# DOS Problem Set 2

1. Assuming an exponential distribution of a random variable, X, with $\lambda$ = 1/4 (we expect on average 1 occurrence per 4 time periods), find the following:

   a) p(x<5)
   b) p(x $\leq$ 5)
   c) p(x>6)
   d) x such that p(x) < 3
   e) x such that p(x) $\geq$ 4

2. Assuming a Poisson distribution of a random variable where $\lambda$ = 2 (we expect on average 2 occurrences per time period), find the following:

   a) p(x>3)
   b) p(x<1)
   c) p(x = 0)
   d) p(x>=4)
   e) p(x>4)
   f) p(x<=4)

3. On the same axis plot a Poisson distribution (use plt.bar) with $\lambda$ = 3 along with a normal distribution (use plt.plot) with $\mu$ = 3, and $\sigma$ = $\sqrt{3}$. (We do this because the mean and variance of a Poisson random variable are always $\lambda$.) Do the same thing using a $\lambda$ of 10 but on a second subplot. Comment on the similarities and differences between the Poisson and normal and between the two plots. Which value of lambda is a better approximation when using the normal?

*Hint: try using np.arange(21) for the x values for the Poisson and np.linspace(0, 20) for the x values for the normal distribution.*

4. Write a function, binest, that takes in two parameters, $\lambda$ and x, where x is the number of events from the Poisson random variable such that you will be finding P(X<=x) exactly and by using the normal approximation.
   Because the Poisson random variable is discrete and the normal is continuous, we need to adjust the cutoff when using the normal approximation bound. In other words, if X ~ Poisson($\lambda$) and Y ~ Normal($\lambda$, $\lambda^2$) then p(x<=4) $\approx$ (y<=4.5) and p(x<4) $\approx$ p(y<=3.5). **Your function should return the exact probability, the approximate probability and the percent error when using the approximation.**

   ```
   def binest(lambda, x):
   ```

5. Suppose an intersection records an accident on average 1 time every 7 days (1/week). Accidents happen independently and randomly. Answer the following:

   a) What's the probability that 2 accidents will happen in any single week?

   b) What's the probability that 0 or 1 accident will happen?

   c) If any intersection records more than 5 accidents in any given week, it's flagged by the state for inspection for possible changes to the traffic flow or additional signage. What's the probability this intersection would get flagged?

   d) Bonus: Calculate the probability that the intersection gets two accidents in a 2-week timespan in two different ways.

> For the first way, keep lambda = 1, and realize there are THREE ways that you can achieve 2 accidents in a 2-week span. Find the probability of each way and then add them.

> For the second way, simply scale lambda up to 2 weeks and compute x = 2.

What do you notice about these two methods? Which is correct?

6. Suppose customers wait in line at a bank on average 10 minutes and are first come first served and customers arrive randomly and independently. Answer the following:

   a) What's the probability that a customer, Chong, waits more than twice the average wait?

   b) What's the probability that a customer, Sydney, waits an average amount of time or less?

   c) The bank manager has been receiving complaints from about 3% of their customers, one named Anthony, who say their wait times are unacceptable. What's the minimum wait time that appears to be generating complaints?

   d) Neil is a customer of a different bank that has an average wait time of 8 minutes. Which is more likely, that Neil waits 8 minutes or less or Sydney waits 10 minutes or less? Explain. Show your justification.