Valuation of Contingent Convertibles with Derivatives

Nicolay Schmitt

Frankfurt School of Finance and Management

A thesis submitted for the degree of

Master of Science in Finance

August 2016

This thesis is dedicated to my parents for their love and support.

Thank you!

Abstract

Financial crises have led to higher regulatory standards on the capital adequacy of banks. They are required to hold more capital with loss absorbance capacities on their balance sheet. In conjunction with this development, contingent convertible bonds (CoCos) have become an attractive instrument for banks to seek new capital. The defining characteristic of CoCos is the automatic conversion into common equity when a predetermined trigger is met. Loss absorbing capital is created, which instantly improves the capital structure of the distressed bank. However, the pricing of these hybrid instruments remains opaque. In this context, the thesis scrutinizes the valuation of CoCos. Three major approaches are examined: the structural approach in accordance to Pennacchi (2010), the credit derivatives approach and the equity derivatives approach both pursuant to De Spiegeleer and Schoutens (2011b). The application covers sensitivity analysis to further understand the dynamics of the different methodologies. Based on a case study the viability of those approaches is evaluated. Software is provided for further replication.

Contents

List of Figures

Chapter 1

Introduction and Motivation

1.1   Introduction

1.2   Literature Overview

1.3   Motivation

1.4   Methodology

Chapter 2

Structure of CoCos

2.1 Description of CoCos

2.2 Payoff and Risk Profile
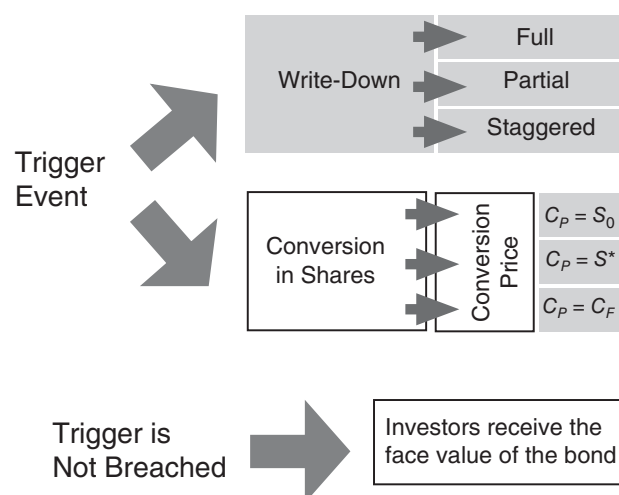
Figure 2.1: Anatomy of Cocos (De Spiegeleer and Schoutens, 2011a)

asdasd

2.3 Conversion Trigger

2.3.1 Market Trigger

2.3.2 Accounting Trigger

2.3.3 Regulatory Trigger

2.3.4 Multivariate Trigger

2.4 Conversion Details

2.4.1 Conversion Fraction

- conversion fraction α

- face value N

- conversion amount $N \times \alpha$

- amount remaining in case of partial equity conversion $N \times (1 - \alpha)$

2.4.2 Conversion Price and Ratio

- conversion rate C_r

- conversion price C_p

- recovery rate R_{CoCo}

- stock price at trigger event S_T^*

- loss attributable to CoCo holders L_{CoCo}

$$C_p = \frac{\alpha N}{C_r} \tag{2.1}$$

$$C_r = \frac{\alpha N}{C_p} \tag{2.2}$$

$$R_{CoCo} = \frac{S_T^*}{C_p} \tag{2.3}$$

$$L_{CoCo} = N - (1 - R_{CoCo}) = N\left(1 - \frac{S_T^*}{C_p}\right) \tag{2.4}$$

$$P_T = \begin{cases} (1 - \alpha)N + C_r S_T^* & \text{if converted} \\ N & \text{if not converted} \end{cases} \tag{2.5}$$

