NCTS-TCA Summer Student Program 2025

Supernovae

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Grab your phone to take photos. 📱

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Try to remember rough values of physical constants.

- Light velocity
  $c := 2.99792485 \times 10^{10}\,\mathrm{cm\,s^{-1}} \sim 3 \times 10^{10}\,\mathrm{cm\,s^{-1}} \sim 10^{10}\,\mathrm{cm\,s^{-1}}$

- Gravitational constant
  $G = 6.67 \times 10^{-8}\,\mathrm{dyn\,cm^2\,g^{-2}} \sim 6 \times 10^{-8}\,\mathrm{in\ cgs} \sim 10^{-7}\,\mathrm{in\ cgs}$

- elementaly charge (1.6e-19C in SI but now cgs.)
  $q = 4.8 \times 10^{-10}\,\mathrm{esu} \sim 5 \times 10^{-10}\,\mathrm{in\ cgs} \sim 10^{-9}\,\mathrm{in\ cgs}$

- Planck constant
  $h = 6.63 \times 10^{-27}\,\mathrm{erg\,s} \sim 6 \times 10^{-27}\,\mathrm{in\ cgs} \sim 10^{-26}\,\mathrm{in\ cgs}$
  $\hbar = h/2\pi = 1.05 \times 10^{-27}\,\mathrm{erg\,s} \sim 1 \times 10^{-27}\mathrm{in\ cgs}$

- Boltzmann constant
  $k_B = 1.38 \times 10^{-16}\,\mathrm{erg\,K^{-1}} \sim 10^{-16}\,\mathrm{in\ cgs}$

Try to remember rough values of electron volts.

- electron volt $\mathrm{eV} = 1.6 \times 10^{-12}\,\mathrm{erg}$
    - eV: typical energy scale of optical/UV photons
    - $\mathrm{keV} = 1.6 \times 10^{-9}\,\mathrm{erg}$: typical energy scale of X-ray emission
    - $\mathrm{MeV} = 1.6 \times 10^{-6}\,\mathrm{erg}$: typical energy scale of nuclear reaction & thermal neutrinos
    - $\mathrm{GeV} = 1.6 \times 10^{-3}\,\mathrm{erg}$: typical energy scale of proton rest mass
    - $\mathrm{TeV} = 1.6\,\mathrm{erg}$: typical energy scale of nonthermal neutrino
    - $\mathrm{PeV} = 1.6 \times 10^{3}\,\mathrm{erg}$: energy scale of “knee” cosmic rays
- Electron rest mass energy $m_e c^2 = 511\,\mathrm{keV} \sim 10^{-6}\,\mathrm{erg}$
- Proton rest mass energy $m_p c^2 = 938\,\mathrm{keV} \sim \mathrm{GeV}$

Try to remember rough values of length scales.

- $\mathrm{AU} = 1.5 \times 10^{13}\,\mathrm{cm} \sim 2 \times 10^{13}\,\mathrm{cm} \sim 10^{13}\,\mathrm{cm}$
- $\mathrm{pc} = 3.09 \times 10^{18}\,\mathrm{cm} \sim 3 \times 10^{18}\,\mathrm{cm} \sim 10^{18.5}\,\mathrm{cm} \sim 10^{18}\,\mathrm{cm}$
- $\mathrm{light\ year} = 9.46 \times 10^{17}\,\mathrm{cm} \sim 10^{18}\,\mathrm{cm} \sim \mathrm{pc}/3$
- Solar radius $R_\odot = 6.96 \times 10^{10}\,\mathrm{cm} \sim 7 \times 10^{10}\,\mathrm{cm} \sim 10^{11}\,\mathrm{cm}$
- Earth radius $R_\oplus = 6.37 \times 10^{8}\,\mathrm{cm} \sim 6 \times 10^{8}\,\mathrm{cm} \sim 10^{9}\,\mathrm{cm}$
- Size of galaxy $\sim 10\,\mathrm{kpc} \sim 3 \times 10^{22}\,\mathrm{cm}$
- Classical electron radius $r_e = e^2/m_e c^2 = 2.82 \times 10^{-13}\,\mathrm{cm}$
- Wavelength of photons with $1\,\mathrm{eV} \sim hc/\lambda \to \lambda \sim 124\,\mathrm{nm}$: UV, optical.

Try to remember rough values of time scales.

- day $= 86{,}400\,\mathrm{sec} \sim 8 \times 10^4\,\mathrm{s} \sim 10^5\,\mathrm{s}$
- year $= 3.154 \times 10^7\,\mathrm{sec} \sim 3 \times 10^7\,\mathrm{s} \sim 10^{7.5}\,\mathrm{s} \sim 10^8\,\mathrm{s}$
- Cosmic age $= 13.8$ billion years $\sim 13.8 \times 10^9 \times 10^{7.5}\,\mathrm{s} \sim 10^{18}\,\mathrm{s}$

Try to remember rough values of mass scales.

- proton mass $m_p = 1.67 \times 10^{-24}\,\mathrm{cm}^2 \sim 10^{-24}\,\mathrm{g}$
- electron mass $m_e = 9.11 \times 10^{-28}\,\mathrm{cm}^2 \sim 9 \times 10^{-28}\,\mathrm{g} \sim 10^{-27}\,\mathrm{g}$
- proton-electron mass ratio $m_p/m_e \sim 1843 \sim 2000$
- Solar mass $M_\odot = 1.99 \times 10^{33}\,\mathrm{g} \sim 2 \times 10^{33}\,\mathrm{g} \sim 10^{33}\,\mathrm{g}$
- Earth mass $M_\oplus = 5.98 \times 10^{27}\,\mathrm{g} \sim 6 \times 10^{27}\,\mathrm{g} \sim 10^{28}\,\mathrm{g}$
- Galaxy mass $M_{\mathrm{gal.}} \sim M_\odot \times 10^{11}$

Try to remember rough values of other scales.

- Stephan-Boltzmann constant $\sigma = ac/4 = 5.67 \times 10^{-5}\,\mathrm{erg\ cm^{-2}\ s^{-1}\ K^{-4}}$
  $\sim 6 \times 10^{-5}$ in cgs $\sim 10^{-4}$ in cgs
- Radiation constant
  $a = \pi^2 k_B^4 / 15 c^3 \hbar^3 = 7.57 \times 10^{-15}\,\mathrm{erg\ cm^{-3}\ K^{-4}} \sim 8 \times 10^{-15}$ in cgs
  $\sim 10^{-14}$ in cgs
- Solar luminosity $L_\odot = 3.8 \times 10^{33}\,\mathrm{erg\ s^{-1}} \sim 4 \times 10^{33}$ in cgs $\sim 10^{33}$ in cgs
- CMB temperature $T_{\mathrm{CMB}} = 2.73\,\mathrm{K}$
- Hubble constant $H_0 \sim 70\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$
- Hubble time $1/H_0 \sim 9.8 \times 10^9$ years $\sim$ cosmic age

Develop the sense of physical quantities.

For the more physical constants, see
https://erksy1.rikkyo.ac.jp/syamada/watarai/bhsim/blackhole/doc_calc_blackhole_accretion/material/phys_const.pdf

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1. What is a supernova

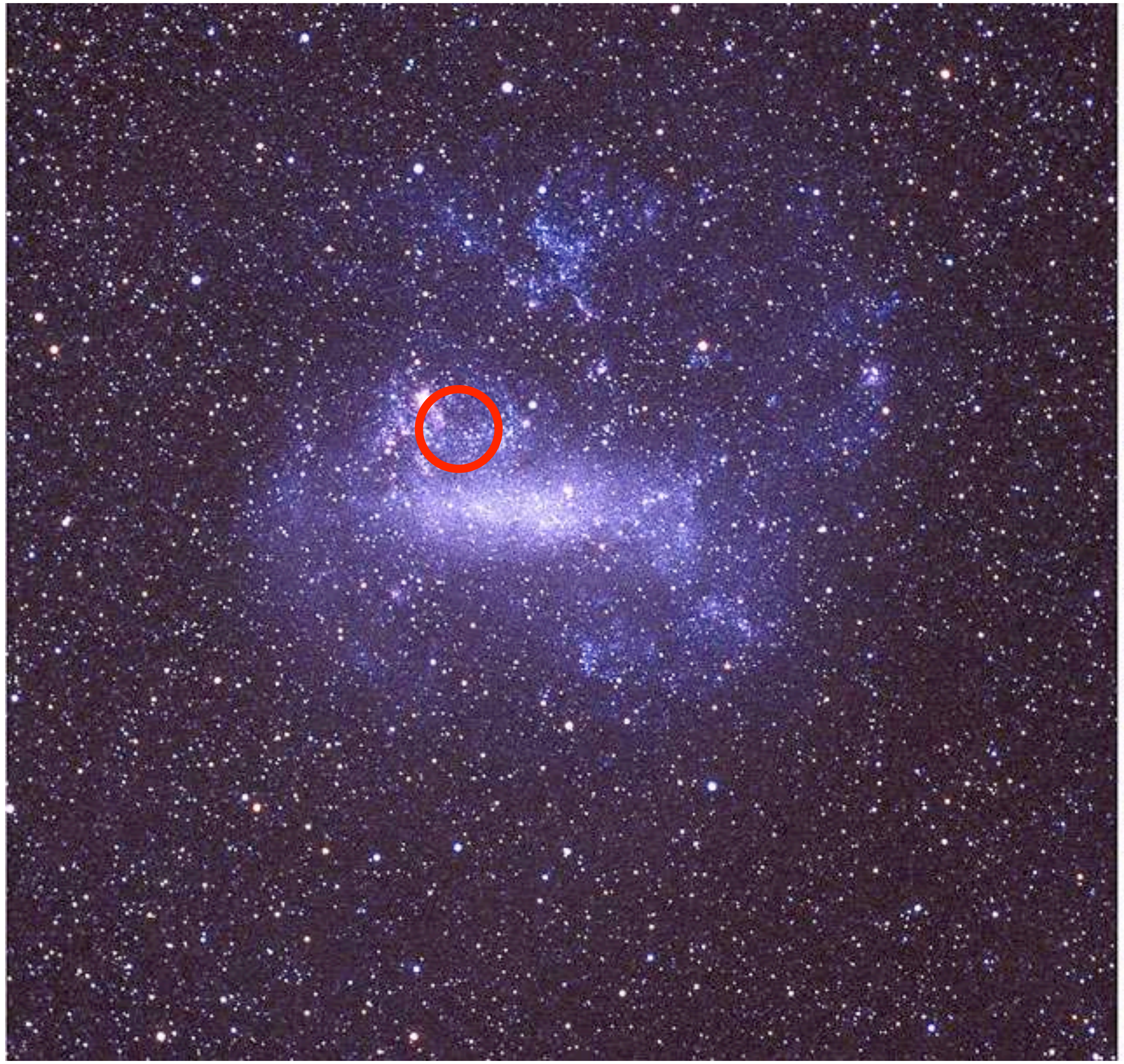
Before

After

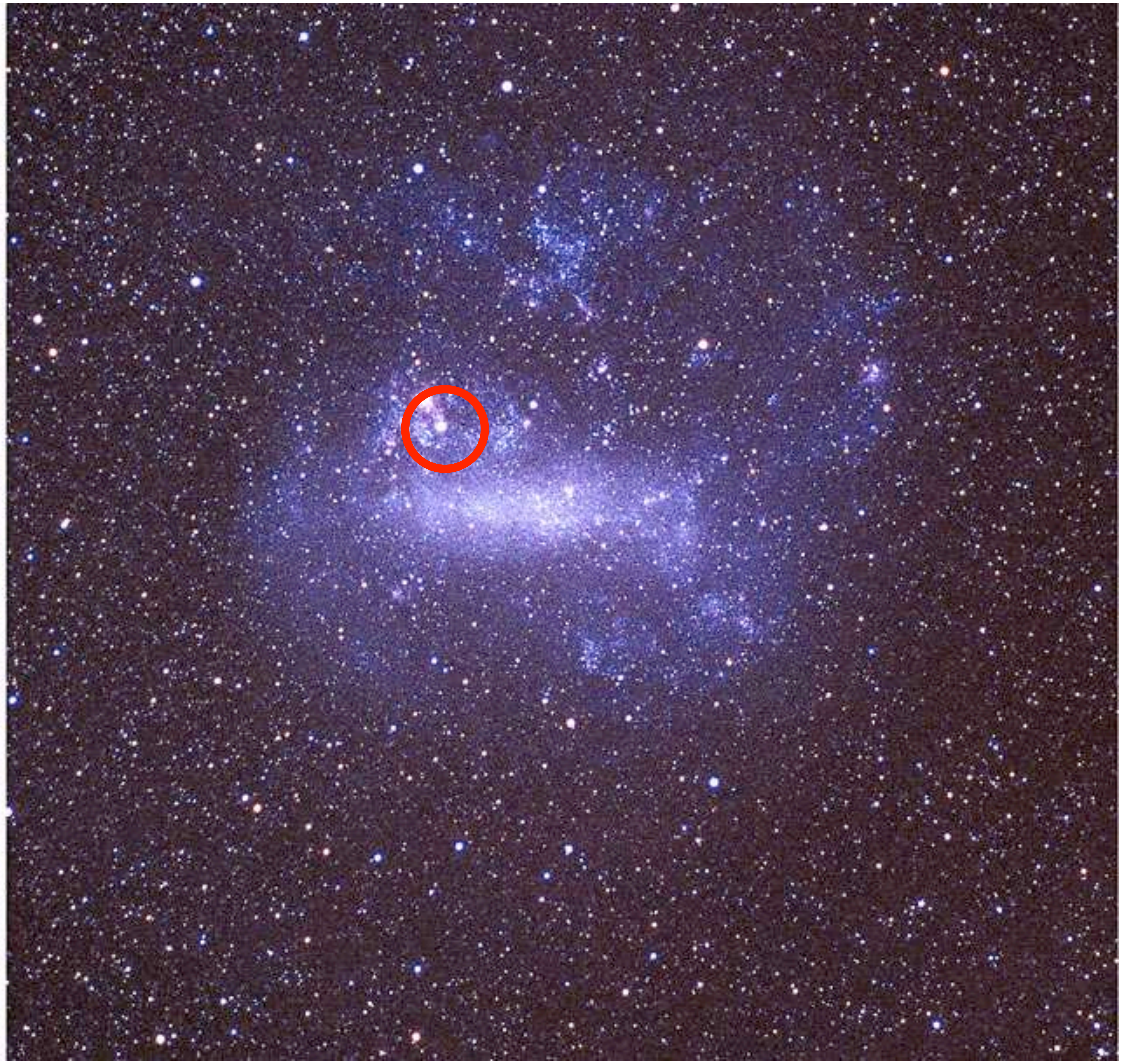
credit:ESO

1. What is a supernova

Before



After



credit:ESO

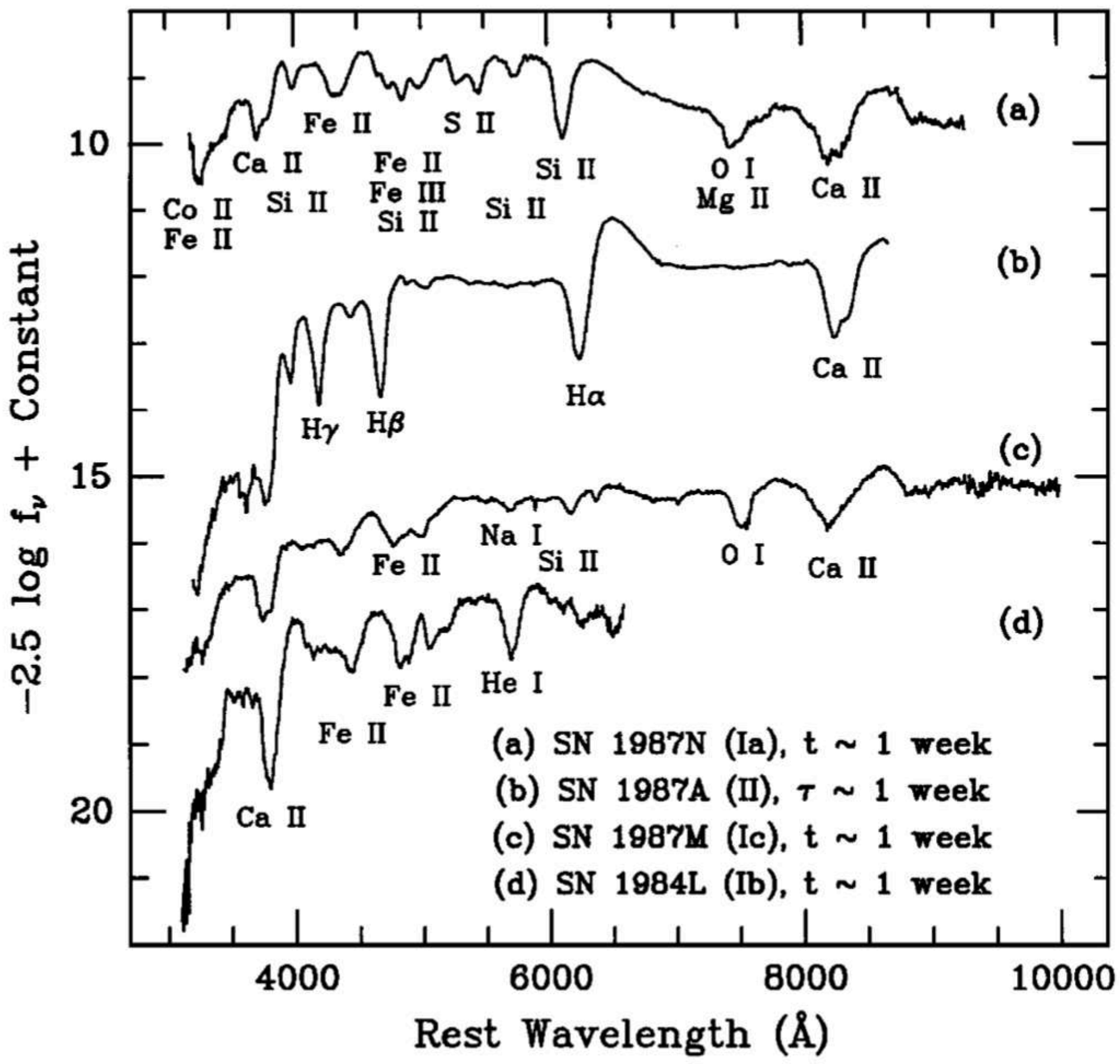
1. What is a supernova

- If we were in the old era, what will we do? Scared? Worship?
- However, we are doing research in the modern era. Our work is to give physical explanation for the sudden appearance of the bright point in the sky.
- Let’s call it supernova.
- Assuming that we have sufficient instruments to observe supernovae, let’s constrain the physical properties of supernovae ***by using order estimate, not the simulation.***

Spectra

- P-Cygni profile: Blueshifted absorption line. Characteristic spectral feature of expanding material (stellar wind, explosion, …etc)

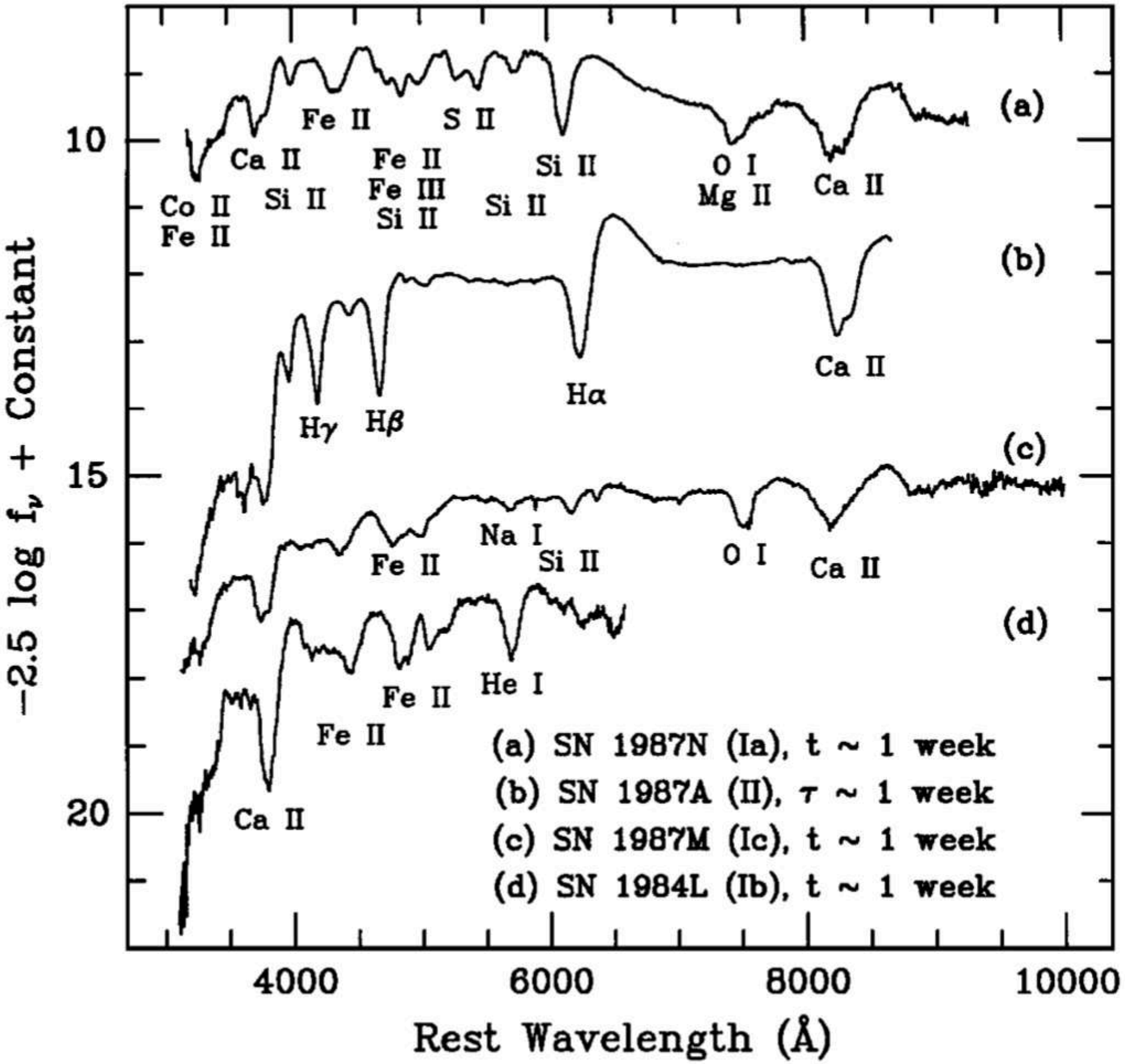
- Blueshifted line can be interpreted as a result of Doppler effect



Filippenko (1997)

Spectra

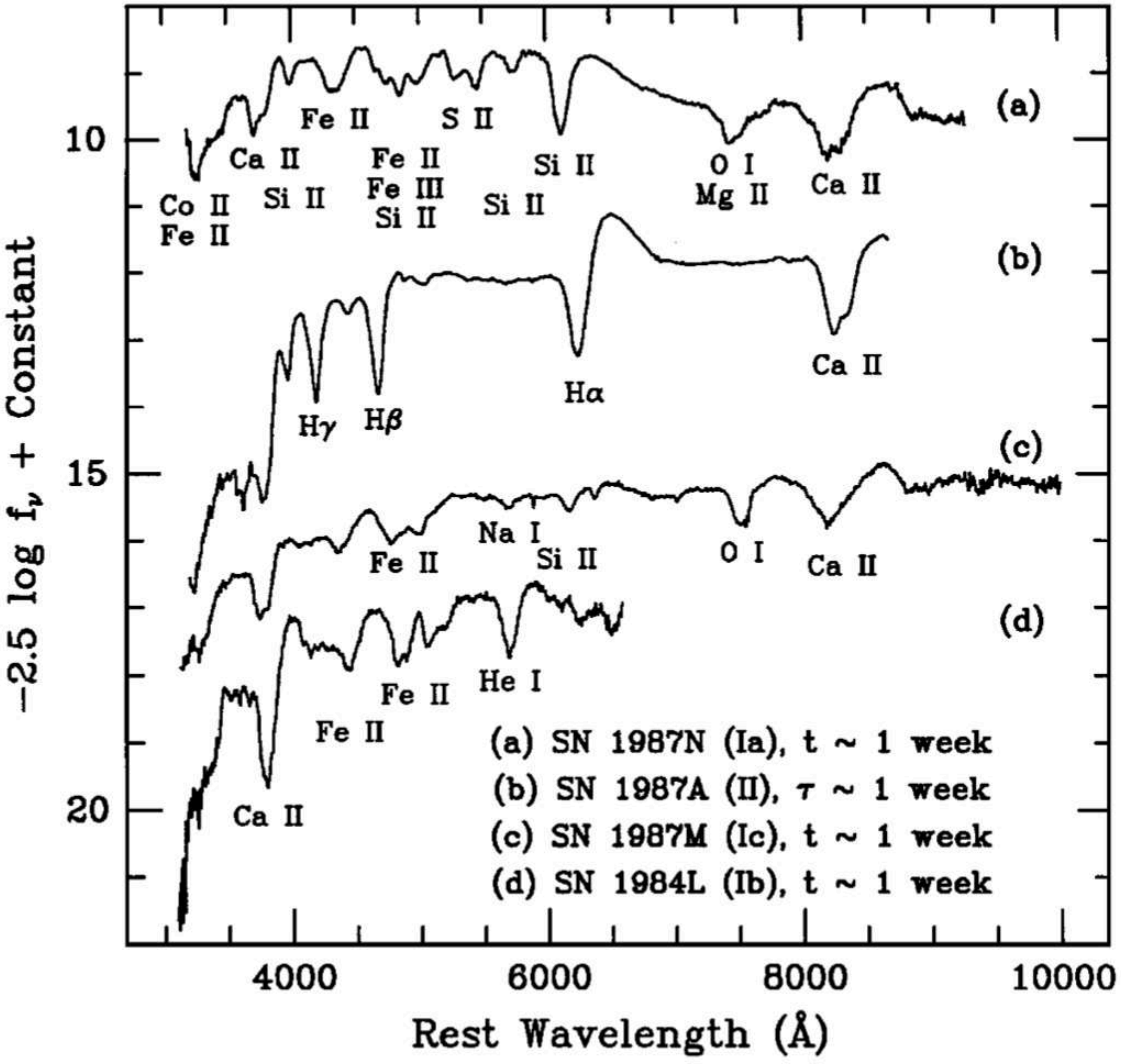
- $$\frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} = 1 - \frac{V}{c}$$
- For $\mathrm{H}\alpha$ emission in this panel, $$\frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} \sim 0.95$$
- $\therefore V \sim 0.05c \sim 10^9\,\mathrm{cm\,s^{-1}} \sim 10^4\,\mathrm{km\,s^{-1}}$
- Supernova possesses material expanding at the speed of orders of $\sim 10^4\,\mathrm{km\,s^{-1}}$. So fast.



