Valuation of Contingent Convertibles with Derivatives

Nicolay Schmitt

Frankfurt School of Finance and Management

A thesis submitted for the degree of

Master of Science in Finance

August 2016

Abstract

Financial crises have led to higher regulatory standards on the capital adequacy of banks. Banks are required to hold more capital with loss absorbance capacities on their balance sheet. In conjunction with this development, contingent convertible bonds (CoCos) have become an attractive instrument for banks to seek new capital. The defining characteristic of CoCos is the automatic conversion into common equity or the principal write-down when a predetermined trigger is met. Loss absorbing capital is created, which instantly improves the capital structure of the distressed bank. However, the pricing of these hybrid instruments remains opaque. In this context, the thesis scrutinizes the valuation of CoCos with equity conversion mechanism. In this context, three dominant approaches are examined: the structural approach in accordance to Pennacchi (2010), the credit derivatives approach and the equity derivatives approach both pursuant to De Spiegeleer and Schoutens (2011b). Additionally, the application covers sensitivity analyses to further understand the dynamics of the different methodologies. Based on a case study of HSBC's perpetual subordinated contingent convertible securities (ISIN US404280AT69) the viability of the approaches is evaluated by analyzing their pricing tracking accuracy. Subsequently, comprehensive software is provided for further applications of the aforementioned pricing approaches.

Contents

1 Introduction **1**
1.1 Background 1
1.2 Previous Studies 3
1.3 Research Methodology 7

2 CoCo Structure **8**
2.1 Definition 8
2.2 Trigger 9
2.3 Loss-Absorption 12

3 Pricing Theories **17**
3.1 Credit Derivative Approach 17
3.2 Equity Derivative Approach 21
3.3 Structural Approach 26

4 Sensitivity Analyses **36**
4.1 Credit Derivative Approach 36
4.2 Equity Derivative Approach 39
4.3 Structural Approach 42

5 Case Study **46**
5.1 Data Description 46
5.2 Parametrization 46
5.3 Comparison 46

6 Conclusion **47**

A Code - Models **48**
A.1 Credit Derivative Approach 48
A.2 Equity Derivative Approach 49
A.3 Structural Approach 51

B Code - Sensitivity Analyses **55**
B.1 Credit Derivative Approach 55
B.2 Equity Derivative Approach 56
B.3 Structural Approach 58

C Code - Case Study **62**
C.1 Parametrization - Structural Approach 62

Bibliography **65**

List of Figures

2.1 Anatomy of CoCos . . . 8

3.1 CoCo price V^{ed} and its components . . . 26
3.2 Jump-diffusion process . . . 28
3.3 Comparison of pricing results . . . 34

4.1 CoCo price pursuant to the credit derivative approach as function of share price and share price volatility . . . 37
4.2 CoCo price pursuant to the credit derivative approach as function of maturity and interest rate . . . 38
4.3 CoCo price pursuant to the credit derivative approach function of trigger price and conversion volatility . . . 38
4.4 CoCo price pursuant to the equity derivative approach as function of share price and share price volatility . . . 39
4.5 CoCo price pursuant to the equity derivative approach as function of maturity and interest rate . . . 40
4.6 CoCo price pursuant to the equity derivative approach as function of trigger price and conversion price . . . 41
4.7 CoCo price pursuant to the structural approach as function of asset-to-deposit ratio and asset volatility . . . 42
4.8 CoCo price pursuant to the structural approach as function of maturity and interest rate . . . 43
4.9 CoCo price pursuant to the structural approach as function of asset-to-deposit ratio and equity-to-deposit threshold . . . 43
4.10 CoCo price pursuant to the structural approach as function of asset-to-deposit ratio and jump intensity . . . 44
4.11 CoCo price pursuant to the structural approach as function of asset-to-deposit ratio and initial ratio of contingent capital's nominal to deposits 44

List of Tables

1.1 Literature overview of valuation approaches for CoCos 3

2.1 CoCo examples with different loss-absorption mechanisms 12

3.1 Parameter classification of the credit derivative approach 20
3.2 Parameter specification of credit derivative approach application . . . 21
3.3 Parameter classification of the equity derivative approach 25
3.4 Parameter specification of equity derivative approach application . . . 25
3.5 Parameter classification of the structural approach 32
3.6 Parameter specification of structural approach application 35

Chapter 1

Introduction

1.1 Background

Investors are overly restrictive in providing liquidity to financial institutions during periods of financial distress. In the past, governments were often in the situation to inject liquidity to financial markets in order to avoid disruptive insolvencies as no other market participant was inclined to do so. Government bailouts, however, externalize the cost of bankruptcy to taxpayers while distorting risk-taking incentives of banking professionals. Contingent convertibles (CoCos) aim to internalize these costs in the capital structure of systemically important financial institutions. CoCos are hybrid financial instruments that absorb losses pursuant to their specifications in case a pre-determined threshold fails to remain above a minimum trigger level. Then, debt automatically morphs to equity which instantly improves a bank's capitalization. Due to their loss-absorption capacity, CoCos are eligible to be categorized as regulatory capital under Basel III. (Avdjiev et al., 2013)

After the global financial crisis of 2008, regulators around the world have been working on two different objectives. On the one hand, they attempt to lower spillover effects on the economy due to bankruptcies of financial institutions. On the other hand, they aim to reduce the individual default probabilities of banks. The latter objective might be attained by ensuring that banks have enough loss absorbing capital on their balance sheet even in though times. (De Spiegeleer and Schoutens, 2011a) In this context, the Basel Committee on Banking Supervision (BCBS) specified that debt instruments are permitted as regulatory capital if losses are absorbed to such an extent that tax payers do not have to bear the costs. (Basel Committee on Banking Supervision, 2010b) Subsequently, this opened the door for CoCos and ever since, the regulatory treatment has been a major driver of past issues.

Besides the reforms of policy makers to raise the quality of regulatory capital, recent studies on CoCos highlight a number of advantages. Albul et al. (2015) show that CoCos lessen financial distress, whether caused by idiosyncratic or systemic shocks. They indicate lower default probabilities of banks and a smaller likelihood of costly bailouts by the public sector. Hilscher and Raviv (2014) support these findings. Additionally, they argue that an appropriate specification of a CoCo's building parts can eliminate the incentives of shareholders to asset-substitution. The problem of asset-substitution arises when managers undertake excessively risky investment decisions to maximize shareholder value at the expense of debtholders. (Bannier, 2011) Moreover, research evinces that CoCos increase the firm value of their issuer as they reduce the cost of capital. (Albul et al., 2015; Von Furstenberg et al., 2011; Barucci and Del Viva, 2012) To sum up, CoCos should be considered in the liability structure of banks from an academic standpoint.

In addition to the positive perception of policy makers and academia, CoCos are well accepted by the financial industry. Banks value that the hybrid instrument enables them to refinance themselves while simultaneously satisfying the regulatory capital requirements at lower costs than with equity. (European Parliament, 2016) Between 2009 and 2015, financial institutions around the world issued CoCos worth USD 446.96 bn in 519 different issues. (Avdjiev et al., 2015) Albeit the amount of CoCos issued is relatively small compared to the market size of other financial products, they were brought into focus in early 2016. At this time, CoCos contributed to increased market volatility in view of some European banks. The relevance of this episode concerning the potential systemic implications should not be whitewashed as it is likely that discussions on regulatory changes will emerge. Having regard to this circumstances, the question arises about the perception of CoCos by investors and the robustness of their pricing models. (European Parliament, 2016)

In this context, an investigation of different valuation concepts for CoCos is highly interesting for both investors as well as supervisory authorities. The paper contributes to a better understanding of relevant concepts. Additionally, the valuation approaches will be applied to a CoCo which was affected by the turbulences in early 2016.

1.2 Previous Studies

Various valuation approaches for CoCos have been developed over time covering different aspects of their nature. The variety of approaches is due to the hybrid character of CoCos which also makes them a highly interesting object of study. Wilkens and Bethke (2014) propose three groups to organize the broad universe: structural approaches, equity derivative approaches and credit derivative approaches. Additionally, Turfus and Shubert (2015) suggest a fourth category: hybrid equity-credit derivative approaches. A comprehensive compilation of relevant studies for each category can be found in table 1.1. Subsequently, the main idea of each type will be explained.

Structural Approaches	Equity Derivative Approaches	Credit Derivative Approaches
Pennacchi (2010)	De Spiegeleer and Schoutens (2011b)	De Spiegeleer and Schoutens (2011b)
Albul et al. (2010)	Henriques and Doctor (2011) as cited by Erismann (2015)	Serjantov (2011) as cited by Wilkens and Bethke (2014)
Madan and Schoutens (2011)	Alvemar and Ericson (2012)	Alvemar and Ericson (2012)
Glasserman and Nouri (2012)	Corcuera et al. (2013)	Erismann (2015)
Alvemar and Ericson (2012)	Corcuera et al. (2014)	
Buergi (2013)	Teneberg (2015)	
Hilscher and Raviv (2014)	Erismann (2015)	
Pennacchi and Tchistyi (2015)		
Cheridito and Xu (2015)	**Hybrid Equity-Credit Derivative Approaches**	
Erismann (2015)	Turfus and Shubert (2015)	
Sundaresan and Wang (2015)		

Table 1.1: Literature overview of valuation approaches for CoCos (Wilkens and Bethke, 2014; Erismann, 2015) with the examined methods.

Structural approaches try to capture all parameters that influence the issuer's ability to pay its liabilities. They are normally built upon a stochastic model which focusses on the variation in asset values relative to debt. (Duffie and Singleton, 2003) By contrast, **equity derivative approaches** emphasize the dependence of a CoCo's state on the share price and use equity derivatives to replicate their payoff. This model type follows the train of thought that the share price is the best proxy to track the solvency of the issuer. **Credit derivative approaches** assume an exogenously specified process for the migration of conversion probabilities. They apply the idea of reduced-form approaches to model the equity conversion intensity process of CoCos in line with a credit default intensity process. The rationale behind this approach is that CoCos are credit-risky instruments as their conversion depends on the issuer's solvency. (Wilkens and Bethke, 2014) **Hybrid equity-credit derivative approaches** capture the advantages of the latter two concepts. They model the share price and the conversion intensity as correlated stochastic processes. (Turfus and Shubert, 2015) A detailed literature review of the various pricing approaches is outlined in the following sections.

Structural Approaches

Structural approaches offer a natural pricing framework for CoCos. They consider a bank's balance sheet structure as the most important value driver. Numerous structural approaches have been proposed in academia. All share common characteristics but vary in their application. (Wilkens and Bethke, 2014) For instance, they are often used to draw policy recommendations. A selection can be found in the following.

The study of Albul et al. (2010) is the first paper to provide analytic propositions to price CoCos by adapting the structural model of Leland (1994). The authors develop implications for the design of CoCos with the objective of maximizing the benefit for the issuer. Interestingly, their analysis is at first not limited to financial institutions. In fact, they argue that CoCos might generally be advantageous for corporates to optimize their capital structure. The authors further recommend the specific use of CoCos as tool for bank regulation. In this context, studies like Madan and Schoutens (2011), Hilscher and Raviv (2014) and Sundaresan and Wang (2015) analyze beneficial structures of CoCos.

In the scientific literature, the work of Pennacchi (2010) is often used as a reference article for structural approaches as he attempts to model the stochastic evolution of a bank's balance sheet to price CoCos (for further details see chapter 3.3). The author is able determine the value of CoCos by applying a jump-diffusion process to account for discontinuous asset returns. Capital ratios with mean-reverting tendency and a stochastic term-structure model shall improve the pricing accuracy. Based on the derived framework the author is able to capture several risk factors that may influence a CoCo's price. However, this is also the main shortcoming of the approach because the author does not address the parametrization of input factors in practice, which is also indicated by the work of Erismann (2015).

Madan and Schoutens (2011) implement a structural model utilizing conic finance theory. Classical Mertonian models (Merton, 1974) assume that assets are risky but liabilities are not. For instance, Alvemar and Ericson (2012) applies such a pure model pursuant to Merton (1974) to price CoCos. In contrast, the model of Madan and Schoutens (2011) assumes that liabilities are risky and correlated to the asset dynamics. The authors abandon the one-price-market idea and assume that bid-ask spreads exist. In addition, they argue that the Core Tier 1 ratio is potentially not optimal as trigger if one considers the presence of risky liabilities. As alternative they propose accounting triggers based on capital shortfall.

Equity Derivative Approaches

Equity derivative approaches are an important category of valuation approaches which consider the share price as the best proxy. Most important approaches will be summarized in the following.

De Spiegeleer and Schoutens (2011b), for example, replicate the payout profile of a CoCo with a portfolio consisting of a straight bond, a knock-in forward and a set of binary down-and-in calls (for further details see chapter 3.3). Under the assumption that a CoCo will not convert to equity, one can assume that a CoCo is equivalent to a straight bond. Though, the knock-in forward simulates the conversion of a straight bond when a predetermined strike price is met. A CoCo investor would receive the shares at maturity if he or she is long a knock-in forward. However, this is a simplification which is reasonable under the assumption that dividend payments are cancelled in times of distress. Additionally, the foregone coupon payments of a straight bond at conversion are modeled with a short position in binary down-and-in calls. One of the main findings is that the assumed Black-Scholes setting does not sufficiently capture tail risks but which are inherent in CoCos.

Other approaches enhance the model dynamics by accounting for jumps and heavy tails. Erismann (2015) and Teneberg (2015) amend the model of De Spiegeleer and Schoutens (2011b) by allowing for discontinuous returns. The calculations with regard to the binary down-and-in calls and the knock-in forward position accommodates a jump-diffusion process. Corcuera et al. (2013) also consider an equity derivative approach that reduces the valuation to a set of barrier options in which the trigger event is determined by the underlying hitting a certain barrier. They use smile conform models, more precisely, an exponential Lévy process incorporating jumps and heavy tails.

Credit Derivative Approaches

The price of a CoCo is directly linked to the issuer's solvency and default probability. Intensity-based credit modeling allows to develop comprehensive pricing approaches. In this connection, one should mention the work of De Spiegeleer and Schoutens (2011b), Serjantov (2011) and Erismann (2015).

De Spiegeleer and Schoutens (2011b) tackle the pricing problem with a credit-derivative approach (for further details see chapter 3.1). Their main contribution lies in the derivation of a closed-form solution of a CoCo's credit spread. In their model the spread follows a function of an exogenously defined trigger probability. The spread compensates for the risk that the CoCo converts to equity implying a loss for each investor. Their approach is an elegant way of bridging the gap between the prediction of conversion and the pricing of conversion risk. Though, the largest shortcoming of the model is that it fails to capture losses from cancelled coupons of triggered CoCos.

Erismann (2015) expands the model of De Spiegeleer and Schoutens (2011b) by assuming that returns follow a jump-diffusion process. The approach models the exposure to return outliers of both signs and amplitudes. Finally, the author demonstrates that his approach is superior to De Spiegeleer and Schoutens (2011b) considering pricing accuracy.

Serjantov (2011) as cited by Wilkens and Bethke (2014) develops a closed form solution to price CoCos. All cashflows are weighted with cumulative survival probabilities. In addition, the approach distinguishes between the conversion ratio without default and the recovery rate at default. The joint probability of both events happening in the same time interval is described with a Gaussian copula. Furthermore, this approach overcomes the shortcoming of the credit derivative approach of De Spiegeleer and Schoutens (2011b) as it explicitly captures coupon payments.

Hybrid Equity-Credit Derivative Approaches

Turfus and Shubert (2015) present a new pricing approach for CoCos. Their starting point is a stochastic model which captures interest rates, share prices and a conversion intensity process. The evolution of the first two is assumed to be determined by diffusive processes. By contrast, the share price is supposed to be governed by a jump-diffusion process which factors into a downward jump when the trigger level is touched. Both the share price and the conversion intensity process are modeled as correlated stochastic processes. For this very reason, the hybrid equity-credit derivative approach may be regarded as an important step forward because two direct benefits arise. On the one hand, the share price at conversion is modeled instead of being an input parameter and on the other hand, both equity and credit risk sensitivity can be estimated individually.

1.3 Research Methodology

The objective of the thesis consists of an examination of three dominant pricing approaches for CoCos similar to the proceedings of Alvemar and Ericson (2012), Erismann (2015) and Wilkens and Bethke (2014). All of them are widely discussed in academic literature as they are often used as basis for further model advancements. The utilized approaches are namely the structural approach of Pennacchi (2010), the equity derivative approach and the credit derivative approach both pursuant to De Spiegeleer and Schoutens (2011b). Hereinafter, chapter 2 provides an overview of the anatomy of CoCos. Characteristic building parts of the financial product will be discussed in detail in order to create an improved understanding of the mechanisms which drive the valuation of this hybrid instrument. Examples of past CoCos issues are highlighted covering the most important variations of the aforementioned design features. On this basis, chapter 3 studies the theoretical concepts behind each of the three valuation approaches. In addition, pricing examples provide an understanding of the data requirements of each model. In chapter 4, sensitivity analysis determine how different values of certain pricing parameters impact the valuation of CoCos. Chapter 5 comprises an empirical analysis on the price tracking accuracy of the aforementioned valuation approaches. Finally, a conclusion is reached in chapter 6.

