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MultiObjective MetaHeuristics

MultiObjective Two-Stage Uncapacitated Facility Location Problems

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Abstract: This technical report summarizes the main stages examined during the project related to the course "MultiObjective MetaHeuristics (MOMH)". The aim is to practice the concepts of MOMH using a case study as support. The problem choosen for 2023-2024 is a Two-Stage Uncapacitated Facility Location Problem with 2 objectives.

1 Two-Stage Uncapacitated Facility Location Problem

This section is based on the paper "A memetic algorithm for solving two variants of the Two-Stage Uncapacitated Facility Location Problem", Stefan Mišković and Zorica Stanimirović, *Information Technology and Control*, volume 42, numero 2, pp. 131-149, 2013, available on madoc.

The Two-Stage Uncapacitated Facility Location Problem (TSUFLP) has important applications in designing telecommunication systems. Given a set of demand points and a set of possible locations for the first and second level concentrators (switches, multiplexors), the goal of the TSUFLP is to define the structure of two level concentrator access network, such that the total cost of establishing such a network is minimized [MS13]. Aside this classic economic objective function, other objectives with other natures may be encountered as reaching an balanced workload between all the concentrators or a fairness distance between the demand points and the first level concentrators. This problem belongs to the class of multiobjective multilevel facility location problem.

We start with a given set of locations of terminals, a set of potential locations for installing the first-level concentrators and a set of potential sites for locating the second-level concentrators. An efficient solution given multiple objectives aims to choose locations on the first and second level for installing certain number of concentrators, and to make necessary assignments of each terminal to an installed first-level concentrator, which further have to be assigned to one of the concentrators established at locations on the second-level. The problem involves a single allocation scheme, which means that each terminal is assigned to exactly one, previously established concentrator on the first-level, while each concentrator is assigned to at most one, previously located concentrator on the second level. All second level concentrators are connected to a central unit (Figure 1).

Many papers in the literature are devoted to the TSUFLP and its variants. The problem can be viewed as an extension of the uncapacitated facility location problem (UFLP) [CLS99] or again as a set packing problem (SPP) [LM09].

1.1 Mathematical formulation

We consider a single objective integer formulation of the TSUFLP given in [CLS99].

Data:

- $I = \{1, ..., nI\}$, a set of locations of terminals;

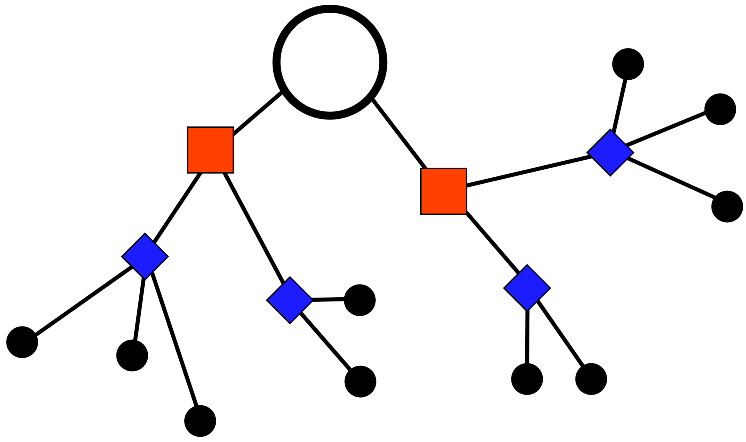

Figure 1: Illustration of an example. Small circles (in black) are terminals, diamond (in blue) are first-level concentrators, squares (in red) are second-level concentrators, large circle (in white) is the central unit. In the terminology of telecommunication networks, this configuration is called *a star/star network* [GLY02].