Filippenko (1997)

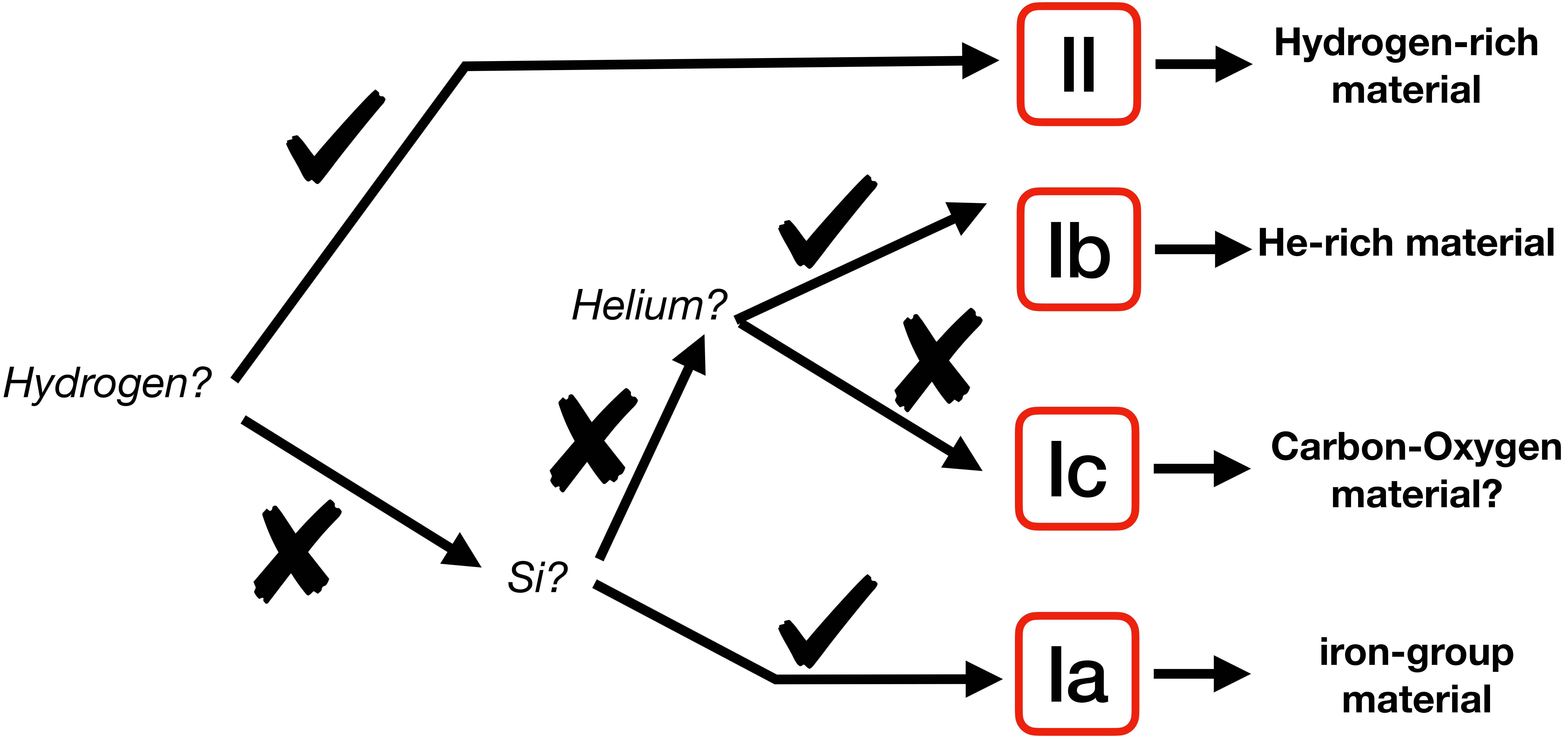
Spectra

- Supernova spectra have diversity.
  - Some have Balmer-series emission (e.g., $\mathrm{H}\alpha, \lambda_{\mathrm{emit}} = 6535\mathrm{\AA}$)
  - Some do not, instead have emission from He
  - Some have strong emission from iron-group elements, particularly of Si
- Tracer of chemical composition of the material.



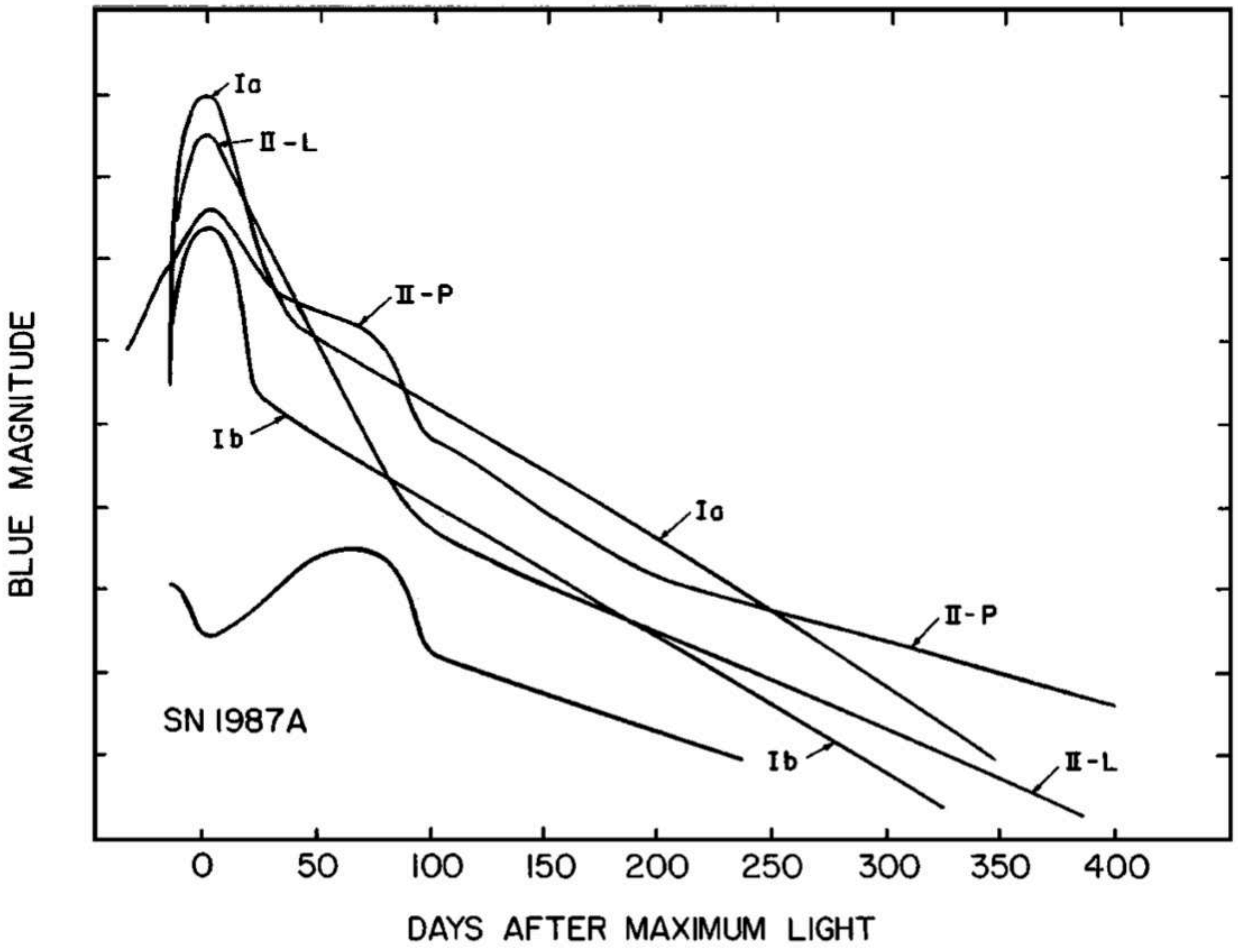
Filippenko (1997)

Supernova spectral classification



Light curve: time evolution of the luminosity

- Luminosity: Total energy of ***photons*** emitted per unit time (unit: erg/s)
- Peak time: $t_p \sim 10 - 100\,\mathrm{days}$
- Peak luminosity: $L_p \sim 10^{42} - 10^{43}\,\mathrm{erg\,s^{-1}} \sim 10^{10} L_\odot$
  - 10^{10} of the Sun ~ 1 galaxy.
- Other characteristic behaviors can be seen depending on SN types, but the peak timescale and luminosity looks important.



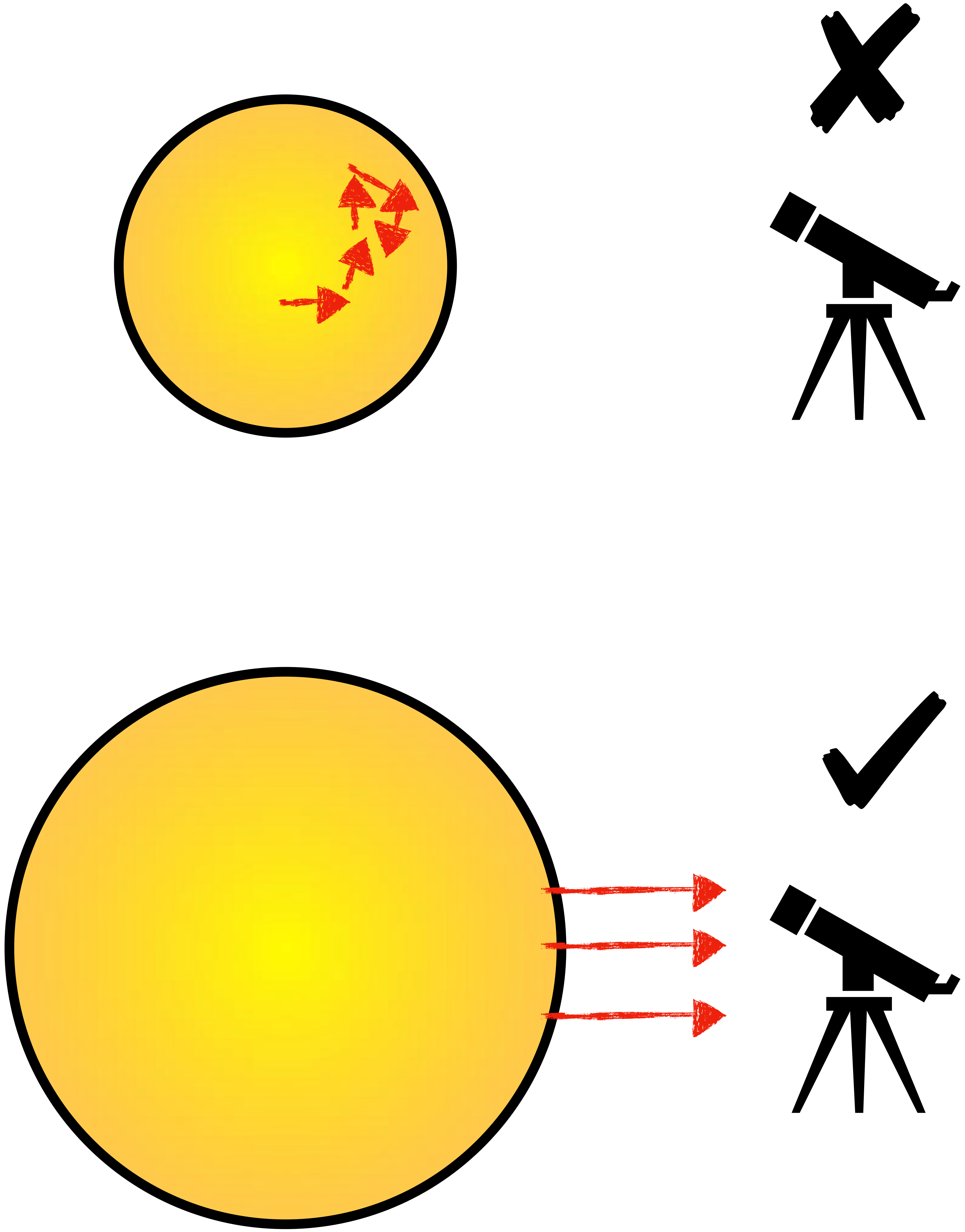
Filippenko (1997)

Peak in the ligut curve

- Initially $t_{\mathrm{exp}} \ll t_{\mathrm{diff}}$, meaning that it takes long time for photons to diffuse into expanding materials. Thus we cannot observe any photons (up).

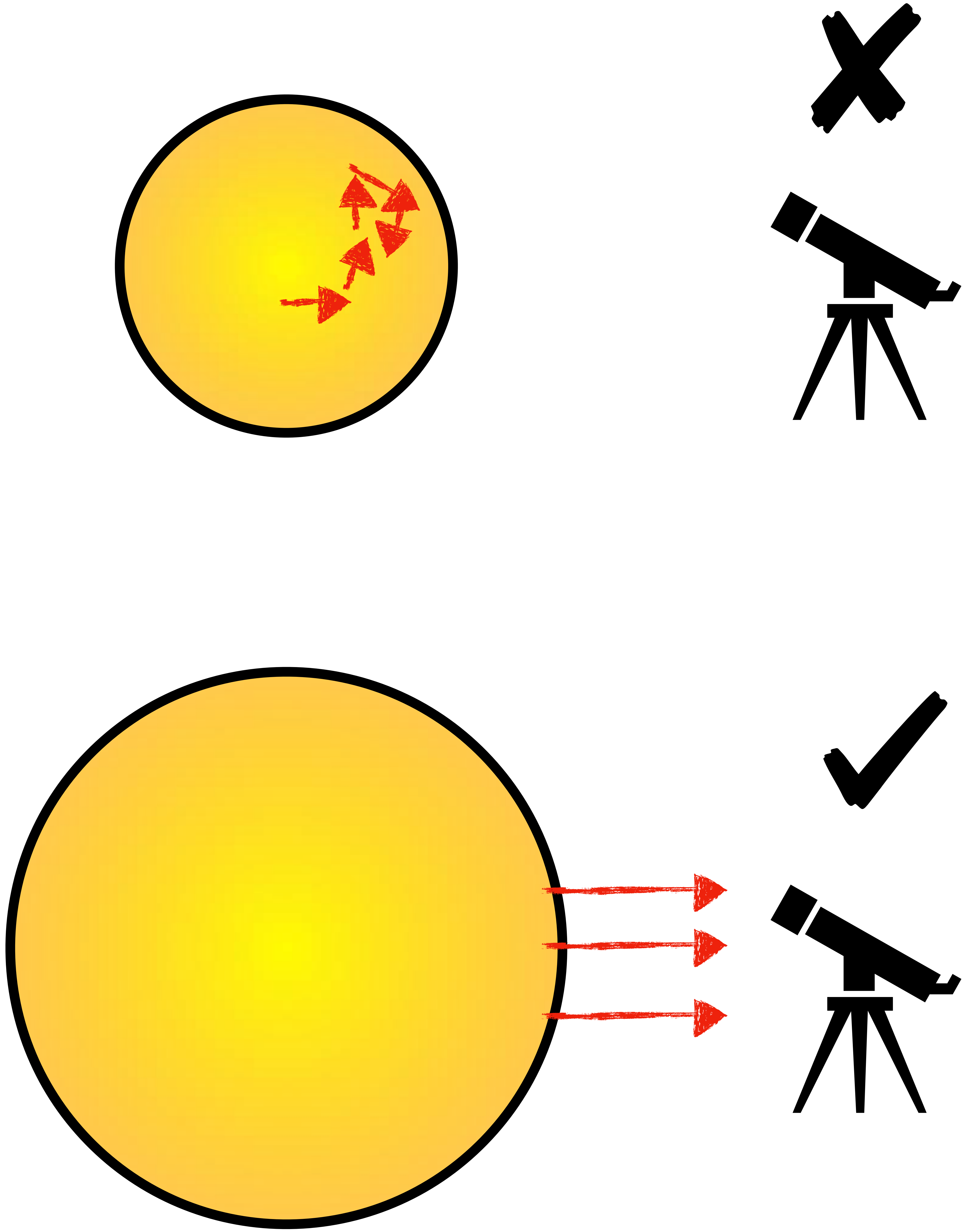
- However, as time goes by, we will have the moment $t_{\mathrm{exp}} \sim t_{\mathrm{diff}}$ $(\sim t_p)$, when photons can escape the expanding material (bottom).

- We can observe photons and they should characterize the peak in the light curve.



Timescales

- Expansion timescale: $t_{\text{exp}} \sim \dfrac{R}{V}$
- diffusion timescale of photons:
$$t_{\text{diff}} \sim R \div \left(\frac{c}{\tau}\right) \sim \frac{\tau R}{c}$$
- From $t_{\text{exp}} \sim t_{\text{diff}}$, $\tau \sim \dfrac{c}{V}$
- condition for the photon leakage



Compute mass & energy budgets.

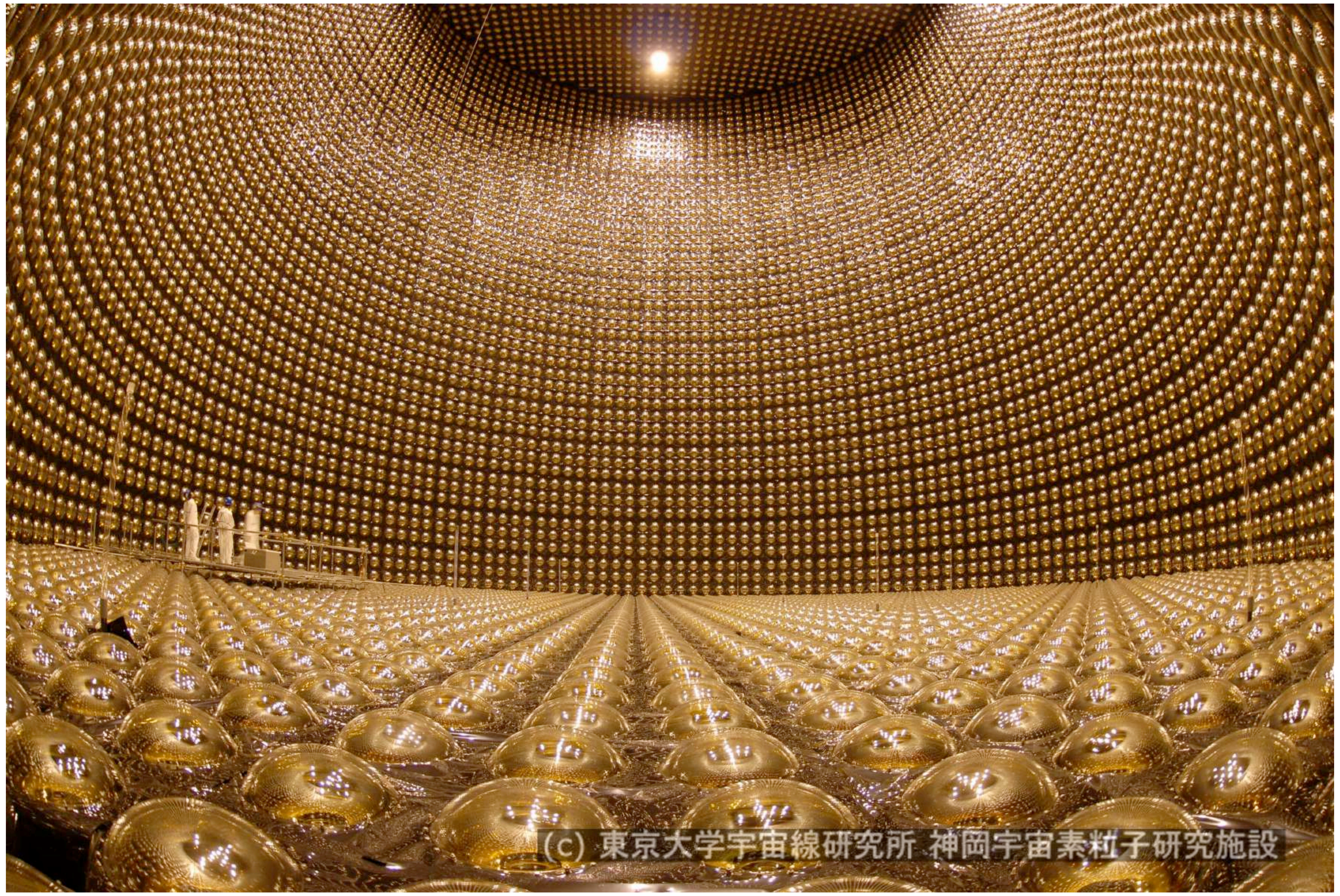
- By definition, $\tau \sim \kappa\rho R$, $\rho \sim \dfrac{M}{R^3}$, $R \sim Vt_p$, and $V \sim \left(\dfrac{E_{\rm kin}}{M}\right)^{1/2}$, so we obtain $t_p \sim \left(\dfrac{\kappa M}{Vc}\right)^{1/2}$
- Substituting $t_p \sim 10^6\,{\rm sec}$, $V \sim 10^9\,{\rm cm\,s^{-1}}$ and $\kappa \sim 0.1\,{\rm g\,cm^{-2}}$, we get $M \sim 1 - 10\,M_\odot$.
- Furthermore, $E_{\rm kin} \sim MV^2 \sim 10^{51}\,{\rm erg}$, $E_{\rm rad} \sim L_p t_p \sim 10^{49}\,{\rm erg}$
- Supernova's energetics seems enormous.
  Expansion of material as massive as the Sun with ~1% of light speed,
  brightening as luminous as a galaxy.

Neutrino

- In 1987, we had a historically nearby supernova event called SN 1987A. Fortunately, neutrino detectors were under operation and Kamiokande in Japan successfully detected 10 neutrinos in the first 10 seconds

- Kamiokande, filled up by water, detected Cherenkov radiation from positrons produced through inverse-beta decay $\bar{\nu}_e + p \longrightarrow n + e^+$.

- Successor: Super Kamiokande (right) & Hyper Kamiokande



credit: ICRR, U.Tokyo

Neutrino

- # of water molecules $N_{\mathrm{H_2O}} \sim \dfrac{2\,\mathrm{kton}}{18 m_p} \sim 10^{32}$
- Neutrino detection $(\sigma(\nu, \mathrm{H_2O}) \sim 10^{-41}\,\mathrm{cm}^2, E_{\nu,\mathrm{one}} \sim 10\,\mathrm{MeV})$
- $\dfrac{N_{\nu,\mathrm{event}}}{\Delta t} \sim N_{\mathrm{H_2O}} n_\nu \sigma(\nu, \mathrm{H_2O}) c \Longrightarrow F_\nu \sim E_{\nu,\mathrm{one}} n_\nu c \sim \dfrac{N_{\nu,\mathrm{event}}/\Delta t}{N_{\mathrm{H_2O}} \sigma(\nu, \mathrm{H_2O})} \sim 10^4\,\mathrm{erg\,s^{-1}\,cm^{-2}}$
- $E_\nu \sim 4\pi D^2 F_\nu \times \Delta t \times (\#\ \mathrm{of\ flavors}) \sim 10^{53}\,\mathrm{erg}\ \ (D = 50\,\mathrm{kpc})$
- Supernova seems to release vast energy via neutrino. What we can see by optical observation is just 1% of E_ν or even less.

What we have learned

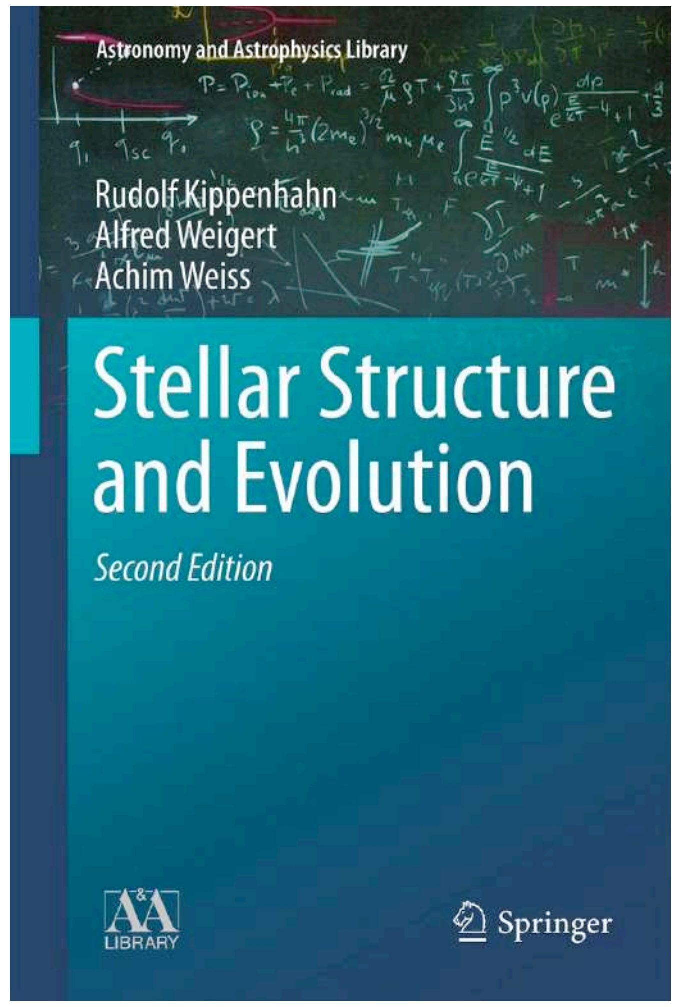
- Supernova is as luminous as a galaxy. $L_{\rm SN} \sim 10^{10} L_\odot \sim L_{\rm galaxy}$
- Supernova has a variety of spectra with various chemical compositions, having a representative blueshifted absorption line characterized by fast expansion. $V_{\rm SN} \sim 10^4\,{\rm km\,s^{-1}}$
- Supernova has a mass comparable to the Sun. $M_{\rm SN} \sim 1 - 10\,M_\odot$
- Supernova releases most of its energy via neutrino.
  $E_{\rm SN} \sim E_\nu + E_{\rm kin} + E_{\rm rad} \sim (0.99 + 0.01 + 0.0001)E_{\rm SN}$

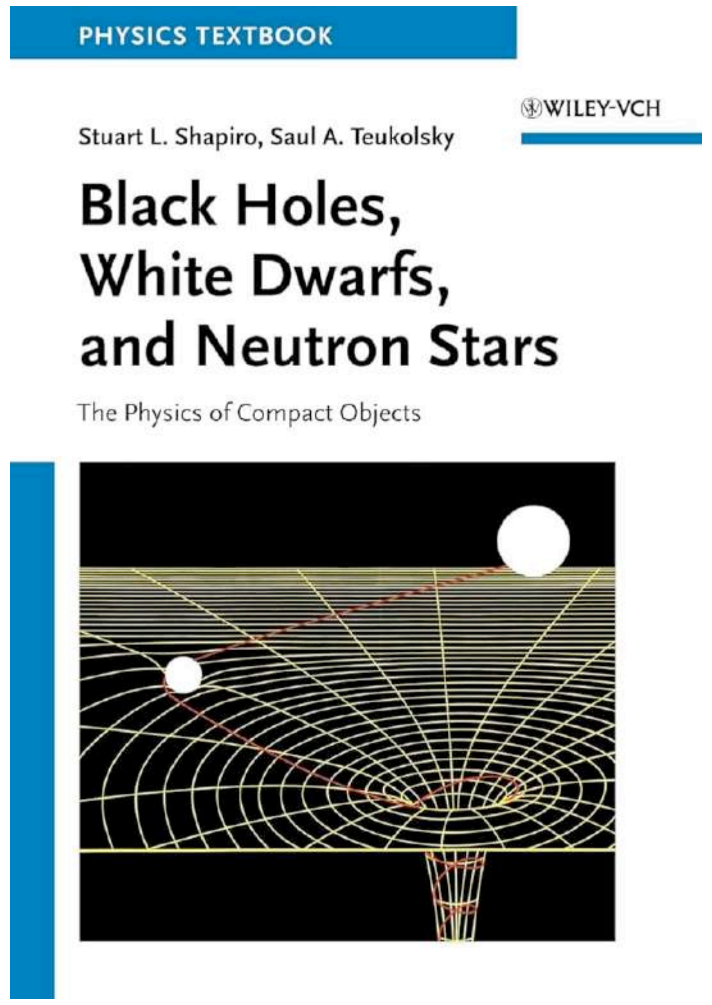
Conclusion: supernova seems to be an explosion of a star. Is that possible?
Let us dig into physical consideration of stellar astrophysics.

