

9.2 TECHNOLOGY CORNER

One-proportion z test on the calculator page 555

TI-Nspire instructions in Appendix B

SECTION 9.2 Exercises

In Exercises 33 and 34, check that the conditions for carrying out a one-sample z test for the population proportion p are met.

- pg 550 33. **Lefties** Simon reads a newspaper report claiming that 12% of all adults in the United States are left-handed. He wonders if 12% of the students at his large public high school are left-handed. Simon chooses an SRS of 100 students and records whether each student is right- or left-handed.

34. **Don't argue!** A Gallup Poll report on a national survey of 1028 teenagers revealed that 72% of teens said they rarely or never argue with their friends.¹² Yvonne wonders whether this national result would be true in her large high school, so she surveys a random sample of 150 students at her school.

In Exercises 35 and 36, explain why we aren't safe carrying out a one-sample z test for the population proportion p .

35. **No test** You toss a coin 10 times to test the hypothesis $H_0: p = 0.5$ that the coin is balanced.
36. **No test** A college president says, "99% of the alumni support my firing of Coach Boggs." You contact an SRS of 200 of the college's 15,000 living alumni to test the hypothesis $H_0: p = 0.99$.
- pg 551 37. **Lefties** Refer to Exercise 33. In Simon's SRS, 16 of the students were left-handed.
- Calculate the test statistic.
 - Find the P -value using Table A. Show this result as an area under a standard Normal curve.
38. **Don't argue!** Refer to Exercise 34. For Yvonne's survey, 96 students in the sample said they seldom or never argue with friends.
- Calculate the test statistic.
 - Find the P -value using Table A. Show this result as an area under a standard Normal curve.
39. **Significance tests** A test of $H_0: p = 0.5$ versus $H_a: p > 0.5$ has test statistic $z = 2.19$.

- What conclusion would you draw at the 5% significance level? At the 1% level?

- If the alternative hypothesis were $H_a: p \neq 0.5$, what conclusion would you draw at the 5% significance level? At the 1% level?

40. **Significance tests** A test of $H_0: p = 0.65$ against $H_a: p < 0.65$ has test statistic $z = -1.78$.

- What conclusion would you draw at the 5% significance level? At the 1% level?

- If the alternative hypothesis were $H_a: p \neq 0.65$, what conclusion would you draw at the 5% significance level? At the 1% level?

- pg 553 41. **Better parking** A local high school makes a change that should improve student satisfaction with the parking situation. Before the change, 37% of the school's students approved of the parking that was provided. After the change, the principal surveys an SRS of 200 of the over 2500 students at the school. In all, 83 students say that they approve of the new parking arrangement. The principal cites this as evidence that the change was effective. Perform a test of the principal's claim at the $\alpha = 0.05$ significance level.

42. **Side effects** A drug manufacturer claims that less than 10% of patients who take its new drug for treating Alzheimer's disease will experience nausea. To test this claim, researchers conduct an experiment. They give the new drug to a random sample of 300 out of 5000 Alzheimer's patients whose families have given informed consent for the patients to participate in the study. In all, 25 of the subjects experience nausea. Use these data to perform a test of the drug manufacturer's claim at the $\alpha = 0.05$ significance level.

43. **Better parking** Refer to Exercise 41.

- Describe a Type I error and a Type II error in this setting, and explain the consequences of each.
- The test has a power of 0.75 to detect that $p = 0.45$. Explain what this means.
- Identify two ways to increase the power in part (b).

44. **Side effects** Refer to Exercise 42.

- Describe a Type I error and a Type II error in this setting, and explain the consequences of each.
- The test has a power of 0.54 to detect that $p = 0.07$. Explain what this means.
- Identify two ways to increase the power in part (b).

45. **Are boys more likely?** We hear that newborn babies are more likely to be boys than girls. Is this true? A random sample of 25,468 firstborn children included 13,173 boys.¹³ Boys do make up more than half of the sample, but of course we don't expect a perfect 50-50 split in a random sample.

- To what population can the results of this study be generalized: all children or all firstborn children? Justify your answer.
- Do these data give convincing evidence that boys are more common than girls in the population? Carry out a significance test to help answer this question.

46. **Fresh coffee** People of taste are supposed to prefer fresh-brewed coffee to the instant variety. On the other hand, perhaps many coffee drinkers just want their caffeine fix. A skeptic claims that only half of all coffee drinkers prefer fresh-brewed coffee. To test this claim, we ask a random sample of 50 coffee drinkers in a small city to take part in a study. Each person tastes two unmarked cups—one containing instant coffee and one containing fresh-brewed coffee—and says which he or she prefers. We find that 36 of the 50 choose the fresh coffee.

