

## SECTION 2.2

## Exercises

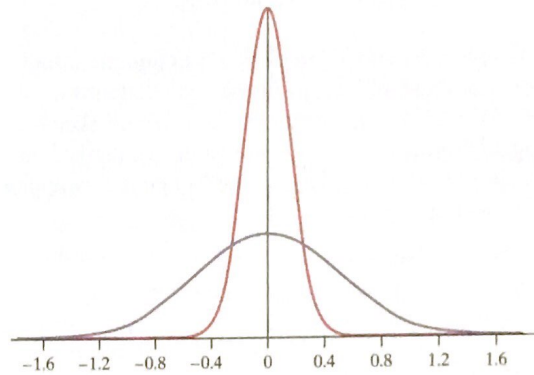
pg 113

41. **Men's heights** The distribution of heights of adult American men is approximately Normal with mean 69 inches and standard deviation 2.5 inches. Draw a Normal curve on which this mean and standard deviation are correctly located. (*Hint:* Draw the curve first, locate the points where the curvature changes, then mark the horizontal axis.)

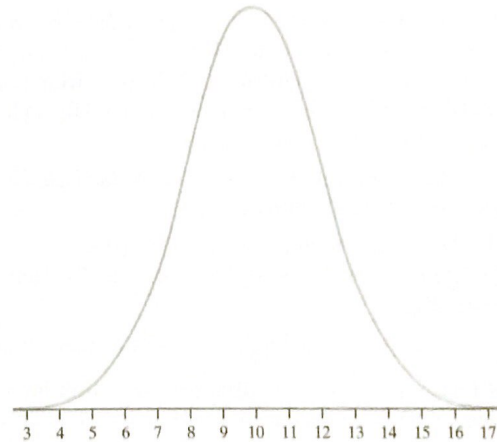
42. **Potato chips** The distribution of weights of 9-ounce bags of a particular brand of potato chips is approximately Normal with mean  $\mu = 9.12$  ounces and standard deviation  $\sigma = 0.05$  ounce. Draw an accurate sketch of the distribution of potato chip bag weights. Be sure to label the mean, as well as the points one, two, and three standard deviations away from the mean on the horizontal axis.

pg 113

43. **Men's heights** Refer to Exercise 41. Use the 68–95–99.7 rule to answer the following questions. Show your work!
- What percent of men are taller than 74 inches?
  - Between what heights do the middle 95% of men fall?
  - What percent of men are between 64 and 66.5 inches tall?
  - A height of 71.5 inches corresponds to what percentile of adult male American heights?
44. **Potato chips** Refer to Exercise 42. Use the 68–95–99.7 rule to answer the following questions. Show your work!
- What percent of bags weigh less than 9.02 ounces?
  - Between what weights do the middle 68% of bags fall?
  - What percent of 9-ounce bags of this brand of potato chips weigh between 8.97 and 9.17 ounces?
  - A bag that weighs 9.07 ounces is at what percentile in this distribution?
45. **Estimating SD** The figure at top right shows two Normal curves, both with mean 0. Approximately what is the standard deviation of each of these curves?



46. **A Normal curve** Estimate the mean and standard deviation of the Normal density curve in the figure below.



For Exercises 47 to 50, use Table A to find the proportion of observations from the standard Normal distribution that satisfies each of the following statements. In each case, sketch a standard Normal curve and shade the area under the curve that is the answer to the question. Use your calculator or the *Normal Curve* applet to check your answers.

47. **Table A practice**

- $z < 2.85$
- $z > 2.85$
- $z > -1.66$
- $-1.66 < z < 2.85$

48. **Table A practice**

- $z < -2.46$
- $z > 2.46$
- $0.89 < z < 2.46$
- $-2.95 < z < -1.27$

## 49. More Table A practice

- pg 117 (a)  $z$  is between  $-1.33$  and  $1.65$   
 (b)  $z$  is between  $0.50$  and  $1.79$

## 50. More Table A practice

- (a)  $z$  is between  $-2.05$  and  $0.78$   
 (b)  $z$  is between  $-1.11$  and  $-0.32$

For Exercises 51 and 52, use Table A to find the value  $z$  from the standard Normal distribution that satisfies each of the following conditions. In each case, sketch a standard Normal curve with your value of  $z$  marked on the axis. Use your calculator or the *Normal Curve* applet to check your answers.

