Valuation of Contingent Convertibles with Derivatives

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A thesis submitted for the degree of

Master of Science in Finance

August 2016

Valuation of Contingent Convertibles with Derivatives

Master's Thesis
at Frankfurt School of Finance and Management

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This thesis is dedicated to my parents for their love and support.
Thank you!

Abstract

Financial crises have led to higher regulatory standards on the capital adequacy of banks. Banks are required to hold more capital with loss absorbance capacities on their balance sheet. In conjunction with this development, contingent convertible bonds (CoCos) have become an attractive instrument for banks to seek new capital. The defining characteristic of CoCos is the automatic conversion into common equity or the principal write-down when a certain ratio meets a predetermined trigger. Loss-absorbing capital is created, which instantly improves the capital structure of the distressed bank. However, the pricing of these hybrid instruments remains opaque. In this context, the thesis scrutinizes the valuation of CoCos with equity conversion mechanism. The paper examines three dominant approaches: the structural approach in accordance to Pennacchi (2010), the credit derivatives approach and the equity derivatives approach both under De Spiegeleer and Schoutens (2012). Additionally, the application covers sensitivity analyses to understand the dynamics of the different methodologies further. Based on a case study of HSBC's perpetual subordinated contingent convertible securities (ISIN US404280AT69) the viability of the approaches is evaluated by analyzing their price tracking accuracy. Subsequently, the comprehensive software provides a basis for further applications of the pricing approaches as mentioned above.

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Chapter 1

Introduction

1.1 Background

Investors are overly restrictive in providing liquidity to financial institutions during periods of financial distress. In the past, governments were often in the situation to inject liquidity to financial markets to avoid disruptive insolvencies as no other market participant was inclined to do so. Government bailouts, however, externalize the cost of bankruptcy to taxpayers while distorting risk-taking incentives of banking professionals. Contingent convertibles (CoCos) aim to internalize these costs in the capital structure of systemically important financial institutions. CoCos are hybrid financial instruments that absorb losses under their specifications in case a pre-determined threshold fails to remain above a minimum trigger level. Then, debt automatically morphs to equity which instantly improves a bank's capitalization. Due to their loss-absorption capacity, CoCos are eligible to be categorized as regulatory capital under Basel III. (Avdjiev et al., 2013)

After the global financial crisis of 2008, regulators around the world have been working on two different objectives. On the one hand, they attempt to lower spillover effects on the economy due to bankruptcies of financial institutions. On the contrary, they aim to reduce the individual default probabilities of banks. One can might attain the latter objective by ensuring that banks have enough loss-absorbing capital on their balance sheet even in tough times. (De Spiegeleer and Schoutens, 2011) In this context, the Basel Committee on Banking Supervision (BCBS) specified that debt instruments are permitted as regulatory capital if they absorb losses to such an extent that taxpayers do not have to bear the costs. (Basel Committee on Banking Supervision, 2010b) Subsequently, this opened the door for CoCos and ever since, the regulatory treatment has been a major driver of past issues.

Besides the reforms of policy makers to raise the quality of regulatory capital, recent studies on CoCos highlight some advantages. Albul et al. (2015) show that CoCos lessen financial distress, whether caused by idiosyncratic or systemic shocks. They indicate lower default probabilities of banks and a smaller likelihood of costly bailouts by the public sector. Hilscher and Raviv (2014) support these findings. Additionally, they argue that an appropriate specification of a CoCo's building parts can eliminate the incentives of shareholders to asset-substitution. The problem of asset-substitution arises when managers undertake excessively risky investment decisions to maximize shareholder value at the expense of debtholders. (Bannier, 2011) Moreover, research evinces that CoCos increase the issuer's firm value as they reduce the cost of capital. (Albul et al., 2015; Von Furstenberg et al., 2011; Barucci and Del Viva, 2012) To sum up, one should consider CoCos in the liability structure of banks from an academic standpoint.

In addition to the positive perception of policy makers and academia, CoCos are well accepted by the financial industry. Banks value that the hybrid instrument enables them to refinance themselves while simultaneously satisfying the regulatory capital requirements at lower costs than with equity. (European Parliament, 2016) Between 2009 and 2015, financial institutions around the world issued CoCos worth USD 446.96 bn in 519 different issues. (Avdjiev et al., 2015) Albeit the amount of CoCos issued is relatively small compared to the market size of other financial products; they were brought into focus in early 2016. At this time, CoCos contributed to increased market volatility of some European banks. One should not whitewash the relevance of this episode concerning the potential systemic implications as it is likely that discussions on regulatory changes will emerge. Moreover, the question arises about the perception of CoCos by investors and the robustness of their pricing models. (European Parliament, 2016)

In this context, an investigation of different valuation concepts for CoCos is highly attractive for both investors as well as supervisory authorities. The paper contributes to a better understanding of relevant concepts. Additionally, the valuation approaches will be applied to a CoCo which was affected by the turbulences in early 2016.

1.2 Previous Studies

Various valuation approaches for CoCos have been developed over time covering different aspects of their nature. The variety of approaches is due to the hybrid character of CoCos which also makes them a highly interesting object of study. Wilkens and Bethke (2014) propose three groups to organize the broad universe: structural approaches, equity derivative approaches and credit derivative approaches. Additionally, Turfus and Shubert (2015) suggest a fourth category: hybrid equity-credit derivative approaches. A comprehensive compilation of relevant studies for each category can be found in table 2.1. Subsequently, the main idea of each type will be explained.

Structural Approaches	Equity Derivative Approaches	Credit Derivative Approaches
Pennacchi (2010)	De Spiegeleer and Schoutens (2012)	De Spiegeleer and Schoutens (2012)
Albul et al. (2010)	Henriques and Doctor (2011) as cited by Erismann (2015)	Serjantov (2011) as cited by Wilkens and Bethke (2014)
Madan and Schoutens (2011)	Alvemar and Ericson (2012)	Alvemar and Ericson (2012)
Glasserman and Nouri (2012)	Corcuera et al. (2013)	Erismann (2015)
Alvemar and Ericson (2012)	Corcuera et al. (2014)	
Buergi (2013)	Teneberg (2015)	
Hilscher and Raviv (2014)	Erismann (2015)	
Pennacchi and Tchistyi (2015)		
Cheridito and Xu (2015)	**Hybrid Equity-Credit Derivative Approaches**	
Erismann (2015)	Turfus and Shubert (2015)	
Sundaresan and Wang (2015)		

Table 1.1: Literature overview of valuation approaches for CoCos (Wilkens and Bethke, 2014; Erismann, 2015) with the examined methods.

Structural approaches try to capture all parameters that influence the issuer's ability to pay its liabilities. They are normally built upon a stochastic model which focusses on the variation in asset values relative to debt. (Duffie and Singleton, 2003) By contrast, **equity derivative approaches** emphasize the dependence of a CoCo's state on the share price and use equity derivatives to replicate their payoff. This model type follows the train of thought that the share price is the best proxy to track the solvency of the issuer. **Credit derivative approaches** assume an exogenously specified process for the migration of conversion probabilities. They apply the idea of reduced-form approaches to model the equity conversion intensity process of CoCos in line with a credit default intensity process. The rationale behind this approach is that CoCos are credit-risky instruments as their conversion depends on the issuer's solvency. (Wilkens and Bethke, 2014) **Hybrid equity-credit derivative approaches** capture the advantages of the latter two concepts. They model the share price and the conversion intensity as correlated stochastic processes. (Turfus and Shubert, 2015) A detailed literature review of the various pricing approaches is outlined in the following sections.

Structural Approaches

Structural approaches offer a natural pricing framework for CoCos. They consider a bank's balance sheet structure as the most important value driver. Numerous structural approaches have been proposed in academia. All share common characteristics but vary in their application. (Wilkens and Bethke, 2014) For instance, they are often used to draw policy recommendations. A selection can be found in the following.

The study of Albul et al. (2010) is the first paper to provide analytic propositions to price CoCos by adapting the structural model of Leland (1994). The authors develop implications for the design of CoCos with the objective of maximizing the benefit for the issuer. Interestingly, their analysis is at first not limited to financial institutions. In fact, they argue that CoCos might generally be advantageous for corporates to optimize their capital structure. The authors further recommend the specific use of CoCos as tool for bank regulation. In this context, studies like Madan and Schoutens (2011), Hilscher and Raviv (2014) and Sundaresan and Wang (2015) analyze beneficial structures of CoCos.

In the scientific literature, the work of Pennacchi (2010) is often used as a reference article for structural approaches as he attempts to model the stochastic evolution of a bank's balance sheet to price CoCos (for further details see chapter 3.3). The author is able determine the value of CoCos by applying a jump-diffusion process to account for discontinuous asset returns. Capital ratios with mean-reverting tendency and a stochastic term-structure model shall improve the pricing accuracy. Based on the derived framework the author is able to capture several risk factors that may influence a CoCo's price. However, this is also the main shortcoming of the approach because the author does not address the parametrization of input factors in practice, which is also indicated by the work of Erismann (2015).

Madan and Schoutens (2011) implement a structural model utilizing conic finance theory. Classical Mertonian models (Merton, 1974) assume that assets are risky but liabilities are not. For instance, Alvemar and Ericson (2012) apply such a pure model pursuant to Merton (1974) to price CoCos. In contrast, the model of Madan and Schoutens (2011) assumes that liabilities are risky and correlated to the asset dynamics. The authors abandon the one-price-market idea and assume that bid-ask spreads exist. In addition, they argue that the Core Tier 1 ratio is potentially not optimal as trigger if one considers the presence of risky liabilities. As alternative they propose accounting triggers based on capital shortfall.

Equity Derivative Approaches

Equity derivative approaches are an important category of valuation approaches which consider the share price as the best proxy. Most important approaches will be summarized in the following.

De Spiegeleer and Schoutens (2012), replicate the payout profile of a CoCo with a portfolio consisting of a straight bond, a knock-in forward and a set of binary down-and-in calls (for further details see chapter 3.3). Under the assumption that a CoCo will not convert to equity, one can assume that a CoCo is equivalent to a straight bond. Though, the knock-in forward simulates the conversion of a straight bond when a predetermined strike price is met. A CoCo investor would receive the shares at maturity if he or she is long a knock-in forward. However, this is a simplification which is reasonable under the assumption that dividend payments are cancelled in times of distress. Additionally, the foregone coupon payments of a straight bond at conversion are modeled with a short position in binary down-and-in calls. One of the main findings is that the assumed Black-Scholes setting does not sufficiently capture tail risks but which are inherent in CoCos.

Other approaches enhance the model dynamics by accounting for jumps and heavy tails. Erismann (2015) and Teneberg (2015) amend the model of De Spiegeleer and Schoutens (2012) by allowing for discontinuous returns. The calculations with regard to the binary down-and-in calls and the knock-in forward position accommodates a jump-diffusion process. Corcuera et al. (2013) also consider an equity derivative approach that reduces the valuation to a set of barrier options in which the trigger event is determined by the underlying hitting a certain barrier. They use smile conform models, more precisely, an exponential Lévy process incorporating jumps and heavy tails.

Credit Derivative Approaches

The price of a CoCo is directly linked to the issuer's solvency and default probability. Intensity-based credit modeling allows to develop comprehensive pricing approaches. In this connection, one should mention the work of De Spiegeleer and Schoutens (2012), Serjantov (2011) and Erismann (2015).

De Spiegeleer and Schoutens (2012) tackle the pricing problem with a credit-derivative approach (for further details see chapter 3.1). Their main contribution lies in the derivation of a closed-form solution of a CoCo's credit spread. In their model the spread follows a function of an exogenously defined trigger probability. The spread compensates for the risk that the CoCo converts to equity implying a loss for each investor. Their approach is an elegant way of bridging the gap between the prediction of conversion and the pricing of conversion risk. Though, the largest shortcoming of the model is that it fails to capture losses from cancelled coupons of triggered CoCos.

Erismann (2015) expands the model of De Spiegeleer and Schoutens (2012) by assuming that returns follow a jump-diffusion process. The approach models the exposure to return outliers of both signs and amplitudes. Finally, the author demonstrates that his approach is superior to De Spiegeleer and Schoutens (2012) considering price tracking accuracy. However, the case study on a HSBC CoCo reveals that the conclusions of Erismann (2015) and Wilkens and Bethke (2014) might not be as reliable as initially thought.

Serjantov (2011) as cited by Wilkens and Bethke (2014) develops a closed form solution to price CoCos. All cashflows are weighted with cumulative survival probabilities. In addition, the approach distinguishes between the conversion ratio without default and the recovery rate at default. The joint probability of both events happening in the same time interval is described with a Gaussian copula. Furthermore, this approach overcomes the shortcoming of the credit derivative approach of De Spiegeleer and Schoutens (2012) as it explicitly captures coupon payments.

Hybrid Equity-Credit Derivative Approaches

Turfus and Shubert (2015) present a new pricing approach for CoCos. Their starting point is a stochastic model which captures interest rates, share prices and a conversion intensity process. The evolution of the first two is assumed to be determined by diffusive processes. By contrast, the share price is supposed to be governed by a jump-diffusion process which factors into a downward jump when the trigger level is touched. Both the share price and the conversion intensity process are modeled as correlated stochastic processes. For this very reason, the hybrid equity-credit derivative approach may be regarded as an important step forward because two direct benefits arise. On the one hand, the share price at conversion is modeled instead of being an input parameter and on the other hand, both equity and credit risk sensitivity can be estimated individually.

1.3 Research Methodology

The objective of the thesis consists of an examination of three dominant pricing approaches for CoCos similar to the proceedings of Alvemar and Ericson (2012), Erismann (2015) and Wilkens and Bethke (2014). All of three valuation models are widely discussed in academic literature as they are often used as basis for further model advancements. The utilized approaches are namely the structural approach of Pennacchi (2010), the equity derivative approach and the credit derivative approach both pursuant to De Spiegeleer and Schoutens (2012). Hereinafter, chapter 2 provides an overview of the anatomy of CoCos. Characteristic building parts of the financial product will be discussed in detail in order to create an improved understanding of the mechanisms which drive the valuation of this hybrid instrument. Examples of past CoCos issues are highlighted covering the most important variations of the aforementioned design features. On this basis, chapter 3 studies the theoretical concepts behind each of the three valuation approaches. In addition, pricing examples provide an understanding of the data requirements of each model. In chapter 4, sensitivity analyses determine how different values of certain pricing parameters impact the valuation of CoCos. Chapter 5 comprises an empirical analysis on the price tracking accuracy of the aforementioned valuation approaches. Finally, a conclusion is reached in chapter 6.

Chapter 2

Introduction

2.1 Background

Investors are overly restrictive in providing liquidity to financial institutions during periods of financial distress. In the past, governments were often in the situation to inject liquidity to financial markets to avoid disruptive insolvencies as no other market participant was inclined to do so. Government bailouts, however, externalize the cost of bankruptcy to taxpayers while distorting risk-taking incentives of banking professionals. Contingent convertibles (CoCos) aim to internalize these costs in the capital structure of systemically important financial institutions. CoCos are hybrid financial instruments that absorb losses under their specifications in case a pre-determined threshold fails to remain above a minimum trigger level. Then, debt automatically morphs to equity which instantly improves a bank's capitalization. Due to their loss-absorption capacity, CoCos are eligible to be categorized as regulatory capital under Basel III. (Avdjiev et al., 2013)

After the global financial crisis, regulators have been working globally on two different objectives. On the one hand, they attempt to lower spillover effects on the economy due to bankruptcies of financial institutions. On the contrary, they aim to reduce the individual default probabilities of banks. One can achieve the latter objective by ensuring that banks have enough loss-absorbing capital on their balance sheet even in tough times. (De Spiegeleer and Schoutens, 2011) In this context, the Basel Committee on Banking Supervision (BCBS) specified that debt instruments are permitted as regulatory capital if they absorb losses to such an extent that taxpayers do not have to bear the costs. (Basel Committee on Banking Supervision, 2010b) Subsequently, this opened the door for CoCos and ever since, the regulatory treatment has been a major driver of past issues.

Besides the reforms of policy makers to raise the quality of regulatory capital, recent studies on CoCos highlight some advantages. Albul et al. (2015) show that CoCos lessen financial distress, whether caused by idiosyncratic or systemic shocks. They indicate lower default probabilities of banks and a smaller likelihood of costly bailouts by the public sector. Hilscher and Raviv (2014) support these findings. Additionally, they argue that an appropriate specification of a CoCo's building parts can eliminate the incentives of shareholders to asset-substitution. The problem of asset-substitution arises when managers undertake excessively risky investment decisions to maximize shareholder value at the expense of debtholders. (Bannier, 2011) Moreover, research evinces that CoCos increase the firm value of their issuer as they reduce the cost of capital. (Albul et al., 2015; Von Furstenberg et al., 2011; Barucci and Del Viva, 2012) To sum up, academia considers CoCos to fit well in their liability structure.

In addition to the positive perception of policy makers and academia, CoCos are well accepted by the financial industry. Banks value that the hybrid instrument enables them to refinance themselves while simultaneously satisfying the regulatory capital requirements at lower costs than with equity. (European Parliament, 2016) Between 2009 and 2015, financial institutions around the world issued CoCos worth USD 446.96 bn in 519 different issues. (Avdjiev et al., 2015) Albeit the amount of CoCos issued is relatively small compared to the market size of other financial products; they were brought into focus in early 2016. At this time, CoCos contributed to increased market volatility given some European banks. Investors cannot whitewash the relevance of this episode concerning the potential systemic implications as it is likely that discussions on regulatory changes will emerge. Having regard to this circumstances, the question arises about the perception of CoCos by investors and the robustness of their pricing models. (European Parliament, 2016)

In this context, an investigation of different valuation concepts for CoCos is highly attractive for both investors as well as supervisory authorities. The paper contributes to an enhanced understanding of relevant concepts. Additionally, the paper applies the valuation approaches to a CoCo which was affected by the turbulences in early 2016.

