Name:_____

Introduction to Cryptography
Euclidean Algorithm Program

(1) Write a function in Python which
   (a) accepts as arguments two integers $a$ and $b$;
   (b) returns a list whose first element is the $gcd(a, b)$, whose second element is the number of iterations needed in the Euclidean algorithm, and whose third element is the time taken to run the iterations.
(2) Use your function to find the greatest common divisor of the pairs of numbers below. Record the computer run time and iteration count for each set.
   (a) $a = 135301852344706746049, b = 947112966412947222343$
   (b) $a = 354224848179261915075, b = 573147844013817084101$
   (c) $a = 573147844013817084101, b = 927372692143078999176$
(3) Write a function in Python which
   (a) accepts a positive integer $n$ as an argument;
   (b) returns a list of all positive integers which are less than $n$ and relatively prime to $n$.
(4) Euler's totient function is the function $\phi(n)$ which gives the number of positive integers less than $n$ which are relatively prime to $n$. Use your function in the previous problem to find the following values.
   (a) $\phi(2)$
   (b) $\phi(3)$
   (c) $\phi(5)$
   (d) $\phi(7)$
   (e) $\phi(11)$
   (f) $\phi(6)$
   (g) $\phi(10)$
   (h) $\phi(14)$
   (i) $\phi(15)$
   (j) $\phi(21)$
   (k) $\phi(33)$
   (l) $\phi(35)$
(5) Conjecture a formula to compute $\phi(p)$ where $p$ is a prime number.
(6) Conjecture a formula to compute $\phi(pq)$ where $p$ and $q$ are distinct prime numbers.