SECTION 9.1 Exercises

In Exercises 1 to 6, each situation calls for a significance test. State the appropriate null hypothesis H_0 and alternative hypothesis H_a in each case. Be sure to define your parameter each time.

1. **Lefties** Simon reads a newspaper report claiming that 12% of all adults in the United States are left-handed. He wonders if 12% of the students at his large public high school are left-handed. Simon chooses an SRS of 100 students and records whether each student is right- or left-handed.

2. **Don't argue!** A Gallup Poll report on a national survey of 1028 teenagers revealed that 72% of teens said they seldom or never argue with their friends.[7] Yvonne wonders whether this national result would be true in her large high school. So she surveys a random sample of 150 students at her school.

3. **Attitudes** The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures students' attitudes toward school and study habits. Scores range from 0 to 200. The mean score for U.S. college students is about 115. A teacher suspects that older students have better attitudes toward school. She gives the SSHA to an SRS of 45 of the over 1000 students at her college who are at least 30 years of age.

pg 531

4. **Anemia** Hemoglobin is a protein in red blood cells that carries oxygen from the lungs to body tissues. People with less than 12 grams of hemoglobin per deciliter of blood (g/dl) are anemic. A public health official in Jordan suspects that Jordanian children are at risk of anemia. He measures a random sample of 50 children.

5. **Cold cabin?** During the winter months, the temperatures at the Colorado cabin owned by the Starnes family can stay well below freezing (32°F or 0°C) for weeks at a time. To prevent the pipes from freezing, Mrs. Starnes sets the thermostat at 50°F. The manufacturer claims that the thermostat allows variation in home temperature of $\sigma = 3$°F. Mrs. Starnes suspects that the manufacturer is overstating how well the thermostat works.

6. **Ski jump** When ski jumpers take off, the distance they fly varies considerably depending on their speed, skill, and wind conditions. Event organizers must position the landing area to allow for differences in the distances that the athletes fly. For a particular competition, the organizers estimate that the variation in distance flown by the athletes will be $\sigma = 10$ meters. An experienced jumper thinks that the organizers are underestimating the variation.

In Exercises 7 to 10, explain what's wrong with the stated hypotheses. Then give correct hypotheses.

7. **Better parking** A change is made that should improve student satisfaction with the parking situation at a local high school. Right now, 37% of students approve of the parking that's provided. The null hypothesis $H_0: p > 0.37$ is tested against the alternative $H_a: p = 0.37$.

8. **Better parking** A change is made that should improve student satisfaction with the parking situation at your school. Right now, 37% of students approve of the parking that's provided. The null hypothesis $H_0: \hat{p} = 0.37$ is tested against the alternative $H_a: \hat{p} \neq 0.37$.

9. **Birth weights** In planning a study of the birth weights of babies whose mothers did not see a doctor before delivery, a researcher states the hypotheses as

$$H_0: \bar{x} = 1000 \text{ grams}$$
$$H_a: \bar{x} < 1000 \text{ grams}$$

10. **Birth weights** In planning a study of the birth weights of babies whose mothers did not see a doctor before delivery, a researcher states the hypotheses as

$$H_0: \mu < 1000 \text{ grams}$$
$$H_a: \mu = 900 \text{ grams}$$

11. **Lefties** Refer to Exercise 1. In Simon's SRS, 16 of the students were left-handed. A significance test yields a P-value of 0.2184.

pg 534

(a) Interpret this result in context.

(b) Do the data provide convincing evidence against the null hypothesis? Explain.

12. **Don't argue!** Refer to Exercise 2. For Yvonne's survey, 96 students in the sample said they rarely or never argue with friends. A significance test yields a P-value of 0.0291.

(a) Interpret this result in context.

(b) Do the data provide convincing evidence against the null hypothesis? Explain.

13. **Attitudes** In the study of older students' attitudes from Exercise 3, the sample mean SSHA score was 125.7 and the sample standard deviation was 29.8. A significance test yields a P-value of 0.0101.

pg 536

(a) Interpret the P-value in context.

(b) What conclusion would you make if $\alpha = 0.05$? If $\alpha = 0.01$? Justify your answer.

14. **Anemia** For the study of Jordanian children in Exercise 4, the sample mean hemoglobin level was 11.3 g/dl and the sample standard deviation was 1.6 g/dl. A significance test yields a P-value of 0.0016.

(a) Interpret the P-value in context.

(b) What conclusion would you make if $\alpha = 0.05$? $\alpha = 0.01$? Justify your answer.

