

Valuation of Contingent Convertibles with Derivatives



Nicolay Schmitt

Frankfurt School of Finance and Management

A thesis submitted for the degree of

Master of Science in Finance

August 2016

This thesis is dedicated to my parents for their love and support.
Thank you!

Abstract

Financial crises have led to higher regulatory standards on the capital adequacy of banks. They are required to hold more capital with loss absorbance capacities on their balance sheet. In conjunction with this development, contingent convertible bonds (CoCos) have become an attractive instrument for banks to seek new capital. The defining characteristic of CoCos is the automatic conversion into common equity when a predetermined trigger is met. Loss absorbing capital is created, which instantly improves the capital structure of the distressed bank. However, the pricing of these hybrid instruments remains opaque. In this context, the thesis scrutinizes the valuation of CoCos. Three dominant approaches are examined: the structural approach in accordance to Pennacchi (2010), the credit derivatives approach and the equity derivatives approach both pursuant to De Spiegeleer and Schoutens (2011b). The application covers sensitivity analysis to further understand the dynamics of the different methodologies. Based on a case study the viability of those approaches is evaluated. Software is provided for further replication.

Contents

1	Introduction and Motivation	1
1.1	Introduction	1
1.2	Literature Overview	1
1.3	Motivation	1
1.4	Methodology	1
2	Structure of CoCos	2
2.1	Description of CoCos	2
2.2	Payoff and Risk Profile	2
2.3	Conversion Trigger	3
2.3.1	Market Trigger	3
2.3.2	Accounting Trigger	3
2.3.3	Regulatory Trigger	3
2.3.4	Multivariate Trigger	3
2.4	Conversion Details	3
2.4.1	Conversion Fraction	3
2.4.2	Conversion Price and Ratio	3
3	Theory of Pricing	4
3.1	Credit Derivative Approach	4
3.1.1	Intensity-based Approach	4
3.1.2	Application to CoCos	5
3.1.3	Data Requirements and Calibration	5
3.1.4	Pricing Example	5
3.2	Equity Derivative Approach	5
3.2.1	Corporate Bonds	6
3.2.2	Binary Options	6
3.2.3	Down-And-In Forward	6

3.2.4	Data Requirements and Calibration	8
3.2.5	Pricing Example	8
3.3	Structural Approach	8
3.3.1	Synthetic Balance Sheet	8
3.3.2	Data Requirements and Calibration	8
3.3.3	Pricing Example	8
4	Dynamics and Sensitivity Analysis	9
4.1	Credit Derivative Approach	9
4.2	Equity Derivative Approach	9
4.3	Structural Approach	9
5	Empirical Analysis and Model Comparison	10
5.1	Data Description	10
5.1.1	Deutsche Bank	10
5.2	Model Parametrization	10
5.3	Model Comparison	10
5.3.1	Qualitative Analysis	10
5.3.2	Quantitative Analysis	10
6	Conclusion	11
A	Sample Title	12
B	Code - Credit Derivative Approach	13
C	Code - Equity Derivative Approach	15
D	Code - Structural Approach	18
	Bibliography	22

List of Figures

2.1	CoCos under Basel III	2
2.2	Anatomy of CoCos	2

Chapter 1

Introduction and Motivation

1.1 Introduction

1.2 Literature Overview

1.3 Motivation

1.4 Methodology

Chapter 2

Structure of CoCos

2.1 Description of CoCos

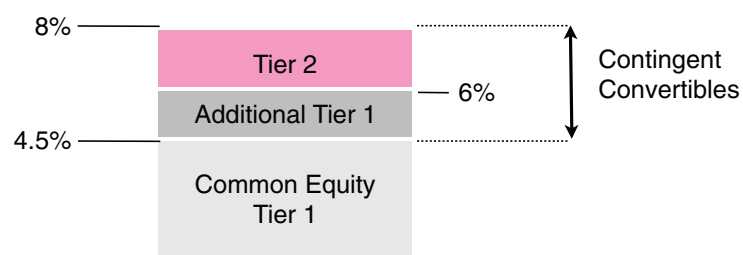


Figure 2.1: CoCos under Basel III (De Spiegeleer and Schoutens, 2011a)

