigger price $S^*$.

## 4.2 Equity Derivative Approach

For the equity derivative approach we conduct the same sensitivity analyses like for the credit derivative approach. Again the aim is to determine how the CoCo price is affected by changes in the model inputs.

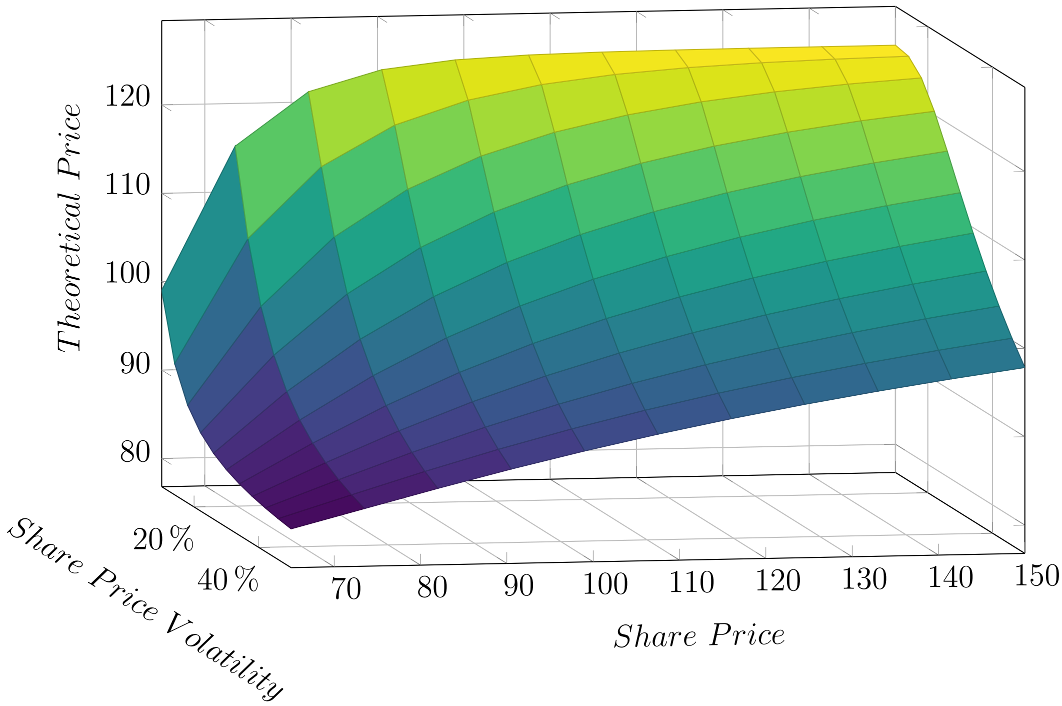


Figure 4.4: CoCo price pursuant to the equity derivative approach as function of share price $S$ and share price volatility $\sigma_E$

Figure 4.4 helps to understand the price dynamics with respect to the share price $S$ and the share price volatility $\sigma_E$. One might argue that with an increasing share price $S$ the distance to the trigger price $S^*$ grows, which in turn reduces the conversion probability of the CoCo. This has a positive impact on the price. The CoCo converges towards a straight bond. Besides, the line of thought for changes of the share price volatility $\sigma_E$ is comparable. With a rising share price volatility $\sigma_E$ the risk increases that the underlying share price $S$ hits the barrier $S^*$ and that CoCo investors face losses.

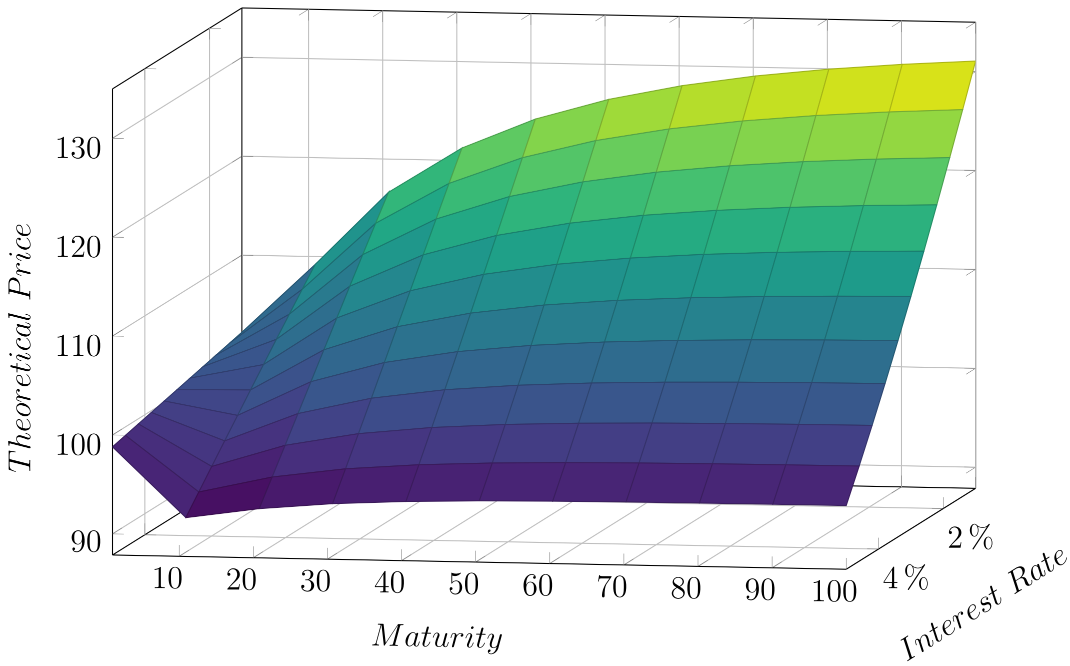


Figure 4.5: CoCo price pursuant to the credit derivative approach as function of maturity $T$ and risk-free interest rate $r$

Figure 4.5 shows the price sensitivity of a CoCo with respect to its maturity $T$ and the interest rate $r$. Considering a straight bond as a major component of a CoCo helps to understand the price dynamics. One can observe an inverse relationship between the CoCo price and the risk-free interest rate. Also, the price sensitivity of the CoCo with respect to the interest rate rises with its maturity. Though, the increase occurs at a decreasing rate except for a maturity smaller than ten years.

Figure 4.6 illustrates the price sensitivity of a CoCo concerning the conversion price $C_p$ and the trigger price $S^*$. It helps to take a closer look at a CoCo's components under the equity derivative approach. The first component namely the long position in a straight bond is not affected by either of the two variables. However, the value of the short position in a set of down-and-in calls is determined by the trigger price $S^*$. Moreover, the trigger price $S^*$ and the conversion price $C_p$ impact the price of

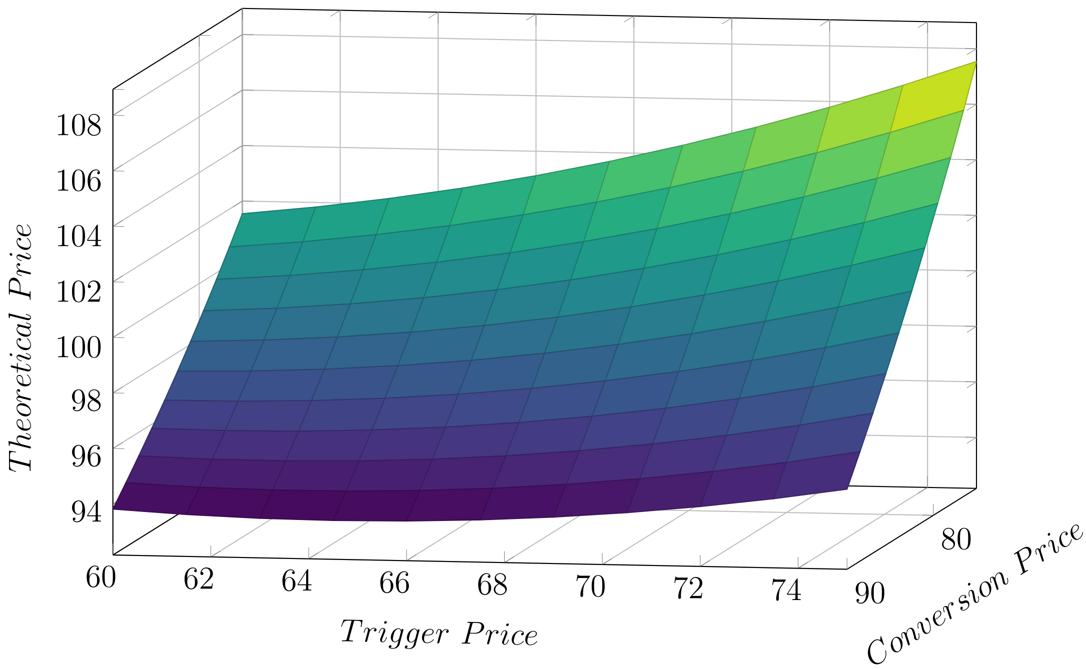


Figure 4.6: CoCo price pursuant to the credit derivative approach as function of trigger price $S^*$ and conversion price $C_p$

the long position in a knock-in forward which consists of a long position in a call and a short position in a put both with strike $C_p$. These two options come into existence if the trigger price $S^*$ is touched. One might argue for a given trigger price $S^*$, the lower the conversion price $C_p$ is, the farther is the knock-in forward in the money. This, in turn, might be associated with a higher CoCo price. Though, these two options come only into existence if the trigger price $S^*$ is met. Hence, the lower $S^*$ is, the lower is the probability that the knock-in forward comes into existence and the lower is the value of the position. Having said that, one might also consider that the lower the trigger price $S^*$ is, the higher is the value of the short position in a set of binary down-and-in calls as it becomes more likely that the underlying share price $S$ fails to remain above the trigger price $S^*$. These are two opposite forces, whereupon the impact of both change with decreasing conversion prices as the influence of the knock-in forward effect becomes dominant.