2.2 Previous Studies

Various valuation approaches for CoCos have been developed over time covering different aspects of their nature. The variety of approaches is due to the hybrid character of CoCos which also makes them a highly interesting object of study. Wilkens and Bethke (2014) propose three groups to organize the broad universe: structural approaches, equity derivative approaches and credit derivative approaches. Additionally, Turfus and Shubert (2015) suggest a fourth category: hybrid equity-credit derivative approaches. A comprehensive compilation of relevant studies for each category can be found in table 2.1. Subsequently, the main idea of each type will be explained.

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Table 2.1: Literature overview of valuation approaches for CoCos (Wilkens and Bethke, 2014; Erismann, 2015) with the examined methods.

Structural approaches try to capture all parameters that influence the issuer's ability to pay its liabilities. They are normally built upon a stochastic model which focusses on the variation in asset values relative to debt. (Duffie and Singleton, 2003) By contrast, **equity derivative approaches** emphasize the dependence of a CoCo's state on the share price and use equity derivatives to replicate their payoff. This model type follows the train of thought that the share price is the best proxy to track the solvency of the issuer. **Credit derivative approaches** assume an exogenously specified process for the migration of conversion probabilities. They apply the idea of reduced-form approaches to model the equity conversion intensity process of CoCos in line with a credit default intensity process. The rationale behind this approach is that CoCos are credit-risky instruments as their conversion depends on the issuer's solvency. (Wilkens and Bethke, 2014) **Hybrid equity-credit derivative approaches** capture the advantages of the latter two concepts. They model the share price and the conversion intensity as correlated stochastic processes. (Turfus and Shubert, 2015) A detailed literature review of the various pricing approaches is outlined in the following sections.

Structural Approaches

Structural approaches offer a natural pricing framework for CoCos. They consider a bank's balance sheet structure as the most important value driver. Academics have proposed numerous structural approaches. All share common characteristics but vary in their application. (Wilkens and Bethke, 2014) For instance, they are often used to draw policy recommendations. The reader can find a selection in the following.

The study of Albul et al. (2010) is the first paper to provide analytic propositions to price CoCos by adapting the structural model of Leland (1994). The authors develop implications for the design of CoCos with the objective of maximizing the benefit for the issuer. Interestingly, their analysis is at first not limited to financial institutions. In fact, they argue that CoCos might be advantageous for corporates to optimize their capital structure. The authors further recommend the particular use of CoCos as a tool for bank regulation. In this context, studies like Madan and Schoutens (2011), Hilscher and Raviv (2014) and Sundaresan and Wang (2015) analyze beneficial structures of CoCos.

In the scientific literature, the work of Pennacchi (2010) is often used as a reference article for structural approaches as he attempts to model the stochastic evolution of a bank's balance sheet to price CoCos (for further details see chapter 3.3). The author can determine the value of CoCos by applying a jump-diffusion process to account for discontinuous asset returns. Capital ratios with the mean-reverting tendency and a stochastic term structure model shall improve the pricing accuracy. Based on the derived framework the author can capture several risk factors that may influence a CoCo's price. However, this is also the main shortcoming of the approach because the author does not address the parametrization of input factors in practice, which is also indicated by the work of Erismann (2015).

Madan and Schoutens (2011) implement a structural model utilizing conic finance theory. Classical Mertonian models (Merton, 1974) assume that assets are risky but liabilities are not. For instance, Alvemar and Ericson (2012) apply such a pure model pursuant to Merton (1974) to price CoCos. In contrast, the model of Madan and Schoutens (2011) assumes that liabilities are risky and correlated to the asset dynamics. The authors abandon the one-price-market idea and assume that bid-ask spreads exist. Also, they argue that the Core Tier 1 ratio is potentially not optimal as a trigger if one considers the presence of risky liabilities. As an alternative, they propose accounting triggers based on the capital shortfall.

Equity Derivative Approaches

Equity derivative approaches are an important category of valuation approaches which consider the share price as the best proxy. One may find a comprehensive overview of relevant methods in the following.

De Spiegeleer and Schoutens (2012), replicate the payout profile of a CoCo with a portfolio consisting of a straight bond, a knock-in forward and a set of binary down-and-in calls (for further details see chapter 3.3). Under the assumption that a CoCo will not convert to equity, one can assume that a CoCo is equivalent to a straight bond. Though, the knock-in forward mimics the conversion of a straight bond when the underlying meets a predetermined strike price. A CoCo investor will receive the shares at maturity if he or she is long a knock-in forward. However, this is a simplification which is reasonable under the assumption the company cancels dividend payments in times of distress. Additionally, the authors model the canceled coupon payments of a straight bond at conversion with a short position in binary down-and-in calls. One of the main findings is that the assumed Black-Scholes setting does not sufficiently capture tail risks but which are inherent in CoCos.

Other approaches enhance the model dynamics by accounting for jumps. Erismann (2015) and Teneberg (2015) amend the model of De Spiegeleer and Schoutens (2012) by allowing for discontinuous returns. The calculations concerning the binary down-and-in calls and the knock-in forward position accommodate a jump-diffusion process. Corcuera et al. (2013) also consider an equity derivative approach that reduces the valuation to a set of barrier options in which the trigger event happens if the underlying hits a particular threshold. They use smile conform models, more precisely, an exponential Lévy process incorporating jumps and thick tails.

Credit Derivative Approaches

The price of a CoCo links directly to the issuer's solvency and default probability. Intensity-based credit modeling allows developing comprehensive pricing methods. In this connection, one should mention the work of De Spiegeleer and Schoutens (2012), Serjantov (2011) and Erismann (2015).

De Spiegeleer and Schoutens (2012) tackle the pricing problem with a credit-derivative approach (for further details see chapter 3.1). Their main contribution lies in the derivation of a closed-form solution of a CoCo's credit spread. In their model, the spread follows a function of an exogenously defined trigger probability. The spread compensates for the risk that the CoCo converts to equity implying a loss for each investor. Their approach is an elegant way of bridging the gap between the prediction of conversion and the pricing of conversion risk. Though, the largest shortcoming of the model is that it fails to capture losses from canceled coupons of triggered CoCos.

Erismann (2015) expands the model of De Spiegeleer and Schoutens (2012) by assuming that returns follow a jump-diffusion process. The approach models the exposure to return outliers of both signs and amplitudes. Finally, the author demonstrates that his approach is superior to De Spiegeleer and Schoutens (2012) considering price tracking accuracy. However, the case study on a HSBC CoCo reveals that the conclusions of Erismann (2015) and Wilkens and Bethke (2014) might not be as reliable as initially thought.

Serjantov (2011) as cited by Wilkens and Bethke (2014) develops a closed form solution to price CoCos which weighs All cash flows with cumulative survival probabilities. Also, the approach distinguishes between the conversion ratio without default and the recovery rate at default. A Gaussian copula describes the joint probability of both events happening in the same time interval. Furthermore, this approach overcomes the shortcoming of the credit derivative approach of De Spiegeleer and Schoutens (2012) as it explicitly captures coupon payments.

Hybrid Equity-Credit Derivative Approaches

Turfus and Shubert (2015) present a new pricing approach for CoCos. Their starting point is a stochastic model which captures interest rates, share prices, and a conversion intensity process. The evolution of the first two is assumed to be determined by diffusive processes. By contrast, a jump-diffusion process is supposed to govern the share price. The stochastic process factors into a downward jump when the trigger level is touched. Correlated stochastic processes model both the stock price and the conversion intensity process. For this very reason, the hybrid equity-credit derivative approach may be regarded as an important step forward because two direct benefits arise. On the one hand, the share price at conversion is modeled instead of being an input parameter and on the contrary, both equity and credit risk sensitivity can be estimated individually.

2.3 Research Methodology

The objective of the thesis consists of an examination of three dominant pricing approaches for CoCos similar to the proceedings of Alvemar and Ericson (2012), Erismann (2015) and Wilkens and Bethke (2014). All of three valuation models are widely discussed in the academic literature as researchers often use them as a basis for further advancements. The utilized approaches are namely the structural approach of Pennacchi (2010), the equity derivative approach and the credit derivative approach both under De Spiegeleer and Schoutens (2012). After this, chapter 2 provides an overview of the anatomy of CoCos. Characteristic building parts of the financial product will be discussed in detail to create an improved understanding of the mechanisms which drive the valuation of this hybrid instrument. Examples of past CoCos issues are highlighted covering the most significant variations of the design features as mentioned above. On this basis, chapter 3 studies the theoretical concepts behind each of the three valuation approaches. Besides, pricing examples provide an understanding of the data requirements of each model. In chapter 4, sensitivity analyses determine how different values of certain pricing parameters impact the valuation of CoCos. Chapter 5 comprises an empirical analysis on the price tracking accuracy of the valuation approaches above. Finally, the reader can find a conclusion in chapter 6.

Chapter 3

Pricing Theories

3.1 Credit Derivative Approach

In financial markets the reduced-form approach is widely used in order to price credit risk. It was originally introduced by Jarrow and Turnbull (1995) respectively Duffie and Singleton (1999). The credit derivative approach applies the reduced-form approach to CoCos. In this context, the derivation of a pricing formula for CoCos follows mainly De Spiegeleer and Schoutens (2012).

3.1.1 Reduced-Form Approach and Credit Triangle

The reduced-form approach is an elegant way of bridging the gap between the prediction of default and the pricing of default risk of a straight bond. In the following, we investigate the link between estimated default intensities and credit spreads under the reduced-form approach. (Lando, 2009)

Let τ denote the random time of default of some company. It is assumed that the distribution of τ has a continuous density function f, so that the distribution function F and the curve of survival probabilities q are related as follows:

$$P(\tau \leq t) = F(t) = 1 - q(t) = \int_0^t f(s)ds, \text{ with } t \geq 0 \tag{3.1}$$

The hazard rate respectively the default intensity λ is defined as follows:

$$\lambda(t) = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} P(\tau \leq t + \Delta | \tau > t) = \frac{F'(t)}{q(t)} = \frac{F'(t)}{1 - F(t)} = -\frac{d}{dt} \log q(t) \tag{3.2}$$

Intuitively, the hazard rate is the default rate per year as of today. Using 3.2 we can derive a formula for the survival probability:

$$q(t) = \exp\left(-\int_0^t \lambda(s)ds\right) \tag{3.3}$$

For our application of the reduced-form approach we assume that the hazard rate $\lambda(t)$ is a deterministic function of time. In reality $\lambda(t)$ is not deterministic but itself stochastic. That fits in with the fact that credit spreads are not static but stochastically varying over time. (Schmidt, 2015) However, we further consider the hazard rate to be constant in order to simplify the problem. Hence, a constant hazard rate $\lambda(t) = \lambda$ implies an exponential distribution of the default time:

$$F(t) = 1 - q(t) = 1 - \exp(-\lambda t) \tag{3.4}$$

Under the idealized assumption of a flat zero interest rate curve, a flat spread curve and continuous spread payments, the default intensity λ can be calculated directly from the credit spread s and the recovery rate R by the rule of thumb formula (Schmidt, 2015), which is also known as credit triangle:

$$\lambda = \frac{s}{1 - R} \tag{3.5}$$

Finally, this relationship makes it possible to determine the default probability F from the credit spread s and vice versa.

3.1.2 Adaption to CoCos

In accordance with the aforementioned reduced-form approach, De Spiegeleer and Schoutens (2012) assume that the probability F^*, which measures the likelihood that a CoCo triggers within the next $T - t$ years, follows similar mechanics as the default probability of a straight bond does. Under the credit derivative approach the probability F^* can be expressed as follows:

$$F^* = 1 - \exp\left[-\lambda_{Trigger}(T - t)\right] \tag{3.6}$$

Additionally, the credit derivative approach models F^* with the first exit time equation used in barrier option pricing under a Black-Scholes setting. (Su and Rieger, 2009) Hence, the probability F^* that the trigger level S^* is touched within the next $T - t$ years is given by the following equation with the continuous dividend yield q, the continuous interest rate r, the drift μ, the volatility σ and the current share price S of the issuing company:

$$F^* = \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) - \mu(T-t)}{\sigma\sqrt{(T-t)}}\right) + \left(\frac{S^*}{S}\right)^{\frac{2\mu}{\sigma^2}} \Phi\left(\frac{\log\left(\frac{S^*}{S}\right) + \mu(T-t)}{\sigma\sqrt{(T-t)}}\right) \tag{3.7}$$

In this regard, a CoCo's credit spread s_{CoCo} can be approximated by the credit triangle, where R_{CoCo} denotes the recovery rate of a CoCo and L_{CoCo} is the loss rate:

$$s_{CoCo} = (1 - R_{CoCo})\,\lambda_{Trigger} = L_{CoCo}\lambda_{Trigger} \tag{3.8}$$

In the trigger event, the face value N converts into C_r shares worth S^*. The loss of a long position in a CoCo is therefore determined by the conversion price C_p:

$$Loss_{CoCo} = N - C_r S^* = N\,(1 - R_{CoCo}) = N\left(1 - \frac{S^*}{C_p}\right) \tag{3.9}$$

By combining 3.6, 3.8 and 3.9 we see that the credit spread s_{CoCo} of a CoCo with maturity T at time t is driven by its major design elements, the trigger level S^* and the conversion price C_p:

$$s_{CoCo_t} = -\frac{\log(1 - F^*)}{(T - t)}\left(1 - \frac{S^*}{C_p}\right) \tag{3.10}$$

Subsequently, a pricing formula for CoCos under the credit derivative approach can be derived. The present value V^{cd} at time t is given by:

$$V_t^{cd} = \sum_{i=1}^{T} c_i \exp\left[-(r + s_{CoCo_t})(t_i - t)\right] + N \exp\left[-(r + s_{CoCo_t})(T - t)\right] \tag{3.11}$$

In summary, the credit derivative approach provides us with a concise method to price CoCos. However, one has to bear in mind its largest shortcoming. Losses from cancelled coupons of triggered CoCos are not taken into account in the valuation. Hence, the credit derivative approach naturally overestimates the price of CoCos, but it equips investors with a simple rule of thumb formula.

3.1.3 Parameter Classification and Adjustment

The credit derivative approach requires several model inputs which can be found in table 3.1. Generally, one can separate three different types. Static inputs comprise parameters that are specified ex ante and are assumed to be constant. Dynamic inputs are updated regularly. Fitting parameters are used to fit the model prices to market prices. (Wilkens and Bethke, 2014)

	Description	Usage	Source
T	Maturity	Static input	Term sheet
N	Notional	Static input	Term sheet
c	Coupon rate	Static input	Term sheet
S_t	Share price	Dynamic input	Market data
S^*	Trigger share price	Fitting parameter	–
C_p	Conversion price	Static input	Term sheet
r	Risk-free interest rate	Dynamic input	Market data
q	Dividend yield	Static input	Market data
σ_E	Share price volatility	Dynamic input	Market data

Table 3.1: Parameter classification of the credit derivative approach (Wilkens and Bethke, 2014)

All static inputs can be found in the term sheets of the respective CoCo. But beyond the model depends upon further dynamic inputs among others the share price S, the risk-free interest rate r, the dividend yield q and the share price volatility σ. The daily share price S is directly observed in the market. The risk-free interest rate r is derived from sovereign bonds with the same maturity and currency. Furthermore, the input parameter q relies on the three year average dividend yield of the issuing company. The share price volatility σ is derived based on a yearly average volatility on a reference stock market index of the last five years similar to Alvemar and Ericson (2012). In addition, the only degree of freedom in the credit derivative approach is the fitting parameter S^* respectively the trigger share price. Because this input variable is neither a static parameter nor a market parameter, S^* is adjusted by minimizing the root mean squared deviation to realized CoCo prices. (Erismann, 2011)

3.1.4 Model Application

In the following a fictive CoCo is priced with the credit derivative approach pursuantly the specifications as stated in table 3.2. After installing both R, a programming language for statistical computing, and Rstudio, an open-source integrated development environment, a reader can easily price the aforementioned CoCo example with the source code of chapter A.1. The CoCo specifications are also used to analyze the price sensitivity in regard to certain input parameters. The results of the sensitivity analysis for the credit derivative approach can be found in section B.1. In addition, this example data set is also used to apply the equity derivative approach and the structural approach.

	Value	**Comment**
T	10yrs	Maturity
N	100%	Nominal
c	6.00%	Coupon rate
S_0	120	Initial share price
S^*	60	Trigger share price
C_p	75	Conversion price
r	3.00%	Risk-free interest rate
q	0.00%	Dividend yield
σ_E	30.00%	Share price volatility

Table 3.2: Specification of input variables for a generic CoCo under the credit derivative approach (Alvemar and Ericson, 2012)

The spread of the CoCo s_{CoCo}, which compensates an investor for the risk of equity conversion, equals 1.46%. Moreover, the price V^{cd} of the fictive CoCo under the credit derivative approach equates to 111.31.