2.2 Payoff and Risk Profile

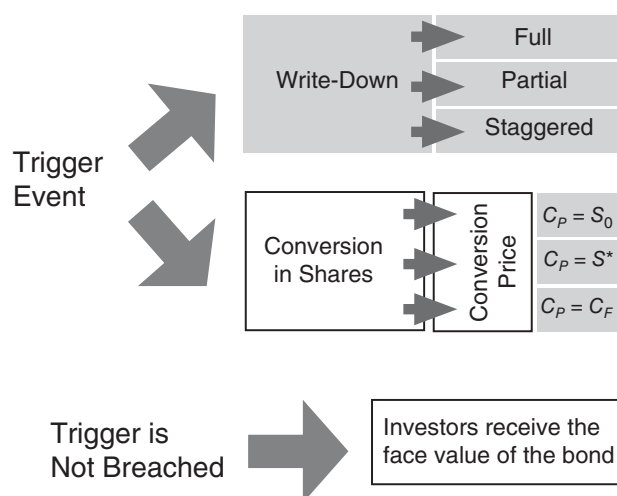


Figure 2.2: Anatomy of CoCos (De Spiegeleer and Schoutens, 2011a)

2.3 Conversion Trigger

2.3.1 Market Trigger

2.3.2 Accounting Trigger

2.3.3 Regulatory Trigger

2.3.4 Multivariate Trigger

2.4 Conversion Details

2.4.1 Conversion Fraction

- conversion fraction α
- face value N
- conversion amount $N \times \alpha$
- amount remaining in case of partial equity conversion $N \times (1 - \alpha)$

2.4.2 Conversion Price and Ratio

- conversion rate C_r
- conversion price C_p
- recovery rate R_{CoCo}
- stock price at trigger event S_T^*
- loss attributable to CoCo holders L_{CoCo}

$$C_p = \frac{\alpha N}{C_r} \tag{2.1}$$

$$C_r = \frac{\alpha N}{C_p} \tag{2.2}$$

$$R_{CoCo} = \frac{S_T^*}{C_p} \tag{2.3}$$

$$L_{CoCo} = N - (1 - R_{CoCo}) = N \left(1 - \frac{S_T^*}{C_p} \right) \tag{2.4}$$

$$P_T = \begin{cases} (1 - \alpha)N + C_r S_T^* & \text{if converted} \\ N & \text{if not converted} \end{cases} \tag{2.5}$$

Chapter 3

Theory of Pricing

3.1 Credit Derivative Approach

3.1.1 Intensity-based Approach

Let τ denote the random time of default of some company. It is assumed that the distribution of τ has a continuous density function f , so that the distribution function F and the curve of survival probabilities q are related as follows:

$$P(\tau \leq t) = F(t) = 1 - q(t) = \int_0^t f(s)ds, \text{ with } t \geq 0. \quad (3.1)$$

The hazard rate respectively the default intensity λ is defined as

$$\lambda(t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} P(\tau \leq t + \Delta | \tau > t) = \frac{F'(t)}{q(t)} = \frac{F'(t)}{1 - F(t)} = -\frac{d}{dt} \log q(t). \quad (3.2)$$

Intuitively, the hazard rate is the default rate per year as of today. Using 3.2 we get

$$q(t) = \exp \left(- \int_0^t \lambda(s)ds \right). \quad (3.3)$$

For our application of the credit derivative approach (De Spiegeleer and Schoutens, 2011b) we assume the hazard rate $\lambda(t)$ is a deterministic function of time. Hence, a constant hazard rate $\lambda(t) = \lambda$ implies an exponential distribution of the default time:

$$F(t) = 1 - q(t) = 1 - \exp(-\lambda t) \quad (3.4)$$

Under the idealized assumption of a flat zero interest rate curve, a flat spread curve and continuous spread payments the default intensity can be calculated directly from the spread by the rule of thumb formula, which is also known as credit triangle:

