Valuation of Contingent Convertibles with Derivatives

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This thesis is dedicated to my parents for their love and support.

Thank you!

Abstract

Financial crises have led to higher regulatory standards on the capital adequacy of banks. They are required to hold more capital with loss absorbance capacities on their balance sheet. In conjunction with this development, contingent convertible bonds (CoCos) have become an attractive instrument for banks to seek new capital. The defining characteristic of CoCos is the automatic conversion into common equity when a predetermined trigger is met. Loss absorbing capital is created, which instantly improves the capital structure of the distressed bank. However, the pricing of these hybrid instruments remains opaque. In this context, the thesis scrutinizes the valuation of CoCos. Three dominant approaches are examined: the structural approach in accordance to Pennacchi (2010), the credit derivatives approach and the equity derivatives approach both pursuant to De Spiegeleer and Schoutens (2011b). The application covers sensitivity analysis to further understand the dynamics of the different methodologies. Based on a case study the viability of those approaches is evaluated. Software is provided for further replication.

Contents

List of Figures

Chapter 1

Introduction and Motivation

1.1 Introduction

1.2 Literature Overview

1.3 Motivation

1.4 Methodology

Chapter 2

Structure of CoCos

2.1 Description of CoCos

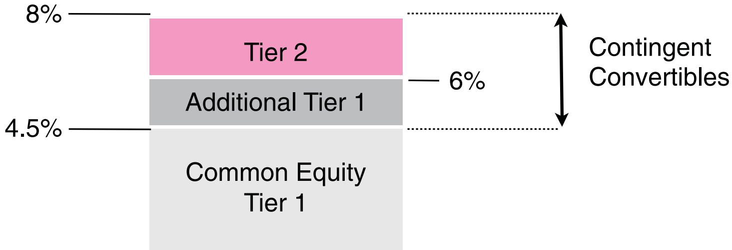

Figure 2.1: CoCos under Basel III (De Spiegeleer and Schoutens, 2011a)

2.2 Payoff and Risk Profile

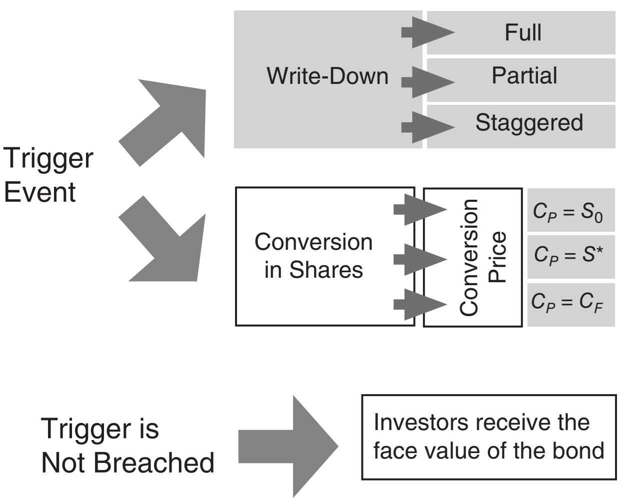

Figure 2.2: Anatomy of CoCos (De Spiegeleer and Schoutens, 2011a)

2.3 Conversion Trigger

2.3.1 Market Trigger

2.3.2 Accounting Trigger

2.3.3 Regulatory Trigger

2.3.4 Multivariate Trigger

2.4 Conversion Details

2.4.1 Conversion Fraction

- conversion fraction α
- face value N
- conversion amount $N \times \alpha$
- amount remaining in case of partial equity conversion $N \times (1 - \alpha)$

2.4.2 Conversion Price and Ratio

- conversion rate C_r
- conversion price C_p
- recovery rate R_{CoCo}
- stock price at trigger event S_T^*
- loss attributable to CoCo holders L_{CoCo}

$$C_p = \frac{\alpha N}{C_r} \tag{2.1}$$

$$C_r = \frac{\alpha N}{C_p} \tag{2.2}$$

$$R_{CoCo} = \frac{S_T^*}{C_p} \tag{2.3}$$

$$L_{CoCo} = N - (1 - R_{CoCo}) = N\left(1 - \frac{S_T^*}{C_p}\right) \tag{2.4}$$

$$P_T = \begin{cases} (1-\alpha)N + C_r S_T^* & \text{if converted} \\ N & \text{if not converted} \end{cases} \tag{2.5}$$

Chapter 3

Theory of Pricing

3.1 Credit Derivative Approach

The derivation mainly follows De Spiegeleer and Schoutens (2011b).