Chapter 3

Theory of Pricing

3.1 Credit Derivative Approach

3.1.1 Intensity-based Approach

$$p^* = 1 - \exp\left(-\lambda_{Trigger} \times T\right) \tag{3.1}$$

$$s_{CoCo} = (1 - R_{CoCo}) \times \lambda_{Trigger} = Loss_{CoCo} \times \lambda_{Trigger} \tag{3.2}$$

$$Loss_{CoCo} = N - C_r \times S^* = N\left(1 - \frac{S^*}{C_P}\right) \tag{3.3}$$

$$R_{CoCo} = \frac{S^*}{C_p} \tag{3.4}$$

$$p^* = \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) - \mu T}{\sigma\sqrt{T}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) + \mu T}{\sigma\sqrt{T}}\right) \tag{3.5}$$

$$\lambda_{Trigger} = -\frac{\log\left(1 - p^*\right)}{T} \tag{3.6}$$

$$s_{CoCo} = -\frac{\log\left(1 - p^*\right)}{T} \times \left(1 - \frac{S^*}{C_p}\right) \tag{3.7}$$

3.1.2 Application to CoCos

3.1.3 Data Requirements and Calibration

3.1.4 Pricing Example

3.2 Equity Derivative Approach

Sources: Erismann (2015), De Spiegeleer and Schoutens (2011b)

$$
\begin{aligned}
P_T &= \mathbb{1}_{\{\tau > T\}} N + \left[(1 - \alpha) N + \frac{\alpha N}{C_p S^*} \right] \mathbb{1}_{\{\tau \le T\}} \\
&= N + \left[\frac{\alpha N}{C_p} S^* - \alpha N \right] \mathbb{1}_{\{\tau \le T\}} \\
&= N + \left[C_r S^* - \alpha N \right] \mathbb{1}_{\{\tau \le T\}} \\
&= N + C_r \left[S^* - \frac{\alpha N}{C_r} \right] \mathbb{1}_{\{\tau \le T\}} \\
&= N + C_r \left[S^* - C_p \right] \mathbb{1}_{\{\tau \le T\}}
\end{aligned}
$$

$$
V_t^{ed} = V_t^{cb} - V_{t_i}^{dibi} + V_t^{difwd} \tag{3.8}
$$

3.2.1 Corporate Bonds

$$
V_t^{cb} = \sum_{i=t}^{T} c_i \exp\left(-rt_i\right) + N \exp\left[-r\left(T - t\right)\right] \tag{3.9}
$$

3.2.2 Binary Options

$$
\begin{aligned}
V_t^{dibi}\left(c_i, S^*, t\right) = \alpha \sum_{i=1}^{k} c_i \exp\left(-rt_i\right) & \left[\Phi\left(-x_{1i} + \sigma\sqrt{t_i}\right) \right. \\
& \left. + \left(\frac{S^*}{S_t}\right)^{2\lambda - 2} \Phi\left(y_{1i} - \sigma\sqrt{t_i}\right) \right]
\end{aligned} \tag{3.10}
$$

with

$$x_{1i} = \frac{\log\left(\dfrac{S_t}{S^*}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i}$$

$$y_{1i} = \frac{\log\left(\dfrac{S^*}{S_t}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i}$$

$$\lambda = \frac{r - q + \dfrac{\sigma^2}{2}}{\sigma^2}$$

3.2.3 Down-And-In Forward

$$\max\left(S_T - K\right) \ \text{if} \ \min_{0 \le t \le T}\left(S_T\right) \le S^* \tag{3.11}$$

$$\max\left(K - S_T\right) \ \text{if} \ \min_{0 \le t \le T}\left(S_T\right) \le S^* \tag{3.12}$$

$$V_t^{dic}\left(S_t, S^*, K\right) = S_t \exp\left[-q\left(T - t\right)\right]\left(\frac{S^*}{S_t}\right)^{2\lambda}\Phi\left(y\right)$$
$$- K \exp\left[-r\left(T - t\right)\right]\left(\frac{S^*}{S_t}\right)^{2\lambda - 2}\Phi\left(y - \sigma\sqrt{T - t}\right) \tag{3.13}$$

with

$$K = C_p$$

$$y = \frac{\log\left(\dfrac{S^{*2}}{S_t K}\right)}{\sigma\sqrt{T - t}} + \lambda\sigma\sqrt{T - t}$$

