



## nonprobsvy – An R package for modern methods for non-probability surveys

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### Abstract

The paper presents the **nonprobsvy** package which implements the state-of-the-art statistical inference methods for non-probability samples. The package implements various approaches that can be categorized into three groups: prediction-based approach, inverse probability weighting and doubly robust approach. On the contrary to the existing packages **nonprobsvy** assumes existence of either full population or probability-based population information and leverage the **survey** package for the inference. The package implements both analytical and bootstrap variance estimation for all of the proposed estimators. In the paper we present the theory behind the package, its functionalities and case study that showcases the usage of the package. The package is aimed at official statisticians, public opinion or market researchers who would like to use non-probability samples (e.g. big data, opt-in web panels, social media) to accurately estimate population characteristics.

*Keywords:* data integration, doubly robust estimation, propensity score estimation, mass imputation, R, Python, **survey**.

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## 1. Introduction

In official statistics, information about the target population and its characteristics is mainly collected through probability surveys, census or is obtained from administrative registers, which covers all (or nearly all) units of the population. However, owing to increasing non-response rates, particularly unit non-response and non-contact, resulting from the growing

respondent burden, as well as rising costs of surveys conducted by National Statistical Institutes (NSIs), non-probability data sources are becoming more popular (Beręsewicz 2017; Beaumont 2020; Biffignandi and Bethlehem 2021). Non-probability surveys, such as opt-in web panels, social media, scanner data, mobile phone data or voluntary register data, are currently being explored for use in the production of official statistics (Citro 2014; Daas, Puts, Buelens, and Hurk 2015), public opinion studies or market research. Since the selection mechanism in these sources is unknown, standard design-based inference methods cannot be directly applied and in case of large datasets may lead to *big data paradox* as described by Meng (2018).

Table 1 compares the basic characteristics of probability and non-probability samples. In particular, what are the advantages and disadvantages of each type of sample with respect to population coverage, bias, variance, costs, and the selection mechanism for observations into the samples. In general, non-probability samples suffers from unknown selection mechanism (i.e. unknown probabilities of inclusion into sample) and under-coverage of certain groups from the population. As a result direct estimation based on these samples are characterised with bias and, in most cases, small variance (for large non-probability surveys) which leads to so called big data paradox. Certainly, cost and timeliness of these surveys is significantly smaller than for non-probability samples.

Factor	Probability sample	Non-probability sample
Selection	Known probabilities	Unknown self-selection
Coverage	Complete	Incomplete
Estimation bias	Unbiased under design	Potential systematic bias
Variance of estimates	Typically high	Typically low
Cost	High	Low
Availability	Long	Rapid

Table 1: Comparison of probability and non-probability samples and its characteristics

To address this problem, several approaches based on inverse probability weighting (IPW; also propensity score weighting/adjustment, Lee (2006)), model-based prediction (in particular, mass imputation estimators; MI) and doubly robust (DR) approach have been proposed for two main scenarios: 1) population-level data are available, either in the form of unit-level data (e.g. from a register covering the whole population) or known population totals/means, and 2) only survey data is available as a source of information about the target population (cf. Elliott and Valliant 2017). Wu (2022) classified these approaches into three groups that require a joint randomization framework involving *probability sampling design* (denoted as  $p$ ) and one of the outcome regression model (denoted as  $\xi$ ) or propensity score model (denoted as  $q$ ). In this approach the IPW estimators are under the  $qp$  framework, the MI estimators are under the  $\xi p$  framework, DR estimators are under the  $qp$  or  $\xi p$  framework.

Most approaches assume that population data are used to reduce the bias of non-probability sampling by a proper reweighting to reproduce known population totals/means (i.e. IPW estimators); by modelling target variable using various techniques (i.e. MI estimators); or combining both approaches (for instance DR estimators, cf. Chen, Li, and Wu (2020); Multi-level Regression and Post-stratification (MRP) also called *Mister-P*, cf. Gelman (1997)). This topic have became very popular and number of new methods were proposed; for instance non-parametric approaches based on nearest neighbours (Yang, Kim, and Hwang 2021), kernel

density estimation (Chen, Yang, and Kim 2022), empirical likelihood (Kim and Morikawa 2023) or quantile balanced (Beręsewicz, Szymkowiak, and Chlebicki 2025) to name few. It should be highlighted that, on contrary to probability samples, there is no single method that can be used for non-probability samples. As shown literature, and thus statistical software, offers various methods as presented in the next section.

### 1.1. Software for non-probability samples

Table 2 presents comparison of availability of various inference methods and functionalities of selected packages. We focused on packages available through CRAN or PyPI (for non-CRAN or non-PyPI software see Cobo, Ferri-García, Rueda-Sánchez, and Rueda (2024)).

In the comparison we have included four packages that particularly focus on non-probability samples: **NonProbEst** (Rueda, Ferri-García, and Castro 2020), **balance** (Sarig, Galili, and Eilat 2023), **inps** (Castro Martín 2024) and our **nonprobsvy**. We have included two packages that implements specific methods: **rstanarm** (MRP; Goodrich, Gabry, Ali, and Brilleman (2024)) and **GJRM** (generalized sample selection models; Marra and Rodicw (2023)).

Functionalities	NonProbEst	rstanarm	GJRM	balance	inps	nonprobsvy
IPW	✓	–	+/-	✓	✓	✓
Calibrated IPW	–	–	–	–	–	✓
MI	✓	–	–	–	–	✓
DR	–	–	–	+/-	✓	✓
MRP	–	✓	–	–	–	–
Sample selection	–	–	✓	–	–	–
Variable selection	✓	–	–	✓	–	✓
Analytical variance	–	–	–	–	–	✓
Bootstrap variance	✓	–	–	–	–	✓
Integration with <b>survey</b> or <b>samplex</b>	–	–	–	–	–	✓

Table 2: Comparison of packages and implemented methods

The **NonProbEst** implements IPW under logistic regression as well as machine learning approaches (e.g. CART, GBM or Neural Networks) however does not provide theoretical justification of using the ML approaches. The **balance** focuses solely on the IPW and uses standard and covariate balancing propensity score.

It should be however noted that implementation of specific approaches may vary significantly as the documentation of specific modules is limited. For instance, **NonProbEst** allows for calibration of IPW weights *after* they are estimated, while **nonprobsvy** does this in one step. On the other hand **NonProbEst** implements various weights based on the estimated propensity scores while **nonprobsvy** supports only two types of IPW weights. Furthermore, it should be noted that **NonProbEst**, **balance** and **inps** implements survey calibration for non-probability sample but it is not theoretically justified as the inclusion probabilities are unknown (on the contrary to probability samples). Only the **nonprobsvy** package leveraged the use of the **survey** package to estimate analytic as well as bootstrap variance of the non-probability estimators.

To our knowledge the **nonprobsvy** is the solely software (open or close) that implements

state-of-the-art methods for non-probability samples.

The remaining part of the paper is as follows. In Section 2 theory of the statistical inference based on non-probability samples is presented. We provide basic set-up and introduce methods in separate subsections. In the paper we use Wu (2022) notation. Section 3 describes the dependencies of the package and the main function with its arguments. Section 4 presents a case study of integration of the Polish Job Vacancy Survey with a voluntary admin data: Central Job Offers Database with an aim on estimating number of companies with at least vacancy offered on a single shift. Section 5 presents classes and the **S3Methods** implemented in the package. Paper finishes with summary and plans for the future works. In the Appendix we present codes for specific approaches discussed in the paper, algorithms and detailed derivations of the implemented methods.

## 2. Methods for non-probability samples

### 2.1. Basic setup

Let  $U = \{1, \dots, N\}$  denote the target population consisting of  $N$  labelled units. Each unit  $i$  has an associated vector of auxiliary variables  $\mathbf{x}_i$  and the study/target variable  $y_i$ . Let  $\{(y_i, \mathbf{x}_i), i \in S_A\}$  be a dataset of a non-probability sample  $S_A$  of size  $n_A$  and let  $\{(\mathbf{x}_i, \pi_i^B), i \in S_B\}$  be a dataset of a probability sample  $S_B$  of size  $n_B$ , where only information about variables  $\mathbf{X}$  and inclusion probabilities  $\pi^B$  are known for all units in the population. Each unit in the sample  $S_B$  has been assigned a design-based weight given by  $d_i^B = 1/\pi_i^B$ .

Let  $R_i = I(i \in S_A)$  be an indicator of inclusion into non-probability sample  $S_A$  defined for all units in the target population. Let  $\pi_i^A = P(R_i = 1 \mid \mathbf{x}_i, y_i) = P(R_i = 1 \mid \mathbf{x}_i)$  be the propensity scores which characterize the sample  $S_A$  inclusion and participation mechanisms. On the contrary to  $\pi_i^B$ , the  $\pi_i^A$  and thus  $d_i^A = 1/\pi_i^A$  are unknown. This description of the data is presented in a more concise form in Table 3.

Sample	ID	Inclusion ( $R$ )	Design weight ( $d$ )	Covariates ( $\mathbf{X}$ )	Study variable ( $Y$ )
Non-probability	1	1	?	✓	✓
sample ( $S_A$ )	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$n_A$	1	?	✓	✓
Probability	1	0	✓	✓	?
sample ( $S_B$ )	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$n_B$	0	✓	✓	?

Table 3: Two sample setting

The goal is to estimate a finite population mean  $\mu_y = \frac{1}{N} \sum_{i=1}^N y_i$  of the target variable  $Y$ .

