




nonprobsvy – An R package for modern methods for non-probability surveys

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Abstract

The abstract of the article.

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1. Introduction

With the availability of large sets of administrative data, voluntary internet panels, social media and big data, inference with non-probability samples is being heavily studied in the statistical literature [Beaumont \(2020\)](#), [Elliott and Valliant \(2017\)](#), [Beręsewicz \(2017\)](#), [Citro \(2014\)](#). Because of their non-statistical character and unknown sampling mechanism, these sources cannot be used directly for estimating population characteristics.

Several inference approaches have been proposed in the literature with respect to data from non-probability samples, which either involve data integration with population level data or probability samples from the same population (for recent review see [Wu \(2022\)](#)). Although probability samples are still the most popular standard among statisticians, the

cost of obtaining them, in terms of time or capital, motivates the use of non-probability samples, which have to overcome other challenges. The first is that such samples generally do not represent the whole population, as can be said of probability samples. Another problem is the lack, or rather the ignorance, of the mechanism for selecting individuals for this type of sample, which does not allow the substantial use of existing statistical methods. For this reason, many different techniques have been proposed in the literature for integrating data in order to infer from available sources of different structural character.

It should be noted that there are several packages that allow the correction of selection bias in nonprobability samples, such as:

- **GJRM** ([Marra and Rodicw \(2023\)](#)) supports generalized joint regression modeling, enabling users to fit complex joint models for multivariate outcomes often needed in handling selection bias in nonprobability samples.
- **NonProbEst** ([Luis Castro Martín and del Mar Rueda \(2020\)](#)) provides methods specifically designed for estimating population parameters from nonprobability samples, employing approaches like propensity score adjustment and calibration weighting to correct for sample selection bias.
- **sampling** ([Tillé and Matei \(2021\)](#)) offers tools for implementing various sampling designs, including stratified, cluster, and unequal probability sampling, which are essential for creating representative samples and reducing selection bias in survey data analysis.

However, these packages do not implement state-of-the-art approaches recently proposed in the literature: [Chen, Li, and Wu \(2020\)](#), [Yang, Kim, and Song \(2020\)](#), [Wu \(2022\)](#) nor do they use the survey package [Lumley \(2004\)](#) for inference.

This paper describes the `nonprobsvy` package for inference with non-probability samples, available from the Comprehensive R Archive Network (CRAN) at [CRAN.R-project](#). Development version of the package can be also found at [github](#).

Table 1 shows the basic characteristics of each of the samples described. In particular, what are the advantages and disadvantages of each type of sample with respect to population coverage, bias, variance, costs, and the selection mechanism for observations into the samples.

2. Methods for non-probability samples

2.1. Basic setup

Let $U = \{1, \dots, N\}$ denote the target population consisting of N labelled units. Each unit i has an associated vector of auxiliary variables \mathbf{x}_i (a realisation of the random vector \mathbf{X}_i

Table 1: Probability and non-probability samples

Factor	Probability sample	Non-probability sample
Selection	Sampling design	Auto-selection
Coverage	Typically good	Certain groups are excluded
Bias	Typically smaller	Large or very large
Variance	Typically larger	Small, or very small
Cost	Large or very large	Typically small

in the super-population) and the study variable y_i (a realisation of the random variable Y_i in the super-population). Let $\{(y_i, \mathbf{x}_i), i \in S_A\}$ be a dataset of a non-probability sample of size n_A and let $\{(\mathbf{x}_i, \pi_i), i \in S_B\}$ be a dataset of a probability sample of size n_B , where only information about variables \mathbf{X} and inclusion probabilities π (which in the super population model are also considered to be random variables) are available. Let R_i be an indicator of inclusion into non-probability sample. Each unit in the sample S_B has been assigned a design-based weight given by $d_i = 1/\pi_i$.

The goal is to estimate a finite population mean $\mu_y = \frac{1}{N} \sum_{i=1}^N y_i$ of the target variable Y . As values of y_i are not observed in the probability sample, it cannot be used to estimate the target quantity. Instead, one could try combining the non-probability and probability samples to estimate μ_y . In this paper we do not consider modifications for the possibly occurring overlap. The above description of the data is presented in a more concise form in Table ??.

Sample	ID	Sample weight $d = \pi^{-1}$	Covariates \mathbf{x}	Study variable y
Non-probability sample (S_A)	1	?	✓	✓
	\vdots	?	\vdots	\vdots
	n_A	?	✓	✓
Probability sample (S_B)	1	✓	✓	?
	\vdots	\vdots	\vdots	?
	n_B	✓	✓	?

Table 2: Two sample setting.

2.2. Mass Imputation estimators

Imputation refers to the process of replacing missing or incomplete data with substituted values. The goal of imputation is to allow for more complete data analysis, as many statistical methods require complete datasets. Mass imputation is the application of imputation techniques to an entire dataset where many observations have missing values for the given variable. Kim, Park, Chen, and Wu (2021), Yang, Kim, and Hwang

(2021), Chlebicki, Chrostowski, and Beręsewicz (2024) propose the following imputation strategies as:

- Model based approach (GLM),
- Nearest neighbour imputation (NN),
- Predictive mean mathing (PMM).

Mass imputation is particularly useful in large datasets where missing data can be widespread, and it seeks to preserve the relationships between variables, thus improving the overall integrity of the data.