- We presented the two cups to each coffee drinker in a random order, so that some people tasted the fresh coffee first, while others drank the instant coffee first. Why do you think we did this?
- Do these results give convincing evidence that coffee drinkers favor fresh-brewed over instant coffee? Carry out a significance test to help answer this question.

47. **Bullies in middle school** A University of Illinois study on aggressive behavior surveyed a random sample of 558 middle school students. When asked to describe their behavior in the last 30 days, 445 students said their behavior included physical aggression, social ridicule, teasing, name-calling, and issuing threats. This behavior was not defined as bullying in the questionnaire.¹⁴ Is this evidence that more than three-quarters of the students at that middle school engage in bullying behavior? To find out, Maurice decides to perform a significance test. Unfortunately, he made a few errors along the way. Your job is to spot the mistakes and correct them.

$$H_0: p = 0.75$$

$$H_a: \hat{p} > 0.797$$

where p = the true mean proportion of middle school students who engaged in bullying.

- A random sample of 558 middle school students was surveyed.
- $558(0.797) = 444.73$ is at least 10.

$$z = \frac{0.75 - 0.797}{\sqrt{\frac{0.797(0.203)}{445}}} = -2.46; P\text{-value} = 2(0.0069) = 0.0138$$

The probability that the null hypothesis is true is only 0.0138, so we reject H_0 . This proves that more than three-quarters of the school engaged in bullying behavior.

48. **Is this coin fair?** The French naturalist Count Buffon (1707–1788) tossed a coin 4040 times. He got 2048 heads. That's a bit more than one-half. Is this evidence that Count Buffon's coin was not balanced? To find out, Luisa decides to perform a significance test. Unfortunately, she made a few errors along the way. Your job is to spot the mistakes and correct them.

$$H_0: \mu > 0.5$$

$$H_a: \bar{x} = 0.5$$

- Independent** $4040(0.5) = 2020$ and $4040(1 - 0.5) = 2020$ are both at least 10.
- Normal** There are at least 40,400 coins in the world.

$$t = \frac{0.5 - 0.507}{\sqrt{\frac{0.5(0.5)}{4040}}} = -0.89; P\text{-value} = 1 - 0.1867 = 0.8133$$

Reject H_0 because the P-value is so large and conclude that the coin is fair.

49. **Teen drivers** A state's Division of Motor Vehicles (DMV) claims that 60% of teens pass their driving test on the first attempt. An investigative reporter examines an SRS of the DMV records for 125 teens; 86 of them passed the test on their first try. Is this good evidence that the DMV's claim is incorrect? Carry out a test at the $\alpha = 0.05$ significance level to help answer this question.

50. **We want to be rich** In a recent year, 73% of first-year college students responding to a national survey identified "being very well-off financially" as an important personal goal. A state university finds that 132 of an SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%? Carry out a test at the $\alpha = 0.05$ significance level to help answer this question.

51. **Teen drivers** Refer to Exercise 49.

- Construct and interpret a 95% confidence interval for the proportion of all teens in the state who passed their driving test on the first attempt.
- Explain what the interval in part (a) tells you about the DMV's claim.

52. **We want to be rich** Refer to Exercise 50.

- Construct and interpret a 95% confidence interval for the true proportion p of all first-year students at the university who would identify being well-off as an important personal goal.
- Explain what the interval in part (a) tells you about whether the national value holds at this university.

53. **Do you Twitter?** In late 2009, the Pew Internet and American Life Project asked a random sample of U.S. adults, "Do you ever . . . use Twitter or another service to share updates about yourself or to see updates about others?" According to Pew, the resulting 95% confidence interval is (0.167, 0.213).¹⁵ Can we use this interval to conclude that the actual proportion of U.S. adults who would say they Twitter differs from 0.20? Justify your answer.

54. **Losing weight** A Gallup Poll found that 59% of the people in its sample said "Yes" when asked, "Would you like to lose weight?" Gallup announced: "For results based on the total sample of national adults, one can say with 95% confidence that the margin of (sampling) error is ± 3 percentage points."¹⁶ Can we use this interval to conclude that the actual proportion of U.S. adults who would say they want to lose weight differs from 0.55? Justify your answer.

55. **Teens and sex** The Gallup Youth Survey asked a random sample of U.S. teens aged 13 to 17 whether they thought that young people should wait to have sex until marriage.¹⁷ The Minitab output below shows the results of a significance test and a 95% confidence interval based on the survey data.

Sample	X	N	Sample p	95% CI	Z-Value	P-Value
1	246	439	0.560364	(0.513935, 0.606794)	2.53	0.011

- Define the parameter of interest.
- Check that the conditions for performing the significance test are met in this case.
- Interpret the P -value in context.
- Do these data give convincing evidence that the actual population proportion differs from 0.5? Justify your answer with appropriate evidence.