$$\lambda = \frac{r - q + \dfrac{\sigma^2}{2}}{\sigma^2}$$

$$V_t^{dip}\left(S_t, S^*, K\right) = S_t \exp\left[-q\left(T - t\right)\right]\left(\frac{S^*}{S_t}\right)^{2\lambda}\left[\Phi\left(y\right) - \Phi\left(y_1\right)\right]$$
$$- K \exp\left[-r\left(T - t\right)\right]\left(\frac{S^*}{S_t}\right)^{2\lambda - 2}\left[\Phi\left(y - \sigma\sqrt{T - t}\right) - \Phi\left(y_1 - \sigma\sqrt{T}\right)\right]$$
$$+ K \exp\left[-r\left(T - t\right)\right]\Phi\left(x_1 + \sigma\sqrt{T - t}\right)$$
$$- S_t \exp\left[-q\left(T - t\right)\right]\Phi\left(-x_1\right) \tag{3.14}$$

with

$$x_1 = \frac{\log\left(\dfrac{S_t}{S^*}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t}$$

$$y_1 = \frac{\log\left(\dfrac{S^*}{S_t}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t}$$

$$\min\left(S_t\right) \leq S^* : P_T = S_T - K = \max\left(S_T - K\right) - \max\left(K - S_T\right) \tag{3.15}$$

$$\min\left(S_t\right) > S^* : P_T = 0 \tag{3.16}$$

$$
\begin{aligned}
V_t^{difwd} = C_r \Bigg[& S_t \exp\left[-q\left(T-t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda} \Phi\left(y_1\right) \\
& - K \exp\left[-r\left(T-t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \Phi\left(y_1 - \sigma\sqrt{T-t}\right) \\
& - K \exp\left[-r\left(T-t\right)\right] \Phi\left(-x_1 - \sigma\sqrt{T-t}\right) \\
& + S_t \exp\left[-q\left(T-t\right)\right] \Phi\left(-x_1\right) \Bigg]
\end{aligned}
\tag{3.17}
$$

with

$$C_r = \frac{\alpha N}{C_p} \tag{3.18}$$

3.2.4 Data Requirements and Calibration

3.2.5 Pricing Example

3.3 Structural Approach

3.3.1 Synthetic Balance Sheet

3.3.2 Data Requirements and Calibration

3.3.3 Pricing Example

Chapter 4

Dynamics and Sensitivity Analysis

4.1   Credit Derivative Approach

4.2   Equity Derivative Approach

4.3   Structural Approach

Chapter 5

Empirical Analysis and Model Comparison

5.1 Data Description

5.1.1 Deutsche Bank

5.2 Model Parametrization

5.3 Model Comparison

5.3.1 Qualitative Analysis

5.3.2 Quantitative Analysis

Chapter 6

Conclusion

Appendix A

Sample Title

Appendix B

Code - Credit Derivative Approach

The following source code is an implementation of the Credit Derivative Approach written in R.

```r
# Price of Contingent Convertible Bond
price_coco_cd <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma){

  spread_coco <- calc_spread_coco(t, T, S_t, S_star, C_p, r, q, sigma)
  V_t_coco <- N * exp(-(r + spread_coco) * (T - t))

  for (t in 1:T){
    V_t_coco <- V_t_coco + c_i * exp(-(r + spread_coco) * t)
  }
  V_t_coco
}

# Calculation of Trigger Probability
calc_p_star <- function(t, T, S_t, S_star, r, q, sigma){
  p_star <- pnorm((log(S_star / S_t) - calc_mu(r, q, sigma) * (T - t)) /
      (sigma * sqrt(T - t))) + (S_star / S_t)^(2 * calc_mu(r, q, sigma) /
      sigma^2) * pnorm((log(S_star / S_t) + calc_mu(r, q, sigma) * (T - t
      )) / (sigma * sqrt(T - t)))
  p_star
}

# Calculation of Drift of Underlying
calc_mu <- function(r, q, sigma){
  mu <- r - q - sigma^2 / 2
  mu
}

# Spread of CoCo Bond
calc_spread_coco <- function(t, T, S_t, S_star, C_p, r, q, sigma){
  spread_coco <- - log(1 - calc_p_star(t, T, S_t, S_star, r, q, sigma))
      / (T - t) * (1 - S_star / C_p)
  spread_coco
}

# Pricing Example
```

```
32  price_coco_cd(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <-
        7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3)
```