3.2 Equity Derivative Approach

In order to assess the value of a CoCo, investors can use a method which depends on equity derivatives. (De Spiegeleer and Schoutens, 2012; De Spiegeleer et al., 2014) The so-called equity derivative approach attempts to compensate for the main drawback of the credit derivative approach, since it takes into account that coupon payments might be cancelled if the trigger of a CoCo was touched.

Subsequently, the valuation can be divided into two steps: In the first step, the value of a CoCo is determined without coupon payments. Such a CoCo is called Zero-Coupon CoCo. In the second step, one has to incorporate the coupon payments in the pricing formula while keeping in mind that they might be knocked out. The closed-form solution is derived under a Black-Scholes setting.

3.2.1 Step One - Zero-Coupon CoCo

To determine the present value of a Zero-Coupon CoCo V^{zcoco} at maturity T we can use equation **??**. The underlying assumption of the equity derivative approach is that the triggering of a CoCo respectively of a Zero-Coupon CoCo is equivalent to the share price falling below the level S^*.

$$
\begin{aligned}
V_T^{zcoco} &= \begin{cases} N & \text{if not triggered} \\ (1-\alpha)N + \frac{\alpha N}{C_p} S^* & \text{if triggered} \end{cases} \\
&= N \mathbb{1}_{\{\tau > T\}} + \left[(1-\alpha) N + \frac{\alpha N}{C_p} S^* \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + \left[\frac{\alpha N}{C_p} S^* - \alpha N \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + \left[C_r S^* - \alpha N \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + C_r \left[S^* - \frac{\alpha N}{C_r} \right] \mathbb{1}_{\{\tau \leq T\}} \\
&= N + C_r \left[S^* - C_p \right] \mathbb{1}_{\{\tau \leq T\}}
\end{aligned} \tag{3.12}
$$

It may be inferred that the financial payoff of equation 3.12 consists of two components (Erismann, 2015): (1) the face value N of a zero bond and (2) a long position in C_r shares generating a payoff only if the CoCo materializes at time τ. This component can be approximated with a knock-in forward. The intuition behind equation 3.12 is that if the share price falls below a certain level S^*, an investor will use the face value N to exercise the knock-in forward. That said, the investor is committed to buy the amount of C_r shares for the price of C_p at maturity T.

Before maturity the present value of a Zero-Coupon CoCo V_t^{zcoco} can be determined by adding up the present value of a zero bond V_t^{zb} and the present value of a knock-in forward V_t^{kifwd}. Hereinafter, the components will be explained briefly.

$$
V_t^{zcoco} = V_t^{zb} + V_t^{kifwd} \tag{3.13}
$$

with

$$
V_t^{zb} = N \exp\left[-r(T-t)\right] \tag{3.14}
$$

Moreover, the long position in shares at time t can be approximated with the respective closed-form solution of a knock-in forward. (Hull, 2006)

$$
\begin{aligned}
V_t^{kifwd} = C_r \Bigg[& S_t \exp\left[-q\left(T-t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda} \Phi\left(y_1\right) \\
& - K \exp\left[-r\left(T-t\right)\right] \left(\frac{S^*}{S_t}\right)^{2\lambda-2} \Phi\left(y_1 - \sigma\sqrt{T-t}\right) \\
& - K \exp\left[-r\left(T-t\right)\right] \Phi\left(-x_1 - \sigma\sqrt{T-t}\right) \\
& + S_t \exp\left[-q\left(T-t\right)\right] \Phi\left(-x_1\right) \Bigg]
\end{aligned} \tag{3.15}
$$

with

$$
\begin{aligned}
C_r &= \frac{\alpha N}{C_p} \\
K &= C_p \\
\lambda &= \frac{r - q + \frac{\sigma^2}{2}}{\sigma^2} \\
x_1 &= \frac{\log\left(\frac{S_t}{S^*}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t} \\
y_1 &= \frac{\log\left(\frac{S^*}{S_t}\right)}{\sigma\sqrt{T-t}} + \lambda\sigma\sqrt{T-t}
\end{aligned}
$$

It is important, however, to recognize that a subtle difference exists between the actual economic payoff of equation 3.12 and its replication with a knock-in forward, since the knock-in forward replicates an economic ownership of shares at maturity T. Though, the triggering of a CoCo forces investors to accept the conversion immediately. This could lead to an economic ownership of shares at trigger time τ and, thus, prior to T. Therefore, one could argue that receiving a knock-in forward in the trigger event disregards the dividends which a shareholder would receive in particular when a CoCo triggers early. De Spiegeleer and Schoutens (2012) argue that dividends can be neglected because distressed banks are likely to behave with great restraint when it comes to dividend payments.

3.2.2 Step Two - Adding Coupons

As mentioned earlier, the first step excludes coupon payments from the valuation. Yet, in this step we want to include them in order to replicate the exact payout profile of a CoCo. Therefore, we replace the zero bond in equation 3.13 with a straight bond with regular coupon payments c. Besides, a third component has to be added which takes into account the foregone coupon payments if the trigger is touched. This can be modeled with a short position in k binary down-and-in calls with maturity t_i. Those binary down-and-in calls are knocked in if the trigger S^* is met and hence, offset all future coupon payments.

The price of a straight bond can be determined with:

$$V_t^{sb} = \sum_{i=1}^{T} c_i \exp\left[-r(t_i - t)\right] + N \exp\left[-r(T - t)\right] \tag{3.16}$$

Furthermore, the formula of Rubinstein and Reiner (1991) can be used to price the down-and-in calls:

$$\begin{aligned} V_t^{bdic} = \alpha \sum_{i=1}^{k} c_i \exp\left(-rt_i\right) \Bigg[& \Phi\left(-x_{1i} + \sigma\sqrt{t_i}\right) \\ & + \left(\frac{S^*}{S_t}\right)^{2\lambda - 2} \Phi\left(y_{1i} - \sigma\sqrt{t_i}\right) \Bigg] \end{aligned} \tag{3.17}$$

with

$$\begin{aligned} x_{1i} &= \frac{\log\left(\dfrac{S_t}{S^*}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ y_{1i} &= \frac{\log\left(\dfrac{S^*}{S_t}\right)}{\sigma\sqrt{t_i}} + \lambda\sigma\sqrt{t_i} \\ \lambda &= \frac{r - q + \dfrac{\sigma^2}{2}}{\sigma^2} \end{aligned}$$

To sum up, the theoretical price of a CoCo V^{ed} at time t pursuant the equity derivative approach consists of three components: (1) a straight bond V^{sb}, (2) a knock-in-forward V^{kifwd} and (3) a set of binary down-and-in calls V^{bdic}:

$$V_t^{ed} = V_t^{sb} + V_t^{kifwd} - V_t^{bdic} \tag{3.18}$$

3.2.3 Parameter Classification and Adjustment

The equity derivative approach requires the same model inputs as the credit derivative approach. All parameters are outlined in table 3.3. The input variables are adjusted in the same way as already described in section 3.1.3.

	Description	Usage	Source
T	Maturity	Static input	Term sheet
N	Notional	Static input	Term sheet
c	Coupon rate	Static input	Term sheet
α	Conversion factor	Static input	Term sheet
S_t	Share price	Dynamic input	Market data
S^*	Share price	Fitting parameter	–
C_p	Conversion price	Static input	Term sheet
r	Risk-free interest rate	Dynamic input	Market data
q	Dividend yield	Static input	Market data
σ_E	Share price volatility	Dynamic input	Market data

Table 3.3: Parameter classification of the equity derivative approach (Wilkens and Bethke, 2014)

3.2.4 Model Application

A fictive CoCo is priced based on the values shown in table 3.4. The source code for the equity derivative approach can be found in chapter A.2.

	Value	Comment
T	10yrs	Maturity
N	100%	Nominal
c	6.00%	Coupon rate
α	1	Conversion factor
S_0	120	Initial share price
S^*	60	Trigger share price
C_p	75	Nominal conversion price
r	3.00%	Risk-free interest rate
q	0.00%	Dividend yield
σ_E	30.00%	Share price volatility

Table 3.4: Specification of input variables for a generic CoCo

The price of the CoCo can be separated into the individual components as presented in diagram 3.1. In that sense, the value of the risk-free coupon bond makes up a major portion of a CoCo's value under the equity derivative approach. The value of the straight bond V^{sb} is equal to 125.14. The short position in the set of binary down-and-in calls V^{bdic} is equivalent to -15.88. Furthermore, the value of the knock-in forward corresponds to -1.80.

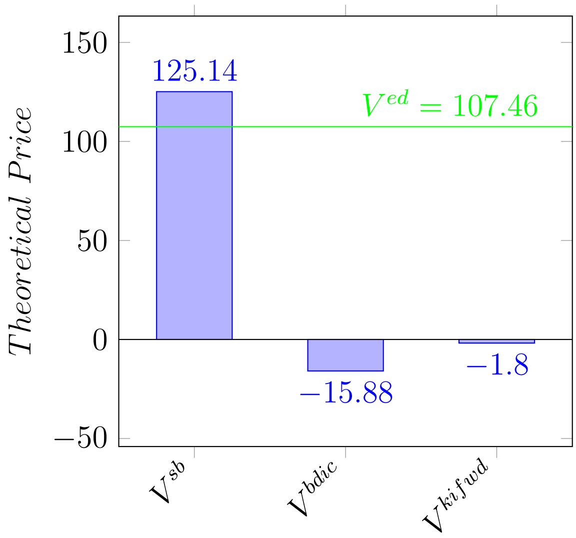

Figure 3.1: CoCo price V^{ed} and separation of its components under the equity derivative approach

Thus, the price of the fictive CoCo V^{ed} under the equity derivative approach is equal to 107.46.

3.3 Structural Approach

A third alternative to price CoCos is the structural approach of Pennacchi (2010). The idea has its roots in the seminal work of Merton (1974), which aims to explain a company's default based on the relationship of its assets and liabilities under a standard Black-Scholes setting. Pennacchi (2010)'s approach expands the idea by modeling the stochastic evolution of a bank's balance sheet respectively of its components. In the following, the assets' rate of return process will be explained. Thereafter, we will outline the assumptions of the model regarding the various liabilities a bank issues to refinance itself including deposits, equity and coupon bonds in the form of CoCos. Lastly, a pricing formula will be illustrated.

3.3.1 Structural Banking Model

Bank Assets and Asset-To-Deposit Ratio

Pennacchi (2010) assumes that a bank holds a portfolio of loans, equities and off-balance sheet positions as assets whose returns follow a jump-diffusion process. The change of this portfolio A_t is determined by the rate of return and the cash in-respectively outflows. In this context, the symbol $*$ is used to point out the change in value of the portfolio which can be quantified by the rate of return, excluding net cashflows. The aforementioned instantaneous rate of return is denoted as dA_t^*/A_t^* and follows a stochastic process as stated below under the risk-neutral probability measure $\mathbb{Q}$:

$$\frac{dA_t^*}{A_t^*} = (r_t - \lambda_t k_t)\, dt + \sigma dz + \left(Y_{q_{t-}} - 1\right) dq_t \tag{3.19}$$

It should be noted that r_t stands for the risk-free interest rate as defined by the Cox et al. (1985) term-structure model which will be discussed shortly. dz is a Brownian motion, whereby σ denotes the volatility of returns of the aforementioned asset portfolio. q_t is a Poisson counting process which increases by one whenever a Poisson-distributed event respectively a jump occurs. Hence, the variable dq_t is one whenever such a jump takes place and zero otherwise. The risk-neutral probability that a jump happens is equal to $\lambda_t dt$ where λ_t stands for the intensity of the jump process. Variable $Y_{q_{t-}}$ is a i.i.d. random variable drawn from $\ln(Y_{q_{t-}}) \sim \Phi\left(\mu_y, \sigma_y^2\right)$ at time t where μ_y stands for the mean jump size and σ_y denotes the standard deviation of jumps. In case the random variable $Y_{q_{t-}}$ is greater than one, an upward shift in the bank's asset value can be observed. If the value is smaller than one a downward jump takes place. Given that the risk-neutral expected proportional jump k_t is defined as $k_t = E_t^{\mathbb{Q}}\left[Y_{q_{t-}} - 1\right]$, one can determine k_t with the following formula: $k_t = \exp(\mu_y + \frac{1}{2}\sigma_y^2) - 1$. Thus, the risk-neutral expected change in A^* from the jump element $(Y_{q_{t-}} - 1)dq_t$ equals $\lambda_t k_t dt$ in dt. To sum up, the value development of a bank's asset portfolio A_t^* follows largely a continuous process. But disruptive jumps may occur as illustrated in the graph 3.2.

The risk-neutral process of bank assets A_t including the net cashflows is equal to the assets' rate of return less interest payments r_t respectively premium payments h_t to deposit holders proportionally to their deposits D_t. Furthermore, one has to subtract the coupon payments c_t to CoCo investors proportionally to the face value B.

$$dA_t = \left[(r_t - \lambda k)\, A_t - (r_t + h_t)\, D_t - c_t B\right] dt + \sigma A_t dz + \left(Y_{q_{t-}} - 1\right) A_t dq \tag{3.20}$$

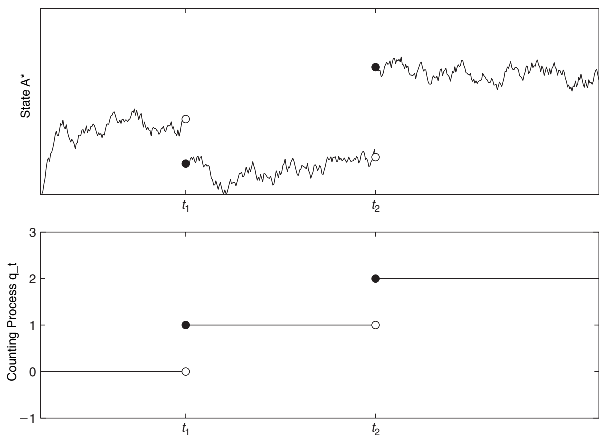

Figure 3.2: The first graph shows two jumps in the state variable A^* at discrete time points. Additionally, the corresponding Poisson counting process q_t is highlighted in the second graph. (Aït-Sahalia and Hansen, 2009)

By substituting variable x_t with A_t/D_t and anticipating the deposit growth process $g\left(x_t - \hat{x}\right)$ to behave as pointed out by equation 3.31, the risk neutral process of the asset-to-deposit ratio equals:

$$\begin{aligned} \frac{dx_t}{x_t} &= \frac{dA_t}{A_t} - \frac{dD_t}{D_t} \\ &= \left[(r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g\left(x_t - \hat{x}\right) \right] dt + \sigma dz + \left(Y_{q_{t-}} - 1 \right) dq_t \end{aligned} \tag{3.21}$$

with

$$b_t = \frac{B}{D_t} \tag{3.22}$$

Lastly, an application of Itô's lemma for jump-diffusion processes leads to the following formula for the asset-to-deposit ratio process:

$$\begin{aligned} d\ln\left(x_t\right) = &\left[(r_t - \lambda k) - \frac{r_t + h_t + c_t b_t}{x_t} - g\left(x_t - \hat{x}\right) - \frac{1}{2}\sigma^2 \right] dt \\ &+ \sigma dz + \ln Y_{q_{t-}} dq_t \end{aligned} \tag{3.23}$$

Default-Free Term Structure

Pennacchi (2010) applies the term-structure specifications of Cox et al. (1985) to model the risk-neutral process of the instantaneous risk-free interest rate dr_t which is defined as follows:

$$dr_t = \kappa\left(\bar{r} - r_t\right) dt + \sigma_r \sqrt{r_t} d\zeta \tag{3.24}$$

Note that κ is the speed of convergence, $\bar{r}$ is the long-run equilibrium interest rate, r_t is the continuous short-term interest rate, σ_r is the instantaneous volatility and $d\zeta$ is a Brownian motion.

A zero bond can be priced using the Cox et al. (1985) specifications under the no-arbitrage assumption. This implies that the price of a risk-free zero bond at time t that pays the amount of 1 currency unit in $\tau = T - t$ is given by:

$$P\left(r_t, \tau\right) = A\left(\tau\right) \exp\left[-B\left(\tau\right) r_t\right] \tag{3.25}$$

with

$$A\left(\tau\right) = \left\{ \frac{2\theta \exp\left[\left(\theta + \kappa\right) \frac{\tau}{2}\right]}{\left(\theta + \kappa\right)\left[\exp\left(\theta\tau\right) - 1\right] + 2\theta} \right\}^{2\kappa\bar{r}/\sigma_r^2}$$

$$B\left(\tau\right) = \frac{2\left[\exp\left(\theta\tau\right) - 1\right]}{\left(\theta + \kappa\right)\left[\exp\left(\theta\tau\right) - 1\right] + 2\theta}$$

$$\theta = \sqrt{\kappa^2 + 2\sigma_r^2}$$

The cost of replication of a risk-free coupon bond that pays a continuous coupon of $c_r dt$ is equal to a set of zero bonds which can be priced with equation 3.25. Therefore, the fair coupon rate c_r of such a coupon bond at time t, which is issued at par, equals:

$$\begin{aligned} c_r &= \frac{1 - A\left(\tau\right) \exp\left[-B\left(\tau\right) r_t\right]}{\int_0^\tau A\left(s\right) \exp\left[-B\left(s\right) r_t\right] ds} \\ &\approx \frac{1 - A\left(\tau\right) \exp\left[-B\left(\tau\right) r_t\right]}{\sum_{i=1}^{i=n} A\left(\Delta t \times i\right) \exp\left[-B\left(\Delta t \times i\right) r_t\right] \Delta t} \end{aligned} \tag{3.26}$$

with

$$n = \frac{\tau}{\Delta t} \tag{3.27}$$

Deposits and Insurance Premium

Bank deposits are not riskless because depositors may suffer losses if a bank's asset value A_t is worth less than the deposits D_t. That said, one can assume that a bank is closed by the deposit insurer when the asset-to-deposit ratio x_t is less or equal to one. A bank might become distressed due to continuous downward movements in its asset value. Then, the bank will be shut down with $A_{t_b} = D_t$ and subsequently, depositors will not face any loss. However, depositors may experience severe losses when a downward jump in asset value happens at a discrete point in time, $\hat{t}$. It may be that the downward jump in asset value exceeds the bank's capital. If such a jump occurs the instantaneous proportional loss to deposits will equal $\left(D_t - Y_{q_{t-}} A_{\hat{t-}}\right) / D_t$.