3.1.1 Intensity-Based Approach

Intensity-based approaches model factors influencing the event of default but usually leave aside the question of the default trigger. However, they are an elegant way of bridging the gap between the prediction of default and the pricing of default risk. The following section highlights the link between estimated default intensities and credit spreads under the intensity-based approach (Lando, 2009) which shall be the basis of the credit risk approach. (De Spiegeleer and Schoutens, 2011b)

Let τ denote the random time of default of some company. It is assumed that the distribution of τ has a continuous density function f, so that the distribution function F and the curve of survival probabilities q are related as follows:

$$P(\tau \leq t) = F(t) = 1 - q(t) = \int_0^t f(s)ds, \text{ with } t \geq 0 \tag{3.1}$$

The hazard rate respectively the default intensity λ is defined as

$$\lambda(t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} P(\tau \leq t + \Delta | \tau > t) = \frac{F'(t)}{q(t)} = \frac{F'(t)}{1 - F(t)} = -\frac{d}{dt} \log q(t) \tag{3.2}$$

Intuitively, the hazard rate is the default rate per year as of today. Using 3.2 we get

$$q(t) = \exp\left(-\int_0^t \lambda(s)ds\right) \tag{3.3}$$

For our application of the credit derivative approach (De Spiegeleer and Schoutens, 2011b) we assume that the hazard rate $\lambda(t)$ is a deterministic function of time. A constant hazard rate $\lambda(t) = \lambda$ implies an exponential distribution of the default time:

$$F(t) = 1 - q(t) = 1 - \exp(-\lambda t) \tag{3.4}$$

In reality $\lambda(t)$ is not deterministic but itself stochastic. That fits in with the fact that credit spreads are not static but stochastically varying over time. (Schmidt, 2015)

Under the idealized assumption of a flat zero interest rate curve, a flat spread curve and continuous spread payments, the default intensity λ can be calculated directly from the credit spread s and the recovery rate R by the rule of thumb formula (Schmidt, 2015), which is also known as credit triangle:

$$\lambda = \frac{s}{1-R} \Leftrightarrow s = \lambda\,(1-R) \tag{3.5}$$

Finally, this relationship makes it possible to determine the default probability F from the credit spread s and vice versa.

3.1.2 Application to CoCos

In line with the intensity-based approach, a hazard rate $\lambda_{Trigger}$ is introduced in order to model the triggering of a CoCo. It can be shown that the probability $F*$

$$F^* = 1 - \exp\left(-\lambda_{Trigger} \times t\right) \tag{3.6}$$

$$s_{CoCo} = (1 - R_{CoCo}) \times \lambda_{Trigger} = Loss_{CoCo} \times \lambda_{Trigger} \tag{3.7}$$

$$Loss_{CoCo} = N - C_r \times S^* = N\left(1 - \frac{S^*}{C_P}\right) \tag{3.8}$$

$$R_{CoCo} = \frac{S^*}{C_p} \tag{3.9}$$

$$p^* = \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) - \mu T}{\sigma\sqrt{T}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) + \mu T}{\sigma\sqrt{T}}\right) \tag{3.10}$$

$$\lambda_{Trigger} = -\frac{\log(1-p^*)}{T} \tag{3.11}$$

$$s_{CoCo} = -\frac{\log(1-p^*)}{T} \times \left(1 - \frac{S^*}{C_p}\right) \tag{3.12}$$

3.1.3 Data Requirements and Calibration

3.1.4 Pricing Example

3.2 Equity Derivative Approach

Sources: Erismann (2015), De Spiegeleer and Schoutens (2011b)

$$\begin{aligned}
P_T &= \mathbb{1}_{\{\tau > T\}} N + \left[(1-\alpha)N + \frac{\alpha N}{C_p S^*}\right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + \left[\frac{\alpha N}{C_p} S^* - \alpha N\right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + \left[C_r S^* - \alpha N\right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + C_r \left[S^* - \frac{\alpha N}{C_r}\right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + C_r \left[S^* - C_p\right] \mathbb{1}_{\{\tau \leq T\}}
\end{aligned}$$

$$V_t^{ed} = V_t^{cb} - V_{t_i}^{dibi} + V_t^{difwd} \tag{3.13}$$

3.2.1 Corporate Bonds

$$V_t^{cb} = \sum_{i=t}^{T} c_i \exp(-r t_i) + N \exp\left[-r(T-t)\right] \tag{3.14}$$