Chapter 2

CoCo Structure

This chapter explains the nature of CoCos, a new member in the family of hybrid securities. In the following, the general structure of these new instruments will be explained (section 2.1) including characteristic design features among others their trigger (section 2.2) and loss-absorption mechanism (section 2.3).

2.1 Definition

The term CoCo is used to describe a new hybrid capital instrument with an automatic conversion mechanism which morphs debt into equity when the financial soundness of the issuer is at stake. A write-down of the notional is also a viable loss-absorption mechanism to recapitalize the distressed bank. Generally, the loss-absorption mechanism is activated if a predefined trigger level is breached. (De Spiegeleer and Schoutens, 2011b; Zähres, 2011). In this context, figure 2.1 provides an overview of major design characteristics. Exactly these anatomic aspects will be explained in the subsequent sections.

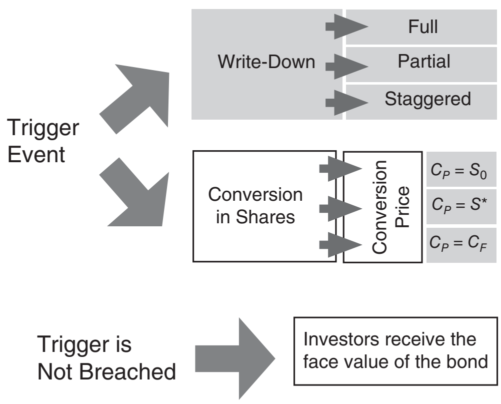

Figure 2.1: Anatomy of CoCos (De Spiegeleer et al., 2014)

CoCos are particularly interesting from the perspective of a regulatory authority because they might mitigate externalization costs of insolvencies and frictions due to spill-over effects. In times of distress, stakeholders might question the financial viability of the respective financial institute. Yet, the major advantage of CoCos is that a distressed bank does not have to approach new investors to issue new capital in extremely though times as everything happens automatically. (De Spiegeleer and Schoutens, 2011b)

In 2009, the Lloyds Banking Group was the first financial institute which issued this new financial instrument. In an exchange offer, they asked hybrid debt holders to swap their holdings into CoCos. (De Spiegeleer and Schoutens, 2011b) In 2010, the Basel Committee on Banking Supervision (BCBS) provided further impetus to the use of this instrument when it disclosed its proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. CoCos fit into their line of argumentation that regulatory capital instruments have to be capable of absorbing financial losses in gone-concern phases. (Basel Committee on Banking Supervision, 2010a)

2.2 Trigger

The trigger is a key design element of a CoCo which initializes the loss-absorption mechanism. In the following sections, four different trigger mechanisms will be described: accounting triggers, market triggers, regulatory triggers and multi-variate triggers.

2.2.1 Accounting Trigger

CoCos with accounting trigger have a loss absorption mechanism which is inherently connected to the financial soundness of a bank's balance sheet. Accounting triggers are built upon capital ratios which compare a bank's regulatory capital with its assets. Capital ratios are an objective indicator for a bank's solvency as they are defined uniformly for all financial institutions by regulatory authorities. (De Spiegeleer et al., 2014) In addition, Pazarbasioglu et al. (2011) note that accounting triggers are easy to price, intuitive and simple to implement. Having said that, one might argue that accounting triggers assess the viability of financial institutions from a perspective that is far-removed from reality. A major objection against accounting triggers follow the line of thought that they just become active long after the need for loss absorbing capital arose because accounting data is published infrequently. (De Spiegeleer and

Schoutens, 2011b) Moreover, as accounting concept, book values are prone to manipulation and managerial dishonesty especially in times of distress. (McDonald, 2013)

Empirical findings bring up a another aspect. Haldane (2011) points out that major financial institutions, which either went bankrupt, were bailed out or were taken over under distress during the global financial crisis, reported similar CET 1 ratios right before the collapse of Lehman Brothers compared to their peers which coped relatively well with the collapse. In this context, Haldane (2011) highlights that market-based solvency measures performed creditably as they showed clear signals of impending distress a year ahead of the bankruptcy of Lehman Brothers. Empirical evidence of the United States Bankruptcy Court (2010) further supports these findings. This leads to the conclusion that CoCos with accounting triggers might not reinforce distressed banks at the right time but instead produce false positives, which means that CoCos of non-distressed banks trigger prematurely. Inefficiencies like higher funding costs could be the consequence. (Pazarbasioglu et al., 2011)

2.2.2 Market Trigger

A market trigger uses directly observable indicators like the issuing company's share price or credit default swap (CDS) spreads while assuming sufficiently efficient markets. The major advantage of those measures is that one can observe and verify them in real-time. (Haldane, 2011) Market triggers are widely discussed in academia and seen as preferable trigger mechanism. Calomiris and Herring (2013) pronounce themselves for using share prices. Besides, Haldane (2011), Pazarbasioglu et al. (2011) and (Calomiris and Herring, 2013) contend to apply market-based capital ratios as trigger indicator. Their line of argumentation is based on some of the best-known examples of corporate defaults, which have been indicated well before by a serious and continuous deterioration of a company's market capitalization.

In contrast, Sundaresan and Wang (2015) argue based on a structural approach that CoCos with market trigger do not lead to a unique share price equilibrium, unless conversion result in a value transfer between shareholders and CoCo investors. Having said that, the design of dilutive conversion ratios to punish bank managers for taking excessive risks creates multiple equilibria which in turn makes CoCos susceptible to market manipulation. The authors conclude that regulation with good intention might cause instability in the market and that the impact of regulation may be limited by the market itself. However, Hilscher and Raviv (2014) demonstrate that an appropriate design of CoCos can mitigate the risk of asset-substitution by exactly offsetting costs and benefits of shareholders when increasing the probability of conversion. However, Pennacchi and Tchistyi (2015) weaken the argumentation of Sundaresan and Wang (2015) as they demonstrate that a unique share price equilibrium exists for CoCos with perpetual maturity independent of their trigger type. The relevance of their findings is emphasized by the fact that 57.1% of CoCos, which have been issued between 2009 and 2015 do not have a set maturity. (European Parliament, 2016)

2.2.3 Regulatory or Non-Viability Trigger

Regulatory or non-viability trigger is a conversion mechanism by which a CoCo is converted into equity at the discretion of the responsible supervisory authority. The rationale behind this approach is that regulators want to limit the impact of any development that could pose a danger to the going-concern of a systemically important bank. (Erismann, 2015) Moreover, this kind of trigger would eliminate the periodicity problem of accounting data and the risk of market manipulation.

Though, it is very difficult for market participants to estimate the conversion probability of a CoCo with regulatory trigger. The valuation of such a hybrid instrument becomes opaque for market participants with limited information. (Alvemar and Ericson, 2012) One can also argue that a CoCo's marketability is weakened because of the greater uncertainty which could ultimately lead to higher funding costs. (De Spiegeleer et al., 2014)

2.2.4 Multi-Variate Trigger

The multi-variate trigger combines an accounting trigger with a systemic trigger which covers severe states of the world. For instance, the Squam Lake Working Group (2009) argues that the implementation of such a dual trigger is preferable as it combines the best of two worlds. The bank-specific trigger serves as direct disciplining mechanism for a bank's management. In parallel, it reduces the political pressure from the regulator who has to decide whether the systemic trigger is met. Moreover, if the conversion of a CoCo has only been linked to a systemic trigger, even well capitalized banks would be forced to convert debt into equity during a systemic crisis. However, this would disincentivize financially sound banks to preserve their status quo.

2.3 Loss-Absorption

As mentioned earlier, banks can decide whether to issue CoCos with an equity conversion- or a write-down mechanism. Conversion into equity means that a certain portion of a CoCo's notional will be converted into equity if a certain trigger event occurs. It is also possible to specify that the notional of a bond suffers a haircut if the issuer decides to use a write-down mechanism. Hereinafter both types will be explained in detail.

	Lloyds	**Credit Suisse**	**Barclays**	**Rabobank**	**ZKB**
Full Name	Enhanced Capital Notes	Tier 2 Buffer Capital Notes	Contingent Capital Notes	Senior Contingent Notes	Subordinated Tier 1 Notes
ISIN	XS0459088281	XS0595225318	US06740L8C27	XS0496281618	CH0143808332
Issue Date	Dec 1, 2009	Feb 24, 2011	Nov 21, 2012	Mar 19, 2010	Jan 31, 2012
Maturity	May 12, 2020	Feb 24, 2041	Nov 21, 2022	Mar 19, 2020	Perpetual
Nominal	GBP 7.5 bn	USD 2 bn	USD 3 bn	EUR 1.25 bn	CHF 590 mn
Callability	n/a	Callable from Aug 24, 2016	n/a	n/a	Callable from Jun 20, 2017
Coupon	7.5884%	7.875%	7.625%	6.875%	3.5%
Write-down	n/a	n/a	Full by (100% of notional)	Partial (75% of notional)	Staggered (multiples of 25% of notional)
Conversion price	Fixed at GBP 0.59 (= S_0)	Floored at lowest of USD 20 and 30 day-VWAP	n/a	n/a	n/a
Trigger	Core Tier 1 capital ratio	CET 1 ratio	CET 1 ratio	Equity capital ratio (Member certificates to risk weighted assets)	CET 1 ratio
Trigger Level	5%	7%	7%	7%	7%

Table 2.1: CoCo examples with different loss-absorption mechanisms (Lloyds, 2009; Credit Suisse, 2011; Barclays, 2010; Rabobank, 2010; Zurich Cantonal Bank, 2013)

In addition, for each of the loss-absorption mechanism selected CoCos are characterized in order to gain a better sense of how these hybrid products are implemented in real-life. Broadly discussed CoCos of well-known financial institutions are picked out, i.a. Lloyds, Credit Suisse, Barclays, Rabobank and Zurich Cantonal Bank. An overview of CoCos which have different characteristics with respect to their loss-absorbency can be found in table 2.1. The first two apply similar equity conversion mechanisms whereas the latter three use different write-downs.

2.3.1 Conversion into Equity

The following sections clarify three of the most common structures which rely on the specification of the conversion price of a CoCo.

In the beginning, a few variables are introduced in order to study the equity conversion mechanism. First, the number of shares which a CoCo holder receives at conversion is given by the conversion rate C_r. Second, the conversion amount αN is determined by the conversion fraction α and the notional N. All of these parameters are defined ex ante. One can now describe the relationship between the implied conversion price C_p and the introduced parameters as follows: (De Spiegeleer et al., 2014)

$$C_p = \frac{\alpha N}{C_r} \tag{2.1}$$

The recovery rate R_{CoCo} is be calculated from the conversion price C_p and the share price S^* at conversion. It becomes immediately evident from equation 2.2 that a CoCo investor is better off if C_p is low since the recovery rate R_{CoCo} becomes higher as more equity is created.

$$R_{CoCo} = \frac{S^*}{C_p} \tag{2.2}$$

If the CoCo converts into equity one can directly determine the occurring damage. The financial loss L_{CoCo} of a CoCo investor can be expressed by the subsequent equation:

$$Loss_{CoCo} = N - C_r S^* = N - (1 - R_{CoCo}) = N\left(1 - \frac{S^*}{C_p}\right) \tag{2.3}$$

A CoCo may or may not trigger throughout its life. Nevertheless, the final payoff V^{CoCo} of an investor at maturity T can be described as follows:

$$V_T^{CoCo} = \begin{cases} N & \text{if not triggered} \\ (1-\alpha)N + \frac{\alpha N}{C_p} S^* & \text{if triggered} \end{cases} \tag{2.4}$$

Floating conversion price $C_p = S^*$

Ex ante an issuer can set a floating conversion price where C_p is equal to S^*. In this regard, S^* is the share price which is observed at conversion. Intuitively, the value of the share price at precisely the trigger time is fairly low because the purpose of a CoCo is to help an undercapitalized bank in difficult times. If the issuer decides to specify a floating conversion price the recovery rate of a CoCo holder will be 100%. However, current shareholders would carry the load of conversion. The main shortcoming of this approach is that regulators would not categorize this instrument to be adequate as regulatory capital instrument. The dilution is potentially unbounded and it is effectively not loss absorbing. (De Spiegeleer et al., 2014)

Fixed conversion price $C_p = S_0$

Fixed conversion means that the conversion price C_p corresponds to the share price at the time of issue S_0. This means that the amount of shares upon conversion is fixed at the bond issue date and that the conversion amount is known beforehand. Unlike the floating conversion price, there is a predetermined limit on the amount of shares converted. (De Spiegeleer et al., 2014) In 2009, Lloyds issued Enhanced Capital Notes. At that time, the company was the first bank to refinance itself with CoCos. The CoCos convert into equity at a fixed conversion price which has been set to equal the share price at issue. The Enhanced Capital Notes trigger if the bank's Core Tier 1 capital (Basel II) fails to remain above the threshold of 5%. (Lloyds, 2009)

Floored conversion price $C_p = \max(S^*, S_F)$

One can also specify a floored conversion price where C_p is equal to $\max(S^*, S_F)$. Hence, the conversion price C_p is either equal to the floored share price S_F or to the share price at conversion S^*. This approach represents a compromise between the aforementioned floating and fixed conversion price. (De Spiegeleer et al., 2014) An example for the floored conversion price are the Tier 2 Buffer Capital Notes of Credit Suisse. The trigger event is specified to convert the Tier 2 Buffer Capital Notes into ordinary shares if the reported risk-based capital ratio is below 7%. The conversion price is floored at the average daily share price of the last 30 days or USD 20. It may also be that the CoCo is converted if the FINMA determines that Credit Suisse needs to be bailed out. (Credit Suisse, 2011)

2.3.2 Write-Down

The implementation of a loss-absorption mechanism with equity conversion mechanism entails several disadvantages which might reduce a CoCo's marketability. Portfolio managers with a mandate for bonds might face hard times to broaden their investment universe because CoCos with equity conversion mechanism are likely to convert into shares. In addition, the write-down mechanism is preferable as investors know beforehand the potential loss. Shareholders could be concerned that their voting rights are diluted when a conversion occurs. Both investors and shareholder have clear incentives to force banks to issue CoCos with write-down mechanism.

Full write-down

The first way is to specify a full write-down of a CoCo's face value in case a certain capital ratio drops below a predetermined level. (De Spiegeleer et al., 2014) In 2012, Barclays launched the first high-trigger total-loss CoCo. The Contingent Capital Notes depreciate to a value of zero should the CET 1 ratio fail to remain above a minimum of 7%. Such a structure assumes already that the bank has a solid buffer before the trigger is actually met. (Barclays, 2010)

Partial write-down

Another approach is that only a certain portion of the notional is wiped out. CoCos with write-off features are particularly suitable for cooperative banks that are deterred by their legal form to issue shares. (De Spiegeleer et al., 2014) In 2010, Rabobank was the first cooperative bank to issue Senior Contingent Notes with a loss-absorption mechanism that imposes a considerable haircut on its principal. One quarter of the notional is reimbursed at the trigger event if the equity capital ratio falls below 7%. The distinct haircut explains its high financing costs. (Rabobank, 2010)

Staggered write-down

The third option consists of a staggered write-down. This means that a CoCo inherits a flexible write-down mechanism. Losses materialize up to the point to enhance a certain capital ratio to a fixed threshold. One might think of a gradual process in which a haircut is imposed in multiples of 10%. (De Spiegeleer et al., 2014) Zurich Cantonal Bank is wholly owned by the Canton of Zurich and hence, it is not listed. The bank has been in a similar situation to improve its regulatory capital as Rabobank. In this situation, the bank issued Subordinated Tier 1 Notes that are exemplary for CoCos with a staggered write-down mechanism. What has been new is that an investor faces a dilution of his or her holdings up to the point where the write-down lifts the regulatory capital up to its minimum level. Haircuts only materialize in multiples of 25%. (Zurich Cantonal Bank, 2013)

Temporary write-down

Theoretically, it is possible that the write-down mechanism which is embedded into a CoCo is temporarily active. This means that the haircut of the notional is reversed upon restoration of financial health of the issuing bank. However, regulators seek a permanent improvement in capitalization on conversion which makes it less likely that a temporarily mechanism is embedded. (Avdjiev et al., 2013) For example, the Italian bank Intesa Sanpaolo issued Additional Tier 1 Notes with a temporarily write-down mechanism. The bond is temporarily written off if the CET 1 ratio fails to remain above a level of 5.125%. (Intesa Sanpaolo, 2011)

Chapter 3

Pricing Theories

3.1 Credit Derivative Approach

In financial markets the reduced-form approach is widely used in order to price credit risk. It was originally introduced by Jarrow and Turnbull (1995) respectively Duffie and Singleton (1999). The credit derivative approach applies the reduced-form approach to CoCos. In this context, the derivation of a pricing formula for CoCos follows mainly De Spiegeleer and Schoutens (2011b).

