Lagrangian Optimization

Original Problem

$$\begin{gathered}\max_X Tr(CX) \\ s.t. Tr(F_i X) = \omega_i \\ Tr(C_j X) \leq v_j \\ X \geq 0\end{gathered}$$

Lagrangian

$$\begin{aligned}\mathcal{L} \; = \quad & Tr(CX) \\ & + \sum_i \lambda_i (Tr(F_i X) - \omega_i) \\ & + \sum_j \mu_j (v_j - Tr(C_j X)) \\ & + Tr(XY)\end{aligned}$$

with $Y \geq 0, \lambda_i \in \mathbb{R}, \mu_j \geq 0$

Lagrangian Problem

$$\max_X \min_{\lambda,\mu,Y} \mathcal{L}$$

Dual Problem

$$\min_{\lambda,\mu,Y} \max_X \mathcal{L}$$