As presented in (Table ??), we do not know the value of the dependent variable Y for the units in the probability sample. In this case, the method will be to impute the values of the explanatory variable for all units in the probability sample. We therefore treat the non-probability sample as a training set that is used to build the imputation model. In this subsection, we distinguish three main methods of mass imputation based on linear models and the k-nearest neighbours algorithm. Other popular methods for estimating the variable Y from the variable \mathbf{X} can also be considered, e.g. machine learning models such as random forests or neural networks.

We can obtain an estimate of the population mean based on known design weights and an imputation model for units from the probability sample:

$$\hat{\mu}_{MI} = \frac{1}{\hat{N}_B} \sum_{i \in S_B} d_i^B \hat{y}, \quad (1)$$

$\hat{N}_B = \sum_{i \in S_B} d_i^B$ and \hat{y} is the estimated value of y for units from probability samples based on mass imputation model.

This estimator can be understood as a version of the Horvitz–Thompson estimator, which are used to estimate mean or total values in the population (based on probability sampling and inclusion probabilities). The only difference is that in our case, instead of the known values of the Y variable, we use its estimated equivalents.

Generalized Linear Models

Let us assume the following parametric model for the sample S_A based on the conditional expected value of the variable Y . Let

$$\mathbb{E}(y_i | \mathbf{x}_i) = m(\mathbf{x}_i, \boldsymbol{\beta}_0) \quad (2)$$

for a certain p -dimensional vector $\boldsymbol{\beta}_0$ and a known m function from a given class of mean functions for generalized linear models.

According to the model described, we have

$$y_i = m(\mathbf{x}_i) + \varepsilon_i, \quad i = 1, 2, \dots, N.$$

We also assume that the random variables ε_i are independent with $\mathbb{E}(\varepsilon_i) = 0$ and $\sigma^2(\varepsilon_i) = \mathbf{v}(\mathbf{x})\sigma^2$. It is assumed that $\mathbf{v}(\mathbf{x})$ has a known value and is homogeneous, i.e. homogeneous, regardless of the sample under study. Let us represent the process of mass imputation of a linear model to a sample S_B . Finally we are interested in finding a vector β solves the following equation:

$$U(\beta) = \frac{1}{n_A} \sum_{i \in S_A} \{y_i - m(\mathbf{x}_i; \beta)\} h(\mathbf{x}_i; \beta) = \mathbf{0}, \quad (3)$$

for some p -dimensional vector of function $h(\mathbf{x}_i; \beta)$, where $h(\mathbf{x}_i; \beta) = \mathbf{x}_i$ might be used for certain applications. The mass imputation process is described in Algorithm 1.

Algorithm 1 Mass imputation based on a generalized linear model

- 1: Estimate the regression model $\mathbb{E}[Y|\mathbf{X} = \mathbf{x}] = m(\mathbf{x}, \beta)$ basing on units from S_A sample.
- 2: For each $i \in S_B$, calculate the imputed value as

$$\hat{y}_i = m(\mathbf{x}_i, \hat{\beta}).$$

2.3. Nearest Neighbour Algorithm

On the other hand, it is also possible to consider a non-parametric model for the problem described, i.e. for each individual from sample S_B , the k -nearest neighbours from sample S_A are found based on the values of the auxiliary vector \mathbf{X} and the corresponding metric. Then, the missing values of the variable Y from sample S_B are replaced by the values (or their mean if more than one neighbour is considered) of this variable for the corresponding neighbours from sample S_A . The algorithm is as follows

Algorithm 2 Mass imputation using the k -nearest-neighbour algorithm

- 1: If $k = 1$, then for each $i \in S_B$ match $\hat{\nu}(i)$ such that $\hat{\nu}(i) = \arg \min_{j \in S_A} d(\mathbf{x}_i, \mathbf{x}_j)$.

- 2: If $k > 1$, then

$$\hat{\nu}(i, z) = \arg \min_{j \in S_A \setminus \bigcup_{t=1}^{z-1} \{\hat{\nu}(i, t)\}} d(\mathbf{x}_i, \mathbf{x}_j)$$

i.e. $\hat{\nu}(i, z)$ is z -th nearest neighbour from the sample.

- 3: For each $i \in S_B$, calculate the imputed value as

$$\hat{y}_i = \frac{1}{k} \sum_{t=1}^k y_{\hat{\nu}(i, t)}.$$

Note that the algorithm differs depending on the number of nearest neighbours chosen. In case $k = 1$ the nearest neighbour value is imputed according to the chosen metric, for example the Euclidean metric. In case $k > 2$ the average of the nearest neighbours values is imputed. The literature indicates that this method suffers from the so-called curse of multidimensionality, i.e. for samples with several explanatory variables, imputation can lead to a large variance in the estimator. On the other hand, the algorithm is easy to interpret and simple to implement.

2.4. Predictive Mean Matching

Predictive mean-matching imputation is a particularly well-known way of dealing with non-response among respondents, and is favoured by statistical offices for compiling a country's official population statistics. It is a version of the k-nearest neighbour algorithm, but instead of looking at the distances between the vectors of the auxiliary variables, it looks at the distance between the functions of the mean vectors. This helps to reduce the curse of multidimensionality and, at the same time, allows the observed values of the explanatory variable or their mean to be calculated. Let us therefore present two algorithms that describe the steps to follow to perform a mass imputation using the mean matching method.

Algorithm 3 $\hat{y} - \hat{y}$ Imputation:

1: Estimate regression model $\mathbb{E}[Y|\mathbf{X} = \mathbf{x}] = m(\mathbf{x}, \boldsymbol{\beta})$.

