

SECTION 7.2

Summary

- When we want information about the population proportion p of successes, we often take an SRS and use the sample proportion \hat{p} to estimate the unknown parameter p . The **sampling distribution** of \hat{p} describes how the statistic varies in all possible samples from the population.
- The **mean** of the sampling distribution of \hat{p} is equal to the population proportion p . That is, \hat{p} is an unbiased estimator of p .
- The **standard deviation** of the sampling distribution of \hat{p} is $\sqrt{p(1-p)/n}$ for an SRS of size n . This formula can be used if the population is at least 10 times as large as the sample (the *10% condition*). The standard deviation of \hat{p} gets smaller as the sample size n gets larger. Because of the square root, a sample four times larger is needed to cut the standard deviation in half.
- When the sample size n is large, the sampling distribution of \hat{p} is close to a Normal distribution with mean p and standard deviation $\sqrt{p(1-p)/n}$. In practice, use this **Normal approximation** when both $np \geq 10$ and $n(1-p) \geq 10$ (the *Normal condition*).

SECTION 7.2

Exercises

27. **The candy machine** Suppose a large candy machine has 45% orange candies. Use Figures 7.11 and 7.12 (page 434) to help answer the following questions.
- Would you be surprised if a sample of 25 candies from the machine contained 8 orange candies (that's 32% orange)? How about 5 orange candies (20% orange)? Explain.
 - Which is more surprising: getting a sample of 25 candies in which 32% are orange or getting a sample of 50 candies in which 32% are orange? Explain.
28. **The candy machine** Suppose a large candy machine has 15% orange candies. Use Figure 7.13 (page 435) to help answer the following questions.
- Would you be surprised if a sample of 25 candies from the machine contained 8 orange candies (that's 32% orange)? How about 5 orange candies (20% orange)? Explain.
 - Which is more surprising: getting a sample of 25 candies in which 32% are orange or getting a sample of 50 candies in which 32% are orange? Explain.
29. **The candy machine** Suppose a large candy machine has 45% orange candies. Imagine taking an SRS of 25 candies from the machine and observing the sample proportion \hat{p} of orange candies.
- What is the mean of the sampling distribution of \hat{p} ? Why?
 - Find the standard deviation of the sampling distribution of \hat{p} . Check to see if the 10% condition is met.
 - Is the sampling distribution of \hat{p} approximately Normal? Check to see if the Normal condition is met.
 - If the sample size were 50 rather than 25, how would this change the sampling distribution of \hat{p} ?

30. **The candy machine** Suppose a large candy machine has 15% orange candies. Imagine taking an SRS of 25 candies from the machine and observing the sample proportion \hat{p} of orange candies.
- What is the mean of the sampling distribution of \hat{p} ? Why?
 - Find the standard deviation of the sampling distribution of \hat{p} . Check to see if the 10% condition is met.
 - Is the sampling distribution of \hat{p} approximately Normal? Check to see if the Normal condition is met.
 - If the sample size were 75 rather than 25, how would this change the sampling distribution of \hat{p} ?
31. **Airport security** The Transportation Security Administration (TSA) is responsible for airport safety. On some flights, TSA officers randomly select passengers for an extra security check before boarding. One such flight had 76 passengers—12 in first class and 64 in coach class. TSA officers selected an SRS of 10 passengers for screening. Let \hat{p} be the proportion of first-class passengers in the sample.
- Is the 10% condition met in this case? Justify your answer.
 - Is the Normal condition met in this case? Justify your answer.
32. **Scrabble** In the game of Scrabble, each player begins by drawing 7 tiles from a bag containing 100 tiles. There are 42 vowels, 56 consonants, and 2 blank tiles in the bag. Cait chooses an SRS of 7 tiles. Let \hat{p} be the proportion of vowels in her sample.
- Is the 10% condition met in this case? Justify your answer.
 - Is the Normal condition met in this case? Justify your answer.

In Exercises 33 and 34, explain why you cannot use the methods of this section to find the desired probability.

33. **Hispanic workers** A factory employs 3000 unionized workers, of whom 30% are Hispanic. The 15-member union executive committee contains 3 Hispanics. What would be the probability of 3 or fewer Hispanics if the executive committee were chosen at random from all the workers?

CHALLENGE: See if you can compute the probability using another method.

34. **Studious athletes** A university is concerned about the academic standing of its intercollegiate athletes. A study committee chooses an SRS of 50 of the 316 athletes to interview in detail. Suppose that 40% of

the athletes have been told by coaches to neglect their studies on at least one occasion. What is the probability that at least 15 in the sample are among this group?