## 4.3 Structural Approach

The structural approach requires different model inputs then the other two approaches. Therefore, the following sensitivity analyses focusses on the initial asset-to-deposit ratio $x_0$, the asset volatility $\sigma_A$, the maturity $T$, the risk-free interest rate $r$, the equity-to-deposit threshold $\bar{e}$, the jump intensity $\lambda$ and the contingent capital's nominal to the initial value of deposits $b_0$.[1]

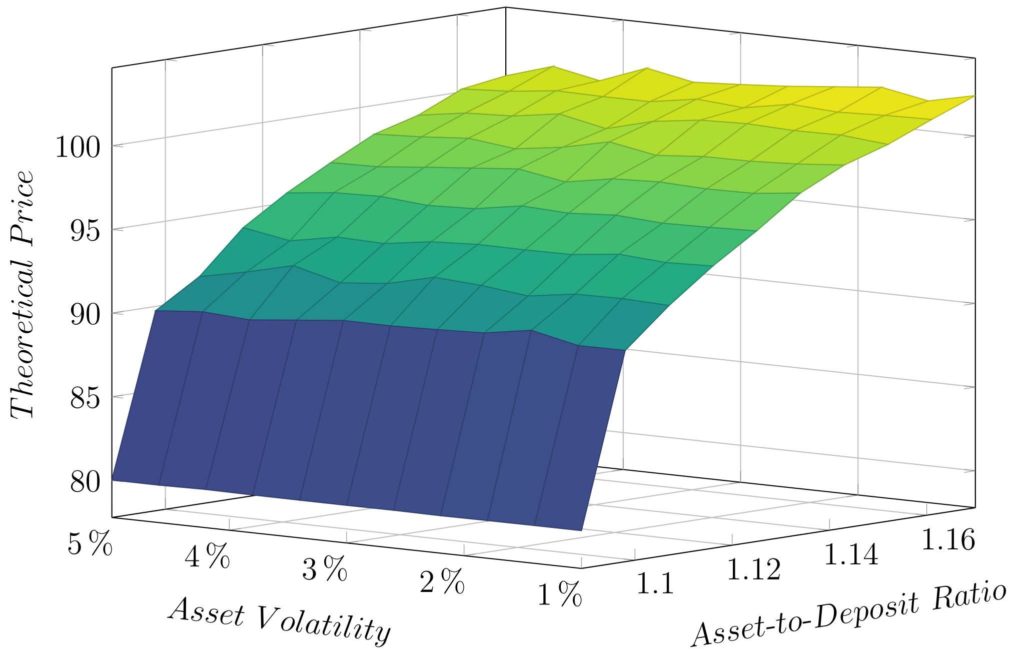


Figure 4.7: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio $x_0$ and asset volatility $\sigma_A$

Figure 4.7 details a CoCo's price reaction to changes of the initial asset-to-deposit ratio $x_0$ and the asset volatility $\sigma_A$. The diagram demonstrates that prices move closely with the asset-to-deposit ratio. This is because financial institutions with higher initial asset-to-deposit ratios are better capitalized which, in turn, reduces the conversion probability. At an asset-to-deposit ratio of 1.09, one can observe that the CoCo bond triggers. Interestingly, the analysis shows that the asset volatility does not influence a CoCo's price. Having said that, one would typically assume that a higher asset volatility is inversely related to the price because the likelihood of conversion rises. The lack of a stable solution for a given asset volatility $\sigma_A$ might be a shortcoming of the structural approach.

[1] The Monte-Carlo simulation, which is used to determine the prices, runs in the Amazon Elastic Compute Cloud (EC2) as the service provides a re-sizable compute capacity which is key to quickly scale the computing requirements particularly for computational intense simulations. If one wants to replicate the simulations, it is recommended to follow the instructions of Shekel (2015) to set up a Rstudio server on Amazon EC2.

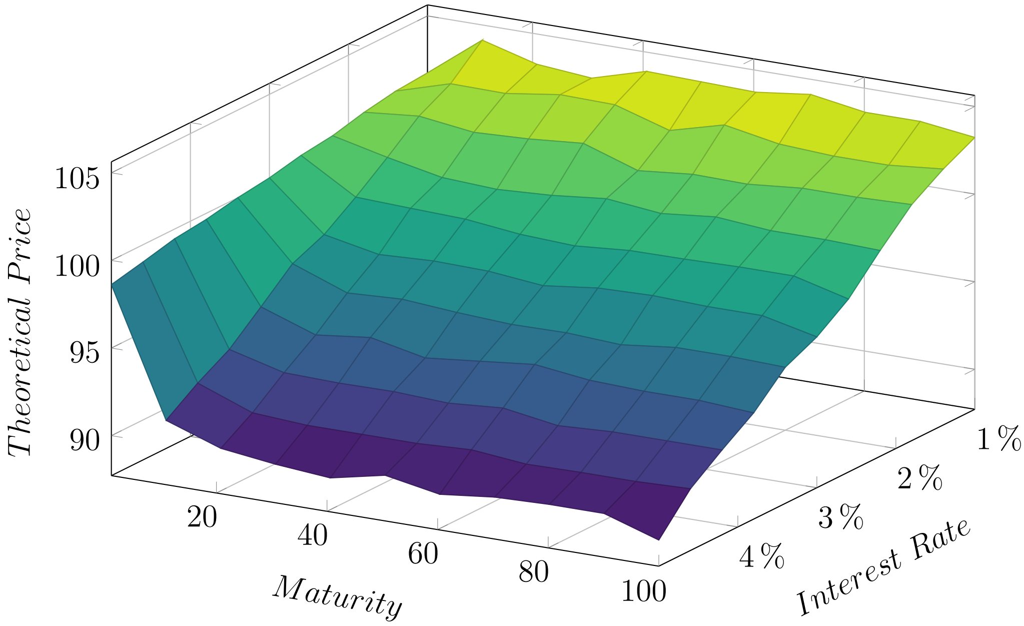


Figure 4.8: CoCo price pursuant to the structural approach as function of maturity $T$ and interest rate $r$

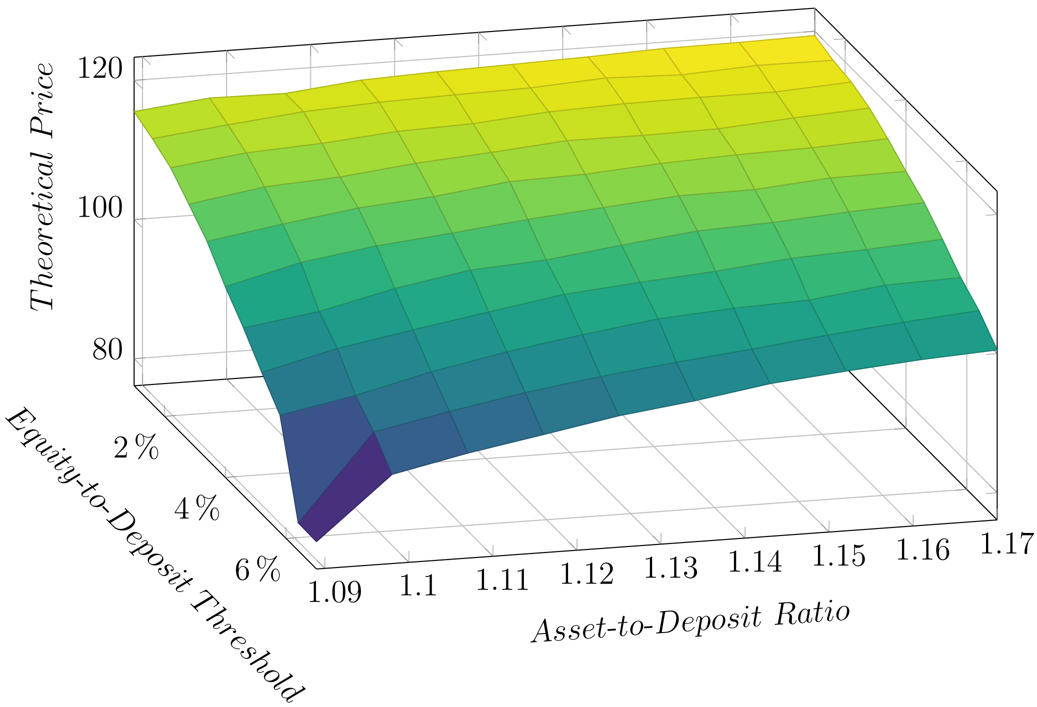


Figure 4.9: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio $x_0$ and equity-to-deposit threshold $\bar{e}$