As values of  $y_i$  are not observed in the probability sample, it cannot be used to estimate the target quantity. Instead, one could try combining the non-probability and probability samples to estimate  $\mu_y$ . Given the absence of a universally accepted methodology for achieving this objective, the assumptions vary considerably, as outlined by Wu (2022). However, the main assumptions that apply to all presented in this section methods are:

- A1  $R_i$  and the study variable  $y_i$  are independent given the set of covariates  $\mathbf{x}_i$ , i.e.,  $(R_i \perp y_i) \mid \mathbf{x}_i$ .
- A2 All the units in the target population have non-zero propensity scores, i.e.,  $\pi_i^A > 0$ ,  $i = 1, 2, \dots, N$ .
- A3 The indicator variables  $R_1, R_2, \dots, R_N$  are independent given the set of auxiliary variables  $(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$ .

In addition, we assume no overlap between  $S_A$  and  $S_B$ , and no measurement error in  $Y_i$  and  $\mathbf{X}_i$  is observed. Setting presented in Table 3 may be also extended to calibrated  $d_i^B$  weights (i.e.  $d_i^B$  adjusted for under-coverage, non-contact or non-response; cf. [Särndal and Lundström \(2005\)](#)) but this requires additional developments in the theory about the consistency of the MI, IPW or DR estimators. In the next sections we briefly present methods that are implemented in the package.

## 2.2. Prediction-based approach

### *General form of prediction estimators*

In prediction approach the following semiparametric model for the finite population, denoted as  $\xi$  is assumed

$$E_\xi(y_i \mid \mathbf{x}_i) = m(\mathbf{x}_i, \boldsymbol{\beta}), \text{ and } V_\xi(y_i \mid \mathbf{x}_i) = v(\mathbf{x}_i) \sigma^2, \quad i = 1, 2, \dots, N, \quad (1)$$

where the mean function  $m(\cdot; \cdot)$  and the variance  $v(\cdot)$  have known forms and it assumed that  $y_i$  are independent given  $\mathbf{x}_i$ . The model (1) is assumed to hold for all units in the non-probability sample  $S_A$ . Parameters of the (1) can be estimated using quasi maximum likelihood estimation method which covers linear and non-linear models such as generalized linear models. Let  $\boldsymbol{\beta}_0$  and  $\sigma_0^2$  be the true values of the model parameters  $\boldsymbol{\beta}$  and  $\sigma_0^2$  under assumed model and the  $\hat{\boldsymbol{\beta}}$  be the quasi maximum likelihood estimator of  $\boldsymbol{\beta}_0$ . Let  $m_i = m(\mathbf{x}_i, \boldsymbol{\beta}_0)$  and  $\hat{m}_i = m(\mathbf{x}_i, \hat{\boldsymbol{\beta}})$  for all units  $i = 1, \dots, N$ . Under this settings, [Wu \(2022\)](#) notes there are two commonly used prediction estimators

$$\hat{\mu}_{y1} = \frac{1}{N} \sum_{i=1}^N \hat{m}_i \quad \text{and} \quad \hat{\mu}_{y2} = \frac{1}{N} \left\{ \sum_{i \in S_A} y_i - \sum_{i \in S_A} \hat{m}_i + \sum_{i=1}^N \hat{m}_i \right\}. \quad (2)$$

Under linear models where  $m(\mathbf{x}_i, \boldsymbol{\beta}) = \mathbf{x}_i' \boldsymbol{\beta}$ , the two estimators (2) reduce to

$$\hat{\mu}_{y1} = \mu_x' \hat{\boldsymbol{\beta}} \quad \text{and} \quad \hat{\mu}_{y2} = \frac{n_A}{N} (\bar{y}_A - \bar{\mathbf{x}}_A' \hat{\boldsymbol{\beta}}) + \mu_x' \hat{\boldsymbol{\beta}}, \quad (3)$$

where  $\mu_x = N^{-1} \sum_{i=1}^N \mathbf{x}_i$  is the vector of the population means of the  $\mathbf{x}$  variables and  $\bar{\mathbf{x}}_A = n_A^{-1} \sum_{i \in S_A} \mathbf{x}_i$  is the vector of the simple means of  $\mathbf{x}$  from the non-probability sample  $S_A$ . If the linear model contains an intercept and  $\hat{\boldsymbol{\beta}}$  is the ordinary least square estimator then  $\hat{\mu}_{y1} = \hat{\mu}_{y2}$ .

This form is appealing as the only requirement is availability of non-probability sample  $S_A$  and reference population means (or totals and population size  $N$ ). If the population means

are unknown they can be replaced by estimates provided by the reference probability sample  $S_B$ :  $\sum_{i=1}^N \hat{m}_i = \sum_{i \in S_B} d_i^B \hat{m}_i$  for (2) and  $\mu_x$  replaced by  $\hat{\mu}_x = \hat{N}_B^{-1} \sum_{i \in S_B} d_i^B x_i$  for (3) where  $\hat{N}_B = \sum_{i \in S_B} d_i^B$ .

### Mass imputation estimators

Model-based prediction estimators of  $\mu_y$  can be treated as *mass imputation estimators* as the information on  $y_i$  is missing entirely in the reference probability sample  $S_B$  (but  $x_i$  is available) and  $y_i$  can be imputed based on the non-probability sample  $S_A$  as  $(y_i, x_i)$  is observed. The general form of the mass imputation (MI) estimator is given by

$$\hat{\mu}_{yMI} = \frac{1}{\hat{N}_B} \sum_{i \in S_B} d_i^B y_i^*, \quad (4)$$

where  $y_i^*$  is the imputed  $y_i$  value and  $\hat{N}_B$  is defined previously. Under deterministic regression imputation the  $\hat{\mu}_{yMI}$  estimator reduce to (2) estimators.

There are several approaches how to impute  $y_i^*$  and in the package we have implemented the following MI estimators: semiparametric approach based on the generalized linear models (hereinafter denoted as MI-GLM), nearest neighbour matching (denoted as MI-NN) and predictive mean matching (MI-PMM).

The properties of the MI-GLM estimator was studied by [Kim, Park, Chen, and Wu \(2021\)](#) additional assumption called *model transportability*: the model which is built for the non-probability sample  $S_A$  can be used for prediction with the reference probability sample  $S_B$ . In the package we allow for the following GLM families: **gaussian**, **binomial** and **poisson**.

The MI-NN estimator was initially proposed by [Rivers \(2007\)](#) under the name *sample matching* and theoretical properties for the MI-NN estimator for large non-probability samples (i.e.  $Nn_A^{-1} = O(1)$ ) was studied by [Yang et al. \(2021\)](#). The basic idea of the NN matching is as follows: 1) for each  $i$  unit in the probability sample  $S_B$  find a donor  $j$  (or donors) in sample  $S_A$  based on some distance between  $x_i$  and  $x_j$ ; 2) use the matched values  $y_j$  from  $S_A$  to impute missing  $y_i$  values in the probability sample  $S_B$ . Imputed values  $y_i^*$  depends on the number of selected neighbours. For  $k = 1$  the closest one is selected and for  $k > 1$  a simple average over a vector of selected  $y$  is calculated. The detailed description is presented in the Algorithm 1 in the Appendix. This algorithm suffers from the curse of dimensionality as proved by for i.e. asymptotic bias of the MI estimator increases as the number of covariates  $x$  increases with a fixed  $k$ . To overcome this issue PMM was proposed

In the PMM matching is done using predicted values of  $\hat{m}_i = m(x_i, \hat{\beta})$  instead of  $x_i$ , thus the NN algorithm is modified as follows: 1) fit the  $m(x_i, \beta)$  to non-probability sample  $S_A$ , 2) assign predicted values  $\hat{m}_i$  to all units in  $S_A$  and  $S_B$ ; 3) match all units from sample  $S_B$  to donor units from sample  $S_A$ . The remaining part is the same as in the NN approach. [Chlebicki, Łukasz Chrostowski, and Beręsewicz \(2024\)](#) studied properties of two variants of the MI-PMM estimator for non-probability samples: matching predicted to predicted ( $\hat{m} - \hat{m}$  matching; MI-PMM A) and predicted to observed ( $\hat{m} - y$  matching; PMM B). Details about the procedure can be found in Algorithm 2 and 3 in the Appendix. [Chlebicki et al. \(2024\)](#) prove consistency of the MI-PMM A estimator under model mis-specification i.e. the assumed model may be different from the true one.

### Variance estimators for the prediction approach

Variance for the MI estimators can be either estimated using analytical or bootstrap approach. Analytical estimator of the variance of the MI-GLM estimator proposed by Kim *et al.* (2021) (p. 950) which contains two components:  $\hat{V}_1$  (based on information from both  $S_A$  and  $S_B$ ) and  $\hat{V}_2$  (based solely on probability sample  $S_B$ ); for the MI-NN estimator for large samples Yang *et al.* (2021) proposed variance estimator based solely on the probability sample  $S_B$  (i.e. design-based variance estimator of the mean) and for small  $S_A$  samples variance estimator can be found in Chlebicki *et al.* (2024); and for the MI-PMM estimators Chlebicki *et al.* (2024) contains formulas for variance estimators for small and large non-probability samples  $S_A$ .

Second approach is based on the bootstrap where each run of the procedure  $b = 1, \dots, B$  can be expressed as follows

1. Independently:
  - draw a simple random sampling with replacement from the non-probability sample  $S_A$ ,
  - draw a sample according to the declared sampling design from the probability sample  $S_B$ .
2. Estimate  $\mu_{yMI}^b$  based on appropriate approach (e.g. MI-GLM, MI-NN or MI-PMM).

After obtaining  $B$  bootstrap estimate estimate variance based on the following equation

$$\hat{V}_{\text{boot}} = \frac{1}{B-1} \sum_{b=1}^B \left( \hat{\mu}_y^b - \hat{\mu}_y \right)^2, \quad (5)$$

where  $\hat{\mu}_y$  is the estimated mean using either MI-GLM, MI-NN or MI-PMM estimator.