$$\lambda = \frac{s}{1 - R}. \quad (3.5)$$

3.1.2 Application to CoCos

$$p^* = 1 - \exp(-\lambda_{Trigger} \times T) \quad (3.6)$$

$$s_{CoCo} = (1 - R_{CoCo}) \times \lambda_{Trigger} = Loss_{CoCo} \times \lambda_{Trigger} \quad (3.7)$$

$$Loss_{CoCo} = N - C_r \times S^* = N \left(1 - \frac{S^*}{C_P}\right) \quad (3.8)$$

$$R_{CoCo} = \frac{S^*}{C_p} \quad (3.9)$$

$$p^* = \Phi \left(\frac{\log \left(\frac{S^*}{S} \right) - \mu T}{\sigma \sqrt{T}} \right) + \left(\frac{S^*}{S} \right)^{\frac{2\mu}{\sigma^2}} \Phi \left(\frac{\log \left(\frac{S^*}{S} \right) + \mu T}{\sigma \sqrt{T}} \right) \quad (3.10)$$

$$\lambda_{Trigger} = -\frac{\log(1 - p^*)}{T} \quad (3.11)$$

$$s_{CoCo} = -\frac{\log(1 - p^*)}{T} \times \left(1 - \frac{S^*}{C_p}\right) \quad (3.12)$$

3.1.3 Data Requirements and Calibration

3.1.4 Pricing Example

3.2 Equity Derivative Approach

Sources: Erismann (2015), De Spiegeleer and Schoutens (2011b)

$$\begin{aligned} P_T &= \mathbb{1}_{\{\tau > T\}} N + \left[(1 - \alpha) N + \frac{\alpha N}{C_p S^*} \right] \mathbb{1}_{\{\tau \leq T\}} \\ &= N + \left[\frac{\alpha N}{C_p} S^* - \alpha N \right] \mathbb{1}_{\{\tau \leq T\}} \\ &= N + [C_r S^* - \alpha N] \mathbb{1}_{\{\tau \leq T\}} \\ &= N + C_r \left[S^* - \frac{\alpha N}{C_r} \right] \mathbb{1}_{\{\tau \leq T\}} \\ &= N + C_r [S^* - C_p] \mathbb{1}_{\{\tau \leq T\}} \end{aligned}$$

$$V_t^{ed} = V_t^{cb} - V_t^{dibi} + V_t^{difwd} \quad (3.13)$$

3.2.1 Corporate Bonds

$$V_t^{cb} = \sum_{i=t}^T c_i \exp(-rt_i) + N \exp[-r(T-t)] \quad (3.14)$$

3.2.2 Binary Options

$$V_t^{dibi}(c_i, S^*, t) = \alpha \sum_{i=1}^k c_i \exp(-rt_i) \left[\Phi(-x_{1i} + \sigma\sqrt{t_i}) + \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \Phi(y_{1i} - \sigma\sqrt{t_i}) \right] \quad (3.15)$$

with

$$\begin{aligned} x_{1i} &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ y_{1i} &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ \lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2} \end{aligned}$$

3.2.3 Down-And-In Forward

$$\max(S_T - K) \text{ if } \min_{0 \leq t \leq T} (S_t) \leq S^* \quad (3.16)$$

$$\max(K - S_T) \text{ if } \min_{0 \leq t \leq T} (S_t) \leq S^* \quad (3.17)$$

$$\begin{aligned} V_t^{dic}(S_t, S^*, K) &= S_t \exp[-q(T-t)] \left(\frac{S^*}{S_t}\right)^{2\lambda} \Phi(y) \\ &\quad - K \exp[-r(T-t)] \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \Phi(y - \sigma\sqrt{T-t}) \end{aligned} \quad (3.18)$$

with

$$\begin{aligned}
K &= C_p \\
y &= \frac{\log\left(\frac{S^{*2}}{S_t K}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \\
\lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}
\end{aligned}$$

$$\begin{aligned}
V_t^{dip}(S_t, S^*, K) &= S_t \exp[-q(T-t)] \left(\frac{S^*}{S_t}\right)^{2\lambda} [\Phi(y) - \Phi(y_1)] \\
&\quad - K \exp[-r(T-t)] \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \left[\Phi\left(y - \sigma\sqrt{T-t}\right) - \Phi\left(y_1 - \sigma\sqrt{T-t}\right)\right] \\
&\quad + K \exp[-r(T-t)] \Phi\left(x_1 + \sigma\sqrt{T-t}\right) \\
&\quad - S_t \exp[-q(T-t)] \Phi(-x_1)
\end{aligned} \tag{3.19}$$