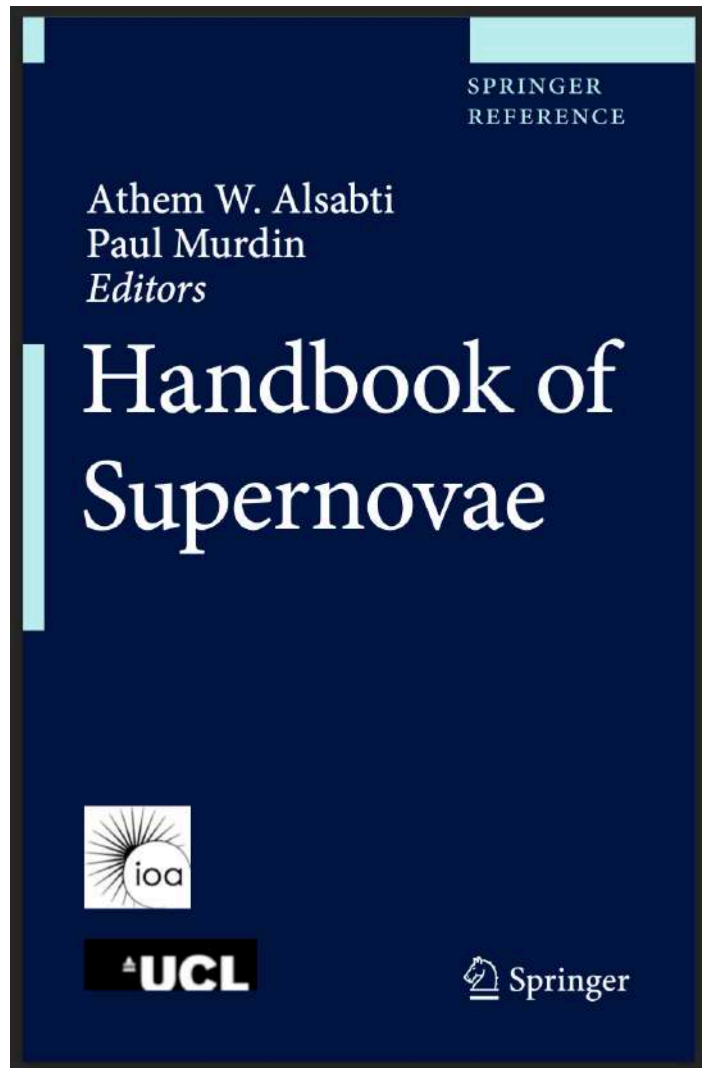
Contents

2. Stability of a star

- In the old era, people believed that the Universe is unchanged.
- Even now, we know that our Sun looks unchanged. (at least in daily life timescale, except surface eruptive phenomena such as solar flares).
- Why do stars look stable? Are they really unchanged?
- Textbooks
    - Stellar Structure and Evolution (Kippenhahn+)
    - Black Holes, White Dwarfs, and Neutron Stars (Shapiro & Teukolsky)
    - Handbook of Supernovae (Springer)

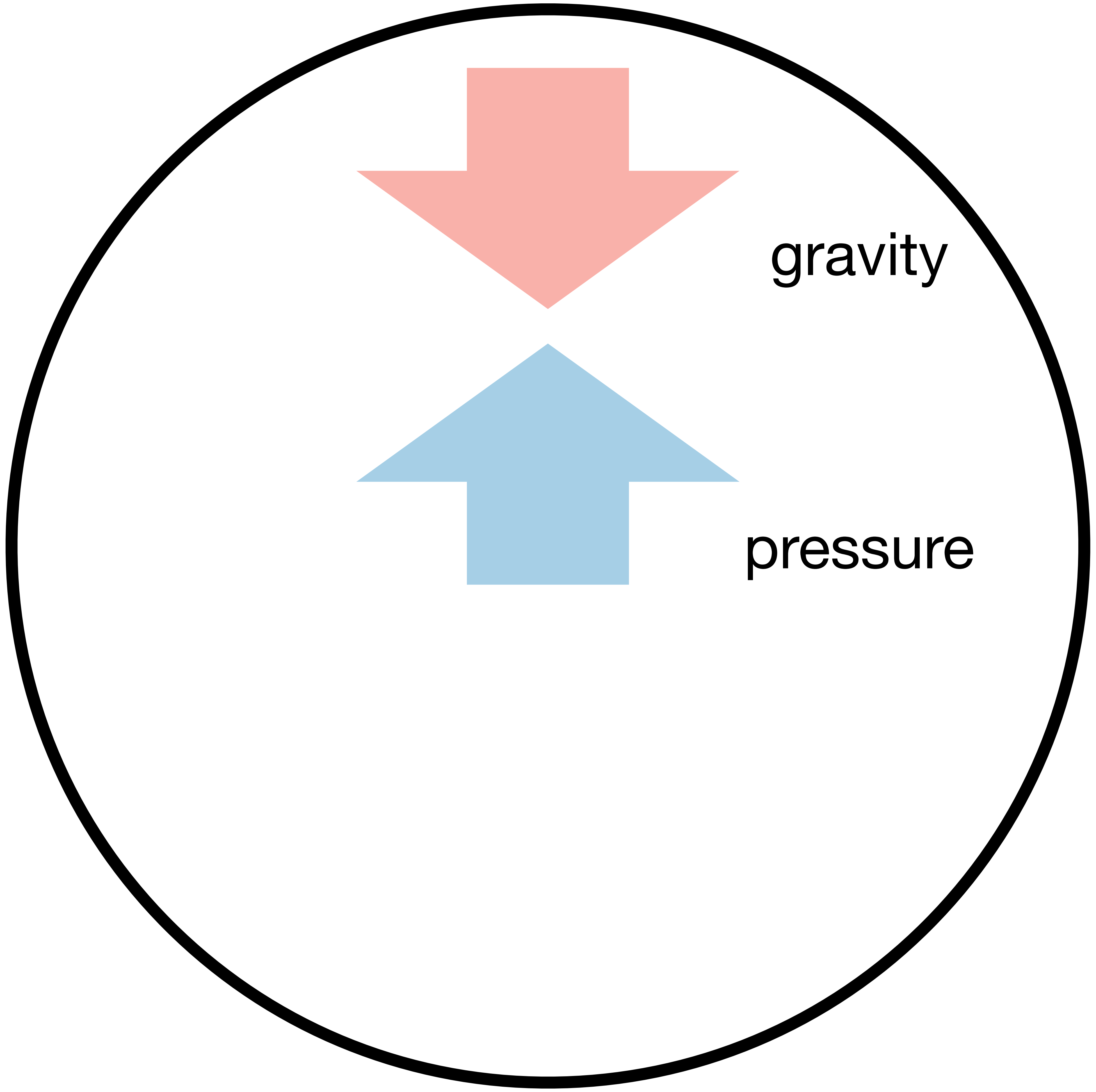






Hydrostatic equilibrium

- Star: self-gravitational system, so spherically symmetric.
- Gas in a star feels gravitational force vertically downward
- Gas does not collapse towards the center, because, gas in a star ***possesses pressure, resisting the gravity***
- The balance of gravity and pressure —> hydrostatic equilibrium



Hydrostatic equilibrium

gravitational force of the mass shell per unit area:

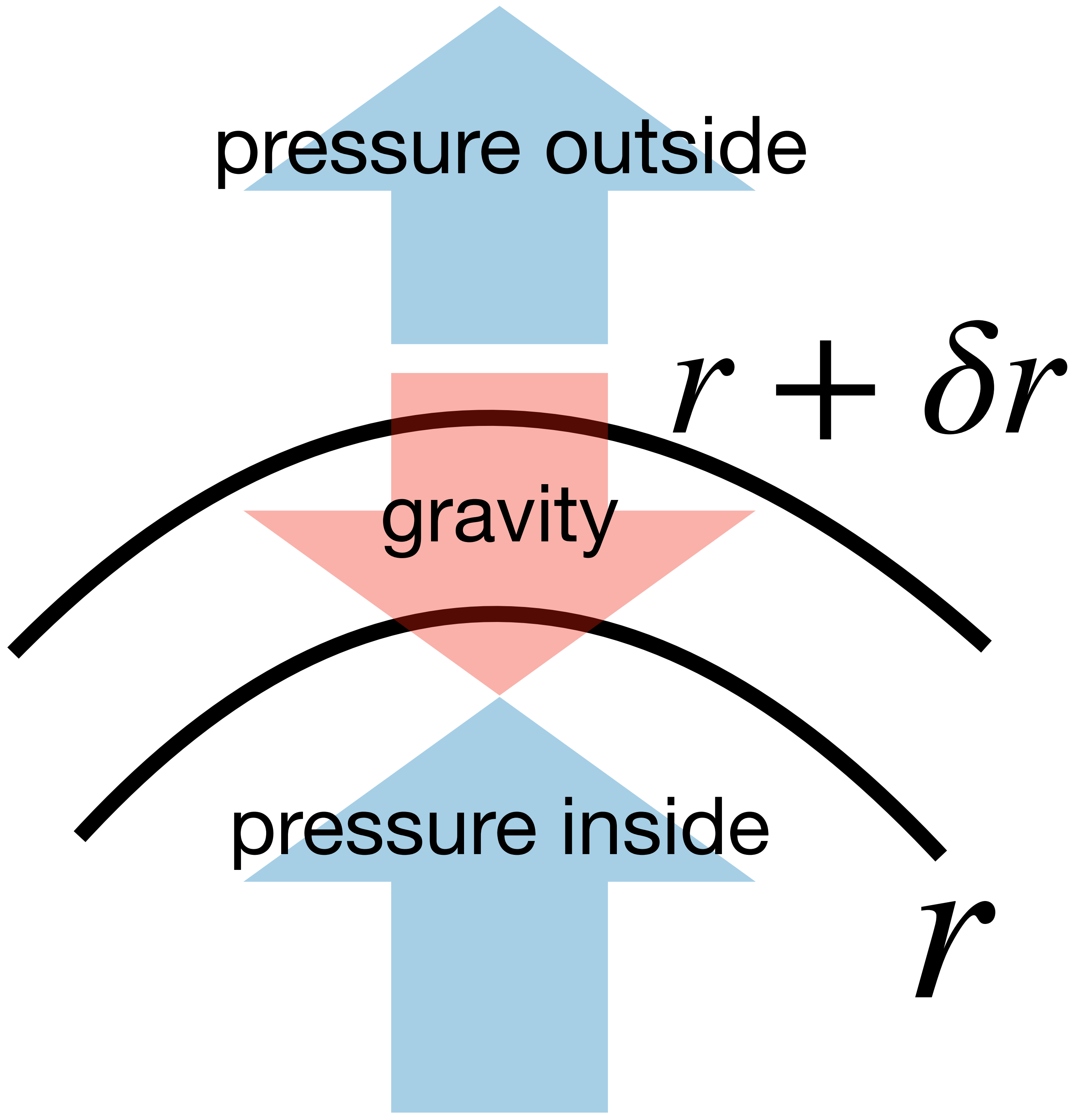
$\rho g \delta r = \dfrac{GM_r \rho}{r^2}\delta r$, where M_r is the mass enclosed within the radius of r

pressure difference of the mass shell:

$$p(r) - p(r+\delta r) \simeq p(r) - \left\{ p(r) + \frac{dp}{dr}\delta r \right\} = -\frac{dp}{dr}\delta r$$

The balance between these two forces results in

$$\frac{dp}{dr} = -\frac{GM_r}{r^2}\rho$$



Hydrostatic equilibrium

Multiply $4\pi r^3 dr$ and integrate the equation in the whole of the star;

$$\int_0^{R_*} 4\pi r^3 \frac{dp}{dr} dr = \int_0^{R_*} \left(-\frac{GM_r}{r^2}\right) \rho 4\pi r^3 dr$$

$$(\mathrm{l.h.s.}) = [4\pi r^3 p]_0^{R_*} - 3\int_0^{R_*} 4\pi r^2 p dr$$

Assuming zero surface pressure $p(r = R_*) = 0$ and $p = (\gamma - 1)\rho e$ where e is the internal energy density per unit mass, we obtain

$$(\mathrm{l.h.s.}) = -3(\gamma - 1)\int_0^{R_*} 4\pi r^2 \rho e dr =: -3(\gamma - 1)U$$

where U is the total internal energy of a star

Hydrostatic equilibrium

Multiply $4\pi r^3 dr$ and integrate the equation in the whole of the star;

$$\int_0^{R_*} 4\pi r^3 \frac{dp}{dr} dr = \int_0^{R_*} \left(-\frac{GM_r}{r^2} \right) \rho 4\pi r^3 dr$$

$$(\mathrm{r.h.s.}) = \int_0^{R_*} \left(-\frac{GM_r}{r} \right) 4\pi r^2 \rho dr = \int_0^{R_*} \left(-\frac{GM_r}{r} \right) dm =: \Omega$$

where Ω is the total gravitational potential energy of a star

In summary,

$$3(\gamma - 1)U + \Omega = 0$$

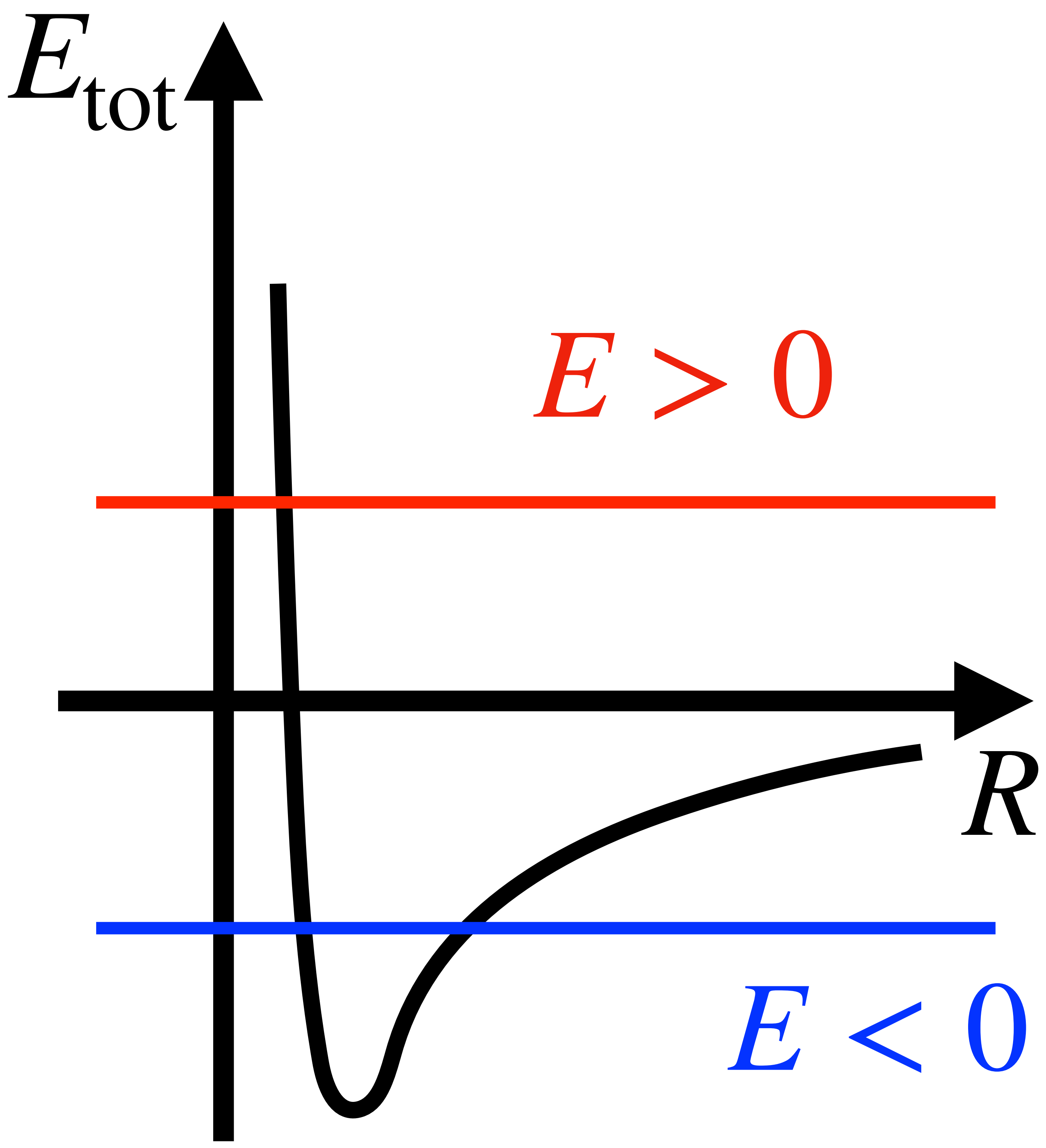
is realized in a star, giving a relationship between the internal energy and gravitational ptential: ***Virial Theorem***

Energetics of self-gravity system

- The total energy of the star is given as

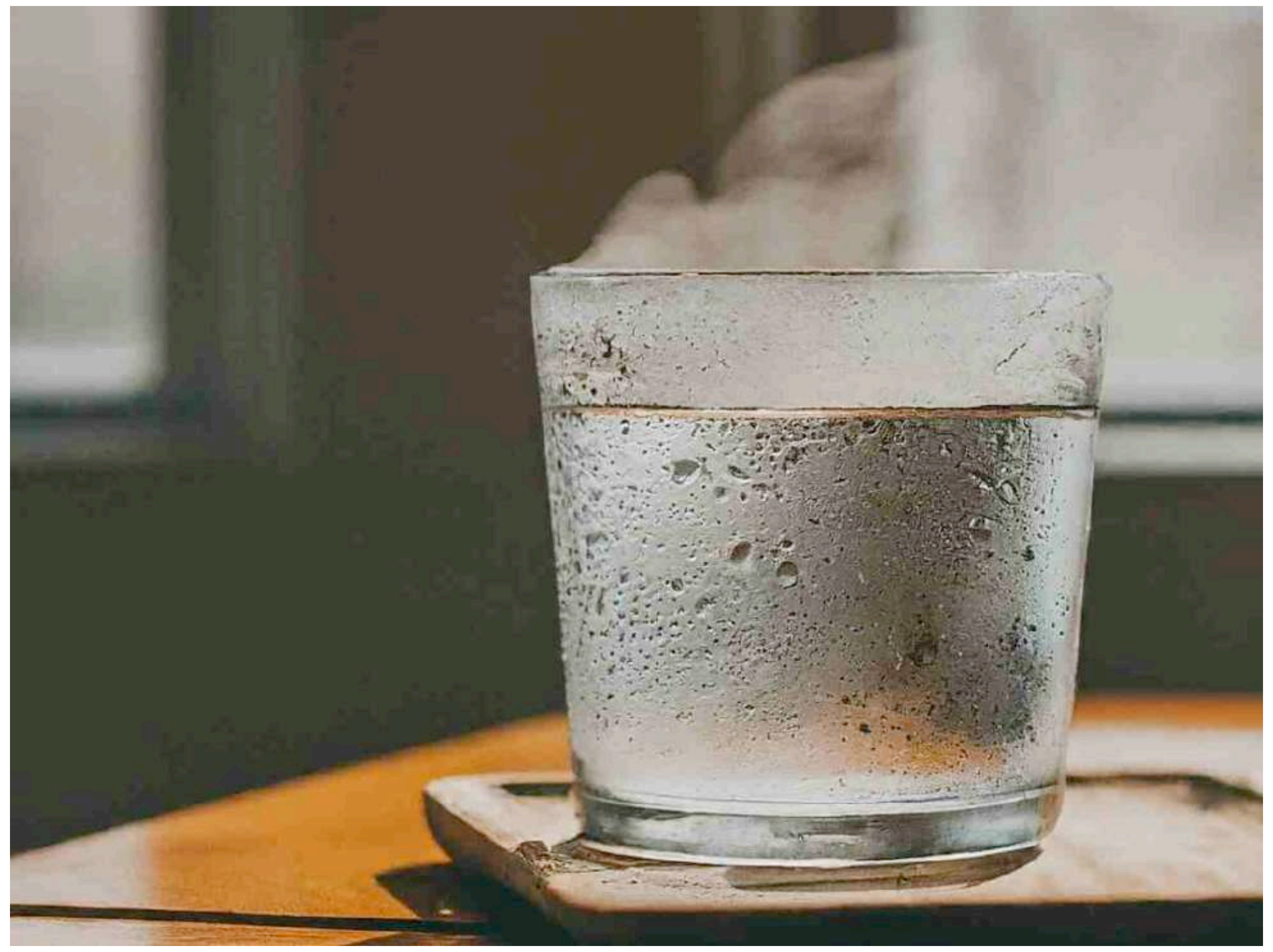
$$E = U + \Omega = (4 - 3\gamma)U = \frac{3\gamma - 4}{3(\gamma - 1)}\Omega < 0 \text{ for } \gamma = 5/3,$$

indicating that the star is “gravitationally bounded” system as long as $\gamma > 4/3$ ($\gamma = 5/3$ for monoatomic molecule).

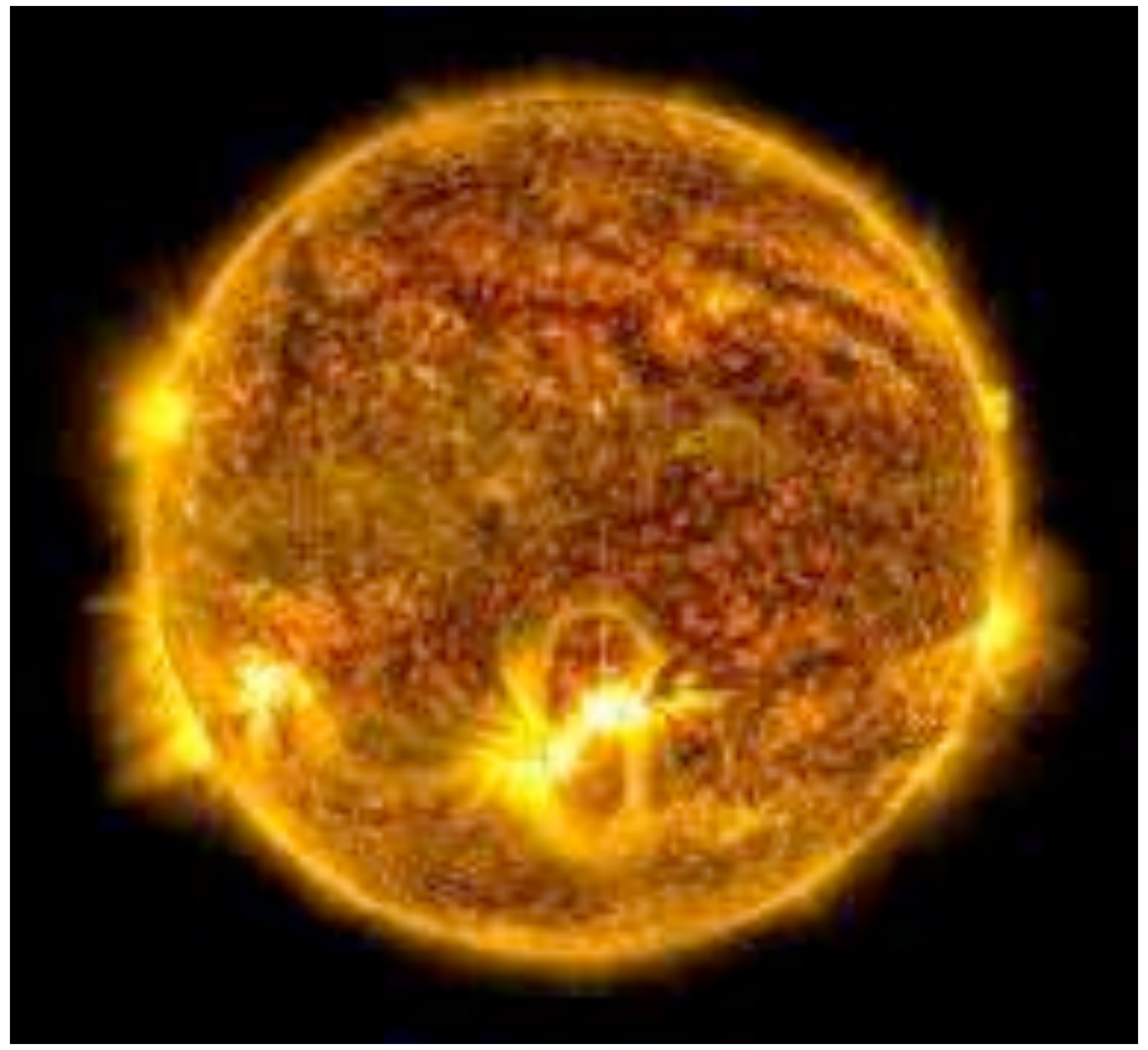


Energetics of self-gravity system

- $L = -\frac{dE}{dt} > 0$, thus $\frac{dU}{dt} > 0, \frac{d\Omega}{dt} < 0$
- Stars lose their energy via radiation from the surface, then the internal energy increases (because of the release of gravitaional energy) and (core) temperature increases.
  - This is opposite to normal material around us.
  - We call it ***Negative specific heat.***



credit: THIP



credit: NASA

Timescales

- If the star had no pressure force, the system collapses following gravity.

- This is called free fall, whose timescale is
$$t_{\rm ff} \sim \frac{r}{\sqrt{GM/r}} \sim \frac{1}{\sqrt{G\rho}} \sim 10^3\,\mathrm{sec}$$ for the Sun.

- That said, a star releases gravitational potential energy via radiation from the surface at the rate of L ($\simeq 4 \times 10^{33}\,\mathrm{erg\,s^{-1}}$ for the Sun).

- The Kelvin-Helmholtz timescale is defined as the timescale at which the system takes to release all of the gravitational potential energy via radiation:
$$t_{\rm KH} \sim \frac{GM^2}{RL} \sim 3 \times 10^7\,\mathrm{years}$$ for the Sun.