- $J = \{1, ..., nJ\}$, a set of possible locations for installing first-level concentrators;
- $K = \{1, ..., nK\}$, a set of possible locations for establishing second-level concentrators;
- c_{ij} is the cost of assigning terminal at location $i \in I$ to a concentrator at location $j \in J$ on the first-level;
- b_{jk} is the cost of installing concentrator at location $j \in J$ on the first-level and connecting it to the concentrator at location $k \in K$ on the second-level;
- s_k is the cost of setting up a concentrator at site $k \in K$ on the second-level.

Decision variables:

- $x_{ij} \in \{0, 1\},\ i \in I,\ j \in J$ is equal to 1 if terminal at $i \in I$ is assigned to concentrator installed at location $j \in J$ on the first level, 0 otherwise;
- $y_{jk} \in \{0, 1\},\ j \in J,\ k \in K$ is equal to 1 if a concentrator at location $j \in J$ on the first level is connected to a concentrator at location $k \in K$ on the second level, 0 otherwise;
- $z_k \in \{0, 1\},\ k \in K$ is equal to 1 if a concentrator is installed at location $k \in K$ on the second level, 0 otherwise;

Formulation:

$$\left[\begin{array}{lllll}
\min & f^1(x,y,z) = & \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{j\in J}\sum_{k\in K} b_{jk}y_{jk} + \sum_{k\in K} s_k z_k & & (1) \\
s/t & & \sum_{j\in J} x_{ij} = 1 & \forall i \in I & (2) \\
 & & x_{ij} \leq \sum_{k\in K} y_{jk} & \forall i \in I,\ \forall j \in J & (3) \\
 & & y_{jk} \leq z_k & \forall j \in J,\ \forall k \in K & (4) \\
 & & \sum_{k\in K} y_{jk} \leq 1 & \forall j \in J & (5) \\
 & & x_{ij} \in \{0,1\} & \forall i \in I,\ \forall j \in J & (6) \\
 & & y_{jk} \in \{0,1\} & \forall j \in J,\ \forall k \in K & (7) \\
 & & z_k \in \{0,1\} & \forall k \in K & (8)
\end{array}\right]$$

where

(1) minimizes the sum of costs of establishing the concentrators at locations on the first and the second levels, plus the costs of assigning terminals to first-level concentrators and assigning first-level concentrators to sedonc level concentrators;

(2,3) each terminal is allocated to exactly one concentrator located on the first level, which is further connected to a concentrator located on the second level;

(4,5) each first level concentrator is assigned to at most one second-level concentrator;

(6,7,8) the variables are binary.

1.2 Example

Consider a network illustrated by Figure 2 with $\mid I \mid = 5$ terminal nodes $(t_1, t_2, t_3, t_4, t_5)$, $\mid J \mid = 3$ potential locations for first-level concentrators (p_1, p_2, p_3) and, $\mid K \mid = 2$ potential locations for second-level concentrators (q_1, q_2).

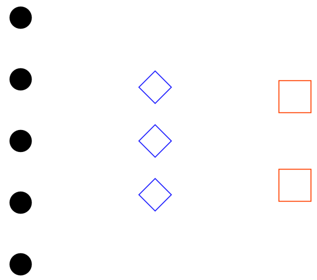

Figure 2: Illustration of an example with 5 terminal nodes, 3 potential locations for first-level concentrators, 2 potential locations for second-level concentrators.

The assignment costs of terminals to first level concentrators are given in Table 1, the costs for installing first-level concentrators and their assigments to second-level concentrators are available in Table 2, and finally the costs of establishing second-level concentrators are equal to $s_1 = 20$ and $s_2 = 16$ respectively.

	p_1	p_2	p_3
t_1	12	22	18
t_2	14	19	20
t_3	20	31	13
t_4	22	21	2
t_5	17	24	9

Table 1: Assignments costs of terminals to first level concentrators.

	q_1	q_2
p_1	29	12
p_2	28	31
p_3	21	13

Table 2: Costs for installing first-level concentrators and their assigments to second-level concentrators.

