Name:______________________

Introduction to Cryptography
Prime and Composite Numbers

(1) Determine whether the following numbers are prime or composite. Record the number of seconds needed to determine each result.
 (a) 116666666666611
 (b) 1000000000100011
 (c) 13000000000000019
 (d) 12345689798654321
(2) Consider an ideal computer with a 4 gigahertz processor which can perform 4 billion arithmetic operations per second. Approximately how many seconds would it take such a computer running your program to verify that a 20 digit number is prime? What about a 200 digit number? How many seconds old is the universe?
(3) Let $\pi(n)$ be the number of prime numbers less than an integer n. For example, $\pi(10) = 4$, and $\pi(20) = 8$. Write a program in Python to compute $\pi(100000)$.
 Note: The **Sieve of Eratosthenes** from class will be useful.
(4) The Prime Number Theorem states that
$$\lim_{n\to\infty} \frac{\pi(n)}{\frac{n}{\ln(n)}} = 1,$$
 which means that $\pi(n)$ is approximately equal to $\frac{n}{\ln(n)}$ whenever n is large. Demonstrate the Prime Number Theorem using a few large numbers n.
(5) Using the Prime Number Theorem, approximate the number of 10 digit prime numbers. Approximate the probability that a US phone number is a prime number.
(6) The twin prime conjecture states that there are infinitely many prime numbers p such that $p+2$ or $p-2$ is also prime. As examples, 3 and 5 are twin primes, so are 17 and 19. Write a program in Python that creates a list of the first 1000 twin primes.