35. **Do you drink the cereal milk?** A *USA Today* Poll asked a random sample of 1012 U.S. adults what they do with the milk in the bowl after they have eaten the cereal. Of the respondents, 67% said that they drink it. Suppose that 70% of U.S. adults actually drink the cereal milk. Let \hat{p} be the proportion of people in the sample who drink the cereal milk.
- What is the mean of the sampling distribution of \hat{p} ? Why?
 - Find the standard deviation of the sampling distribution of \hat{p} . Check to see if the 10% condition is met.
 - Is the sampling distribution of \hat{p} approximately Normal? Check to see if the Normal condition is met.
 - Find the probability of obtaining a sample of 1012 adults in which 67% or fewer say they drink the cereal milk. Do you have any doubts about the result of this poll?
36. **Do you go to church?** The Gallup Poll asked a random sample of 1785 adults whether they attended church or synagogue during the past week. Of the respondents, 44% said they did attend. Suppose that 40% of the adult population actually went to church or synagogue last week. Let \hat{p} be the proportion of people in the sample who attended church or synagogue.
- What is the mean of the sampling distribution of \hat{p} ? Why?
 - Find the standard deviation of the sampling distribution of \hat{p} . Check to see if the 10% condition is met.
 - Is the sampling distribution of \hat{p} approximately Normal? Check to see if the Normal condition is met.
 - Find the probability of obtaining a sample of 1785 adults in which 44% or more say they attended church or synagogue last week. Do you have any doubts about the result of this poll?
37. **Do you drink the cereal milk?** What sample size would be required to reduce the standard deviation of the sampling distribution to one-half the value you found in Exercise 35(b)? Justify your answer.
38. **Do you go to church?** What sample size would be required to reduce the standard deviation of the sampling distribution to one-third the value you found in Exercise 36(b)? Justify your answer.

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STEP 4
39. **Students on diets** A sample survey interviews an SRS of 267 college women. Suppose (as is roughly true) that 70% of college women have been on a diet within the past 12 months. What is the probability that 75% or more of the women in the sample have been on a diet? Follow the four-step process.

STEP 4
40. **Who owns a Harley?** Harley-Davidson motorcycles make up 14% of all the motorcycles registered in the United States. You plan to interview an SRS of 500 motorcycle owners. How likely is your sample to contain 20% or more who own Harleys? Follow the four-step process.

STEP 4
41. **On-time shipping** Your mail-order company advertises that it ships 90% of its orders within three working days. You select an SRS of 100 of the 5000 orders received in the past week for an audit. The audit reveals that 86 of these orders were shipped on time.
(a) If the company really ships 90% of its orders on time, what is the probability that the proportion in an SRS of 100 orders is as small as the proportion in your sample or smaller? Follow the four-step process.

(b) A critic says, "Aha! You claim 90%, but in your sample the on-time percentage is lower than that. So the 90% claim is wrong." Explain in simple language why your probability calculation in (a) shows that the result of the sample does not refute the 90% claim.

42. **Underage drinking** The Harvard College Alcohol Study finds that 67% of college students support efforts to "crack down on underage drinking." The study took a random sample of almost 15,000 students, so the population proportion who support a crackdown is close to $p = 0.67$.⁵ The administration of a local college surveys an SRS of 100 students and finds that 62 support a crackdown on underage drinking.

STEP 4
(a) Suppose that the proportion of all students attending this college who support a crackdown is 67%, the same as the national proportion. What is the probability that the proportion in an SRS of 100 students is as small as or smaller than the result of the administration's sample? Follow the four-step process.

(b) A writer in the college's student paper says that "support for a crackdown is lower at our school than nationally." Write a short letter to the editor explaining why the survey does not support this conclusion.

Multiple choice: Select the best answer for Exercises 43 to 46, which refer to the following setting. The magazine *Sports Illustrated* asked a random sample of 750 Division I college athletes, "Do you believe performance-enhancing drugs are a problem in college sports?" Suppose that 30% of all Division I athletes think that these drugs are a problem. Let \hat{p} be the sample proportion who say that these drugs are a problem.

43. The sampling distribution of \hat{p} has mean
(a) 225. (c) 0.017. (e) none of these.
(b) 0.30. (d) 0.

44. The standard deviation of the sampling distribution is about
(a) 0.0006. (c) 0.017. (e) none of these.
(b) 0.033. (d) 1.

45. Decreasing the sample size from 750 to 375 would multiply the standard deviation by
(a) 2. (c) $1/2$. (e) none of these.
(b) $\sqrt{2}$. (d) $1/\sqrt{2}$.

46. The sampling distribution of \hat{p} is approximately Normal because
(a) there are at least 7570 Division I college athletes.
(b) $np = 225$ and $n(1 - p) = 525$.
(c) a random sample was chosen.
(d) a large sample size like $n = 750$ guarantees it.
(e) the sampling distribution of \hat{p} always has this shape.

47. **Sharing music online** (5.2) A sample survey reports that 29% of Internet users download music files online, 21% share music files from their computers, and 12% both download and share music.⁶ Make a Venn diagram that displays this information. What percent of Internet users neither download nor share music files?

48. **California's endangered animals** (4.1) The California Department of Fish and Game publishes a list of the state's endangered animals. The reptiles on the list are given across the bottom of the page.

(a) Describe how you would use Table D at line 111 to choose an SRS of 3 of these reptiles to study.

(b) Use your method from part (a) to select your sample. Identify the reptiles you chose.

Desert tortoise

Olive Ridley sea turtle

Island night lizard

Flat-tailed horned lizard

Green sea turtle

Leatherback sea turtle

Alameda whip snake

Southern rubber boa

Loggerhead sea turtle

Barefoot banded gecko

Coachella Valley fringe-toed lizard

Blunt-nosed leopard lizard

Giant garter snake

San Francisco garter snake