The optimal solution of this example is illustrated in Figure 3. Concentrators p_1 and p_3 are installed on the first level, while concentrator q_2 is established on the second level. Terminal nodes t_1 and t_2 are assigned to p_1, while t_3, t_4 and t_5 are assigned to p_3. Both first-level concentrators p_1 and p_3 are allocated to the second-level concentrator q_2. The objective function that corresponds to this optimal solution takes the values of 91.

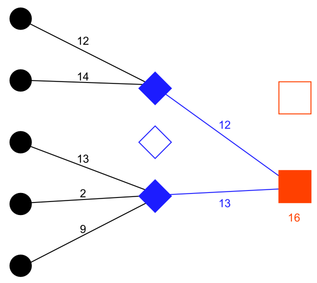

Figure 3: Illustration of an example with 5 terminal nodes, 3 potential locations for first-level concentrators, 2 potential locations for second-level concentrators.

2 Location objectives

Various objective functions in facility location problems are described in the literature. Among the most usual:

- Minimizing the total setup cost, the fixed cost, total annual operating cost,
- Minimizing the longest distance from the existing facilities,
- Maximizing service, responsiveness,
- Minimizing average time/ distance traveled, maximum time/ distance traveled,
- Minimizing the number of located facilities,
- etc.

Eiselt and Laporte [EL95] have identified three type of location objectives:

- Pull objectives (efficienty): desirable facilities to locate close to the customers;
- Push objectives (risk): undesirable facilities to locate;
- Balance objectives (equity): the main goal is to achieve "equity" for serving fairly the customers or the workload of facilities

Environmental and social objectives are now more and more taking into account [HMN09]. They are based on [FSA10]:

- energy cost,
- land use and construction cost,
- congestion,
- noise,
- quality of life,
- pollution,
- etc.

Taking into account multiple objectives for facility location problems is a growing area (8 references reported in [EG00], more than 100 references in [FSA10]).

3 Variants of TSUFLP to consider

The canonical formulation of the TSUFLP given in Section 1 may be enriched in many ways (see e.g. [GLY02] for a survey of facility location problems in telecommunication).

1. A concentrator (at the first-level) is equipped with a limited number of ports. Knowing that each terminal connected to a concentrator uses one port, a limited number of terminals can be connected to one concentrator. Let us define Q the maximal number of ports available with one concentrator, a capacity constraint may be added to the model, giving a model with *capacitated concentrators*.

2. For several reasons, the number of concentrators installed at the first level can be limited. Let us define C the maximal number of concentrators, a capacity constraint may be added to the model, giving a model with *limited concentrators*.

3. Different types of concentrators (at the first-level) can be considered giving a model with *multitype concentrators*. For each candidate location for concentrators there is a given set of concentrator types with a different capacity (number of ports) and cost. The decision involved are to determine the location and type of concentrators and to assign the terminal to the concentrator subject to capacity constraints.

4. In a standard configuration, each terminal is assigned to a single concentrator. To increase the reliability of the network, one terminal can be assigned to more than one concentrator giving a model with *multiple homing*. Here we consider each terminal is assigned to two concentrators, one for the primary coverage and the second for the secondary or backup coverage in case of disturbance on the primary coverage.

4 Objectives for TSUFLP to consider as second objective

Alongside an usual economic objective to minimize, we consider a second objective among the those described hereafter.

1. The objective where we seek to minimize the maximum distance between terminals and their nearest first-level concentrators leads to the objective of *the p-center Problem*, which is *a pull objective*.

2. For quality of service reasons, the network administrator may wish an "equity" on the first-level concentrators. A balanced workload on the concentrators can be expressed as to get the same charge on all concentrators, which is *a balancing objective*.

3. With the perspective of expending the network in the future, the network operator may wish a well dispersion of the concentrators (at both levels), which is *a push objective* (see e.g. [EN91] available on madoc for a detailed study on dispersion objectives).