with

$$\begin{aligned}
x_1 &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \\
y_1 &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t}
\end{aligned}$$

$$\min(S_t) \leq S^* : P_T = S_T - K = \max(S_T - K) - \max(K - S_T) \tag{3.20}$$

$$\min(S_t) > S^* : P_T = 0 \tag{3.21}$$

$$\begin{aligned}
V_t^{difwd} = C_r \Bigg[& S_t \exp [-q (T - t)] \left(\frac{S^*}{S_t} \right)^{2\lambda} \Phi (y_1) \\
& - K \exp [-r (T - t)] \left(\frac{S^*}{S_t} \right)^{2\lambda-2} \Phi \left(y_1 - \sigma \sqrt{T - t} \right) \\
& - K \exp [-r (T - t)] \Phi \left(-x_1 - \sigma \sqrt{T - t} \right) \\
& + S_t \exp [-q (T - t)] \Phi (-x_1) \Bigg] \tag{3.22}
\end{aligned}$$

with

$$C_r = \frac{\alpha N}{C_p} \tag{3.23}$$

3.2.4 Data Requirements and Calibration

3.2.5 Pricing Example

3.3 Structural Approach

3.3.1 Synthetic Balance Sheet

3.3.2 Data Requirements and Calibration

3.3.3 Pricing Example

Chapter 4

Dynamics and Sensitivity Analysis

4.1 Credit Derivative Approach

4.2 Equity Derivative Approach

4.3 Structural Approach

Chapter 5

Empirical Analysis and Model Comparison

5.1 Data Description

5.1.1 Deutsche Bank

5.2 Model Parametrization

5.3 Model Comparison

5.3.1 Qualitative Analysis

5.3.2 Quantitative Analysis

Chapter 6

Conclusion

Appendix A

Sample Title

Appendix B

Code - Credit Derivative Approach

The following source code is an implementation of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
1 # Price of Contingent Convertible Bond
2 price_coco_cd <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma){
3
4   spread_coco <- calc_spread_coco(t, T, S_t, S_star, C_p, r, q, sigma)
5   V_t_coco <- N * exp(-(r + spread_coco) * (T - t))
6
7   for (t in 1:T){
8     V_t_coco <- V_t_coco + c_i * exp(-(r + spread_coco) * t)
9   }
10  V_t_coco
11 }
12
13 # Calculation of Trigger Probability
14 calc_p_star <- function(t, T, S_t, S_star, r, q, sigma){
15   p_star <- pnorm((log(S_star / S_t) - calc_mu(r, q, sigma) * (T - t)) /
16     (sigma * sqrt(T - t))) + (S_star / S_t)^(2 * calc_mu(r, q, sigma) /
17     sigma^2) * pnorm((log(S_star / S_t) + calc_mu(r, q, sigma) * (T - t)
18     )) / (sigma * sqrt(T - t)))
19   p_star
20 }
21
22 # Calculation of Drift of Underlying
23 calc_mu <- function(r, q, sigma){
24   mu <- r - q - sigma^2 / 2
25   mu
26 }
27
28 # Spread of CoCo Bond
29 calc_spread_coco <- function(t, T, S_t, S_star, C_p, r, q, sigma){
30   spread_coco <- - log(1 - calc_p_star(t, T, S_t, S_star, r, q, sigma))
31     / (T - t) * (1 - S_star / C_p)
32   spread_coco
33 }
34
35 # Pricing Example
```