15. **Is this what P means?** When asked to explain the meaning of the P-value in Exercise 13, a student says, "This means there is only probability 0.01 that the null hypothesis is true." Explain clearly why the student's explanation is wrong.

16. **Statistical significance** Asked to explain the meaning of "statistically significant at the $\alpha = 0.05$ level," a student says, "This means that the probability that the null hypothesis is true is less than 0.05." Is this explanation correct? Why or why not?

17. **Statistical significance** Explain in plain language why a significance test that is significant at the 1% level must always be significant at the 5% level. If a test is significant at the 5% level, what can you say about its significance at the 1% level?

18. ***P*-values and statistical significance** Write a few sentences comparing what the P-values in Exercises 13 and 14 tell you about statistical significance in each of the two studies.

Exercises 19 and 20 refer to the following setting. Slow response times by paramedics, firefighters, and policemen can have serious consequences for accident victims. In the case of life-threatening injuries, victims generally need medical attention within 8 minutes of the accident. Several cities have begun to monitor emergency response times. In one such city, the mean response time to all accidents involving life-threatening injuries last year was $\mu = 6.7$ minutes. Emergency personnel arrived within 8 minutes after 78% of all calls involving life-threatening injuries last year. The city manager shares this information and encourages these first responders to "do better." At the end of the year, the city manager selects an SRS of 400 calls involving life-threatening injuries and examines the response times.

19. **Awful accidents**

pg 538

(a) State hypotheses for a significance test to determine whether the average response time has decreased. Be sure to define the parameter of interest.

(b) Describe a Type I error and a Type II error in this setting, and explain the consequences of each.

(c) Which is more serious in this setting: a Type I error or a Type II error? Justify your answer.

20. **Awful accidents**

(a) State hypotheses for a significance test to determine whether first responders are arriving within 8 minutes of the call more often. Be sure to define the parameter of interest.

(b) Describe a Type I error and a Type II error in this setting and explain the consequences of each.

(c) Which is more serious in this setting: a Type I error or a Type II error? Justify your answer.

(d) If you sustain a life-threatening injury due to a vehicle accident, you want to receive medical treatment as quickly as possible. Which of the two significance tests—$H_0: \mu = 6.7$ versus $H_a: \mu < 6.7$ or the one from part (a) of this exercise—would you be more interested in? Justify your answer.

21. **Opening a restaurant** You are thinking about opening a restaurant and are searching for a good location. From research you have done, you know that the mean income of those living near the restaurant must be over $85,000 to support the type of upscale restaurant you wish to open. You decide to take a simple random sample of 50 people living near one potential location. Based on the mean income of this sample, you will decide whether to open a restaurant there.[8]

(a) State appropriate null and alternative hypotheses. Be sure to define your parameter.

(b) Describe a Type I and a Type II error, and explain the consequences of each.

(c) If you had to choose one of the "standard" significance levels for your significance test, would you choose $\alpha = 0.01$, 0.05, or 0.10? Justify your choice.

22. **Blood pressure screening** Your company markets a computerized device for detecting high blood pressure. The device measures an individual's blood pressure once per hour at a randomly selected time throughout a 12-hour period. Then it calculates the mean systolic (top number) pressure for the sample of measurements. Based on the sample results, the device determines whether there is significant evidence that the individual's actual mean systolic pressure is greater than 130. If so, it recommends that the person seek medical attention.

(a) State appropriate null and alternative hypotheses in this setting. Be sure to define your parameter.

(b) Describe a Type I and a Type II error, and explain the consequences of each.

(c) The blood pressure device can be adjusted to decrease one error probability at the cost of an increase in the other error probability. Which error probability would you choose to make smaller, and why?

23. **Error probabilities** You read that a statistical test at significance level $\alpha = 0.05$ has power 0.78. What are the probabilities of Type I and Type II errors for this test?

24. **Error probabilities** You read that a statistical test at the $\alpha = 0.01$ level has probability 0.14 of making a Type II error when a specific alternative is true. What is the power of the test against this alternative?

25. **Power** A drug manufacturer claims that fewer than 10% of patients who take its new drug for treating Alzheimer's disease will experience nausea. To test this claim, a significance test is carried out of

$$H_0: p = 0.10$$

$$H_a: p < 0.10$$

You learn that the power of this test at the 5% significance level against the alternative $p = 0.08$ is 0.64.

(a) Explain in simple language what "power = 0.64" means in this setting.

(b) You could get higher power against the same alternative with the same α by changing the number of measurements you make. Should you make more measurements or fewer to increase power? Explain.