The fair deposit insurance premium h_t for deposit holders can be derived with equation 3.28. The equation illustrates that h_t is closely related to the asset-to-deposit ratio x_t:

$$h_t = \lambda \left[\Phi\left(-d_1\right) - x_{t^-} \exp\left(\mu_y + \frac{1}{2}\sigma_y^2 \right) \Phi\left(-d_2\right) \right] \tag{3.28}$$

with

$$d_1 = \frac{\ln\left(x_{t^-}\right) + \mu_y}{\sigma_y} \tag{3.29}$$

$$d_2 = d_1 + \sigma_y \tag{3.30}$$

The model assumes that a bank pays continuously a total interest and deposit premium of $\left(r_t + h_t\right) D_t dt$ to each depositor. Hence, one might recognize that the deposits change only because of comparatively higher deposit inflows than outflows. Empirical research of Adrian and Shin (2010) suggests that banks have a target capital ratio and that deposit growth is positively related to the current asset-to deposit ratio:

$$\frac{dD_t}{D_t} = g\left(x_t - \hat{x}\right) dt \tag{3.31}$$

$\hat{x} > 1$ is a bank's target asset-to-deposit ratio with g being a positive constant. Whenever the actual asset-to-deposit ratio is higher than its target, $x_t > \hat{x}$, a bank will shrink its balance sheet. Thus, the deposit growth rate $g\left(x_t - \hat{x}\right)$ in the time interval dt, leads to a mean-reverting tendency for the asset-to-deposit ratio x_t.

Equity and Conversion Threshold

As stated originally, the conversion of a CoCo at time t_c occurs when the asset-to-deposit ratio x_{t_c} meets the trigger level $\bar{x}_{t_c}$. The conversion threshold can also be expressed relative to the original equity-to-deposits ratio $\bar{e}$. This is favourable because the equity value is directly observable in the market whereas the asset value is not. The relationship between the equity threshold $\bar{e}$ and the asset-to-deposit threshold $\bar{x}_{t_c}$ can be summarized as follows:

$$\bar{e} = \frac{E_{t_c}}{D_{t_c}} = \frac{A_{t_c} - D_{t_c} - pB}{D_{t_c}} = \bar{x}_{t_c} - 1 - pb_{t_c} \tag{3.32}$$

Hence, it is possible to specify exactly the conversion trigger of a CoCo bond. This will be important for the valuation part.

CoCos

The valuation of a CoCo can be accomplished with a Monte Carlo simulation of both the asset and the deposit process. Along the asset-to-deposit ratio process, the CoCo pays coupons and the nominal at maturity unless the CoCo has not been triggered. If the trigger event occurs the conversion amount is paid out. (Wilkens and Bethke, 2014) The price of the CoCo V^{st} is equal to the risk-neutral expectation of the aforementioned cashflows as derived by Pennacchi (2010):

$$V_0^{st} = E_0^{\mathbb{Q}} \left[\int_0^T \exp\left(- \int_0^t r_s ds \right) v\left(t\right) dt \right] \tag{3.33}$$

Please note $v(t)$ stands for a coupon payment at date t which equals $c_t B$ as long as the CoCo has not been triggered. If the CoCo does not convert until maturity T, a final payout of B will be performed. However, if the CoCo triggers early at time t_c, there is the one-time cashflow of pB. Parameter p determines the maximum conversion amount of new equity per par value of contingent capital. Thereafter, $v(t)$ is zero.

3.3.2 Parameter Classification and Adjustment

The biggest challenge of the structural approach is the accurate estimation of its input parameters. A reliable estimation is not straightforward as most of the variables are not directly observable in the market. (De Spiegeleer et al., 2014) A complete overview of all input variables is presented in table 3.5.

In general, one can distinguish three parameter types. The first group comprises parameters which are directly observable in the market respectively in a CoCo's term sheet. The second category encompasses variables that are linked to a bank's balance sheet or strategy and are thus semi-observable. Finally, the third group covers parameters which are in fact not observable and are determined based on expert judgement or calibration to market data. (Wilkens and Bethke, 2014) Hereinafter, major input variables and their adjustment will be described.

	Description	Usage	Source
T	Maturity	Static input	Term sheet
B	Notional	Static input	Term sheet
c	Coupon rate	Static input	Term sheet
p	Conversion factor	Static input	
$\hookrightarrow S^*$	Trigger share price	Fitting parameter	–
$\hookrightarrow C_p$	Conversion price	Static input	Term sheet
x_t	Asset-to-deposit ratio	Dynamic input	
$\hookrightarrow S_t$	Share price	Dynamic input	Market data
$\hookrightarrow n_t$	Number of shares	Dynamic input	Market data
$\hookrightarrow D_t$	Deposit value	Dynamic input	Balance sheet
$\hat{x}$	Target asset-to-deposit ratio	Static input	
$\hookrightarrow A_{\text{Target}}$	Target asset value	Static input	Term sheet
$\hookrightarrow D_{\text{Target}}$	Target deposit value	Static input	Term sheet
g	Mean-reversion speed	Static input	Expert judgm.
σ_A	Annual asset return volatility	Dynamic input	
$\hookrightarrow D_t$	Deposit value	Dynamic input	Balance sheet
$\hookrightarrow S_t$	Share price	Dynamic input	Market data
$\hookrightarrow n_t$	Number of shares	Dynamic input	Market data
$\hookrightarrow \sigma_E$	Historic share price volatility	Static input	Market data
$\hookrightarrow r_t$	Risk-free interest rate	Dynamic input	Market data
λ	Jump intensity in asset return process	Static input	Expert judgm.
μ_y	Mean jump size in asset return process	Static input	
σ_y	Jump volatility in asset return process	Static input	
$\hookrightarrow S_{\text{past}}$	Historic share price data	Static input	Market data
r_t	Risk-free interest rate	Dynamic input	Market data
$\bar{r}$	Long-term risk-free interest rate	Static input	
σ_r	Interest rate volatility	Static input	
κ	Speed of convergence	Static input	
$\hookrightarrow r_{\text{past}}$	Set of historic risk-free interest rate data	Static input	Market data
ρ	Correlation between Brownian motion for asset returns and interest rate process	Static input	
$\hookrightarrow S_{\text{past}}$	Historic share price data	Static input	Market data
$\hookrightarrow r_{\text{past}}$	Historic risk-free interest rate data	Static input	Market data
$\bar{e}$	Conversion threshold of the market value of original shareholders' equity to deposit value	Static input	
$\hookrightarrow S^*$	Trigger share price	Fitting parameter	–
$\hookrightarrow n_0$	Initial number of shares	Static input	Term sheet
$\hookrightarrow D_0$	Initial deposit value	Static input	Term sheet
b_0	Ratio of the contingent capital's nominal to the initial value of deposits	Dynamic input	
$\hookrightarrow D_t$	Initial deposit value	Dynamic input	Balance sheet
$\hookrightarrow B$	Contingent capital nominal	Static input	Term sheet

Table 3.5: Parameter classification of the structural approach

Bank Assets and Asset-To-Deposit Ratio

The structural approach models the development of the asset-to-deposit ratio x_t over time. On this account, the paper attempts to obtain x_t with equity and deposit estimates. The equity component is equivalent to the daily observable market capitalization $S_t n_t$. Moreover, deposit values are inferred from a bank's quarterly published balance sheet data while assuming that all liabilities are deposits. The deposit level D_t is interpolated between the disclosure of financial statements. (Wilkens and Bethke, 2014) Hence one can determine the asset-to-deposit ratio with the following equation: $x_t = (S_t n_t + D_t)/D_t$. Furthermore, the target asset-to-deposit ratio $\hat{x}$ is driven by the strategy of the issuing bank and can be described as follows: $\hat{x} = A_{\text{Target}}/D_{\text{Target}}$.

The asset volatility σ_A is also an important input variable of the asset process but is not observable on a daily basis. Therefore, the paper evaluates the asset volatility based on its relation to the share price volatility σ_E as described by Merton (1974). The source code can be found in section C.1.2. Moreover, the structural model assumes that asset returns follow a jump-diffusion process. This implies that the probability of extreme asset returns is larger than predicted by normally distributed asset returns. To estimate the parameters that govern the distribution of the jump size, the paper assumes that historical share price returns are a reliable proxy for asset jumps. The mean jump size μ_y and the jump volatility σ_y are estimated based on threshold exceedance methods and assumptions on the jump intensity λ, which is in turn used to specify the number of exceedances over a threshold. (Longin and Solnik, 2001) The implementation is presented in section C.1.3.

Default-Free Term Structure

For the Cox et al. (1985) model pricing parameters are estimated with the approach of Remillard (2013b). The approach calibrates the long-term risk-free interest rate $\bar{r}$, the interest rate volatility σ_r and the speed of convergence κ. To do so, maximum likelihood techniques for dependent observations are applied. The approach takes a series of daily sovereign bond yields as input which have maturities of one, three, five and ten years. The implementation of this approach can be found in section C.1.1. Additionally, the correlation between the geometric Brownian motion for asset returns and the interest rate process ρ is approximated with the five year daily correlation between a sovereign bond of the same tenure respectively denomination and a related stock market index.

Deposits and Insurance Premium

A reasonable mean-reversion speed g for deposits is assumed to be equal to 0.5. The assumption follows the assessment of Pennacchi (2010). He argues that this estimate for g gives a plausible deposit's half time of around 3 years. The deposit's half-time in turn describes the time it takes for the deposit value to move half the distance towards its target value.

Equity and Conversion Threshold

Similar to the credit and equity derivative approach the conversion factor p depends on the trigger share price S^* and the conversion price C_p: $p = S^*/C_p$. Moreover, the conversion threshold $\bar{e}$ of a CoCo also relies on the trigger share price S^* as outlined by $\bar{e} = E_{\text{Trigger}}/D_0 = nS^*/D_0$.

3.3.3 Model Application

The parameters shown in table 3.6 serve to price a generic CoCo pursuant to the structural approach. The pricing results can be found in figure 3.3.

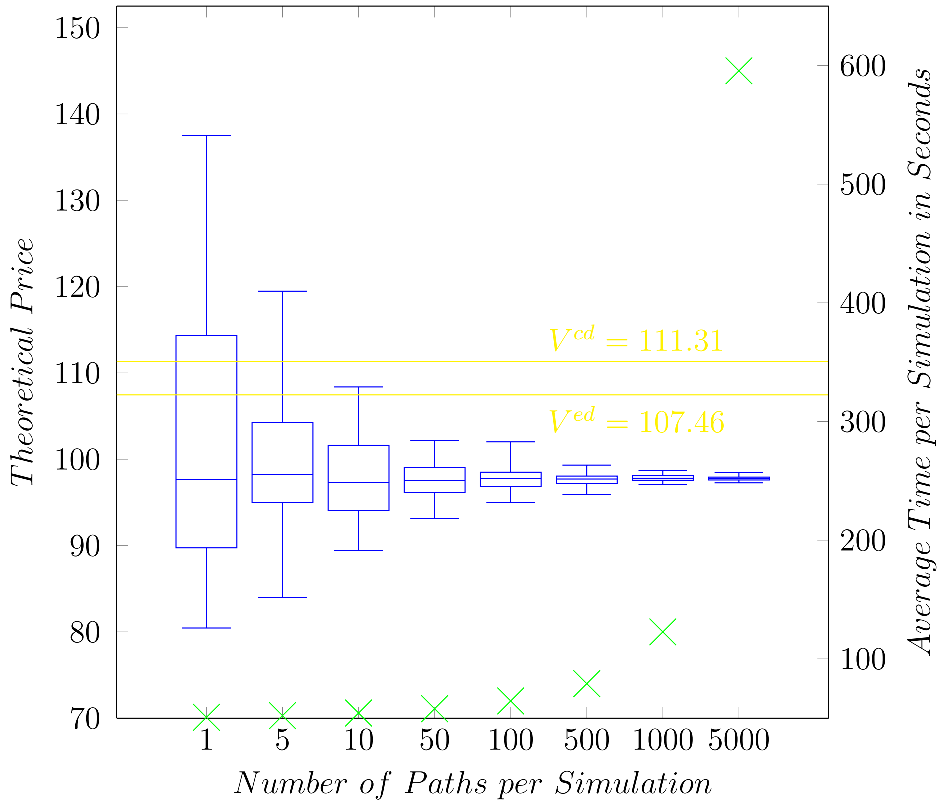

Figure 3.3: Comparison of pricing results for a generic CoCo

Running the monte-carlo simulation with different path numbers per simulation leads to a broad range of prices. As shown by the blue whisker plots one can directly see the minimum, the 25%-percentile, the median, the 75%-percentile and the maximum for each path number. One hundred simulations have been conducted for each boxplot. It becomes apparent that the price stability is tightly connected to the average time per simulation as shown by the green crosses. For 5000 paths per simulation one can derive a median price of V^{sa} of 97.77. The derived value is significantly lower than those of the credit derivative approach V^{cd} and the equity derivative approach V^{ed} which are highlighted in yellow. This might be the case due to the fact that the structural approach accounts for discontinuous returns.

	Value	Comment
T	10yrs	Maturity
B	100.00%	Notional
c	6.00%	Coupon rate
p	0.8	Conversion factor
$\hookrightarrow S^*$	60	Trigger share price
$\hookrightarrow C_p$	75	Conversion price
x_0	1.1364	Initial asset-to-deposit ratio
$\hookrightarrow S_0$	120	Initial share price
$\hookrightarrow n$	1	Number of shares
$\hookrightarrow D_0$	880	Initial deposit value
$\hat{x}$	1.12	Target asset-to-deposit ratio
$\hookrightarrow A_{\text{Target}}$	1000	Target asset value
$\hookrightarrow D_{\text{Target}}$	892.86	Target deposit value
g	10	Mean-reversion speed
σ_A	3.63%	Asset volatility
$\hookrightarrow D_0$	880	Initial deposit value
$\hookrightarrow n$	1	Number of shares
$\hookrightarrow S_0$	120	Initial share price
$\hookrightarrow \sigma_E$	30%	Historic share price volatility
$\hookrightarrow r_0$	3.00%	Initial risk-free interest rate
λ	2	Jump intensity in asset return process
μ_y	0	Mean jump size in asset return process
σ_y	2.00%	Jump volatility in asset return process
$\hookrightarrow S_{\text{past}}$		Historic share price data
r_0	3.00%	Risk-free interest rate
$\bar{r}$	6.00%	Long-term risk-free interest rate
σ_r	5.00%	Interest rate volatility
κ	4.00%	Speed of convergence
$\hookrightarrow r_{\text{past}}$		Set of historic risk-free interest rate data
ρ	50%	Correlation between Brownian motion for asset returns and interest rate process
$\hookrightarrow S_{\text{past}}$		Historic share price data
$\hookrightarrow r_{\text{past}}$		Historic risk-free interest rate data
$\bar{e}$	6.81%	Conversion threshold of the market value of shareholders' equity to original deposit value
$\hookrightarrow S^*$	60	Trigger share price
$\hookrightarrow n$	1	Number of shares
$\hookrightarrow D_0$	880	Initial deposit value
b_0	3.41%	Ratio of contingent capital's nominal to the initial deposit value
$\hookrightarrow D_0$	880	Initial deposit value
$\hookrightarrow CC$	30	Contingent capital value

Table 3.6: Specification of input variables for a generic CoCo under the structural approach

Chapter 4

Sensitivity Analyses

In the following sections the price sensitivity with respect to certain input variables of all three valuation approaches will be examined. Sensitivity analyses are especially useful to quantify the impact a variable has on the actual pricing result if it varies from what was originally assumed. This is particularly interesting as all three pricing approaches rest on different theoretical concepts. Therefore, the paper outlines a set of scenarios. In order to ensure comparability, all analyses are based on the same aforementioned fictive CoCo example.

4.1 Credit Derivative Approach

For the credit derivative approach we investigate a set of scenarios with regard to the underlying share price S, the share price volatility σ_E, a CoCo's maturity T, the risk-free interest rate r, the trigger share price S^* and the conversion price C_p. The same scenarios are also analyzed when considering the equity derivative approach because of the congruence of input parameters. Similar results are found for these two approaches.