Appendix C

Code - Equity Derivative Approach

The following source code is an implementation of the Equity Derivative Approach written in R.

```r
# Price of Contingent Convertible Bond
price_coco_ed <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma,
    alpha){
  V_t_ed <- price_cb(t, T, c_i, r, N) - price_dibi(t, T, S_t, S_star, c_
    i, r, q, sigma, alpha) + price_difwd(t, T, S_t, S_star, C_p, r, N, q
    , sigma, alpha)

  return(V_t_ed)
}

# Price of Corporate Bond
price_cb <- function(t, T, c_i, r, N){
  V_t_cb <- N * exp(-r * (T - t))

  for (t in 1:T){
  V_t_cb <- V_t_cb + c_i * exp(-r * t)
  }

  return(V_t_cb)
}

# Price of Binary Option
price_dibi <- function(t, T, S_t, S_star, c_i, r, q, sigma, alpha){
  V_t_dibi <- 0

  i <- t
  k <- T

  for (i in 1:k) {
  V_t_dibi <- V_t_dibi + c_i * exp(- r * i) * (pnorm(- calc_x_1_i(S_t, S
    _star, sigma, r, q, i) + sigma * sqrt(i)) + (S_star / S_t)^(2 * calc
    _lambda(r, q, sigma) - 2) * pnorm ( calc_y_1_i(S_t, S_star, sigma, r
    , q, i) - sigma * sqrt(i)))
  }

```

```r
30    V_t_dibi <- alpha * V_t_dibi
31
32    return(V_t_dibi)
33  }
34
35  # Price of Down-And-In Forward
36  price_difwd <- function(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha){
37    V_t_difwd <- calc_conversion_rate(C_p, N, alpha) * (S_t * exp(- q * (T
        - t)) * (S_star / S_t) ^ (2 * calc_lambda(r, q, sigma)) * pnorm(
        calc_y_1(t, T, S_t, S_star, r, q, sigma)) - C_p * exp(- r * (T - t))
        * (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm(calc_y_
        1(t, T, S_t, S_star, r, q, sigma) - sigma * sqrt(T - t)) - C_p * exp
        (- r * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma) +
        sigma * sqrt(T - t)) + S_t * exp(- q * (T - t)) * pnorm(- calc_x_1(
        t, T, S_t, S_star, r, q, sigma)))
38
39    return(V_t_difwd)
40  }
41
42  # Calculation of Conversion Rate
43  calc_conversion_rate <- function(C_p, N, alpha){
44    C_r <- alpha * N / C_p
45
46    return(C_r)
47  }
48
49  # Calculation of additional Parameters
50  calc_x_1_i <- function(S_t, S_star, sigma, r, q, t_i){
51    x_1_i <- log(S_t / S_star) / (sigma * sqrt(t_i)) + calc_lambda(r, q,
        sigma) * sigma * sqrt(t_i)
52
53    return(x_1_i)
54  }
55
56  calc_y_1_i <- function(S_t, S_star, sigma, r, q, t_i){
57    y_1_i <- log(S_star / S_t) / (sigma * sqrt(t_i)) + calc_lambda(r, q,
        sigma) * sigma * sqrt(t_i)
58
59    return(y_1_i)
60  }
61
62  calc_lambda <- function(r, q, sigma){
63    lambda <- (r - q + sigma^2 / 2) / sigma^2
64
65    return(lambda)
66  }
67
68  calc_x_1 <- function(t, T, S_t, S_star, r, q, sigma){
69    x_1 <- log(S_t / S_star) / (sigma * sqrt(T - t)) + calc_lambda(r, q,
        sigma) * sigma * sqrt(T - t)
70
71    return(x_1)
72  }
73
```

```
74 calc_y_1 <- function(t, T, S_t, S_star, r, q, sigma){
75   y_1 <- log(S_star / S_t) / (sigma * sqrt(T - t)) + calc_lambda(r, q,
      sigma) * sigma * sqrt(T - t)
76
77   return(y_1)
78 }
79
80 # Pricing Example
81 price_coco_ed(t <- 0, T <- 5, S_t <- 40, S_star <- 20, C_p <- 25, c_i <-
      7, r <- 0.03, N <- 100, q <- 0, sigma <- 0.3, alpha <- 1)
```