```
32 price_coco_cd(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <-  
    7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3)
```

Appendix C

Code - Equity Derivative Approach

The following source code is an implementation of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
1 # Price of Contingent Convertible Bond
2 price_coco_ed <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma,
3   alpha){
4   V_t_ed <- price_cb(t, T, c_i, r, N) - price_dibi(t, T, S_t, S_star, c_
5     i, r, q, sigma, alpha) + price_difwd(t, T, S_t, S_star, C_p, r, N, q
6     , sigma, alpha)
7
8   return(V_t_ed)
9 }
10
11 # Price of Corporate Bond
12 price_cb <- function(t, T, c_i, r, N){
13   V_t_cb <- N * exp(-r * (T - t))
14
15   for (t in 1:T){
16     V_t_cb <- V_t_cb + c_i * exp(-r * t)
17   }
18
19   return(V_t_cb)
20 }
21
22 # Price of Binary Option
23 price_dibi <- function(t, T, S_t, S_star, c_i, r, q, sigma, alpha){
24   V_t_dibi <- 0
25
26   i <- t
27   k <- T
28
29   for (i in 1:k) {
30     V_t_dibi <- V_t_dibi + c_i * exp(- r * i) * (pnorm(- calc_x_1_i(S_t, S
31       _star, sigma, r, q, i) + sigma * sqrt(i)) + (S_star / S_t)^(2 * calc
32       _lambda(r, q, sigma) - 2) * pnorm ( calc_y_1_i(S_t, S_star, sigma, r
33       , q, i) - sigma * sqrt(i)))
34   }
35 }
```

```

30 V_t_dibi <- alpha * V_t_dibi
31
32 return(V_t_dibi)
33 }
34
35 # Price of Down-And-In Forward
36 price_difwd <- function(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha){
37   V_t_difwd <- calc_conversion_rate(C_p, N, alpha) * (S_t * exp(- q * (T
    - t)) * (S_star / S_t) ^ (2 * calc_lambda(r, q, sigma)) * pnorm(
    calc_y_1(t, T, S_t, S_star, r, q, sigma)) - C_p * exp(- r * (T - t))
    * (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm(calc_y_
    1(t, T, S_t, S_star, r, q, sigma) - sigma * sqrt(T - t)) - C_p * exp
    (- r * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma) +
    sigma * sqrt(T - t)) + S_t * exp(- q * (T - t)) * pnorm(- calc_x_1(
    t, T, S_t, S_star, r, q, sigma)))
38
39   return(V_t_difwd)
40 }
41
42 # Calculation of Conversion Rate
43 calc_conversion_rate <- function(C_p, N, alpha){
44   C_r <- alpha * N / C_p
45
46   return(C_r)
47 }
48
49 # Calculation of additional Parameters
50 calc_x_1_i <- function(S_t, S_star, sigma, r, q, t_i){
51   x_1_i <- log(S_t / S_star) / (sigma * sqrt(t_i)) + calc_lambda(r, q,
    sigma) * sigma * sqrt(t_i)
52
53   return(x_1_i)
54 }
55
56 calc_y_1_i <- function(S_t, S_star, sigma, r, q, t_i){
57   y_1_i <- log(S_star / S_t) / (sigma * sqrt(t_i)) + calc_lambda(r, q,
    sigma) * sigma * sqrt(t_i)
58
59   return(y_1_i)
60 }
61
62 calc_lambda <- function(r, q, sigma){
63   lambda <- (r - q + sigma^2 / 2) / sigma^2
64
65   return(lambda)
66 }
67
68 calc_x_1 <- function(t, T, S_t, S_star, r, q, sigma){
69   x_1 <- log(S_t / S_star) / (sigma * sqrt(T - t)) + calc_lambda(r, q,
    sigma) * sigma * sqrt(T - t)
70
71   return(x_1)
72 }
73

```

```

74 calc_y_1 <- function(t, T, S_t, S_star, r, q, sigma){
75   y_1 <- log(S_star / S_t) / (sigma * sqrt(T - t)) + calc_lambda(r, q,
      sigma) * sigma * sqrt(T - t)
76
77   return(y_1)
78 }
79
80 # Pricing Example
81 price_coco_ed(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <-
      7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3, alpha <- 1)