3.1.1 Reduced-Form Approach and Credit Triangle

The reduced-form approach is an elegant way of bridging the gap between the prediction of default and the pricing of default risk of a straight bond. In the following, we investigate the link between estimated default intensities and credit spreads under the reduced-form approach. (Lando, 2009)

Let τ denote the random time of default of some company. It is assumed that the distribution of τ has a continuous density function f, so that the distribution function F and the curve of survival probabilities q are related as follows:

$$P(\tau \leq t) = F(t) = 1 - q(t) = \int_0^t f(s)ds, \text{ with } t \geq 0 \tag{3.1}$$

The hazard rate respectively the default intensity λ is defined as follows:

$$\lambda(t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} P(\tau \leq t + \Delta | \tau > t) = \frac{F'(t)}{q(t)} = \frac{F'(t)}{1 - F(t)} = -\frac{d}{dt} \log q(t) \tag{3.2}$$

Intuitively, the hazard rate is the default rate per year as of today. Using 3.2 we can derive a formula for the survival probability:

$$q(t) = \exp\left(-\int_0^t \lambda(s)ds\right) \tag{3.3}$$

For our application of the reduced-form approach we assume that the hazard rate $\lambda(t)$ is a deterministic function of time. In reality $\lambda(t)$ is not deterministic but itself stochastic. That fits in with the fact that credit spreads are not static but stochastically varying over time. (Schmidt, 2015) However, we further consider the hazard rate to be constant in order to simplify the problem. Hence, a constant hazard rate $\lambda(t) = \lambda$ implies an exponential distribution of the default time:

$$F(t) = 1 - q(t) = 1 - \exp(-\lambda t) \tag{3.4}$$

Under the idealized assumption of a flat zero interest rate curve, a flat spread curve and continuous spread payments, the default intensity λ can be calculated directly from the credit spread s and the recovery rate R by the rule of thumb formula (Schmidt, 2015), which is also known as credit triangle:

$$\lambda = \frac{s}{1-R} \tag{3.5}$$

Finally, this relationship makes it possible to determine the default probability F from the credit spread s and vice versa.

3.1.2 Adaption to CoCos

In accordance with the aforementioned reduced-form approach, De Spiegeleer and Schoutens (2011b) assume that the probability F^*, which measures the likelihood that a CoCo triggers within the next $T-t$ years, follows similar mechanics as the default probability of a straight bond does. Under the credit derivative approach the probability F^* can be expressed as follows:

$$F^* = 1 - \exp\left[-\lambda_{Trigger}(T-t)\right] \tag{3.6}$$

Additionally, the credit derivative approach models F^* with the first exit time equation used in barrier option pricing under a Black-Scholes setting. (Su and Rieger, 2009) Hence, the probability F^* that the trigger level S^* is touched within the next $T-t$ years is given by the following equation with the continuous dividend yield q, the continuous interest rate r, the drift μ, the volatility σ and the current share price S of the issuing company:

$$F^* = \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) - \mu(T-t)}{\sigma\sqrt{(T-t)}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) + \mu(T-t)}{\sigma\sqrt{(T-t)}}\right) \tag{3.7}$$

In this regard, a CoCo's credit spread s_{CoCo} can be approximated by the credit triangle, where R_{CoCo} denotes the recovery rate of a CoCo and L_{CoCo} is the loss rate:

$$s_{CoCo} = (1 - R_{CoCo})\,\lambda_{Trigger} = L_{CoCo}\lambda_{Trigger} \tag{3.8}$$

In the trigger event, the face value N converts into C_r shares worth S^*. The loss of a long position in a CoCo is therefore determined by the conversion price C_p:

$$Loss_{CoCo} = N - C_r S^* = N\,(1 - R_{CoCo}) = N\left(1 - \frac{S^*}{C_p}\right) \tag{3.9}$$

By combining 3.6, 3.8 and 3.9 we see that the credit spread s_{CoCo} of a CoCo with maturity T at time t is driven by its major design elements, the trigger level S^* and the conversion price C_p:

$$s_{CoCo_t} = -\frac{\log(1 - F^*)}{(T - t)}\left(1 - \frac{S^*}{C_p}\right) \tag{3.10}$$

Subsequently, a pricing formula for CoCos under the credit derivative approach can be derived. The present value V^{cd} at time t is given by:

$$V_t^{cd} = \sum_{i=1}^{T} c_i \exp\left[-(r + s_{CoCo_t})(t_i - t)\right] + N \exp\left[-(r + s_{CoCo_t})(T - t)\right] \tag{3.11}$$

In summary, the credit derivative approach provides us with a concise method to price CoCos. However, one has to bear in mind its largest shortcoming. Losses from cancelled coupons of triggered CoCos are not taken into account in the valuation. Hence, the credit derivative approach naturally overestimates the price of CoCos, but it equips investors with a simple rule of thumb formula.

3.1.3 Parameter Classification and Adjustment

The credit derivative approach requires several model inputs which can be found in table 3.1. Generally, one can separate three different types. Static inputs comprise parameters that are specified ex ante and are assumed to be constant. Dynamic inputs are updated regularly. Fitting parameters are used to fit the model prices to market prices. (Wilkens and Bethke, 2014)

	Description	Usage	Source
T	Maturity	Static input	Term sheet
N	Notional	Static input	Term sheet
c	Coupon rate	Static input	Term sheet
S_0	Initial share price	Dynamic input	Market data
S^*	Trigger share price	Fitting parameter	–
C_p	Conversion price	Static input	Term sheet
r	Risk-free interest rate	Dynamic input	Market data
q	Dividend yield	Dynamic input	Market data
σ	Share price volatility	Dynamic input	Market data

Table 3.1: Parameter classification of the credit derivative approach (Wilkens and Bethke, 2014)

All static inputs can be found in the term sheets of the respective CoCo. But beyond the model depends upon further dynamic inputs among others the share price S, the risk-free interest rate r, the dividend yield q and the share price volatility σ. The daily share price S is directly observed in the market. The risk-free interest rate r is derived from sovereign bonds with the same maturity and currency. Furthermore, the input parameter q relies on the three year average dividend yield of the issuing company. The share price volatility σ is derived based on a yearly average volatility on a reference stock market index of the last five years similar to Alvemar and Ericson (2012). In addition, the only degree of freedom in the credit derivative approach is the fitting parameter S^* respectively the trigger share price. Because this input variable is neither a static parameter nor a market parameter, S^* is adjusted by minimizing the root mean squared deviation to realized CoCo prices. (Erismann, 2011)

3.1.4 Model Application

In the following a fictive CoCo is priced with the credit derivative approach pursuantly the specifications as stated in table 3.2. After installing both R, a programming language for statistical computing, and Rstudio, an open-source integrated development environment, a reader can easily price the aforementioned CoCo example with the source code of chapter A.1. The CoCo specifications are also used to analyze the price sensitivity in regard to certain input parameters. The results of the sensitivity analysis for the credit derivative approach can be found in section B.1. In addition, this example data set is also used to apply the equity derivative approach and the structural approach.

	Value	Comment
T	10yrs	Maturity
N	100%	Nominal
c	6.00%	Coupon rate
S_0	120	Initial share price
S^*	60	Trigger share price
C_p	75	Conversion price
r	3.00%	Risk-free interest rate
q	0.00%	Dividend yield
σ_E	30.00%	Share price volatility

Table 3.2: Parameter specification of credit derivative approach application (Alvemar and Ericson, 2012)

The spread of the CoCo s_{CoCo}, which compensates an investor for the risk of equity conversion, equals 1.46%. Moreover, the price V^{cd} of the fictive CoCo under the credit derivative approach equates to 111.31.

3.2 Equity Derivative Approach

In order to assess the value of a CoCo, investors can use a method which depends on equity derivatives. (De Spiegeleer and Schoutens, 2011b; De Spiegeleer et al., 2014) The so-called equity derivative approach attempts to compensate for the main drawback of the credit derivative approach, since it takes into account that coupon payments might be cancelled if the trigger of a CoCo was touched.

Subsequently, the valuation can be divided into two steps: In the first step, the value of a CoCo is determined without coupon payments. Such a CoCo is called Zero-Coupon CoCo. In the second step, one has to incorporate the coupon payments in the pricing formula while keeping in mind that they might be knocked out. The closed-form solution is derived under a Black-Scholes setting.

3.2.1 Step One - Zero-Coupon CoCo

To determine the present value of a Zero-Coupon CoCo V^{zcoco} at maturity T we can use equation 2.4. The underlying assumption of the equity derivative approach is that the triggering of a CoCo respectively of a Zero-Coupon CoCo is equivalent to the share price falling below the level S^*.

$$\begin{aligned}
V_T^{zcoco} &= \begin{cases} N & \text{if not triggered} \\ (1-\alpha)N + \frac{\alpha N}{C_p}S^* & \text{if triggered} \end{cases} \\
&= N\mathbb{1}_{\{\tau>T\}} + \left[(1-\alpha)\,N + \frac{\alpha N}{C_p}S^*\right]\mathbb{1}_{\{\tau\leq T\}} \\
&= N + \left[\frac{\alpha N}{C_p}S^* - \alpha N\right]\mathbb{1}_{\{\tau\leq T\}} \\
&= N + \left[C_r S^* - \alpha N\right]\mathbb{1}_{\{\tau\leq T\}} \\
&= N + C_r\left[S^* - \frac{\alpha N}{C_r}\right]\mathbb{1}_{\{\tau\leq T\}} \\
&= N + C_r\left[S^* - C_p\right]\mathbb{1}_{\{\tau\leq T\}} \qquad (3.12)
\end{aligned}$$

It may be inferred that the financial payoff of equation 3.12 consists of two components (Erismann, 2015): (1) the face value N of a zero bond and (2) a long position in C_r shares generating a payoff only if the CoCo materializes at time τ. This component can be approximated with a knock-in forward. The intuition behind equation 3.12 is that if the share price falls below a certain level S^*, an investor will use the face value N to exercise the knock-in forward. That said, the investor is committed to buy the amount of C_r shares for the price of C_p at maturity T.

Before maturity the present value of a Zero-Coupon CoCo V^{zcoco} can be determined by adding up the present value of a zero bond V^{zb} and the present value of a knock-in forward V_t^{kifwd}. Hereinafter, the components will be explained briefly.

$$V_t^{zcoco} = V_t^{zb} + V_t^{kifwd} \qquad (3.13)$$

with

$$V_t^{zb} = N\exp\left[-r(T-t)\right] \qquad (3.14)$$

Moreover, the long position in shares at time t can be approximated with the respective closed-form solution of a knock-in forward. (Hull, 2006)

$$\begin{aligned} V_t^{kifwd} = C_r \Bigg[& S_t \exp\left[-q\left(T-t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda} \Phi\left(y_1\right) \\ & - K \exp\left[-r\left(T-t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \Phi\left(y_1 - \sigma\sqrt{T-t}\right) \\ & - K \exp\left[-r\left(T-t\right)\right] \Phi\left(-x_1 - \sigma\sqrt{T-t}\right) \\ & + S_t \exp\left[-q\left(T-t\right)\right] \Phi\left(-x_1\right) \Bigg] \end{aligned} \tag{3.15}$$

with

$$\begin{aligned} C_r &= \frac{\alpha N}{C_p} \\ K &= C_p \\ \lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2} \\ x_1 &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \\ y_1 &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \end{aligned}$$

It is important, however, to recognize that a subtle difference exists between the actual economic payoff of equation 3.12 and its replication with a knock-in forward, since the knock-in forward replicates an economic ownership of shares at maturity T. Though, the triggering of a CoCo forces investors to accept the conversion immediately. This could lead to an economic ownership of shares at trigger time τ and, thus, prior to T. Therefore, one could argue that receiving a knock-in forward in the trigger event disregards the dividends which a shareholder would receive in particular when a CoCo triggers early. De Spiegeleer and Schoutens (2011b) argue that dividends can be neglected because distressed banks are likely to behave with great restraint when it comes to dividend payments.

3.2.2 Step Two - Adding Coupons

As mentioned earlier, the first step excludes coupon payments from the valuation. Yet, in this step we want to include them in order to replicate the exact payout profile of a CoCo. Therefore, we replace the zero bond in equation 3.13 with a straight bond with regular coupon payments c. Besides, a third component has to be added which takes into account the foregone coupon payments if the trigger is touched. This can be modeled with a short position in k binary down-and-in calls with maturity t_i. Those binary down-and-in calls are knocked in if the trigger S^* is met and hence, offset all future coupon payments.

The price of a straight bond can be determined with:

$$V_t^{sb} = \sum_{i=1}^{T} c_i \exp\left[-r(t_i - t)\right] + N \exp\left[-r(T - t)\right] \tag{3.16}$$

Furthermore, the formula of Rubinstein and Reiner (1991) can be used to price the down-and-in calls:

$$\begin{aligned} V_t^{bdic} = \alpha \sum_{i=1}^{k} c_i \exp\left(-rt_i\right) \Bigg[& \Phi\left(-x_{1i} + \sigma\sqrt{t_i}\right) \\ & + \left(\frac{S^*}{S_t}\right)^{2\lambda - 2} \Phi\left(y_{1i} - \sigma\sqrt{t_i}\right) \Bigg] \end{aligned} \tag{3.17}$$

with

$$\begin{aligned} x_{1i} &= \frac{\log\left(\dfrac{S_t}{S^*}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ y_{1i} &= \frac{\log\left(\dfrac{S^*}{S_t}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ \lambda &= \frac{r - q + \dfrac{\sigma^2}{2}}{\sigma^2} \end{aligned}$$

To sum up, the theoretical price of a CoCo V^{ed} at time t pursuant the equity derivative approach consists of three components: (1) a straight bond V^{sb}, (2) a knock-in-forward V^{kifwd} and (3) a set of binary down-and-in calls V^{bdic}:

$$V_t^{ed} = V_t^{sb} + V_t^{kifwd} - V_t^{bdic} \tag{3.18}$$

3.2.3 Parameter Classification and Adjustment

The equity derivative approach requires the same model inputs as the credit derivative approach. All parameters are outlined in table 3.3. All parameters are adjusted in the same way as already described in section 3.1.3.

	Description	**Usage**	**Source**
T	Maturity	Static input	Term sheet
N	Notional	Static input	Term sheet
c	Coupon rate	Static input	Term sheet
α	Conversion factor	Static input	Term sheet
S_0	Initial share price	Dynamic input	Market data
S^*	Share price	Fitting parameter	–
C_p	Conversion price	Static input	Term sheet
r	Risk-free interest rate	Dynamic input	Market data
q	Dividend yield	Dynamic input	Market data
σ_E	Share price volatility	Dynamic input	Market data

Table 3.3: Parameter classification of the equity derivative approach (Wilkens and Bethke, 2014)

3.2.4 Model Application

A fictive CoCo is priced based on the values shown in table 3.4. The source code for the equity derivative approach can be found in chapter A.2.

	Value	**Comment**
T	10yrs	Maturity
N	100%	Nominal
c	6.00%	Coupon rate
α	1	Conversion factor
S_0	120	Initial share price
S^*	60	Trigger share price
C_p	75	Nominal conversion price
r	3.00%	Risk-free interest rate
q	0.00%	Dividend yield
σ_E	30.00%	Share price volatility

Table 3.4: Parameter specification of the equity derivative approach application

The price of the CoCo can be separated into the individual components as presented in diagram 3.1. In that sense, the value of the risk-free coupon bond makes up a major portion of a CoCo's value under the equity derivative approach. The value of the straight bond V^{sb} is equal to 125.14. The short position in the set of binary down-and-in calls V^{bdic} is equivalent to -15.88. Furthermore, the value of the knock-in forward corresponds to -1.80.

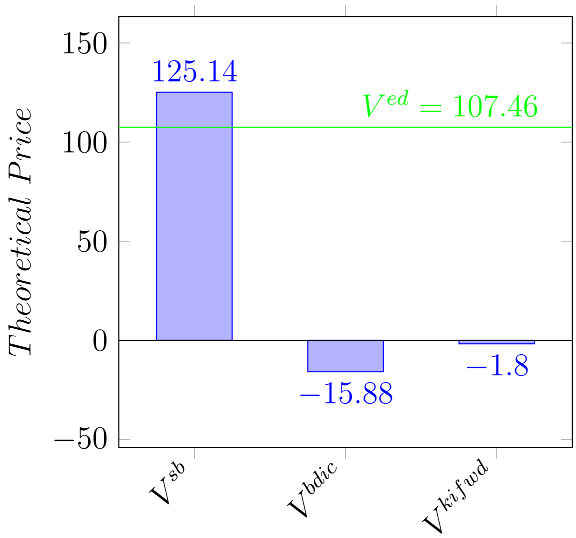

Figure 3.1: CoCo price V^{ed} and its components under the equity derivative approach

Thus, the price of the fictive CoCo V^{ed} under the equity derivative approach is equal to 107.46.