## 51. Working backward

- (a) The 10th percentile.  
 (b) 34% of all observations are greater than  $z$ .

## 52. Working backward

- (a) The 63rd percentile.  
 (b) 75% of all observations are greater than  $z$ .

- pg 120–122 53. Length of pregnancies The length of human pregnancies from conception to birth varies according to a distribution that is approximately Normal with mean 266 days and standard deviation 16 days. For each part, follow the four-step process.



- (a) At what percentile is a pregnancy that lasts 240 days (that's about 8 months)?  
 (b) What percent of pregnancies last between 240 and 270 days (roughly between 8 months and 9 months)?  
 (c) How long do the longest 20% of pregnancies last?



54. IQ test scores Scores on the Wechsler Adult Intelligence Scale (a standard IQ test) for the 20 to 34 age group are approximately Normally distributed with  $\mu = 110$  and  $\sigma = 25$ . For each part, follow the four-step process.

- (a) At what percentile is an IQ score of 150?  
 (b) What percent of people aged 20 to 34 have IQs between 125 and 150?  
 (c) MENSA is an elite organization that admits as members people who score in the top 2% on IQ tests. What score on the Wechsler Adult Intelligence Scale would an individual have to earn to qualify for MENSA membership?

55. I think I can! An important measure of the performance of a locomotive is its "adhesion," which is the locomotive's pulling force as a multiple of its weight. The adhesion of one 4400-horsepower diesel locomotive varies in actual use according to a Normal distribution with mean  $\mu = 0.37$  and standard deviation

$\sigma = 0.04$ . For each part that follows, sketch and shade an appropriate Normal distribution. Then show your work.

(a) For a certain small train's daily route, the locomotive needs to have an adhesion of at least 0.30 for the train to arrive at its destination on time. On what proportion of days will this happen? Show your method.

(b) An adhesion greater than 0.50 for the locomotive will result in a problem because the train will arrive too early at a switch point along the route. On what proportion of days will this happen? Show your method.

(c) Compare your answers to (a) and (b). Does it make sense to try to make one of these values larger than the other? Why or why not?

56. Put a lid on it! At some fast-food restaurants, customers who want a lid for their drinks get them from a large stack left near straws, napkins, and condiments. The lids are made with a small amount of flexibility so they can be stretched across the mouth of the cup and then snugly secured. When lids are too small or too large, customers can get very frustrated, especially if they end up spilling their drinks. At one particular restaurant, large drink cups require lids with a "diameter" of between 3.95 and 4.05 inches. The restaurant's lid supplier claims that the diameter of their large lids follows a Normal distribution with mean 3.98 inches and standard deviation 0.02 inches. Assume that the supplier's claim is true.

- (a) What percent of large lids are too small to fit? Show your method.  
 (b) What percent of large lids are too big to fit? Show your method.  
 (c) Compare your answers to (a) and (b). Does it make sense for the lid manufacturer to try to make one of these values larger than the other? Why or why not?

57. I think I can! Refer to Exercise 55. The locomotive's manufacturer is considering two changes that could reduce the percent of times that the train arrives late. One option is to increase the mean adhesion of the locomotive. The other possibility is to decrease the variability in adhesion from trip to trip, that is, to reduce the standard deviation.  
 (a) If the standard deviation remains at  $\sigma = 0.04$ , to what value must the manufacturer change the mean adhesion of the locomotive to reduce its proportion of late arrivals to less than 2% of days? Show your work.  
 (b) If the mean adhesion stays at  $\mu = 0.37$ , how much must the standard deviation be decreased to ensure that the train will arrive late less than 2% of the time? Show your work.



