SECTION 5.3 Exercises

63. **Get rich** A survey of 4826 randomly selected young adults (aged 19 to 25) asked, "What do you think are the chances you will have much more than a middle-class income at age 30?" The two-way table shows the responses.[14] Choose a survey respondent at random.

	Sex		
Opinion	**Female**	**Male**	**Total**
Almost no chance	96	98	194
Some chance but probably not	426	286	712
A 50-50 chance	696	720	1416
A good chance	663	758	1421
Almost certain	486	597	1083
Total	**2367**	**2459**	**4826**

(a) Given that the person selected is male, what's the probability that he answered "almost certain"?

(b) If the person selected said "some chance but probably not," what's the probability that the person is female?

64. **A *Titanic* disaster** In 1912 the luxury liner *Titanic*, on its first voyage across the Atlantic, struck an iceberg and sank. Some passengers got off the ship in lifeboats, but many died. The two-way table gives information about adult passengers who lived and who died, by class of travel. Suppose we choose an adult passenger at random.

	Survival Status	
Class of Travel	**Survived**	**Died**
First class	197	122
Second class	94	167
Third class	151	476

(a) Given that the person selected was in first class, what's the probability that he or she survived?

(b) If the person selected survived, what's the probability that he or she was a third-class passenger?

65. **Get rich** Refer to Exercise 63.

(a) Find P("a good chance" | female).

(b) Find P("a good chance").

(c) Use your answers to (a) and (b) to determine whether the events "a good chance" and "female" are independent. Explain your reasoning.

66. **A *Titanic* disaster** Refer to Exercise 64.

(a) Find P(survived | second class).

(b) Find P(survived).

(c) Use your answers to (a) and (b) to determine whether the events "survived" and "second class" are independent. Explain your reasoning.

67. **Sampling senators** The two-way table describes the members of the U.S. Senate in a recent year. Suppose we select a senator at random. Consider events D: is a democrat, and F: is female.

	Male	**Female**
Democrats	47	13
Republicans	36	4

(a) Find $P(D \mid F)$. Explain what this value means.

(b) Find $P(F \mid D)$. Explain what this value means.

68. **Who eats breakfast?** The two-way table describes the 595 students who responded to a school survey about eating breakfast. Suppose we select a student at random. Consider events B: eats breakfast regularly, and M: is male.

	Male	**Female**	**Total**
Eats breakfast regularly	190	110	300
Doesn't eat breakfast regularly	130	165	295
Total	**320**	**275**	**595**

(a) Find $P(B \mid M)$. Explain what this value means.

(b) Find $P(M \mid B)$. Explain what this value means.

69. **Sampling senators** Refer to Exercise 67. Are events D and F independent? Justify your answer.

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70. **Who eats breakfast?** Refer to Exercise 68. Are events B and M independent? Justify your answer.

71. **Foreign-language study** Choose a student in grades 9 to 12 at random and ask if he or she is studying a language other than English. Here is the distribution of results:

Language:	Spanish	French	German	All others	None
Probability:	0.26	0.09	0.03	0.03	0.59

(a) What's the probability that the student is studying a language other than English?

(b) What is the conditional probability that a student is studying Spanish given that he or she is studying some language other than English?

72. **Income tax returns** Here is the distribution of the adjusted gross income (in thousands of dollars) reported on individual federal income tax returns in a recent year:

Income:	<25	25–49	50–99	100–499	≥500
Probability:	0.431	0.248	0.215	0.100	0.006

(a) What is the probability that a randomly chosen return shows an adjusted gross income of $50,000 or more?

(b) Given that a return shows an income of at least $50,000, what is the conditional probability that the income is at least $100,000?

73. **Tall people and basketball players** Select an adult at random. Define events T: person is over 6 feet tall, and B: person is a professional basketball player. Rank the following probabilities from smallest to largest. Justify your answer.