Appendix D

Code - Structural Approach

The following source code is an implementation of the Structural Approach written in R.

```r
# Price of Contingent Convertible Bond
price_coco_sa <- function(T , npath , rho , kappa , r_bar , r0, sigma_r,
    mu_Y, sigma_Y, lambda , g, x_hat , b0, p, e_bar , sigma_x , x0_low , x0_
    high , x0_nint , B, c_low , c_high , c_nint ){
  n <- T * 250
  dt <- T / n

  result <- sim_corrProcess (T, npath , rho , n, dt)
  dW_1 <- result $dW_1
  dW_2corr <- result $dW_2corr

  r <- sim_interestrate (kappa , r_bar , r0 , sigma_r , dW_2corr , n, npath ,
    dt)

  V_t_sa <- get_price (npath , n, dt , dW_1, dW_2corr , r, mu_Y, sigma_Y,
    lambda , g, x_hat , b0, p, e_bar , sigma_x , x0_low , x0_high , x0_nint , B
    , c_low , c_high , c_nint ) * 100
  return (V_t_sa)
}

sim_corrProcess <- function (T, npath , rho , n, dt){
  vect <- c(1, rho , rho , 1)
  RHO <- matrix (vect , nrow = 2)
  chol_RHO <- t(chol(RHO))

  # Create two Brownian Motions
  dW_1 <- matrix (1, n, npath )
  dW_2 <- matrix (1, n, npath )

  for(j in 1:npath )
  {
    dW_1[ , j] <- rnorm (n) * sqrt (dt)
    dW_2[ , j] <- rnorm (n) * sqrt (dt)
  }
```

```r
31    # Create Correlated Process based on Brownian Motions using Cholesky-
         Decomposition
32    dW_2corr <- matrix(1, n, npath)
33    for(j in 1:npath)
34    {
35      for(i in 1:n)
36      {
37        dW_2corr[i, j] <- dW_1[i, j] * chol_RHO[2, 1] + dW_2[i, j] * chol_
      RHO[2, 2]
38      }
39    }
40
41    return(list("dW_1" = dW_1, "dW_2corr" = dW_2corr))
42  }
43
44  # Create Interest Rate Process
45  sim_interestrate <- function(kappa, r_bar, r0, sigma_r, dW_2corr, n,
         npath, dt){
46    r <- matrix(r0, n + 1, npath)
47
48    for(j in 1:npath)
49    {
50      for(i in 1:n)
51      {
52        r[i + 1, j] <- r[i, j] + kappa * (r_bar - r[i, j]) * dt + sigma_r
      * sqrt(r[i, j]) * dW_2corr[i, j]
53      }
54    }
55
56    return(r)
57  }
58
59  get_price <- function(npath, n, dt, dW_1, dW_2corr, r, mu_Y, sigma_Y,
         lambda, g, x_hat, b0, p, e_bar, sigma_x, x0_low, x0_high, x0_nint, B
         , c_low, c_high, c_nint){
60
61    c_fit_matrix <- matrix(0, x0_nint, length(lambda))
62
63    for(w in 1:length(lambda))
64    {
65      # Create parametres for jump process
66      phi <- matrix(rbinom( n%*%npath, 1, dt * lambda[w]), n, npath)
67      ln_Y <- matrix(rnorm(n%*%npath, mu_Y, sigma_Y), n, npath)
68
69      b <- matrix(b0, n + 1, npath)
70      x_bar0 <- 1 + e_bar + p * b0
71      x_bar <- matrix(x_bar0, n + 1, npath)
72
73      h <- matrix(1, n, npath)
74
75      k <- exp(mu_Y + 0.5 * sigma_Y^2) - 1
76
77      c <- seq(c_low, c_high, length = c_nint)
78      x0 <- seq(x0_low, x0_high, length = x0_nint)
```

```r
for(l in 1:x0_nint) # Wieso?
{
  for(m in 1:c_nint) # Wieso?
  {
    x <- matrix(x0[l],n+1,npath)
    ln_x0 <- matrix(log(x0[l]),n+1,npath)
    ln_x <- ln_x0
    binom_c <- matrix(1,n+1,npath)

    for(j in 1:npath)