2: Impute

$$\hat{y}_i = m(\mathbf{x}_i, \hat{\boldsymbol{\beta}}), \hat{y}_j = m(\mathbf{x}_j, \hat{\boldsymbol{\beta}})$$

for $i \in S_B, j \in S_A$ and assign each $i \in S_B$ to $\hat{\nu}(i)$, where

$$\hat{\nu}(i) = \arg \min_{j \in S_A} \|\hat{y}_i - \hat{y}_j\|$$

or

$$\hat{\nu}(i) = \arg \min_{j \in S_A} d(\hat{y}_i, \hat{y}_j)$$

if d is not induced by the norm.

3: If $k > 1$, then:

$$\hat{\nu}(i, z) = \arg \min_{\substack{j \in S_A \setminus \bigcup_{t=1}^{z-1} \{\hat{\nu}(i, t)\}}} d(\hat{y}_i, \hat{y}_j)$$

e.g., $\hat{\nu}(i, z)$ is z -th nearest neighbor from a sample.

4: For $i \in S_B$, calculate imputation value as

$$\hat{y}_i = \frac{1}{k} \sum_{t=1}^k y_{\hat{\nu}(i, t)}.$$

Algorithm 4 $\hat{y} - y$ Imputation:

- 1: Estimate regression $\mathbb{E}[Y|\mathbf{X} = \mathbf{x}] = m(\mathbf{x}, \beta)$.
- 2: Impute $\hat{y}_i = m(\mathbf{x}_i, \hat{\beta})$ for $i \in S_B$ and assign each $i \in S_B$ do $\hat{\nu}(i)$, where $\hat{\nu}(i) = \arg \min_{j \in S_A} \|\hat{y}_i - y_j\|$ or $\hat{\nu}(i) = \arg \min_{j \in S_A} d(\hat{y}_i, y_j)$ if d not induced by the norm.
- 3: If $k > 1$, then:

$$\hat{\nu}(i, z) = \arg \min_{j \in S_A \setminus \bigcup_{t=1}^{z-1} \{\hat{\nu}(i, t)\}} d(\hat{y}_i, y_j).$$

- 4: For each $i \in S_B$ calculate imputation value as

$$\hat{y}_i = \frac{1}{k} \sum_{t=1}^k y_{\hat{\nu}(i, t)}.$$

As can be seen, the difference between the two algorithms is due to step 2. In the first approach, we compare \hat{y} from samples S_A and S_B . The second, on the other hand, compares \hat{y} from sample S_B with the known y from sample S_A . It is worth noting that proof of the consistency of these estimators can be found in [Chlebicki et al. \(2024\)](#).

2.5. Inverse Probability Weighting estimators

The main disadvantage of non-probability sampling is the unknown selection mechanism for a unit to be included in the sample. This is why we talk about the so-called biased sample problem. The inverse probability approach is based on the assumption that a reference probability sample is available and therefore we can estimate the propensity score of the selection mechanism. In recent years, a number of articles have addressed this issue. [Chen et al. \(2020\)](#) propose maximum likelihood estimation approach for estimating propensity scores for selection mechanism. [Wu \(2022\)](#) present the approach based on generalized estimating equations, this method is also mentioned in [Yang et al. \(2020\)](#). On the other hand calibration approach for quantiles was explained [Beręsewicz and Szymkowiak \(2024\)](#) and [Sant'Anna, Song, and Xu \(2022\)](#) present the approach based on maximize the covariate distribution balance among different treatment groups.

In the formal framework, let us introduce the following assumptions for propensity score model, which will imply a number of properties derived in the thesis.

- (A1) The selection indicator R_i^A and explanatory variable y_i are independent.
- (A2) All units have a so-called non-probability sample propensity score, which is non-zero, i.e. $\pi_i^A > 0$, where $\pi_i^A = P_q(R_i^A = 1 | \mathbf{x}_i, y_i)$, where q refers to the model for the selection mechanism for the non-probability sample (propensity score model).
- (A3) Indicator variables R_i^A and R_j^A are independent with $i \neq j$.

The estimated propensity score is used to construct an inverse probability weighting estimator of the population mean of the form

$$\hat{\mu}_{IPW} = \frac{1}{\hat{N}^A} \sum_{i \in S_A} \frac{y_i}{\hat{\pi}_i^A}. \quad (4)$$

where $\hat{N}^A = \sum_{i \in S_A} \hat{d}_i^A = \sum_{i \in S_A} \frac{1}{\hat{\pi}_i^A}$.

Maximum Likelihood Estimation

Consider the following likelihood function

$$\begin{aligned} \ell(\boldsymbol{\theta}) &= \sum_{i=1}^N \left\{ R_i^A \log \pi_i^A + (1 - R_i^A) \log (1 - \pi_i^A) \right\} \\ &= \sum_{i \in S_A} \log \left\{ \frac{\pi(\mathbf{x}_i, \boldsymbol{\theta})}{1 - \pi(\mathbf{x}_i, \boldsymbol{\theta})} \right\} + \sum_{i=1}^N \log \{1 - \pi(\mathbf{x}_i, \boldsymbol{\theta})\} \end{aligned} \quad (5)$$

In practice, a function of this form cannot be used because we do not observe all units from the population. Hence, the second component of the function is replaced by the Horvitz-Thompson estimator, which is used when having access to the design weights for the units in the sample. In our case, these will be the weights d_i^B for the units in the sample S_B . We then have

$$\ell^*(\boldsymbol{\theta}) = \sum_{i \in S_A} \log \left\{ \frac{\pi(\mathbf{x}_i, \boldsymbol{\theta})}{1 - \pi(\mathbf{x}_i, \boldsymbol{\theta})} \right\} + \sum_{i \in S_B} d_i^B \log \{1 - \pi(\mathbf{x}_i, \boldsymbol{\theta})\}. \quad (6)$$

Our objective is to find the maximum likelihood estimator $\hat{\pi}_i^A = \pi(\mathbf{x}_i, \hat{\boldsymbol{\theta}})$, such that $\hat{\boldsymbol{\theta}}$ maximises the function defined above.