Figure 4.8 exemplifies the CoCo price as function of maturity $T$ and the interest rate $r$. As can be seen from the diagram, the CoCo price decreases with rising interest rates. Though, there is a difference between the structural approach and the other two approaches because the price sensitivity of the CoCo decreases with rising maturities. At a maturity of around thirty years, the price sensitivity concerning the interest rate does not change anymore. Moreover, the price does not react to changes in the maturity.

As expected, figure 4.9 shows that the CoCo price falls with a rising equity-to-deposit threshold, which is set to be the conversion threshold for the structural approach. The explanation is similar to that of the trigger price $S^*$ under the equity derivative approach. The conversion probability rises because the equity-to-deposit threshold approaches the current share price. Furthermore, one can observe that the CoCo triggers at the combination of an asset-to-deposit ratio of 1.09 and an equity-to-deposit threshold of 6.00%. Similar dynamics are also observable in figure 4.10. By increasing the ratio of initial contingent-capital's nominal to deposits ratio, one can recognize that the valuation comes down as leverage increases.

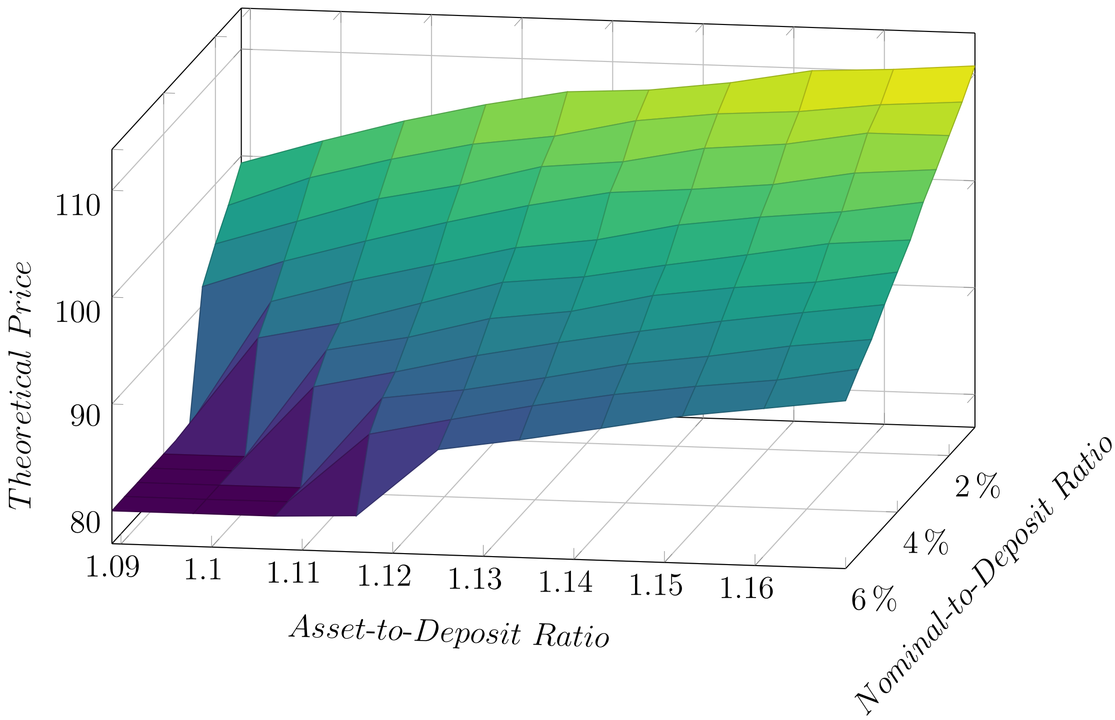


Figure 4.10: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio $x_0$ and initial ratio of contingent capital's nominal to the initial value of deposits $b_0$

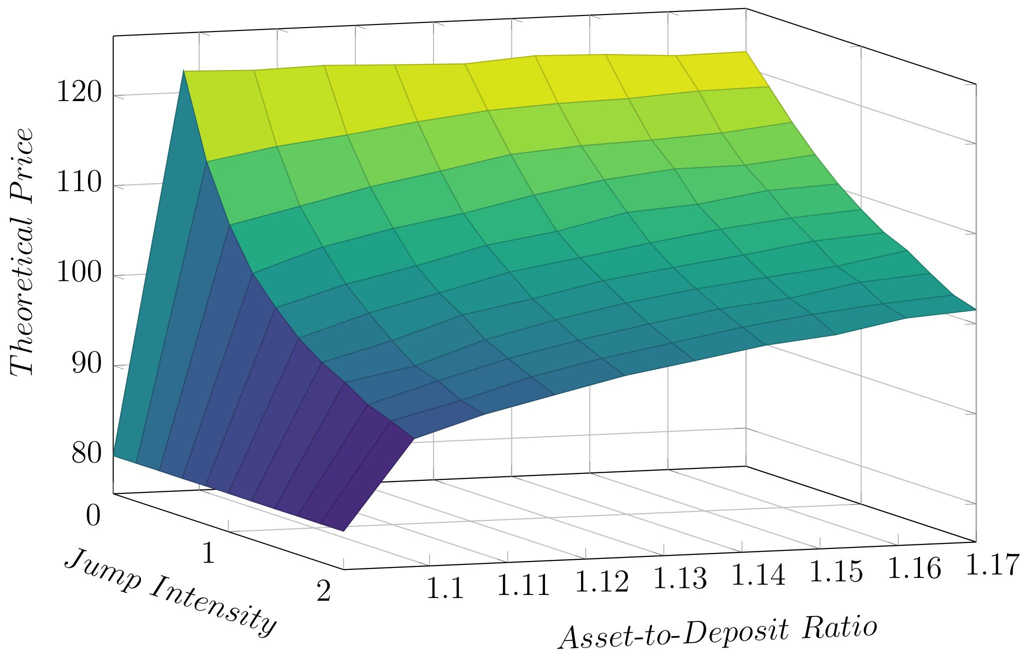


Figure 4.11: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio $x_0$ and jump intensity $\lambda$

Figure 4.11 draws attention to the importance of a correct estimation of the jump intensity $\lambda$ due to its significant impact on a CoCo's valuation. Rising jump intensities imply that more return jumps of both signs are expected to occur over the course of one year. This entails a higher tail risk which depresses the value of a CoCo but with a decreasing rate. At low asset-to-deposit ratios, one can observe that the depreciation of a CoCo can be serious. Undercapitalized banks bear high conversion risks. The sensitivity analysis indicates that the other two approaches may underestimate the risk of severe events as they do not factor in discontinuous returns in their model.

# Chapter 5

# Case Study

All three approaches will be applied to a AT1 CoCo of HSBC which was issued in early 2015. The case study helps to evaluate the models concerning their price tracking accuracy.

## 5.1 CoCo Example

On March 30, 2015, HSBC issued its Perpetual Subordinated Contingent Convertible Securities. The aggregate principal amount from the issuance of the CoCo sums up to USD 2.475bn. HSBC intended to strengthen its capital base with the proceeds of the issuance. The interest on the CoCo will be a rate per annum equal to 6.375%. Besides, the CoCo pays its coupon semi-annually. The conversion price is fixed ex-ante at USD 4.03488. The trigger event occurs if the CET1 ratio fails to remain above a threshold of 7.0% as of any business day on which HSBC calculates the CET1 ratio. If the CoCo breaches the threshold, it converts automatically to equity.