```

Appendix D

Code - Structural Approach

The following source code is an implementation of the Structural Approach (Pennacchi, 2010) written in R.

```
1 # Price of Contingent Convertible Bond
2 price_coco_sa <- function(T , npath , rho , kappa , r_bar, r0, sigma_r,
   mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_x, x0_low, x0_
   high, x0_nint, B, c_low, c_high, c_nint){
3   n <- T * 250
4   dt <- T / n
5
6   result <- sim_corrProcess(T, npath, rho, n, dt)
7   dW_1 <- result$dW_1
8   dW_2corr <- result$dW_2corr
9
10  r <- sim_interestrates(kappa, r_bar, r0, sigma_r, dW_2corr, n, npath,
   dt)
11
12  V_t_sa <- get_price(npath, n, dt, dW_1, dW_2corr, r, mu_Y, sigma_Y,
   lambda, g, x_hat, b0, p, e_bar, sigma_x, x0_low, x0_high, x0_nint, B
   , c_low, c_high, c_nint) * 100
13  return(V_t_sa)
14 }
15
16 sim_corrProcess <- function(T, npath, rho, n, dt){
17   vect <- c(1, rho, rho, 1)
18   RHO <- matrix(vect, nrow = 2)
19   chol_RHO <- t(chol(RHO))
20
21   # Create two Brownian Motions
22   dW_1 <- matrix(1, n, npath)
23   dW_2 <- matrix(1, n, npath)
24
25   for(j in 1:npath)
26   {
27     dW_1[, j] <- rnorm(n) * sqrt(dt)
28     dW_2[, j] <- rnorm(n) * sqrt(dt)
29   }
30 }
```



```

31 # Create Correlated Process based on Brownian Motions using Cholesky-
    Decomposition
32 dW_2corr <- matrix(1, n, npath)
33 for(j in 1:npath)
34 {
35   for(i in 1:n)
36   {
37     dW_2corr[i, j] <- dW_1[i, j] * chol_RHO[2, 1] + dW_2[i, j] * chol_
        RHO[2, 2]
38   }
39 }
40
41 return(list("dW_1" = dW_1, "dW_2corr" = dW_2corr))
42 }
43
44 # Create Interest Rate Process
45 sim_interestrates <- function(kappa, r_bar, r0, sigma_r, dW_2corr, n,
    npath, dt){
46   r <- matrix(r0, n + 1, npath)
47
48   for(j in 1:npath)
49   {
50     for(i in 1:n)
51     {
52       r[i + 1, j] <- r[i, j] + kappa * (r_bar - r[i, j]) * dt + sigma_r
        * sqrt(r[i, j]) * dW_2corr[i, j]
53     }
54   }
55
56   return(r)
57 }
58
59 get_price <- function(npath, n, dt, dW_1, dW_2corr, r, mu_Y, sigma_Y,
    lambda, g, x_hat, b0, p, e_bar, sigma_x, x0_low, x0_high, x0_nint, B
    , c_low, c_high, c_nint){
60
61   c_fit_matrix <- matrix(0, x0_nint, length(lambda))
62
63   for(w in 1:length(lambda))
64   {
65     # Create parametres for jump process
66     phi <- matrix(rbinom( n%*%npath, 1, dt * lambda[w]), n, npath)
67     ln_Y <- matrix(rnorm(n%*%npath, mu_Y, sigma_Y), n, npath)
68
69     b <- matrix(b0, n + 1, npath)
70     x_bar0 <- 1 + e_bar + p * b0
71     x_bar <- matrix(x_bar0, n + 1, npath)
72
73     h <- matrix(1, n, npath)
74
75     k <- exp(mu_Y + 0.5 * sigma_Y^2) - 1
76
77     c <- seq(c_low, c_high, length = c_nint)
78     x0 <- seq(x0_low, x0_high, length = x0_nint)

