Name:_____

Introduction to Cryptography
Fibonacci Numbers

(1) Fibonacci Numbers are the sequence defined recursively by

$$F_0 = 1$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2} \text{ where } n \geq 2$$

(a) Write a function in Python that accepts an integer $n$ as an input and returns the value of $F_n$.

(b) Using your Fibonacci number generator, demonstrate that

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \frac{1+\sqrt{5}}{2}$$

Note: $\frac{1+\sqrt{5}}{2}$ is also called the Golden Ratio.

(c) Write a program to make a list of all prime Fibonacci numbers less than one million.

(d) Write a function in Python that returns the number of digits in an integer $n$. Use your function to find the smallest Fibonacci number with 1000 digits.

(e) Compute $gcd(F_3, F_2)$, $gcd(F_4, F_3)$, $gcd(F_5, F_4)$, and $gcd(F_6, F_5)$. Make a conjecture for the value of $gcd(F_{n+1}, F_n)$.