Generalized Estimating Equations

Equations of the type $\mathbf{U}(\boldsymbol{\theta}) = \mathbf{0}$, where $\mathbf{U}(\boldsymbol{\theta}) = \frac{\partial}{\partial \boldsymbol{\theta}} \ell^*(\boldsymbol{\theta})$, obtained from the maximum likelihood estimation can be replaced by a system of generalized estimating equations of the form

$$\mathbf{G}(\boldsymbol{\theta}) = \sum_{i \in S_A} h(\mathbf{x}_i, \boldsymbol{\theta}) - \sum_{i \in S_B} d_i^B \pi(\mathbf{x}_i, \boldsymbol{\theta}) h(\mathbf{x}_i, \boldsymbol{\theta}) = \mathbf{0}, \quad (7)$$

where $h(\mathbf{x}_i, \boldsymbol{\theta})$ is a certain continuous function. In the literature, the most commonly considered functions are $h(\mathbf{x}_i, \boldsymbol{\theta}) = \mathbf{x}_i$ and $h(\mathbf{x}_i, \boldsymbol{\theta}) = \mathbf{x}_i \pi(\mathbf{x}_i, \boldsymbol{\theta})^{-1}$. Note that if the function h is equal to the vector of observed characteristics \mathbf{x} , then \mathbf{G} is reduced to

$$\mathbf{G}(\boldsymbol{\theta}) = \sum_{i \in S_A} \mathbf{x}_i - \sum_{i \in S_B} d_i^B \pi(\mathbf{x}_i, \boldsymbol{\theta}) \mathbf{x}_i.$$

In the next subsection we will proof that this is disorted version of MLE approach with π_i^A modelling by logistic regression. If we use the second form of the function h we get

the following form of the function \mathbf{G}

$$\mathbf{G}(\boldsymbol{\theta}) = \sum_{i \in S_A} \frac{\mathbf{x}_i}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} - \sum_{i \in S_B} d_i^B \mathbf{x}_i.$$

The advantage of this method is the ability to estimate with global values of the variables (e.g. from external sources) instead of a probability sample. Note that this is allowed by the second form of the \mathbf{G} function. Its second term is nothing more than the estimated sums of the \mathbf{x} variables. On the other hand, empirical studies suggest that the process of solving this type of equation may be less stable than the maximum likelihood method. In other words, the iterative algorithm for finding zeros that satisfy equation (7) may not converge. In the `nonprobsvy` package the propensity scores can be modelled using three different link functions: logistic, complementary log-log and probit. The logistic regression is the most commonly used for modelling probabilities. This method is based on the so-called sigmoidal function and for our setting it has the form It satisfies certain properties to model the probability. For our scheme, this will be the probability of belonging to the non-probability sample. Another approach to modelling binary variables and also probabilities is probit regression. It is based on the standard normal distribution and as logit is also symmetric around $p = 0.5$. On the other hand regression with the cloglog model is particularly useful if we are modelling rare phenomena, i.e. the probabilities will oscillate around the values 1 and 0. Compared to the sigmoidal function and the distribution, the cloglog function is more asymmetric towards the value 0.5.

Table 3: MLE Functions and Gradients for Different Link Functions

Link	MLE Function	Gradient
logit	$\sum_{i \in S_A} \mathbf{x}_i^\top \boldsymbol{\theta}$ $- \sum_{i \in S_B} d_i^B \log [1 + \exp(\mathbf{x}_i^\top \boldsymbol{\theta})]$	$\sum_{i \in S_A} \mathbf{x}_i$ $- \sum_{i \in S_B} d_i^B \pi(\mathbf{x}_i, \boldsymbol{\theta}) \mathbf{x}_i$
probit	$\sum_{i \in S_A} \log \left(\frac{\Phi(\mathbf{x}_i^\top \boldsymbol{\theta})}{1 - \Phi(\mathbf{x}_i^\top \boldsymbol{\theta})} \right)$ $+ \sum_{i \in S_B} d_i^B \log [1 - \Phi(\mathbf{x}_i^\top \boldsymbol{\theta})]$	$\sum_{i \in S_A} \frac{\phi(\mathbf{x}_i^\top \boldsymbol{\theta})}{\Phi(\mathbf{x}_i^\top \boldsymbol{\theta})[1 - \Phi(\mathbf{x}_i^\top \boldsymbol{\theta})]} \mathbf{x}_i$ $- \sum_{i \in S_B} d_i^B \frac{\phi(\mathbf{x}_i^\top \boldsymbol{\theta})}{1 - \Phi(\mathbf{x}_i^\top \boldsymbol{\theta})} \mathbf{x}_i$
cloglog	$\sum_{i \in S_A} \{ \log [1 - \exp(-\exp(\mathbf{x}_i^\top \boldsymbol{\theta}))] \}$ $+ \exp(\mathbf{x}_i^\top \boldsymbol{\theta}) - \sum_{i \in S_B} d_i^B \exp(\mathbf{x}_i^\top \boldsymbol{\theta})$	$\sum_{i \in S_A} \frac{\exp(\mathbf{x}_i^\top \boldsymbol{\theta}) \mathbf{x}_i}{\pi(\mathbf{x}_i, \boldsymbol{\theta})}$ $- \sum_{i \in S_B} d_i^B \exp(\mathbf{x}_i^\top \boldsymbol{\theta}) \mathbf{x}_i$

2.6. Doubly Robust estimators

The inverse probability weighting and mass imputation estimators are sensible on misspecified models for propensity score and outcome variable respectively. For this purpose so called doubly-robust methods, which take into account these problems, are presented.