Pull, push, and balancing objectives plus the economic objective are encountered in most of the real problems and they are by nature conflictual. It means that objective are combined with desirable and undesirable effects (example, a good dispersion of the concentrators increases the cost due the augmented distance that may occur in the allocation terminal-concentrator).

5 A Scatter Search metaheuristic for solving MOCO problems

Scatter Search is a population based metaheuristic originaly proposed by F. Glover in 1997 for single objective optimization problems. In the years 2000, scatter search has been proposed for computing an approximation of Y_N (see e.g Nebro et al. in 2005, Beausoleil in 2006, etc.).

The multiobjective metaheuristic considered in this project is a scatter search based-metaheuristic introduced in 2021 [LSSOL21], and published in the paper "A New Scatter Search Design for Multiobjective Combinatorial Optimization with an Application to Facility Location", A. D. López-Sanchez, J. Sánchez-Oro, M. Laguna, *INFORMS Journal on Computing*, volume 33, number 2, pages 629-642, 2021, available on madoc. These authors provide on github (at `https://github.com/jesussanchezoro/ProblemInstances`) the numerical instances used in this paper.

6 Project: guidelines and productions

On base of two objective functions to optimize, the goal reached in this project is to compute the set of nondominated points Y_N and one corresponding complete set of efficient solutions X_E for instances of enriched TSUFLP. As the UFLP has been proved NP-hard [KP83], the use of metaheuristics in the presence of large numerical instances appears as a resonable choice. Here, a multiobjective metaheuristic based on the scatter search principle has to be developed, aiming to compute efficiently a set Y_{PN} presenting excellent values on quality indicators.

6.1 Step 1: Establish your MOCO problem to study

The 14 students will constitues 7 groups of 2 students. Starting from the TSUFLP described in Section 1, each group has to establish its MOCO problem to study among the following combinations, giving at the end 7 different problems:

Variants	$group_1$	$group_2$	$group_3$	$group_4$	$group_5$	$group_6$	$group_7$
capacitated concentrators	.	.	.	.	.	.	.
limited concentrators	.	.	X	.	.	.	.
multitype concentrators	.	.	.	.	.	.	.
multiple homing	.	.	.	.	.	.	.
pull objective	.	.	.	.	.	.	.
balancing objective	.	.	X	.	.	.	.
push objective	.	.	.	.	.	.	.

Table 3: Table defining the bi-objective location problem based-TSUFLP studied by a group.

The example of choices indicated by `X` reported in Table 3 correspond to the TSUFLP with limited concentrators considering the cost objective and the balancing objective.

6.2 Step 2: Master the MOMH to use

Read carefully the MOMH described in [LSSOL21]. This MOMH has to be applied for solving your MOCO problem.

6.3 Step 3: Apply the MOMH to your MOCO problem

The MOMH has to be instantiated on your MOCO problem, implemented and tested on various numerical instances. Julia language is recommended for your implementation, but C/C++ is also accepted. The dataset discussed in [LSSOL21] and available on github may serve as starting point for building your datasets.

6.4 Step 3: Numerical experiment

Report a numerical comparison between the algorithm that you have established and one of the two situations: (1) the results obtained with an ϵ-constraint method (e.g. the method available into vOptSolver) or (2) the result obtained with NSGA-II [DPAM02] in using your own implementation of this NSGA-II or an available implementation in Julia (see e.g. the fresh implementation at `https://github.com/jMetal/MetaJul`). Indicators such as CPUtime consumed and standard quality measures are expected in your analysis to discuss about the effectiveness of your code.

6.5 Step 4: Submit your project

Your productions will be composed of an archive with (1) your codes ready to use (a readme has to be provided to explain how to build your executable if your code is implemented in C/C++, how to run your code), (2) your datasets, (3) your results, (4) your report, (5) your talk.

6.6 Step 5: Presentation of your project

You will deliver a talk of 10 minutes presenting your main contributions and your results.

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