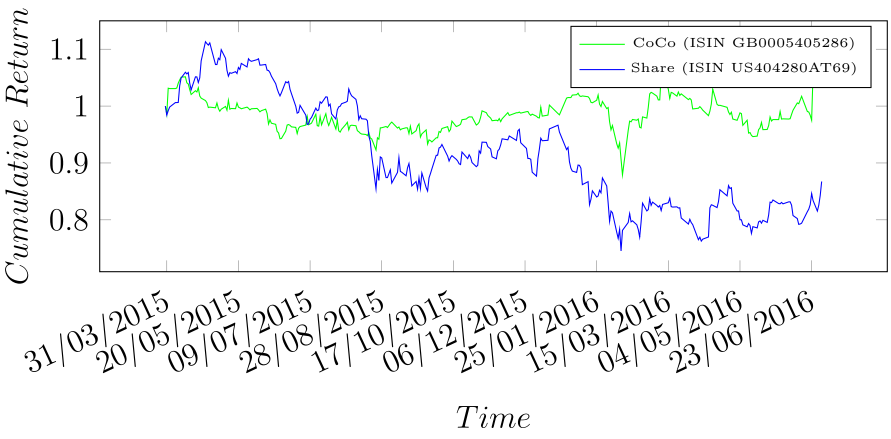


Figure 5.1: Cumulative return of HSBC's share and CoCo adjusted for GBP/USD currency effects

Figure 5.1 gives a first impression of both the cumulative return development of the reference share price (ISIN US404280AT69) and the respective CoCo (ISIN GB0005405286). The pricing period ranges from March 31, 2015, to June 30, 2016. The mean price of the CoCo during this time intervall equals GBP 66.25, whereas the minimum price is GBP 56.88. The maximum price amounts to GBP 74.12. The biggest price jump can be found in early 2016 when turmoils arose regarding the healthiness of the entire European banking system. Many CoCos of other financial institutions reacted similarly.

## 5.2 Methodology

The aim of the case study is to determine how different the predicted are compared to observed prices. This is called price tracking accuracy. The mean absolute error (MAE) and the root mean squared error (RMSE) are calculated for the model estimates in the observation period to evaluate the price tracking accuracy of all three approaches. Input parameters are estimated as described in previous chapters. Subsequently, a brute force algorithm is applied to minimize the RMSE of each approach by varying the model implied trigger share price $S^*$. The approach is computationally intense. Further algorithm optimizations could improve the time-consuming pricing process. After deriving a value for the model implied trigger price $S^*$ for each approach, the MAE is calculated.

The RMSE represents the standard deviation of the differences between predicted CoCo values $\hat{y}_i$ and observed values $y_i$ for a number of trading days $i$. With the following equation the RMSE can be derived:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (\hat{y}_i - y_i)^2}{n}} \tag{5.1}$$

By contrast, the MAE is the arithmetical average of the absolute differences between predicted CoCo values $\hat{y}_i$ and observed market values $y_i$ up to a certain time point $i$. The MAE is given by:

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |\hat{y}_i - y_i| \tag{5.2}$$

The obtained values will help to determine the appropriateness of the models to price existing CoCos over a certain period.

## 5.3 Valuation Results

In the following, the valuation results on HSBC's CoCo are analyzed visually and numerically. In that sense, figure 5.2 illustrates the valuation results and the observed CoCo prices.

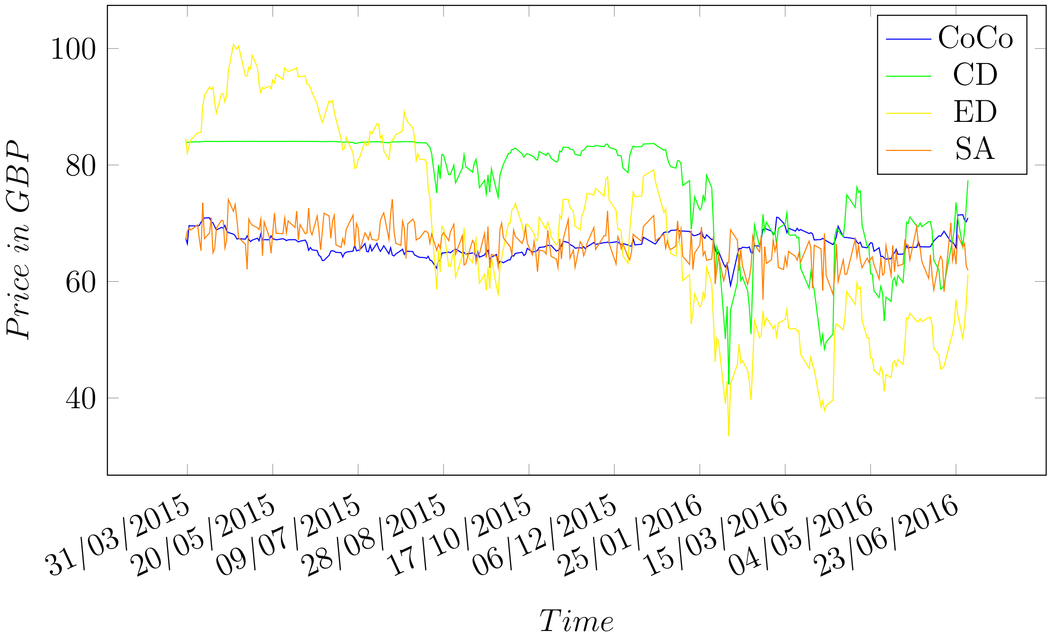


Figure 5.2: Simulation results for the CoCo of HSBC

What becomes apparent is that the credit derivative approach and equity derivative approach under- respectively overestimate the price of the CoCo for long time periods. It is interesting to observe that the are higher for the credit derivative approach compared to the estimates of the equity derivative approach. This finding has also been describe theoretically. The structural approach, on the other side, follows more or less the CoCo price. The lack of stability of the price estimates for the structural approach may be explained by the fact that only one hundred simulations have been conducted in the Monte-Carlo simulation per trading day. Due to the RMSE brute force algorithm and finite assumptions on the maturity, significant computational capacities are necessary to derive stable prices.

Table 5.1 supports the initial visual impression that the structural approach is superior in tracking the actual CoCo price as the RMSE equals 0.1012 and the MAE is 6.52. The other two methods do not appear to track the price of the CoCo adequately which is indicated by high RMSE and MAE values. The model implied conversion price of the structural approach is equal to 306.12. In contrast, the credit derivative method assumes a significantly lower trigger price of 215.03 which is different to the price of 318.67 that is implied by the equity derivative approach.

| | **Trigger price $S^*$** | **RMSE** | **MAE** |
|---|---|---|---|
| Credit Derivative Approach | 215.03 | 0.2031 | 12.34 |
| Equity Derivative Approach | 318.67 | 0.1974 | 11.65 |
| Structural Approach | 306.41 | 0.1012 | 6.52 |

Table 5.1: Case study results and model implied trigger price $S^*$

For the observation period, it seems that the structural approach performs best to track the price of the perpetual CoCo. Price differences between the estimated and observed prices might occur because none of the approaches accounts for the callability feature. Though, the empirical results cannot be generalized based on one CoCo. But one can summarize that the structural approach has been dominant to price the CoCo example in the observation period.

# Chapter 6

# Conclusion

The thesis has the objective to compare different valuation methods for CoCos. Three pricing approaches have been selected: the credit derivative approach, the equity derivative approach (De Spiegeleer and Schoutens, 2012) and the structural approach (Pennacchi, 2010). All approaches are brought into context to the current state of research. Apart from this, the paper provides comprehensive explanations of the theoretical concepts behind the valuation approaches. Also, the models are applied consistently to a generic CoCo to understand their parametrization and implementation complexity. An application to a real-world CoCo of HSBC allows for further insights.

The first model, the credit derivative approach, is theoretically an elegant way to price CoCos. This is partly because the parametrization to market data is straightforward and fast calculations are guaranteed. However, conceptual weaknesses are detectable. A closer look into the model dynamics suggests that the model does not account for discontinuous returns. Hence, inherent tail risks are potentially underestimated. This seems to be confirmed by the fact that the estimated price of the generic CoCo is significantly higher than those of the equity derivative approach and the structural approach. Also, losses from canceled coupons of triggered CoCos are not taken into account in the valuation which might lead to an overestimation. From a practical viewpoint, the model does not inherit an equity spot process and therefore, equity risks cannot be determined. (Turfus and Shubert, 2015)

The equity derivative approach has strengths similar to those of the credit derivative approach. Though, they share conceptual flaws. They take the stock price at conversion as model input and not as stochastic output although it is very likely that equity jumps occur when the CoCo is triggered. (Turfus and Shubert, 2015) Furthermore, the equity derivative approach might underestimate the value of dividend payments to CoCo investors after the conversion has happened. One might also argue that credit risk calculations are not possible since the approach does not account for them. (Turfus and Shubert, 2015)

The structural approach complies very well with the hybrid nature of CoCos as it attempts to model the dynamics of the entire balance sheet. Tail risks are taken into consideration by factoring in a jump diffusion process to overcome the artificial simplification of continuous returns under a Black-Scholes setting. Though, several model inputs are necessary to apply the valuation approach to real world examples. An accurate estimation proves itself to be tough since certain parameters are not directly observable in the market or updated infrequently. The case study, however, reveals that a precise parameterization is indispensable. Also, good algorithm design skills are necessary to cut the calculation time of the Monte-Carlo simulation.

| | **CD** | **ED** | **SA** |
|---|---|---|---|
| Price tracking accuracy | low | low | medium |
| Parametrization complexity | low | low | high |
| Calculation time | low | low | high |

Table 6.1: Evaluation of pricing approaches with regard to price tacking accuracy, parametrization complexity and calculation time

Table 6.1 summarizes the results of the thesis. It becomes apparent that the parameterization is far more complicated fo the structural approach compared to the other two approaches. Besides, the calculation time and associated cost are significantly higher for the structural approach. But the advantage is that the price tracking results are better over time. As already mentioned, the applied pricing process might only be a heuristic that worked well for the HSBC CoCo in the observation period. Further empirical evidence should be conducted. A shortcoming of all approaches is that they do not answer the question how confident they are about their own price estimates.