The proposed estimation procedure addresses the challenge of combining data from non-probability and probability survey samples. Traditional semiparametric models, often applied to such problems, are not directly usable in this context due to the distinct nature of the two samples. Instead, a joint randomization framework is employed, integrating semiparametric models for propensity scores with outcome regression for the nonprobability sample and design-based inference from the probability sample. This framework leads to a doubly robust (DR) estimation approach, which is effective in the presence of model misspecifications.

Inverse Probability Weighted (IPW) estimators are sensitive to misspecified propensity score models, particularly when propensity scores are very small. To improve robustness and efficiency, the doubly robust method incorporates a prediction model for the response variable. Moreover, even if one of the models is misspecified, the DR estimator remains consistent, showcasing the “double robustness” property.

Joint Randomization Approach

The joint randomization approach combines two processes: the selection mechanism of a non-probability sample, modelled by propensity scores, and the design-based inference from a probability sample.

The response y_i is predicted using a regression model $m(\mathbf{x}_i, \beta)$ (or NN/PMM methods), where β is estimated from the non-probability sample. With known design weights d_i^B for $i \in S_B$ we can define the DR estimator as

$$\hat{\mu}_{\text{DR}} = \frac{1}{\hat{N}^A} \sum_{i \in S_A} d_i^A \{y_i - m(\mathbf{x}_i, \hat{\beta})\} + \frac{1}{\hat{N}^B} \sum_{i \in S_B} d_i^B m(\mathbf{x}_i, \hat{\beta}), \quad (8)$$

where $d_i^A = \pi(\mathbf{x}_i, \theta)^{-1}$, $\hat{N}^A = \sum_{i \in S_A} d_i^A$ and $\hat{N}^B = \sum_{i \in S_B} d_i^B$.

It remains consistent if either the propensity score model $\pi(\mathbf{x}_i, \theta)$ or the outcome regression model $m(\mathbf{x}_i, \beta)$ is correctly specified.

The joint randomization approach ensures robustness by accounting for randomness in both the non-probability sample through $\pi(\mathbf{x}_i, \theta)$ and the probability sample through design-based inference.

Minimization of the bias for doubly robust methods

By reducing the variance of the estimators, for example by variable selection, we cannot control the bias of the estimator, which may increase. Therefore, according to [Yang et al. \(2020\)](#), the idea is to determine the equations leading to the estimation of the β and θ parameters based on the bias of the population mean estimator. In contrast to the joint randomization approach, this method allows for the estimation of the parameters β and θ in a single step, rather than in two separate steps.

We will first present the bias of the doubly robust estimator and then, using optimisation techniques, discuss the equations leading to its minimization. Thus we have

$$\begin{aligned} \text{bias}(\hat{\mu}_{DR}) &= |\hat{\mu}_{DR} - \mu| \\ &= \frac{1}{N} \sum_{i=1}^N \left\{ \frac{R_i^A}{\pi_i^A(\mathbf{x}_i^T \boldsymbol{\theta})} - 1 \right\} \{y_i - m(\mathbf{x}_i^T \boldsymbol{\beta})\} \\ &\quad + \frac{1}{N} \sum_{i=1}^N (R_i^B d_i^B - 1) m(\mathbf{x}_i^T \boldsymbol{\beta}) \end{aligned} \quad (9)$$

To minimize $\text{bias}(\hat{\mu}_{DR})^2$ let us calculate the gradient of the square of the bias at $(\boldsymbol{\beta}, \boldsymbol{\theta})$. We then have

$$\frac{\partial \text{bias}(\hat{\mu}_{DR})^2}{\partial (\boldsymbol{\beta}^T, \boldsymbol{\theta}^T)^T} = 2 \text{bias}(\hat{\mu}_{DR}) J(\boldsymbol{\theta}, \boldsymbol{\beta}),$$

where

$$J(\boldsymbol{\theta}, \boldsymbol{\beta}) = \begin{pmatrix} J_1(\boldsymbol{\theta}, \boldsymbol{\beta}) \\ J_2(\boldsymbol{\theta}, \boldsymbol{\beta}) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^N R_i^A \left\{ \frac{1}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} - 1 \right\} \{y_i - m(\mathbf{x}_i, \boldsymbol{\beta})\} \mathbf{x}_i \\ \sum_{i=1}^N \frac{R_i^A}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} \frac{\partial m(\mathbf{x}_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} - \sum_{i \in S_B} d_i^B \frac{\partial m(\mathbf{x}_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} \end{pmatrix},$$

which leads to the problem of solving the following system of equations

$$\begin{pmatrix} \sum_{i=1}^N R_i^A \left\{ \frac{1}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} - 1 \right\} \{y_i - m(\mathbf{x}_i, \boldsymbol{\beta})\} \mathbf{x}_i \\ \sum_{i=1}^N \frac{R_i^A}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} \frac{\partial m(\mathbf{x}_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} - \sum_{i \in S_B} d_i^B \frac{\partial m(\mathbf{x}_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} \end{pmatrix} = \mathbf{0}, \quad (10)$$

which can be solved using Newton–Raphson optimization method.