Varying share prices S and share price volatilities σ_E have a decisive impact on CoCo prices. The effect of both variables is presented in figure 4.1. The diagram shows that higher share price levels lead to higher CoCo prices. This can be justified with the fact that it becomes less likely that the share price S falls below a predetermined trigger share price S^*. Hence, the probability that investors have to face losses due to an equity conversion is lower. The compensation for that risk respectively the conversion spreads s_{CoCo} decrease. With surging share price volatilities σ_E CoCo investors demand higher yields to compensate for rising conversion probabilities. For that reason, the conversion spread s_{CoCo} increases which in turn leads to lower CoCo

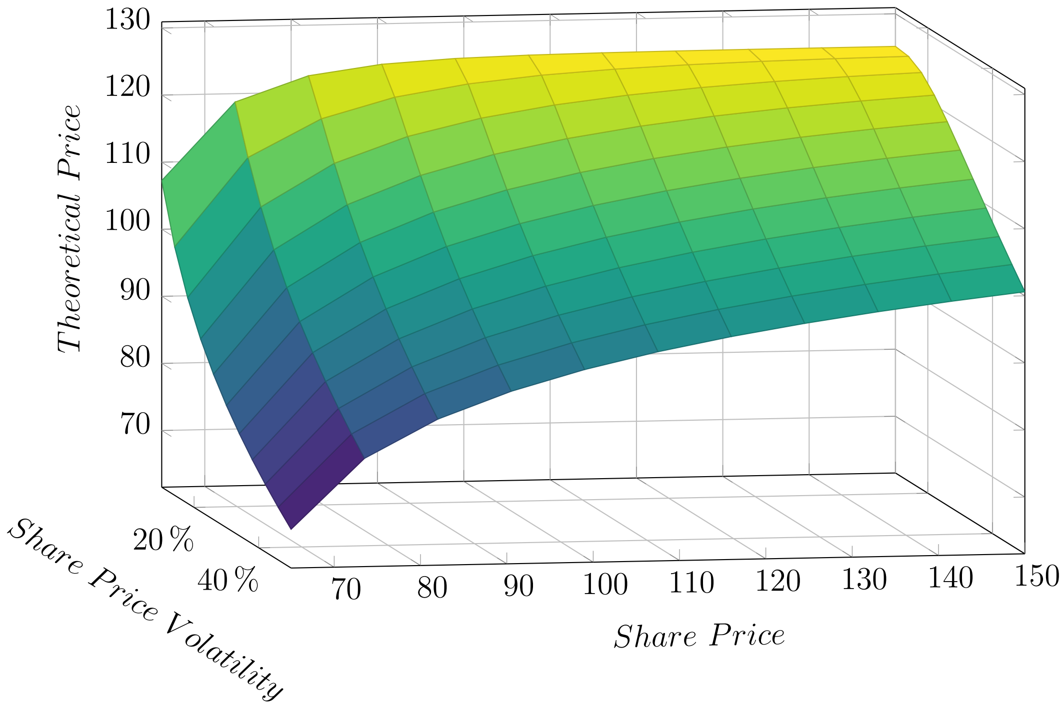

Figure 4.1: CoCo price pursuant to the credit derivative approach as function of share price S and share price volatility σ_E

prices. However, the influence of this effect diminishes with rising share prices S as it becomes generally less likely that the share price falls below the predefined trigger share price S^*.

Furthermore, figure 4.2 shows the reaction of the CoCo price due to changes in maturity T and varying levels of the risk-free interest rate r. One can observe that an increase of the risk-free interest rate leads to lower CoCo prices. By contrast, for rising maturities one can generally observe higher CoCo prices, except for the combination of high interest rates and a maturity of ten years. In this scenario the risk-free interest rate effect outweighs the maturity effect. In addition, when analyzing the development of the conversion spread s_{CoCo} one can also see that the conversion spread is at its maximum for a maturity of ten years. However, the influence of the parameter T on the value of a CoCo increases significantly with lower interest rate levels. The highest CoCo price can be found for the combination of low interest rates and high maturities. This can be explained with compounding effects and comparatively higher discount factors in particular for the notional. The effect reduces in significance with decreasing interest rates.

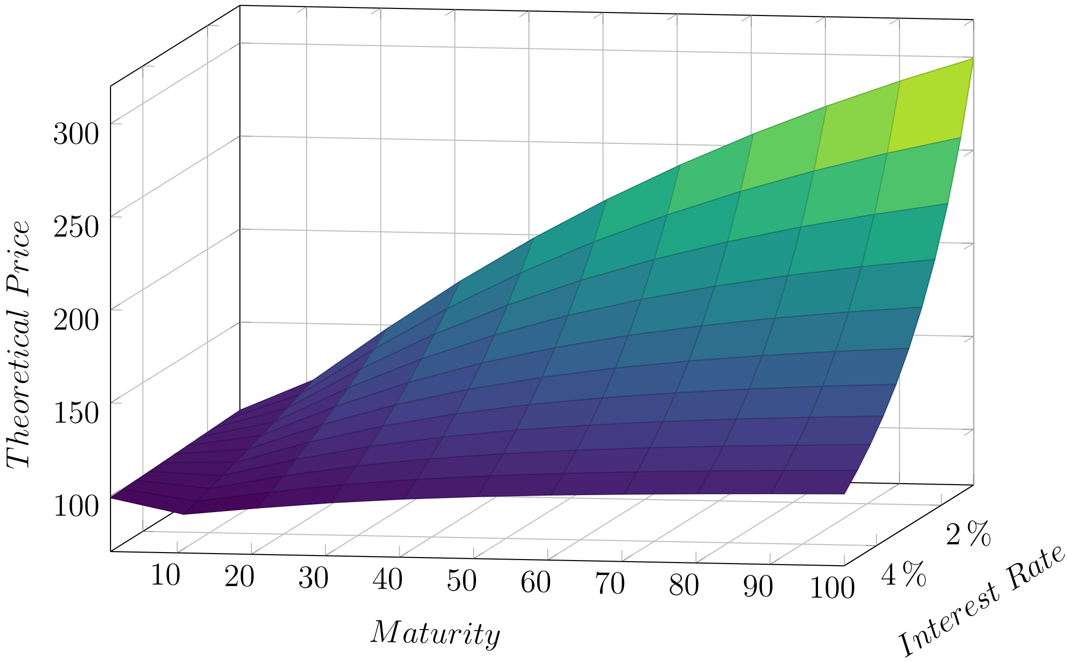

Figure 4.2: CoCo price pursuant to the credit derivative approach as function of maturity T and risk-free interest rate r

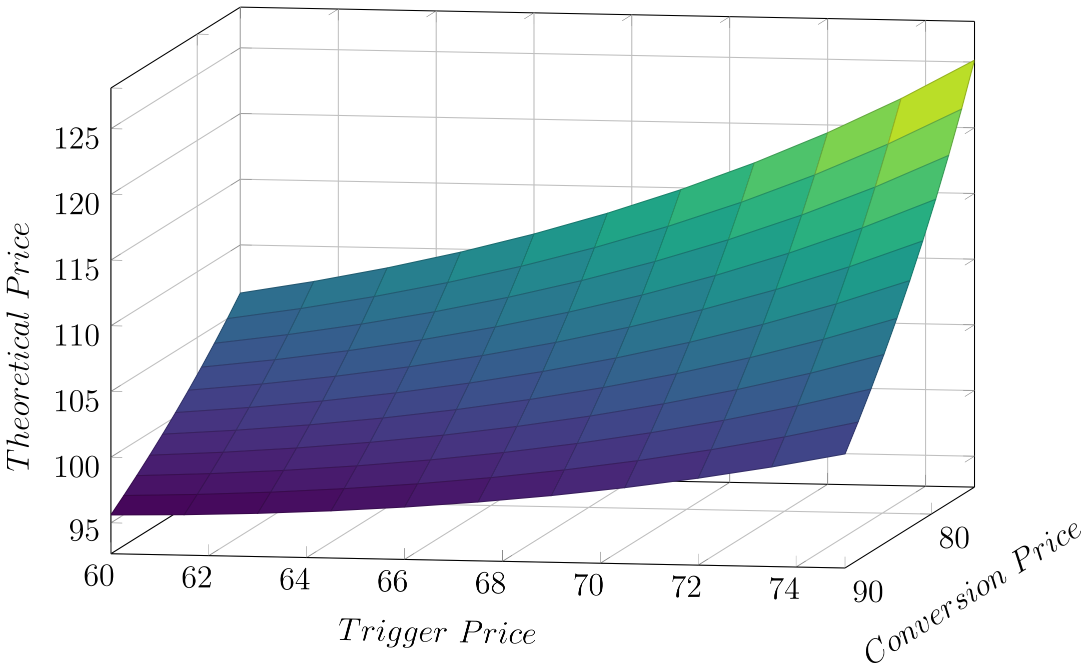

Figure 4.3: CoCo price pursuant to the credit derivative approach as function of trigger price S^* and conversion price C_p

Figure 4.3 illustrates the sensitivity of the CoCo price with respect to the trigger price S^* and the conversion price C_p. The graph reveals that lower conversion prices result in higher CoCo prices while keeping the trigger price constant. This is due to the fact that the recovery rate R_{CoCo} rises. The opposite effect is visible for low trigger prices while holding the conversion price constant. A low trigger price implies that the likelihood of conversion is lower due to a higher distance between the actual share price S and the trigger price S^*.

4.2 Equity Derivative Approach

For equity derivative approach we conduct the same sensitivity analyses much like for the credit derivative approach. Again the aim is to determine how the CoCo price is affected by changes in the model inputs.

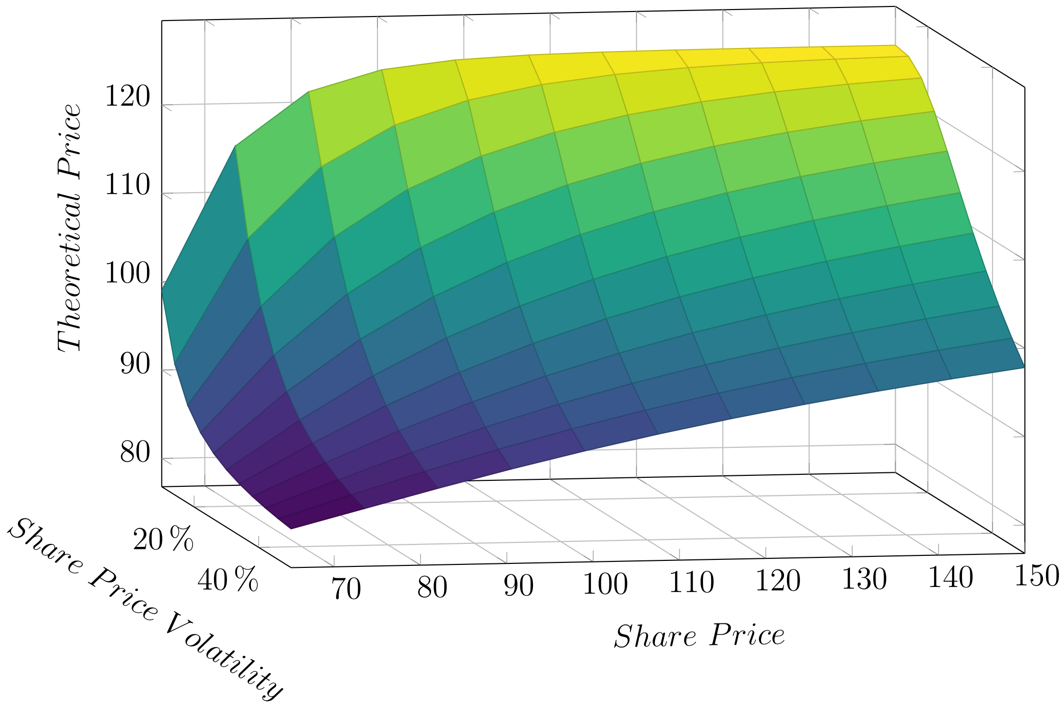

Figure 4.4: CoCo price pursuant to the equity derivative approach as function of share price S and share price volatility σ_E

Figure 4.4 helps to understand the price dynamics with respect to the share price S and the share price volatility σ_E. Generally, one might argue that with increasing share price S the distance to the trigger price S^* grows, which in turn reduces the conversion probability of the CoCo. This has a positive impact on the price. The

CoCo becomes more similar to a straight bond. Besides, the line of thought for changes of the share price volaitlity σ_E is comparable. With a rising share price volatility σ_E the risk increases that the underlying share price S hits the barrier S^* and that the CoCo investor faces a loss. Moreover, this effect can be explained with vega respectively the falling values of the short position in several down-and-in calls and the put option of the synthetic forward. Both are used under the equity derivative approach to replicate the payoff of a CoCo.

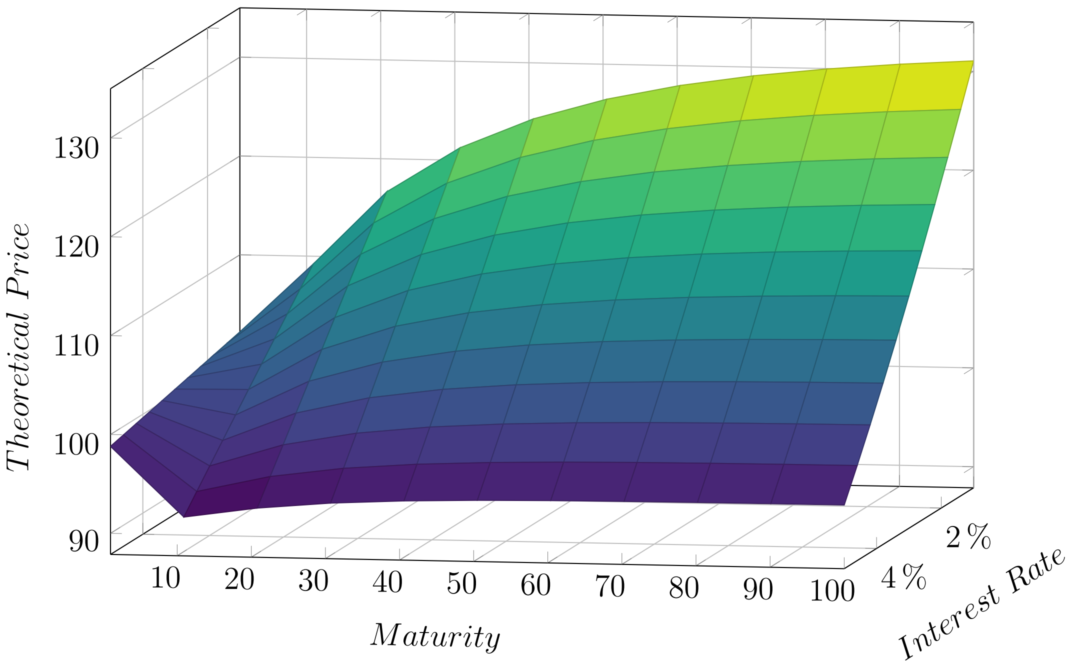

Figure 4.5: CoCo price pursuant to the credit derivative approach as function of maturity T and risk-free interest rate r

Figure 4.5 illustrates the price sensitivity of a CoCo with respect to its maturity T and the interest rate r. Considering a straight bond as major component of a CoCo helps to understand the shown price dynamics. One can observe an inverse relationship of the CoCo price and the risk-free interest rate. In addition, the price sensitivity of the CoCo with respect to the interest rate rises with its maturity. Though, the increase occurs at a decreasing rate except for a maturity smaller than ten years.

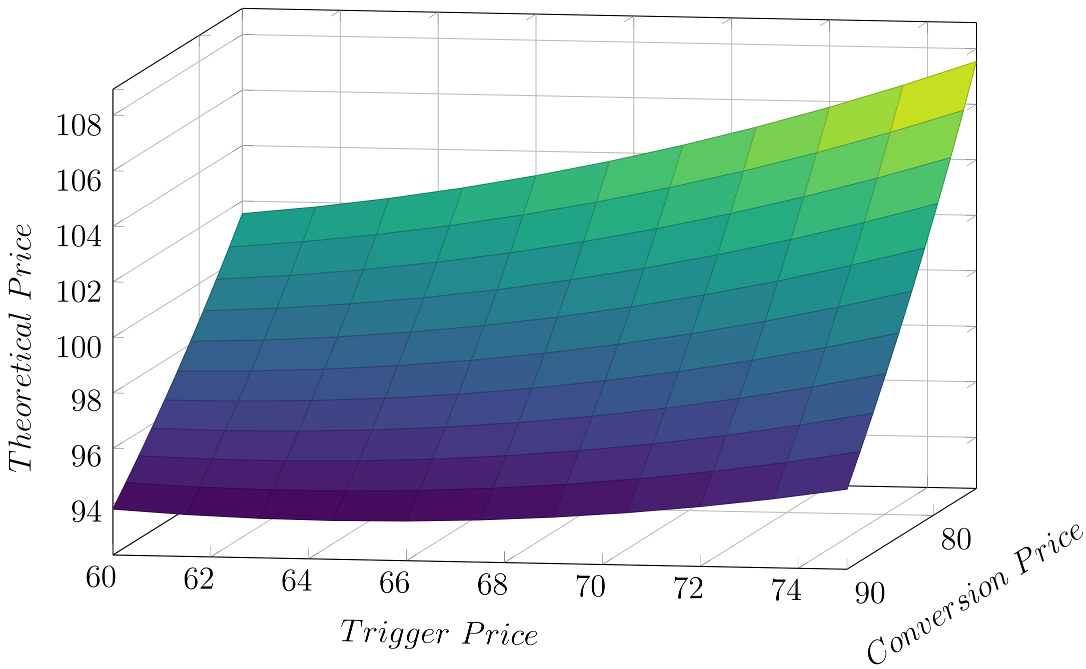

Figure 4.6: CoCo price pursuant to the credit derivative approach as function of trigger price S^* and conversion price C_p

Figure 4.6 illustrates the price sensitivity of a CoCo concerning the conversion price C_p and the trigger price S^*. In order to understand the dynamics in detail, it is advantageous to take a look at a CoCo's components under the equity derivative approach. The first component namely the long position in a straight bond is not affected by either of the variables. However, the value of the short position in a set of down-and-in calls is determined by the trigger price S^*. Moreover, the trigger price S^* and the conversion price C_p influence the price of the long position in a knock-in forward which consists of a long position in a call and a short position in a put both with strike C_p. These two options come into existence if the trigger price S^* is met. One might argue for a given trigger price S^*, the lower the conversion price C_p is the farer the knock-in forward is in the money. This in turn might be associated with a higher CoCo price. Though, these two options come only into existence if the trigger price S^* is met. Hence, the lower S^* is the lower is the probability that the knock-in forward comes into existence and the lower is the value of the position. Having said that, one has also to consider that the lower the trigger price S^* is the higher is the value of the short position in a set of binary down-and-in calls as it becomes more likely that the underlying share price S fails to remain above the trigger price S^*. These are two opposite forces, whereupon the impact of both change with decreasing conversion prices as the influence of the knock-in forward effect becomes dominant.