Timescales

- $t_{\mathrm{KH}} \sim \dfrac{GM^2}{RL} \sim 3 \times 10^7\,\mathrm{years}$ for the Sun
- Too short! At least, we need $\gg 10^8\,\mathrm{years}$ to realize our earth environment (e.g,. biology, geology). We need another energy source to make the Sun stably alive longer.
- Energy source: Nuclear reaction in the core.
  e.g., Hydrogen burning: $4\,^1\mathrm{H} \longrightarrow {}^4\mathrm{He} + \gamma, E_\gamma \sim 7 \times 10^{-3} m_p c^2 \sim 26.73\,\mathrm{MeV}$
- Nuclear burning timescale
  $t_{\mathrm{nuc}} \sim \dfrac{7 \times 10^{-3} Mc^2}{L} \sim 10^{10}\,\mathrm{years}$ for the Sun. Enough.
- Note: decreasing entropy → Increasing temperature → More nuclear burning.

Summary

- Stars are stable because they realize ***hydrostatic equilibrium***: the balance between gravitational force and pressure.

- Stars ***release their energy via radiation*** from the surface. Having negative specific heat.

- Stars generate ***new energy via nuclear reaction*** at the center. Nuclear reaction produces vast energy, making the evolutionary timescale of stars as long as ~10^{10} years or so. That's why we can recognize the Sun stable.

- Stars' characteristic timescales are $t_{\rm ff} \ll t_{\rm KH} \ll t_{\rm nuc}$. So, stars ***evolve "slowly" via nuclear reactions, so, they are not "unchanged"***.
  How do stars evolve with the long time?

Contents

3. Evolution of a star

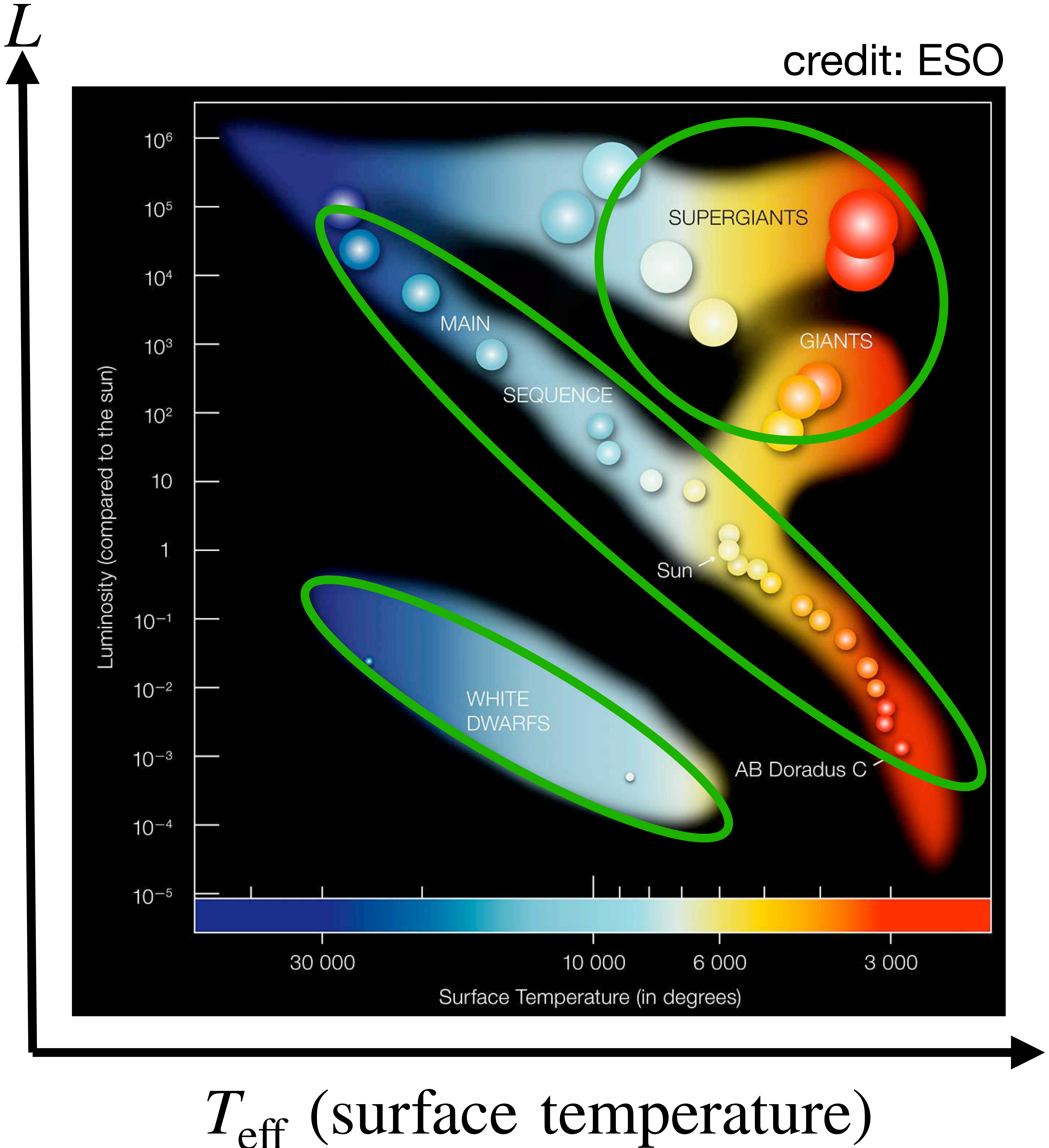
- Hertzsprung-Russel (HR) diagram: distribution of stars in luminosity-effective temperature plane

- $L = 4\pi R_*^2 \sigma_{SB} T_{\rm eff}^4$, so HR diagram gives the luminosity, color, and radius of stars.

    - White dwarf ($R_* \sim 0.01 R_\odot$)

    - Main sequence ($R_* \sim R_\odot$)

    - Giants ($R_* \sim 1000 R_\odot$)

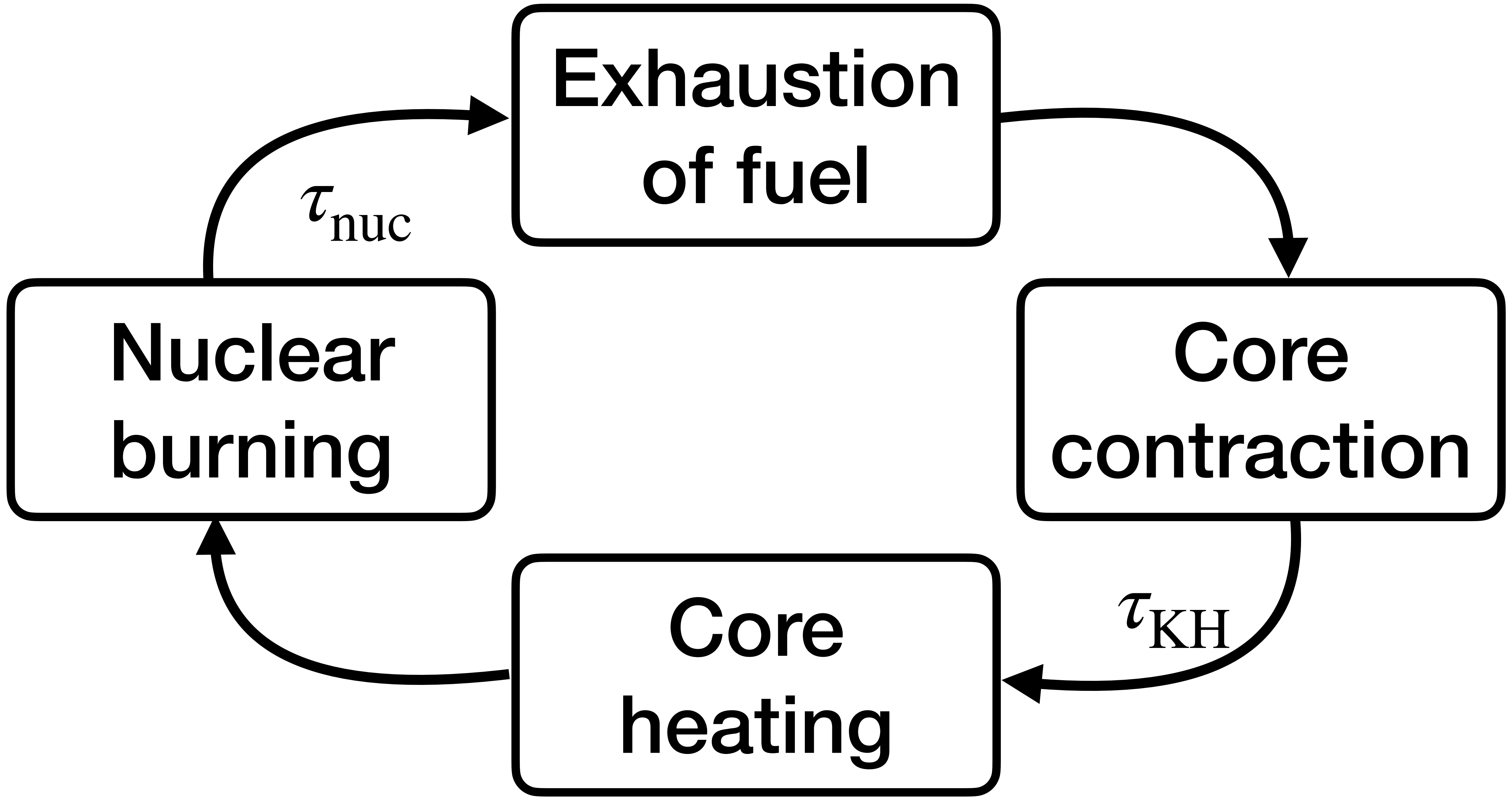


Key physics: pressure source and nuclear burning

- ***Pressure source***
  - ***Ideal gas*** $p_{\mathrm{gas}} = \dfrac{\rho k_B T}{\mu m_u}$ $\rightarrow$ $\gamma = \dfrac{5}{3}$
  - Non-relativistic electron degeneracy pressure $p_{\mathrm{non-rela,deg}} = K_{\mathrm{non-rela}} n_e^{5/3}$ $\rightarrow$ $\gamma = \dfrac{5}{3}$
  - ***Relativistic electron degeneracy pressure*** $p_{\mathrm{rela,deg}} = K_{\mathrm{rela}} n_e^{4/3}$ $\rightarrow$ $\gamma = \dfrac{4}{3}$
  - Radiation pressure $p_{\mathrm{rad}} = \dfrac{aT^4}{3}$ $\rightarrow$ $\gamma = \dfrac{4}{3}$
- **Nuclear burning**
  - H → He → C → Ne → O → Si

Chart of stellar evolution

The system cycles
as long as $p_{\rm gas} \sim (\text{gravity})$

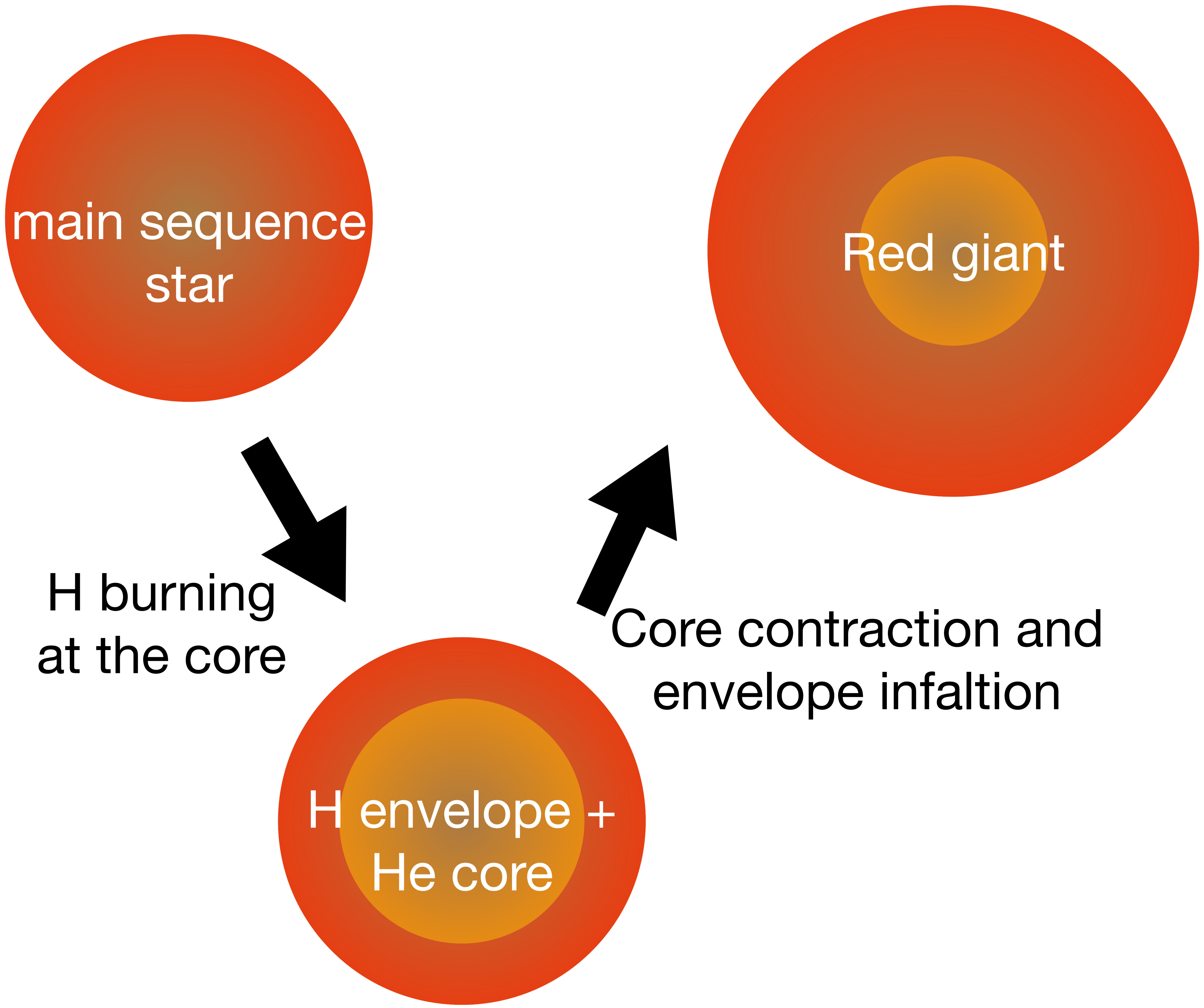


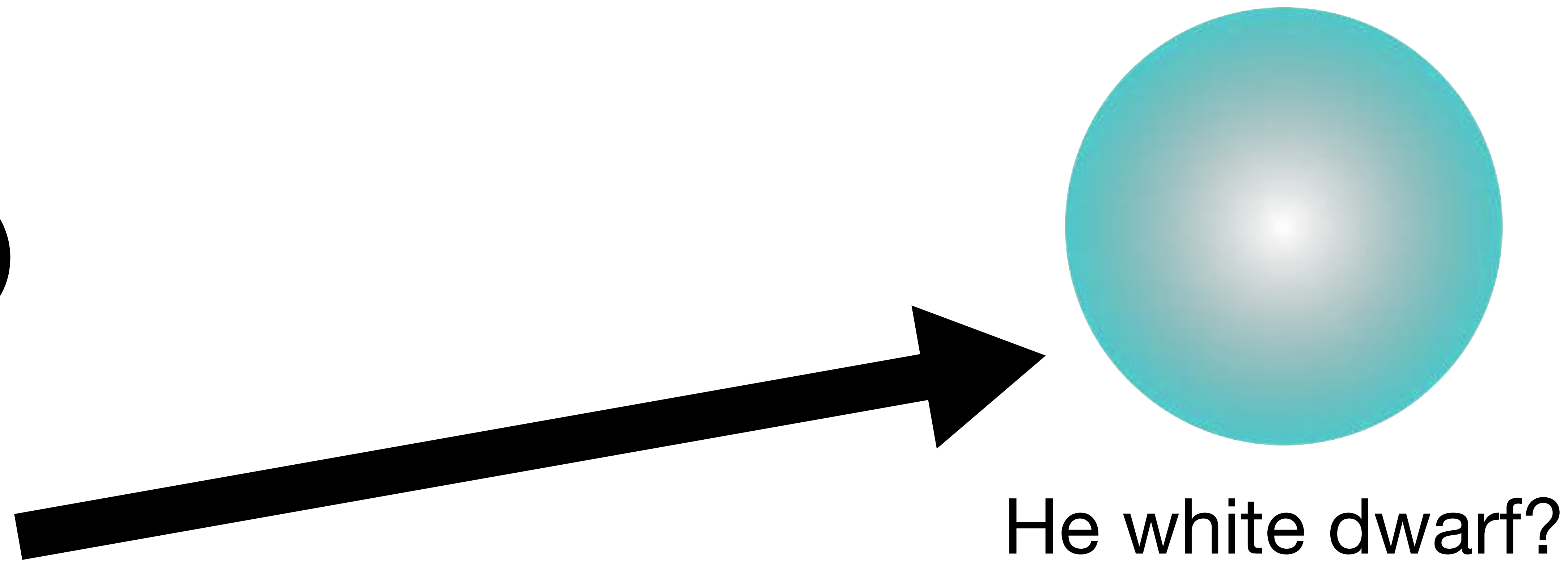
Editing after the figure
in S35 of Kippenhahn's textbook

Chart of stellar evolution

Editing after the figure
in S35 of Kippenhahn's textbook

Low-mass stars $(M \lesssim 8\, M_{\odot})$

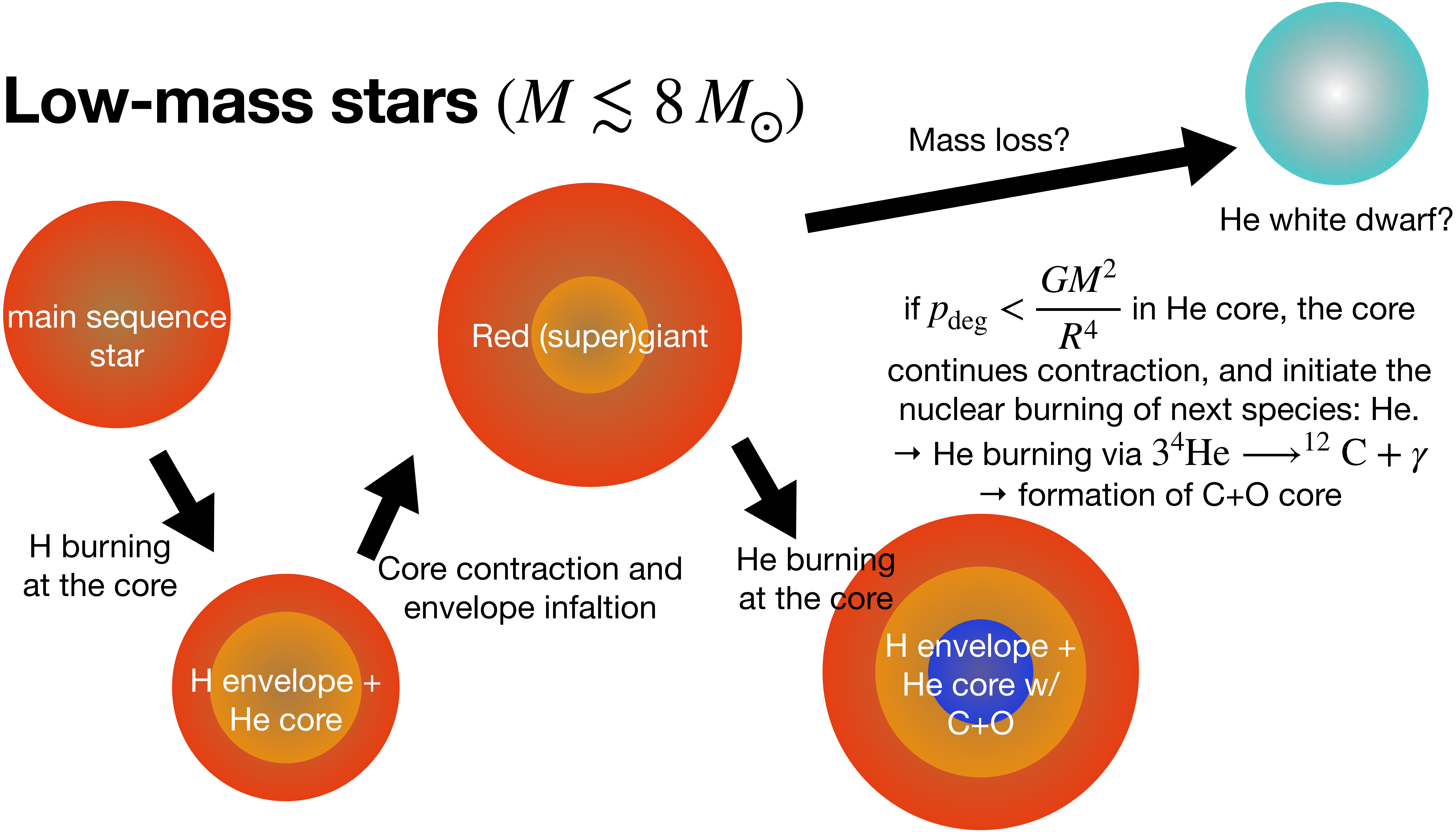




if $p_{\rm deg} > \dfrac{GM^2}{R^4}$ in He core, the core will stop contraction, while the envelope will be removed by intense mass loss.
→ formation of a He white dwarf

Note: since $t_{\rm nuc} > t_{\rm cosmic}$, this kind of He dwarf does not exist at present.

Low-mass stars $(M \lesssim 8\, M_\odot)$



if $p_{\rm deg} < \dfrac{GM^2}{R^4}$ in He core, the core continues contraction, and initiate the nuclear burning of next species: He.

→ He burning via $3\,{}^4\mathrm{He} \longrightarrow {}^{12}\mathrm{C} + \gamma$

→ formation of C+O core

Low-mass stars $(M \lesssim 8\,M_\odot)$

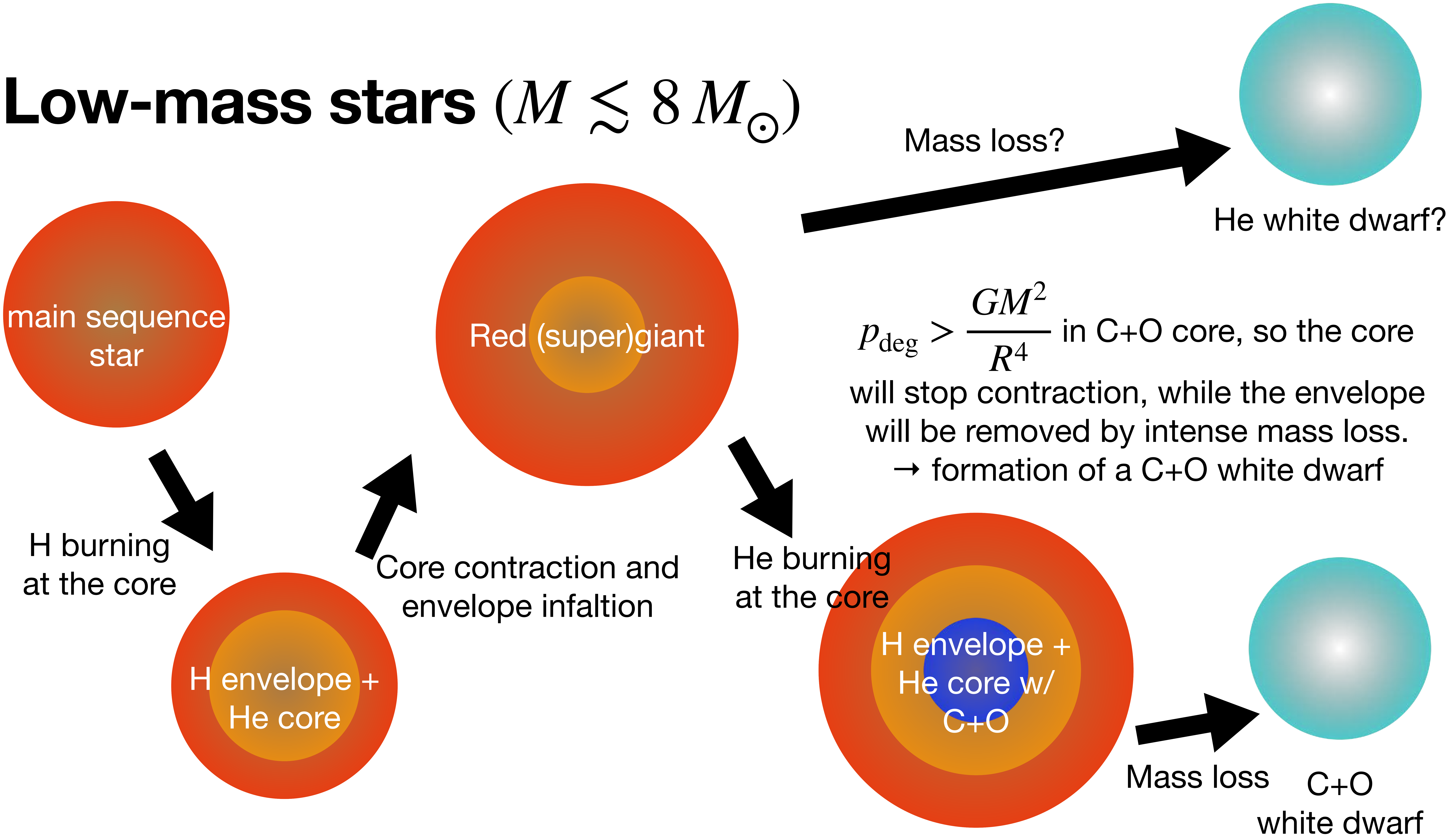
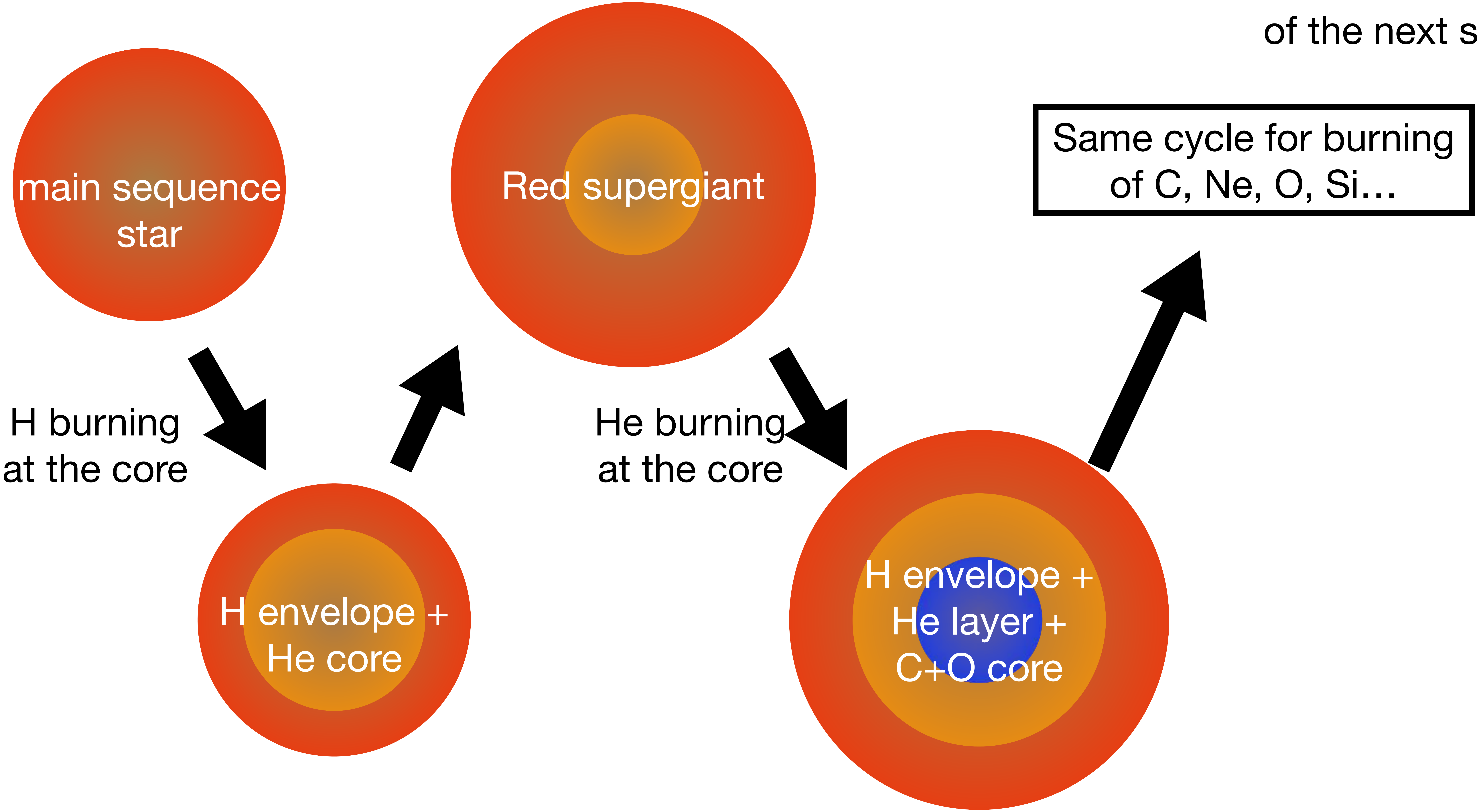


