SECTION 7.3 Exercises

49. **Songs on an iPod** David's iPod has about 10,000 songs. The distribution of the play times for these songs is heavily skewed to the right with a mean of 225 seconds and a standard deviation of 60 seconds. Suppose we choose an SRS of 10 songs from this population and calculate the mean play time $\bar{x}$ of these songs. What are the mean and the standard deviation of the sampling distribution of $\bar{x}$? Explain.

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50. **Making auto parts** A grinding machine in an auto parts plant prepares axles with a target diameter $\mu = 40.125$ millimeters (mm). The machine has some variability, so the standard deviation of the diameters is $\sigma = 0.002$ mm. The machine operator inspects a random sample of 4 axles each hour for quality control purposes and records the sample mean diameter $\bar{x}$. Assuming that the process is working properly, what are the mean and standard deviation of the sampling distribution of $\bar{x}$? Explain.

51. **Songs on an iPod** Refer to Exercise 49. How many songs would you need to sample if you wanted the standard deviation of the sampling distribution of $\bar{x}$ to be 30 seconds? Justify your answer.

52. **Making auto parts** Refer to Exercise 50. How many axles would you need to sample if you wanted the standard deviation of the sampling distribution of $\bar{x}$ to be 0.0005 mm? Justify your answer.

53. **Larger sample** Suppose that the blood cholesterol level of all men aged 20 to 34 follows the Normal distribution with mean $\mu = 188$ milligrams per deciliter (mg/dl) and standard deviation $\sigma = 41$ mg/dl.

(a) Choose an SRS of 100 men from this population. What is the sampling distribution of $\bar{x}$?

(b) Find the probability that $\bar{x}$ estimates μ within ± 3 mg/dl. (This is the probability that $\bar{x}$ takes a value between 185 and 191 mg/dl.) Show your work.

(c) Choose an SRS of 1000 men from this population. Now what is the probability that $\bar{x}$ falls within ± 3 mg/dl of μ? Show your work. In what sense is the larger sample "better"?

54. **Stop the car!** A car company has found that the lifetime of its disc brake pads varies from car to car according to a Normal distribution with mean $\mu = 55{,}000$ miles and standard deviation $\sigma = 4500$ miles. The company installs a new brand of brake pads on an SRS of 8 cars.

(a) If the new brand has the same lifetime distribution as the previous type of brake pad, what is the sampling distribution of the mean lifetime $\bar{x}$?

(b) The average life of the pads on these 8 cars turns out to be $\bar{x} = 51{,}800$ miles. Find the probability that the sample mean lifetime is 51,800 miles or less if the lifetime distribution is unchanged. What conclusion would you draw?

55. **Bottling cola** A bottling company uses a filling machine to fill plastic bottles with cola. The bottles are supposed to contain 300 milliliters (ml). In fact, the contents vary according to a Normal distribution with mean $\mu = 298$ ml and standard deviation $\sigma = 3$ ml.

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(a) What is the probability that an individual bottle contains less than 295 ml? Show your work.

(b) What is the probability that the mean contents of six randomly selected bottles is less than 295 ml? Show your work.

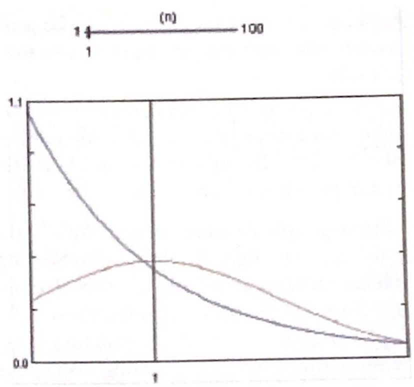
56. **ACT scores** The composite scores of individual students on the ACT college entrance examination in 2009 followed a Normal distribution with mean 21.1 and standard deviation 5.1.

(a) What is the probability that a single student randomly chosen from all those taking the test scores 23 or higher? Show your work.

(b) Now take an SRS of 50 students who took the test. What is the probability that the mean score $\bar{x}$ of these students is 23 or higher? Show your work.

57. **What does the CLT say?** Asked what the central limit theorem says, a student replies, "As you take larger and larger samples from a population, the histogram of the sample values looks more and more Normal." Is the student right? Explain your answer.

58. **The CLT applet** Go to the textbook Web site (www.whfreeman.com/tps4e) and click on "Statistical Applets." Launch the *Central Limit Theorem* applet. You should see a screen like the one shown here. Click and drag the slider to change the sample size, and watch how the density curve for the sampling distribution changes with it. Write a few sentences describing what is happening.



59. **Songs on an iPod** Refer to Exercise 49.

(a) Explain why you cannot safely calculate the probability that the mean play time $\bar{x}$ is more than 4 minutes (240 seconds) for an SRS of 10 songs.

(b) Suppose we take an SRS of 36 songs instead. Explain how the central limit theorem allows us to find the probability that the mean play time is more than 240 seconds. Then calculate this probability. Show your work.