3.2.2 Binary Options

$$\begin{aligned}
V_t^{dibi}(c_i, S^*, t) = \alpha \sum_{i=1}^{k} c_i \exp(-r t_i) \bigg[& \Phi\left(-x_{1i} + \sigma\sqrt{t_i}\right) \\
& + \left(\frac{S^*}{S_t}\right)^{2\lambda - 2} \Phi\left(y_{1i} - \sigma\sqrt{t_i}\right) \bigg]
\end{aligned} \tag{3.15}$$

with

$$
\begin{aligned}
x_{1i} &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\
y_{1i} &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\
\lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}
\end{aligned}
$$

3.2.3 Down-And-In Forward

$$
\max\left(S_T - K\right) \ \text{ if } \min_{0\le t\le T}\left(S_T\right) \le S^* \tag{3.16}
$$

$$
\max\left(K - S_T\right) \ \text{ if } \min_{0\le t\le T}\left(S_T\right) \le S^* \tag{3.17}
$$

$$
\begin{aligned}
V_t^{dic}\left(S_t, S^*, K\right) = {} & S_t \exp\left[-q\left(T-t\right)\right]\left(\frac{S^*}{S_t}\right)^{2\lambda}\Phi\left(y\right) \\
& - K\exp\left[-r\left(T-t\right)\right]\left(\frac{S^*}{S_t}\right)^{2\lambda-2}\Phi\left(y - \sigma\sqrt{T-t}\right)
\end{aligned} \tag{3.18}
$$

with

$$
\begin{aligned}
K &= C_p \\
y &= \frac{\log\left(\frac{S^{*2}}{S_t K}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \\
\lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2}
\end{aligned}
$$

$$
\begin{aligned}
V_t^{dip}\left(S_t, S^*, K\right) = {} & S_t \exp\left[-q\left(T-t\right)\right]\left(\frac{S^*}{S_t}\right)^{2\lambda}\left[\Phi\left(y\right) - \Phi\left(y_1\right)\right] \\
& - K\exp\left[-r\left(T-t\right)\right]\left(\frac{S^*}{S_t}\right)^{2\lambda-2}\left[\Phi\left(y - \sigma\sqrt{T-t}\right) - \Phi\left(y_1 - \sigma\sqrt{T}\right)\right] \\
& + K\exp\left[-r\left(T-t\right)\right]\Phi\left(x_1 + \sigma\sqrt{T-t}\right) \\
& - S_t\exp\left[-q\left(T-t\right)\right]\Phi\left(-x_1\right)
\end{aligned} \tag{3.19}
$$

with

$$x_1 = \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t}$$
$$y_1 = \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t}$$

$$\min(S_t) \leq S^* : P_T = S_T - K = \max(S_T - K) - \max(K - S_T) \tag{3.20}$$

$$\min(S_t) > S^* : P_T = 0 \tag{3.21}$$

$$\begin{aligned} V_t^{difwd} = C_r \Bigg[& S_t \exp\left[-q(T-t)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda} \Phi(y_1) \\ & - K \exp\left[-r(T-t)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \Phi\left(y_1 - \sigma\sqrt{T-t}\right) \\ & - K \exp\left[-r(T-t)\right] \Phi\left(-x_1 - \sigma\sqrt{T-t}\right) \\ & + S_t \exp\left[-q(T-t)\right] \Phi(-x_1) \Bigg] \end{aligned} \tag{3.22}$$

with

$$C_r = \frac{\alpha N}{C_p} \tag{3.23}$$

3.2.4 Data Requirements and Calibration

3.2.5 Pricing Example

3.3 Structural Approach

"Structural credit pricing models are based on modeling the stochastic evolution of the balance sheet of the issuer, with default when the issuer is unable or unwilling to meet its obligations." (Duffie and Singleton, 2003)