Chart of stellar evolution

Editing after the figure
in S35 of Kippenhahn's textbook

High-mass stars $(M \gtrsim 8\,M_\odot)$

$p_{\rm deg} < \dfrac{GM^2}{R^4}$ in all phases, so repeats the core contraction and nuclear burning of the next species…



High-mass stars $(M \gtrsim 8\,M_\odot)$

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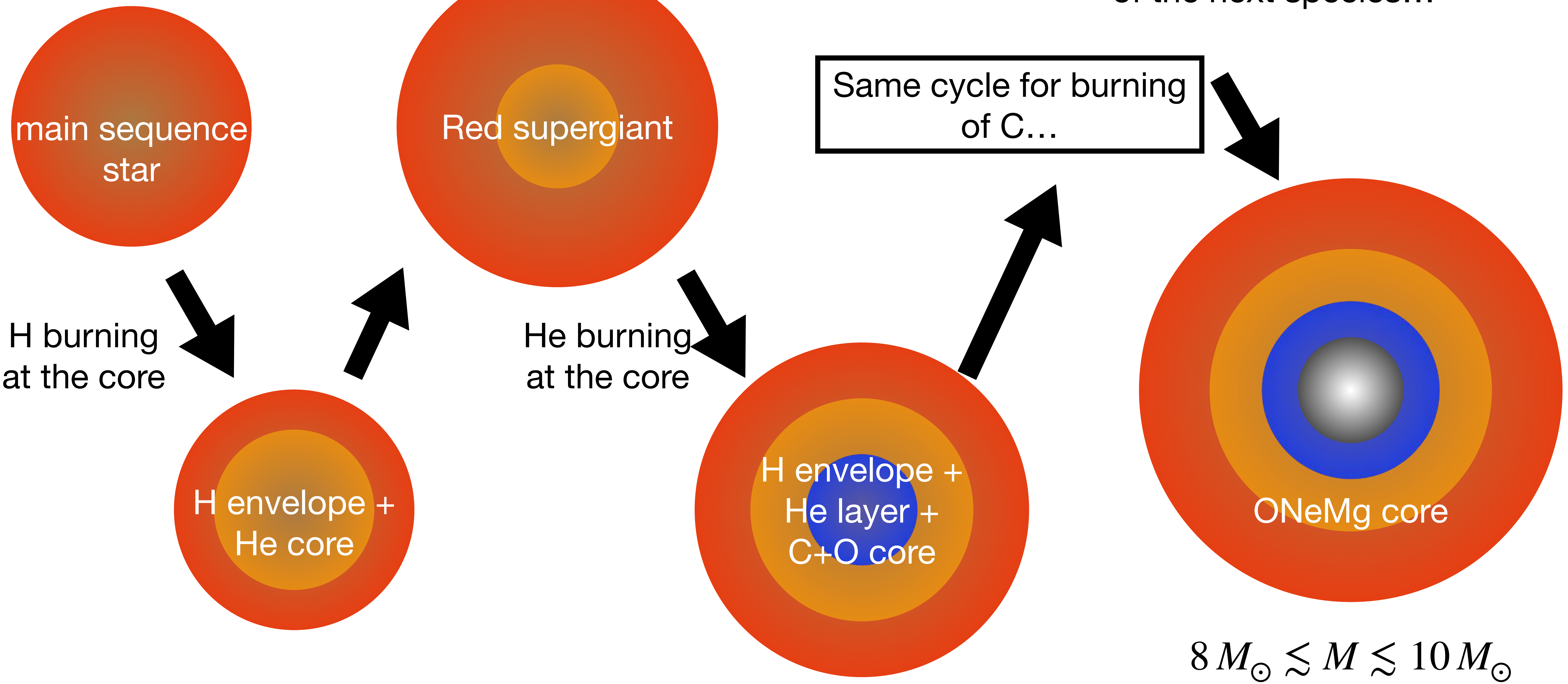
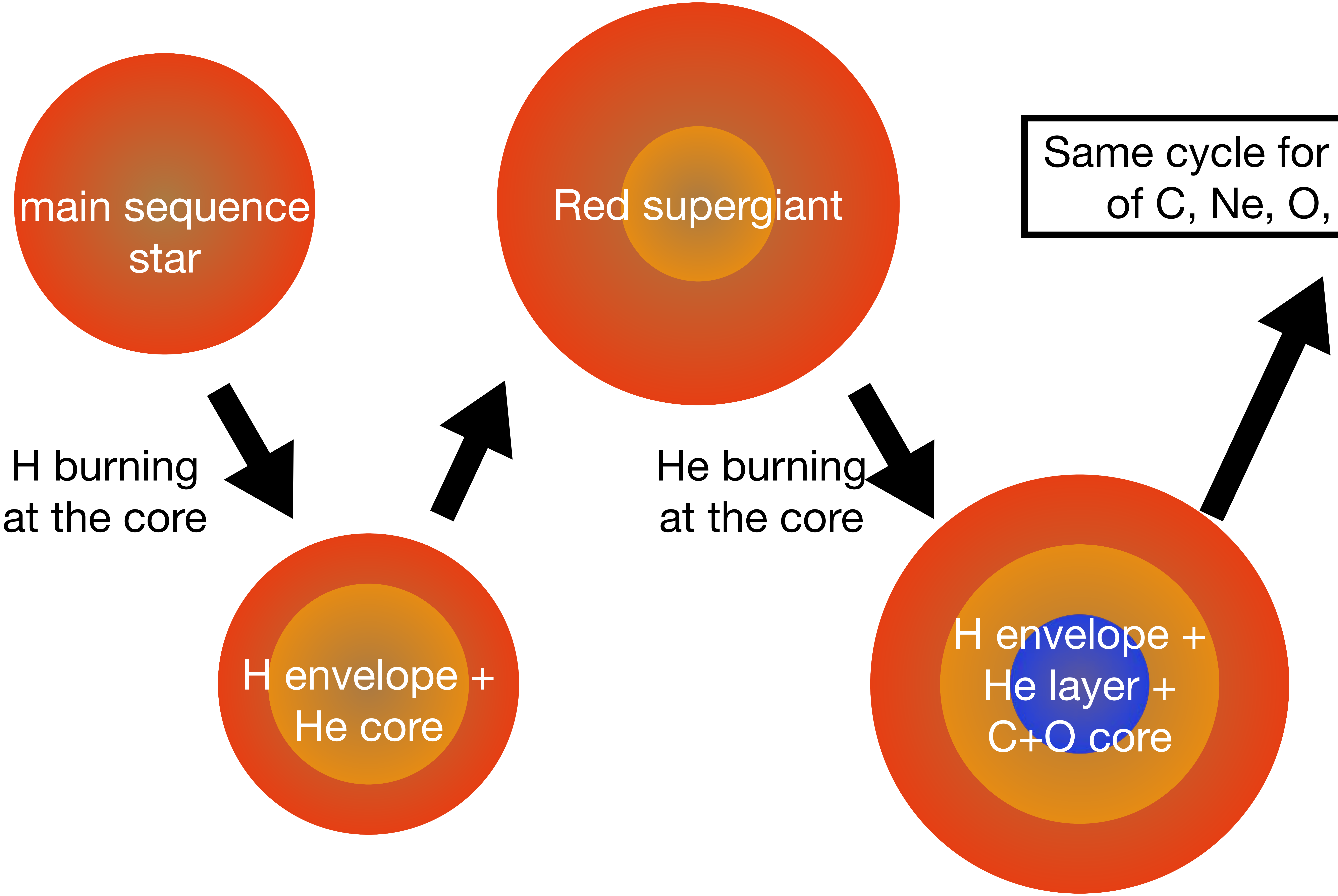


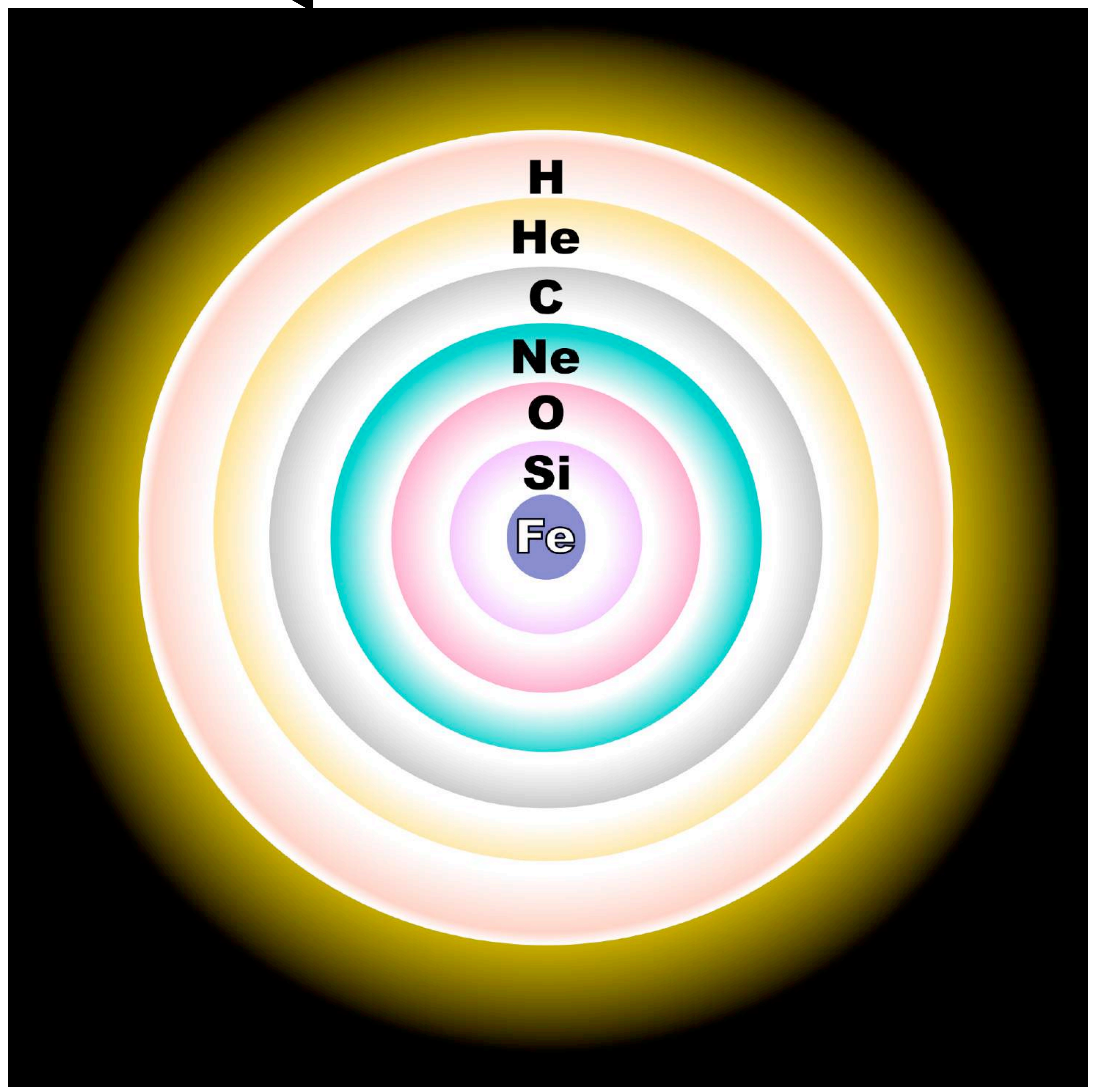
High-mass stars ($M \gtrsim 8\,M_\odot$)

$p_{\rm deg} < \dfrac{GM^2}{R^4}$ in all phases, so repeats the core contraction and nuclear burning of the next species…

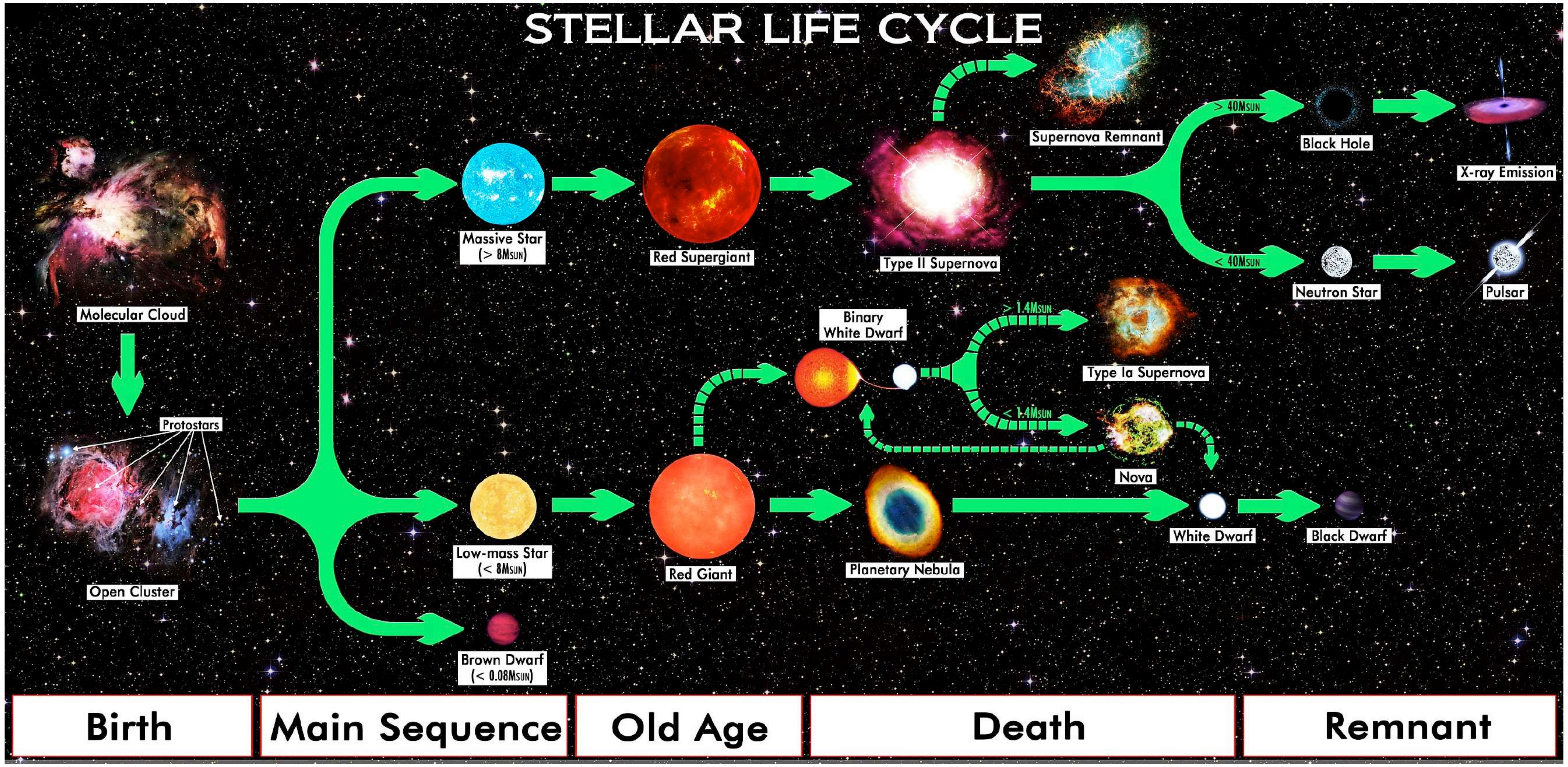


Same cycle for burning of C, Ne, O, Si…

From Wikipedia (Stellar evolution)



Onion-like structure $M \gtrsim 10\,M_\odot$



Wikipedia
(stellar evolution)

- A low mass stars $(M \lesssim 8\,M_\odot)$ evolves into a white dwarf (WD)
  -some of WDs in a binary can cause Type Ia SNe

- A high mass stars $(M \gtrsim 8\,M_\odot)$ forms onion-like structured core
  -the core collapses at the final moment, and we can observe it as core-collapse SNe

Advanced: how to numerically model a star

Given $dm = 4\pi r^2 \rho dr$,

- $\dfrac{\partial r}{\partial m} = \dfrac{1}{4\pi r^2 \rho}$: mass conservation
- $\dfrac{\partial p}{\partial m} = -\dfrac{GM_r}{4\pi r^4 \rho}$: momentum conservation $\Longleftrightarrow$ hydrostatic equilibrium
- $\dfrac{\partial L}{\partial m} = \epsilon_{\mathrm{n}} - \epsilon_\nu - T\dfrac{\partial S}{\partial t}$: energy conservation $\Longleftrightarrow$ 1st thermodynamics law
- $\dfrac{\partial T}{\partial m} = -\dfrac{GmT}{4\pi r^4 P}\nabla$: energy transportation

Advanced: how to numerically model a star

Sometimes we can treat other physical process such as

- mass loss from the surface $\dot{M}$
- convection (mass exchange) $\alpha_{\rm MLT}$
- overshooting
- energy loss by neutrino
- chemical composition (tightly related to nuclear reaction) X_j
- …etc

Nowadays people tend to rely on the open source code of stellar evolution, the representative is MESA (Modules for Experiments in Stellar Astrophysics)
details: https://docs.mesastar.org/en/latest/

Contents

Revisiting Virial Theorem

$$E = U + \Omega = (4 - 3\gamma)U = \frac{3\gamma - 4}{3(\gamma - 1)}\Omega < 0 \text{ for } \gamma = 5/3$$

Q. What happens if $\gamma \sim 4/3...$?

$$E = U + \Omega = (4 - 3\gamma)U = \frac{3\gamma - 4}{3(\gamma - 1)}\Omega \simeq 0 \text{ for } \gamma \simeq 4/3$$

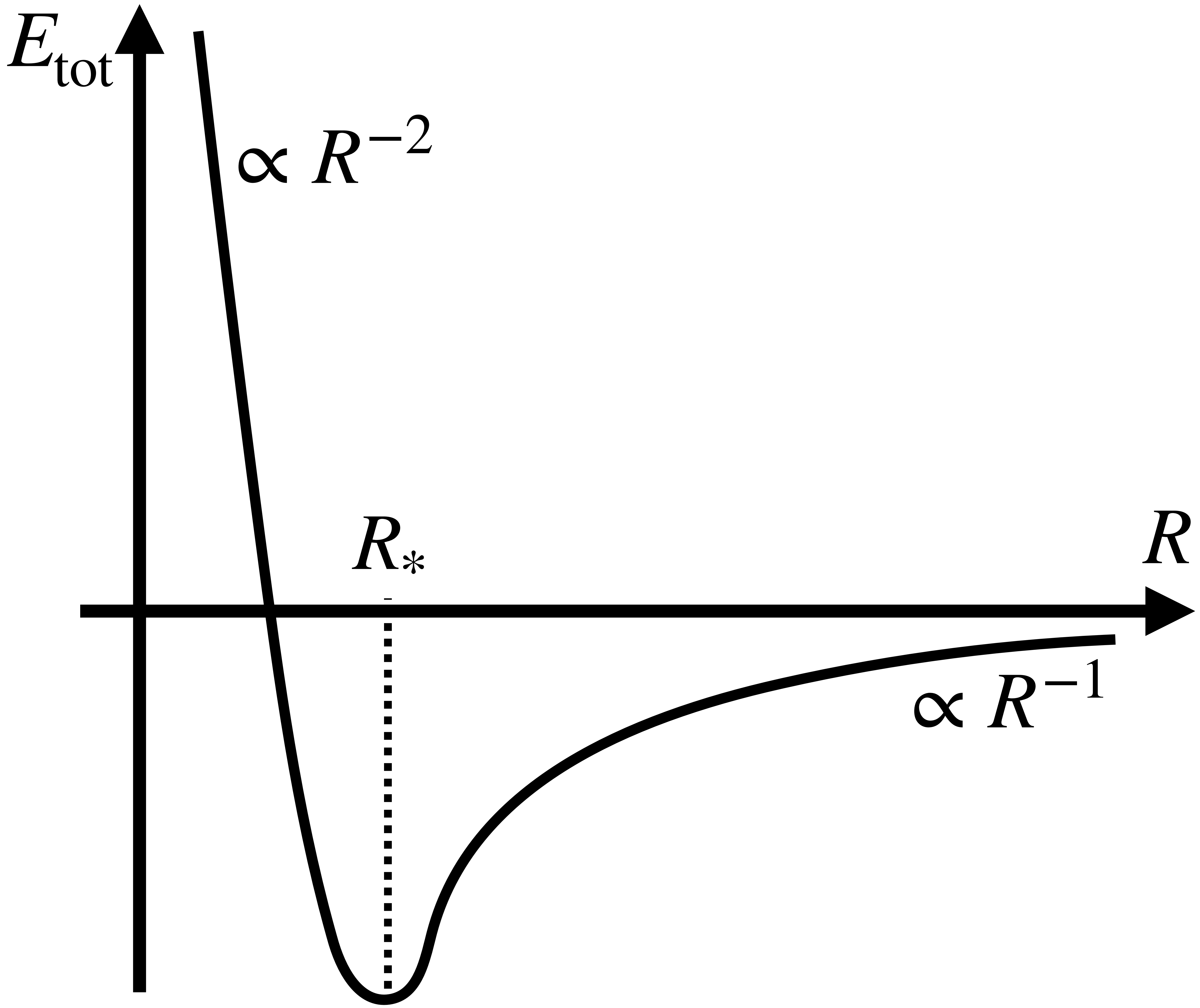
A. Yes, the star with $\gamma \sim 4/3$ is not stable.

Stars with $\gamma = 5/3$ are stable

- The system stabilizes towards the state with minimum total energy ($\simeq$ principle of least action, Landau's textbook)
- Assuming $p = K\rho^\gamma$, we obtain $E_{\rm int} = \dfrac{p}{(\gamma-1)\rho} m_B \sim \dfrac{K}{(\gamma-1)} \rho^{\gamma-1}$
- In case of a main-sequence star $\gamma = 5/3$, thus $E_{\rm int} \sim \dfrac{3K}{2}\dfrac{M^{2/3}}{R^2}$
- In addition, $E_{\rm grav} = -\dfrac{GMm_B}{R}$
- $\therefore E_{\rm tot} = E_{\rm int} + E_{\rm grav} \sim \dfrac{3K}{2}\dfrac{M^{3/2}}{R^2} - \dfrac{GMm_B}{R}$

Stars with $\gamma = 5/3$ are stable

- Given M, the star can have a local minimum radius R_* where the total energy can be minimized.
- If we consider the perturbation of $R_* \to R_* + \delta R$,
  - $\delta R > 0 \to$ Gravity shrinks the system $\to$ return to R_*
  - $\delta R < 0 \to$ pressure expands the system $\to$ return to R_*



Stars with $\gamma = 4/3$ are unstable

- Again, the system stabilizes towards the state with minimum total energy ($\simeq$ principle of least action, Landau's textbook)

- In case of ***relativistically*** degererated stars, $p = \hbar c n_e^{4/3}$ ($\gamma = 4/3$), thus the degeneracy energy per electron is
$$E_{\rm deg} \sim p \times 1/\rho \times m_B \sim \hbar c n_e^{1/3} \sim \frac{\hbar c M^{1/3}}{\mu_e^{1/3} m_u^{1/3} R}$$

- $$\therefore E_{\rm tot} = E_{\rm deg} + E_{\rm grav} \sim (\hbar c \mu_e^{-1/3} m_u^{-1/3} M^{1/3} - GM\mu_e m_u)\frac{1}{R}$$

Stars with $\gamma = 4/3$ are unstable

(Note: $E = (4 - 3\gamma)U = 0$)

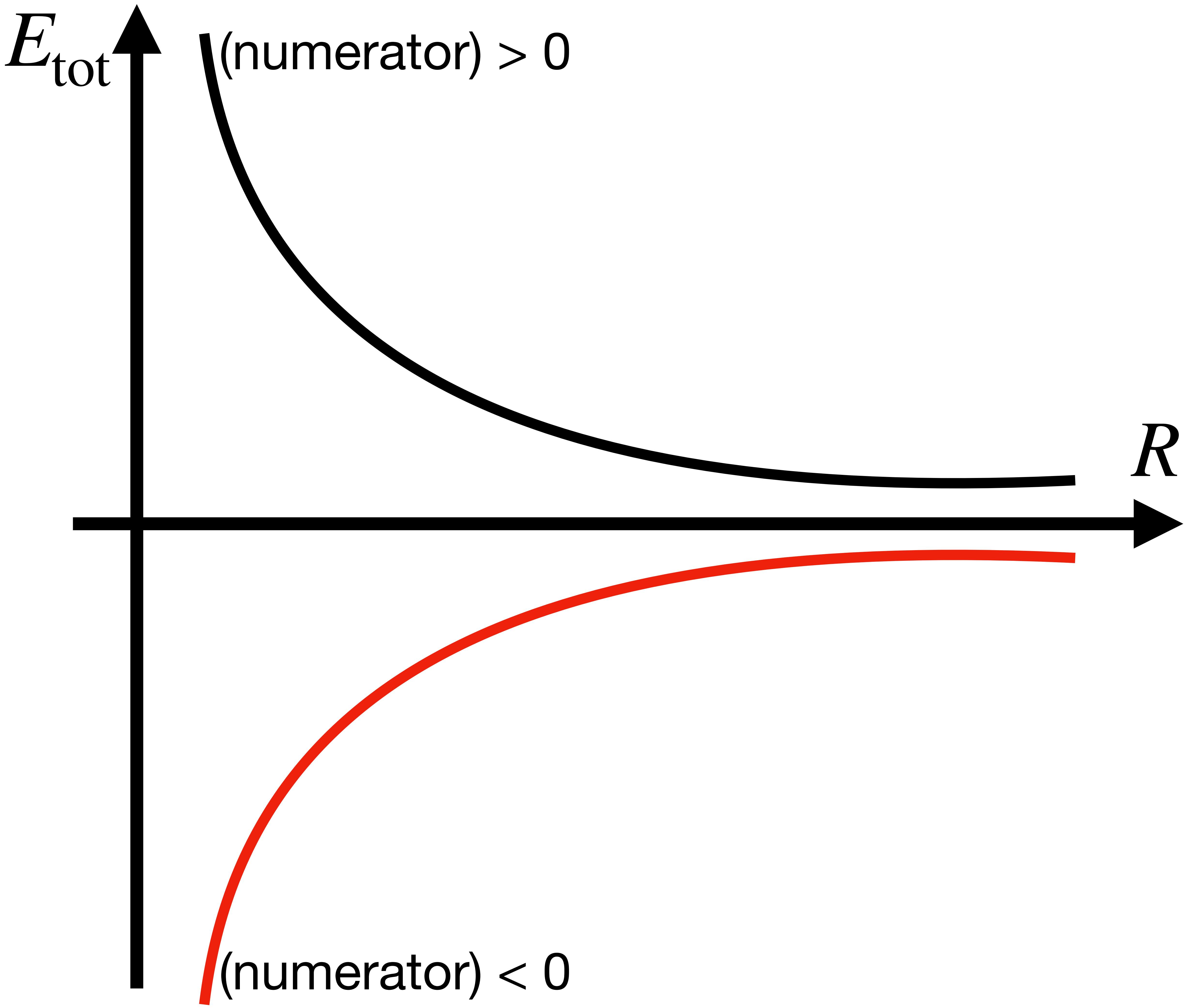
- The total energy has no local minimum radius! Particularly if $E_{\rm tot} < 0$, the system tries to shrink infinitely → collapse

→ No more massive stars can alive if it is relativistically degerated gas.