60. **Lightning strikes** The number of lightning strikes on a square kilometer of open ground in a year has mean 6 and standard deviation 2.4. (These values are typical of much of the United States.) The National Lightning Detection Network (NLDN) uses automatic sensors to watch for lightning in a random sample of 10 one-square-kilometer plots of land.

(a) What are the mean and standard deviation of $\bar{x}$, the sample mean number of strikes per square kilometer?

(b) Explain why you cannot safely calculate the probability that $\bar{x} < 5$ based on a sample of size 10.

(c) Suppose the NLDN takes a random sample of $n = 50$ square kilometers instead. Explain how the central limit theorem allows us to find the probability that the mean number of lightning strikes per square kilometer is less than 5. Then calculate this probability. Show your work.

61. **Airline passengers get heavier** In response to the increasing weight of airline passengers, the Federal Aviation Administration (FAA) in 2003 told airlines to assume that passengers average 190 pounds in the summer, including clothes and carry-on baggage. But passengers vary, and the FAA did not specify a standard deviation. A reasonable standard deviation is 35 pounds. Weights are not Normally distributed, especially when the population includes both men and women, but they are not very non-Normal. A commuter plane carries 30 passengers.

(a) Explain why you cannot calculate the probability that a randomly selected passenger weighs more than 200 pounds.

(b) Find the probability that the total weight of the passengers on a full flight exceeds 6000 pounds. Show your work. (*Hint:* To apply the central limit theorem, restate the problem in terms of the mean weight.)

62. **How many people in a car?** A study of rush-hour traffic in San Francisco counts the number of people in each car entering a freeway at a suburban interchange. Suppose that this count has mean 1.5 and standard deviation 0.75 in the population of all cars that enter at this interchange during rush hours.

(a) Could the exact distribution of the count be Normal? Why or why not?

(b) Traffic engineers estimate that the capacity of the interchange is 700 cars per hour. Find the probability that 700 cars will carry more than 1075 people. Show your work. (*Hint:* Restate this event in terms of the mean number of people $\bar{x}$ per car.)

63. **More on insurance** An insurance company knows that in the entire population of homeowners, the mean annual loss from fire is $\mu = \$250$ and the standard deviation of the loss is $\sigma = \$300$. The distribution of losses is strongly right-skewed: many policies have \$0 loss, but a few have large losses. If the company sells 10,000 policies, can it safely base its rates on the assumption that its average loss will be no greater than \$275? Follow the four-step process.

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STEP 4

64. **Bad carpet** The number of flaws per square yard in a type of carpet material varies with mean 1.6 flaws per square yard and standard deviation 1.2 flaws per square yard. The population distribution cannot be Normal, because a count takes only whole-number values. An inspector studies 200 square yards of the material, records the number of flaws found in each square yard, and calculates $\bar{x}$, the mean number of flaws per square yard inspected. Find the probability that the mean number of flaws exceeds 2 per square yard. Follow the four-step process.

STEP 4

Multiple choice: Select the best answer for Exercises 65 to 68.

Exercises 65 and 66 refer to the following setting. Scores on the mathematics part of the SAT exam in a recent year were roughly Normal with mean 515 and standard deviation 114. You choose an SRS of 100 students and average their SAT Math scores. Suppose that you do this many, many times.

65. The mean of the average scores you get should be close to

(a) 515.
(b) $515/100 = 5.15$.
(c) $515/\sqrt{100} = 51.5$.
(d) 0.
(e) none of these.

66. The standard deviation of the average scores you get should be close to

(a) 114.
(b) $114/100 = 1.14$.
(c) $114/\sqrt{100} = 11.4$.
(d) 1.
(e) none of these.

67. A newborn baby has extremely low birth weight (ELBW) if it weighs less than 1000 grams. A study of the health of such children in later years examined a random sample of 219 children. Their mean weight at birth was $\bar{x} = 810$ grams. This sample mean is an *unbiased estimator* of the mean weight μ in the population of all ELBW babies, which means that

(a) in all possible samples of size 219 from this population, the mean of the values of $\bar{x}$ will equal 810.

(b) in all possible samples of size 219 from this population, the mean of the values of $\bar{x}$ will equal μ.

(c) as we take larger and larger samples from this population, $\bar{x}$ will get closer and closer to μ.

(d) in all possible samples of size 219 from this population, the values of $\bar{x}$ will have a distribution that is close to Normal.

(e) the person measuring the children's weights does so without any systematic error.

68. The number of hours a light bulb burns before failing varies from bulb to bulb. The distribution of burnout times is strongly skewed to the right. The central limit theorem says that

(a) as we look at more and more bulbs, their average burnout time gets ever closer to the mean μ for all bulbs of this type.

(b) the average burnout time of a large number of bulbs has a distribution of the same shape (strongly skewed) as the population distribution.

(c) the average burnout time of a large number of bulbs has a distribution with similar shape but not as extreme (skewed, but not as strongly) as the population distribution.

(d) the average burnout time of a large number of bulbs has a distribution that is close to Normal.

(e) the average burnout time of a large number of bulbs has a distribution that is exactly Normal.

Exercises 69 to 72 refer to the following setting. In the language of government statistics, you are "in the labor force" if you are available for work and either working or actively