The above approaches are applied when unit-level data from the probability sample  $S_B$  are available. If this is not the case and only population means (or totals and population size) are available we use the first component  $\hat{V}_1$  of the Kim *et al.* (2021) variance estimator (estimated population quantities are replaced with known ones). For the variance of the MI-NN and MI-PMM estimators we only use bootstrap approach with known population means. Note, that current version of the **nonprobsvy** does not support the replicated weights in the probability sample  $S_B$  for any of the estimators discussed in this section.

### 2.3. Inverse Probability Weighting

Another popular approach is inverse probability weighting (IPW) that involves estimation of propensity scores denoted as  $\pi_i^A = P(i \in S_A)$  which denotes inclusion and participation probabilities. Similarly as for the prediction approach we have two variants of the IPW estimators given by

$$\hat{\mu}_{IPW1} = \frac{1}{N} \sum_{i \in S_A} \frac{y_i}{\hat{\pi}_i^A} \quad \text{and} \quad \hat{\mu}_{IPW2} = \frac{1}{\hat{N}^A} \sum_{i \in S_A} \frac{y_i}{\hat{\pi}_i^A}, \quad (6)$$

where the  $\hat{\mu}_{IPW1}$  is a version of the Horvitz-Thompson estimator and the  $\hat{\mu}_{IPW2}$  is the Hájek estimator with the estimated population size given by  $\hat{N}^A = \sum_{i \in S_A} \frac{1}{\hat{\pi}_i^A}$ . This estimator in the



case of non-probability samples was discussed by .. and several approaches on using propensity scores and alternative versions of the IPW estimators were discussed by ... Recently, [Chen et al. \(2020\)](#) studied properties of the IPW estimators, proved their consistency as well as derived closed form estimators. [Wu \(2022, section 4.2\)](#) argue that  $\hat{\mu}_{IPW2}$  estimator performs better than  $\hat{\mu}_{IPW1}$  even if the population size is known.

The construction of the IPW estimator involved two steps: 1) estimation of the propensity scores; and 2) deriving  $d_i^A$ , which in our case are equal to  $1/\pi_i^A$ . To estimate the propensity scores  $\pi_i^A = \pi(\mathbf{x}_i, \gamma)$  one can use the likelihood approach under assumption that the information about  $\mathbf{x}_i$  are available for each unit in the population

$$\ell(\gamma) = \log \left\{ \prod_{i=1}^N (\pi_i^A)^{R_i} (1 - \pi_i^A)^{1-R_i} \right\} = \sum_{i \in S_A} \log \left\{ \frac{\pi(\mathbf{x}_i, \gamma)}{1 - \pi(\mathbf{x}_i, \gamma)} \right\} + \sum_{i=1}^N \log \{1 - \pi(\mathbf{x}_i, \gamma)\}. \quad (7)$$

In practice, a function of this form cannot be used because we do not observe all units from the population. More realistic approach involves a reference probability sample  $S_B$  and as a result the second component of the (7) is replaced forming pseudo log-likelihood function

$$\ell^*(\gamma) = \sum_{i \in S_A} \log \left\{ \frac{\pi(\mathbf{x}_i, \gamma)}{1 - \pi(\mathbf{x}_i, \gamma)} \right\} + \sum_{i \in S_B} d_i^B \log \{1 - \pi(\mathbf{x}_i, \gamma)\}. \quad (8)$$

The maximum pseudo-likelihood estimator  $\hat{\gamma}$  can be obtained as the solution to the pseudo score equation which under logit function assumed for  $\pi_i^A$  is given by

$$\mathbf{U}(\gamma) = \sum_{i \in S_A} \mathbf{x}_i - \sum_{i \in S_B} d_i^B \pi(\mathbf{x}_i, \gamma) \mathbf{x}_i. \quad (9)$$

In general, the pseudo score functions  $\mathbf{U}(\gamma)$  at the true values of the model parameters  $\gamma_0$  are unbiased under the joint  $qp$  randomization in the sense that  $E_{qp} \{\mathbf{U}(\gamma_0)\} = \mathbf{0}$ , which implies that the estimator  $\hat{\alpha}$  is  $qp$ -consistent for  $\gamma_0$  ([Wu 2022](#)).

The system (9) can be replaced by a system of general estimation equations. Let  $\mathbf{h}(\mathbf{x}, \gamma)$  be a user-specified vector of functions with the same dimension of  $\gamma$  and  $\mathbf{G}(\gamma)$  is defined as

$$\mathbf{G}(\gamma) = \sum_{i \in S_A} \mathbf{h}(\mathbf{x}_i, \gamma) - \sum_{i \in S_B} d_i^B \pi(\mathbf{x}_i, \gamma) \mathbf{h}(\mathbf{x}_i, \gamma), \quad (10)$$

then solving  $\mathbf{G}(\gamma) = \mathbf{0}$  with the chosen parametric form of  $\pi_i^A$  and the choosen  $\mathbf{h}(\mathbf{x}, \gamma)$  leads to the consistent estimator of  $\hat{\gamma}$ . In the literature, the most commonly considered functions are  $\mathbf{h}(\mathbf{x}_i, \gamma) = \mathbf{x}_i$  and  $\mathbf{h}(\mathbf{x}_i, \gamma) = \mathbf{x}_i \pi(\mathbf{x}_i, \gamma)^{-1}$ . Note that if the function  $\mathbf{h}_i = \mathbf{x}_i$ , then  $\mathbf{G}$  is reduced to

$$\mathbf{G}(\gamma) = \sum_{i \in S_A} \mathbf{x}_i - \sum_{i \in S_B} d_i^B \pi(\mathbf{x}_i, \gamma) \mathbf{x}_i.$$

If we use the second form of the function  $\mathbf{h}$  we get the following form of the function  $\mathbf{G}$

$$\mathbf{G}(\theta) = \sum_{i \in S_A} \frac{\mathbf{x}_i}{\pi(\mathbf{x}_i, \gamma)} - \sum_{i \in S_B} d_i^B \mathbf{x}_i, \quad (11)$$



which can be viewed as calibrated IPW. Moreover, using (11) requires the knowledge of population totals for the auxiliary variables  $\mathbf{x}$ .

#### *Variance estimators for the inverse probability weighting approach*

Chen *et al.* (2020, section 3.2) derived asymptotic variance estimators for both IPW estimators presented in (6) and presented the plug-in variance estimator for the  $\hat{\mu}_{IPW2}$  estimator assuming logistic regression. In the package we have implemented this approach for `logit`, `probit` and `cloglog` link functions. We refer the reader to the Wu (2022, section 6.2) and Chrostowski (2024, chapter 3) for more details on how this estimators are derived based on the general estimating equations approach.

Another approach is to use bootstrap which essentially is the same as the one presented in the `\eqref{eq-var-bootstrap}` where the  $\hat{\mu}_y$  is replaced by one of the (6) estimators.

## 2.4. Doubly Robust approach

The inverse probability weighting and mass imputation estimators are sensible on misspecified models for propensity score and outcome variable respectively. For this purpose so called doubly-robust methods, which take into account these problems, are presented.

The proposed estimation procedure addresses the challenge of combining data from nonprobability and probability survey samples. Traditional semiparametric models, often applied to such problems, are not directly usable in this context due to the distinct nature of the two samples. Instead, a joint randomization framework is employed, integrating semiparametric models for propensity scores with outcome regression for the nonprobability sample and design-based inference from the probability sample. This framework leads to a doubly robust (DR) estimation approach, which is effective in the presence of model misspecifications.

Inverse Probability Weighted (IPW) estimators are sensitive to misspecified propensity score models, particularly when propensity scores are very small. To improve robustness and efficiency, the doubly robust method incorporates a prediction model for the response variable. Moreover, even if one of the models is misspecified, the DR estimator remains consistent, showcasing the “double robustness” property.

#### *Joint Randomization Approach*

The joint randomization approach combines two processes: the selection mechanism of a non-probability sample, modelled by propensity scores, and the design-based inference from a probability sample.

The response  $y_i$  is predicted using a regression model  $m(\mathbf{x}_i, \boldsymbol{\beta})$  (or NN/PMM methods), where  $\boldsymbol{\beta}$  is estimated from the non-probability sample. With known design weights  $d_i^B$  for  $i \in S_B$  we can define the DR estimator as

$$\hat{\mu}_{DR} = \frac{1}{\hat{N}^A} \sum_{i \in S_A} d_i^A \{y_i - m(\mathbf{x}_i, \hat{\boldsymbol{\beta}})\} + \frac{1}{\hat{N}^B} \sum_{i \in S_B} d_i^B m(\mathbf{x}_i, \hat{\boldsymbol{\beta}}), \quad (12)$$

where  $d_i^A = \pi(\mathbf{x}_i, \boldsymbol{\theta})^{-1}$ ,  $\hat{N}^A = \sum_{i \in S_A} d_i^A$  and  $\hat{N}^B = \sum_{i \in S_B} d_i^B$ .

It remains consistent if either the propensity score model  $\pi(\mathbf{x}_i, \boldsymbol{\theta})$  or the outcome regression model  $m(\mathbf{x}_i, \boldsymbol{\beta})$  is correctly specified.

The joint randomization approach ensures robustness by accounting for randomness in both the non-probability sample through  $\pi(\mathbf{x}_i, \boldsymbol{\theta})$  and the probability sample through design-based inference.