# Appendix A

# Code - Models

## A.1 Credit Derivative Approach

The following source code is an implementation of the credit derivative approach (De Spiegeleer and Schoutens, 2012) written in R.

```
# Price of Contingent Convertible Bond
price_coco_cd <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma){

  spread_coco <- calc_spread_coco(t, T, S_t, S_star, C_p, r, q, sigma)
  V_t_coco <- N * exp(-(r + spread_coco) * (T - t))

  for (time in seq((t+0.5), T, 0.5)){
    V_t_coco <- V_t_coco + c_i * exp(-(r + spread_coco) * time)
  }
  return(V_t_coco)
}

# Calculation of Trigger Probability
calc_p_star <- function(t, T, S_t, S_star, r, q, sigma){
  p_star <- pnorm((log(S_star / S_t) - calc_mu(r, q, sigma) * (T - t)) / (sigma * sqrt(T - t))) + (S_star / S_t)^(2 * calc_mu(r, q, sigma) / sigma^2) * pnorm((log(S_star / S_t) + calc_mu(r, q, sigma) * (T - t)) / (sigma * sqrt(T - t)))
  return(p_star)
}

# Calculation of Drift of Underlying
calc_mu <- function(r, q, sigma){
  mu <- r - q - sigma^2 / 2
  return(mu)
}

# Spread of CoCo Bond
calc_spread_coco <- function(t, T, S_t, S_star, C_p, r, q, sigma){
  spread_coco <- - log(1 - calc_p_star(t, T, S_t, S_star, r, q, sigma)) / (T - t) * (1 - S_star / C_p)
  return(spread_coco)
```

```r
}

# Pricing Example
#price_coco_cd(t <- 0, T <- 10, S_t <- 120, S_star <- 60, C_p <- 75, c_i <- 6.00, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3)
```

## A.2 Equity Derivative Approach

The following source code is an implementation of the equity derivative approach (De Spiegeleer and Schoutens, 2012) written in R.

```r
# Price of Contingent Convertible Bond
price_coco_ed <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma, alpha){
  V_t_ed <- price_cb(t, T, c_i, r, N) - price_dibi(t, T, S_t, S_star, c_i, r, q, sigma, alpha) + price_difwd(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha)

  return(V_t_ed)
}

# Price of Corporate Bond
price_cb <- function(t, T, c_i, r, N){
  V_t_cb <- N * exp(-r * (T - t))

  for (time in seq((t+1), T, 1)){
  V_t_cb <- V_t_cb + c_i * exp(-r * time)
  }

  return(V_t_cb)
}

# Price of Binary Option
price_dibi <- function(t, T, S_t, S_star, c_i, r, q, sigma, alpha){
  V_t_dibi <- 0

  i <- t
  k <- T

  for (i in seq((t+1), k, 1)) {
  V_t_dibi <- V_t_dibi + c_i * exp(- r * i) * (pnorm(- calc_x_1_i(S_t, S_star, sigma, r, q, i) + sigma * sqrt(i)) + (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm ( calc_y_1_i(S_t, S_star, sigma, r, q, i) - sigma * sqrt(i)))
  }

  V_t_dibi <- alpha * V_t_dibi

  return(V_t_dibi)
}

# Price of Down-And-In Forward
```

```
price_difwd <- function(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha){
  V_t_difwd <- calc_conversion_rate(C_p, N, alpha) * (S_t * exp(- q * (T - t)) * (S_star / S_t) ^ (2 * calc_lambda(r, q, sigma)) * pnorm(calc_y_1(t, T, S_t, S_star, r, q, sigma)) - C_p * exp(- r * (T - t)) * (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm(calc_y_1(t, T, S_t, S_star, r, q, sigma) - sigma * sqrt(T - t)) - C_p * exp(- r * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma) + sigma * sqrt(T - t)) + S_t * exp(- q * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma)))

  return(V_t_difwd)
}

# Calculation of Conversion Rate
calc_conversion_rate <- function(C_p, N, alpha){
  C_r <- alpha * N / C_p

  return(C_r)
}

# Calculation of additional Parameters
calc_x_1_i <- function(S_t, S_star, sigma, r, q, t_i){
  x_1_i <- log(S_t / S_star) / (sigma * sqrt(t_i)) + calc_lambda(r, q, sigma) * sigma * sqrt(t_i)

  return(x_1_i)
}

calc_y_1_i <- function(S_t, S_star, sigma, r, q, t_i){
  y_1_i <- log(S_star / S_t) / (sigma * sqrt(t_i)) + calc_lambda(r, q, sigma) * sigma * sqrt(t_i)

  return(y_1_i)
}

calc_lambda <- function(r, q, sigma){
  lambda <- (r - q + sigma^2 / 2) / sigma^2

  return(lambda)
}

calc_x_1 <- function(t, T, S_t, S_star, r, q, sigma){
  x_1 <- log(S_t / S_star) / (sigma * sqrt(T - t)) + calc_lambda(r, q, sigma) * sigma * sqrt(T - t)

  return(x_1)
}

calc_y_1 <- function(t, T, S_t, S_star, r, q, sigma){
  y_1 <- log(S_star / S_t) / (sigma * sqrt(T - t)) + calc_lambda(r, q, sigma) * sigma * sqrt(T - t)

  return(y_1)
}
```

```

# Pricing Example
#price_coco_ed(t <- 0, T <- 10, S_t <- 120, S_star <- 60, C_p <- 75, c_i
    <- 6.00, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
```