- (c) Which of the two options in (a) and (b) do you think is preferable? Justify your answer. (Be sure to consider the effect of these changes on the percent of days that the train arrives early to the switch point.)
58. **Put a lid on it!** Refer to Exercise 56. The supplier is considering two changes to reduce the percent of its large-cup lids that are too small to less than 1%. One strategy is to adjust the mean diameter of its lids. Another option is to alter the production process, thereby decreasing the standard deviation of the lid diameters.
- (a) If the standard deviation remains at  $\sigma = 0.02$  inches, at what value should the supplier set the mean diameter of its large-cup lids to ensure that less than 1% are too small to fit? Show your method.
- (b) If the mean diameter stays at  $\mu = 3.98$  inches, what value of the standard deviation will result in less than 1% of lids that are too small to fit? Show your method.
- (c) Which of the two options in (a) and (b) do you think is preferable? Justify your answer. (Be sure to consider the effect of these changes on the percent of lids that are too large to fit.)
59. **Deciles** The deciles of any distribution are the points that mark off the lowest 10% and the highest 10%. The deciles of a density curve are therefore the points with area 0.1 and 0.9 to their left under the curve.
- (a) What are the deciles of the standard Normal distribution?
- (b) The heights of young women are approximately Normal with mean 64.5 inches and standard deviation 2.5 inches. What are the deciles of this distribution? Show your work.
60. **Outliers** The percent of the observations that are classified as outliers by the  $1.5 \times IQR$  rule is the same in any Normal distribution. What is this percent? Show your method clearly.
61. **Flight times** An airline flies the same route at the same time each day. The flight time varies according to a Normal distribution with unknown mean and standard deviation. On 15% of days, the flight takes more than an hour. On 3% of days, the flight lasts 75 minutes or more. Use this information to determine the mean and standard deviation of the flight time distribution.
62. **Brush your teeth** The amount of time Ricardo spends brushing his teeth follows a Normal distribution with unknown mean and standard deviation. Ricardo spends less than one minute brushing his teeth about 40% of the time. He spends more than

two minutes brushing his teeth 2% of the time. Use this information to determine the mean and standard deviation of this distribution.

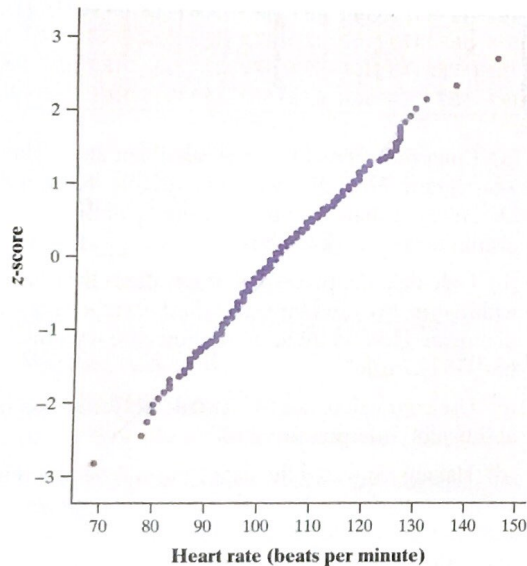
63. **Sharks** Here are the lengths in feet of 44 great white sharks:<sup>12</sup>