$P(T)$ $\quad P(B)$ $\quad P(T \mid B)$ $\quad P(B \mid T)$

74. **Teachers and college degrees** Select an adult at random. Define events A: person has earned a college degree, and T: person's career is teaching. Rank the following probabilities from smallest to largest. Justify your answer.

$P(A)$ $\quad P(T)$ $\quad P(A \mid T)$ $\quad P(T \mid A)$

75. **Rolling dice** Suppose you roll two fair, six-sided dice—one red and one green. Are the events "sum is 7" and "green die shows a 4" independent? Justify your answer.

76. **Rolling dice** Suppose you roll two fair, six-sided dice—one red and one green. Are the events "sum is 8" and "green die shows a 4" independent? Justify your answer.

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77. **Box of chocolates** According to Forrest Gump, "Life is like a box of chocolates. You never know what you're gonna get." Suppose a candy maker offers a special "Gump box" with 20 chocolate candies that look the same. In fact, 14 of the candies have soft centers and 6 have hard centers. Choose 2 of the candies from a Gump box at random.

(a) Draw a tree diagram that shows the sample space of this chance process.

(b) Find the probability that one of the chocolates has a soft center and the other one doesn't.

78. **Inspecting switches** A shipment contains 10,000 switches. Of these, 1000 are bad. An inspector draws 2 switches at random, one after the other.

(a) Draw a tree diagram that shows the sample space of this chance process.

(b) Find the probability that both switches are defective.

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79. **Free downloads?** Illegal music downloading has become a big problem: 29% of Internet users download music files, and 67% of downloaders say they don't care if the music is copyrighted.[15] What percent of Internet users download music and don't care if it's copyrighted? Write the information given in terms of probabilities, and use the general multiplication rule.

80. **At the gym** Suppose that 10% of adults belong to health clubs, and 40% of these health club members go to the club at least twice a week. What percent of all adults go to a health club at least twice a week? Write the information given in terms of probabilities, and use the general multiplication rule.

81. **Going pro** Only 5% of male high school basketball, baseball, and football players go on to play at the college level. Of these, only 1.7% enter major league professional sports. About 40% of the athletes who compete in college and then reach the pros have a career of more than 3 years.[16] What is the probability that a high school athlete who plays basketball, baseball, or football competes in college and then goes on to have a pro career of more than 3 years? Show your work.

82. **Teens online** We saw in an earlier example (page 319) that 93% of teenagers are online and that 55% of online teens have posted a profile on a social-networking site. Of online teens with a profile, 76% have placed comments on a friend's blog. What percent of all teens are online, have a profile, and comment on a friend's blog? Show your work.

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83. STEP 4 **Fill 'er up!** In a recent month, 88% of automobile drivers filled their vehicles with regular gasoline, 2% purchased midgrade gas, and 10% bought premium gas.[17] Of those who bought regular gas, 28% paid with a credit card; of customers who bought midgrade and premium gas, 34% and 42%, respectively, paid with a credit card. Suppose we select a customer at random. Draw a tree diagram to represent this situation. What's the probability that the customer paid with a credit card? Use the four-step process to guide your work.

84. STEP 4 **Urban voters** The voters in a large city are 40% white, 40% black, and 20% Hispanic. (Hispanics may be of any race in official statistics, but here we are speaking of political blocks.) A mayoral candidate anticipates attracting 30% of the white vote, 90% of the black vote, and 50% of the Hispanic vote. Draw a tree diagram to represent this situation. What percent of the overall vote does the candidate expect to get? Use the four-step process to guide your work.

85. **Fill 'er up** Refer to Exercise 83. Given that the customer paid with a credit card, find the probability that she bought premium gas.

86. **Urban voters** In the election described in Exercise 84, if the candidate's predictions come true, what percent of her votes come from black voters? (Write this as a conditional probability and use the definition of conditional probability.)