### *Minimization of the bias for doubly robust methods*

By reducing the variance of the estimators, for example by variable selection, we cannot control the bias of the estimator, which may increase. Therefore, according to Yang, Kim, and Song (2020), the idea is to determine the equations leading to the estimation of the  $\boldsymbol{\beta}$  and  $\boldsymbol{\theta}$  parameters based on the bias of the population mean estimator. In contrast to the joint randomization approach, this method allows for the estimation of the parameters  $\boldsymbol{\beta}$  and  $\boldsymbol{\theta}$  in a single step, rather than in two separate steps.

We will first present the bias of the doubly robust estimator and then, using optimisation techniques, discuss the equations leading to its minimization. Thus we have

$$\begin{aligned} \text{bias}(\hat{\mu}_{DR}) &= |\hat{\mu}_{DR} - \mu| \\ &= \frac{1}{N} \sum_{i=1}^N \left\{ \frac{R_i^A}{\pi_i^A(\mathbf{x}_i^T \boldsymbol{\theta})} - 1 \right\} \{y_i - m(\mathbf{x}_i^T \boldsymbol{\beta})\} \\ &\quad + \frac{1}{N} \sum_{i=1}^N (R_i^B d_i^B - 1) m(\mathbf{x}_i^T \boldsymbol{\beta}) \end{aligned} \quad (13)$$

To minimize  $\text{bias}(\hat{\mu}_{DR})^2$  let us calculate the gradient of the square of the bias at  $(\boldsymbol{\beta}, \boldsymbol{\theta})$ . We then have

$$\frac{\partial \text{bias}(\hat{\mu}_{DR})^2}{\partial (\boldsymbol{\beta}^T, \boldsymbol{\theta}^T)^T} = 2 \text{bias}(\hat{\mu}_{DR}) J(\boldsymbol{\theta}, \boldsymbol{\beta}),$$

where

$$J(\boldsymbol{\theta}, \boldsymbol{\beta}) = \begin{pmatrix} J_1(\boldsymbol{\theta}, \boldsymbol{\beta}) \\ J_2(\boldsymbol{\theta}, \boldsymbol{\beta}) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^N R_i^A \left\{ \frac{1}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} - 1 \right\} \{y_i - m(\mathbf{x}_i, \boldsymbol{\beta})\} \mathbf{x}_i \\ \sum_{i=1}^N \frac{R_i^A}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} \frac{\partial m(\mathbf{x}_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} - \sum_{i \in S_B} d_i^B \frac{\partial m(\mathbf{x}_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} \end{pmatrix},$$

which leads to the problem of solving the following system of equations

$$\begin{pmatrix} \sum_{i=1}^N R_i^A \left\{ \frac{1}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} - 1 \right\} \{y_i - m(\mathbf{x}_i, \boldsymbol{\beta})\} \mathbf{x}_i \\ \sum_{i=1}^N \frac{R_i^A}{\pi(\mathbf{x}_i, \boldsymbol{\theta})} \frac{\partial m(\mathbf{x}_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} - \sum_{i \in S_B} d_i^B \frac{\partial m(\mathbf{x}_i, \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} \end{pmatrix} = \mathbf{0}, \quad (14)$$

which can be solved using Newton–Raphson optimization method.

### *Variance estimators for the doubly robust approach*

## **2.5. Variable selection algorithms**

When dealing with multivariate data with a large number of features, it is recommended to select variables for estimation that are statistically significant for the model under consideration. Yang and Kim (2020) point that using variable selection techniques during estimation

is crucial, especially when dealing with high-dimensional data. Variable selection not only improves model stability and computational feasibility, but also reduces variance, which can increase when irrelevant auxiliary variables are included. Including irrelevant variables increases the complexity of the model and makes the estimation process more error-prone and unstable. Therefore, variable selection is key to ensure robust and efficient estimation. Very popular in the statistical literature are variable selection methods such as *Least Absolute Shrinkage and Selection Operator* (**LASSO**), *Smoothly Clipped Absolute Deviation* (**SCAD**) or *Minimax Concave Penalty* (**MCP**), which, thanks to appropriate loss functions, degenerate the coefficients in variables that have no significant effect on the dependent variable. In this way, the result obtained, for example, a linear regression equation, is based only on the features selected by the model. The selection procedure works in a similar way for non-probability methods using external data sources such as sample or population totals or averages. In particular, the technique is divided into two steps. In the first, we select the relevant variables using an appropriately constructed loss function and the estimating equations used. In the second case, we construct the equations on the basis of the derived biases of the relevant estimator, whose value we calculate only on the basis of the selected characteristics. In the case of data integration based on probability and nonprobability samples, the selection of variables is part of a two-step process leading to the estimation of the mean, where in the first step statistically significant variables are selected and in the second step the model is rebuilt. For the first step, a penalized logistic regression model has been proposed to estimate propensity scores (Yang *et al.* (2020)), but this approach can be extended to other linking models such as clog-log and probit functions. For a parametric model-based mass imputation, penalized OLS (*Ordinary Least Squared*) is considered. It is worth mentioning that Yang and Kim (2020), in their article on this topic, used the SCAD method (*Smoothly Clipped Absolute Deviation*), but we extend it on other selection techniques such as LASSO and MCP.

### 3. The main function

#### 3.1. The nonprob function

The **nonprobsvy** package is built around the **nonprob** function. The main design objective was to make using **nonprob** as similar as possible to standard R functions for fitting statistical models, such as `stats::glm`, while incorporating survey design features from the **survey** package. The most important arguments are given in Table 4 and the obligatory ones are `data`, while `selection`, `outcome`, or `target` must be specified depending on the chosen method.

The **nonprob** function is used specify inference methods through specifying `selection` and `outcome` arguments. If out of these two `selection` is specified than the IPW estimators are used, if only the `outcome` then the MI approach is used and if both are specified the DR approach is applied. The package allows to provide either reference population data (via the `pop_totals`, or `pop_means` and `pop_size`) or a probability sample declared by the `svydesign` argument (`svydesign2` class of from the **survey** package). Selection of the specific inference method is done through `method_selection`, `method_outcome`, `family_outcome`, `control_selection` and `control_outcome` arguments. Specification of variance estimation method is done via the `control_inference` argument.

Argument	Description
<code>data</code>	<code>data.frame</code> with data from the non-probability sample
<code>selection</code>	<code>formula</code> for the selection (propensity) equation
<code>outcome</code>	<code>formula</code> for the outcome equation
<code>target</code>	<code>formula</code> with target variables
<code>svydesign</code>	Optional <code>svydesign2</code> object
<code>pop_totals</code> , <code>pop_means</code> , <code>pop_size</code>	Optional named <code>vector</code> with population totals or means of the covariates and population size
<code>method_selection</code>	Link function for the IPW approach ( <code>"logit"</code> , <code>"probit"</code> , <code>"cloglog"</code> )
<code>method_outcome</code>	Specification of the MI approach (one of <code>c("glm", "nn", "pmm")</code> )
<code>family_outcome</code>	The GLM family for the MI approach (one of <code>c("gaussian", "binomial", "poisson")</code> )
<code>subset</code>	Optional <code>vector</code> specifying a subset of observations to be used in the fitting process
<code>strata</code>	Optional <code>vector</code> specifying strata
<code>weights</code>	Optional <code>vector</code> of prior weights to be used in the fitting process
<code>na_action</code>	<code>function</code> indicating what should happen when the data contain NA's
<code>control_selection</code> , <code>control_outcome</code> , <code>control_inference</code>	Control parameters for selection and outcome model and the variance estimation via the <code>controlSel</code> , <code>controlOut</code> and <code>controlInf</code> functions respectively
<code>start_selection</code> , <code>start_outcome</code>	Optional <code>vector</code> with starting values for the parameters of the selection and the outcome equation
<code>verbose</code>	Logical value indicating if verbose output should be printed
<code>se</code>	Logical value indicating whether to calculate and return the standard error of the estimated mean
<code>...</code>	Additional optional arguments

Table 4: `nonprob` function arguments description

In addition to using the survey package for design-based inference when probability samples are available, it also supports the various methods for estimating propensity scores and outcome models described in this thesis, such as logistic regression, complementary log-log models, probit models, generalized linear models, nearest neighbour algorithms and predictive mean matching.

Resulting object of class `nonprobsvy` is a list that contains the following (most important) elements:

- `data` – an `data.frame` containing non-probability sample.
- `X` – a `matrix` containing both samples,
- `y` – a list containing all variables declared in either `target` or `outcome` arguments,
- `R` – a numeric `vector` informing about inclusion into non-probability sample,
- `weights` – propensity score weights or `NULL` (for the MI estimators),

- `output` - a `data.frame` containing point and standard error estimates,
- `outcome` - a list of results for each `outcome` models,
- `selection` - a list of results for the `selection` model.
- `svydesign` - a `svydesign2` object passed by the `svydesign` argument.

After this neat description of the main functionality of the package, we will move on to some examples of its use. We will show how to define the given arguments in order to obtain estimates of interest as a result. We will be less interested in the results than in the way they are presented. There will be room in the following chapters for an analysis of simulations and applications of the package to the real world. We will focus on the three main estimators, as function calls for other functionalities such as variable selection, other linking functions or mass imputation methods.

### 3.2. Controlling inference methods and the variance estimation

We provide three control function that allow users to specify the exact inference methods and the variance estimation. The `controlSel` function provides essential control parameters for fitting the selection model in the `nonprob` function. It allows users to select between the MLE or GEE (calibrated) approach through `est_method_sel` (and the type with the `h` argument), specify the optimizer (`optimizer`) and the which variable selection should be applied (using different penalty functions like SCAD, lasso, and MCP through `penalty`) along with parameters (e.g. number of folds via the `nfolds` argument). The package uses the `nleqslv` package and fitting parameters (arguments starting with the `nleqslv*`).