3.3.1 Synthetic Balance Sheet

3.3.2 Data Requirements and Calibration

3.3.3 Pricing Example

Chapter 4

Dynamics and Sensitivity Analysis

4.1 Credit Derivative Approach

4.2 Equity Derivative Approach

4.3 Structural Approach

Chapter 5

Empirical Analysis and Model Comparison

5.1 Data Description

5.1.1 Deutsche Bank

5.2 Model Parametrization

5.3 Model Comparison

5.3.1 Qualitative Analysis

5.3.2 Quantitative Analysis

Chapter 6

Conclusion

Appendix A

Sample Title

Appendix B

Code - Credit Derivative Approach

The following source code is an implementation of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
# Price of Contingent Convertible Bond
price_coco_cd <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma){

  spread_coco <- calc_spread_coco(t, T, S_t, S_star, C_p, r, q, sigma)
  V_t_coco <- N * exp(-(r + spread_coco) * (T - t))

  for (t in 1:T){
    V_t_coco <- V_t_coco + c_i * exp(-(r + spread_coco) * t)
  }
  V_t_coco
}

# Calculation of Trigger Probability
calc_p_star <- function(t, T, S_t, S_star, r, q, sigma){
  p_star <- pnorm((log(S_star / S_t) - calc_mu(r, q, sigma) * (T - t)) /
    (sigma * sqrt(T - t))) + (S_star / S_t)^(2 * calc_mu(r, q, sigma) /
    sigma^2) * pnorm((log(S_star / S_t) + calc_mu(r, q, sigma) * (T - t
   )) / (sigma * sqrt(T - t)))
  p_star
}

# Calculation of Drift of Underlying
calc_mu <- function(r, q, sigma){
  mu <- r - q - sigma^2 / 2
  mu
}

# Spread of CoCo Bond
calc_spread_coco <- function(t, T, S_t, S_star, C_p, r, q, sigma){
  spread_coco <- - log(1 - calc_p_star(t, T, S_t, S_star, r, q, sigma))
    / (T - t) * (1 - S_star / C_p)
  spread_coco
}

# Pricing Example
```

```
price_coco_cd(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <-
    7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3)
```

Appendix C

Code - Equity Derivative Approach

The following source code is an implementation of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
# Price of Contingent Convertible Bond
price_coco_ed <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma,
    alpha){
  V_t_ed <- price_cb(t, T, c_i, r, N) - price_dibi(t, T, S_t, S_star, c_
    i, r, q, sigma, alpha) + price_difwd(t, T, S_t, S_star, C_p, r, N, q
    , sigma, alpha)

  return(V_t_ed)
}

# Price of Corporate Bond
price_cb <- function(t, T, c_i, r, N){
  V_t_cb <- N * exp(-r * (T - t))

  for (t in 1:T){
  V_t_cb <- V_t_cb + c_i * exp(-r * t)
  }

  return(V_t_cb)
}

# Price of Binary Option
price_dibi <- function(t, T, S_t, S_star, c_i, r, q, sigma, alpha){
  V_t_dibi <- 0

  i <- t
  k <- T

  for (i in 1:k) {
  V_t_dibi <- V_t_dibi + c_i * exp(- r * i) * (pnorm(- calc_x_1_i(S_t, S
    _star, sigma, r, q, i) + sigma * sqrt(i)) + (S_star / S_t)^(2 * calc
    _lambda(r, q, sigma) - 2) * pnorm ( calc_y_1_i(S_t, S_star, sigma, r
    , q, i) - sigma * sqrt(i)))
  }