3. Package contents and implementation

All of the methods described in this paper have been implemented in the R package **nonprobsvy** Łukasz Chrostowski and Chlebicki (2024). In this chapter, we will show you how to use the main **nonprob** function of the package and what its main features are. The package has been written to be as compatible as possible with the survey package for inference for probability samples. Namely, the first step to use the **nonprobsvy** package is to define an object using the **sydesign** function that stores the probability sample **data.frame** and other objects, such as design weights. This is a negligible step if, instead of the probability sample, we have access to the values of the vector of sums of variables in the population. It is also worth mentioning that in order to speed up the calculations in the case of variable selection, part of the package, or more precisely the whole variable selection algorithm, was written in C++ using the **Rcpp** (Eddelbuettel, Francois, Allaire, Ushey, Kou, Russell, Ucar, Bates, and Chambers (2024)) package, which allows the C++ code to be called in the R environment. Moreover, the package is supported by

other R packages such as **foreach** [Folashade Daniel Hong Ooi and Weston \(2023\)](#) (looping construct), **maxLik** [Henningesen and Toomet \(2023\)](#) (maximum likelihood estimation), **Matrix** [Bates, Maechler *et al.* \(2023\)](#) (matrix operations), **MASS** [Ripley, Venables *et al.* \(2023\)](#) (statistical functions and datasets), **ncvreg** [Breheny and Huang \(2023\)](#) (regularization methods), **mathjaxr** [Epskamp \(2023\)](#) (rendering equations in documentation), **nleqslv** [Groemping \(2023\)](#) (solving nonlinear equations), and **doParallel** [Steve Weston and Tenenbaum \(2022\)](#) (parallel computing).

3.1. Arguments

Below is the definition of most of the arguments we can pass to the function. These are described in more detail in the documentation on the [CRAN](#) platform.

Table 4: nonprob function arguments description

Argument	Description	Default
data	data.frame with data from the non-probability sample.	-
selection	formula for the selection (propensity) equation.	NULL
outcome	formula for the outcome equation.	NULL
target	formula with target variables.	NULL
svydesign	Optional svydesign object containing probability sample and design weights.	NULL
pop_totals	Optional named vector with population totals of the covariates.	NULL
pop_means	Optional named vector with population means of the covariates.	NULL
pop_size	Optional double with population size.	NULL
method_selection	Character string specifying the method for propensity score estimation (e.g., "logit").	"logit"
method_outcome	Character string specifying the method for response variable estimation (e.g., "glm").	"glm"
family_outcome	Character string describing the error distribution and link function to be used in the model (e.g., "gaussian").	"gaussian"
subset	Optional vector specifying a subset of observations to be used in the fitting process.	NULL
strata	Optional vector specifying strata.	NULL
weights	Optional vector of prior weights to be used in the fitting process.	NULL
na_action	function indicating what should happen when the data contain NA's.	NULL
control_selection	list indicating parameters to use in fitting selection model for propensity scores.	controlSel()
control_outcome	list indicating parameters to use in fitting model for outcome variable.	controlOut()
control_inference	list indicating parameters to use in inference based on probability and non-probability samples.	controlInf()
start_selection	Optional vector with starting values for the parameters of the selection equation.	NULL
start_outcome	Optional vector with starting values for the parameters of the outcome equation.	NULL
verbose	Logical value indicating if verbose output should be printed.	FALSE
x	Logical value indicating whether to return the model matrix of covariates as part of the output.	TRUE
y	Logical value indicating whether to return the vector of outcome variable as part of the output.	TRUE
se	Logical value indicating whether to calculate and return the standard error of the estimated mean.	FALSE
...	Additional optional arguments.	

In addition to using the survey package for design-based inference when probability samples are available, it also supports the various methods for estimating propensity scores and outcome models described in this thesis, such as logistic regression, complementary log-log models, probit models, generalized linear models, nearest neighbour algorithms and predictive mean matching.

After this neat description of the main functionality of the package, we will move on to some examples of its use. We will show how to define the given arguments in order to obtain estimates of interest as a result. We will be less interested in the results than in the way they are presented. There will be room in the following chapters for an analysis of simulations and applications of the package to the real world. We will focus on the three main estimators, as function calls for other functionalities such as variable selection, other linking functions or mass imputation methods.

4. Package Usage

We apply our methods to integrate administrative and survey data about job vacancies in Poland. The goal is to estimate the percentage of single shift job vacancies according to available data sources. We defined our outcome variable Y as follows: *whether the vacancy notice was for single-shift work*. Now we present the description of data used.

The first source is the Job Vacancy Survey (JVS, also known as the Labour Demand Survey) with a sample of 6523 units. The data include “the number of employed persons, as well as the number and structure of job vacancies, including newly created and vacant positions reported to employment offices. Information on newly created and eliminated jobs” (Central Statistical Office of Poland). The survey is conducted using a representative method. The sample is drawn separately for units employing more than 9 people and for units employing up to 9 people. The sampling frame for this survey is the Statistical Units Database. It includes information about NACE (The Nomenclature of Economic Activities) (19 levels), region (16 levels), sector (2 levels), size (3 levels) and the number of employees according to administrative data integrated by Statistics Poland (RE).

```
R> jvs <- read.csv("data-raw/jvs.csv",
+                 colClasses = c("character", "numeric",
+                               rep("character", 3), "numeric"))
R> head(jvs)
```

	id	private	size	nace	region	weight
1	a9cc990df6a99ab215a1bc13f51d4825c7d52d18	0	L	O	14	1
2	c9dbaf50890165ebe810aa770de0e9df903dc35b	0	L	O	24	6
3	718e0bba42bcec6ed98f9690db6d26cb7b93c880	0	L	R.S	14	1
4	532a1879a692b9d7bbb7282ba757d028156ef341	0	L	R.S	14	1
5	0b6b623fa45e257284a3049d097af322841337e3	0	L	R.S	22	1
6	c855a825e80866c00c7513721d5fcb38929f3cd6	0	M	R.S	26	1

```
R> jvs_svy <- svydesign(ids = ~ 1,
+                      weights = ~ weight,
+                      strata = ~ size + nace + region,
+                      data = jvs)
```