- From (numerator) > 0, we get

$$M_{\rm Ch} \sim \left(\frac{\hbar c}{G}\right)^{3/2} \mu_e^{-2} m_u^{-2} = 1.4 \left(\frac{\mu_e}{2}\right)^{-2} M_\odot$$

- Chandrasekhar mass.
- The limiting mass of white dwarfs.



Explosion of a white dwarf

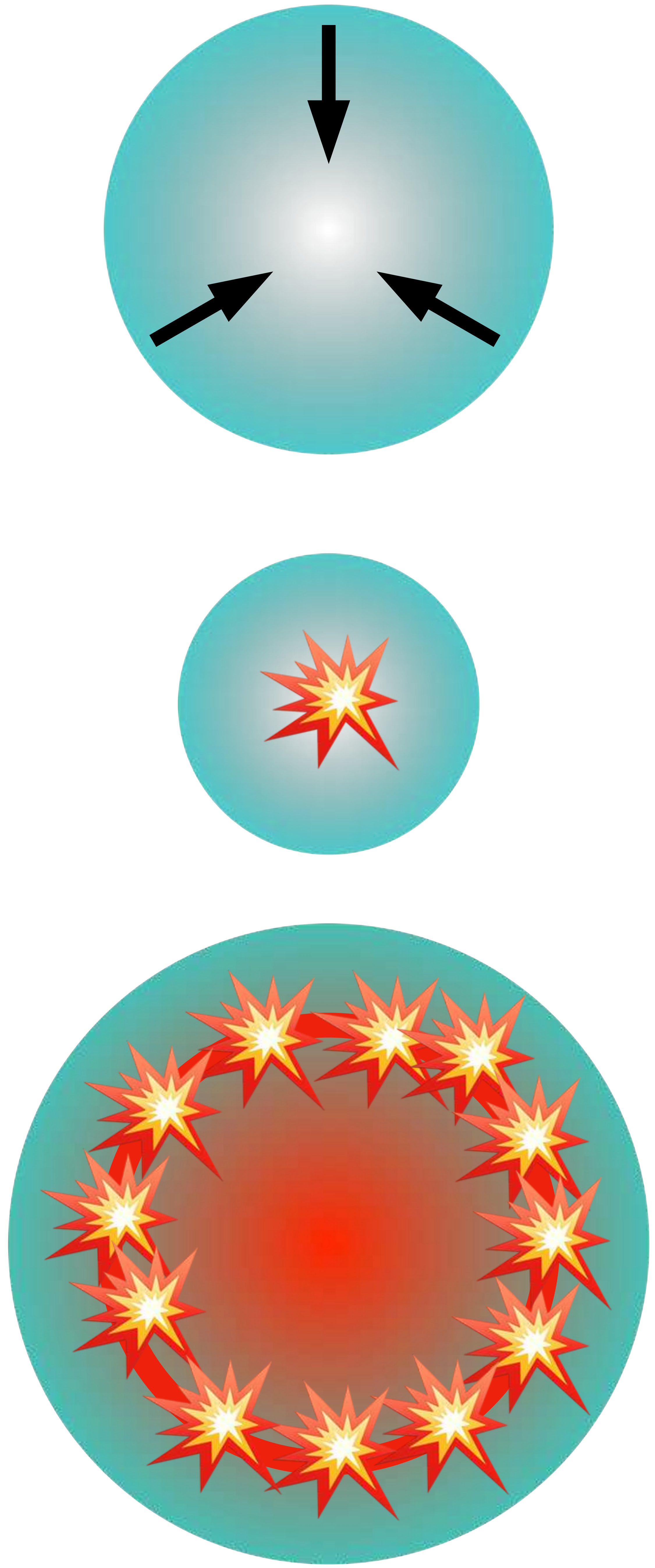
- If a white dwarf is weighted up to Chandrasekhar mass by some reason, it collapses towards center

- → high density

- → initiation of nuclear burning of C and O to 56Ni
  ~ combustion

$$E_{\mathrm{expl.}} \sim \left(m_{\mathrm{C}} - \frac{12}{56} m_{\mathrm{Ni}}\right) c^2 \times \frac{1.4\, M_\odot}{12 m_u} \sim 10^{51}\,\mathrm{erg} > E_{\mathrm{grav}}$$

- Explosion of a white dwarf.
  We call it Thermonuclear explosion; Type Ia supernova



Advanced: Polytrope sphere to derive $M_{\rm Ch} = 1.4(\mu_e/2)^{-2}M_\odot$

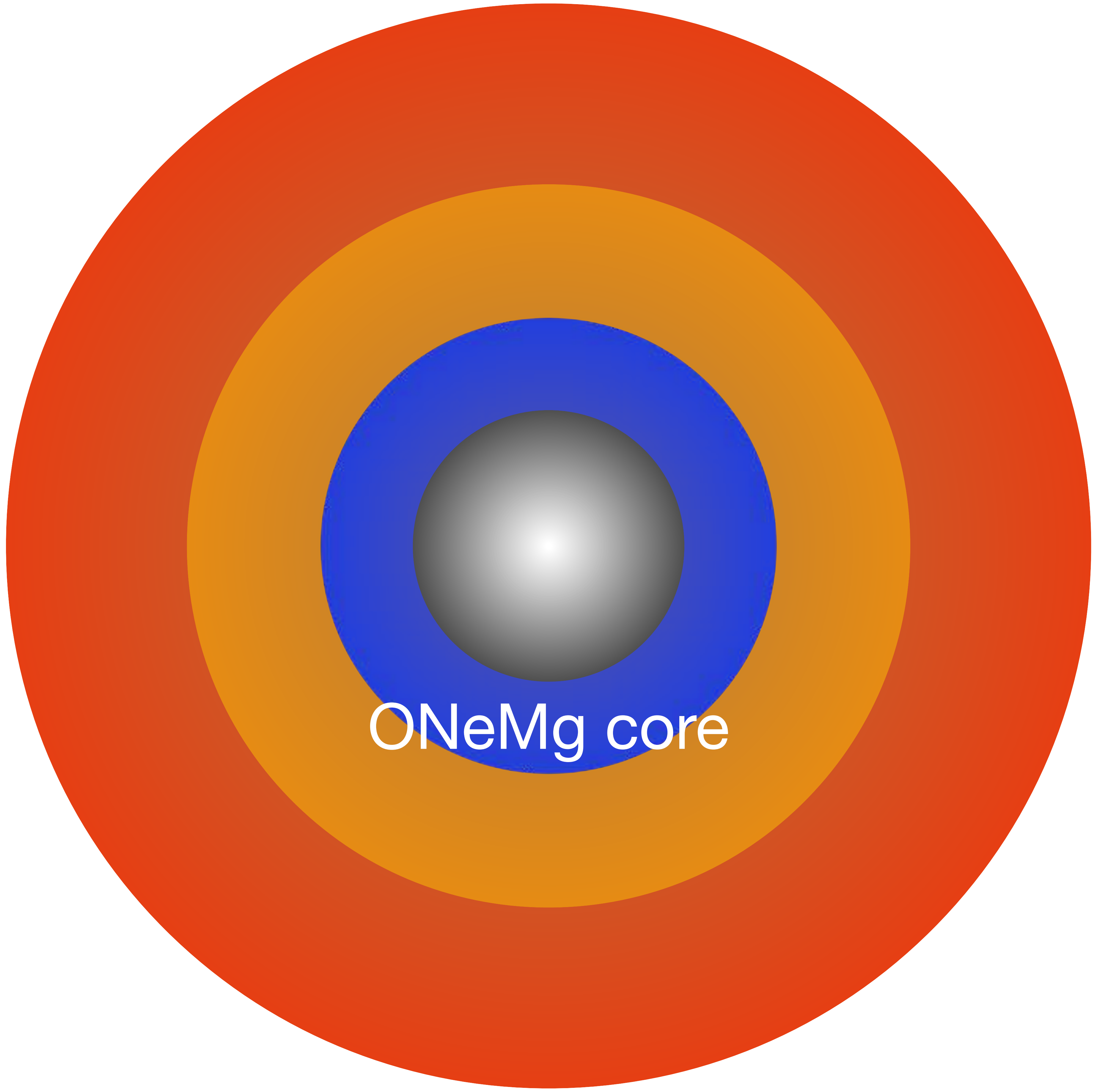
- From $\dfrac{dp}{dr} = -\dfrac{GM_r}{r}\rho$ and $p = K\rho^{1+1/n}$, we can deduce $\dfrac{d}{dr}\left(\dfrac{r^2}{\rho}\dfrac{dp}{dr}\right) = -4\pi G r^2 \rho$
- Assuming $\rho = \rho_c \theta(\xi)^n$ and $p = p_c \theta(\xi)^{n+1}$ where $r = \alpha\xi$ with $\alpha = \mathrm{const.}$, the equation can be $\dfrac{1}{\xi^2}\dfrac{d}{d\xi}\left(\xi^2 \dfrac{d\theta}{d\xi}\right) = -\theta^n$: ***Lane-Emden equation.***
- Numerically integrate to obtain the shape of $\theta(\xi)$ if $n \neq 0, 1, 5$.
- The total mass can be $M = \displaystyle\int_0^R 4\pi r^2 \rho dr$.
  Particularly for $n = 3$, M is derived to be independent of R.

Final moment of massive stars: core collapse

- For stars with $8\,M_\odot \lesssim M \lesssim 10\,M_\odot$, the star forms ONeMg core at the center ($M_\mathrm{core} \sim 1.37 M_\odot \sim M_\mathrm{Ch}$)

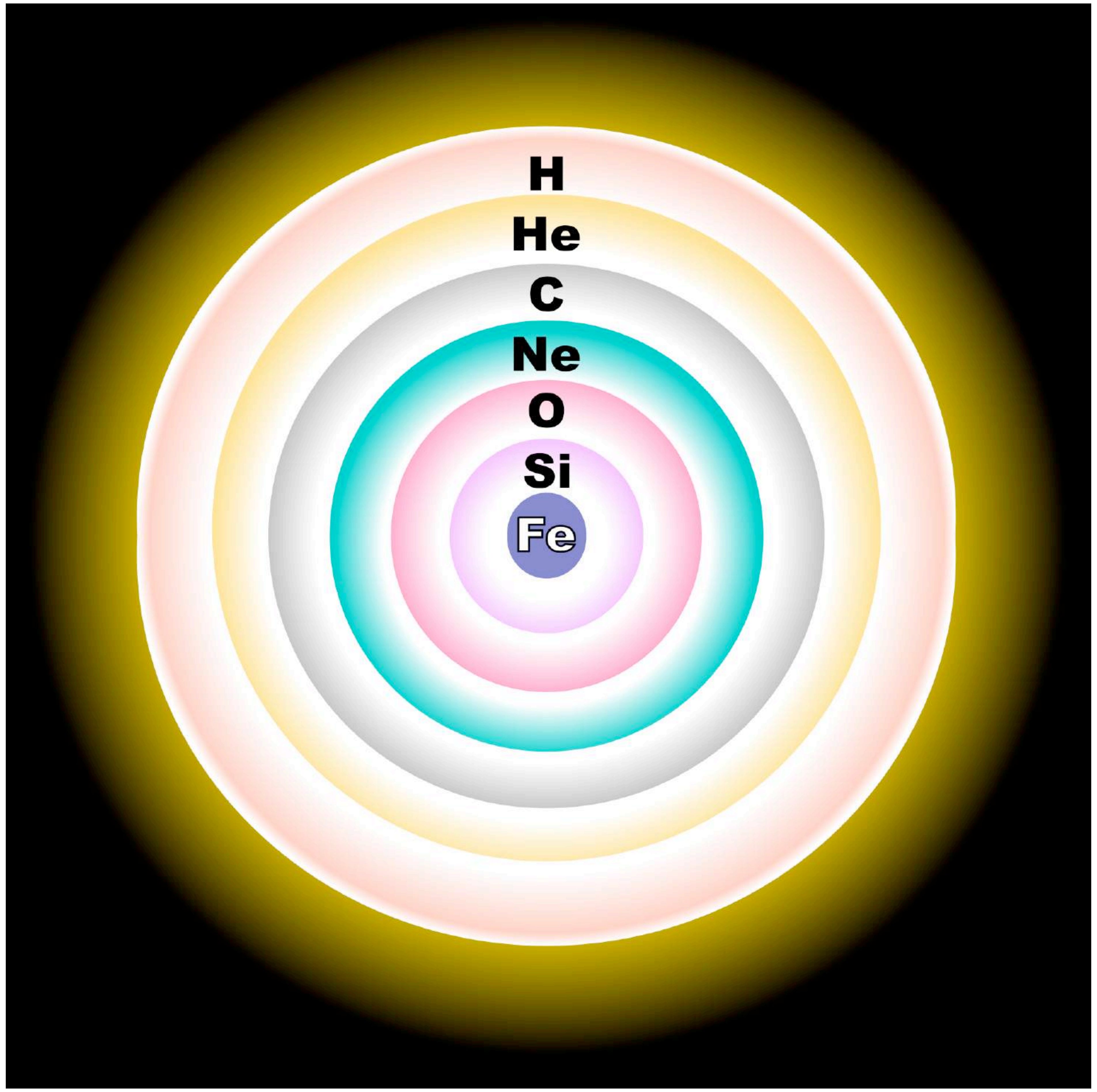
- If the electron degenerates relativistically electron capture proceeds,
$$^{24}\mathrm{Mg} + \mathrm{e}^- \longrightarrow ^{24}\mathrm{Na} + \nu_\mathrm{e}$$
$$^{24}\mathrm{Na} + \mathrm{e}^- \longrightarrow ^{24}\mathrm{Ne} + \nu_\mathrm{e}$$
$$^{20}\mathrm{Ne} + \mathrm{e}^- \longrightarrow ^{20}\mathrm{F} + \nu_\mathrm{e}$$
$$^{20}\mathrm{F} + \mathrm{e}^- \longrightarrow ^{20}\mathrm{O} + \nu_\mathrm{e}$$
$$^{16}\mathrm{O} + \mathrm{e}^- \longrightarrow ^{16}\mathrm{Ne} + \nu_\mathrm{e}$$
reducing the number of electrons → ***reducing pressure***

- $\therefore p \ll \dfrac{GM^2}{R^4}$: Core collapse.



Final moment of massive stars: core collapse

- For stars with $M \gtrsim 10\,M_\odot$, the star forms Fe core at the center $(T \sim \mathrm{GeV})$
- Fe has maximum binding energy per nucleon → No nuclear burning reaction.
- Then Fe nuclei will be photodissociated into p+n via $^{56}\mathrm{Fe} \longrightarrow 13^4\mathrm{He} + 4\mathrm{n} - 124.4\,\mathrm{MeV}$ and $^4\mathrm{He} \longrightarrow 2\mathrm{p} + 2\mathrm{n} - 28.3\,\mathrm{MeV}$
  — endothemic reaction, ***reducing pressure***
- In addition, electron capture $\mathrm{p} + \mathrm{e}^- \longrightarrow n + \nu_e$ ***reduces degeneracy pressure***.
- $\therefore p \ll \dfrac{GM^2}{R^4}$: Core collapse.

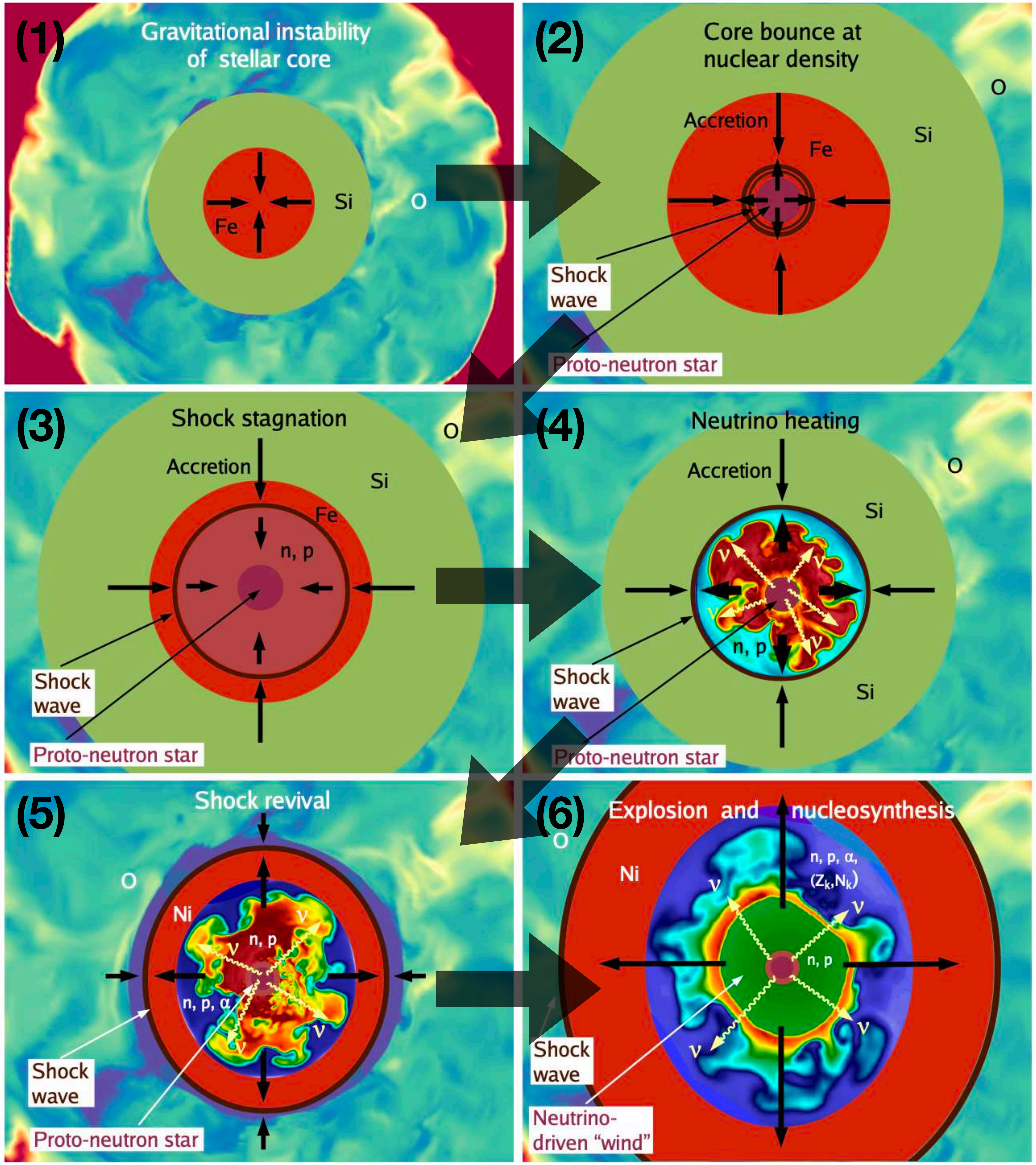


From Wikipedia (Stellar evolution)

Janka17

Neutrino-driven explosion

1. Gravitational collapse of the core
2. Formation of Proto-neutron star and core bounce → shock launch
3. Shock stagnation
4. Neutrino heating
5. Shock revival
6. Successful explosion & nucleosynthesis (especially producing ^{56}Ni)



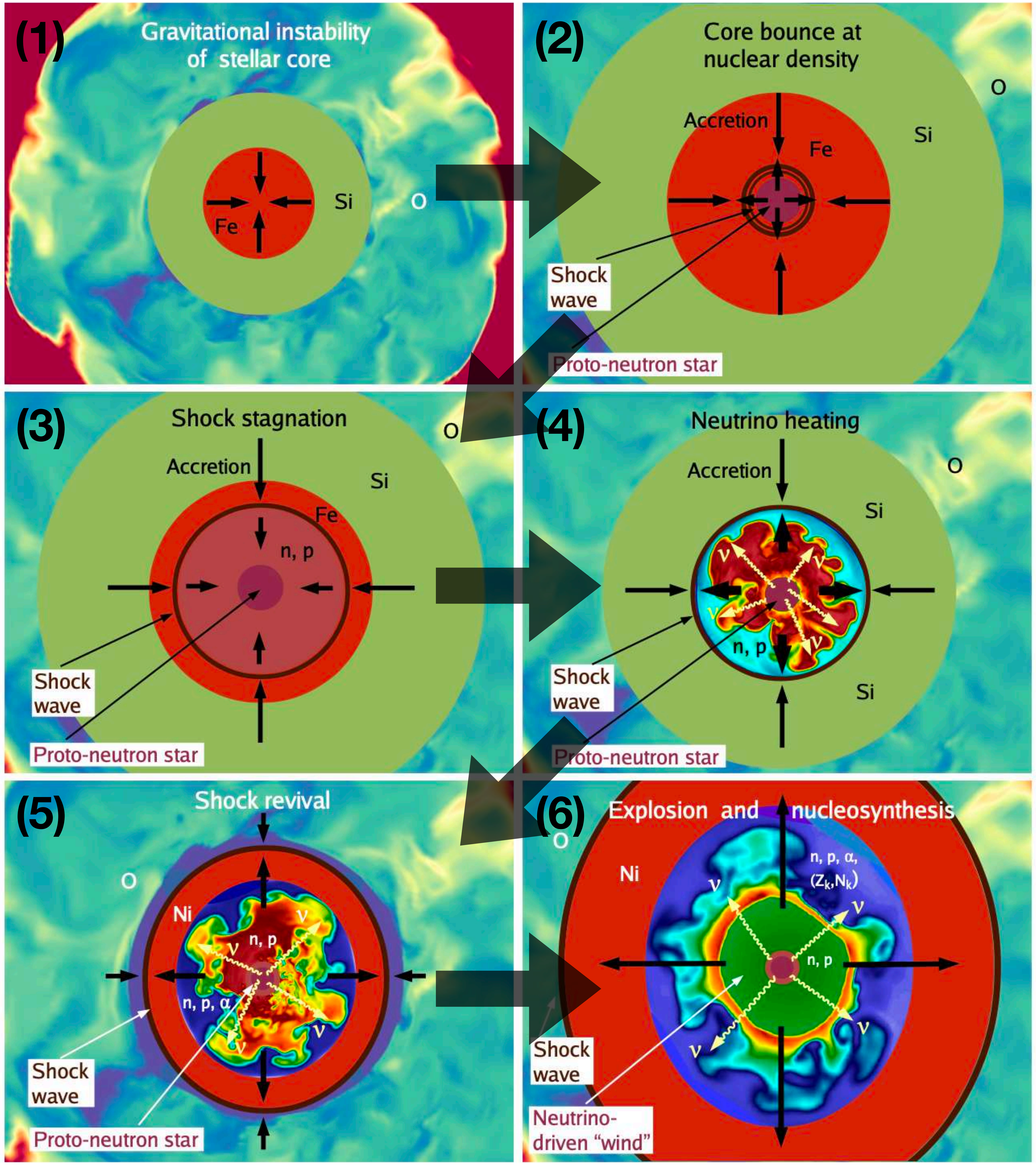
Janka17

Neutrino-driven explosion

$E_\nu \sim 10^{53}$ erg can be derived from the released gravitational potential:

$$\Delta E_{\rm grav} \sim -\cancel{\frac{GM_*^2}{R_*}} + \frac{GM_{\rm NS}^2}{R_{\rm NS}} \sim 10^{53}\,{\rm erg}$$

But, $E_{\rm kin} \sim E_\nu \times 0.01$ ← requiring high precision to test.
We still do not have clear answer to reproduce this efficiency. Multi-D effect? B-field? Rotation? Progenitor model? Anything else?



Analogy: ping pong receiving wind

Without wind

With wind

Special thanks: Yuya Fukuhara

Supernova spectral classification

Contents

Light curve variety

- $E_\nu \sim 10^{53}$ erg: release of gravitational potential of massive star's core
- $E_{\rm kin} \sim 10^{51}$ erg: thermonuclear burning runaway in Type Ia SN, and conversion from neutrino energy budget in core-collapse SNe
- $E_{\rm rad} \sim 10^{49}$ erg: What we see. Where the radiation is coming from?