(c) If you decide to use $\alpha = 0.01$ in place of $\alpha = 0.05$, with no other changes in the test, will the power increase or decrease? Justify your answer.

(d) If you shift your interest to the alternative $p = 0.07$ with no other changes, will the power increase or decrease? Justify your answer.

26. **What is power?** You manufacture and sell a liquid product whose electrical conductivity is supposed to be 5. You plan to make 6 measurements of the conductivity of each lot of product. If the product meets specifications, the mean of many measurements will be 5. You will therefore test

$$H_0: \mu = 5$$

$$H_a: \mu \neq 5$$

If the true conductivity is 5.1, the liquid is not suitable for its intended use. You learn that the power of your test at the 5% significance level against the alternative $\mu = 5.1$ is 0.23.

(a) Explain in simple language what "power = 0.23" means in this setting.

(b) You could get higher power against the same alternative with the same α by changing the number of measurements you make. Should you make more measurements or fewer to increase power?

(c) If you decide to use $\alpha = 0.10$ in place of $\alpha = 0.05$, with no other changes in the test, will the power increase or decrease? Justify your answer.

(d) If you shift your interest to the alternative $\mu = 5.2$, with no other changes, will the power increase or decrease? Justify your answer.

Multiple choice: Select the best answer for Exercises 27 to 30.

27. Experiments on learning in animals sometimes measure how long it takes mice to find their way through a maze. The mean time is 18 seconds for one particular maze. A researcher thinks that a loud noise will cause the mice to complete the maze faster. She measures how long each of 10 mice takes with a noise as stimulus. The sample mean is $\bar{x} = 16.5$ seconds. The appropriate hypotheses for the significance test are

(a) $H_0: \mu = 18$; $H_a: \mu \neq 18$.

(b) $H_0: \mu = 16.5$; $H_a: \mu < 18$.

(c) $H_0: \mu < 18$; $H_a: \mu = 18$.

(d) $H_0: \mu = 18$; $H_a: \mu < 18$.

(e) $H_0: \bar{x} = 18$; $H_a: \bar{x} < 18$.

28. You use technology to carry out a significance test and get a P-value of 0.031. The correct conclusion is

(a) accept H_a at the $\alpha = 0.05$ significance level.

(b) reject H_0 at the $\alpha = 0.05$ significance level.

(c) reject H_0 at the $\alpha = 0.01$ significance level.

(d) fail to reject H_0 at the $\alpha = 0.05$ significance level.

(e) fail to reject H_a at the $\alpha = 0.05$ significance level.

29. Vigorous exercise helps people live several years longer (on average). Whether mild activities like slow walking extend life is not clear. Suppose that the added life expectancy from regular slow walking is just 2 months. A statistical test is more likely to find a significant increase in mean life expectancy if

(a) it is based on a very large random sample and a 5% significance level is used.

(b) it is based on a very large random sample and a 1% significance level is used.

(c) it is based on a very small random sample and a 5% significance level is used.

(d) it is based on a very small random sample and a 1% significance level is used.

(e) the size of the sample doesn't have any effect on the significance of the test.

30. A researcher plans to conduct a significance test at the $\alpha = 0.01$ significance level. She designs her study to have a power of 0.90 at a particular alternative value of the parameter of interest. The probability

that the researcher will commit a Type II error for the particular alternative value of the parameter at which she computed the power is

(a) 0.01. (b) 0.10. (c) 0.89. (d) 0.90. (e) 0.99.

31. **Women in math** (5.3) Of the 16,701 degrees in mathematics given by U.S. colleges and universities in a recent year, 73% were bachelor's degrees, 21% were master's degrees, and the rest were doctorates. Moreover, women earned 48% of the bachelor's degrees, 42% of the master's degrees, and 29% of the doctorates.[9]

(a) How many of the mathematics degrees given in this year were earned by women? Justify your answer.

(b) Are the events "degree earned by a woman" and "degree was a master's degree" independent? Justify your answer using appropriate probabilities.

(c) If you choose 2 of the 16,701 mathematics degrees at random, what is the probability that at least 1 of the 2 degrees was earned by a woman? Show your work.

32. **Explaining confidence** (8.2) Here is an explanation from a newspaper concerning one of its opinion polls. Explain briefly but clearly in what way this explanation is incorrect.

For a poll of 1,600 adults, the variation due to sampling error is no more than three percentage points either way. The error margin is said to be valid at the 95 percent confidence level. This means that, if the same questions were repeated in 20 polls, the results of at least 19 surveys would be within three percentage points of the results of this survey.