```

```

79   for(l in 1:x0_nint) # Wieso?
80   {
81     for(m in 1:c_nint) # Wieso?
82     {
83       x <- matrix(x0[l],n+1,npath)
84       ln_x0 <- matrix(log(x0[l]),n+1,npath)
85       ln_x <- ln_x0
86       binom_c <- matrix(1,n+1,npath)
87
88       for(j in 1:npath)
89       {
90         for(i in 1:n)
91         {
92           d_1 <- (ln_x[i, j] + mu_Y) / sigma_Y
93           d_2 <- d_1 + sigma_Y
94
95           h[i, j] <- lambda[w] * (pnorm(- d_1) - exp(ln_x[i, j]) *
96             exp(mu_Y + 0.5 * sigma_Y^2) * pnorm(-d_2))
97
98           b[i + 1, j] <- b[i, j] * exp(- g[w] * (exp(ln_x[i, j]) - x_
99             hat) * dt)
100
101           ln_x[i + 1, j] <- ln_x[i, j] + ( (r[i, j] - lambda[w] * k) -
102             (r[i, j] + h[i, j] + c[m] * b[i, j]) / exp(ln_x[i, j]) - g[w] * (
103             exp(ln_x[i, j]) - x_hat) - 0.5 * sigma_x^2) * dt + sigma_x * sqrt(dt
104             ) * dW_1[i, j] + ln_Y[i,j] * phi[i, j]
105
106           x[i + 1, j] <- exp(ln_x[i + 1, j])
107
108           x_bar[i + 1, j] <- 1 + e_bar + p * b[i + 1, j]
109
110           if(x[i + 1, j] >= x_bar[i + 1, j] && binom_c[i, j] > 0.5)
111           {
112             binom_c[i + 1, j] <- 1
113           }else
114           {
115             binom_c[i + 1, j] <- 0
116           }
117         }
118       }
119
120       payments <- matrix(c(rep(c[m] * dt, n - 1), B), n, npath) *
121       binom_c[1:n, ]
122
123       for(j in 1:npath){
124         for(i in 2:n){
125           if(payments[i, j] == 0 && p * b[sum(binom_c[, j]) + 1, j]
126             <= x[sum(binom_c[, j]) + 1, j] - 1 ){
127             payments[i, j] <- p * B
128             break
129           }
130           else if(payments[i, j] == 0 && 0 < x[sum(binom_c[, j]) + 1,
131             j] - 1 && x[sum(binom_c[, j]) + 1, j] - 1 < p * b[sum(binom_c[, j]

```

```

125     )) + 1, j)) {
126         payments[i, j] <- (x[sum(binom_c[, j]) + 1, j] - 1) * B /
127         b[sum(binom_c[, j]) + 1, j]
128         break
129     }
130     else {
131         payments[i, j] <- payments[i, j]
132     }
133 }
134 vec_disc_v <- rep(0, npath)
135 for(j in 1:npath)
136 {
137     disc_v <- 0
138     int_r <- 0
139     for(i in 1:n)
140     {
141         int_r <- int_r + r[i, j] * dt
142         disc_v <- disc_v + exp(- int_r) * payments[i, j]
143     }
144     vec_disc_v[j] <- disc_v
145 }
146
147 V_t_sa <- mean(vec_disc_v)
148
149 return(V_t_sa)
150 }
151 }
152 }
153 }
154
155 # Pricing Example
156 price_coco_sa(T = 5, npath = 2, rho = - 0.2, kappa = 0.114, r_bar =
    0.069, r0 = 0.035, sigma_r = 0.07, mu_Y = -0.01, sigma_Y = 0.02,
    lambda = c(1), g = c(0.5), x_hat = 1.1, b0 = 0.04, p = 1, e_bar =
    0.02, sigma_x = 0.02, x0_low = 1.15, x0_high = 1.15, x0_nint = 10, B
    = 1, c_low = 0.05, c_high = 0.05, c_nint = 10)

```

References

- Jan De Spiegeleer and Wim Schoutens. *The handbook of convertible bonds: pricing, strategies and risk management*, volume 581. John Wiley & Sons, 2011a.
- Jan De Spiegeleer and Wim Schoutens. Pricing contingent convertibles: a derivatives approach. *Available at SSRN 1795092*, 2011b.
- Marc Erismann. *Pricing Contingent Convertible Capital-A Theoretical and Empirical Analysis of Selected Pricing Models*. PhD thesis, University of St. Gallen, 2015.
- George Pennacchi. A structural model of contingent bank capital. Working Paper 1004, Federal Reserve Bank of Cleveland, 2010. URL <https://ideas.repec.org/p/fip/fedcwp/1004.html>.