    {
      for(i in 1:n)
      {
        d_1 <- (ln_x[i, j] + mu_Y) / sigma_Y
        d_2 <- d_1 + sigma_Y

        h[i, j] <- lambda[w] * (pnorm( - d_1) - exp(ln_x[i, j]) *
exp(mu_Y + 0.5 * sigma_Y^2) * pnorm(-d_2))

        b[i + 1, j] <- b[i, j] * exp(- g[w] * (exp(ln_x[i, j]) - x_
hat) * dt)

        ln_x[i + 1, j] <- ln_x[i, j] + ( (r[i, j] - lambda[w] * k) -
  (r[i, j] + h[i, j] + c[m] * b[i, j]) / exp(ln_x[i, j]) - g[w] * (
exp(ln_x[i, j]) - x_hat) - 0.5 * sigma_x^2) * dt + sigma_x * sqrt(dt
) * dW_1[i, j] + ln_Y[i,j] * phi[i, j]

        x[i + 1, j] <- exp(ln_x[i + 1, j])

        x_bar[i + 1, j] <- 1 + e_bar + p * b[i + 1, j]

        if(x[i + 1, j] >= x_bar[i + 1, j] && binom_c[i, j] > 0.5)
        {
          binom_c[i + 1, j] <- 1
        }else
        {
          binom_c[i + 1, j] <- 0
        }
      }
    }

    payments <- matrix(c(rep(c[m] * dt, n - 1), B), n, npath) *
binom_c[1:n, ]

    for(j in 1:npath){
      for(i in 2:n){
        if(payments[i, j] == 0 && p * b[sum(binom_c[ , j]) + 1, j]
<= x[sum(binom_c[ , j]) + 1, j] - 1 ){
          payments[i, j] <- p * B
          break
        }
        else if(payments[i, j] == 0 && 0 < x[sum(binom_c[ , j]) + 1,
j] - 1 && x[sum(binom_c[ , j]) + 1, j] - 1 < p * b[sum(binom_c[ , j
```

```
        ]) + 1, j]){
125           payments[i, j] <- (x[sum(binom_c[ , j]) + 1, j] - 1) * B /
    b[sum(binom_c[ , j]) + 1, j]
126           break
127         }
128         else{
129           payments[i, j] <- payments[i, j]
130         }
131       }
132     }
133     vec_disc_v <- rep(0, npath)
134     for(j in 1:npath)
135     {
136       disc_v <- 0
137       int_r <- 0
138
139       for(i in 1:n)
140       {
141         int_r <- int_r + r[i, j] * dt
142         disc_v <- disc_v + exp(- int_r) * payments[i, j]
143       }
144       vec_disc_v[j] <- disc_v
145     }
146
147     V_t_sa <- mean(vec_disc_v)
148
149     return(V_t_sa)
150   }
151   }
152   }
153 }
154
155 # Pricing Example
156 price_coco_sa(T = 5, npath = 2, rho = - 0.2, kappa = 0.114, r_bar =
    0.069, r0 = 0.035, sigma_r = 0.07, mu_Y = −0.01, sigma_Y = 0.02,
    lambda = c(1), g = c(0.5), x_hat = 1.1, b0 = 0.04, p = 1, e_bar =
    0.02, sigma_x = 0.02, x0_low = 1.15, x0_high = 1.15, x0_nint = 10, B
    = 1, c_low = 0.05, c_high = 0.05, c_nint = 10)
```

References

Jan De Spiegeleer and Wim Schoutens. *The handbook of convertible bonds: pricing, strategies and risk management*, volume 581. John Wiley & Sons, 2011a.

Jan De Spiegeleer and Wim Schoutens. Pricing contingent convertibles: a derivatives approach. *Available at SSRN 1795092*, 2011b.

Marc Erismann. *Pricing Contingent Convertible Capital-A Theoretical and Empirical Analysis of Selected Pricing Models*. PhD thesis, University of St. Gallen, 2015.

George Pennacchi. A structural model of contingent bank capital. Working Paper 1004, Federal Reserve Bank of Cleveland, 2010. URL https://ideas.repec.org/p/fip/fedcwp/1004.html.