87. **Medical risks** Morris's kidneys are failing, and he is awaiting a kidney transplant. His doctor gives him this information for patients in his condition: 90% survive the transplant and 10% die. The transplant succeeds in 60% of those who survive, and the other 40% must return to kidney dialysis. The proportions who survive five years are 70% for those with a new kidney and 50% for those who return to dialysis.

(a) Make a tree diagram to represent this setting.

(b) Find the probability that Morris will survive for five years. Show your work.

88. **Winning at tennis** A player serving in tennis has two chances to get a serve into play. If the first serve goes out of bounds, the player serves again. If the second serve is also out, the player loses the point. Here are probabilities based on four years of the Wimbledon Championship:[18]

$$P(\text{1st serve in}) = 0.59 \quad P(\text{win point} \mid \text{1st serve in}) = 0.73$$

$$P(\text{2nd serve in} \mid \text{1st serve out}) = 0.86$$

$$P(\text{win point} \mid \text{1st serve out and 2nd serve in}) = 0.59$$

(a) Make a tree diagram for the results of the two serves and the outcome (win or lose) of the point.

(b) What is the probability that the serving player wins the point? Show your work.

89. **Bright lights?** A string of Christmas lights contains 20 lights. The lights are wired in series, so that if any light fails the whole string will go dark. Each light has probability 0.02 of failing during a 3-year period. The lights fail independently of each other. Find the probability that the string of lights will remain bright for 3 years.

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90. **Common names** The Census Bureau says that the 10 most common names in the United States are (in order) Smith, Johnson, Williams, Brown, Jones, Miller, Davis, Garcia, Rodriguez, and Wilson. These names account for 9.6% of all U.S. residents. Out of curiosity, you look at the authors of the textbooks for your current courses. There are 9 authors in all. Would you be surprised if none of the names of these authors were among the 10 most common? (Assume that authors' names are independent and follow the same probability distribution as the names of all residents.)

91. **Universal blood donors** People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7.2% of the American population have O-negative blood. If 10 people appear at random to give blood, what is the probability that at least 1 of them is a universal donor? Follow the four-step process.

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92. **Lost Internet sites** Internet sites often vanish or move, so that references to them can't be followed. In fact, 13% of Internet sites referenced in major scientific journals are lost within two years after publication.[19] If a paper contains seven Internet references, what is the probability that at least one of them doesn't work two years later? Follow the four-step process.

93. **Late shows** Some TV shows begin after their scheduled times when earlier programs run late. According to a network's records, about 3% of its shows start late. To find the probability that three consecutive shows on this network start on time, can we multiply (0.97)(0.97)(0.97)? Why or why not?

94. **Late flights** An airline reports that 85% of its flights arrive on time. To find the probability that its next four flights into LaGuardia Airport all arrive on time, can we multiply (0.85)(0.85)(0.85)(0.85)? Why or why not?

95. **MySpace versus Facebook** A recent survey suggests that 85% of college students have posted a profile on Facebook, 54% use MySpace regularly, and 42% do both. Suppose we select a college student at random and learn that the student has a profile on Facebook. Find the probability that the student uses MySpace regularly. Show your work.

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96. **Mac or PC?** A recent census at a major university revealed that 40% of its students mainly used Macintosh computers (Macs). The rest mainly used PCs. At the time of the census, 67% of the school's students were undergraduates. The rest were graduate students. In the census, 23% of the respondents were graduate students who said that they used PCs as their primary computers. Suppose we select a student at random from among those who were part of the census and learn that the student mainly uses a Mac. Find the probability that this person is a graduate student. Show your work.