4.3 Structural Approach

The structural approach requires different model inputs in comparison to the other two approaches. Therefore, the focus of the following sensitivity analyses is concentrated on the initial asset-to-deposit ratio x_0, the asset volatility σ_A, maturity T, the risk-free interest rate r, the equity-to-deposit threshold $\bar{e}$, the jump intensity λ and the contingent capital's nominal to the initial value of deposits b_0.[1]

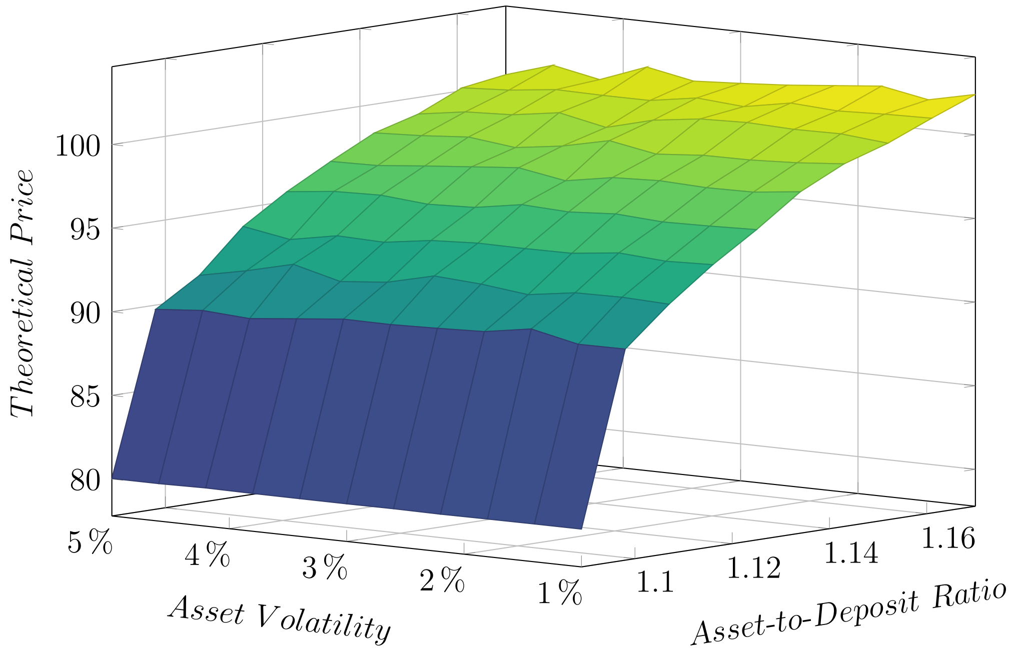

Figure 4.7: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio x_0 and asset volatility σ_A

Figure 4.7 details a CoCo's price reaction to changes of the initial asset-to-deposit ratio x_0 and the asset volatility σ_A. The diagram demonstrates that prices move closely with the asset-to-deposit ratio. This is due to the fact that financial institutions with higher initial asset-to-deposit ratios are better capitalized which reduces the conversion probability. At an asset-to-deposit ratio of 1.09, one can observe that the CoCo bond triggers. Interestingly, the analysis shows that the asset volatility does not influence a CoCo's price. Having said that, one would normally assume that a higher asset volatility is inversely related to the price because the likelihood

[1]The Monte-Carlo simulation which is used to determine the prices runs in the Amazon Elastic Compute Cloud (EC2) as the service provides a re-sizable compute capacity which is key to quickly scale the computing requirements. If one wants to replicate the simulations it is recommended to follow the instructions of Shekel (2015) to set up a Rstudio server on Amazon EC2.

of conversion rises. The lack of a stable solution for a given asset volatility σ_A might be a shortcoming of the structural approach.

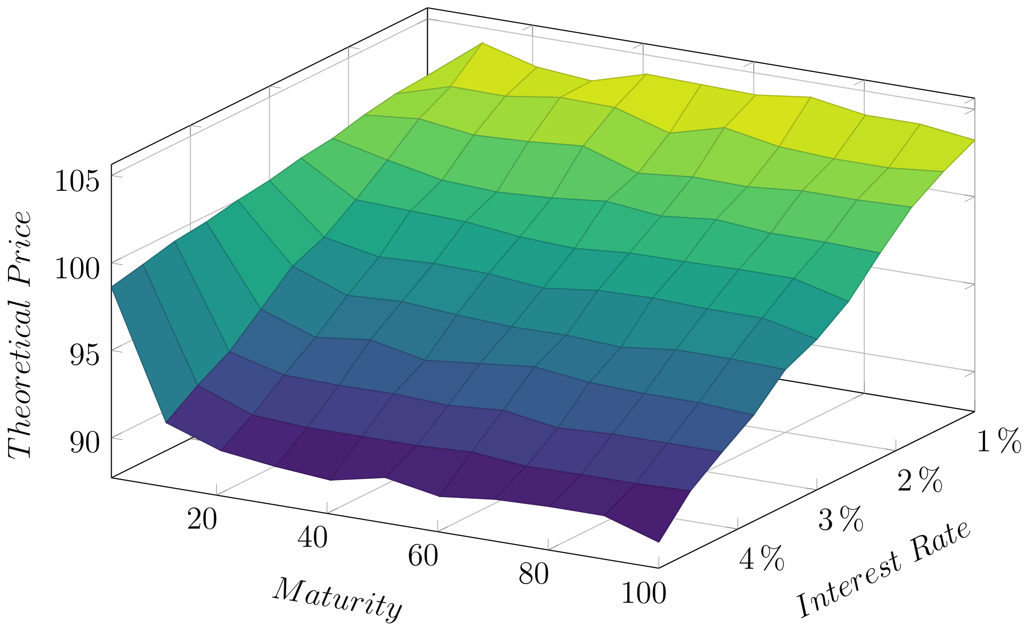

Figure 4.8: CoCo price pursuant to the structural approach as function of maturity T and interest rate r

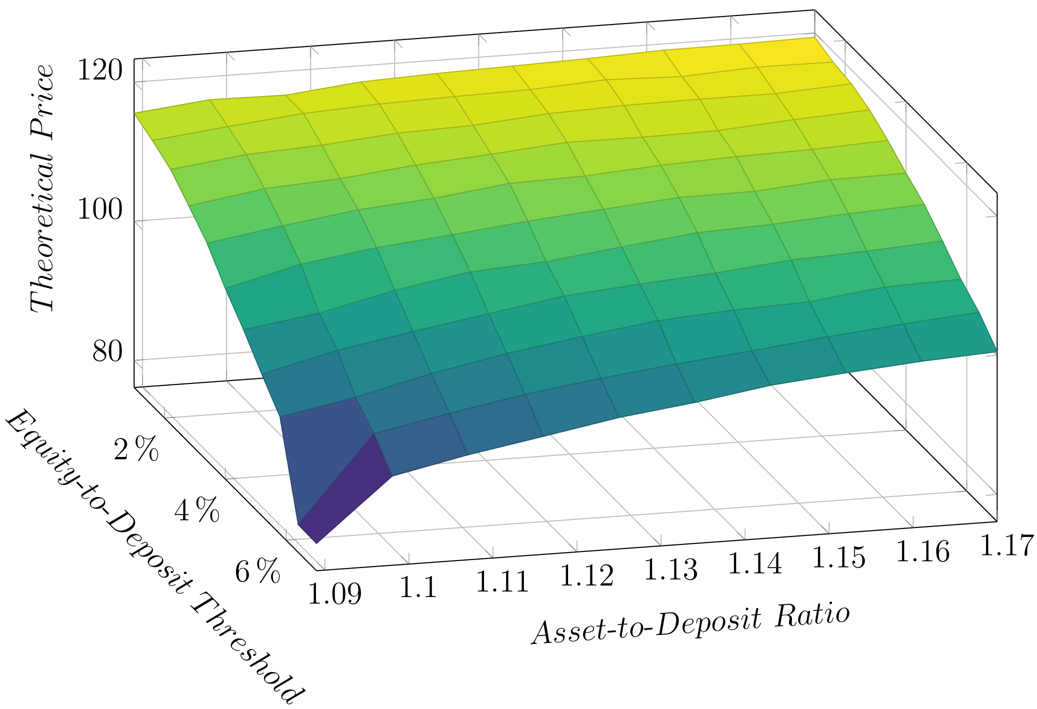

Figure 4.9: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio x_0 and equity-to-deposit threshold $\bar{e}$

Figure 4.8 exemplifies the CoCo price as function of maturity T and the interest rate r. As can be seen from the diagram, the CoCo price decreases with rising interest rates. Though, there is a difference between the structural approach and the other two approaches because the price sensitivity of the CoCo decreases with rising maturities. At a maturity of around thirty years the price sensitivity with respect to the interest rate does not change anymore. Moreover, the price does not react to changes of the maturity.

As expected figure 4.9 that the CoCo price falls with a rising equity-to-deposit threshold, which is set to be the conversion threshold for the structural approach. The explanation is similar to that of the trigger price S^* under the equity or credit derivative approach. Generally, the conversion probability rises because the equity-to-deposit threshold approaches the current share price. Furthermore, one can observe that the CoCo trigger at the combination of asset-to-deposit ratio of 1.09 and an equity-to-deposit threshold of 6.00%. Similar dynamics are also observable in figure 4.10. By increasing the ratio of initial contingent-capital's nominal to deposits ratio the one can recognize that the valuation comes down as leverage increases.

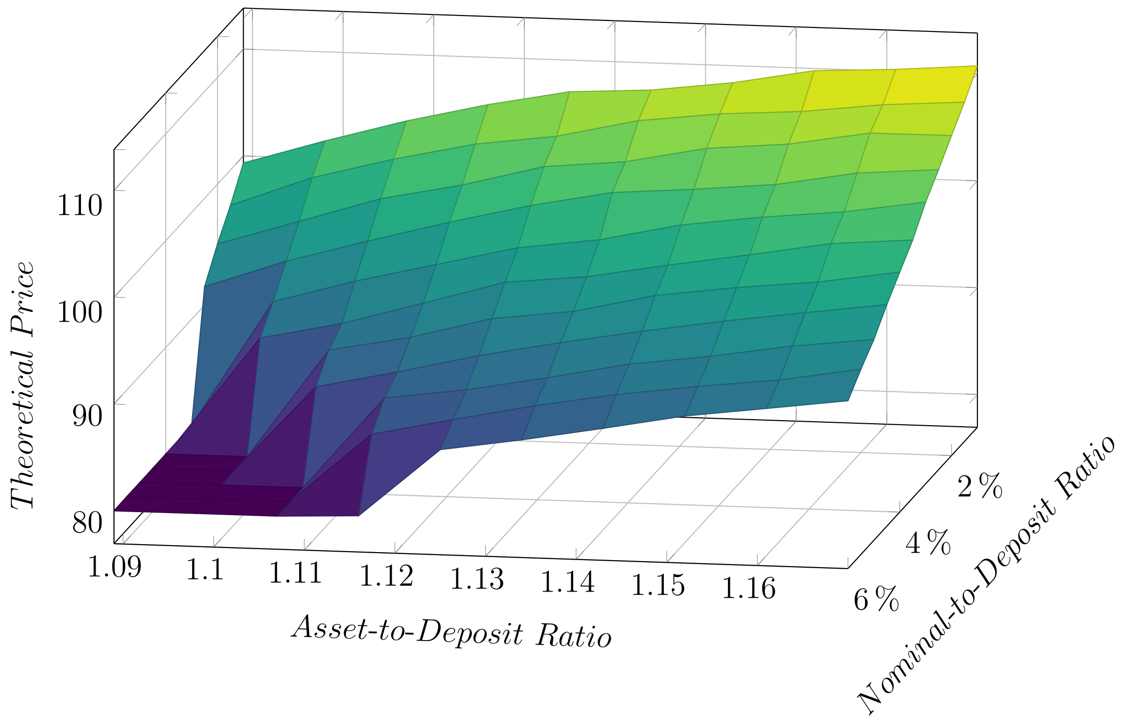

Figure 4.10: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio x_0 and initial ratio of contingent capital's nominal to the initial value of deposits b_0

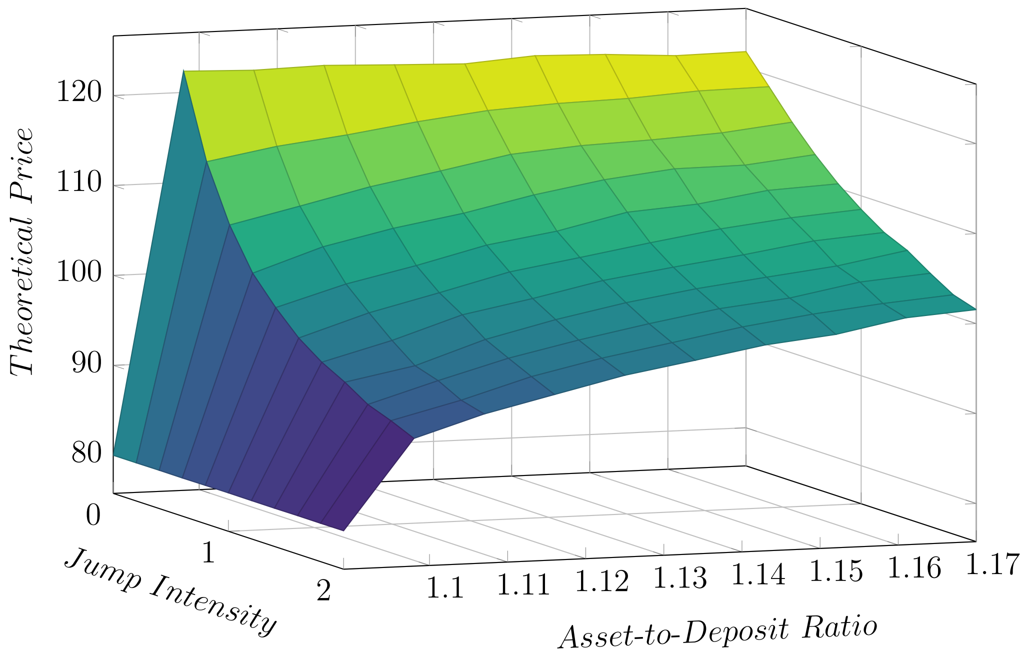

Figure 4.11: CoCo price pursuant to the structural approach as function of initial asset-to-deposit ratio x_0 and jump intensity λ

Figure 4.11 draws attention to the importance of a correct estimation of the jump intensity λ because of its significant impact on a CoCo's valuation. Rising jump intensities mean that more return jumps of both signs are expected to occur over the course of one year. This entails a higher tail risk which depresses the value of a CoCo but with a decreasing rate. The sensitivity analysis indicates, that the other two approaches underestimate the risk of severe events as they do not factor in discontinuous returns in their model. At low asset-to-deposit ratios one can observe that the depreciation of a CoCo can be severe. Undercapitalized banks bear high conversion risks.

Chapter 5

Case Study

In the following, all three approaches will be applied to a AT1 CoCo of HSBC which was issued in early 2015. The case study helps to evaluate the models with respect to their pricing tracking accuracy, parametrization complexity and calculation time.

5.1 CoCo Example

On March 30, 2015, HSBC issued its Perpetual Subordinated Contingent Convertible Securities. The aggregate principal amount from the issuance of the CoCo sums up to USD 2.475bn. HSBC intends to further strengthen its capital base with the proceeds of the issuance. The interest on the CoCo will be a rate per annum equal to 6.375%. Besides, the CoCo pays its coupon semi-annually. The conversion price is fixed ex ante at USD 4.03488. The trigger event occur if the CET1 fails to remain above a threshold of 7.0% as of any business day on which HSBC calculates the CET1 ratio. If the CoCo breaches the threshold, it converts automatically to equity.