```

```
  V_t_dibi <- alpha * V_t_dibi

  return(V_t_dibi)
}

# Price of Down-And-In Forward
price_difwd <- function(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha){
  V_t_difwd <- calc_conversion_rate(C_p, N, alpha) * (S_t * exp(- q * (T - t)) * (S_star / S_t) ^ (2 * calc_lambda(r, q, sigma)) * pnorm(calc_y_1(t, T, S_t, S_star, r, q, sigma)) - C_p * exp(- r * (T - t)) * (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm(calc_y_1(t, T, S_t, S_star, r, q, sigma) - sigma * sqrt(T - t)) - C_p * exp(- r * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma) + sigma * sqrt(T - t)) + S_t * exp(- q * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma)))

  return(V_t_difwd)
}

# Calculation of Conversion Rate
calc_conversion_rate <- function(C_p, N, alpha){
  C_r <- alpha * N / C_p

  return(C_r)
}

# Calculation of additional Parameters
calc_x_1_i <- function(S_t, S_star, sigma, r, q, t_i){
  x_1_i <- log(S_t / S_star) / (sigma * sqrt(t_i)) + calc_lambda(r, q, sigma) * sigma * sqrt(t_i)

  return(x_1_i)
}

calc_y_1_i <- function(S_t, S_star, sigma, r, q, t_i){
  y_1_i <- log(S_star / S_t) / (sigma * sqrt(t_i)) + calc_lambda(r, q, sigma) * sigma * sqrt(t_i)

  return(y_1_i)
}

calc_lambda <- function(r, q, sigma){
  lambda <- (r - q + sigma^2 / 2) / sigma^2

  return(lambda)
}

calc_x_1 <- function(t, T, S_t, S_star, r, q, sigma){
  x_1 <- log(S_t / S_star) / (sigma * sqrt(T - t)) + calc_lambda(r, q, sigma) * sigma * sqrt(T - t)

  return(x_1)
}

```

```r
calc_y_1 <- function(t, T, S_t, S_star, r, q, sigma){
  y_1 <- log(S_star / S_t) / (sigma * sqrt(T - t)) + calc_lambda(r, q,
    sigma) * sigma * sqrt(T - t)

  return(y_1)
}

# Pricing Example
price_coco_ed(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <-
    7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3, alpha <- 1)
```