## A.3 Structural Approach

The following source code is an implementation of the structural approach (Pennacchi, 2010) with support from Quantnet (2014) in translating the source code of Pennacchi (2010) from GAUSS to R.

```
# Price of Contingent Convertible Bond
price_coco_sa <- function(T , nsimulations , rho , kappa , r_bar, r0,
    sigma_r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0,
    B, coupon){

  ndays <- T * 250
  dt <- T / ndays

  # Get Brownian motions
  result <- sim_corrProcess(T, nsimulations, rho, ndays, dt)
  dz_1 <- result$dz_1
  dz_2corr <- result$dz_2corr

  # Simulate Cox et al. (1985) term-structure process
  r <- sim_interestrate(kappa, r_bar, r0, sigma_r, dz_2corr, ndays,
    nsimulations, dt)

  # Simulate price of contingent convertible bond with a Monte-Carlo
    simulation
  V_t_sa <- get_price(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y,
    sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0, B, coupon) *
    100

  return(V_t_sa)
}

# Create correlated Brownian motions for asset and interest rate process
sim_corrProcess <- function(T, nsimulations, rho, ndays, dt){

  # Compute the Choleski factorization of a real symmetric positive-
    definite square matrix.
  chol_RHO <- t(chol(matrix(c(1, rho, rho, 1), nrow = 2)))

  # Random generation for the normal distribution with mean equal to 0
    and standard deviation equal to 1
  dz_1 <- matrix(1, ndays, nsimulations)
  dz_2 <- matrix(1, ndays, nsimulations)
  for(j in 1:nsimulations)
  {
    dz_1[ , j] <- rnorm(ndays) * sqrt(dt)
```

```r
    dz_2[ , j] <- rnorm(ndays) * sqrt(dt)
  }

  # Create correlated Brownian motions using Cholesky-decomposition for the Cox et al. (1985) term-structure process
  dz_2corr <- matrix(1, ndays, nsimulations)
  for(j in 1:nsimulations)
  {
    for(i in 1:ndays)
    {
      dz_2corr[i, j] <- dz_1[i, j] * chol_RHO[2, 1] + dz_2[i, j] * chol_RHO[2, 2]
    }
  }

  return(list("dz_1" = dz_1, "dz_2corr" = dz_2corr))
}

# Simulate Cox et al. (1985) term-structure process
sim_interestrate <- function(kappa, r_bar, r0, sigma_r, dz_2corr, ndays, nsimulations, dt){
  r <- matrix(r0, ndays + 1, nsimulations)

  for(j in 1:nsimulations)
  {
    for(i in 1:ndays)
    {
      r[i + 1, j] <- r[i, j] + kappa * (r_bar - r[i, j]) * dt + sigma_r * sqrt(abs(r[i, j])) * dz_2corr[i, j]
    }
  }

  return(r)
}

get_price <- function(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0, B, coupon){

  # Define parametres
  phi <- matrix(rbinom( ndays %*% nsimulations, 1, dt * lambda), ndays, nsimulations)

  ln_Y <- matrix(rnorm(ndays %*% nsimulations, mu_Y, sigma_Y), ndays, nsimulations)

  # Ratio of contingent capital's nominal to the value of deposits
  b <- matrix(b0, ndays + 1, nsimulations)

  h <- matrix(1, ndays, nsimulations)

  # Paramter for jump diffusion process
  k <- exp(mu_Y + 0.5 * sigma_Y^2) - 1

  # Target asset-to-deposit ratio
```

```r
x_bar0 <- 1 + e_bar + p * b0
x_bar <- matrix(x_bar0, ndays + 1, nsimulations)

# Asset-to-deposit ratio
x <- matrix(x0, ndays + 1, nsimulations)
ln_x0 <- matrix(log(x0), ndays + 1, nsimulations)
ln_x <- ln_x0

trigger_dummy <- matrix(1, ndays + 1, nsimulations)

# Simulate asset-to-deposit ratio and trigger events

for(j in 1:nsimulations)
{
  for(i in 1:ndays)
  {
    d_1 <- (ln_x[i, j] + mu_Y) / sigma_Y
    d_2 <- d_1 + sigma_Y

    h[i, j] <- lambda * (pnorm( - d_1) - exp(ln_x[i, j]) * exp(mu_Y +
  0.5 * sigma_Y^2) * pnorm(-d_2))

    b[i + 1, j] <- b[i, j] * exp(- g * (exp(ln_x[i, j]) - x_hat) * dt)

    ln_x[i + 1, j] <- ln_x[i, j] + ( (r[i, j] - lambda * k) - (r[i, j]
   + h[i, j] + coupon * b[i, j]) / exp(ln_x[i, j]) - g * (exp(ln_x[i,
  j]) - x_hat) - 0.5 * sigma_A^2) * dt + sigma_A * sqrt(dt) * dz_1[i,
  j] + ln_Y[i,j] * phi[i, j]

    x[i + 1, j] <- exp(ln_x[i + 1, j])

    x_bar[i + 1, j] <- 1 + e_bar + p * b[i + 1, j]

    if(is.na(trigger_dummy[i, j]) == TRUE){
      trigger_dummy[i, j] <- trigger_dummy[i-1, j]
    }

    if(x[i + 1, j] >= x_bar[i + 1, j] && trigger_dummy[i, j] > 0.5)
    {
      trigger_dummy[i + 1, j] <- 1
    }else
    {
      trigger_dummy[i + 1, j] <- 0
    }
  }
}

cashflows <- matrix(c(rep(coupon * dt, ndays - 1), B), ndays,
  nsimulations) * trigger_dummy[1:ndays, ]

# Determine cashflows for each simulation
for(j in 1:nsimulations){
  for(i in 2:ndays){
```

```r
      if(cashflows[i, j] == 0 && p * b[sum(trigger_dummy[ , j]) + 1, j] <= x[sum(trigger_dummy[ , j]) + 1, j] - 1 ){
        cashflows[i, j] <- p * B
        break
      }
      else if(cashflows[i, j] == 0 && 0 < x[sum(trigger_dummy[ , j]) + 1, j] - 1 && x[sum(trigger_dummy[ , j]) + 1, j] - 1 < p * b[sum(trigger_dummy[ , j]) + 1, j]){
        cashflows[i, j] <- (x[sum(trigger_dummy[ , j]) + 1, j] - 1) * B / b[sum(trigger_dummy[ , j]) + 1, j]
        break
      }
      else{
        cashflows[i, j] <- cashflows[i, j]
      }
    }
  }
  list_discounted_cashflows <- rep(0, nsimulations)

  # Discount cashflows for each simulation
  for(j in 1:nsimulations)
  {
    disc_cashflows <- 0
    int_r <- 0

    for(i in 1:ndays)
    {
      int_r <- int_r + r[i, j] * dt
      disc_cashflows <- disc_cashflows + exp(- int_r) * cashflows[i, j]
    }
    list_discounted_cashflows[j] <- disc_cashflows
  }

  # Calculate arithmetic average over all simulations as present value of contingent convertibles bond
  V_t_sa <- mean(list_discounted_cashflows)

  return(V_t_sa)
}

# Pricing Example
# price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.1494, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_A <- 0.0367, x0 <- 1.1364, B <- 1, coupon <- 0.06)
```

# Appendix B

# Code - Sensitivity Analyses

## B.1 Credit Derivative Approach

The following source code is an implementation of the sensitivity analysis of the credit derivative approach (De Spiegeleer and Schoutens, 2012) written in R.

```r
source('CreditDerivativeApproach.R')

# CoCo price V^cd as function of share price S and volatility sigma
createData_CD_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- S_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- S_increment, S_star <- 60, C_p <- 75, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- sigma_increment)
      data[counter, 3] <- sigma_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_S_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^cd as function of maturity T and risk-free interest rate r
createData_CD_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
```

```r
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- T_increment, S_t <- 100, S_star <- 60, C_p <- 75, c_i <- 6, r <- r_increment, N <- 100, q <- 0.00, sigma <- 0.3)
      data[counter, 3] <- r_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_T_r_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^cd as function of trigger price S^* and conversion price C_p
createData_CD_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S_star_max-S_star_min)/10)))
  {
    for(C_p_increment in seq(from=C_p_min, to=C_p_max, by=((C_p_max-C_p_min)/10)))
    {
      data[counter, 1] <- S_star_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- 100, S_star <- S_star_increment, C_p <- C_p_increment, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3)
      data[counter, 3] <- C_p_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_Sstar_Cp_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

createData_CD_S_sigma(65.01, 150, 0.1, 0.5)
createData_CD_T_r(1, 100, 0.01, 0.05)
createData_CD_Sstar_Cp(60, 75, 75, 90)
```

## B.2 Equity Derivative Approach

The following source code is an implementation of the sensitivity analysis of the equity derivative approach (De Spiegeleer and Schoutens, 2012) written in R.

```r
source('EquityDerivativeApproach.R')

# CoCo price V^ed as function of share price S and volatility sigma
createData_ED_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))
```

```
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- S_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- 10, S_t <- S_increment, S_star <- 60, C_p <- 75, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- sigma_increment, alpha <- 1)
      data[counter, 3] <- sigma_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_S_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^ed as function of maturity T and risk-free interest rate r
createData_ED_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- T_increment, S_t <- 100, S_star <- 60, C_p <- 75, c_i <- 6, r <- r_increment, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
      data[counter, 3] <- r_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_T_r_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^ed as function of trigger price S^* and conversion price C_p
createData_ED_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S_star_max-S_star_min)/10)))
  {
    for(C_p_increment in seq(from=C_p_min, to=C_p_max, by=((C_p_max-C_p_min)/10)))
    {
      data[counter, 1] <- S_star_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- 10, S_t <- 100, S_star <- S_star_increment, C_p <- C_p_increment, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
      data[counter, 3] <- C_p_increment
```

```
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_Sstar_Cp_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

createData_ED_S_sigma(65.01, 150, 0.1, 0.5)
createData_ED_T_r(1, 100, 0.01, 0.05)
createData_ED_Sstar_Cp(60, 75, 75, 90)
```