The `controlOut` to fine-tune various aspects of the estimation process, including the variable selection methods (through different penalty options like SCAD, LASSO, and MCP with their respective tuning parameters), and detailed configuration for NN and PMM approaches (using parameters like `predictive_match`, `pmm_weights`, and `pmm_k_choice`).

Finally, the `controlInf` function configures the parameters for variance estimation in the `nonprob` function. It allows user to specify whether the analytical or bootstrap approach should be used (the `var_method` argument), whether the variable selection should be applied (the `vars_selection` argument) and what type of bootstrap should be applied for the probability sample (the `rep_type` argument). This function also allow to specify the how the inference for the DR approach should be taken: if a union or a division of variables after variable selection was applied (the `bias_inf` argument) and if the bias correction should be applied (the `bias_correction` argument).

### 3.3. Design of the package

- we do not provide ways to assess the models: this should be done prior the estimation stage and there are plenty of packages to do so
- we provide some basic functionalities such as `hatvalues` or `cooks.distance` but in a limited form

In the next sections we present a case study on integration of non-probability sample with a reference probability sample. We will present various estimators and compare them. Finally, we present more advanced options of the package.

## 4. Data analysis example

### 4.1. Description of the data

Before we explain the case study let's first load the package.

```
R> library(nonprobsvy) ## for estimation
R> library(ggplot2) ## for visualisation
```

The goal of the case study is to integrate administrative (**admin**) and survey (**jvs**) data about job vacancies in Poland. The first source is the Job Vacancy Survey (JVS) with a sample of 6,523 units. The survey is based on a probability sample drawn according to proportional-to-size stratified sampling design. The details regarding the survey can be found in [Statistics Poland \(2021\)](#). The dataset contains about The Nomenclature of Economic Activities (NACE; 14 levels, **nace** column), **region** (16 levels), sector (2 levels, **private** column), size of the entity (3 levels: Small, Medium and Large) and the final weight (i.e. design weight corrected for non-contact and non-response).

```
R> data(jvs)
R> head(jvs)
```

	id	private	size	nace	region	weight
1	j_1	0	L	0	14	1
2	j_2	0	L	0	24	6
3	j_3	0	L	R.S	14	1
4	j_4	0	L	R.S	14	1
5	j_5	0	L	R.S	22	1
6	j_6	0	M	R.S	26	1

As the package leverages the **survey** package functionalities we need to define the **svydesign2** object via the **svydesign** function as presented below. The dataset does not contain the true stratification variable we use a simplified version by specifying `~ size + nace + region` and we do not know have information on the non-response and its correction we simply assume that the **weight** is the calibrated weight that sums up to the population size.

```
R> jvs_svy <- svydesign(ids = ~ 1,
+                      weights = ~ weight,
+                      strata = ~ size + nace + region,
+                      data = jvs)
```

The second source is the Central Job Offers Database (CBOP), which is a register of all vacancies submitted to Public Employment Offices (see <https://oferty.praca.gov.pl>).

We treat this as the *non-probability sample* because is voluntary administrative data and inclusion mechanism is unknown. This dataset was prepared in such way that the records out of scope (either by the definition of vacancy or population of entities) were excluded. The dataset contains the same variables as JVS with one additional `single_shift` which is our target variable defined as: *whether a company seeks at least one employee for a single-shift job*. The goal of this case study is to estimate *the share of companies that seeks employees for a single-shift job* in Poland in a given quarter.

```
R> data(admin)
R> head(admin)
```

	id	private	size	nace	region	single_shift
1	j_1	0	L	P	30	FALSE
2	j_2	0	L	O	14	TRUE
3	j_3	0	L	O	04	TRUE
4	j_4	0	L	O	24	TRUE
5	j_5	0	L	O	04	TRUE
6	j_6	1	L	C	28	FALSE

Please note that, this paper does not aim to provide full tutorial on using non-probability samples for statistical inference. Thus, we skipped the part of aligning variables to meet the same definitions, assessing how strong is the relation between auxiliary variables, target variable and selection mechanism and distribution mis-matches between both samples. In the examples below we assume that there is no overlap between two sources and the naïve, reference estimate, given by a simple mean of the `single_shift` column of `admin` equals to 66.1%.

## 4.2. Estimation

### *Propensity score approach*

First, we start with the IPW approach with two possible estimation methods MLE (standard) and GEE (calibrated to the estimated survey totals). We start by calling the `nonprob` function where we define the `selection` argument responsible for the formulae for the inclusion variables, the `target` argument which specifies the variable of interest `single_shift`. The rest refer to the `svydesign` object, dataset and specification of the link function (`method_selection`).

```
R> ipw_est1 <- nonprob(
+   selection = ~ region + private + nace + size,
+   target = ~ single_shift,
+   svydesign = jvs_svy,
+   data = admin,
+   method_selection = "logit" ## this is the default
+ )
```

In order to get the basic information about the estimated target quantity we can use the `print` method the object. It provides the call and the estimated mean, standard error (SE) and 95% confidence interval (`lower_bound` and `upper_bound`).



```
R> ipw_est1
```

Call:

```
nonprob(data = admin, selection = ~region + private + nace +
  size, target = ~single_shift, svydesign = jvs_svy, method_selection = "logit")
```

Estimated population mean with overall std.err and confidence interval:

	mean	SE	lower_bound	upper_bound
single_shift	0.7083228	0.009436907	0.6898268	0.7268188

If we are interested in a detailed information about the model we can use the `summary` method.

```
R> summary(ipw_est1)
```

Call:

```
nonprob(data = admin, selection = ~region + private + nace +
  size, target = ~single_shift, svydesign = jvs_svy, method_selection = "logit")
```

```
-----
Estimated population mean: 0.7083 with overall std.err of: 0.009437
And std.err for nonprobability and probability samples being respectively:
0.003958 and 0.008567
```

95% Confidence interval for population mean:

	lower_bound	upper_bound
single_shift	0.6898268	0.7268188

Based on: Inverse probability weighted method

For a population of estimate size: 52898.13

Obtained on a nonprobability sample of size: 9344

With an auxiliary probability sample of size: 6523

Regression coefficients:

For glm regression on selection variable:

	Estimate	Std. Error	z value	P(> z )	
(Intercept)	-0.65278	0.07498	-8.706	< 2e-16	***
region04	0.83780	0.07121	11.765	< 2e-16	***
region06	0.19954	0.07245	2.754	0.00589	**
region08	0.10481	0.08911	1.176	0.23950	
region10	-0.15756	0.06408	-2.459	0.01393	*
region12	-0.60987	0.06029	-10.115	< 2e-16	***

region14	-0.84150	0.05419	-15.530	< 2e-16	***
region16	0.76386	0.08660	8.821	< 2e-16	***
region18	1.17811	0.07142	16.495	< 2e-16	***
region20	0.22252	0.09261	2.403	0.01627	*
region22	-0.03753	0.06039	-0.621	0.53438	
region24	-0.40670	0.05474	-7.430	1.09e-13	***
region26	0.20287	0.08489	2.390	0.01685	*
region28	0.57863	0.06797	8.513	< 2e-16	***
region30	-0.61021	0.05908	-10.328	< 2e-16	***
region32	0.32744	0.06957	4.706	2.52e-06	***
private	0.05899	0.05880	1.003	0.31571	
naceD.E	0.77274	0.10033	7.702	1.34e-14	***
naceF	-0.37783	0.04271	-8.847	< 2e-16	***
naceG	-0.33370	0.03788	-8.809	< 2e-16	***
naceH	-0.65175	0.05977	-10.904	< 2e-16	***
naceI	0.41179	0.05726	7.191	6.41e-13	***
naceJ	-1.42639	0.13622	-10.471	< 2e-16	***
naceK.L	0.06171	0.07981	0.773	0.43941	
naceM	-0.40678	0.06741	-6.034	1.60e-09	***
naceN	0.80035	0.06733	11.888	< 2e-16	***
naceO	-0.69355	0.09460	-7.331	2.28e-13	***
naceP	1.25095	0.07647	16.359	< 2e-16	***
naceQ	0.30287	0.06799	4.455	8.41e-06	***
naceR.S	0.22228	0.06975	3.187	0.00144	**
sizeM	-0.36413	0.03444	-10.574	< 2e-16	***
sizeS	-1.02916	0.03504	-29.369	< 2e-16	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

-----

Weights:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1.169	2.673	4.333	5.661	7.178	49.951

-----

Residuals:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-0.8552	-0.2308	0.5393	0.3080	0.7987	0.9800

AIC: 43894.82

BIC: 44140.32

Log-Likelihood: -21915.41 on 15835 Degrees of freedom

This displays information on the datasets used for estimation (probability and non-probability sample). The estimated regression coefficients are also shown, in this case for the logit model for propensity score model (section **Regression coefficients**). In addition, for diagnostic purposes, we have access to the distribution of the weights calculated from the inclusion

probabilities (section `Weights`), the distribution of the residuals from the model (section `Residuals`), as well as the values of the AIC, BIC statistics in the case of models based on MLE.