3.3 Structural Approach

A third alternative to price CoCos is the structural approach of Pennacchi (2010). The idea has its roots in the seminal work of Merton (1974), which aims to explain a company's default based on the relationship of its assets and liabilities under a standard Black-Scholes setting. Pennacchi (2010)'s approach expands the idea by modeling the stochastic evolution of a bank's balance sheet respectively of its components. In the following, the assets' rate of return process will be explained. Thereafter, we will outline the assumptions of the model regarding the various liabilities a bank issues to refinance itself including deposits, equity and coupon bonds in the form of CoCos. Lastly, a pricing formula will be illustrated.

3.3.1 Structural Banking Model

Bank Assets and Asset-To-Deposit Ratio

Pennacchi (2010) assumes that a bank holds a portfolio of loans, equities and off-balance sheet positions as assets whose returns follow a jump-diffusion process. The change of this portfolio A_t is determined by the rate of return and the cash in- respectively outflows. In this context, the symbol $*$ is used to point out the change in value of the portfolio which can be quantified by the rate of return, excluding net cashflows. The aforementioned instantaneous rate of return is denoted as dA_t^*/A_t^* and follows a stochastic process as stated below under the risk-neutral probability measure $\mathbb{Q}$:

$$\frac{dA_t^*}{A_t^*} = (r_t - \lambda_t k_t)\, dt + \sigma dz + \left(Y_{q_{t-}} - 1\right) dq_t \tag{3.19}$$

It should be noted that r_t stands for the risk-free interest rate as defined by the Cox et al. (1985) term-structure model which will be discussed shortly. dz is a Brownian motion, whereby σ denotes the volatility of returns of the aforementioned asset portfolio. q_t is a Poisson counting process which increases by one whenever a Poisson-distributed event respectively a jump occurs. Hence, the variable dq_t is one whenever such a jump takes place and zero otherwise. The risk-neutral probability that a jump happens is equal to $\lambda_t dt$ where λ_t stands for the intensity of the jump process. Variable $Y_{q_{t-}}$ is a i.i.d. random variable drawn from $\ln(Y_{q_{t-}}) \sim \Phi\left(\mu_y, \sigma_y^2\right)$ at time t where μ_y stands for the mean jump size and σ_y denotes the standard deviation of jumps. In case the random variable $Y_{q_{t-}}$ is greater than one, an upward shift in the bank's asset value can be observed. If the value is smaller than one a downward jump takes place. Given that the risk-neutral expected proportional jump k_t is defined as $k_t = E_t^{\mathbb{Q}}\left[Y_{q_{t-}} - 1\right]$, one can determine k_t with the following formula: $k_t = \exp(\mu_y + \frac{1}{2}\sigma_y^2) - 1$. Thus, the risk-neutral expected change in A^* from the jump element $(Y_{q_{t-}} - 1)dq_t$ equals $\lambda_t k_t dt$ in dt. To sum up, the value development of a bank's asset portfolio A_t^* follows largely a continuous process. But disruptive jumps may occur as illustrated below in the graph 3.2.

The risk-neutral process of bank assets A_t including the net cashflows is equal to the assets' rate of return less interest payments r_t respectively premium payments h_t to deposit holders proportionally to their deposits D_t. Furthermore, one has to subtract the coupon payments c_t to CoCo investors proportionally to the face value B.

$$dA_t = [(r_t - \lambda k)\, A_t - (r_t + h_t)\, D_t - c_t B]\, dt + \sigma A_t dz + \left(Y_{q_{t-}} - 1\right) A_t dq \tag{3.20}$$

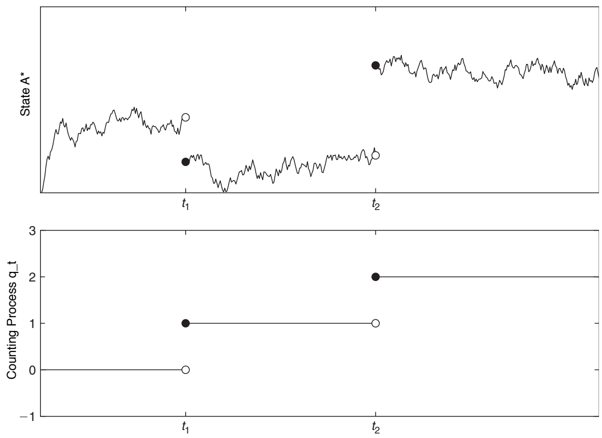

Figure 3.2: The first graph shows two jumps in the state variable A^* at discrete time points. Additionally, the corresponding Poisson counting process q_t is highlighted in the second graph. (Aït-Sahalia and Hansen, 2009)

By substituting variable x_t with A_t/D_t and anticipating the deposit growth process $g\left(x_t - \hat{x}\right)$ as pointed out by equation 3.31, the risk neutral process of the asset-to-deposit ratio equals:

$$\begin{aligned} \frac{dx_t}{x_t} &= \frac{dA_t}{A_t} - \frac{dD_t}{D_t} \\ &= \left[(r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g\left(x_t - \hat{x}\right)\right] dt + \sigma dz + \left(Y_{q_{t-}} - 1\right) dq_t \end{aligned} \tag{3.21}$$

with

$$b_t = \frac{B}{D_t} \tag{3.22}$$

Lastly, an application of Itô's lemma for jump-diffusion processes leads to the following formula for the asset-to-deposit ratio process:

$$\begin{aligned} d\ln\left(x_t\right) = &\left[(r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g\left(x_t - \hat{x}\right) - \frac{1}{2}\sigma^2\right] dt \\ &+ \sigma dz + \ln Y_{q_{t-}} dq_t \end{aligned} \tag{3.23}$$

Default-Free Term Structure

Pennacchi (2010) applies the term-structure specifications of Cox et al. (1985) to model the risk-neutral process of the instantaneous risk-free interest rate dr_t which is defined as follows:

$$dr_t = \kappa\left(\bar{r} - r_t\right) dt + \sigma_r \sqrt{r_t} d\zeta \tag{3.24}$$

Note that κ is the speed of convergence, $\bar{r}$ is the long-run equilibrium interest rate, r_t is the continuous short-term interest rate, σ_r is the instantaneous volatility and $d\zeta$ is a Brownian motion.

A zero bond can be priced using the Cox et al. (1985) specifications under the no-arbitrage assumption. This implies that the price of a risk-free zero bond at time t that pays the amount of 1 currency unit in $\tau = T - t$ is given by:

$$P(r_t, \tau) = A(\tau)\exp\left[-B(\tau) r_t\right] \tag{3.25}$$

with

$$A(\tau) = \left\{ \frac{2\theta \exp\left[(\theta + \kappa)\dfrac{\tau}{2}\right]}{(\theta + \kappa)\left[\exp(\theta\tau) - 1\right] + 2\theta} \right\}^{2\kappa\bar{r}/\sigma_r^2}$$

$$B(\tau) = \frac{2\left[\exp(\theta\tau) - 1\right]}{(\theta + \kappa)\left[\exp(\theta\tau) - 1\right] + 2\theta}$$

$$\theta = \sqrt{\kappa^2 + 2\sigma_r^2}$$

The cost of replication of a risk-free coupon bond that pays a continuous coupon of $c_r dt$ is equal to a set of zero bonds which can be priced with equation 3.25. Therefore, the fair coupon rate c_r of such a coupon bond at time t, which is issued at par, equals:

$$\begin{aligned} c_r &= \frac{1 - A(\tau)\exp\left[-B(\tau) r_t\right]}{\int_0^\tau A(s)\exp\left[-B(s) r_t\right] ds} \\ &\approx \frac{1 - A(\tau)\exp\left[-B(\tau) r_t\right]}{\sum_{i=1}^{i=n} A(\Delta t \times i)\exp\left[-B(\Delta t \times i) r_t\right]\Delta t} \end{aligned} \tag{3.26}$$

with

$$n = \frac{\tau}{\Delta t} \tag{3.27}$$

Deposits and Insurance Premium

Bank deposits are not riskless because depositors may suffer losses if a bank's asset value A_t is worth less than the deposits D_t. That said, one can assume that a bank is closed by the deposit insurer when the asset-to-deposit ratio x_t is less or equal to one. A bank might become distressed due to continuous downward movements in its asset value. Then, the bank will be shut down with $A_{t_b} = D_t$ and subsequently, depositors will not face any loss. However, depositors may experience severe losses when a downward jump in asset value happens at a discrete point in time, $\hat{t}$. It may be that the downward jump in asset value exceeds the bank's capital. If such a jump occurs the instantaneous proportional loss to deposits will equal $\left(D_t - Y_{q_{t^-}} A_{\hat{t^-}}\right)/D_t$.

The fair deposit insurance premium h_t for deposit holders can be derived with equation 3.28. The equation illustrates that h_t is closely related to the asset-to-deposit ratio x_t:

$$h_t = \lambda \left[\Phi\left(-d_1\right) - x_{t^-} \exp\left(\mu_y + \frac{1}{2}\sigma_y^2\right) \Phi\left(-d_2\right) \right] \tag{3.28}$$

with

$$d_1 = \frac{\ln\left(x_{t^-}\right) + \mu_y}{\sigma_y} \tag{3.29}$$

$$d_2 = d_1 + \sigma_y \tag{3.30}$$

The model assumes that a bank pays continuously a total interest and deposit premium of $\left(r_t + h_t\right) D_t dt$ to each depositor. Hence, one might recognize that the deposits change only because of comparatively higher deposit inflows than outflows. Empirical research of Adrian and Shin (2010) suggests that banks have a target capital ratio and that deposit growth is positively related to the current asset-to deposit ratio:

$$\frac{dD_t}{D_t} = g\left(x_t - \hat{x}\right) dt \tag{3.31}$$

$\hat{x} > 1$ is a bank's target asset-to-deposit ratio with g being a positive constant. Whenever the actual asset-to-deposit ratio is higher than its target, $x_t > \hat{x}$, a bank will shrink its balance sheet. Thus, the deposit growth rate $g\left(x_t - \hat{x}\right)$ in the time interval dt, leads to a mean-reverting tendency for the asset-to-deposit ratio x_t.

Equity and Conversion Threshold

As stated originally, the conversion of a CoCo at time t_c occurs when the asset-to-deposit ratio x_{t_c} meets the trigger level $\bar{x}_{t_c}$. The conversion threshold can also be expressed relative to the original equity-to-deposits ratio $\bar{e}$. This is favourable because the equity value is directly observable in the market whereas the asset value is not. The relationship between the equity threshold $\bar{e}$ and the asset-to-deposit threshold $\bar{x}_{t_c}$ can be summarized as follows:

$$\bar{e} = \frac{E_{t_c}}{D_{t_c}} = \frac{A_{t_c} - D_{t_c} - pB}{D_{t_c}} = \bar{x}_{t_c} - 1 - pb_{t_c} \tag{3.32}$$

Hence, it is possible to specify exactly the conversion trigger of a CoCo bond. This will be important for the valuation part.

CoCos

The valuation of a CoCo can be accomplished with a Monte Carlo simulation of both the asset and the deposit process. Along the asset-to-deposit ratio process, the CoCo pays coupons and the nominal at maturity unless the CoCo has not been triggered. If the trigger event occurs the conversion amount is paid out. (Wilkens and Bethke, 2014) The price of the CoCo V^{st} is equal to the risk-neutral expectation of the aforementioned cashflows as derived by Pennacchi (2010):

$$V_0^{st} = E_0^{\mathbb{Q}} \left[\int_0^T \exp\left(- \int_0^t r_s ds \right) v\left(t\right) dt \right] \tag{3.33}$$

Please note $v(t)$ stands for a coupon payment at date t which equals $c_t B$ as long as the CoCo has not been triggered. If the CoCo does not convert until maturity T, a final payout of B will be performed. However, if the CoCo triggers early at time t_c, there is the one-time cashflow of pB. Parameter p determines the maximum conversion amount of new equity per par value of contingent capital. Thereafter, $v(t)$ is zero.

3.3.2 Parameter Classification and Adjustment

The biggest challenge of the structural approach is the accurate estimation of its input parameters. A reliable estimation is not straightforward as most of the variables are not directly observable in the market. (De Spiegeleer et al., 2014) A complete overview of all input variables is presented in table 3.5.

In general, one can distinguish three parameter types. The first group comprises parameters which are directly observable in the market respectively in a CoCo's term sheet. The second category encompasses variables that are linked to a bank's balance sheet or strategy and are thus semi-observable. Finally, the third group covers parameters which are in fact not observable and are determined based on expert judgement or calibration to market data. (Wilkens and Bethke, 2014) Hereinafter, major input variables and their adjustment will be described.

	Description	Usage	Source
T	Maturity	Static input	Term sheet
B	Notional	Static input	Term sheet
c	Coupon rate	Static input	Term sheet
p	Conversion factor	Static input	
$\hookrightarrow S^*$	Trigger share price	Static input	Term sheet
$\hookrightarrow C_p$	Conversion price	Static input	Term sheet
x_t	Asset-to-deposit ratio	Dynamic input	
$\hookrightarrow S_t$	Share price	Dynamic input	Market data
$\hookrightarrow n_t$	Number of shares	Dynamic input	Market data
$\hookrightarrow D_t$	Deposit value	Dynamic input	Balance sheet
$\hat{x}$	Target asset-to-deposit ratio	Static input	
$\hookrightarrow A_{\text{Target}}$	Target asset value	Static input	Term sheet
$\hookrightarrow D_{\text{Target}}$	Target deposit value	Static input	Term sheet
g	Mean-reversion speed	Static input	Expert judgm.
σ_A	Annual asset return volatility	Dynamic input	
$\hookrightarrow D_t$	Deposit value	Dynamic input	Balance sheet
$\hookrightarrow S_t$	Share price	Dynamic input	Market data
$\hookrightarrow n_t$	Number of shares	Dynamic input	Market data
$\hookrightarrow \sigma_E$	Historic share price volatility	Static input	Market data
$\hookrightarrow r_t$	Risk-free interest rate	Dynamic input	Market data
λ	Jump intensity in asset return process	Static input	Expert judgm.
μ_y	Mean jump size in asset return process	Static input	
σ_y	Jump volatility in asset return process	Static input	
$\hookrightarrow S_{\text{past}}$	Historic share price data	Static input	Market data
r_t	Risk-free interest rate	Dynamic input	Market data
$\bar{r}$	Long-term risk-free interest rate	Static input	
σ_r	Interest rate volatility	Static input	
κ	Speed of convergence	Static input	
$\hookrightarrow r_{\text{past}}$	Set of historic risk-free interest rate data	Static input	Market data
ρ	Correlation between Brownian motion for asset returns and interest rate process	Static input	
$\hookrightarrow S_{\text{past}}$	Historic share price data	Static input	Market data
$\hookrightarrow r_{\text{past}}$	Historic risk-free interest rate data	Static input	Market data
$\bar{e}$	Conversion threshold of the market value of original shareholders' equity to deposit value	Static input	
$\hookrightarrow S^*$	Trigger share price	Static input	Term sheet
$\hookrightarrow n_0$	Initial number of shares	Static input	Term sheet
$\hookrightarrow D_0$	Initial deposit value	Static input	Term sheet
b_0	Ratio of the contingent capital's nominal to the initial value of deposits	Dynamic input	
$\hookrightarrow D_t$	Initial deposit value	Dynamic input	Balance sheet
$\hookrightarrow B$	Contingent capital nominal	Static input	Term sheet

Table 3.5: Parameter classification of the structural approach

Bank Assets and Asset-To-Deposit Ratio

The structural approach models the development of the asset-to-deposit ratio x_t over time. On this account, the paper attempts to obtain x_t with equity and deposit estimates. The equity component is equivalent to the daily observable market capitalization $S_t n_t$. Moreover, deposit values are inferred from a bank's quarterly published balance sheet data while assuming that all liabilities are deposits. The deposit level D_t is interpolated between the disclosure of financial statements. (Wilkens and Bethke, 2014) Hence one can determine the asset-to-deposit ratio with the following equation: $x_t = (S_t n_t + D_t)/D_t$. Furthermore, the target asset-to-deposit ratio $\hat{x}$ is driven by the strategy of the issuing bank and can be described as follows: $\hat{x} = A_{\text{Target}}/D_{\text{Target}}$.

The asset volatility σ_A is also an important input variable of the asset process but is not observable on a daily basis. Therefore, the paper evaluates the asset volatility based on its relation to the share price volatility σ_E as described by Merton (1974). The source code can be found in section C.1.2. Moreover, the structural model assumes that asset returns follow a jump-diffusion process. This implies that the probability of extreme asset returns is larger than predicted by normally distributed asset returns. To estimate the parameters that govern the distribution of the jump size, the paper assumes that historical share price returns are a reliable proxy for asset jumps. The mean jump size μ_y and the jump volatility σ_y are estimated based on threshold exceedance methods and assumptions on the jump intensity λ, which is in turn used to specify the number of exceedances over a threshold. (Longin and Solnik, 2001) The implementation is presented in section C.1.3.