56. **Reporting cheating** What proportion of students are willing to report cheating by other students? A student project put this question to an SRS of 172 undergraduates at a large university: "You witness two students cheating on a quiz. Do you go to the professor?" The Minitab output below shows the results of a significance test and a 95% confidence interval based on the survey data.¹⁸

Sample	X	N	Sample p	95% CI	Z-Value	P-Value
1	19	172	0.110465	(0.063619, 0.157312)	-1.45	0.146

- Define the parameter of interest.
- Check that the conditions for performing the significance test are met in this case.
- Interpret the P -value in context.
- Do these data give convincing evidence that the actual population proportion differs from 0.15? Justify your answer with appropriate evidence.

Multiple choice: Select the best answer for Exercises 57 to 60.

57. After once again losing a football game to the archrival, a college's alumni association conducted a survey to see if alumni were in favor of firing the coach. An SRS of 100 alumni from the population of all living alumni was taken, and 64 of the alumni in the sample were in favor of firing the coach. Suppose you wish to see if a majority of living alumni are in favor of firing the coach. The appropriate test statistic is

$$\begin{array}{ll}
 \text{(a) } z = \frac{0.64 - 0.5}{\sqrt{\frac{0.64(0.36)}{100}}} & \text{(d) } z = \frac{0.64 - 0.5}{\sqrt{\frac{0.64(0.36)}{64}}} \\
 \text{(b) } t = \frac{0.64 - 0.5}{\sqrt{\frac{0.64(0.36)}{100}}} & \text{(e) } z = \frac{0.5 - 0.64}{\sqrt{\frac{0.5(0.5)}{100}}} \\
 \text{(c) } z = \frac{0.64 - 0.5}{\sqrt{\frac{0.5(0.5)}{100}}} &
 \end{array}$$

58. Which of the following is *not* a condition for performing a significance test about an unknown population proportion p ?

- The data should come from a random sample or randomized experiment.
- Individual measurements should be independent of one another.
- The population distribution should be approximately Normal, unless the sample size is large.

- (d) Both np and $n(1 - p)$ should be at least 10.
 (e) If you are sampling without replacement from a finite population, then you should sample no more than 10% of the population.

59. The z statistic for a test of $H_0: p = 0.4$ versus $H_a: p > 0.4$ is $z = 2.43$. This test is

- (a) not significant at either $\alpha = 0.05$ or $\alpha = 0.01$.
 (b) significant at $\alpha = 0.05$ but not at $\alpha = 0.01$.
 (c) significant at $\alpha = 0.01$ but not at $\alpha = 0.05$.
 (d) significant at both $\alpha = 0.05$ and $\alpha = 0.01$.
 (e) inconclusive because we don't know the value of \hat{p} .

60. Which of the following 95% confidence intervals would lead us to reject $H_0: p = 0.30$ in favor of $H_a: p \neq 0.30$ at the 5% significance level?

- (a) (0.29, 0.38) (c) (0.27, 0.31) (e) None of these
 (b) (0.19, 0.27) (d) (0.24, 0.30)

61. **Packaging CDs (6.2, 5.3)** A manufacturer of compact discs (CDs) wants to be sure that their CDs will fit inside the plastic cases they have bought for packaging. Both the CDs and the cases are circular. According to the supplier, the plastic cases vary Normally with mean diameter $\mu = 4.2$ inches and standard deviation $\sigma = 0.05$ inches. The CD manufacturer decides to produce CDs with mean diameter $\mu = 4$ inches. Their diameters follow a Normal distribution with $\sigma = 0.1$ inches.

(a) Let X = the diameter of a randomly selected CD and Y = the diameter of a randomly selected case. Describe the shape, center, and spread of the distribution of the random variable $X - Y$. What is the importance of this random variable to the CD manufacturer?

(b) Compute the probability that a randomly selected CD will fit inside a randomly selected case.

(c) The production process actually runs in batches of 100 CDs. If each of these CDs is paired with a randomly chosen plastic case, find the probability that all the CDs fit in their cases.



62. **Cash to find work? (5.2)** Will cash bonuses speed the return to work of unemployed people? The Illinois Department of Employment Security designed an experiment to find out. The subjects were 10,065 people aged 20 to 54 who were filing claims for unemployment insurance. Some were offered \$500 if they found a job within 11 weeks and held it for at least 4 months. Others could tell potential employers that the state would pay the employer \$500 for hiring them. A control group got neither kind of bonus.¹⁹

(a) Describe a completely randomized design for this experiment.

(b) How will you label the subjects for random assignment? Use Table D at line 127 to choose the first 3 subjects for the first treatment.

(c) Explain the purpose of a control group in this setting.