pg 127

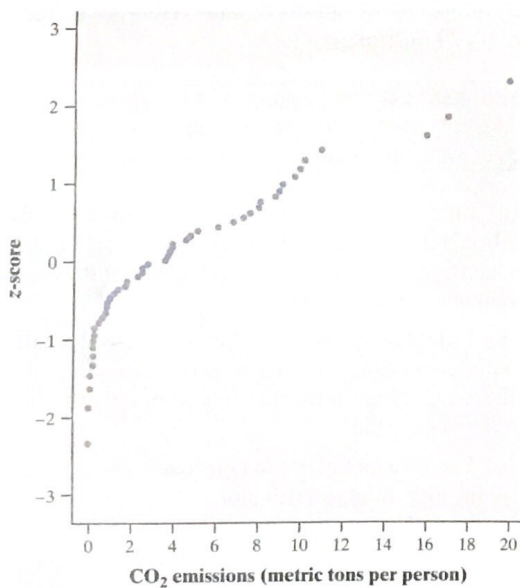
18.7	12.3	18.6	16.4	15.7	18.3	14.6	15.8	14.9	17.6	12.1
16.4	16.7	17.8	16.2	12.6	17.8	13.8	12.2	15.2	14.7	12.4
13.2	15.8	14.3	16.6	9.4	18.2	13.2	13.6	15.3	16.1	13.5
19.1	16.2	22.8	16.8	13.6	13.2	15.7	19.7	18.7	13.2	16.8

- (a) Enter these data into your calculator and make a histogram. Then calculate one-variable statistics. Describe the shape, center, and spread of the distribution of shark lengths.
- (b) Calculate the percent of observations that fall within one, two, and three standard deviations of the mean. How do these results compare with the 68–95–99.7 rule?
- (c) Use your calculator to construct a Normal probability plot. Interpret this plot.
- (d) Having inspected the data from several different perspectives, do you think these data are approximately Normal? Write a brief summary of your assessment that combines your findings from (a) through (c).
64. **Density of the earth** In 1798, the English scientist Henry Cavendish measured the density of the earth several times by careful work with a torsion balance. The variable recorded was the density of the earth as a multiple of the density of water. Here are Cavendish's 29 measurements:<sup>13</sup>
- |      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|
| 5.50 | 5.61 | 4.88 | 5.07 | 5.26 | 5.55 | 5.36 | 5.29 | 5.58 | 5.65 |
| 5.57 | 5.53 | 5.62 | 5.29 | 5.44 | 5.34 | 5.79 | 5.10 | 5.27 | 5.39 |
| 5.42 | 5.47 | 5.63 | 5.34 | 5.46 | 5.30 | 5.75 | 5.68 | 5.85 |      |
- (a) Enter these data into your calculator and make a histogram. Then calculate one-variable statistics. Describe the shape, center, and spread of the distribution of density measurements.
- (b) Calculate the percent of observations that fall within one, two, and three standard deviations of the mean. How do these results compare with the 68–95–99.7 rule?
- (c) Use your calculator to construct a Normal probability plot. Interpret this plot.
- (d) Having inspected the data from several different perspectives, do you think these data are approximately Normal? Write a brief summary of your assessment that combines your findings from (a) through (c).

65. **Runners' heart rates** The figure below is a Normal probability plot of the heart rates of 200 male runners after six minutes of exercise on a treadmill.<sup>14</sup> The distribution is close to Normal. How can you see this? Describe the nature of the small deviations from Normality that are visible in the plot.



66. **Carbon dioxide emissions** The figure below is a Normal probability plot of the emissions of carbon dioxide per person in 48 countries.<sup>15</sup> In what ways is this distribution non-Normal?



67. **Normal is only approximate: ACT scores** Scores on the ACT test for the 2007 high school graduating class had mean 21.2 and standard deviation 5.0. In all, 1,300,599 students in this class took the test. Of these, 149,164 had scores higher than 27 and another 50,310 had scores exactly 27. ACT scores are always whole numbers. The exactly Normal  $N(21.2, 5.0)$  distribution can include any value, not just whole numbers. What's more, there is *no* area exactly above 27 under the smooth Normal curve. So ACT scores can be only approximately Normal. To illustrate this fact, find