Default-Free Term Structure

For the Cox et al. (1985) model pricing parameters are estimated with the approach of Remillard (2013b). The approach calibrates the long-term risk-free interest rate $\bar{r}$, the interest rate volatility σ_r and the speed of convergence κ. To do so, maximum likelihood techniques for dependent observations are applied. The approach takes a series of daily sovereign bond yields as input which have maturities of one, three, six and twelve months. The implementation of this approach can be found in section C.1.1. Additionally, the correlation between the geometric Brownian motion for asset returns and the interest rate process ρ is approximated with the five year daily correlation between a sovereign bond of the same tenure respectively denomination and a related stock market index.

Deposits and Insurance Premium

A reasonable mean-reversion speed g for deposits is assumed to be equal to 0.5. The assumption follows the assessment of Pennacchi (2010). He argues that this estimate for g gives a plausible deposit's half time of around 3 years. The deposit's half-time in turn describes the time it takes for the deposit value to move half the distance towards its target value.

Equity and Conversion Threshold

Similar to the credit and equity derivative approach the conversion factor p depends on the trigger share price S^* and the conversion price C_p: $p = S^*/C_p$. Moreover, the conversion threshold $\bar{e}$ of a CoCo also relies on the trigger share price S^* as outlined by $\bar{e} = E_{\text{Trigger}}/D_0 = nS^*/D_0$.

3.3.3 Model Application

The generic CoCo parameters serve to calculate a pricing example under the structural approach. However, given the model's complexity additional input variables are required as outlined in table 3.6. Running the monte-carlo simulation with these inputs yields a price V^{sa} of 108.65. The derived value is comparable to those of the credit derivative approach V^{cd} and the equity derivative approach V^{ed}.

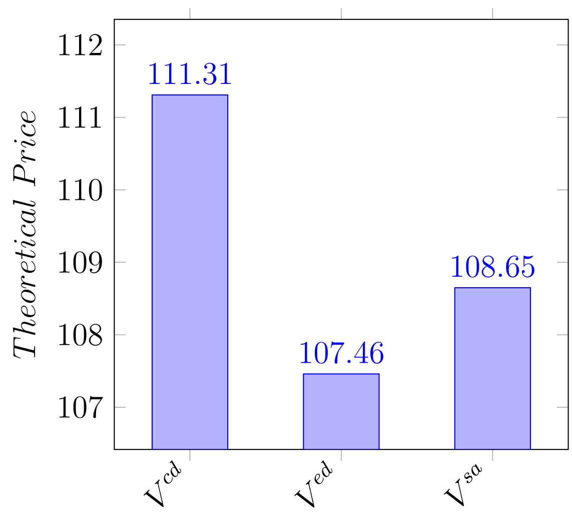

Figure 3.3: Comparison of pricing results

	Value	Comment
T	10yrs	Maturity
B	100.00%	Notional
c	6.00%	Coupon rate
p	0.8	Conversion factor
$\hookrightarrow S^*$	60	Trigger share price
$\hookrightarrow C_p$	75	Conversion price
x_0	1.1364	Initial asset-to-deposit ratio
$\hookrightarrow S_0$	120	Initial share price
$\hookrightarrow n$	1	Number of shares
$\hookrightarrow D_0$	880	Initial deposit value
$\hat{x}$	1.12	Target asset-to-deposit ratio
$\hookrightarrow A_{\text{Target}}$	1000	Target asset value
$\hookrightarrow D_{\text{Target}}$	892.86	Target deposit value
g	10	Mean-reversion speed
σ_A	3.63%	Asset volatility
$\hookrightarrow D_0$	880	Initial deposit value
$\hookrightarrow n$	1	Number of shares
$\hookrightarrow S_0$	120	Initial share price
$\hookrightarrow \sigma_E$	30%	Historic share price volatility
$\hookrightarrow r_0$	3.00%	Initial risk-free interest rate
λ	2	Jump intensity in asset return process
μ_y	0	Mean jump size in asset return process
σ_y	2.00%	Jump volatility in asset return process
$\hookrightarrow S_{\text{past}}$		Historic share price data
r_0	3.00%	Risk-free interest rate
$\bar{r}$	6.00%	Long-term risk-free interest rate
σ_r	5.00%	Interest rate volatility
κ	4.00%	Speed of convergence
$\hookrightarrow r_{\text{past}}$		Set of historic risk-free interest rate data
ρ	50%	Correlation between Brownian motion for asset returns and interest rate process
$\hookrightarrow S_{\text{past}}$		Historic share price data
$\hookrightarrow r_{\text{past}}$		Historic risk-free interest rate data
$\bar{e}$	6.81%	Conversion threshold of the market value of shareholders' equity to original deposit value
$\hookrightarrow S^*$	60	Trigger share price
$\hookrightarrow n$	1	Number of shares
$\hookrightarrow D_0$	880	Initial deposit value
b_0	3.41%	Ratio of contingent capital's nominal to the initial deposit value
$\hookrightarrow D_0$	880	Initial deposit value
$\hookrightarrow CC$	30	Contingent capital value

Table 3.6: Parameter specification of structural approach application

Chapter 4

Sensitivity Analyses

In the following sections the price sensitivity with respect to certain input variables of all three valuation approaches will be examined. Sensitivity analyses are especially useful to quantify the impact a variable has on the actual pricing result if it varies from what was originally assumed. This is particularly interesting as all three pricing approaches rest on different theoretical concepts. Therefore, the paper outlines a set of scenarios. In order to ensure comparability, all analyses are based on the same aforementioned fictive CoCo example.

4.1 Credit Derivative Approach

For the credit derivative approach we investigate a set of scenarios with regard to the underlying share price S, the share price volatility σ_E, a CoCo's maturity T, the risk-free interest rate r, the trigger share price S^* and the conversion price C_p. The same scenarios are also analyzed when considering the equity derivative approach because of the congruence of input parameters. Similar results are found for these two approaches.

Varying share prices S and share price volatilities σ_E have a decisive impact on CoCo prices. The effect of both variables is presented in figure 4.1. The diagram shows that higher share price levels lead to higher CoCo prices. This can be justified with the fact that it becomes less likely that the share price S falls below a predetermined trigger share price S^*. Hence, the probability that investors have to face losses due to an equity conversion is lower. The compensation for that risk respectively the conversion spreads s_{CoCo} decrease. With surging share price volatilities σ_E CoCo investors demand higher yields to compensate for rising conversion probabilities. For that reason, the conversion spread s_{CoCo} increases which in turn leads to lower CoCo

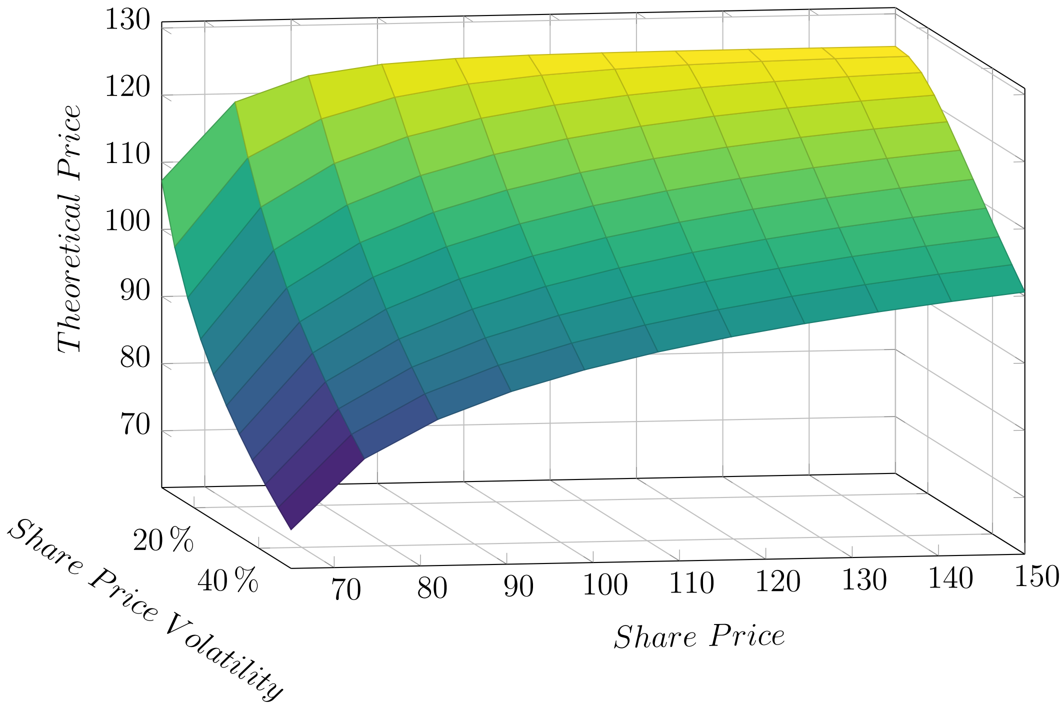

Figure 4.1: CoCo price pursuant to the credit derivative approach as function of share price S and share price volatility σ_E

prices. However, the influence of this effect diminishes with rising share prices S as it becomes generally less likely that the share price falls below the predefined trigger share price S^*.

Furthermore, figure 4.2 shows the reaction of the CoCo price due to changes in maturity T and varying levels of the risk-free interest rate r. One can observe that an increase of the risk-free interest rate leads to lower CoCo prices. By contrast, for rising maturities one can generally observe higher CoCo prices, except for the combination of high interest rates and a maturity of ten years. In this scenario the risk-free interest rate effect outweighs the maturity effect. In addition, when analyzing the development of the conversion spread s_{CoCo} one can also see that the conversion spread is at its maximum for a maturity of ten years. However, the influence of the parameter T on the value of a CoCo increases significantly with lower interest rate levels. The highest CoCo price can be found for the combination of low interest rates and high maturities. This can be explained with compounding effects and comparatively higher discount factors in particular for the notional. The effect reduces in significance with decreasing interest rates.

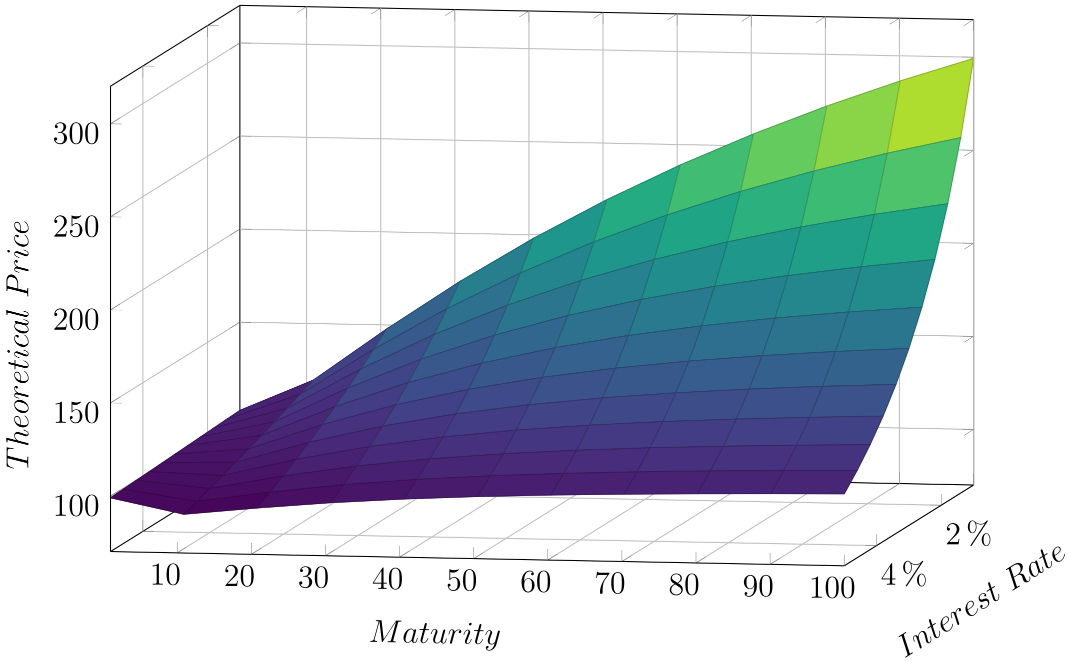

Figure 4.2: CoCo price pursuant to the credit derivative approach as function of maturity T and risk-free interest rate r

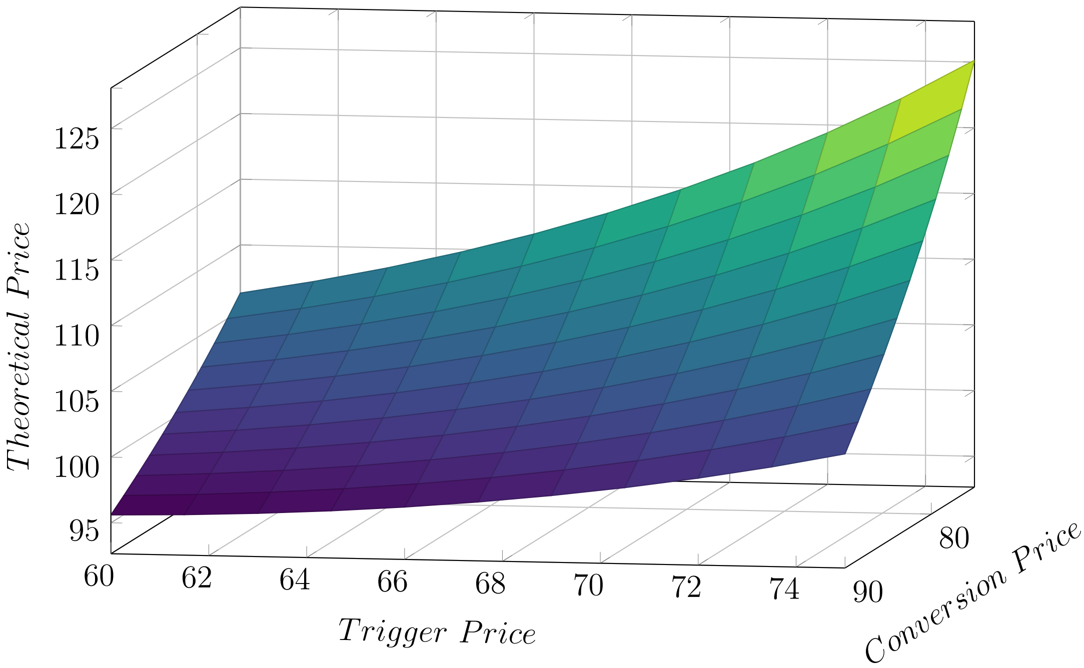

Figure 4.3: CoCo price pursuant to the credit derivative approach as function of trigger price S^* and conversion price C_p

Figure 4.3 illustrates the sensitivity of the CoCo price with respect to the trigger price S^* and the conversion price C_p. The graph reveals that lower conversion prices result in higher CoCo prices while keeping the trigger price constant. This is due to the fact that the recovery rate R_{CoCo} rises. The opposite effect is visible for low trigger prices while holding the conversion price constant. A low trigger price implies that the likelihood of conversion is lower due to a higher distance between the actual share price S and the trigger price S^*.

4.2 Equity Derivative Approach

For equity derivative approach we conduct the same sensitivity analyses much like for the credit derivative approach. Again the aim is to determine how the CoCo price is affected by changes in the model inputs.