97. **Lactose intolerance** Lactose intolerance causes difficulty in digesting dairy products that contain lactose (milk sugar). It is particularly common among people of African and Asian ancestry. In the United States (ignoring other groups and people who consider themselves to belong to more than one race), 82% of the population is white, 14% is black, and 4% is Asian. Moreover, 15% of whites, 70% of blacks, and 90% of Asians are lactose intolerant.[20]

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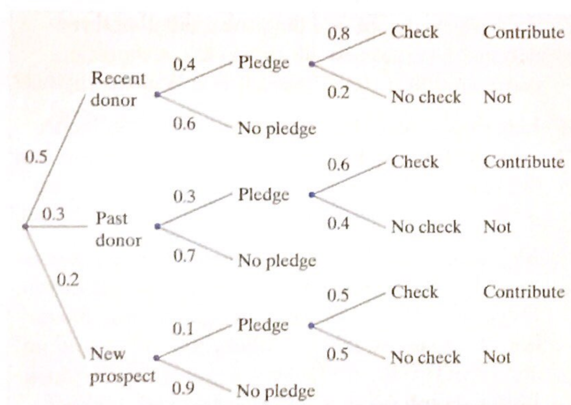
(a) What percent of the entire population is lactose intolerant?

(b) What percent of people who are lactose intolerant are Asian?

98. **Fundraising by telephone** Tree diagrams can organize problems having more than two stages. The figure shows probabilities for a charity calling potential donors by telephone.[21] Each person called is either a recent donor, a past donor, or a new prospect. At the next stage, the person called either does or does not pledge to contribute, with conditional probabilities that depend on the donor class the person belongs to. Finally, those who make a pledge either do or don't actually make a contribution.

(a) What percent of calls result in a contribution?

(b) What percent of those who contribute are recent donors?



99. **HIV testing** Enzyme immunoassay (EIA) tests are used to screen blood specimens for the presence of antibodies to HIV, the virus that causes AIDS. Antibodies indicate the presence of the virus. The test is quite accurate but is not always correct. Here are approximate probabilities of positive and negative EIA outcomes when the blood tested does and does not actually contain antibodies to HIV:[22]

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	Test Result	
	+	−
Antibodies present	0.9985	0.0015
Antibodies absent	0.006	0.994

Suppose that 1% of a large population carries antibodies to HIV in their blood.

(a) Draw a tree diagram for selecting a person from this population (outcomes: antibodies present or absent) and for testing his or her blood (outcomes: EIA positive or negative).

(b) What is the probability that the EIA test is positive for a randomly chosen person from this population?

(c) What is the probability that a person has the antibody given that the EIA test is positive?

100. **Testing the test** Are false positives too common in some medical tests? Researchers conducted an experiment involving 250 patients with a medical condition and 750 other patients who did not have the medical condition. The medical technicians who were reading the test results were unaware that they were subjects in an experiment.

(a) Technicians correctly identified 240 of the 250 patients with the condition. They also identified 50 of the healthy patients as having the condition. What were the false positive and false negative rates for the test?

(b) Given that a patient got a positive test result, what is the probability that the patient actually had the medical condition? Show your work.

101. **The probability of a flush** A poker player holds a flush when all 5 cards in the hand belong to the same suit. We will find the probability of a flush when 5 cards are dealt. Remember that a deck contains 52 cards, 13 of each suit, and that when the deck is well shuffled, each card dealt is equally likely to be any of those that remain in the deck.

(a) We will concentrate on spades. What is the probability that the first card dealt is a spade? What is the conditional probability that the second card is a spade given that the first is a spade?

(b) Continue to count the remaining cards to find the conditional probabilities of a spade on the third, the fourth, and the fifth card given in each case that all previous cards are spades.

(c) The probability of being dealt 5 spades is the product of the five probabilities you have found. Why? What is this probability?

(d) The probability of being dealt 5 hearts or 5 diamonds or 5 clubs is the same as the probability of being dealt 5 spades. What is the probability of being dealt a flush?

102. **The geometric distributions** You are tossing a pair of fair, six-sided dice in a board game. Tosses are independent. You land in a danger zone that requires you to roll doubles (both faces showing the same number of spots) before you are allowed to play again. How long will you wait to play again?

(a) What is the probability of rolling doubles on a single toss of the dice? (If you need review, the possible outcomes appear in Figure 5.2 (page 300). All 36 outcomes are equally likely.)