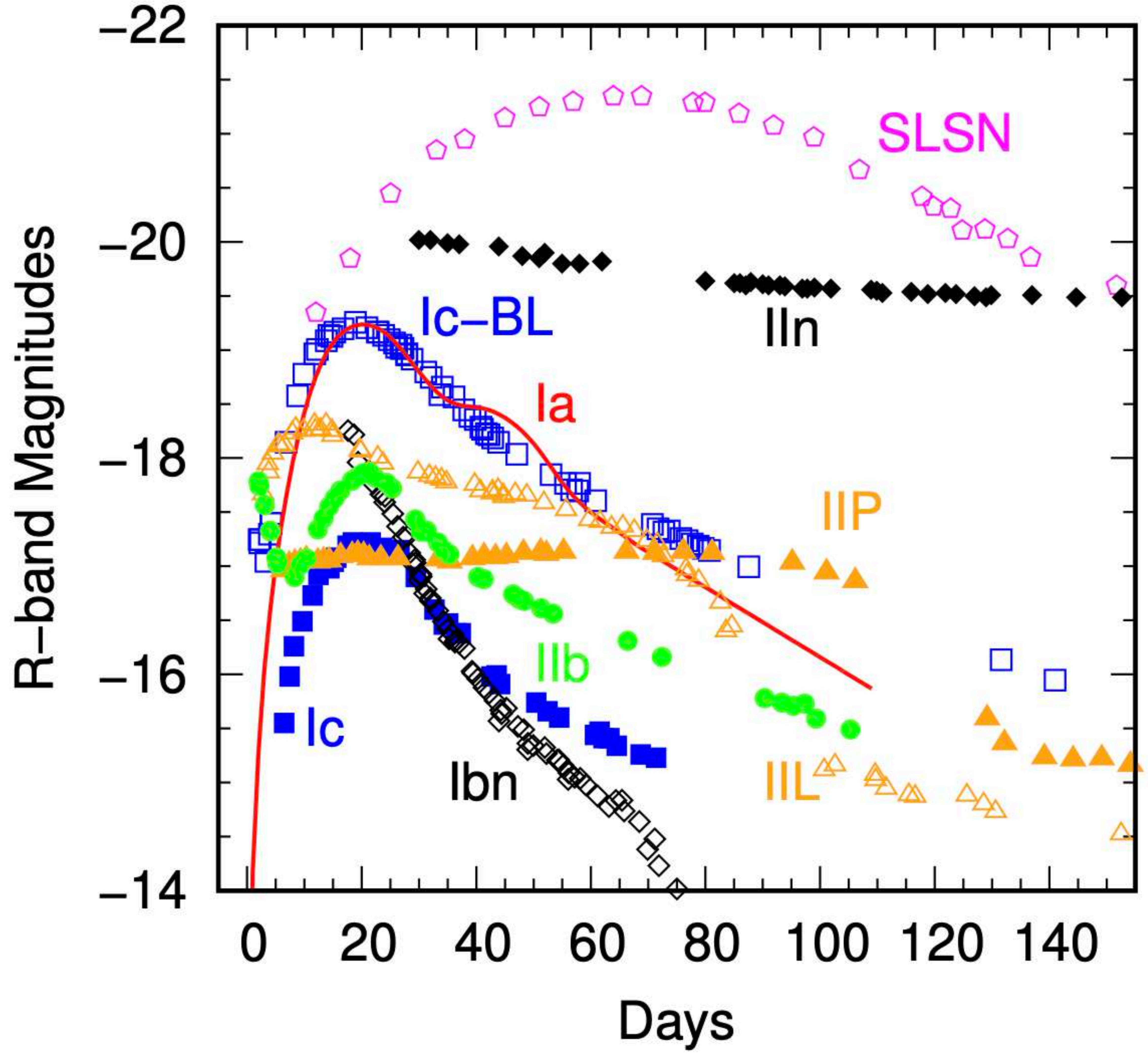
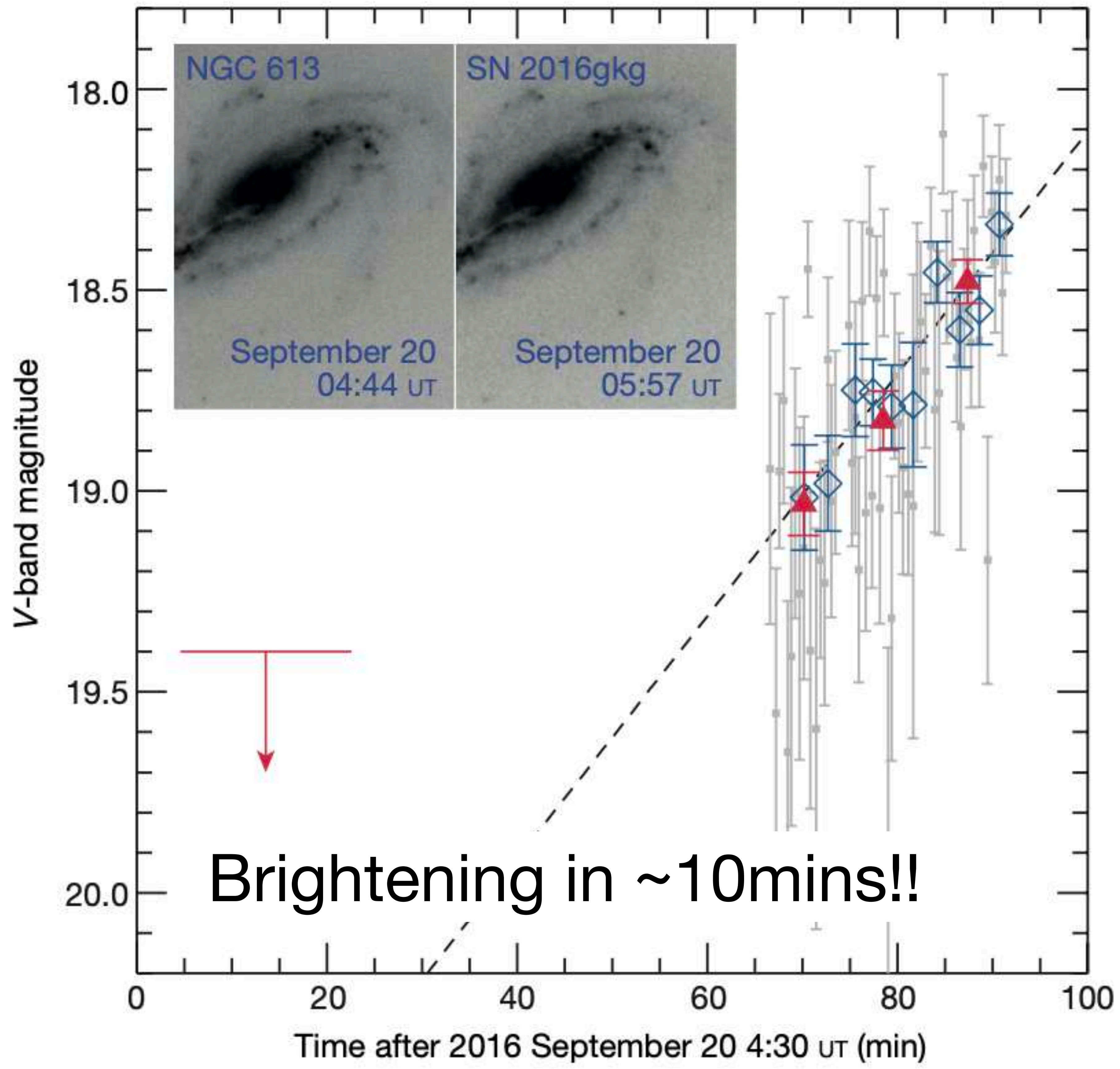


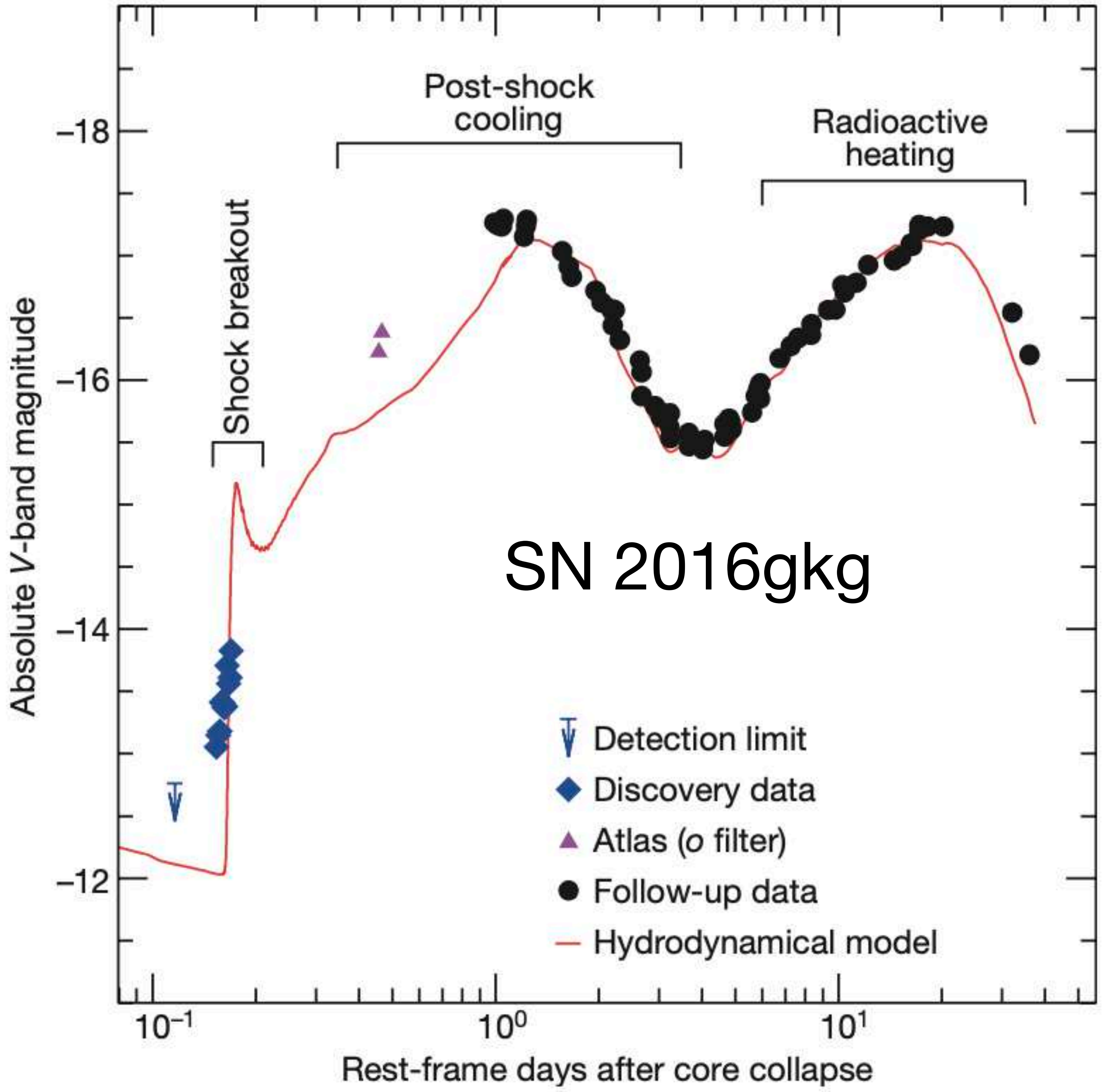
Fig. 10 Examples of optical (*R*-band) light curves of SNe of different types. It is the compilation of the data from the following sources: [122, 70, 50, 114, 136, 149, 21, 168, 150].

Maeda22

Emission source: Shock breakout

- Photons released when the shock reaches the progenitor surface





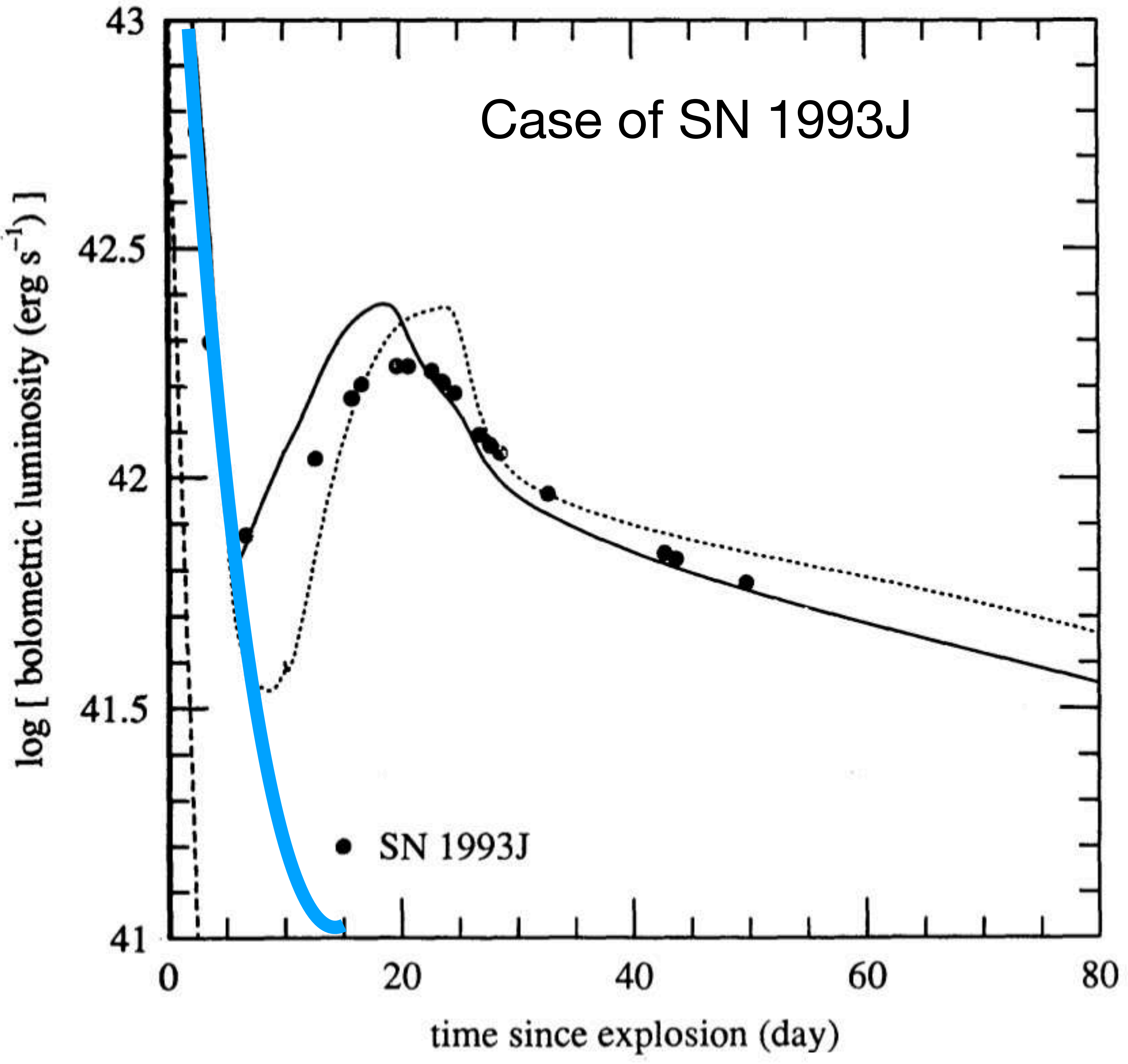
What we find: progenitor star, nature of the shock

Bersten+18

Emission source: Adiabatic expansion

- For the photon gas, $P_{\rm rad} = aT^4/3$. So $P \propto \rho^\gamma$ $(\gamma = 4/3) \Longrightarrow T \propto R^{-1}$
- $L = 4\pi R_{\rm ph}^2 \sigma_{\rm SB} T_{\rm eff}^4$, coming from the material expanding.
    - Initial phase of some types of SNe
    - SN II-P (seen as a plateau when $L \sim \rm const$. due to H-recombination)

What we find: progenitor size

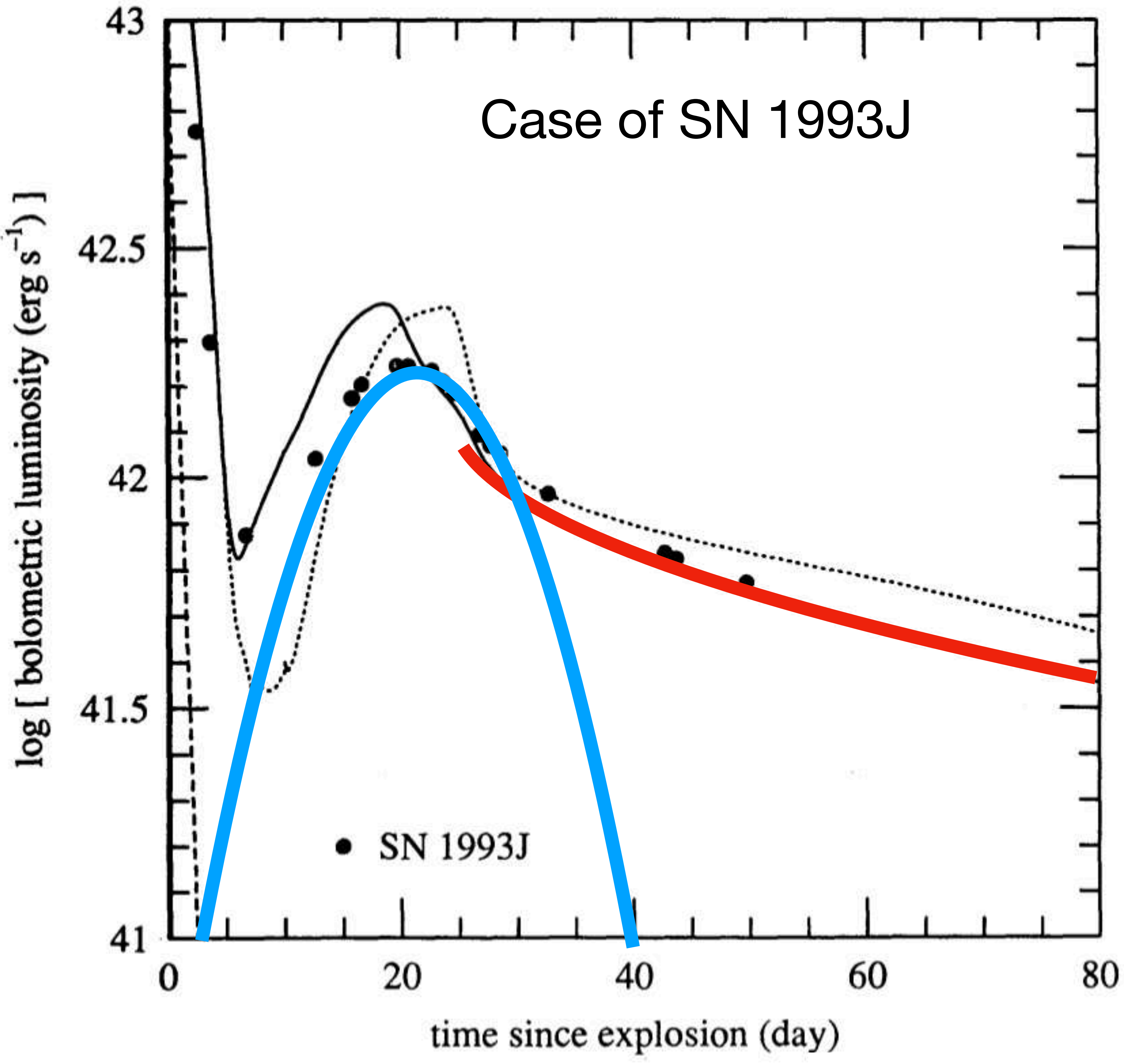


Shigeyama+94

Emission source: Radioactive decay

- $^{56}\mathrm{Ni} \xrightarrow{6.1\mathrm{d}} {}^{56}\,\mathrm{Co} \xrightarrow{77.2\mathrm{d}} {}^{56}\,\mathrm{Fe}$
- Arnett rule (Arnett 82)
    - $L(^{56}\mathrm{Ni}) \sim 6.5 \times 10^{43} \exp\left(\frac{-t}{8.8\ \mathrm{days}}\right) \frac{M(^{56}\mathrm{Ni})}{M_\odot}\ \mathrm{erg\ s^{-1}}$
    - $L(^{56}\mathrm{Co}) \sim 1.5 \times 10^{43} \exp\left(\frac{-t}{111.3\ \mathrm{days}}\right) \frac{M(^{56}\mathrm{Ni})}{M_\odot}(D_\gamma + f_{\mathrm{e+}})\ \mathrm{erg\ s^{-1}}$
- Uncertainty: gamma-ray trapping

What we find: kinematics of ejecta and ^{56}Ni (nucleosynthesis)



Shigeyama+94

Emission source: Interaction with circumstellar medium

SN ejecta colliding with surrounding circumstellar medium (CSM)

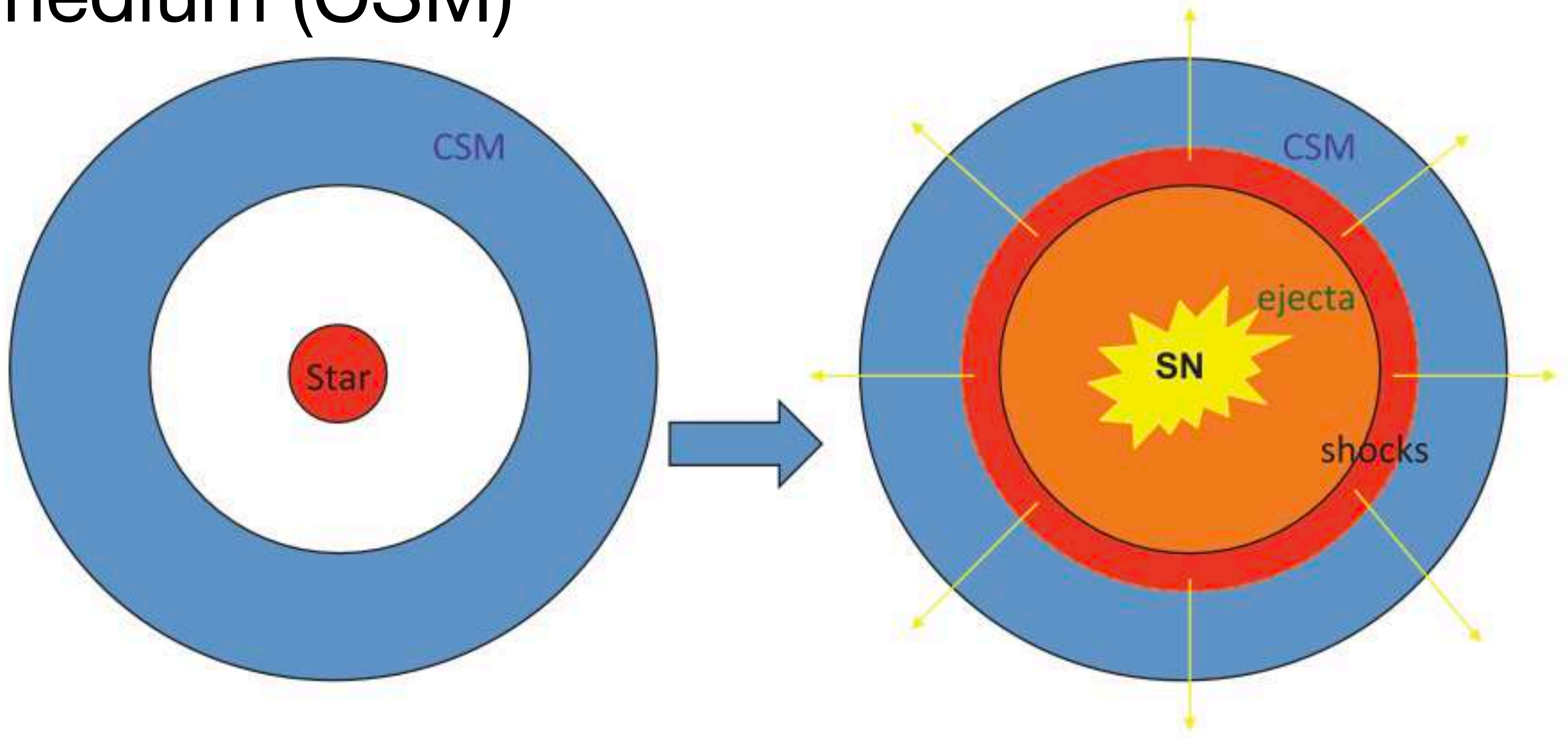
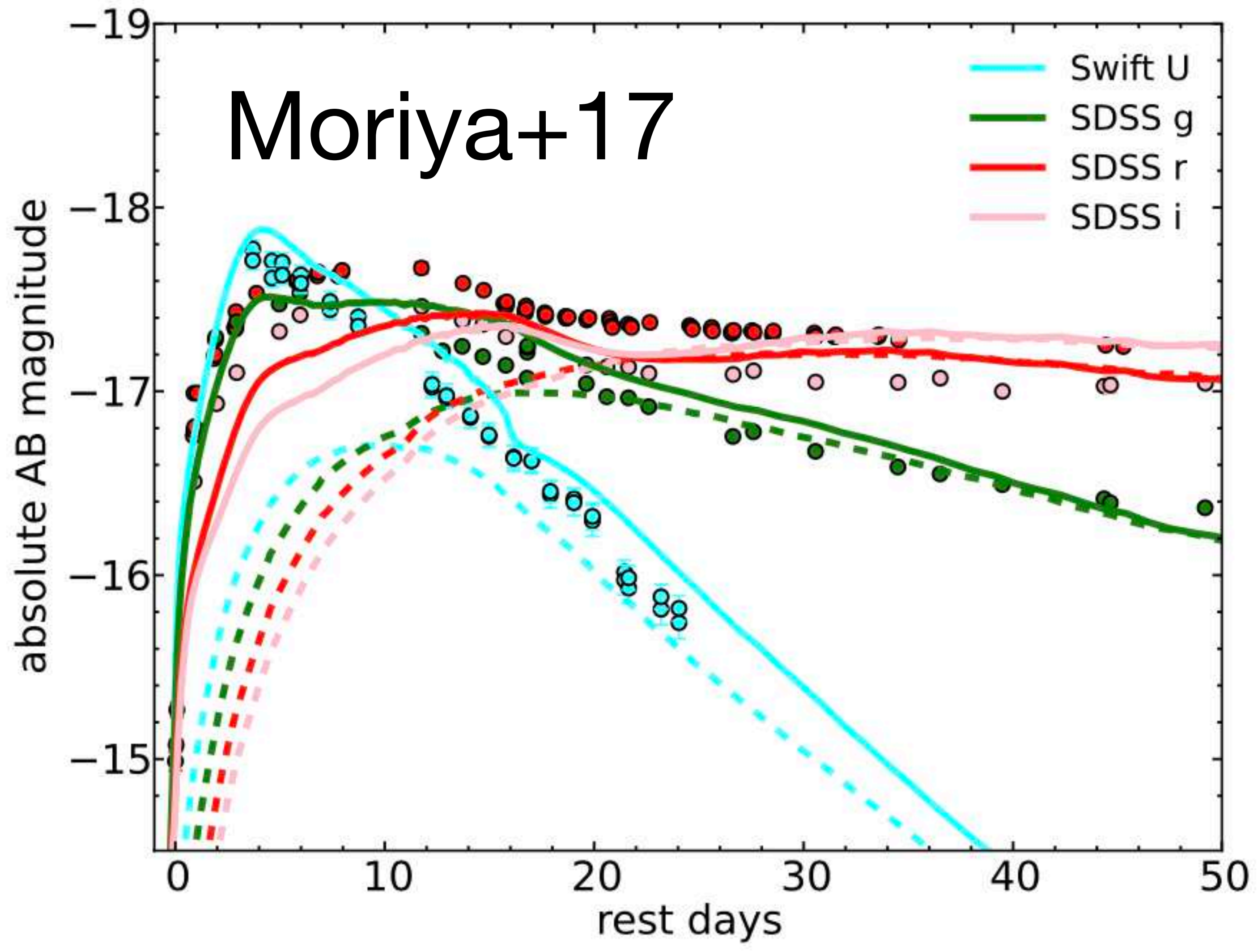


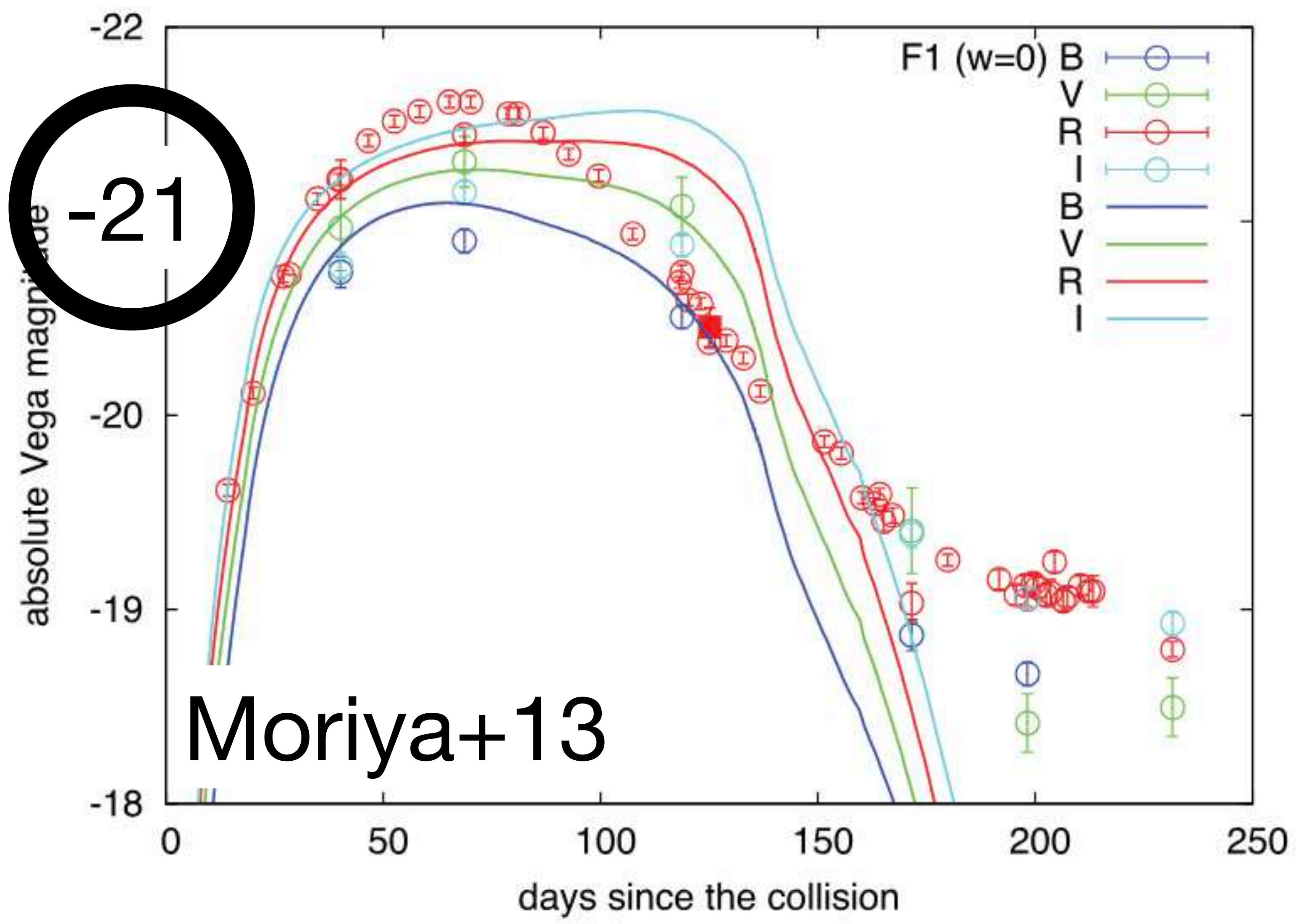
Figure 1. The schematic picture of the interaction-powered SN scenario.