The second source is the Central Job Offers Database (CBOP), which is a register of all vacancies submitted to Public Employment Offices and can be accessed via CBOP API. It contains job offers submitted by employers looking for new employees. If an employer is seeking new workers for their business, they can approach the County Employment Office (PUP) and submit the appropriate application. CBOP also contains information about unit identifiers (REGON and NIP), so we were able to link units to the sampling frame to obtain auxiliary variables with the same definitions as those used in the survey. Beyond that it contains `single_shift` outcome variable.

```
R> admin <- read.csv("data-raw/admin.csv",
+                   colClasses = c("character", "numeric",
+                                   rep("character", 3), "logical")
+                   )
R> head(admin)
```

	id	private	size	nace	region
1	7fddce081cbb1dd5da072e1683e9bfd20acab593	0	L	P	30
2	3d40ca689a4dca7d774981dc7db408301bf7f192	0	L	O	14
3	2b5a57b0c2f03c2b252e559bcd77d52789d33d9c	0	L	O	04
4	f2b18f5ef4386e70206d64810b5d3b7e0654918b	0	L	O	24
5	bf17263f8fa3a9ff12d7ff60c10c4e426aeeb36e	0	L	O	04
6	10fe847b7d19284e9e310105c5acfaf8f1ccbc37	1	L	C	28

	single_shift
1	FALSE
2	TRUE
3	TRUE
4	TRUE
5	TRUE
6	FALSE

4.1. Standard Estimation

We will begin by showing how to use the package for estimation with the standard methods described in Chapter 2. We will start by calling the `nonprob` function, in the case of analysis with the IPW method.

```
R> est1_logit <- nonprob(
+   selection = ~ region + private + nace + size,
```

```
+ target = ~ single_shift,
+ svydesign = jvs_svy,
+ data = admin,
+ method_selection = "logit"
+ )
```

To do this, we define the formulation of the independent variables included in the estimation of the probability of inclusion in the selection argument, as well as the name of the variable for which we are estimating the population mean (target argument). In addition, we apply the svydesign object containing the probability sample and non-probability data (data). We estimate the probability using a logit linking function.

```
R> summary(est1_logit)
```

Call:

```
nonprob(data = admin, selection = ~region + private + nace +
  size, target = ~single_shift, svydesign = jvs_svy, method_selection = "logit")
```

```
Estimated population mean: 0.7083 with overall std.err of: 0.01063
And std.err for nonprobability and probability samples being respectively:
0.0063 and 0.008567
```

95% Confidence interval for population mean:

```
          lower_bound upper_bound
single_shift  0.6874801  0.7291656
```

```
Based on: Inverse probability weighted method
For a population of estimate size: 52898.13
Obtained on a nonprobability sample of size: 9344
With an auxiliary probability sample of size: 6523
```

Regression coefficients:

For glm regression on selection variable:

	Estimate	Std. Error	z value	P(> z)
(Intercept)	-0.65278	0.07498	-8.706	< 2e-16 ***
region04	0.83780	0.07121	11.765	< 2e-16 ***
region06	0.19954	0.07245	2.754	0.00589 **
region08	0.10481	0.08911	1.176	0.23950

region10	-0.15756	0.06408	-2.459	0.01393	*
region12	-0.60987	0.06029	-10.115	< 2e-16	***
region14	-0.84150	0.05419	-15.530	< 2e-16	***
region16	0.76386	0.08660	8.821	< 2e-16	***
region18	1.17811	0.07142	16.495	< 2e-16	***
region20	0.22252	0.09261	2.403	0.01627	*
region22	-0.03753	0.06039	-0.621	0.53438	
region24	-0.40670	0.05474	-7.430	1.09e-13	***
region26	0.20287	0.08489	2.390	0.01685	*
region28	0.57863	0.06797	8.513	< 2e-16	***
region30	-0.61021	0.05908	-10.328	< 2e-16	***
region32	0.32744	0.06957	4.706	2.52e-06	***
private	0.05899	0.05880	1.003	0.31571	
naceD.E	0.77274	0.10033	7.702	1.34e-14	***
naceF	-0.37783	0.04271	-8.847	< 2e-16	***
naceG	-0.33370	0.03788	-8.809	< 2e-16	***
naceH	-0.65175	0.05977	-10.904	< 2e-16	***
naceI	0.41179	0.05726	7.191	6.41e-13	***
naceJ	-1.42639	0.13622	-10.471	< 2e-16	***
naceK.L	0.06171	0.07981	0.773	0.43941	
naceM	-0.40678	0.06741	-6.034	1.60e-09	***
naceN	0.80035	0.06733	11.888	< 2e-16	***
naceO	-0.69355	0.09460	-7.331	2.28e-13	***
naceP	1.25095	0.07647	16.359	< 2e-16	***
naceQ	0.30287	0.06799	4.455	8.41e-06	***
naceR.S	0.22228	0.06975	3.187	0.00144	**
sizeM	-0.36413	0.03444	-10.574	< 2e-16	***
sizeS	-1.02916	0.03504	-29.369	< 2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Weights:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1.169	2.673	4.333	5.661	7.178	49.951