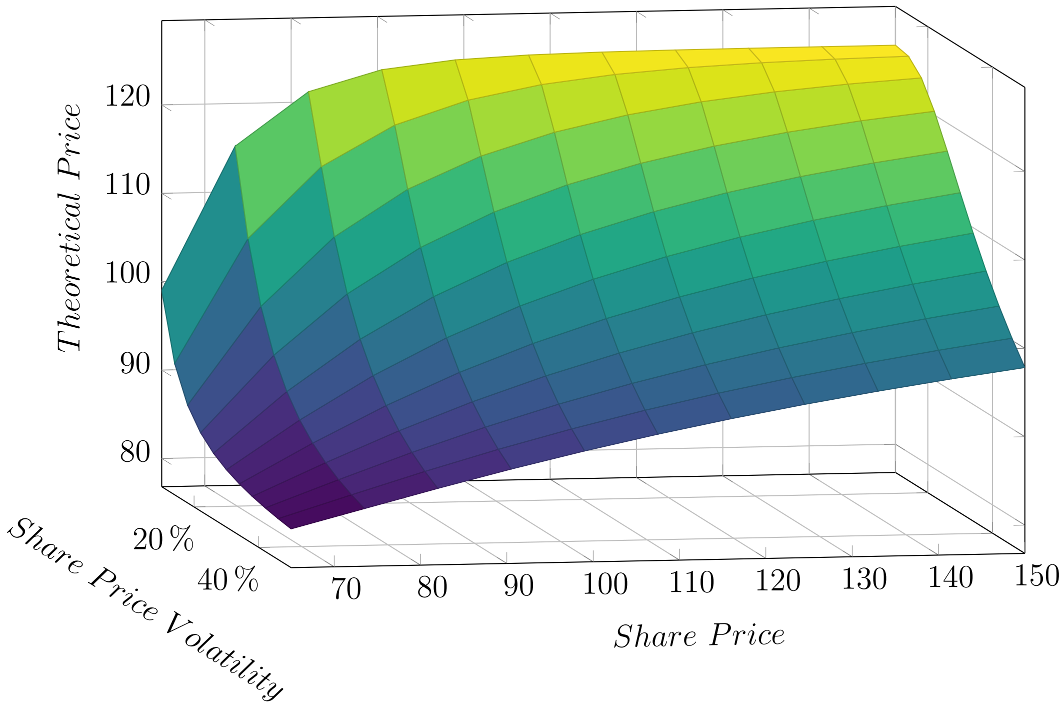

Figure 4.4: CoCo price pursuant to the equity derivative approach as function of share price S and share price volatility σ_E

Figure 4.4 helps to understand the price dynamics with respect to the share price S and the share price volatility σ_E. Generally, one might argue that with increasing share price S the distance to the trigger price S^* grows, which in turn reduces the conversion probability of the CoCo. This has a positive impact on the price. The

CoCo becomes more similar to a straight bond. Besides, the line of thought for changes of the share price volaitlity σ_E is comparable. With a rising share price volatility σ_E the risk increases that the underlying share price S hits the barrier S^* and that the CoCo investor faces a loss. Moreover, this effect can be explained with vega respectively the falling values of the short position in several down-and-in calls and the put option of the synthetic forward. Both are used under the equity derivative approach to replicate the payoff of a CoCo.

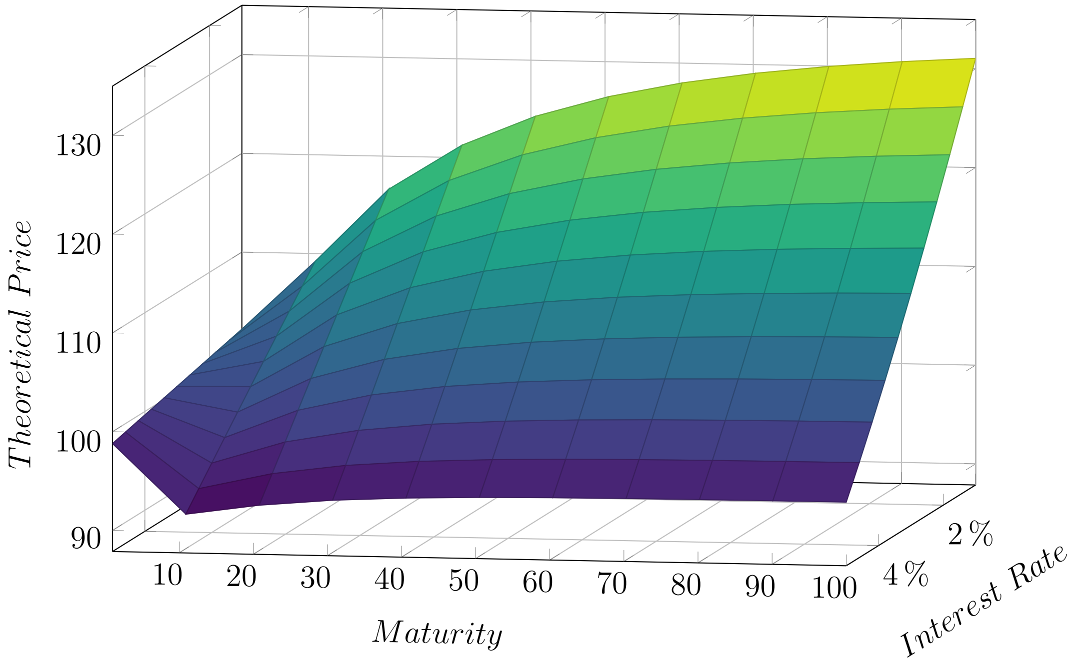

Figure 4.5: CoCo price pursuant to the credit derivative approach as function of maturity T and risk-free interest rate r

Figure 4.5 illustrates the price sensitivity of a CoCo with respect to its maturity T and the interest rate r. Considering a straight bond as major component of a CoCo helps to understand the shown price dynamics. One can observe an inverse relationship of the CoCo price and the risk-free interest rate. In addition, the price sensitivity of the CoCo with respect to the interest rate rises with its maturity. Though, the increase occurs at a decreasing rate except for a maturity smaller than ten years.

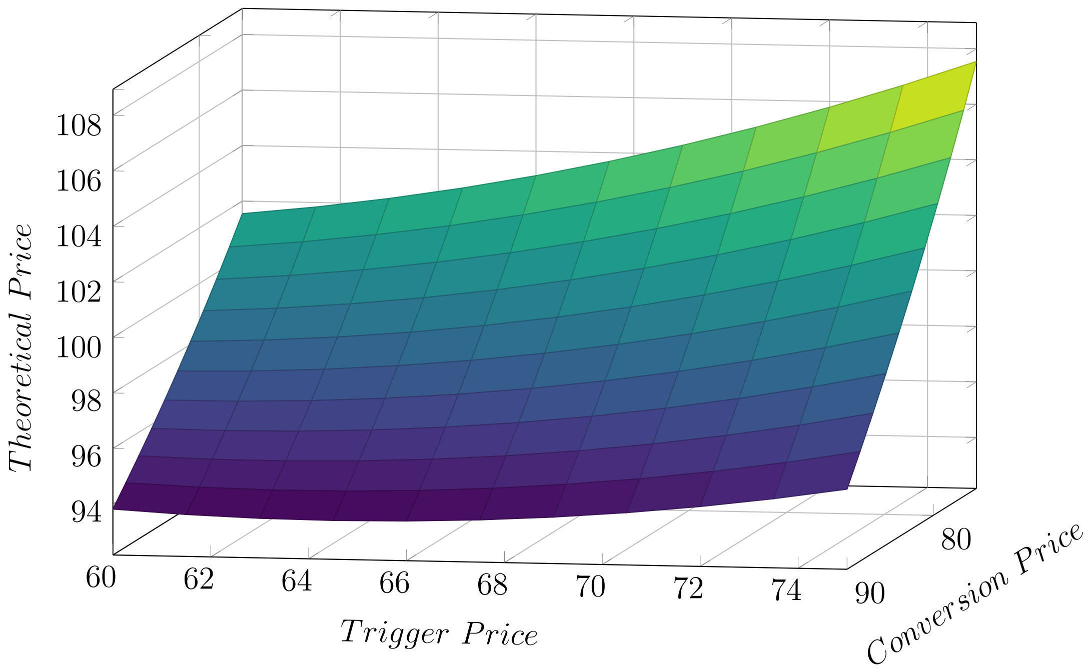

Figure 4.6: CoCo price pursuant to the credit derivative approach as function of trigger price S^* and conversion price C_p

Figure 4.6 illustrates the price sensitivity of a CoCo concerning the conversion price C_p and the trigger price S^*. In order to understand the dynamics in detail, it is advantageous to take a look at a CoCo's components under the equity derivative approach. The first component namely the long position in a straight bond is not affected by either of the variables. However, the value of the short position in a set of down-and-in calls is determined by the trigger price S^*. Moreover, the trigger price S^* and the conversion price C_p influence the price of the long position in a knock-in forward which consists of a long position in a call and a short position in a put both with strike C_p. These two options come into existence if the trigger price S^* is met. One might argue for a given trigger price S^*, the lower the conversion price C_p is the farer the knock-in forward is in the money. This in turn might be associated with a higher CoCo price. Though, these two options come only into existence if the trigger price S^* is met. Hence, the lower S^* is the lower is the probability that the knock-in forward comes into existence and the lower is the value of the position. Having said that, one has also to consider that the lower the trigger price S^* is the higher is the value of the short position in a set of binary down-and-in calls as it becomes more likely that the underlying share price S fails to remain above the trigger price S^*. These are two opposite forces, whereupon the impact of both change with decreasing conversion prices as the influence of the knock-in forward effect becomes dominant.

4.3 Structural Approach

The structural approach requires different model inputs in comparison to the other two approaches. Therefore, the focus of the following sensitivity analyses is concentrated on the initial asset-to-deposit ratio x_0, the asset volatility σ_A, maturity T, the risk-free interest rate r, the equity-to-deposit threshold $\bar{e}$, the jump intensity λ and the contingent capital's nominal to the initial value of deposits b_0.[1]

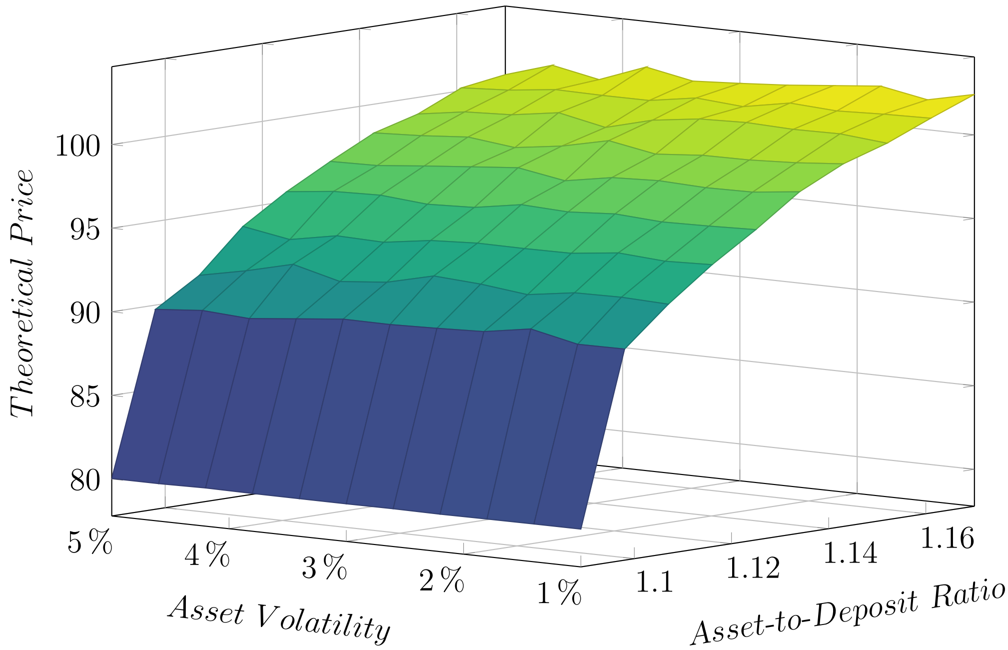

Figure 4.7: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio x_0 and asset volatility σ_A

- figure 4.7

[1]The Monte-Carlo simulation which is used to determine the prices runs in the Amazon Elastic Compute Cloud (EC2) as the service provides a re-sizable compute capacity which is key to quickly scale the computing requirements. If one wants to replicate the simulations it is recommended to follow the instructions of Shekel (2015) to set up a Rstudio server on Amazon EC2.

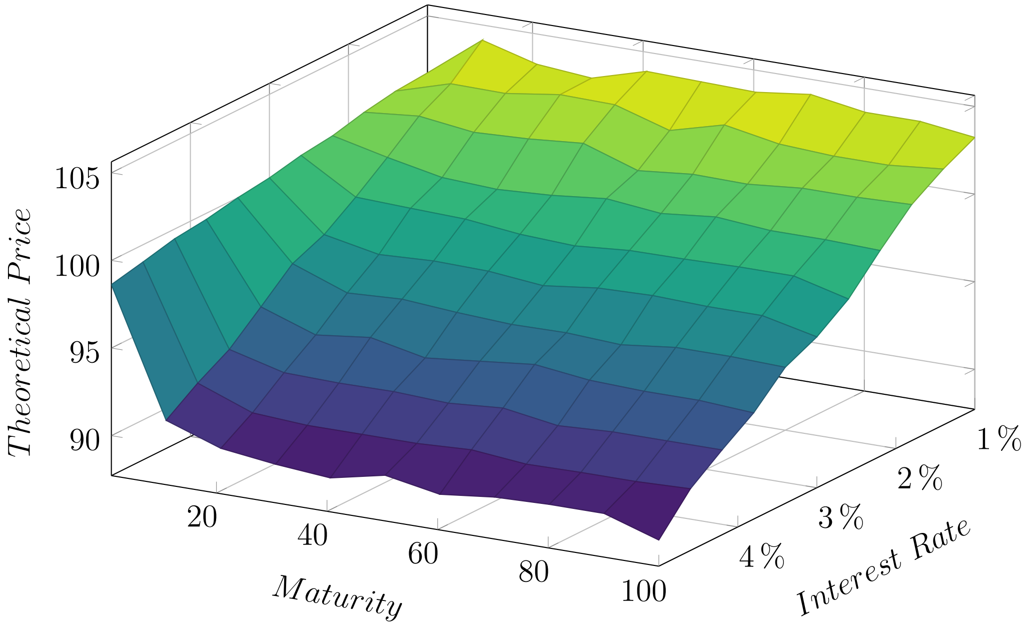

Figure 4.8: CoCo price pursuant to the structural approach as function of maturity T and interest rate r

- figure 4.8

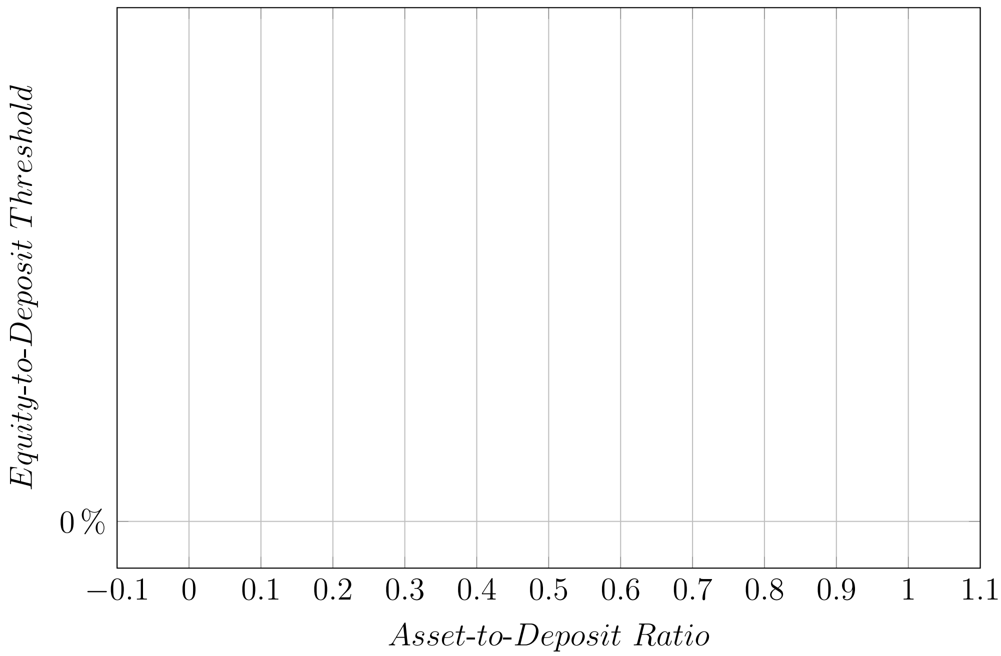

Figure 4.9: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio x_t and equity-to-deposit threshold $\bar{e}$

- figure 4.9

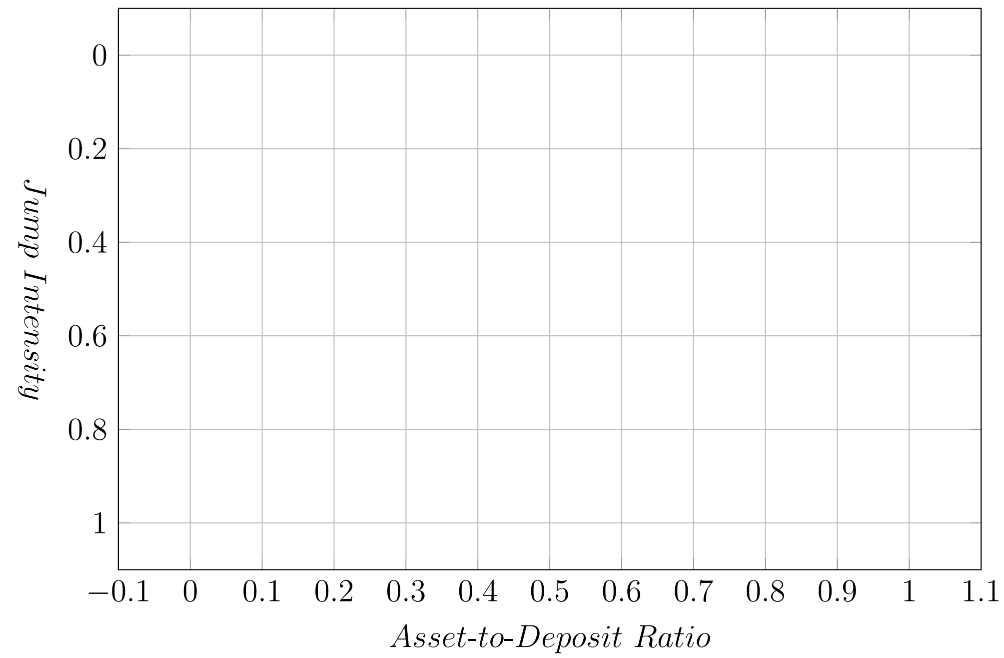

Figure 4.10: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio x_0 and jump intensity λ