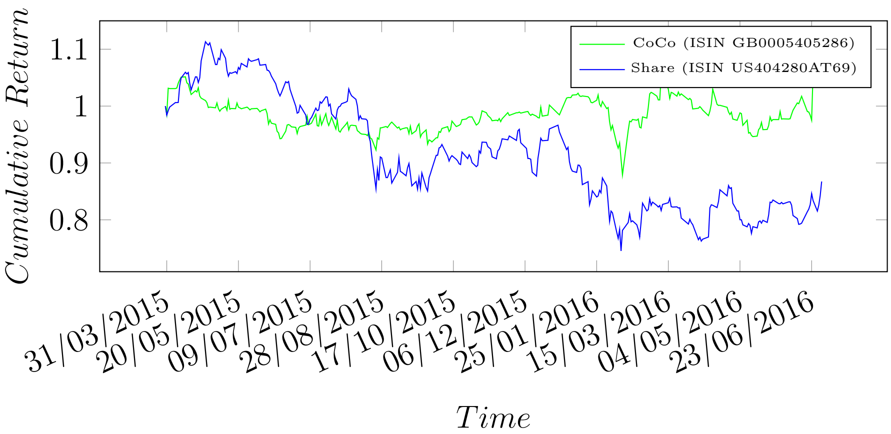

Figure 5.1: Cumulative return of HSBC's share and CoCo adjusted for GBP/USD currency effects

Figure 5.1 gives a first impression of both the return development of the reference share price (ISIN US404280AT69) and the respective CoCo (ISIN GB0005405286). As illustrated the pricing period ranges from March 31, 2015 to June, 30, 2016. The mean price of the CoCo during this period equals GBP 66.25, whereas the minimum price is GBP 56.88 respectively the maximum price amounts to GBP 74.12.

5.2 Methodology

To evaluate the price tracking accuracy of the approaches, the mean absolute error (MAE) and the root mean squared error (RMSE) are calculated for the model estimates in the aforementioned observation period. Subsequently, a brute force algorithm is applied to minimize the RMSE of each approach by varying the model implied trigger share price S^*. The approach is computationally intense and time consuming. Further research could be conducted to find other algorithms to optimize this process. After deriving a value for the model implied trigger price S^* for each approach, the MAE is calculated.

The MAE is the arithmetical average of the absolute difference between the predicted CoCo value $\hat{y}_i$ and the observed market value y_i. The MAE is given by:

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |\hat{y}_i - y_i| \tag{5.1}$$

By contrast, the RMSE represents the standard deviation of the differences between the predicted CoCo value $\hat{y}_i$ and the observed value y_i. With the following equation the value can be derived:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (\hat{y}_i - y_i)^2}{n}} \tag{5.2}$$

The derived values will help to determine the appropriateness of the models to price existing CoCos.

5.3 Valuation Results

In the following, the valuation results with respect to HSBC's CoCo will analyzed. Figure 5.2 visualizes the valuation results of the target approaches. It is interesting to see how the price estimates change by each approach. Qualitatively speaking it seems that the credit and equity derivative approach over- respectively underestimate the price of the CoCo. On the other side, the structural approach varies around the actual CoCo price.

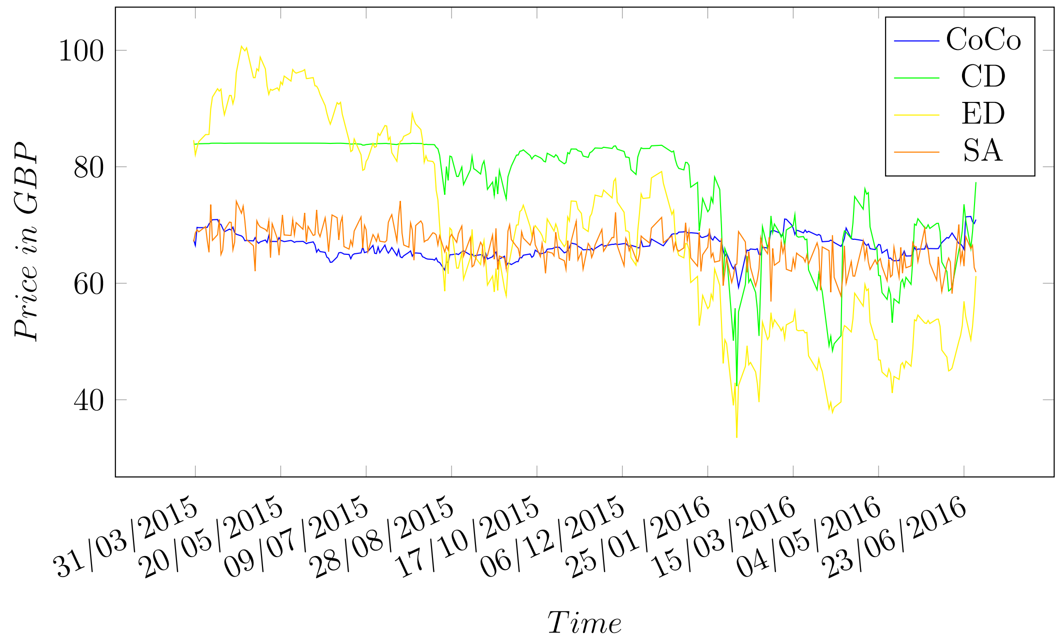

Figure 5.2: Simulation results for the CoCo of HSBC

Table 5.1 emphasizes the first visual thought as the structural approach seems to best track the actual CoCo price with a RMSE of 0.1012 and a MAE of 6.52. The other two approaches do not seem to really track the price of the CoCo. However, the model implied conversion price of the CoCo under the structural approach equals 306.12.

	Trigger price S^*	**RMSE**	**MAE**
Credit Derivative Approach	215.03	0.2031	12.34
Equity Derivative Approach	318.67	0.1974	11.65
Structural Approach	306.41	0.1012	6.52

Table 5.1: Parameter classification of the credit derivative approach

5.4 Final Remarks

	CD	ED	SA
Price tracking accuracy	low	low	medium
Parametrization complexity	low	low	high
Calculation time	low	low	high

Table 5.2: Evaluation of pricing approaches with regard to price tacking accuracy, parametrization complexity and calculation time

Chapter 6

Conclusion

The thesis has the objective to compare different valuation methods for CoCos. Three pricing approaches have been selected: the credit derivative approach, the equity derivative approach (De Spiegeleer and Schoutens, 2012) and the structural approach (Pennacchi, 2010). All approaches are brought into context to the current state of research. Apart from this, the paper provides comprehensive explanations of the theoretical concepts behind the three approaches. In addition, the models are applied consistently to a generic CoCo in order to understand their parametrization and the complexity of their implementation. An application to a real-world CoCo of HSBC allows for further insights.

The first model, the credit derivative approach, is an elegant way to price CoCos. This is partly because the parametrization to market data is straightforward and quick calculations are guaranteed. However, conceptual weakness are detectable. A closer look into the model dynamics suggests that it does not account for discontinuous returns. Inherent tail risks of CoCos are potentially underestimated. This appears to be confirmed by the fact that prices estimated by credit derivative approach are significantly higher than those of the structural approach. In addition, losses from cancelled coupons of triggered CoCos are not taken into account which also leads to an overestimation of the price. From a practical point of view, the model does not inherit an equity spot process and therefore, equity risks cannot be determined. (Turfus and Shubert, 2015)

The equity derivative approach has strengths similar to that of the credit derivative approach. Though, they also share the same conceptual flaw. They take the stock price at conversion as model input and not as stochastic output although it is very likely that equity jumps occur when the CoCo is triggered. (Turfus and Shubert,

2015) Potentially, the equity derivative approach underestimates the value of dividend payments to CoCo investors after conversion has happend, albeit the risk might be small. One might also argue that credit risk calculations are not possible since the approach does not account for them. (Turfus and Shubert, 2015)

The structural approach complies very well with the hybrid nature of CoCos as it attempts to model the dynamics of the entire balance sheet. Tail risks are also taken into account by factoring in a jump diffusion process to overcome the artificial simplification of continuous returns under a Black-Scholes setting. Though, several model inputs are necessary to apply the valuation approach to real world examples. An accurate estimation proves itself to be very difficult since certain parameters are not directly observable in the market or are only updated infrequently. The case study, however, reveals that a precise parametrization is indispensable. In addition, good algorithm design skills are necessary to cut the calculation time of the Monte-Carlo simulation.

Appendix A

Code - Models

A.1 Credit Derivative Approach

The following source code is an implementation of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2012) written in R.

```
# Price of Contingent Convertible Bond
price_coco_cd <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma){

  spread_coco <- calc_spread_coco(t, T, S_t, S_star, C_p, r, q, sigma)
  V_t_coco <- N * exp(-(r + spread_coco) * (T - t))

  for (time in seq((t+0.5), T, 0.5)){
    V_t_coco <- V_t_coco + c_i * exp(-(r + spread_coco) * time)
  }
  return(V_t_coco)
}

# Calculation of Trigger Probability
calc_p_star <- function(t, T, S_t, S_star, r, q, sigma){
  p_star <- pnorm((log(S_star / S_t) - calc_mu(r, q, sigma) * (T - t)) / (sigma * sqrt(T - t))) + (S_star / S_t)^(2 * calc_mu(r, q, sigma) / sigma^2) * pnorm((log(S_star / S_t) + calc_mu(r, q, sigma) * (T - t)) / (sigma * sqrt(T - t)))
  return(p_star)
}

# Calculation of Drift of Underlying
calc_mu <- function(r, q, sigma){
  mu <- r - q - sigma^2 / 2
  return(mu)
}

# Spread of CoCo Bond
calc_spread_coco <- function(t, T, S_t, S_star, C_p, r, q, sigma){
  spread_coco <- - log(1 - calc_p_star(t, T, S_t, S_star, r, q, sigma)) / (T - t) * (1 - S_star / C_p)
  return(spread_coco)
```

```
}

# Pricing Example
#price_coco_cd(t <- 0, T <- 10, S_t <- 120, S_star <- 60, C_p <- 75, c_i <- 6.00, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3)
```

A.2 Equity Derivative Approach

The following source code is an implementation of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2012) written in R.

```
# Price of Contingent Convertible Bond
price_coco_ed <- function(t, T, S_t, S_star, C_p, c_i, r, N, q, sigma, alpha){
  V_t_ed <- price_cb(t, T, c_i, r, N) - price_dibi(t, T, S_t, S_star, c_i, r, q, sigma, alpha) + price_difwd(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha)

  return(V_t_ed)
}

# Price of Corporate Bond
price_cb <- function(t, T, c_i, r, N){
  V_t_cb <- N * exp(-r * (T - t))

  for (time in seq((t+1), T, 1)){
  V_t_cb <- V_t_cb + c_i * exp(-r * time)
  }

  return(V_t_cb)
}

# Price of Binary Option
price_dibi <- function(t, T, S_t, S_star, c_i, r, q, sigma, alpha){
  V_t_dibi <- 0

  i <- t
  k <- T

  for (i in seq((t+1), k, 1)) {
  V_t_dibi <- V_t_dibi + c_i * exp(- r * i) * (pnorm(- calc_x_1_i(S_t, S_star, sigma, r, q, i) + sigma * sqrt(i)) + (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm ( calc_y_1_i(S_t, S_star, sigma, r, q, i) - sigma * sqrt(i)))
  }

  V_t_dibi <- alpha * V_t_dibi

  return(V_t_dibi)
}

# Price of Down-And-In Forward
```

```
price_difwd <- function(t, T, S_t, S_star, C_p, r, N, q, sigma, alpha){
  V_t_difwd <- calc_conversion_rate(C_p, N, alpha) * (S_t * exp(- q * (T - t)) * (S_star / S_t) ^ (2 * calc_lambda(r, q, sigma)) * pnorm(calc_y_1(t, T, S_t, S_star, r, q, sigma)) - C_p * exp(- r * (T - t)) * (S_star / S_t)^(2 * calc_lambda(r, q, sigma) - 2) * pnorm(calc_y_1(t, T, S_t, S_star, r, q, sigma) - sigma * sqrt(T - t)) - C_p * exp(- r * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma) + sigma * sqrt(T - t)) + S_t * exp(- q * (T - t)) * pnorm(- calc_x_1(t, T, S_t, S_star, r, q, sigma)))

  return(V_t_difwd)
}

# Calculation of Conversion Rate
calc_conversion_rate <- function(C_p, N, alpha){
  C_r <- alpha * N / C_p

  return(C_r)
}

# Calculation of additional Parameters
calc_x_1_i <- function(S_t, S_star, sigma, r, q, t_i){
  x_1_i <- log(S_t / S_star) / (sigma * sqrt(t_i)) + calc_lambda(r, q, sigma) * sigma * sqrt(t_i)

  return(x_1_i)
}

calc_y_1_i <- function(S_t, S_star, sigma, r, q, t_i){
  y_1_i <- log(S_star / S_t) / (sigma * sqrt(t_i)) + calc_lambda(r, q, sigma) * sigma * sqrt(t_i)

  return(y_1_i)
}

calc_lambda <- function(r, q, sigma){
  lambda <- (r - q + sigma^2 / 2) / sigma^2

  return(lambda)
}

calc_x_1 <- function(t, T, S_t, S_star, r, q, sigma){
  x_1 <- log(S_t / S_star) / (sigma * sqrt(T - t)) + calc_lambda(r, q, sigma) * sigma * sqrt(T - t)

  return(x_1)
}

calc_y_1 <- function(t, T, S_t, S_star, r, q, sigma){
  y_1 <- log(S_star / S_t) / (sigma * sqrt(T - t)) + calc_lambda(r, q, sigma) * sigma * sqrt(T - t)

  return(y_1)
}
```

```
# Pricing Example
#price_coco_ed(t <- 0, T <- 10, S_t <- 120, S_star <- 60, C_p <- 75, c_i <- 6.00, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
```

A.3 Structural Approach

The following source code is an implementation of the Structural Approach (Pennacchi, 2010) with support from Quantnet (2014) in translating the source code of Pennacchi (2010) from GAUSS to R.

```
# Price of Contingent Convertible Bond
price_coco_sa <- function(T , nsimulations , rho , kappa , r_bar, r0, sigma_r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0, B, coupon){

  ndays <- T * 250
  dt <- T / ndays

  # Get Brownian motions
  result <- sim_corrProcess(T, nsimulations, rho, ndays, dt)
  dz_1 <- result$dz_1
  dz_2corr <- result$dz_2corr

  # Simulate Cox et al. (1985) term-structure process
  r <- sim_interestrate(kappa, r_bar, r0, sigma_r, dz_2corr, ndays, nsimulations, dt)

  # Simulate price of contingent convertible bond with a Monte-Carlo simulation
  V_t_sa <- get_price(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0, B, coupon) * 100

  return(V_t_sa)
}

# Create correlated Brownian motions for asset and interest rate process
sim_corrProcess <- function(T, nsimulations, rho, ndays, dt){

  # Compute the Choleski factorization of a real symmetric positive-definite square matrix.
  chol_RHO <- t(chol(matrix(c(1, rho, rho, 1), nrow = 2)))

  # Random generation for the normal distribution with mean equal to 0 and standard deviation equal to 1
  dz_1 <- matrix(1, ndays, nsimulations)
  dz_2 <- matrix(1, ndays, nsimulations)
  for(j in 1:nsimulations)
  {
    dz_1[ , j] <- rnorm(ndays) * sqrt(dt)
```

```
    dz_2[ , j] <- rnorm(ndays) * sqrt(dt)
  }

  # Create correlated Brownian motions using Cholesky-decomposition for the Cox et al. (1985) term-structure process
  dz_2corr <- matrix(1, ndays, nsimulations)
  for(j in 1:nsimulations)
  {
    for(i in 1:ndays)
    {
      dz_2corr[i, j] <- dz_1[i, j] * chol_RHO[2, 1] + dz_2[i, j] * chol_RHO[2, 2]
    }
  }

  return(list("dz_1" = dz_1, "dz_2corr" = dz_2corr))
}

# Simulate Cox et al. (1985) term-structure process
sim_interestrate <- function(kappa, r_bar, r0, sigma_r, dz_2corr, ndays, nsimulations, dt){
  r <- matrix(r0, ndays + 1, nsimulations)

  for(j in 1:nsimulations)
  {
    for(i in 1:ndays)
    {
      r[i + 1, j] <- r[i, j] + kappa * (r_bar - r[i, j]) * dt + sigma_r * sqrt(abs(r[i, j])) * dz_2corr[i, j]
    }
  }

  return(r)
}

get_price <- function(nsimulations, ndays, dt, dz_1, dz_2corr, r, mu_Y, sigma_Y, lambda, g, x_hat, b0, p, e_bar, sigma_A, x0, B, coupon){

  # Define parametres
  phi <- matrix(rbinom( ndays %*% nsimulations, 1, dt * lambda), ndays, nsimulations)

  ln_Y <- matrix(rnorm(ndays %*% nsimulations, mu_Y, sigma_Y), ndays, nsimulations)

  # Ratio of contingent capital's nominal to the value of deposits
  b <- matrix(b0, ndays + 1, nsimulations)

  h <- matrix(1, ndays, nsimulations)

  # Paramter for jump diffusion process
  k <- exp(mu_Y + 0.5 * sigma_Y^2) - 1

  # Target asset-to-deposit ratio
```

```r
x_bar0 <- 1 + e_bar + p * b0
x_bar <- matrix(x_bar0, ndays + 1, nsimulations)

# Asset-to-deposit ratio
x <- matrix(x0, ndays + 1, nsimulations)
ln_x0 <- matrix(log(x0), ndays + 1, nsimulations)
ln_x <- ln_x0

trigger_dummy <- matrix(1, ndays + 1, nsimulations)

# Simulate asset-to-deposit ratio and trigger events

for(j in 1:nsimulations)
{
  for(i in 1:ndays)
  {
    d_1 <- (ln_x[i, j] + mu_Y) / sigma_Y
    d_2 <- d_1 + sigma_Y

    h[i, j] <- lambda * (pnorm( - d_1) - exp(ln_x[i, j]) * exp(mu_Y +
  0.5 * sigma_Y^2) * pnorm(-d_2))

    b[i + 1, j] <- b[i, j] * exp(- g * (exp(ln_x[i, j]) - x_hat) * dt)

    ln_x[i + 1, j] <- ln_x[i, j] + ( (r[i, j] - lambda * k) - (r[i, j]
   + h[i, j] + coupon * b[i, j]) / exp(ln_x[i, j]) - g * (exp(ln_x[i,
  j]) - x_hat) - 0.5 * sigma_A^2) * dt + sigma_A * sqrt(dt) * dz_1[i,
  j] + ln_Y[i,j] * phi[i, j]

    x[i + 1, j] <- exp(ln_x[i + 1, j])

    x_bar[i + 1, j] <- 1 + e_bar + p * b[i + 1, j]

    if(is.na(trigger_dummy[i, j]) == TRUE){
      trigger_dummy[i, j] <- trigger_dummy[i-1, j]
    }

    if(x[i + 1, j] >= x_bar[i + 1, j] && trigger_dummy[i, j] > 0.5)
    {
      trigger_dummy[i + 1, j] <- 1
    }else
    {
      trigger_dummy[i + 1, j] <- 0
    }
  }
}

cashflows <- matrix(c(rep(coupon * dt, ndays - 1), B), ndays,
  nsimulations) * trigger_dummy[1:ndays, ]

# Determine cashflows for each simulation
for(j in 1:nsimulations){
  for(i in 2:ndays){
```

```
      if(cashflows[i, j] == 0 && p * b[sum(trigger_dummy[ , j]) + 1, j] <= x[sum(trigger_dummy[ , j]) + 1, j] - 1 ){
        cashflows[i, j] <- p * B
        break
      }
      else if(cashflows[i, j] == 0 && 0 < x[sum(trigger_dummy[ , j]) + 1, j] - 1 && x[sum(trigger_dummy[ , j]) + 1, j] - 1 < p * b[sum(trigger_dummy[ , j]) + 1, j]){
        cashflows[i, j] <- (x[sum(trigger_dummy[ , j]) + 1, j] - 1) * B / b[sum(trigger_dummy[ , j]) + 1, j]
        break
      }
      else{
        cashflows[i, j] <- cashflows[i, j]
      }
    }
  }
  list_discounted_cashflows <- rep(0, nsimulations)

  # Discount cashflows for each simulation
  for(j in 1:nsimulations)
  {
    disc_cashflows <- 0
    int_r <- 0

    for(i in 1:ndays)
    {
      int_r <- int_r + r[i, j] * dt
      disc_cashflows <- disc_cashflows + exp(- int_r) * cashflows[i, j]
    }
    list_discounted_cashflows[j] <- disc_cashflows
  }

  # Calculate arithmetic average over all simulations as present value of contingent convertibles bond
  V_t_sa <- mean(list_discounted_cashflows)

  return(V_t_sa)
}

# Pricing Example
# price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.1494, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_A <- 0.0367, x0 <- 1.1364, B <- 1, coupon <- 0.06)
```