## B.3 Structural Approach

The following source code is an implementation of the sensitivity analysis of the structural approach (Pennacchi, 2010) written in R.

```
source('StructuralApproach.R')

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and volatility sigma
createData_SA_x0_sigma <- function(x0_min, x0_max, sigma_min, sigma_max) {
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_x <- sigma_increment, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- sigma_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_x0_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^sa as function of maturity T and risk-free interest rate r
```

```
createData_SA_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_sa(T <- T_increment, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- r_increment, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_x <- 0.0363, x0 <- 1.1364, B <- 1, coupon <- 0.06)
      data[counter, 3] <- r_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_T_r_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^sa as function of initial asset-to-deposit ratio x_0 and equity-to-deposit threshold bar_e
createData_SA_x0_ebar <- function(x0_min, x0_max, ebar_min, ebar_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(ebar_increment in seq(from=ebar_min, to=ebar_max, by=((ebar_max-ebar_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- ebar_increment, sigma_x <- 0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- ebar_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
```

```
  write.table(data, file = "createData_SA_x0_ebar_31Aug2016.txt", row.
    names = FALSE, quote=FALSE)
}

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and
    jump intensity in asset return process lambda
createData_SA_x0_lambda <- function(x0_min, x0_max, lambda_min, lambda_
    max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/
    10)))
  {
    for(lambda_increment in seq(from=lambda_min, to=lambda_max, by=((
    lambda_max-lambda_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000,
    rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <-
    0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- lambda_increment, g
    <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681,
    sigma_x <- 0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- lambda_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_x0_lambda_31Aug2016.txt", row.
    names = FALSE, quote=FALSE)
}

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and
    initial ratio of contingent capital to deposits b0
createData_SA_x0_b0 <- function(x0_min, x0_max, b0_min, b0_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/
    10)))
  {
    for(b0_increment in seq(from=b0_min, to=b0_max, by=((b0_max-b0_min)/
    10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000,
    rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <-
    0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <-
     1.12, b0 <- b0_increment, p <- 0.8, e_bar <- 0.0681, sigma_x <-
    0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- b0_increment
```

```
            print('---')
            print(data[counter, 1])
            print(data[counter, 2])
            print(data[counter, 3])

            counter <- counter + 1
        }
    }
    write.table(data, file = "createData_SA_x0_b0_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

createData_SA_x0_sigma(1.08, 1.17, 0.01, 0.05)
createData_SA_T_r(1, 100, 0.01, 0.05)
createData_SA_x0_ebar(1.08, 1.17, 0.01, 0.07)
createData_SA_x0_lambda(1.08, 1.17, 0, 2)
createData_SA_x0_b0(1.08, 1.17, 0.01, 0.06)
```

# Appendix C

# Code - Case Study

## C.1 Parametrization - Structural Approach

The following source code is usd to calculate important model inputs. The source code is written in R.

```
source('estimateCIRParameter.R')
source('estimateMertonParameter.R')
source('estimateJumpParameter.R')

calibrate_CIR <- function(cir_data){
  cir_parameters <- estimate_CIR_parameters(cir.data)

  kappa <- cir_parameters$kappa
  r_bar <- cir_parameters$r_bar
  sigma_r <- cir_parameters$sigma_r

  return(cir_parameters)
}

calibrate_Merton <- function(deposits, marketcap, r, volatility_equity){
  merton_parameters <- estimate_Merton_parameters(deposits, marketcap, r
    , volatility_equity)

  asset_volatility <- merton_parameters$asset_volatility
  asset_value <- merton_parameters$asset_value

  return(merton_parameters)
}

calibrate_Jump <- function(returns, jump_Intesity){
  jump_parameters <- estimate_Jump_parameters(returns, jump_Intensity)

  mean_jump <- jump_parameters$mean_jump
  sd_jump <- jump_parameters$sd_jump

  return(jump_parameters)
}
```

### C.1.1 Cox et al. (1985) Model

The following source code is an implementation of the method as described by Remillard (2013b). The method is used to calibrate the Cox et al. (1985) model which is used for the structural approach pursuant to Pennacchi (2010) based on historical yield curve data. The software is an adaption of the source code as provided by Remillard (2013a).

```
require(SMFI5)

estimate_CIR_parameters <- function(data, method = 'Hessian', days = 360
    , significanceLevel = 0.95){
  # Estimation of parameters of Cox-Ingersoll-Ross 1985 model
  R   <- data[,1]
  tau <- data[,2]
  h <- 1 / days

  # Estimation of starting parameters corresponding to those of a Feller
    process
  phi0 <- acf(R, 1, plot = FALSE)
  kappa0 <- - log(phi0[1]$acf) / h
  r_bar0  <- mean(R)
  sigma_r0 <- sd(R) * sqrt(2 * kappa0 / r_bar0)

  theta0 <- c(log(kappa0), log(r_bar0), log(sigma_r0), 0, 0)

  # Maximization of the log-Likelihood
  n <- length(R)
  optim.results <- optim(theta0, function(x) sum(LogLikCIR(x, R, tau,
    days, n)), hessian = TRUE)
  theta <- optim.results$par
  kappa <- exp(theta[1])
  r_bar  <- exp(theta[2])
  sigma_r <- exp(theta[3])

  return(list("kappa" = kappa, "r_bar" = r_bar, "sigma_r" = sigma_r))
}
```

### C.1.2 Merton (1974) Model

The following source code is an implementation of the Merton (1974) model which is used to estimate the asset volatility. The code is written in R. Similar techniques are applied by Stackoverflow (2011).

```
require(nleqslv)

estimate_Merton_parameters <- function(deposits, marketcap, r,
    volatility_equity){
  # Estimation of asset value and asset volatility pursuant to Merton
    1974 model
```

```
  data <- new.env()

  for(i in 1:nrow(marketcap)){
    fnewton <- function(x){
      values <- numeric(2)
      d1 <- (log(x[1]/deposits[i])+(r[i]+x[2]^2/2))/x[2]
      d2 <- d1-x[2]

      values[1] <- marketcap[i]- (x[1]*pnorm(d1) - exp(-r[i])*deposits[i]*pnorm(d2))
      values[2]<-volatility_equity[i]*marketcap[i]-pnorm(d1)*x[2]*x[1]
      return(values)
    }
    xstart <- c(marketcap[i]+deposits[i], volatility_equity[i])
    data$asset_new_value[i]<- nleqslv(xstart, fnewton, method="Newton")$x[1]
    data$asset_new_volat[i]<-nleqslv(xstart, fnewton, method="Newton")$x[2]
  }
  return(list("asset_volatility" = data$asset_new_volat, "asset_value" = data$asset_new_value))
}
```

### C.1.3 Jump-Diffusion Process

The following source code is used to estimate the jump parameters after estimating the jump intensity. The code is written in R.

```
library(fExtremes)

estimate_Jump_parameters <- function(returns, jump_Intensity){
  # Estimation of mean jump size and standard deviation of jumps

  positivThreshold <- findThreshold(returns, n = jump_Intensity / 2)
  negativThreshold <- - findThreshold(-returns, n = jump_Intensity / 2)

  jumps <- rbind(positivThreshold, negativThreshold)
  mean_jump <- mean(jumps)
  sd_jump <- sd(jumps)

  return(list("mean_jump" = mean_jump, "sd_jump" = sd_jump))
}
```

# References

Tobias Adrian and Hyun Song Shin. Liquidity and leverage. *Journal of Financial Intermediation*, 19(3):418–437, 2010.

Yacine Aït-Sahalia and Lars Peter Hansen. *Handbook of financial econometrics: tools and techniques*, volume 1. Elsevier, 2009.

Boris Albul, Dwight M Jaffee, and Alexei Tchistyi. Contingent convertible bonds and capital structure decisions. *Coleman Fung Risk Management Research Center*, 2010.

Boris Albul, Dwight M Jaffee, and Alexei Tchistyi. Contingent convertible bonds and capital structure decisions. *Available at SSRN 2772612*, 2015.

Per Alvemar and Philip Ericson. Modelling and pricing contingent convertibles. *University of Gothenburg*, 2012.

Stefan Avdjiev, Anastasia V Kartasheva, and Bilyana Bogdanova. Cocos: a primer. *Available at SSRN 2326334*, 2013.

Stefan Avdjiev, Patrick Bolton, Wei Jiang, Anastasia Kartasheva, and Bilyana Bogdanova. Coco bond issuance and bank funding costs. *BIS and Columbia University working paper*, 2015.

Christina Bannier. Definition of risk shifting, 2011. URL http://lexicon.ft.com/Term?term=risk-shifting.

Barclays. USD 3,000,000,000 7.625 per cent. Contingent Capital Notes due November 2022 Barclays Bank, 2010. URL https://www.home.barclays/content/dam/barclayspublic/docs/InvestorRelations/esma/capital-securities-documentation/tier-2-securities/contingent-tier-2/7625-Contingent-Capital-Notes-due-November-2022-Prospectus-PDF-992KB.pdf.

Emilio Barucci and Luca Del Viva. Countercyclical contingent capital. *Journal of Banking and Finance*, 36(6):1688–1709, 2012.

Basel Committee on Banking Supervision. Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. *Bank for International Settlements*, 2010a.

Basel Committee on Banking Supervision. Basel committee issues final elements of the reforms to raise the quality of regulatory capital. *Bank for International Settlements*, 2010b.

Dirk Bleich. Contingent convertible bonds and the stability of bank funding: the case of partial writedown. Discussion Paper 28/2014, Deutsche Bundesbank Research Centre, 2014. URL https://ideas.repec.org/p/zbw/bubdps/282014.html.