Appendix D

Code - Structural Approach

The following source code is an implementation of the Structural Approach (Pennacchi, 2010) with support from Quantnet (2011) in translating the source code of Pennacchi (2010) from GAUSS to R.

```
# Price of Contingent Convertible Bond
price_coco_sa <- function(T , npath , rho , kappa , r_bar, r0, sigma_r,
    mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_x, x0_low, x0_
    high, x0_nint, B, c_low, c_high, c_nint){
  n <- T * 250
  dt <- T / n

  result <- sim_corrProcess(T, npath, rho, n, dt)
  dW_1 <- result$dW_1
  dW_2corr <- result$dW_2corr

  r <- sim_interestrate(kappa, r_bar, r0, sigma_r, dW_2corr, n, npath,
    dt)

  V_t_sa <- get_price(npath, n, dt, dW_1, dW_2corr, r, mu_Y, sigma_Y,
    lambda, g, x_hat, b0, p, e_bar, sigma_x, x0_low, x0_high, x0_nint, B
    , c_low, c_high, c_nint) * 100
  return(V_t_sa)
}

sim_corrProcess <- function(T, npath, rho, n, dt){
  vect <- c(1, rho, rho, 1)
  RHO <- matrix(vect, nrow = 2)
  chol_RHO <- t(chol(RHO))

  # Create two Brownian Motions
  dW_1 <- matrix(1, n, npath)
  dW_2 <- matrix(1, n, npath)

  for(j in 1:npath)
  {
    dW_1[ , j] <- rnorm(n) * sqrt(dt)
    dW_2[ , j] <- rnorm(n) * sqrt(dt)
  }
```

```

  # Create Correlated Process based on Brownian Motions using Cholesky-Decomposition
  dW_2corr <- matrix(1, n, npath)
  for(j in 1:npath)
  {
    for(i in 1:n)
    {
      dW_2corr[i, j] <- dW_1[i, j] * chol_RHO[2, 1] + dW_2[i, j] * chol_RHO[2, 2]
    }
  }

  return(list("dW_1" = dW_1, "dW_2corr" = dW_2corr))
}

# Create Interest Rate Process
sim_interestrate <- function(kappa, r_bar, r0, sigma_r, dW_2corr, n, npath, dt){
  r <- matrix(r0, n + 1, npath)

  for(j in 1:npath)
  {
    for(i in 1:n)
    {
      r[i + 1, j] <- r[i, j] + kappa * (r_bar - r[i, j]) * dt + sigma_r * sqrt(r[i, j]) * dW_2corr[i, j]
    }
  }

  return(r)
}

get_price <- function(npath, n, dt, dW_1, dW_2corr, r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_x, x0_low, x0_high, x0_nint, B, c_low, c_high, c_nint){

  c_fit_matrix <- matrix(0, x0_nint, length(lambda))

  for(w in 1:length(lambda))
  {
    # Create parametres for jump process
    phi <- matrix(rbinom( n%*%npath, 1, dt * lambda[w]), n, npath)
    ln_Y <- matrix(rnorm(n%*%npath, mu_Y, sigma_Y), n, npath)

    b <- matrix(b0, n + 1, npath)
    x_bar0 <- 1 + e_bar + p * b0
    x_bar <- matrix(x_bar0, n + 1, npath)

    h <- matrix(1, n, npath)

    k <- exp(mu_Y + 0.5 * sigma_Y^2) - 1

    c <- seq(c_low, c_high, length = c_nint)
```

```
  x0 <- seq(x0_low, x0_high, length = x0_nint)

  for(l in 1:x0_nint)
  {
    for(m in 1:c_nint)
    {
      x <- matrix(x0[l],n+1,npath)
      ln_x0 <- matrix(log(x0[l]),n+1,npath)
      ln_x <- ln_x0
      binom_c <- matrix(1,n+1,npath)

      for(j in 1:npath)
      {
        for(i in 1:n)
        {
          d_1 <- (ln_x[i, j] + mu_Y) / sigma_Y
          d_2 <- d_1 + sigma_Y

          h[i, j] <- lambda[w] * (pnorm( - d_1) - exp(ln_x[i, j]) * exp(mu_Y + 0.5 * sigma_Y^2) * pnorm(-d_2))

          b[i + 1, j] <- b[i, j] * exp(- g[w] * (exp(ln_x[i, j]) - x_hat) * dt)

          ln_x[i + 1, j] <- ln_x[i, j] + ( (r[i, j] - lambda[w] * k) - (r[i, j] + h[i, j] + c[m] * b[i, j]) / exp(ln_x[i, j]) - g[w] * (exp(ln_x[i, j]) - x_hat) - 0.5 * sigma_x^2) * dt + sigma_x * sqrt(dt) * dW_1[i, j] + ln_Y[i,j] * phi[i, j]

          x[i + 1, j] <- exp(ln_x[i + 1, j])

          x_bar[i + 1, j] <- 1 + e_bar + p * b[i + 1, j]

          if(x[i + 1, j] >= x_bar[i + 1, j] && binom_c[i, j] > 0.5)
          {
            binom_c[i + 1, j] <- 1
          }else
          {
            binom_c[i + 1, j] <- 0
          }
        }
      }

      payments <- matrix(c(rep(c[m] * dt, n - 1), B), n, npath) * binom_c[1:n, ]

      for(j in 1:npath){
        for(i in 2:n){
          if(payments[i, j] == 0 && p * b[sum(binom_c[ , j]) + 1, j] <= x[sum(binom_c[ , j]) + 1, j] - 1 ){
            payments[i, j] <- p * B
            break
          }
```

```
            else if(payments[i, j] == 0 && 0 < x[sum(binom_c[ , j]) + 1, j] - 1 && x[sum(binom_c[ , j]) + 1, j] - 1 < p * b[sum(binom_c[ , j]) + 1, j]){
              payments[i, j] <- (x[sum(binom_c[ , j]) + 1, j] - 1) * B / b[sum(binom_c[ , j]) + 1, j]
              break
            }
            else{
              payments[i, j] <- payments[i, j]
            }
          }
        }
        vec_disc_v <- rep(0, npath)
        for(j in 1:npath)
        {
          disc_v <- 0
          int_r <- 0

          for(i in 1:n)
          {
            int_r <- int_r + r[i, j] * dt
            disc_v <- disc_v + exp(- int_r) * payments[i, j]
          }
          vec_disc_v[j] <- disc_v
        }

        V_t_sa <- mean(vec_disc_v)

        return(V_t_sa)
      }
    }
  }
}

# Pricing Example
price_coco_sa(T = 5, npath = 2, rho = - 0.2, kappa = 0.114, r_bar = 0.069, r0 = 0.035, sigma_r = 0.07, mu_Y = -0.01, sigma_Y = 0.02, lambda = c(1), g = c(0.5), x_hat = 1.1, b0 = 0.04, p = 1, e_bar = 0.02, sigma_x = 0.02, x0_low = 1.15, x0_high = 1.15, x0_nint = 10, B = 1, c_low = 0.05, c_high = 0.05, c_nint = 10)
```

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