Appendix B

Code - Sensitivity Analyses

B.1 Credit Derivative Approach

The following source code is an implementation of the sensitivity analysis of the Credit Derivative Approach (De Spiegeleer and Schoutens, 2012) written in R.

```
source('CreditDerivativeApproach.R')

# CoCo price V^cd as function of share price S and volatility sigma
createData_CD_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- S_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- S_increment, S_star <- 60, C_p <- 75, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- sigma_increment)
      data[counter, 3] <- sigma_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_S_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^cd as function of maturity T and risk-free interest rate r
createData_CD_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
```

```
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- T_increment, S_t <-
    100, S_star <- 60, C_p <- 75, c_i <- 6, r <- r_increment, N <- 100,
    q <- 0.00, sigma <- 0.3)
      data[counter, 3] <- r_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_T_r_31Aug2016.txt", row.names
  = FALSE, quote=FALSE)
}

# CoCo price V^cd as function of trigger price S^* and conversion price
    C_p
createData_CD_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_
    max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S_
  star_max-S_star_min)/10)))
  {
    for(C_p_increment in seq(from=C_p_min, to=C_p_max, by=((C_p_max-C_p_
  min)/10)))
    {
      data[counter, 1] <- S_star_increment
      data[counter, 2] <- price_coco_cd(t <- 0, T <- 10, S_t <- 100, S_
  star <- S_star_increment, C_p <- C_p_increment, c_i <- 6, r <- 0.03,
  N <- 100, q <- 0.00, sigma <- 0.3)
      data[counter, 3] <- C_p_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_CD_Sstar_Cp_31Aug2016.txt", row.
  names = FALSE, quote=FALSE)
}

createData_CD_S_sigma(65.01, 150, 0.1, 0.5)
createData_CD_T_r(1, 100, 0.01, 0.05)
createData_CD_Sstar_Cp(60, 75, 75, 90)
```

B.2 Equity Derivative Approach

The following source code is an implementation of the sensitivity analysis of the Equity Derivative Approach (De Spiegeleer and Schoutens, 2012) written in R.

```
source('EquityDerivativeApproach.R')

# CoCo price V^ed as function of share price S and volatility sigma
createData_ED_S_sigma <- function(S_min, S_max, sigma_min, sigma_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_increment in seq(from=S_min, to=S_max, by=((S_max-S_min)/10)))
```

```
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- S_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- 10, S_t <- S_increment, S_star <- 60, C_p <- 75, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- sigma_increment, alpha <- 1)
      data[counter, 3] <- sigma_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_S_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^ed as function of maturity T and risk-free interest rate r
createData_ED_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- T_increment, S_t <- 100, S_star <- 60, C_p <- 75, c_i <- 6, r <- r_increment, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
      data[counter, 3] <- r_increment
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_T_r_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^ed as function of trigger price S^* and conversion price C_p
createData_ED_Sstar_Cp <- function(S_star_min, S_star_max, C_p_min, C_p_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(S_star_increment in seq(from=S_star_min, to=S_star_max, by=((S_star_max-S_star_min)/10)))
  {
    for(C_p_increment in seq(from=C_p_min, to=C_p_max, by=((C_p_max-C_p_min)/10)))
    {
      data[counter, 1] <- S_star_increment
      data[counter, 2] <- price_coco_ed(t <- 0, T <- 10, S_t <- 100, S_star <- S_star_increment, C_p <- C_p_increment, c_i <- 6, r <- 0.03, N <- 100, q <- 0.00, sigma <- 0.3, alpha <- 1)
      data[counter, 3] <- C_p_increment
```

```
      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_ED_Sstar_Cp_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

createData_ED_S_sigma(65.01, 150, 0.1, 0.5)
createData_ED_T_r(1, 100, 0.01, 0.05)
createData_ED_Sstar_Cp(60, 75, 75, 90)
```

B.3 Structural Approach

The following source code is an implementation of the sensitivity analysis of the Structural Approach (Pennacchi, 2010) written in R.

```
source('StructuralApproach.R')

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and volatility sigma
createData_SA_x0_sigma <- function(x0_min, x0_max, sigma_min, sigma_max) {
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(sigma_increment in seq(from=sigma_min, to=sigma_max, by=((sigma_max-sigma_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_x <- sigma_increment, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- sigma_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_x0_sigma_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^sa as function of maturity T and risk-free interest rate r
```

```
createData_SA_T_r <- function(T_min, T_max, r_min, r_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(T_increment in seq(from=T_min, to=T_max, by=((T_max-T_min)/10)))
  {
    for(r_increment in seq(from=r_min, to=r_max, by=((r_max-r_min)/10)))
    {
      data[counter, 1] <- T_increment
      data[counter, 2] <- price_coco_sa(T <- T_increment, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- r_increment, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_x <- 0.0363, x0 <- 1.1364, B <- 1, coupon <- 0.06)
      data[counter, 3] <- r_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_T_r_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^sa as function of initial asset-to-deposit ratio x_0 and equity-to-deposit threshold bar_e
createData_SA_x0_ebar <- function(x0_min, x0_max, ebar_min, ebar_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(ebar_increment in seq(from=ebar_min, to=ebar_max, by=((ebar_max-ebar_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- ebar_increment, sigma_x <- 0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- ebar_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
```

```
  write.table(data, file = "createData_SA_x0_ebar_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and jump intensity in asset return process lambda
createData_SA_x0_lambda <- function(x0_min, x0_max, lambda_min, lambda_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(lambda_increment in seq(from=lambda_min, to=lambda_max, by=((lambda_max-lambda_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- lambda_increment, g <- 0.5, x_hat <- 1.12, b0 <- 0.0341, p <- 0.8, e_bar <- 0.0681, sigma_x <- 0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- lambda_increment

      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_x0_lambda_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

# CoCo price V^st as function of initial asset-to-deposit ratio x_0 and initial ratio of contingent capital to deposits b0
createData_SA_x0_b0 <- function(x0_min, x0_max, b0_min, b0_max){
  data <- matrix(1, 121, 3)
  counter <- 1
  for(x0_increment in seq(from=x0_min, to=x0_max, by=((x0_max-x0_min)/10)))
  {
    for(b0_increment in seq(from=b0_min, to=b0_max, by=((b0_max-b0_min)/10)))
    {
      data[counter, 1] <- x0_increment
      data[counter, 2] <- price_coco_sa(T <- 10, nsimulations <- 5000, rho <- 0.5, kappa <- 0.04, r_bar <- 0.06, r0 <- 0.03, sigma_r <- 0.05, mu_Y <- 0.00, sigma_Y <- 0.02, lambda <- 2, g <- 0.5, x_hat <- 1.12, b0 <- b0_increment, p <- 0.8, e_bar <- 0.0681, sigma_x <- 0.0363, x0 <- x0_increment, B <- 1, coupon <- 0.06)
      data[counter, 3] <- b0_increment
```

```
      print('___')
      print(data[counter, 1])
      print(data[counter, 2])
      print(data[counter, 3])

      counter <- counter + 1
    }
  }
  write.table(data, file = "createData_SA_x0_b0_31Aug2016.txt", row.names = FALSE, quote=FALSE)
}

createData_SA_x0_sigma(1.08, 1.17, 0.01, 0.05)
createData_SA_T_r(1, 100, 0.01, 0.05)
createData_SA_x0_ebar(1.08, 1.17, 0.01, 0.07)
createData_SA_x0_lambda(1.08, 1.17, 0, 2)
createData_SA_x0_b0(1.08, 1.17, 0.01, 0.06)
```

Appendix C

Code - Case Study

C.1 Parametrization - Structural Approach

The following source code is an implementation Merton (1974) Model which is used to estimate the asset volatility written in R.

```
source('estimateCIRParameter.R')
source('estimateMertonParameter.R')
source('estimateJumpParameter.R')

calibrate_CIR <- function(cir_data){
  cir_parameters <- estimate_CIR_parameters(cir.data)

  kappa <- cir_parameters$kappa
  r_bar <- cir_parameters$r_bar
  sigma_r <- cir_parameters$sigma_r

  return(cir_parameters)
}

calibrate_Merton <- function(deposits, marketcap, r, volatility_equity){
  merton_parameters <- estimate_Merton_parameters(deposits, marketcap, r
    , volatility_equity)

  asset_volatility <- merton_parameters$asset_volatility
  asset_value <- merton_parameters$asset_value

  return(merton_parameters)
}

calibrate_Jump <- function(returns, jump_Intesity){
  jump_parameters <- estimate_Jump_parameters(returns, jump_Intensity)

  mean_jump <- jump_parameters$mean_jump
  sd_jump <- jump_parameters$sd_jump

  return(jump_parameters)
}
```

```
# CIR data
# Input data: [R,tau] (n x 2), with R: annual bonds yields in percentage and tau: maturities in years
data <- read.csv2("data/spot_interest rate.csv", header = TRUE, sep=";", dec=",", as.is=TRUE)
data[[1]] <- as.Date(data[[1]])
data <- data[rowSums(is.na(data)) == 0,]
data <- data[data$date>="2015-03-30" & data$date<="2015-03-30", ]

tau <- cbind(rep(1, nrow(data)), rep(2, nrow(data)), rep(3, nrow(data)), rep(10, nrow(data)))
tau <- as.vector(tau)

data <- cbind(data$X1.0, data$X2.0, data$X5.0, data$X10.0)
data <- as.vector(data)

cir.data <- cbind(data, tau)
cir.data <- calibrate_CIR(cir.data)

kappa <- cir.data$kappa
r_bar <- cir.data$r_bar
sigma_r <- cir.data$sigma_r


# Merton
sharepriceGBP <- read.csv2("data/final_shareprice_HSBC.csv", header = TRUE, sep=";", dec=".", as.is=TRUE)
sharepriceGBP[[1]] <- as.Date(sharepriceGBP[[1]])
sharepriceGBP <- sharepriceGBP[sharepriceGBP$date>="2014-03-29" & sharepriceGBP$date<="2016-03-29", ]
sharepriceGBP <- na.locf(sharepriceGBP)
returns <- matrix(nrow=nrow(sharepriceGBP)-1, ncol = 1)
for(z in 1:(nrow(sharepriceGBP)-1)){
  r[z,1] <- as.numeric(sharepriceGBP$shareprice[z]) / as.numeric(sharepriceGBP$shareprice[z + 1]) - 1
}


deposits <- matrix(1:1010, ncol = 1)
marketcap <- matrix(2:1011, ncol = 1)
r <- c(matrix(0.01, nrow = 253), matrix(0.02, nrow = 252), matrix(0.03, nrow = 254), matrix(0.04, nrow = 251))




volatility_equity <- matrix(0.1, nrow = 1010)

estimate_Merton_parameters(deposits, marketcap, r, volatility_equity)

# Jumps
returns <- as.timeSeries(data("bmw"))$SS.1
estimate_Jump_parameters(returns, 2)
```

C.1.1 Cox et al. (1985) Model

The following source code is an implementation of the method as described by Remillard (2013b). The method is used to calibrate the Cox et al. (1985) model which is used for the structural approach pursuant to Pennacchi (2010) based on historical data. The software is an adaption of the source code as provided by Remillard (2013a).

```
require(SMFI5)

estimate_CIR_parameters <- function(data, method = 'Hessian', days = 360
    , significanceLevel = 0.95){
  # Estimation of parameters of Cox-Ingersoll-Ross 1985 model
  R   <- data[,1]
  tau <- data[,2]
  h <- 1 / days

  # Estimation of starting parameters corresponding to those of a Feller
     process
  phi0 <- acf(R, 1, plot = FALSE)
  kappa0 <- - log(phi0[1]$acf) / h
  r_bar0  <- mean(R)
  sigma_r0 <- sd(R) * sqrt(2 * kappa0 / r_bar0)

  theta0 <- c(log(kappa0), log(r_bar0), log(sigma_r0), 0, 0)

  # Maximization of the log-Likelihood
  n <- length(R)
  optim.results <- optim(theta0, function(x) sum(LogLikCIR(x, R, tau,
    days, n)), hessian = TRUE)
  theta <- optim.results$par
  kappa <- exp(theta[1])
  r_bar  <- exp(theta[2])
  sigma_r <- exp(theta[3])

  return(list("kappa" = kappa, "r_bar" = r_bar, "sigma_r" = sigma_r))
}
```

C.1.2 Merton (1974) Model

The following source code is an implementation of the Merton (1974) model which is used to estimate the asset volatility. The code is written in R. Similar techniques are applied by Stackoverflow (2011).

```
require(nleqslv)

estimate_Merton_parameters <- function(deposits, marketcap, r,
    volatility_equity){
  # Estimation of asset value and asset volatility pursuant to Merton
    1974 model
```

```
  data <- new.env()

  for(i in 1:nrow(marketcap)){
    fnewton <- function(x){
      values <- numeric(2)
      d1 <- (log(x[1]/deposits[i])+(r[i]+x[2]^2/2))/x[2]
      d2 <- d1-x[2]

      values[1] <- marketcap[i]- (x[1]*pnorm(d1) - exp(-r[i])*deposits[i]*pnorm(d2))
      values[2]<-volatility_equity[i]*marketcap[i]-pnorm(d1)*x[2]*x[1]
      return(values)
    }
    xstart <- c(marketcap[i]+deposits[i],volatility_equity[i])
    data$asset_new_value[i]<- nleqslv(xstart, fnewton, method="Newton")$x[1]
    data$asset_new_volat[i]<-nleqslv(xstart, fnewton, method="Newton")$x[2]
  }
  return(list("asset_volatility" = data$asset_new_volat, "asset_value" = data$asset_new_value))
}
```

C.1.3 Jump-Diffusion Process

The following source code is used to estimate the jump parameters after estimating the jump intensity. The code is written in R.

```
library(fExtremes)

estimate_Jump_parameters <- function(returns, jump_Intensity){
  # Estimation of mean jump size and standard deviation of jumps

  positivThreshold <- findThreshold(returns, n = jump_Intensity / 2)
  negativThreshold <- - findThreshold(-returns, n = jump_Intensity / 2)

  jumps <- rbind(positivThreshold, negativThreshold)
  mean_jump <- mean(jumps)
  sd_jump <- sd(jumps)

  return(list("mean_jump" = mean_jump, "sd_jump" = sd_jump))
}
```

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Frankfurt am Main — August 17, 2016