- figure 4.10

Figure 4.11: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio x_0 and initial ratio of contingent capital's nominal to the initial value of deposits b_0

- figure 4.11

Chapter 5

Case Study

5.1 Data Description

5.1.1 HSBC

5.2 Parametrization

5.3 Comparison

5.3.1 Qualitative Analysis

5.3.2 Quantitative Analysis

Chapter 6

Conclusion

Appendix A

Code - Models

A.1 Credit Derivative Approach

The following source code is an implementation of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
# Price of Contingent Convertible Bond
price_coco_cd <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma){

  spread_coco <- calc_spread_coco(t, T, S_t, S_star, C_p, r, q, sigma)
  V_t_coco <- N * exp(-(r + spread_coco) * (T - t))

  for (time in (t+1):T){
    V_t_coco <- V_t_coco + c_i * exp(-(r + spread_coco) * time)
  }
  return(V_t_coco)
}

# Calculation of Trigger Probability
calc_p_star <- function(t, T, S_t, S_star, r, q, sigma){
  p_star <- pnorm((log(S_star / S_t) - calc_mu(r, q, sigma) * (T - t)) / (sigma * sqrt(T - t))) + (S_star / S_t)^(2 * calc_mu(r, q, sigma) / sigma^2) * pnorm((log(S_star / S_t) + calc_mu(r, q, sigma) * (T - t)) / (sigma * sqrt(T - t)))
  return(p_star)
}

# Calculation of Drift of Underlying
calc_mu <- function(r, q, sigma){
  mu <- r - q - sigma^2 / 2
  return(mu)
}

# Spread of CoCo Bond
calc_spread_coco <- function(t, T, S_t, S_star, C_p, r, q, sigma){
  spread_coco <- - log(1 - calc_p_star(t, T, S_t, S_star, r, q, sigma)) / (T - t) * (1 - S_star / C_p)
  return(spread_coco)
```

```
}

# Pricing Example
#price_coco_cd(t <- 0, T <- 10, S_t <- 120, S_star <- 60, C_p <- 75, c_i
    <- 6.00, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3)
```

A.2 Equity Derivative Approach

The following source code is an implementation of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
# Price of Contingent Convertible Bond
price_coco_ed <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma,
    alpha){
  V_t_ed <- price_cb(t, T, c_i, r, N) - price_dibi(t, T, S_t, S_star, c_
    i, r, q, sigma, alpha) + price_difwd(t, T, S_t, S_star, C_p, r, N, q
    , sigma, alpha)

  return(V_t_ed)
}

# Price of Corporate Bond
price_cb <- function(t, T, c_i, r, N){
  V_t_cb <- N * exp(-r * (T - t))

  for (time in (t+1):T){
  V_t_cb <- V_t_cb + c_i * exp(-r * time)
  }

  return(V_t_cb)
}

# Price of Binary Option
price_dibi <- function(t, T, S_t, S_star, c_i, r, q, sigma, alpha){
  V_t_dibi <- 0

  i <- t
  k <- T

  for (i in (t+1):k) {
  V_t_dibi <- V_t_dibi + c_i * exp(- r * i) * (pnorm(- calc_x_1_i(S_t, S
    _star, sigma, r, q, i) + sigma * sqrt(i)) + (S_star / S_t)^(2 * calc
    _lambda(r, q, sigma) - 2) * pnorm ( calc_y_1_i(S_t, S_star, sigma, r
    , q, i) - sigma * sqrt(i)))
  }

  V_t_dibi <- alpha * V_t_dibi

  return(V_t_dibi)
}

# Price of Down-And-In Forward
```

```r
price_difwd <- function(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha){
  V_t_difwd <- calc_conversion_rate(C_p, N, alpha) * (S_t * exp(- q * (T - t)) * (S_star / S_t) ^ (2 * calc_lambda(r, q, sigma)) * pnorm(calc_y_1(t, T, S_t, S_star, r, q, sigma)) - C_p * exp(- r * (T - t)) * (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm(calc_y_1(t, T, S_t, S_star, r, q, sigma) - sigma * sqrt(T - t)) - C_p * exp(- r * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma) + sigma * sqrt(T - t)) + S_t * exp(- q * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma)))

  return(V_t_difwd)
}

# Calculation of Conversion Rate
calc_conversion_rate <- function(C_p, N, alpha){
  C_r <- alpha * N / C_p

  return(C_r)
}

# Calculation of additional Parameters
calc_x_1_i <- function(S_t, S_star, sigma, r, q, t_i){
  x_1_i <- log(S_t / S_star) / (sigma * sqrt(t_i)) + calc_lambda(r, q, sigma) * sigma * sqrt(t_i)

  return(x_1_i)
}

calc_y_1_i <- function(S_t, S_star, sigma, r, q, t_i){
  y_1_i <- log(S_star / S_t) / (sigma * sqrt(t_i)) + calc_lambda(r, q, sigma) * sigma * sqrt(t_i)

  return(y_1_i)
}

calc_lambda <- function(r, q, sigma){
  lambda <- (r - q + sigma^2 / 2) / sigma^2

  return(lambda)
}

calc_x_1 <- function(t, T, S_t, S_star, r, q, sigma){
  x_1 <- log(S_t / S_star) / (sigma * sqrt(T - t)) + calc_lambda(r, q, sigma) * sigma * sqrt(T - t)

  return(x_1)
}

calc_y_1 <- function(t, T, S_t, S_star, r, q, sigma){
  y_1 <- log(S_star / S_t) / (sigma * sqrt(T - t)) + calc_lambda(r, q, sigma) * sigma * sqrt(T - t)

  return(y_1)
}
```

```
# Pricing Example
#price_coco_ed(t <- 0, T <- 10, S_t <- 120, S_star <- 60, C_p <- 75, c_i
    <- 6.00, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
```

A.3 Structural Approach

The following source code is an implementation of the Structural Approach (Pennacchi, 2010) with support from Quantnet (2011) in translating the source code of Pennacchi (2010) from GAUSS to R.

```
# Price of Contingent Convertible Bond
price_coco_sa <- function(T , nsimulations , rho , kappa , r_bar, r0,
    sigma_r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0,
     B, coupon){

  ndays <- T * 250
  dt <- T / ndays

  # Get Brownian motions
  result <- sim_corrProcess(T, nsimulations, rho, ndays, dt)
  dz_1 <- result$dz_1
  dz_2corr <- result$dz_2corr

  # Simulate Cox et al. (1985) term-structure process
  r <- sim_interestrate(kappa, r_bar, r0, sigma_r, dz_2corr, ndays,
    nsimulations, dt)

  # Simulate price of contingent convertible bond with a Monte-Carlo
    simulation
  V_t_sa <- get_price(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y,
    sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0, B, coupon) *
    100

  return(V_t_sa)
}

# Create correlated Brownian motions for asset and interest rate process
sim_corrProcess <- function(T, nsimulations, rho, ndays, dt){

  # Compute the Choleski factorization of a real symmetric positive-
    definite square matrix.
  chol_RHO <- t(chol(matrix(c(1, rho, rho, 1), nrow = 2)))

  # Random generation for the normal distribution with mean equal to 0
    and standard deviation equal to 1
  dz_1 <- matrix(1, ndays, nsimulations)
  dz_2 <- matrix(1, ndays, nsimulations)
  for(j in 1:nsimulations)
  {
    dz_1[ , j] <- rnorm(ndays) * sqrt(dt)
```

```
    dz_2[ , j] <- rnorm(ndays) * sqrt(dt)
  }

  # Create correlated Brownian motions using Cholesky-decomposition for the Cox et al. (1985) term-structure process
  dz_2corr <- matrix(1, ndays, nsimulations)
  for(j in 1:nsimulations)
  {
    for(i in 1:ndays)
    {
      dz_2corr[i, j] <- dz_1[i, j] * chol_RHO[2, 1] + dz_2[i, j] * chol_RHO[2, 2]
    }
  }

  return(list("dz_1" = dz_1, "dz_2corr" = dz_2corr))
}

# Simulate Cox et al. (1985) term-structure process
sim_interestrate <- function(kappa, r_bar, r0, sigma_r, dz_2corr, ndays, nsimulations, dt){
  r <- matrix(r0, ndays + 1, nsimulations)

  for(j in 1:nsimulations)
  {
    for(i in 1:ndays)
    {
      r[i + 1, j] <- r[i, j] + kappa * (r_bar - r[i, j]) * dt + sigma_r * sqrt(abs(r[i, j])) * dz_2corr[i, j]
    }
  }

  return(r)
}

get_price <- function(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0, B, coupon){

  # Define parametres
  phi <- matrix(rbinom( ndays %*% nsimulations, 1, dt * lambda), ndays, nsimulations)

  ln_Y <- matrix(rnorm(ndays %*% nsimulations, mu_Y, sigma_Y), ndays, nsimulations)

  # Ratio of contingent capital's nominal to the value of deposits
  b <- matrix(b0, ndays + 1, nsimulations)

  h <- matrix(1, ndays, nsimulations)

  # Paramter for jump diffusion process
  k <- exp(mu_Y + 0.5 * sigma_Y^2) - 1

  # Target asset-to-deposit ratio
```

```
x_bar0 <- 1 + e_bar + p * b0
x_bar <- matrix(x_bar0, ndays + 1, nsimulations)

# Asset-to-deposit ratio
x <- matrix(x0, ndays + 1, nsimulations)
ln_x0 <- matrix(log(x0), ndays + 1, nsimulations)
ln_x <- ln_x0

trigger_dummy <- matrix(1, ndays + 1, nsimulations)

# Simulate asset-to-deposit ratio and trigger events

for(j in 1:nsimulations)
{
  for(i in 1:ndays)
  {
    d_1 <- (ln_x[i, j] + mu_Y) / sigma_Y
    d_2 <- d_1 + sigma_Y

    h[i, j] <- lambda * (pnorm( - d_1) - exp(ln_x[i, j]) * exp(mu_Y +
  0.5 * sigma_Y^2) * pnorm(-d_2))

    b[i + 1, j] <- b[i, j] * exp(- g * (exp(ln_x[i, j]) - x_hat) * dt)

    ln_x[i + 1, j] <- ln_x[i, j] + ( (r[i, j] - lambda * k) - (r[i, j]
   + h[i, j] + coupon * b[i, j]) / exp(ln_x[i, j]) - g * (exp(ln_x[i,
  j]) - x_hat) - 0.5 * sigma_A^2) * dt + sigma_A * sqrt(dt) * dz_1[i,
  j] + ln_Y[i,j] * phi[i, j]

    x[i + 1, j] <- exp(ln_x[i + 1, j])

    x_bar[i + 1, j] <- 1 + e_bar + p * b[i + 1, j]

    if(is.na(trigger_dummy[i, j]) == TRUE){
      trigger_dummy[i, j] <- trigger_dummy[i-1, j]
    }

    if(x[i + 1, j] >= x_bar[i + 1, j] && trigger_dummy[i, j] > 0.5)
    {
      trigger_dummy[i + 1, j] <- 1
    }else
    {
      trigger_dummy[i + 1, j] <- 0
    }
  }
}

cashflows <- matrix(c(rep(coupon * dt, ndays - 1), B), ndays,
  nsimulations) * trigger_dummy[1:ndays, ]

# Determine cashflows for each simulation
for(j in 1:nsimulations){
  for(i in 2:ndays){
```

```r
      if(cashflows[i, j] == 0 && p * b[sum(trigger_dummy[ , j]) + 1, j] <= x[sum(trigger_dummy[ , j]) + 1, j] - 1 ){
        cashflows[i, j] <- p * B
        break
      }
      else if(cashflows[i, j] == 0 && 0 < x[sum(trigger_dummy[ , j]) + 1, j] - 1 && x[sum(trigger_dummy[ , j]) + 1, j] - 1 < p * b[sum(trigger_dummy[ , j]) + 1, j]){
        cashflows[i, j] <- (x[sum(trigger_dummy[ , j]) + 1, j] - 1) * B / b[sum(trigger_dummy[ , j]) + 1, j]
        break
      }
      else{
        cashflows[i, j] <- cashflows[i, j]
      }
    }
  }
  list_discounted_cashflows <- rep(0, nsimulations)

  # Discount cashflows for each simulation
  for(j in 1:nsimulations)
  {
    disc_cashflows <- 0
    int_r <- 0

    for(i in 1:ndays)
    {
      int_r <- int_r + r[i, j] * dt
      disc_cashflows <- disc_cashflows + exp(- int_r) * cashflows[i, j]
    }
    list_discounted_cashflows[j] <- disc_cashflows
  }

  # Calculate arithmetic average over all simulations as present value of contingent convertibles bond
  V_t_sa <- mean(list_discounted_cashflows)

  return(V_t_sa)
}

# Pricing Example
# price_coco_sa(T <- 10, nsimulations <- 1, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.1494, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_A <- 0.0367, x0 <- 1.1364, B <- 1, coupon <- 0.06)
```

Appendix B

Code - Sensitivity Analyses

B.1 Credit Derivative Approach

The following source code is an implementation of the sensitivity analysis of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```
source('CreditDerivativeApproach.R')

# CoCo price V^cd as function of share price S and volatility sigma
createData_CD_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- S_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- S_increment, S_star <- 60, C_p <- 75, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- sigma_increment)
      data[counter, 3] <- sigma_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_S_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^cd as function of maturity T and risk-free interest rate r
createData_CD_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
```

```r
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- T_increment, S_t <- 100, S_star <- 60, C_p <- 75, c_i <- 6, r <- r_increment, N <- 100, q <- 0.00, sigma <- 0.3)
      data[counter, 3] <- r_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_T_r_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^cd as function of trigger price S^* and conversion price C_p
createData_CD_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S_star_max-S_star_min)/10)))
  {
    for(C_p_increment in seq(from=C_p_min, to=C_p_max, by=((C_p_max-C_p_min)/10)))
    {
      data[counter, 1] <- S_star_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- 100, S_star <- S_star_increment, C_p <- C_p_increment, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3)
      data[counter, 3] <- C_p_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_Sstar_Cp_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

createData_CD_S_sigma(65.01, 150, 0.1, 0.5)
createData_CD_T_r(1, 100, 0.01, 0.05)
createData_CD_Sstar_Cp(60, 75, 75, 90)
```

B.2 Equity Derivative Approach

The following source code is an implementation of the sensitivity analysis of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2011b) written in R.

```r
source('EquityDerivativeApproach.R')

# CoCo price V^ed as function of share price S and volatility sigma
createData_ED_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))
```

```
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- S_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- 10, S_t <- S_increment, S_star <- 60, C_p <- 75, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- sigma_increment, alpha <- 1)
      data[counter, 3] <- sigma_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_S_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^ed as function of maturity T and risk-free interest rate r
createData_ED_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- T_increment, S_t <- 100, S_star <- 60, C_p <- 75, c_i <- 6, r <- r_increment, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
      data[counter, 3] <- r_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_T_r_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^ed as function of trigger price S^* and conversion price C_p
createData_ED_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S_star_max-S_star_min)/10)))
  {
    for(C_p_increment in seq(from=C_p_min, to=C_p_max, by=((C_p_max-C_p_min)/10)))
    {
      data[counter, 1] <- S_star_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- 10, S_t <- 100, S_star <- S_star_increment, C_p <- C_p_increment, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
      data[counter, 3] <- C_p_increment
```

```
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_Sstar_Cp_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

createData_ED_S_sigma(65.01, 150, 0.1, 0.5)
createData_ED_T_r(1, 100, 0.01, 0.05)
createData_ED_Sstar_Cp(60, 75, 75, 90)
```

B.3 Structural Approach

The following source code is an implementation of the sensitivity analysis of the Structural Approach (Pennacchi, 2010) written in R.