If we are interested in the calibrated IPW, one needs to define a `controlSel` function in the `control_selection` argument with the `est_method_sel` argument equal to `gee` (the default is `mle`) and set the value of `h` (as dicussed in the `??` section).

```
R> ipw_est2 <- nonprob(
+   selection = ~ region + private + nace + size,
+   target = ~ single_shift,
+   svydesign = jvs_svy,
+   data = admin,
+   method_selection = "logit",
+   control_selection = controlSel(h = 1, est_method_sel = "gee")
+ )
```

Results are comparable to the standard IPW point estimate (70.4 vs 70.8) while the standard error is lower higher.

```
R> ipw_est2
```

Call:

```
nonprob(data = admin, selection = ~region + private + nace +
  size, target = ~single_shift, svydesign = jvs_svy, method_selection = "logit",
  control_selection = controlSel(h = 1, est_method_sel = "gee"))
```

Estimated population mean with overall std.err and confidence interval:

	mean	SE	lower_bound	upper_bound
single_shift	0.7041796	0.01169878	0.6812504	0.7271088

The calibrated IPW significantly improves the balance as can be accessed by the `nonprobsvycheck` function:

```
R> data.frame(ipw_mle=nonprobsvycheck(~size, ipw_est1, 1)$balance,
+   ipw_gee=nonprobsvycheck(~size, ipw_est2, 1)$balance)
```

	ipw_mle	ipw_gee
sizeL	-367.6	0
sizeM	-228.4	0
sizeS	1624.1	0

Notice that, neither in the package nor this paper we focus a detailed description of the post-hoc results, such as covariate balance. This can be done via existing CRAN packages, for instance using the `bal.tab` function from the **cobalt** package ([Greifer 2024](#)).

*Prediction-based approach*

If a user is interested in a prediction-based approach, in particular mass imputation estimators, then should specify the argument `outcome` as a formulae (similarly as in the `glm` function). We allow single outcome (specified as  $y \sim x_1 + x_2 + \dots + x_k$ ) or multiple outcomes (as  $y_1 + y_2 + y_2 \sim x_1 + x_2 + \dots + x_k$ ). Note that if the `outcome` argument is specified then there is no need to specify `target` argument. By default GLM type of the MI estimator is assumed (i.e. `method_outcome="glm"`). In the code below we present possible way to declare this type of the MI estimator.

```
R> mi_est1 <- nonprob(
+   outcome = single_shift ~ region + private + nace + size,
+   svydesign = jvs_svy,
+   data = admin,
+   method_outcome = "glm",
+   family_outcome = "binomial"
+ )
R>
R> mi_est1
```

Call:

```
nonprob(data = admin, outcome = single_shift ~ region + private +
  nace + size, svydesign = jvs_svy, method_outcome = "glm",
  family_outcome = "binomial")
```

Estimated population mean with overall std.err and confidence interval:

	mean	SE	lower_bound	upper_bound
single_shift	0.7032081	0.01120231	0.681252	0.7251642

If a user is interested in the nearest neighbours MI estimator one can specify `method_outcome = "nn"` for the nearest neighbours search using all variables specified in the `outcome` argument, or `method_outcome = "pmm"` if is interested in predictive mean matching. In both cases we are using  $k = 5$  nearest neighbours (i.e. `controlOut(k=5)`). For the NN MI estimator there is no need to specify the `family_outcome` argument as no model is estimated underneath. For both approaches we use the **RANN** package (Jefferis, Kemp, Arya, and Mount 2024).

```
R> mi_est2 <- nonprob(
+   outcome = single_shift ~ region + private + nace + size,
+   svydesign = jvs_svy,
+   data = admin,
+   method_outcome = "nn",
+   control_outcome = controlOut(k=5)
+ )
R>
R> mi_est3 <- nonprob(
```

```

+   outcome = single_shift ~ region + private + nace + size,
+   svydesign = jvs_svy,
+   data = admin,
+   method_outcome = "pmm",
+   family_outcome = "binomial",
+   control_outcome = controlOut(k=5)
+ )

```

Results of both estimators seems to be similar, but it should be noted that the NN MI estimator suffers from the curse of dimensionality so one should trust more the PMM MI estimator.

```
R> mi_est2$output
```

```

              mean      SE
single_shift 0.6799537 0.01575574

```

```
R> mi_est3$output
```

```

              mean      SE
single_shift 0.7450896 0.01527651

```

As discussed in Section 2 both IPW and MI estimators are asymptotically unbiased only when the model and auxiliary variables are correctly specified. To overcome this problem we focus now on the doubly robust estimators.

### *The doubly robust approach*

To indicate that the doubly robust estimation should be used user needs to specify both the `selection` and `outcome` arguments. These formulas can be specified with the same or varying number of auxiliary variables. We also allow, similarly as in the MI approach, multiple outcomes. In the following example code we specified the non-calibrated IPW and the GLM MI estimator.

```

R> dr_est1 <- nonprob(
+   selection = ~ region + private + nace + size,
+   outcome = single_shift ~ region + private + nace + size,
+   svydesign = jvs_svy,
+   data = admin,
+   method_selection = "logit",
+   method_outcome = "glm",
+   family_outcome = "binomial"
+ )
R> dr_est1

```

Call:

```
nonprob(data = admin, selection = ~region + private + nace +
  size, outcome = single_shift ~ region + private + nace +
  size, svydesign = jvs_svy, method_selection = "logit", method_outcome = "glm",
  family_outcome = "binomial")
```

Estimated population mean with overall std.err and confidence interval:

	mean	SE	lower_bound	upper_bound
single_shift	0.7034644	0.01131974	0.6812781	0.7256507

Detailed results can be obtained by using `summary` function which prints both set of coefficients for the outcome and selection models. We omit this output due to limited space of the paper. Finally, we can use bias minimisation approach as proposed by [Yang \*et al.\* \(2020\)](#) by specifying `control_inference = controlInf(bias_correction = TRUE)` argument. This part is implemented in the **Rcpp** ([Eddelbuettel, Francois, Allaire, Ushey, Kou, Russell, Ucar, Bates, and Chambers 2024](#)) and **RcppArmadillo** ([Eddelbuettel and Sanderson 2014](#)) packages for performance.

```
R> dr_est2 <- nonprob(
+   selection = ~ region + private + nace + size,
+   outcome = single_shift ~ region + private + nace + size,
+   svydesign = jvs_svy,
+   data = admin,
+   method_selection = "logit",
+   method_outcome = "glm",
+   family_outcome = "binomial",
+   control_inference = controlInf(bias_correction = TRUE)
+ )
R> dr_est2
```

Call:

```
nonprob(data = admin, selection = ~region + private + nace +
  size, outcome = single_shift ~ region + private + nace +
  size, svydesign = jvs_svy, method_selection = "logit", method_outcome = "glm",
  family_outcome = "binomial", control_inference = controlInf(bias_correction = TRUE))
```

Estimated population mean with overall std.err and confidence interval:

	mean	SE	lower_bound	upper_bound
single_shift	0.7043248	0.01128182	0.6822129	0.7264368

### 4.3. Comparison of estimates

Finally, as there is no single method for non-probability samples we suggest to compare results in a single table or a plot. In the Figure ... we presents point estimates along with 95%

confidence intervals. The various estimators show interesting patterns compared to the naive estimate (red dashed line). MI estimators demonstrate notably different behaviours: while PMM produces the highest point estimate with the widest confidence interval, NN yields the lowest estimate, close to the naive value. The other estimators - MI (GLM), IPW (both MLE and GEE), and DR (with and without bias minimization) – cluster together with similar point estimates and confidence interval widths, suggesting some consensus in their bias correction. These methods all indicate a population parameter higher than the naive estimate, but their relative consistency, except for the extreme estimates from MI (PMM) and MI (NN), provides some confidence in their bias correction capabilities.

```
R> dr_summary <- rbind(cbind(ipw_est1$output, ipw_est1$confidence_interval),
+                      cbind(ipw_est2$output, ipw_est2$confidence_interval),
+                      cbind(mi_est1$output, mi_est1$confidence_interval),
+                      cbind(mi_est2$output, mi_est2$confidence_interval),
+                      cbind(mi_est3$output, mi_est3$confidence_interval),
+                      cbind(dr_est1$output, dr_est1$confidence_interval),
+                      cbind(dr_est2$output, dr_est2$confidence_interval))
R> rownames(dr_summary) <- NULL
R> dr_summary$est <- c("IPW (MLE)", "IPW (GEE)", "MI (GLM)", "MI (NN)",
+                    "MI (PMM)", "DR", "DR (BM)")
R> ggplot(data = dr_summary,
+         aes(y = est, x = mean, xmin = lower_bound, xmax = upper_bound)) +
+   geom_point() +
+   geom_vline(xintercept = mean(admin$single_shift),
+             linetype = "dotted", color = "red") +
+   geom_errorbar() +
+   labs(x = "Point estimator and confidence interval", y = "Estimators")
```

#### 4.4. Advanced usage

##### *Bootstrap Approach for Variance Estimation*

In the package we allow user to estimate variance of the mean using analytical (default) or bootstrap approach. In case of analytical variance estimators we use the estimators proposed in the papers described in the Section 2. Users may disable standard error calculation using `nonprob(se=FALSE)`. The bootstrap approach implemented in the package refers to two samples:

- non-probability – we currently support only simple random sampling with replacement,
- probability – we support all the approaches implemented in the `as.svrepdesign` and we refer the reader to the help file of this function. Currently we do not support the

The bootstrap approach is done in the following way: 1) we independently draw the same number of  $B$  bootstrap samples from non-probability and probability survey; 2) we estimate population mean based on selected method (e.g. the DR approach); and 3) calculate bootstrap standard error using the following formulae



