CMPSCI 240
Reasoning Under Uncertainty
Discussion 8

The writers for *Saturday Night Live* are analyzing some classic *SNL* sketches from the 1970s to figure out which combinations of actors worked best together to create the funniest sketches.

In a sample of 10 classic sketches, viewers thought 6 were funny and 4 were not funny. In the 6 funny sketches, Dan Aykroyd appeared in 4, John Belushi in 2, and Jane Curtin in 5. In the 4 non-funny sketches, Dan Aykroyd appeared in 3, John Belushi in 1, and Jane Curtin in 2.

The writers assume that the presence or absence of any actor is conditionally independent of the presence or absence of any other actor, given that the sketch is already known to be funny or not.

The writers have just created a new sketch involving Dan Aykroyd and Jane Curtin but not John Belushi. **What is the maximum a posteriori hypothesis regarding whether this sketch will be funny or not. Then find the posterior probability that this sketch will be funny.**

Note: You should smooth the probabilities of the features given the classes, but not the priors.

Solution:

$P(\text{funny}) = 6/10 = 3/5$ $\qquad$ $P(\text{not funny}) = 4/10 = 2/5$

$P(A \mid \text{funny}) = (4+1)/(6+2) = 5/8$ $\qquad$ $P(A \mid \text{not funny}) = (3+1)/(4+2) = 4/6$

$P(B \mid \text{funny}) = (2+1)/(6+2) = 3/8$ $\qquad$ $P(B \mid \text{not funny}) = (1+1)/(4+2) = 2/6$

$P(C \mid \text{funny}) = (5+1)/(6+2) = 6/8$ $\qquad$ $P(C \mid \text{not funny}) = (2+1)/(4+2) = 3/6$

MAP hypothesis:

$$\begin{aligned} P(A \cap B^c \cap C \mid \text{funny})P(\text{funny}) &= P(A \mid \text{funny})P(B^c \mid \text{funny})P(C \mid \text{funny})P(\text{funny}) \\ &= (5/8)(5/8)(6/8)(3/5) \approx 0.176 \\ P(A \cap B^c \cap C \mid \text{not funny})P(\text{not funny}) &= P(A \mid \text{not funny})P(B^c \mid \text{not funny})P(C \mid \text{not funny})P(\text{funny}) \\ &= (4/6)(4/6)(3/6)(2/5) \approx 0.089 \end{aligned}$$

MAP hypothesis = **funny**, because $0.176 > 0.089$.

$P(\text{funny} \mid A \cap B^c \cap C) = 0.176/(0.176 + 0.089) = 0.664$.