```
source('StructuralApproach.R')

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and volatility sigma
createData_SA_x0_sigma <- function(x0_min, x0_max, sigma_min, sigma_max) {
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_x <- sigma_increment, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- sigma_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_x0_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^sa as function of maturity T and risk-free interest rate r
```

```
createData_SA_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_sa(T <- T_increment, nsimulations
   <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- r_increment
   , sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <-
   0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_x
    <- 0.0363, x0 <- 1.1364, B <- 1, coupon <- 0.06)
      data[counter, 3] <- r_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_T_r_31Aug2016.txt", row.names
   = FALSE, quote=FALSE)
}

# CoCo price V^sa as function of initial asset-to-deposit ratio x_0 and
    equity-to-deposit threshold bar_e
createData_SA_x0_ebar <- function(x0_min, x0_max, ebar_min, ebar_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/
   10)))
  {
    for(ebar_increment in seq(from=ebar_min, to=ebar_max, by=((ebar_max-
   ebar_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000,
   rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <-
   0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <-
    1.12, b0 <- 0.0341, p <- 0.8, e_bar <- ebar_increment, sigma_x <-
   0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- ebar_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
```

```
  write.table(data, file = "createData_SA_x0_ebar_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and jump intensity in asset return process lambda
createData_SA_x0_lambda <- function(x0_min, x0_max, lambda_min, lambda_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(lambda_increment in seq(from=lambda_min, to=lambda_max, by=((lambda_max-lambda_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- lambda_increment, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_x <- 0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- lambda_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_x0_lambda_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and initial ratio of contingent capital to deposits b0
createData_SA_x0_b0 <- function(x0_min, x0_max, b0_min, b0_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(b0_increment in seq(from=b0_min, to=b0_max, by=((b0_max-b0_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- b0_increment, p <- 0.8, e_bar <- 0.0681, sigma_x <- 0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- b0_increment
```

```
      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_x0_b0_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

createData_SA_x0_sigma(1.08, 1.17, 0.01, 0.05)
createData_SA_T_r(1, 100, 0.01, 0.05)
createData_SA_x0_ebar(1.08, 1.17, 0.01, 0.07)
createData_SA_x0_lambda(1.08, 1.17, 0, 2)
createData_SA_x0_b0(1.08, 1.17, 0.01, 0.06)
```

Appendix C

Code - Case Study

C.1 Parametrization - Structural Approach

```
source('estimateCIRParameter.R')
source('estimateMertonParameter.R')
source('estimateJumpParameter.R')

calibrate_CIR <- function(cir_data){
  cir_parameters <- estimate_CIR_parameters(cir.data)

  kappa <- cir_parameters$kappa
  r_bar <- cir_parameters$r_bar
  sigma_r <- cir_parameters$sigma_r

  return(cir_parameters)
}

calibrate_Merton <- function(deposits, marketcap, r, volatility_equity){
  merton_parameters <- estimate_Merton_parameters(deposits, marketcap, r
    , volatility_equity)

  asset_volatility <- merton_parameters$asset_volatility
  asset_value <- merton_parameters$asset_value

  return(merton_parameters)
}

calibrate_Jump <- function(returns, jump_Intesity){
  jump_parameters <- estimate_Jump_parameters(returns, jump_Intensity)

  mean_jump <- jump_parameters$mean_jump
  sd_jump <- jump_parameters$sd_jump

  return(jump_parameters)
}

# CIR
```

```
# Input data: [R,tau] (n x 2), with R: annual bonds yields in percentage
    and tau: maturities in years
data(data.cir)
cir_data <- data.cir

calibrate_CIR(cir_data)

# Merton
deposits <- matrix(1:1010, ncol = 1)
marketcap <- matrix(2:1011, ncol = 1)
r <- c(matrix(0.01, nrow = 253), matrix(0.02, nrow = 252), matrix(0.03,
    nrow = 254), matrix(0.04, nrow = 251))
volatility_equity <- matrix(0.1, nrow = 1010)

estimate_Merton_parameters(deposits, marketcap, r, volatility_equity)

# Jumps
returns <- as.timeSeries(data("bmw"))$SS.1
estimate_Jump_parameters(returns, 2)
```

C.1.1 Cox et al. (1985) Model

The following source code is an implementation of the method as described by Remillard (2013b). The method is used to calibrate the Cox et al. (1985) model which is used for the structural approach pursuant to Pennacchi (2010) based on historical data. The software is an adaption of the source code as provided by Remillard (2013a).

```
require(SMFI5)

estimate_CIR_parameters <- function(data, method = 'Hessian', days = 360
    , significanceLevel = 0.95){
  # Estimation of parameters of Cox-Ingersoll-Ross 1985 model
  R   <- data[,1]
  tau <- data[,2]
  h <- 1 / days

  # Estimation of starting parameters corresponding to those of a Feller
    process
  phi0 <- acf(R, 1, plot = FALSE)
  kappa0 <- - log(phi0[1]$acf) / h
  r_bar0  <- mean(R)
  sigma_r0 <- sd(R) * sqrt(2 * kappa0 / r_bar0)

  theta0 <- c(log(kappa0), log(r_bar0), log(sigma_r0), 0, 0)

  # Maximization of the log-Likelihood
  n <- length(R)
  optim.results <- optim(theta0, function(x) sum(LogLikCIR(x, R, tau,
    days, n)), hessian = TRUE)
  theta <- optim.results$par
  kappa <- exp(theta[1])
```

```
  r_bar  <- exp(theta[2])
  sigma_r <- exp(theta[3])

  return(list("kappa" = kappa, "r_bar" = r_bar, "sigma_r" = sigma_r))
}
```

C.1.2 Merton (1974) Model

```
require(nleqslv)

estimate_Merton_parameters <- function(deposits, marketcap, r,
    volatility_equity){
  # Estimation of asset value and asset volatility pursuant to Merton
    1974 model
  data <- new.env()

  for(i in 1:nrow(marketcap)){
    fnewton <- function(x){
      values <- numeric(2)
      d1 <- (log(x[1]/deposits[i])+(r[i]+x[2]^2/2))/x[2]
      d2 <- d1-x[2]

      values[1] <- marketcap[i]- (x[1]*pnorm(d1) - exp(-r[i])*deposits[i
    ]*pnorm(d2))
      values[2]<-volatility_equity[i]*marketcap[i]-pnorm(d1)*x[2]*x[1]
      return(values)
    }
    xstart <- c(marketcap[i]+deposits[i], volatility_equity[i])
    data$asset_new_value[i]<- nleqslv(xstart, fnewton, method="Newton")$
    x[1]
    data$asset_new_volat[i]<-nleqslv(xstart, fnewton, method="Newton")$x
    [2]
  }
  return(list("asset_volatility" = data$asset_new_volat, "asset_value" =
     data$asset_new_value))
}
```

C.1.3 Jump-Diffusion Process

```
library(fExtremes)

estimate_Jump_parameters <- function(returns, jump_Intensity){
  # Estimation of mean jump size and standard deviation of jumps

  positivThreshold <- findThreshold(returns, n = jump_Intensity / 2)
  negativThreshold <- - findThreshold(-returns, n = jump_Intensity / 2)

  jumps <- rbind(positivThreshold, negativThreshold)
  mean_jump <- mean(jumps)
  sd_jump <- sd(jumps)

  return(list("mean_jump" = mean_jump, "sd_jump" = sd_jump))
}
```

References

Tobias Adrian and Hyun Song Shin. Liquidity and leverage. *Journal of Financial Intermediation*, 19(3):418–437, 2010.

Yacine Aït-Sahalia and Lars Peter Hansen. *Handbook of Financial Econometrics, Vol 1: Tools and Techniques*, volume 1. Elsevier, 2009.

Boris Albul, Dwight M Jaffee, and Alexei Tchistyi. Contingent convertible bonds and capital structure decisions. *Coleman Fung Risk Management Research Center*, 2010.

Boris Albul, Dwight M Jaffee, and Alexei Tchistyi. Contingent convertible bonds and capital structure decisions. *Available at SSRN 2772612*, 2015.

Per Alvemar and Philip Ericson. Modelling and pricing contingent convertibles. *University of Gothenburg*, 2012.

Stefan Avdjiev, Anastasia V Kartasheva, and Bilyana Bogdanova. Cocos: a primer. *Available at SSRN 2326334*, 2013.

Stefan Avdjiev, Patrick Bolton, Wei Jiang, Anastasia Kartasheva, and Bilyana Bogdanova. Coco bond issuance and bank funding costs. *BIS and Columbia University working paper*, 2015.

Christina Bannier. Definition of risk shifting, 2011. URL http://lexicon.ft.com/Term?term=risk-shifting.

Barclays. Usd 3,000,000,000 7.625 per cent. contingent capital notes due november 2022 barclays bank plc, 2010. URL https://www.home.barclays/content/dam/barclayspublic/docs/InvestorRelations/esma/capital-securities-documentation/tier-2-securities/contingent-tier-2/7625-Contingent-Capital-Notes-due-November-2022-Prospectus-PDF-992KB.pdf.

Emilio Barucci and Luca Del Viva. Countercyclical contingent capital. *Journal of Banking and Finance*, 36(6):1688–1709, 2012.

Basel Committee on Banking Supervision. Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. *Bank for International Settlements*, 2010a.

Basel Committee on Banking Supervision. Basel committee issues final elements of the reforms to raise the quality of regulatory capital. *Bank for International Settlements*, 2010b.

Markus PH Buergi. Pricing contingent convertibles: a general framework for application in practice. *Financial Markets and Portfolio Management*, 27(1):31–63, 2013.

Charles W Calomiris and Richard J Herring. How to design a contingent convertible debt requirement that helps solve our too-big-to-fail problem. *Journal of Applied Corporate Finance*, 25(2):39–62, 2013.

Patrick Cheridito and Zhikai Xu. Pricing and hedging cocos. *Available at SSRN 2201364*, 2015.

José Manuel Corcuera, Jan De Spiegeleer, Albert Ferreiro-Castilla, Andreas E Kyprianou, Dilip B Madan, and Wim Schoutens. Pricing of contingent convertibles under smile conform models. *The Journal of Credit Risk*, 9(3):121, 2013.

José Manuel Corcuera, Jan De Spiegeleer, José Fajardo, Henrik Jönsson, Wim Schoutens, and Arturo Valdivia. Close form pricing formulas for coupon cancellable cocos. *Journal of Banking & Finance*, 42:339–351, 2014.

John C Cox, Jonathan E Ingersoll Jr, and Stephen A Ross. A theory of the term structure of interest rates. *Econometrica: Journal of the Econometric Society*, pages 385–407, 1985.

Credit Suisse. Usd 2,000,000,000 7.875 per cent. tier 2 buffer capital notes due 2041, 2011. URL https://www.credit-suisse.com/media/assets/corporate/docs/about-us/investor-relations/regulatory-disclosures/t2-xs0595225318.pdf.

Jan De Spiegeleer and Wim Schoutens. *The handbook of convertible bonds: pricing, strategies and risk management*, volume 581. John Wiley & Sons, 2011a.

Jan De Spiegeleer and Wim Schoutens. Pricing contingent convertibles: a derivatives approach. *Available at SSRN 1795092*, 2011b.

Jan De Spiegeleer, Wim Schoutens, and Cynthia Van Hulle. *The Handbook of Hybrid Securities: convertible bonds, coco bonds and bail-in.* John Wiley & Sons, 2014.

Darrell Duffie and Kenneth J Singleton. Modeling term structures of defaultable bonds. *Review of Financial studies*, 12(4):687–720, 1999.

Darrell Duffie and Kenneth J Singleton. *Credit risk pricing, measurement, and management.* Princeton University Press, 2003.

Marc Erismann. *Analytical propositions to evaluate contingent convertible capital.* PhD thesis, Master's thesis, University of St. Gallen, 2011.

Marc Erismann. *Pricing Contingent Convertible Capital-A Theoretical and Empirical Analysis of Selected Pricing Models.* PhD thesis, University of St. Gallen, 2015.

European Parliament. Contingent convertible securities, is a storm brewing?, 2016. URL http://www.europarl.europa.eu/RegData/etudes/BRIE/2016/582011/EPRS_BRI(2016)582011_EN.pdf.

Paul Glasserman and Behzad Nouri. Contingent capital with a capital-ratio trigger. *Management Science*, 58(10):1816–1833, 2012.

Andrew G Haldane. Capital discipline. In *American Economic Association Meeting*, volume 9, 2011.

Roberto Henriques and Saul Doctor. Making cocos work: Structural and pricing considerations for contingent capital securities. *European Credit Research, JP Morgan, February*, 15, 2011.

Jens Hilscher and Alon Raviv. Bank stability and market discipline: The effect of contingent capital on risk taking and fault probability. *Journal of Corporate Finance*, 29:542–560, 2014.

John C Hull. *Options, futures, and other derivatives.* Pearson Education, 2006.

Intesa Sanpaolo. Eur 1,250,000,000 7 per cent additional tier 1 notes, 2011. URL http://www.group.intesasanpaolo.com/scriptIsir0/si09/contentData/view/20160120_AT1_Euro_Prospectus.pdf?id=CNT-05-00000003D473C&ct=application/pdf.

Robert A Jarrow and Stuart M Turnbull. Pricing derivatives on financial securities subject to credit risk. *The Journal of Finance*, 50(1):53–85, 1995.

David Lando. *Credit risk modeling: theory and applications*. Princeton University Press, 2009.

Hayne E Leland. Corporate debt value, bond covenants, and optimal capital structure. *The Journal of Finance*, 49(4):1213–1252, 1994.

Lloyds. Enhanced capital notes, 2009. URL http://www.lloydsbankinggroup.com/globalassets/documents/investors/2009/non-us_eom.pdf.

Francois Longin and Bruno Solnik. Extreme correlation of international equity markets. *The journal of finance*, 56(2):649–676, 2001.

Dilip B Madan and Wim Schoutens. Conic coconuts: the pricing of contingent capital notes using conic finance. *Mathematics and Financial Economics*, 4(2):87–106, 2011.

Robert L McDonald. Contingent capital with a dual price trigger. *Journal of Financial Stability*, 9(2):230–241, 2013.

Robert C Merton. On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29(2):449–470, 1974.

Ceyla Pazarbasioglu, Ms Jian-Ping Zhou, Vanessa Le Lesle, and Michael Moore. *Contingent capital: economic rationale and design features*. International Monetary Fund, 2011.

George Pennacchi. A structural model of contingent bank capital. Working Paper 1004, Federal Reserve Bank of Cleveland, 2010. URL https://ideas.repec.org/p/fip/fedcwp/1004.html.

George Pennacchi and Alexei Tchistyi. A reexamination of contingent convertibles with stock price triggers. *Available at SSRN 2773335*, 2015.

Quantnet. Translate gauss code to r code, 2011. URL www.quantnet.com/threads/translate-gauss-code-to-r-code.7784/.

Rabobank. Rabobank nederland cooperatieve centrale raiffeisen-boerenleenbank b.a. eur 1,250,000,000 6.875 per cent. senior contingent notes due 2020, 2010. URL https://www.afm.nl/registers/emissies_documents/4220.pdf.

Bruno Remillard. Estimation of the parameters of the cir model, 2013a. URL https://github.com/cran/SMFI5/blob/master/R/est.cir.R.

Bruno Remillard. *Statistical Methods for Financial Engineering*. CRC Press, 2013b.

Mark Rubinstein and Eric Reiner. Unscrambling the binary code. *Risk Magazine*, 4 (9):20, 1991.

Wolfgang Schmidt. Credit risk, default models and credit derivatives. Frankfurt School of Finance and Management, 2015.

Andrei Serjantov. On practical pricing hybrid capital securities. In *Global Derivative Trading ad Risk Management Meeting*, 2011.

Liad Shekel. Rstudio server on amazon ec2, 2015. URL www.r-israel.com/wp-content/uploads/2015/06/Rstudio-in-AWS-16_9.pdf.

Squam Lake Working Group. An expedited resolution mechanism for distressed financial firms: Regulatory hybrid securities. *Council on Foreign Relations*, 10, 2009.

Lujing Su and Marc Olivier Rieger. How likely is it to hit a barrier? theoretical and emperical estimates. Technical report, Technical Report Working Paper, 2009.

Suresh Sundaresan and Zhenyu Wang. On the design of contingent capital with a market trigger. *The Journal of Finance*, 70(2):881–920, 2015.

Henrik Teneberg. Pricing contingent convertibles using equity derivatives jump diffusion approach. *Master thesis*, 2015.

Colin Turfus and Alexander Shubert. Analytic pricing of coco bonds. *Deutsche Bank and J.P. Morgan working paper*, 2015.

Southern District of New York United States Bankruptcy Court. Report of anton r. valukas, examiner. *Lehman Brothers Holdings Inc., et al., Debtors*, 11, 2010.

George M Von Furstenberg et al. *Contingent capital to strengthen the private safety net for financial institutions: CoCos to the rescue?* Dt. Bundesbank, Press and Public Relations Division, 2011.

Sascha Wilkens and Nastja Bethke. Contingent convertible (coco) bonds: A first empirical assessment of selected pricing models. *Financial Analysts Journal*, 70(2): 59–77, 2014.

Meta Zähres. Contingent convertibles: Bank bonds take on a new look. *Deutsche Bank Research, Financial Market Special, EU Monitor*, 79, 2011.

Zurich Cantonal Bank. Nachrangige tier-1-anleihe chf 590 mio., 2013. URL https://www.zkb.ch/media/dok/corporate/investor-relations/produktinformationsblatt-nachrangige-tier1-anleihe.pdf.