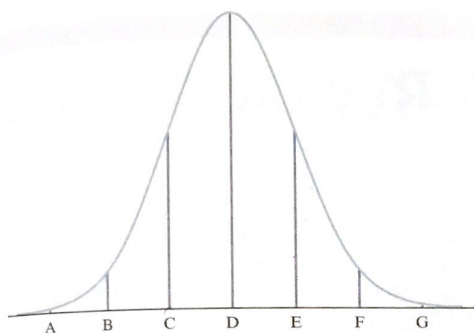
- the percent of 2007 ACT scores greater than 27.
  - the percent of 2007 ACT scores greater than or equal to 27.
  - the percent of observations from the  $N(21.2, 5.0)$  distribution that are greater than 27. (The percent greater than or equal to 27 is the same, because there is no area exactly over 27.)
68. **Weights aren't Normal** The heights of people of the same gender and similar ages follow Normal distributions reasonably closely. Weights, on the other hand, are not Normally distributed. The weights of women aged 20 to 29 have mean 141.7 pounds and median 133.2 pounds. The first and third quartiles are 118.3 pounds and 157.3 pounds. What can you say about the shape of the weight distribution? Why?

*Multiple choice: Select the best answer for Exercises 69 to 74.*

69. Which of the following is *least* likely to have a nearly Normal distribution?
- Heights of all female students taking STAT 001 at State Tech.
  - IQ scores of all students taking STAT 001 at State Tech.
  - SAT Math scores of all students taking STAT 001 at State Tech.
  - Family incomes of all students taking STAT 001 at State Tech.
  - All of (a)–(d) will be approximately Normal.

*Exercises 70 to 72 refer to the following setting. The weights of laboratory cockroaches follow a Normal distribution with mean 80 grams and standard deviation 2 grams. The figure below is the Normal curve for this distribution of weights.*

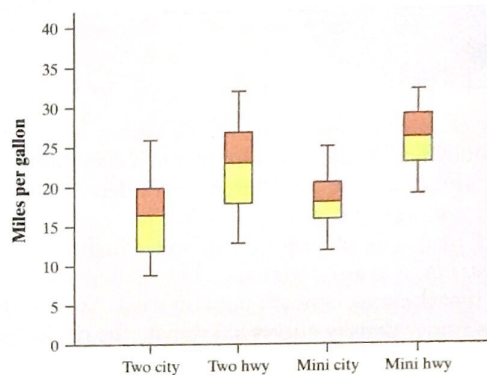




70. Point C on this Normal curve corresponds to  
 (a) 84 grams. (c) 78 grams. (e) 74 grams.  
 (b) 82 grams. (d) 76 grams.
71. About what percent of the cockroaches have weights between 76 and 84 grams?  
 (a) 99.7% (c) 68% (e) 34%  
 (b) 95% (d) 47.5%
72. About what percent of the cockroaches have weights less than 78 grams?  
 (a) 34% (c) 16% (e) none of these  
 (b) 32% (d) 2.5%
73. The proportion of observations from a standard Normal distribution with values less than 1.15 is  
 (a) 0.1251. (c) 0.8749. (e) none of these.  
 (b) 0.8531. (d) 0.8944
74. The proportion of observations from a standard Normal distribution with values larger than  $-0.75$  is  
 (a) 0.2266. (c) 0.7734. (e) none of these  
 (b) 0.7422. (d) 0.8023.

75. Gas it up! (1.3) Interested in a sporty car? Worried that it might use too much gas? The Environmental Protection Agency lists most such vehicles in its "two-seater" or "minicompact" categories. The figure

shows boxplots for both city and highway gas mileages for our two groups of cars. Write a few sentences comparing these distributions.



76. Python eggs (1.1) How is the hatching of water python eggs influenced by the temperature of the snake's nest? Researchers assigned newly laid eggs to one of three temperatures: hot, neutral, or cold. Hot duplicates the extra warmth provided by the mother python, and cold duplicates the absence of the mother. Here are the data on the number of eggs and the number that hatched:<sup>16</sup>

	Cold	Neutral	Hot
Number of eggs	27	56	104
Number hatched	16	38	75

- (a) Make a two-way table of temperature by outcome (hatched or not).  
 (b) Calculate the percent of eggs in each group that hatched. The researchers believed that eggs would not hatch in cold water. Do the data support that belief?