Markus PH Buergi. Pricing contingent convertibles: a general framework for application in practice. *Financial Markets and Portfolio Management*, 27(1):31–63, 2013.

Charles W Calomiris and Richard J Herring. How to design a contingent convertible debt requirement that helps solve our too-big-to-fail problem. *Journal of Applied Corporate Finance*, 25(2):39–62, 2013.

Patrick Cheridito and Zhikai Xu. Pricing and hedging cocos. *Available at SSRN 2201364*, 2015.

José Manuel Corcuera, Jan De Spiegeleer, Albert Ferreiro-Castilla, Andreas E Kyprianou, Dilip B Madan, and Wim Schoutens. Pricing of contingent convertibles under smile conform models. *The Journal of Credit Risk*, 9(3):121, 2013.

José Manuel Corcuera, Jan De Spiegeleer, José Fajardo, Henrik Jönsson, Wim Schoutens, and Arturo Valdivia. Close form pricing formulas for coupon cancellable cocos. *Journal of Banking & Finance*, 42:339–351, 2014.

John C Cox, Jonathan E Ingersoll Jr, and Stephen A Ross. A theory of the term structure of interest rates. *Econometrica: Journal of the Econometric Society*, pages 385–407, 1985.

Credit Suisse. USD 2,000,000,000 7.875 per cent. Tier 2 Buffer Capital Notes due 2041, 2011. URL https://www.credit-suisse.com/media/assets/corporate/docs/about-us/investor-relations/regulatory-disclosures/t2-xs0595225318.pdf.

Jan De Spiegeleer and Wim Schoutens. *The handbook of convertible bonds: pricing, strategies and risk management*, volume 581. John Wiley & Sons, 2011.

Jan De Spiegeleer and Wim Schoutens. Pricing contingent convertibles: a derivatives approach. *Journal of Derivatives*, 20:27–36, 2012.

Jan De Spiegeleer, Wim Schoutens, and Cynthia Van Hulle. *The handbook of hybrid securities: convertible bonds, coco bonds and bail-in*. John Wiley & Sons, 2014.

Darrell Duffie and Kenneth J Singleton. Modeling term structures of defaultable bonds. *Review of Financial studies*, 12(4):687–720, 1999.

Darrell Duffie and Kenneth J Singleton. *Credit risk pricing, measurement, and management*. Princeton University Press, 2003.

Marc Erismann. *Analytical propositions to evaluate contingent convertible capital*. PhD thesis, Master's thesis, University of St. Gallen, 2011.

Marc Erismann. *Pricing Contingent Convertible Capital-A Theoretical and Empirical Analysis of Selected Pricing Models*. PhD thesis, University of St. Gallen, 2015.

European Parliament. Contingent convertible securities, is a storm brewing, 2016. URL http://www.europarl.europa.eu/RegData/etudes/BRIE/2016/582011/EPRS_BRI(2016)582011_EN.pdf.

Paul Glasserman and Behzad Nouri. Contingent capital with a capital-ratio trigger. *Management Science*, 58(10):1816–1833, 2012.

Andrew G Haldane. Capital discipline. In *American Economic Association Meeting*, volume 9, 2011.

Roberto Henriques and Saul Doctor. Making cocos work: structural and pricing considerations for contingent capital securities. *European Credit Research, JP Morgan, February*, 15, 2011.

Jens Hilscher and Alon Raviv. Bank stability and market discipline: the effect of contingent capital on risk taking and fault probability. *Journal of Corporate Finance*, 29:542–560, 2014.

John C Hull. *Options, futures, and other derivatives*. Pearson Education, 2006.

Intesa Sanpaolo. EUR 1,250,000,000 7 per cent Additional Tier 1 Notes, 2011. URL http://www.group.intesasanpaolo.com/scriptIsir0/si09/contentData/view/20160120_AT1_Euro_Prospectus.pdf?id=CNT-05-00000003D473C&ct=application/pdf.

Robert A Jarrow and Stuart M Turnbull. Pricing derivatives on financial securities subject to credit risk. *The Journal of Finance*, 50(1):53–85, 1995.

David Lando. *Credit risk modeling: theory and applications*. Princeton University Press, 2009.

Hayne E Leland. Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance*, 49(4):1213–1252, 1994.

Lloyds. Enhanced capital notes, 2009. URL http://www.lloydsbankinggroup.com/globalassets/documents/investors/2009/non-us_eom.pdf.

Francois Longin and Bruno Solnik. Extreme correlation of international equity markets. *The Journal of Finance*, 56(2):649–676, 2001.

Dilip B Madan and Wim Schoutens. Conic coconuts: the pricing of contingent capital notes using conic finance. *Mathematics and Financial Economics*, 4(2):87–106, 2011.

Robert L McDonald. Contingent capital with a dual price trigger. *Journal of Financial Stability*, 9(2):230–241, 2013.

Robert C Merton. On the pricing of corporate debt: the risk structure of interest rates. *The Journal of Finance*, 29(2):449–470, 1974.

Ceyla Pazarbasioglu, Ms Jian-Ping Zhou, Vanessa Le Lesle, and Michael Moore. *Contingent capital: economic rationale and design features*. International Monetary Fund, 2011.

George Pennacchi. A structural model of contingent bank capital. Working Paper 1004, Federal Reserve Bank of Cleveland, 2010. URL https://ideas.repec.org/p/fip/fedcwp/1004.html.

George Pennacchi and Alexei Tchistyi. A reexamination of contingent convertibles with stock price triggers. *Available at SSRN 2773335*, 2015.

Quantnet. For loop using nleqslv package in r, 2014. URL http://stackoverflow.com/questions/23618277/for-loop-using-nleqslv-package-in-r.

Rabobank. Rabobank Nederland Cooperatieve Centrale Raiffeisen-Boerenleenbank EUR 1,250,000,000 6.875 per cent. Senior Contingent Notes due 2020, 2010. URL https://www.afm.nl/registers/emissies_documents/4220.pdf.

Bruno Remillard. Estimation of the parameters of the cir model, 2013a. URL https://github.com/cran/SMFI5/blob/master/R/est.cir.R.

Bruno Remillard. *Statistical methods for financial engineering*. CRC Press, 2013b.

Mark Rubinstein and Eric Reiner. Unscrambling the binary code. *Risk Magazine*, 4 (9):20, 1991.

Wolfgang Schmidt. Credit risk, default models and credit derivatives. Frankfurt School of Finance and Management, 2015.

Andrei Serjantov. On practical pricing hybrid capital securities. In *Global Derivative Trading ad Risk Management Meeting*, 2011.

Liad Shekel. Rstudio server on Amazon EC2, 2015. URL www.r-israel.com/wp-content/uploads/2015/06/Rstudio-in-AWS-16_9.pdf.

Squam Lake Working Group. An expedited resolution mechanism for distressed financial firms: Regulatory hybrid securities. *Council on Foreign Relations*, 10, 2009.

Stackoverflow. Translate GAUSS code to R code, 2011. URL www.quantnet.com/threads/translate-gauss-code-to-r-code.7784/.

Lujing Su and Marc Olivier Rieger. How likely is it to hit a barrier? Theoretical and empirical estimates. Technical report, 2009.

Suresh Sundaresan and Zhenyu Wang. On the design of contingent capital with a market trigger. *The Journal of Finance*, 70(2):881–920, 2015.

Henrik Teneberg. Pricing contingent convertibles using equity derivatives jump diffusion approach. *Master thesis*, 2015.

Colin Turfus and Alexander Shubert. Analytic pricing of coco bonds. *Deutsche Bank and J.P. Morgan working paper*, 2015.

Southern District of New York United States Bankruptcy Court. Report of Anton R. Valukas, examiner. *Lehman Brothers Holdings Inc., et al., Debtors*, 11, 2010.

George M Von Furstenberg et al. *Contingent capital to strengthen the private safety net for financial institutions: CoCos to the rescue?* Dt. Bundesbank, Press and Public Relations Division, 2011.

Sascha Wilkens and Nastja Bethke. Contingent convertible (coco) bonds: a first empirical assessment of selected pricing models. *Financial Analysts Journal*, 70(2): 59–77, 2014.

Meta Zähres. Contingent convertibles: bank bonds take on a new look. *Deutsche Bank Research, Financial Market Special, EU Monitor*, 79, 2011.

Zurich Cantonal Bank. Nachrangige Tier-1-Anleihe CHF 590 Mio., 2013. URL https://www.zkb.ch/media/dok/corporate/investor-relations/produktinformationsblatt-nachrangige-tier1-anleihe.pdf.

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The paper in this or similar form has never been submitted as an assessed piece of work in or outside of Germany. It also has not yet been published.

Frankfurt am Main — August 19, 2016