Covariate balance:

(Intercept)	region04	region06	region08	region10	region12
1028.12621	-2134.53715	674.22292	-567.99552	2183.91092	2625.05778
region14	region16	region18	region20	region22	region24
5591.76507	-10057.99095	485.50485	-575.81701	3046.75848	3556.72624
region26	region28	region30	region32	private	naceD.E

```

-6137.08004    579.81140    4063.88117   -3833.66622   46302.39060  -46796.28976
      naceF      naceG      naceH      naceI      naceJ      naceK.L
  200.06842  13948.81746  -5572.84664  -11450.33115  -2455.39528  -1200.06938
      naceM      naceN      naceO      naceP      naceQ      naceR.S
  1727.57310    29.82868  -1121.28241   -131.62619    580.73294    575.02916
      sizeM      sizeS
  11575.60069  29388.14082
-----

```

Residuals:

```

      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.8552 -0.2308   0.5393   0.3080   0.7987   0.9800

```

AIC: 43894.82

BIC: 44140.32

Log-Likelihood: -21915.41 on 15835 Degrees of freedom

By calling the summary method on an object of class `nonprobsvy`, we can approach, among other things ...

To call the calibration method, we define a `controlSel` function in the `control_selection` argument with the `est_method_sel` argument equal to 'gee' (default 'mle') and set the value of `h` (corresponding to the `h`-function with ??).

```

R> est2_logit <- nonprob(
+   selection = ~ region + private + nace + size,
+   target = ~ single_shift,
+   svydesign = jvs_svy,
+   data = admin,
+   method_selection = "logit",
+   control_selection = controlSel(h = 1, est_method_sel = "gee")
+ )

```

Another family of estimators (Mass Imputation) requires a slightly different function call. As you might have noticed, the definition of which estimation method we want to use is not defined in the argument of the `nonprob` function, but in the result of the combination of arguments we have defined (specifically formulas). Now the argument in which we define the formula is `outcome`, which mirrors the definition of formulas for models in functions from the `glm` family (for example).

```

R> est5_glm <- nonprob(
+   outcome = single_shift ~ region + private + nace + size,
+   svydesign = jvs_svy,
+   data = admin,

```

```
+ method_outcome = "glm",
+ family_outcome = "gaussian"
+ )

R> est8_dr1 <- nonprob(
+ selection = ~ region + private + nace + size,
+ outcome = single_shift ~ region + private + nace + size,
+ svydesign = jvs_svy,
+ data = admin,
+ method_selection = "logit",
+ method_outcome = "glm",
+ family_outcome = "binomial"
+ )
```

4.2. Bootstrap Approach for Variance Estimation

```
R> est3_logit <- nonprob(
+ selection = ~ region + private + nace + size,
+ target = ~ single_shift,
+ svydesign = jvs_svy,
+ data = admin,
+ method_selection = "logit",
+ control_inference = controlInf(var_method = "bootstrap", num_boot = 50),
+ verbose = F,
+ )
```

4.3. Variable Selection Algorithms

5. Classes and S3methods

6. Summary and future work

The **nonprobsvy** package is an R tool developed to address challenges associated with making inferences from non-probability samples. As non-probability data sources such as administrative data, voluntary online panels, and social media data become increasingly available, statisticians are faced with the problem of how to integrate these sources with traditional probability samples to produce reliable population estimates. This package provides a suite of methods specifically designed to handle the unique characteristics of non-probability samples, where selection mechanisms are often unknown or biased.

nonprobsvy supports a range of estimation techniques including mass imputation, inverse probability weighting (IPW), and doubly robust (DR) methods. Each of these methods allows researchers to correct for selection bias in non-probability samples by leveraging auxiliary data from probability samples or known population totals. The mass imputation approach imputes values for the outcome variable in the probability sample using models trained on the non-probability sample. The IPW estimator uses estimated propensity scores to weight non-probability samples in a way that aligns them more closely with the population. The DR estimator combines both approaches, resulting in an estimator that remains consistent if either the outcome model or the propensity score model is correctly specified, thus providing a level of robustness against model misspecification.

Designed to integrate with the widely used **survey** package in R, **nonprobsvy** enables researchers to define survey designs, specify selection and outcome models, and estimate population parameters using both probability and non-probability samples. The package supports advanced modeling options, including generalized linear models, nearest neighbor algorithms, and predictive mean matching, and allows for customization of tuning parameters, variable selection, and variance estimation methods (e.g., bootstrap).

Overall, **nonprobsvy** is a comprehensive toolkit for researchers and practitioners working with mixed data sources. It simplifies the implementation of complex statistical methods for non-probability samples, making it easier to produce robust, reliable estimates even when working with non-representative data. By providing a unified framework for inference with non-probability samples, **nonprobsvy** contributes to the growing field of data integration and enhances the ability of statisticians to make informed decisions from diverse data sources.

A natural extension of this study is that consistency can be maintained when the design weights d_i are replaced by calibration weights, which is often the case when working with survey sample datasets. In addition, other methods of estimating propensity scores proposed in the literature can be considered, such as estimation using the empirical likelihood approach. It is also worth exploring the situation where the two probability and non-probability samples overlap, i.e. there are units present in both datasets. This type of problem will be the focus of the author’s future work on non-probability sampling, in particular its integration with other statistical data sources.

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