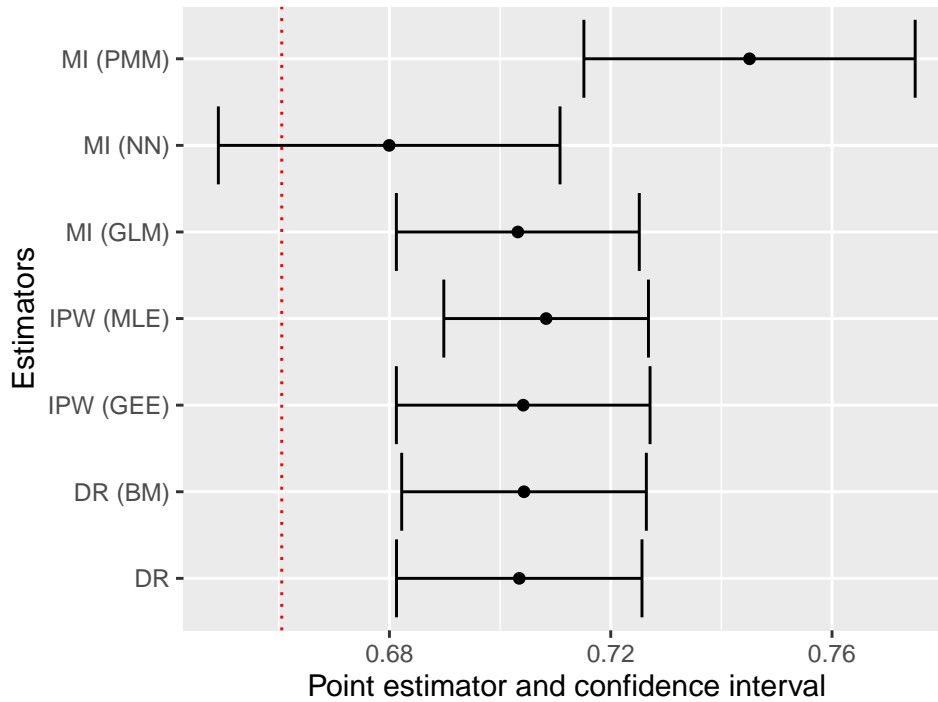


Figure 1: Comparison of estimates of the share of job vacancies offered on a single-shift

(15)

To specify the bootstrap approach one should use `controlInf()` function with `var_method = "bootstrap"`. Controlling the bootstrap method for probability sample is done by `rep_type` argument which passes the method to the `as.svrepdesign` function. The number of iterations is set in the `num_boot` argument (default 100). If the samples are large or the estimation method is complicated (e.g. involves variable selection) one can set `verbose=TRUE` to track the progress. By default results of bootstrap are stored in the `boot_sample` element of the resulting list (to disable this `keep_boot` should be set to `FALSE`). The following code provides an example of using the IPW approach with the bootstrap approach specified by the argument `control_inference` of the `nonprob` function.

```
R> ipw_est1_boot <- nonprob(
+   selection = ~ region + private + nace + size,
+   target = ~ single_shift,
+   svydesign = jvs_svy,
+   data = admin,
+   method_selection = "logit",
+   control_inference = controlInf(var_method = "bootstrap", num_boot = 50),
+   verbose = F
+ )
```

Next, we compare the estimated standard error with the analytical one below.

```
R> rbind(ipw_est1$output,
+        ipw_est1_boot$output)

              mean          SE
single_shift  0.7083228 0.009436907
single_shift1 0.7083228 0.009339383
```

To assess the samples one can access the `boot_sample` element of the output list of the `nonprob` function. Note that this is returned as `matrix` because we allow multiple `target` variables.

```
R> head(ipw_est1_boot$boot_sample, n=3)

      single_shift
[1,]    0.7078424
[2,]    0.7070039
[3,]    0.7047423
```

### *Variable Selection Algorithms*

In this section we briefly present how to use variable selection algorithms. In order to specify that a variable selection algorithm should be used one should specify the `control_inference = controlInf(vars_selection = TRUE)` argument. Then, the user should either leave the default or specify the parameters for the outcome via the `controlOut` function or selection outcome (`controlSel`). Both function have the same parameters:

- `penalty` – The penanlization function used during variables selection (possible values: `c("SCAD", "lasso", "MCP")`)
- `nlambda` – The number of  $\lambda$  values. Default is 50.
- `lambda_min` – The smallest value for  $\lambda$ , as a fraction of `lambda.max`. Default is .001.
- `lambda` – A user specified vector of `lambdas` (only for the `controlSel` function).
- `nfolds` – The number of folds for cross validation. Default is 10.
- `a_SCAD`, `a_MCP` – The tuning parameter of the SCAD and MCP penalty for selection model. Default is 3.7 and 3 respectively.

For the MI approach we leverage the `ncvreg` package ([Breheny and Huang 2011](#)) as it is solely package that uses the SCAD method in R. While for the IPW and DR approaches we developed our own codes in C++ via the `Rcpp` package. In the code below we apply variable selection for the MI GLM estimator using only 5 folds, 25 possible values of  $\lambda$  parameters and apply the LASSO penalty.

```
R> mi_est1_sel <- nonprob(
+   outcome = single_shift ~ region + private + nace + size,
+   svydesign = jvs_svy,
+   data = admin,
+   method_outcome = "glm",
+   family_outcome = "binomial" ,
+   control_outcome = controlOut(nfolds = 5, nlambdas = 25, penalty = "lasso"),
+   control_inference = controlInf(vars_selection = TRUE),
+   verbose = TRUE
+ )
```

```
Starting CV fold #1
Starting CV fold #2
Starting CV fold #3
Starting CV fold #4
Starting CV fold #5
```

In this case study the MI GLM estimator with variable selection yields almost the same results as the approach without it. Point estimates and standard errors differ at the fourth and third digit respectively.

```
R> rbind("MI without var sel"=mi_est1$output,
+       "MI with var sel"=mi_est1_sel$output)
```

	mean	SE
MI without var sel	0.7032081	0.01120231
MI with var sel	0.7034021	0.01119051

## 5. Classes and S3methods

## 6. Summary and future work

The **nonprobsvy** package provides a comprehensive R toolkit for addressing inference challenges with non-probability samples by integrating them with probability samples or known population totals/means. As non-probability data sources like administrative data, voluntary online panels, and social media data become increasingly available, statisticians need robust methods to produce reliable population estimates. The package implements *state-of-the-art* approaches including mass imputation, inverse probability weighting (IPW), and doubly robust (DR) methods, each designed to correct selection bias by leveraging auxiliary data. By providing a unified framework and integration with the **survey** package, **nonprobsvy** makes complex statistical methods for non-probability samples more accessible, enabling researchers to produce robust estimates even when working with non-representative data.

There are several avenues for future development of the **nonprobsvy** package. A key priority is implementing model-based calibration and additional methods for estimating propensity

scores and weights. The package currently assumes no overlap between probability and non-probability samples, so accounting for potential overlap (e.g., in big data sources and registers) is another important extension. Additional planned developments include handling non-ignorable sample selection through sample selection models, maintaining consistency with calibration weights, and supporting multiple non-probability samples for data integration from various sources.

Further methodological extensions under consideration include empirical likelihood approaches for doubly/multiply robust estimation, integration of machine learning methods like debiased/double machine learning from causal inference, handling measurement error in big data variables, and expanding the bootstrap approach beyond simple random sampling with replacement. The package will also be extended to work with the `svyrep.design` class from the **survey** package and the **svrep** package. These developments will enhance **nonprobsvy**'s capabilities for handling complex survey data structures and modern estimation challenges.

## 7. Acknowledgements

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Łukasz Chrostowski is the main developer and maintainer of the package. He was also responsible for the first draft of the paper as it is based on his Master's thesis (available at <https://github.com/ncn-foreigners/graduation-theses>). Piotr Chlebicki contributed to the package and implemented PMM estimators. Maciej Beręsewicz was responsible for the initial idea and the design of the package, testing, reviewing and small contributions code and prepared the final manuscript.

We would like to thank ...

## A. List of symbols

Symbol	Description
$U$	Target population of size $N$
$N$	Population size
$\mathbf{x}_i$	Vector of auxiliary variables for unit $i$
$y_i$	Study variable for unit $i$
$S_A$	Non-probability sample
$S_B$	Probability sample
$n_A$	Size of non-probability sample
$n_B$	Size of probability sample
$\pi_i$	Inclusion probability for unit $i$ in probability sample
$d_i$	Design weight ( $1/\pi_i$ ) for unit $i$
$R_i$	Indicator of inclusion into non-probability sample
$\mu_y$	Population mean of target variable $Y$
$\pi_i^A$	Propensity score for unit $i$ in non-probability sample
$m(\mathbf{x}_i, \boldsymbol{\beta})$	Regression model for outcome variable
$\hat{\mu}_{IPW}$	Inverse probability weighting estimator
$\hat{\mu}_{MI}$	Mass imputation estimator
$\hat{\mu}_{DR}$	Doubly robust estimator
$\boldsymbol{\theta}$	Parameter vector for selection model
$\boldsymbol{\beta}$	Parameter vector for outcome model
$\hat{y}_i$	Imputed value for unit $i$
$\hat{N}^A$	Estimated size based on non-probability sample
$\hat{N}^B$	Estimated size based on probability sample

Table 5: List of symbols and their descriptions

## B. Algorithms

---

**Algorithm 1:** Mass imputation using the k-nearest-neighbour algorithm

---

- 1: If  $k = 1$ , then for each  $i \in S_B$  match  $\hat{\nu}(i)$  such that  $\hat{\nu}(i) = \arg \min_{j \in S_A} d(\mathbf{x}_i, \mathbf{x}_j)$ .
- 2: If  $k > 1$ , then

$$\hat{\nu}(i, z) = \arg \min_{j \in S_A \setminus \bigcup_{t=1}^{z-1} \{\hat{\nu}(i, t)\}} d(\mathbf{x}_i, \mathbf{x}_j)$$

i.e.  $\hat{\nu}(i, z)$  is  $z$ -th nearest neighbour from the sample.;

- 3: For each  $i \in S_B$ , calculate the imputed value as

$$\hat{y}_i = \frac{1}{k} \sum_{t=1}^k y_{\hat{\nu}(i, t)}.$$


---

---

**Algorithm 2:**  $\hat{y} - \hat{y}$  Imputation:

---

- 1: Estimate regression model  $\mathbb{E}[Y|\mathbf{X} = \mathbf{x}] = m(\mathbf{x}, \beta)$ ;
- 2: Impute

$$\hat{y}_i = m(\mathbf{x}_i, \hat{\beta}), \hat{y}_j = m(\mathbf{x}_j, \hat{\beta})$$

for  $i \in S_B, j \in S_A$  and assign each  $i \in S_B$  to  $\hat{\nu}(i)$ , where

$$\hat{\nu}(i) = \arg \min_{j \in S_A} \|\hat{y}_i - \hat{y}_j\|$$

or

$$\hat{\nu}(i) = \arg \min_{j \in S_A} d(\hat{y}_i, \hat{y}_j)$$

if  $d$  is not induced by the norm.;

- 3: If  $k > 1$ , then:

$$\hat{\nu}(i, z) = \arg \min_{j \in S_A \setminus \bigcup_{t=1}^{z-1} \{\hat{\nu}(i, t)\}} d(\hat{y}_i, \hat{y}_j)$$

e.g.,  $\hat{\nu}(i, z)$  is  $z$ -th nearest neighbor from a sample.;

- 4: For  $i \in S_B$ , calculate imputation value as

$$\hat{y}_i = \frac{1}{k} \sum_{t=1}^k y_{\hat{\nu}(i, t)}.$$


---

---

**Algorithm 3:**  $\hat{y} - y$  Imputation:

---

- 1: Estimate regression  $\mathbb{E}[Y|\mathbf{X} = \mathbf{x}] = m(\mathbf{x}, \beta)$ ;
- 2: Impute  $\hat{y}_i = m(\mathbf{x}_i, \hat{\beta})$  for  $i \in S_B$  and assign each  $i \in S_B$  do  $\hat{\nu}(i)$ , where  
 $\hat{\nu}(i) = \arg \min_{j \in S_A} \|\hat{y}_i - y_j\|$  or  $\hat{\nu}(i) = \arg \min_{j \in S_A} d(\hat{y}_i, y_j)$  if  $d$  not induced by the norm.;
- 3: If  $k > 1$ , then:

$$\hat{\nu}(i, z) = \arg \min_{\substack{z-1 \\ j \in S_A \setminus \bigcup_{t=1} \{\hat{\nu}(i, t)\}}} d(\hat{y}_i, y_j).$$

- 4: For each  $i \in S_B$  calculate imputation value as

$$\hat{y}_i = \frac{1}{k} \sum_{t=1}^k y_{\hat{\nu}(i, t)}.$$


---



## C. Codes for specific methods

In this section we provide list of methods along with the appropriate set up of the `nonprob` function. We divide this section into two groups: 1) unit-level data from the reference probability sample is available; 2) only a vector of population totals or means (with population size) is available.

### C.1. Unit-level probability sample is available

#### 1. Mass imputation based on regression imputation

```
R> nonprob(
+   outcome = y ~ x1 + x2,
+   data = nonprob,
+   svydesign = prob,
+   method_outcome = "glm",
+   family_outcome = "gaussian"
+ )
```

#### 2. Mass imputation based on nearest neighbour imputation

```
R> nonprob(
+   outcome = y ~ x1 + x2,
+   data = nonprob,
+   svydesign = prob,
+   method_outcome = "nn",
+   family_outcome = "gaussian",
+   control_outcome = controlOutcome(k = 2)
+ )
```

#### 3. Mass imputation based on predictive mean matching

```
R> nonprob(
+   outcome = y ~ x1 + x2,
+   data = nonprob,
+   svydesign = prob,
+   method_outcome = "pmm",
+   family_outcome = "gaussian"
+ )
```

#### 4. Mass imputation based on regression imputation with variable selection (LASSO)

```
R> nonprob(
+   outcome = y ~ x1 + x2,
+   data = nonprob,
+   svydesign = prob,
```

```
+ method_outcome = "pmm",
+ family_outcome = "gaussian",
+ control_outcome = controlOut(penalty = "lasso"),
+ control_inference = controlInf(vars_selection = TRUE)
+ )
```

#### 5. Inverse probability weighting (MLE)

```
R> nonprob(
+ selection = ~ x1 + x2,
+ target = ~ y,
+ data = nonprob,
+ svydesign = prob,
+ method_selection = "logit"
+ )
```

#### 6. Inverse probability weighting with calibration constraint (GEE)

```
R> nonprob(
+ selection = ~ x1 + x2,
+ target = ~ y,
+ data = nonprob,
+ svydesign = prob,
+ method_selection = "logit",
+ control_selection = controlSel(est_method_sel = "gee", h = 1)
+ )
```

#### 7. Inverse probability weighting with calibration constraint (GEE) with variable selection (SCAD)

```
R> nonprob(
+ selection = ~ x1 + x2,
+ target = ~ y,
+ data = nonprob,
+ svydesign = prob,
+ method_outcome = "pmm",
+ family_outcome = "gaussian",
+ control_inference = controlInf(vars_selection = TRUE)
+ )
```

#### 8. Doubly robust estimator

```
R> nonprob(
+ selection = ~ x1 + x2,
+ outcome = y ~ x1 + x2,
+ data = nonprob,
```

```
+ svydesign = prob,
+ method_outcome = "glm",
+ family_outcome = "gaussian"
+ )
```

#### 9. Doubly robust estimator with variable selection (SCAD) and bias minimization

```
R> nonprob(
+ selection = ~ x1 + x2,
+ outcome = y ~ x1 + x2,
+ data = nonprob,
+ svydesign = prob,
+ method_outcome = "glm",
+ family_outcome = "gaussian",
+ control_inference = controlInf(
+   vars_selection = TRUE,
+   bias_correction = TRUE
+ )
+ )
```

## C.2. Only population totals or means are available

### Example declarations

#### 1. Mass imputation based on regression imputation

```
R> nonprob(
+ outcome = y ~ x1 + x2,
+ data = nonprob,
+ pop_totals = c(`(Intercept)` = N,
+               x1 = tau_x1,
+               x2 = tau_x2),
+ method_outcome = "glm",
+ family_outcome = "gaussian"
+ )
```

#### 2. Inverse probability weighting

```
R> nonprob(
+ selection = ~ x1 + x2,
+ target = ~ y,
+ data = nonprob,
+ pop_totals = c(`(Intercept)` = N,
+               x1 = tau_x1,
+               x2 = tau_x2),
+ method_selection = "logit"
+ )
```

## 3. Inverse probability weighting with calibration constraint

```
R> nonprob(  
+   selection = ~ x1 + x2,  
+   target = ~ y,  
+   data = nonprob,  
+   pop_totals = c(`(Intercept)` = N,  
+                   x1 = tau_x1,  
+                   x2 = tau_x2),  
+   method_selection = "logit",  
+   control_selection = controlSel(est_method_sel = "gee", h = 1)  
+ )
```

## 4. Doubly robust estimator

```
R> nonprob(  
+   selection = ~ x1 + x2,  
+   outcome = y ~ x1 + x2,  
+   pop_totals = c(`(Intercept)` = N,  
+                   x1 = tau_x1,  
+                   x2 = tau_x2),  
+   method_outcome = "glm",  
+   family_outcome = "gaussian"  
+ )
```

## D. Detailed derivations

Table 6: MLE Functions and Gradients for Different Link Functions

Link	MLE Function	Gradient
logit	$\sum_{i \in S_A} \mathbf{x}_i^\top \boldsymbol{\theta}$	$\sum_{i \in S_A} \mathbf{x}_i$
	$-\sum_{i \in S_B} d_i^B \log [1 + \exp(\mathbf{x}_i^\top \boldsymbol{\theta})]$	$-\sum_{i \in S_B} d_i^B \pi(\mathbf{x}_i, \boldsymbol{\theta}) \mathbf{x}_i$
probit	$\sum_{i \in S_A} \log \left( \frac{\Phi(\mathbf{x}_i^\top \boldsymbol{\theta})}{1 - \Phi(\mathbf{x}_i^\top \boldsymbol{\theta})} \right)$	$\sum_{i \in S_A} \frac{\phi(\mathbf{x}_i^\top \boldsymbol{\theta})}{\Phi(\mathbf{x}_i^\top \boldsymbol{\theta})[1 - \Phi(\mathbf{x}_i^\top \boldsymbol{\theta})]} \mathbf{x}_i$
	$+\sum_{i \in S_B} d_i^B \log [1 - \Phi(\mathbf{x}_i^\top \boldsymbol{\theta})]$	$-\sum_{i \in S_B} d_i^B \frac{\phi(\mathbf{x}_i^\top \boldsymbol{\theta})}{1 - \Phi(\mathbf{x}_i^\top \boldsymbol{\theta})} \mathbf{x}_i$
cloglog	$\sum_{i \in S_A} \{ \log [1 - \exp(-\exp(\mathbf{x}_i^\top \boldsymbol{\theta}))] \}$	$\sum_{i \in S_A} \frac{\exp(\mathbf{x}_i^\top \boldsymbol{\theta}) \mathbf{x}_i}{\pi(\mathbf{x}_i, \boldsymbol{\theta})}$
	$+\exp(\mathbf{x}_i^\top \boldsymbol{\theta}) - \sum_{i \in S_B} d_i^B \exp(\mathbf{x}_i^\top \boldsymbol{\theta})$	$-\sum_{i \in S_B} d_i^B \exp(\mathbf{x}_i^\top \boldsymbol{\theta}) \mathbf{x}_i$

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