Murase+14

What we find: circumstellar medium
→ mass-loss history of the progenitor



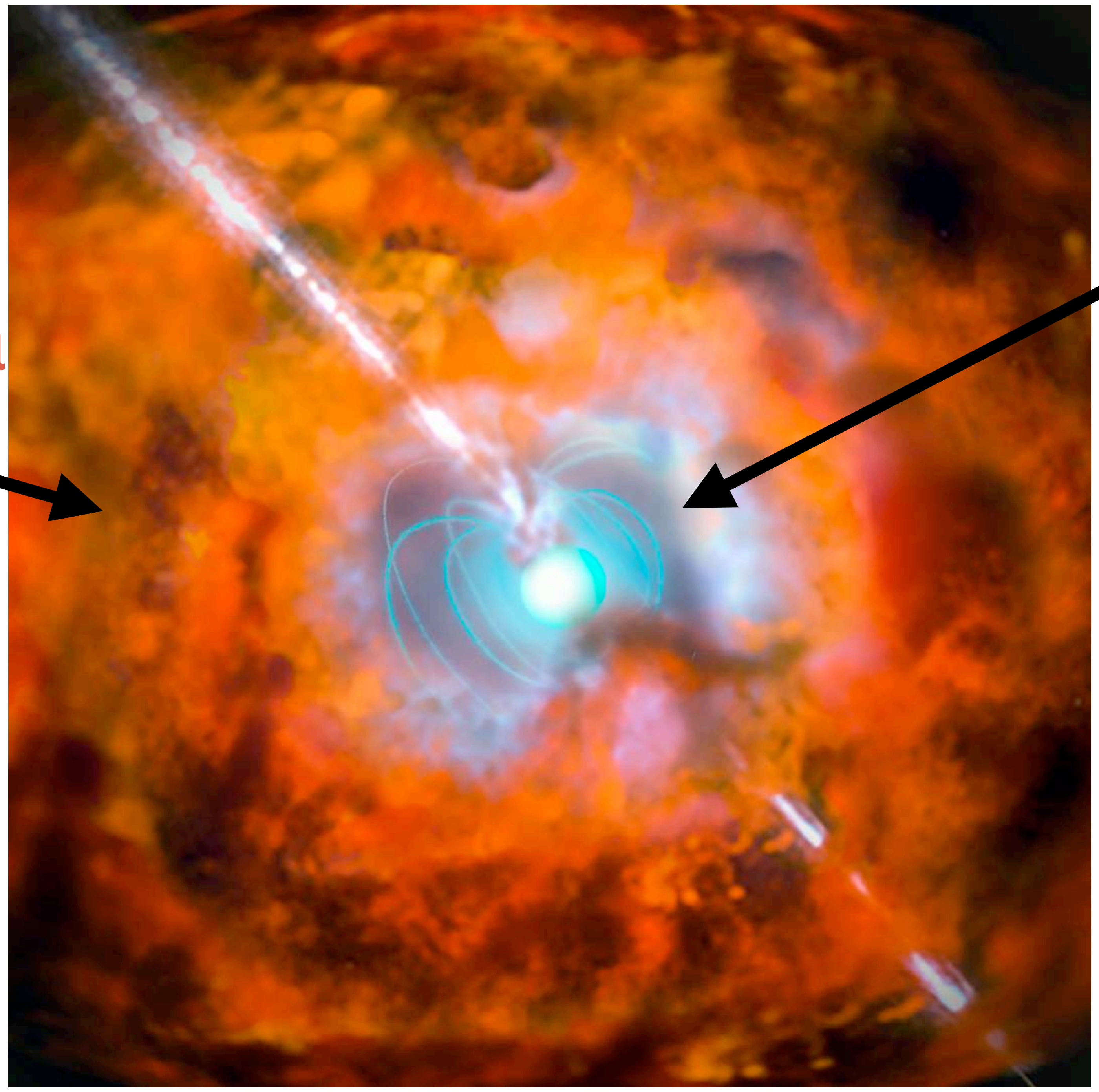
SN II



SN IIn

Emission source: Energy deposition from the central object

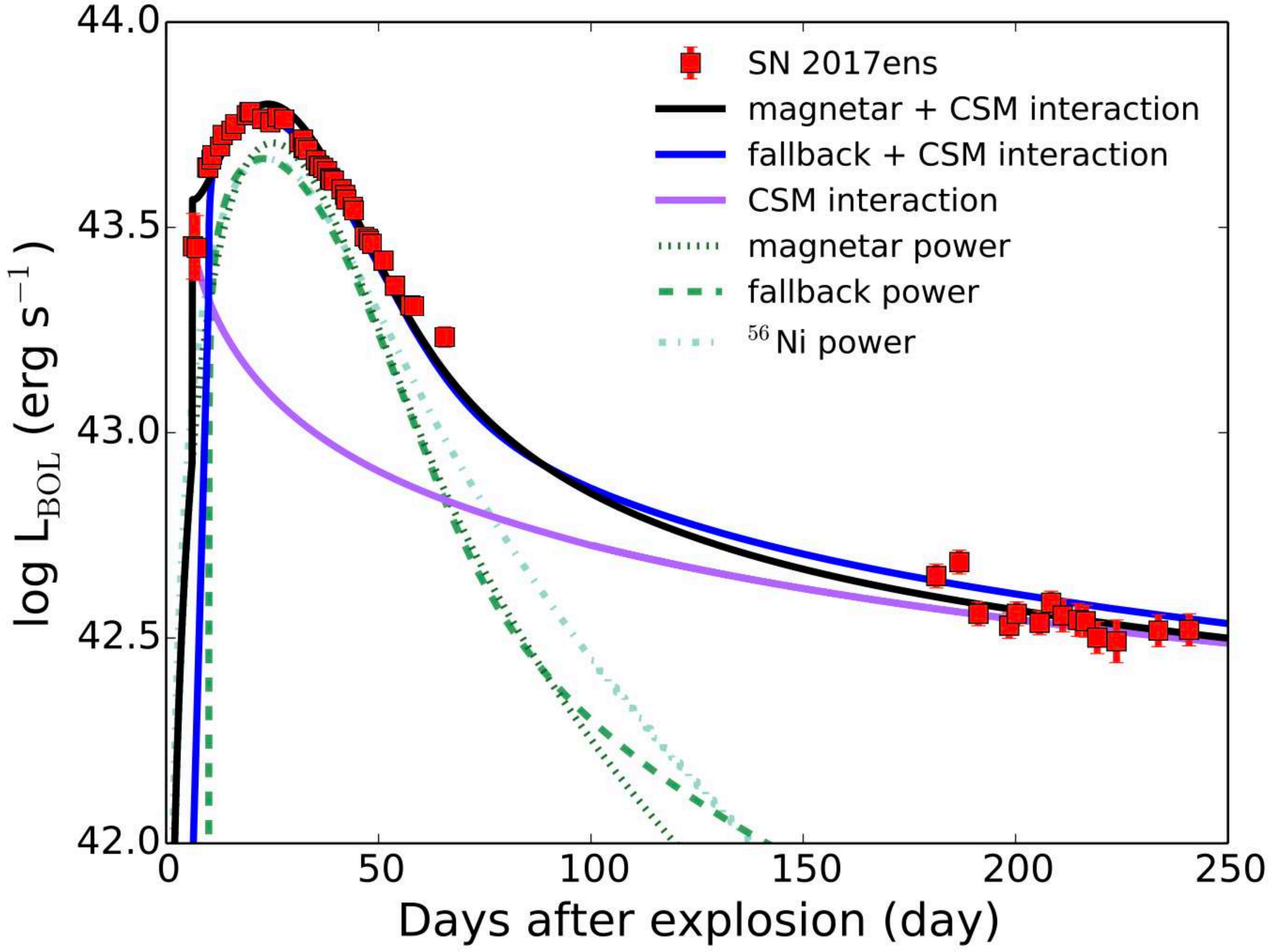
SN ejecta energized by radiation from the rapidly rotatin magnetized neutron star



SN ejecta

Newborn magnetar
(B~10^{15} G, P~ms)

credit: ESO



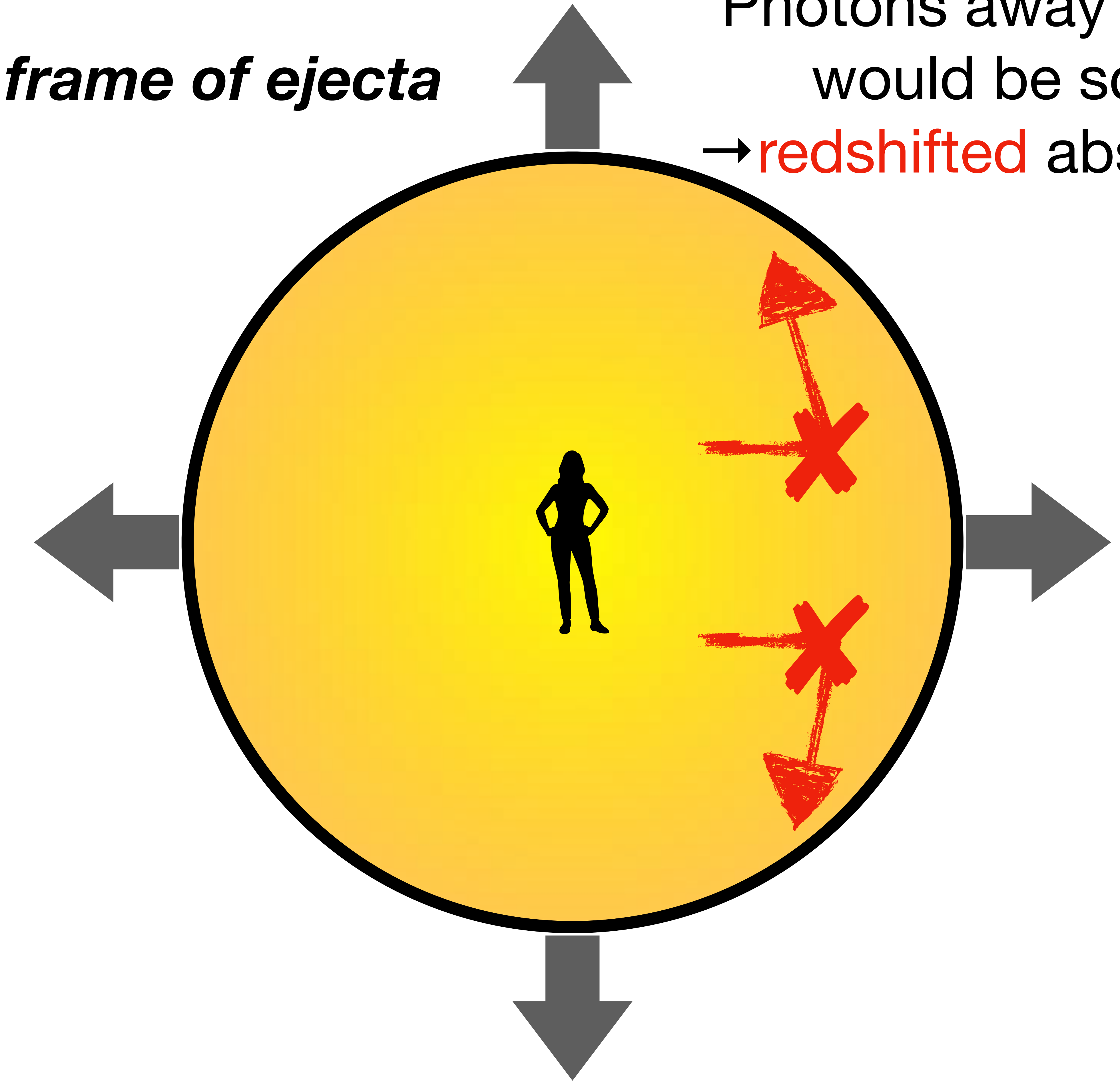
Chen+18
(Contact Janet in NCU for details)

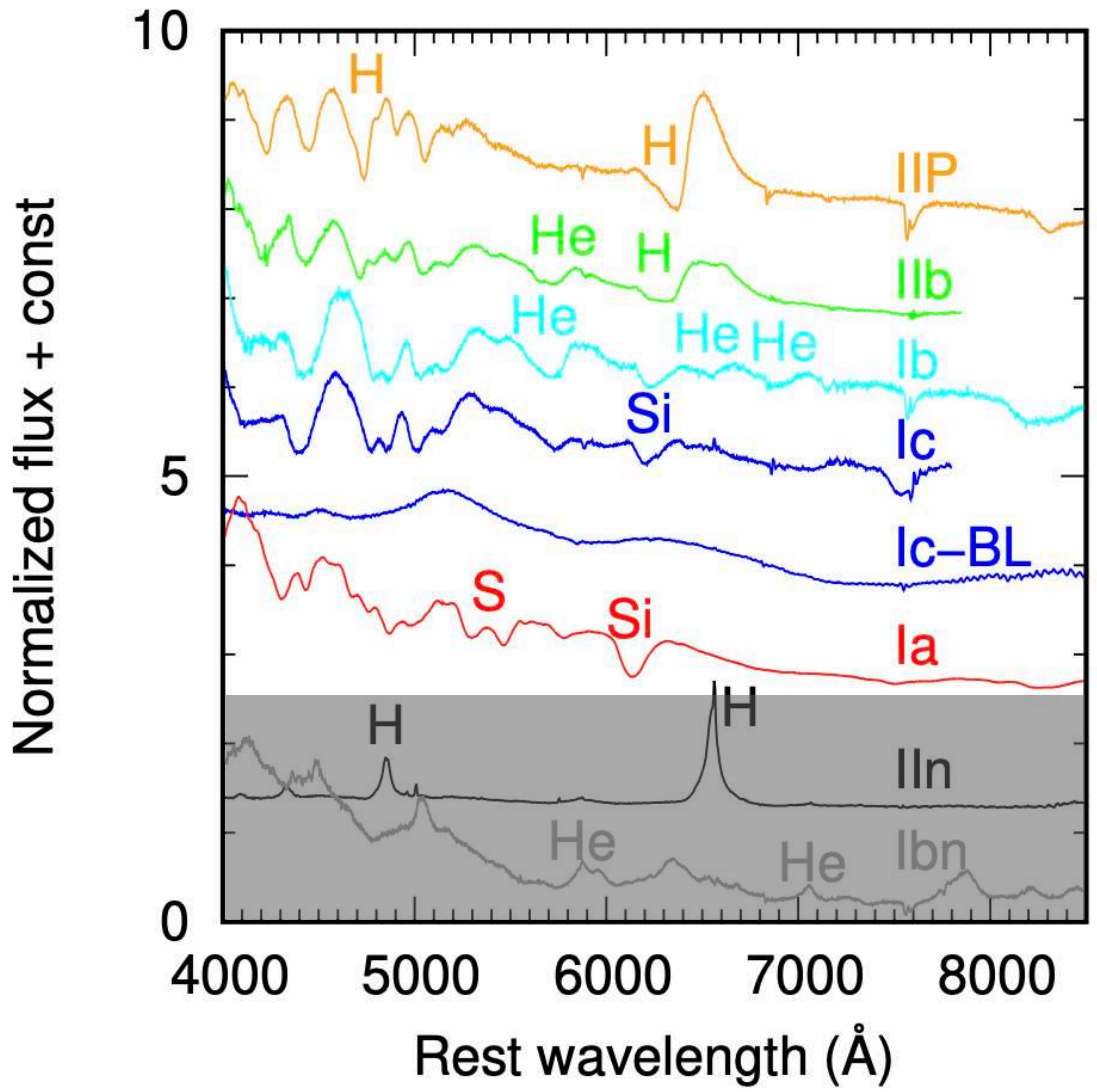
What we find: nature of the newborn object, explosion mechanism

Spectra — without CSM (P-Cygni profile)

Rest frame of ejecta

Photons away towards us
would be scattered
→redshifted absorption line



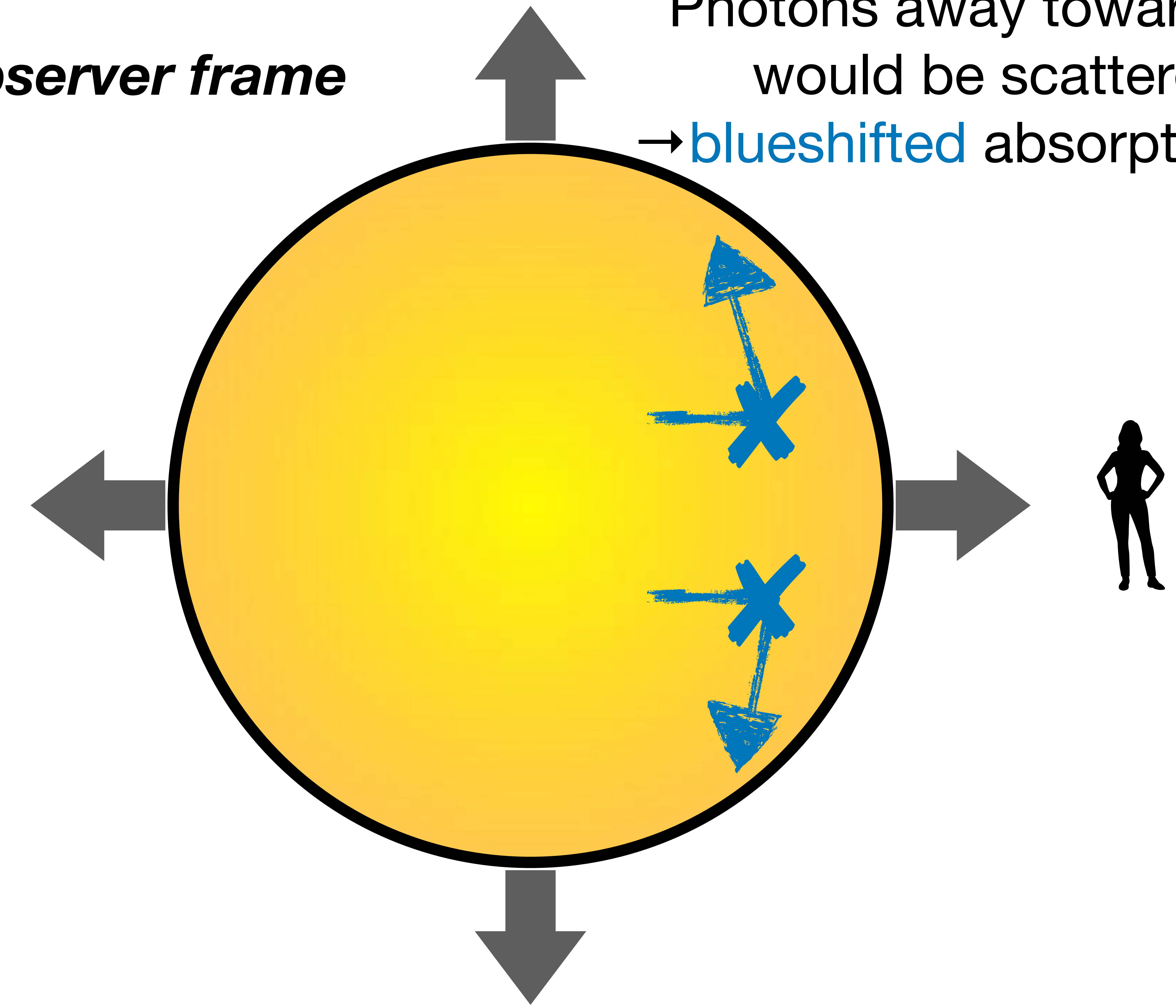


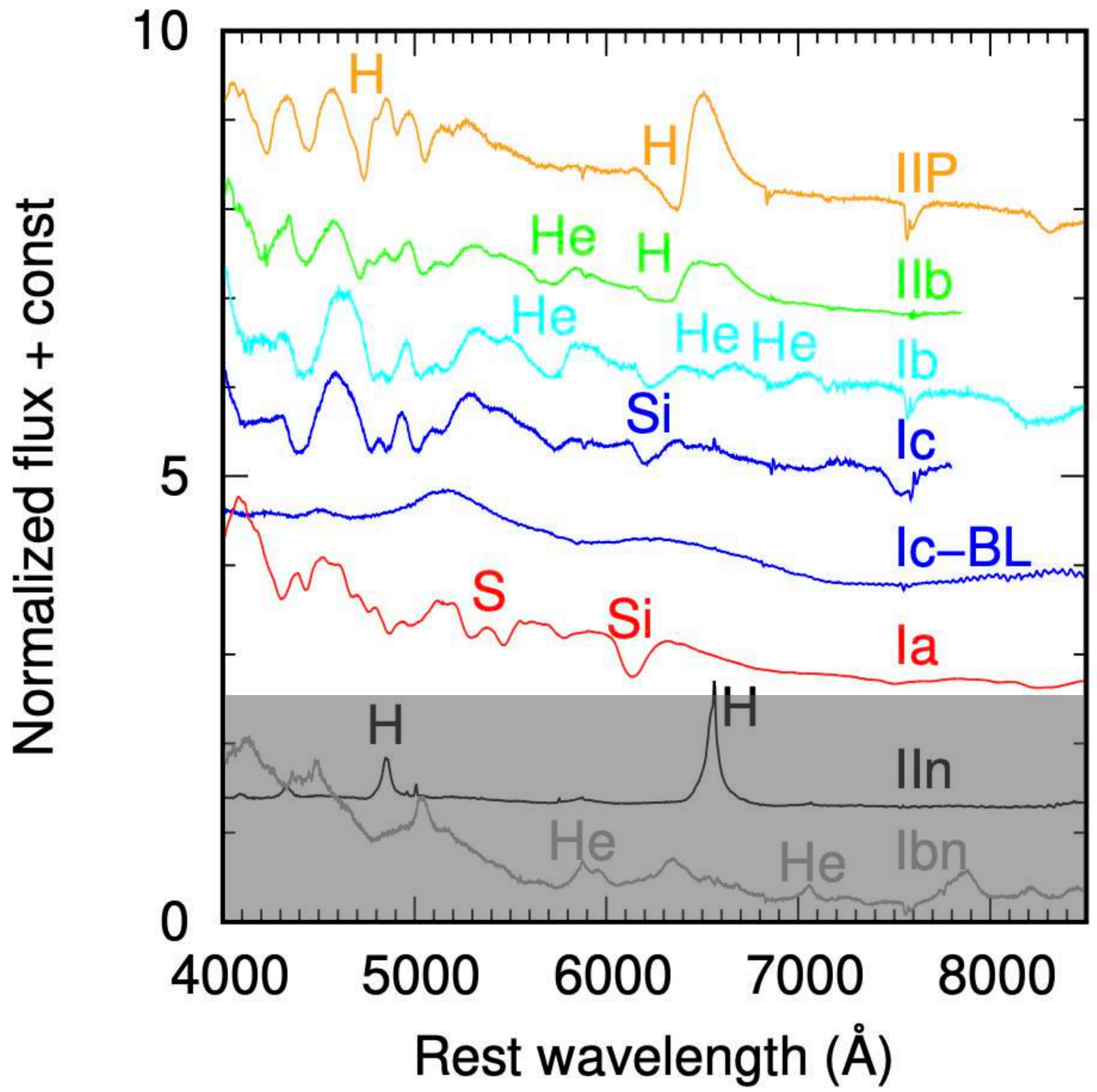
Maeda22

Spectra — without CSM (P-Cygni profile)

Observer frame

Photons away towards us
would be scattered
→blueshifted absorption line





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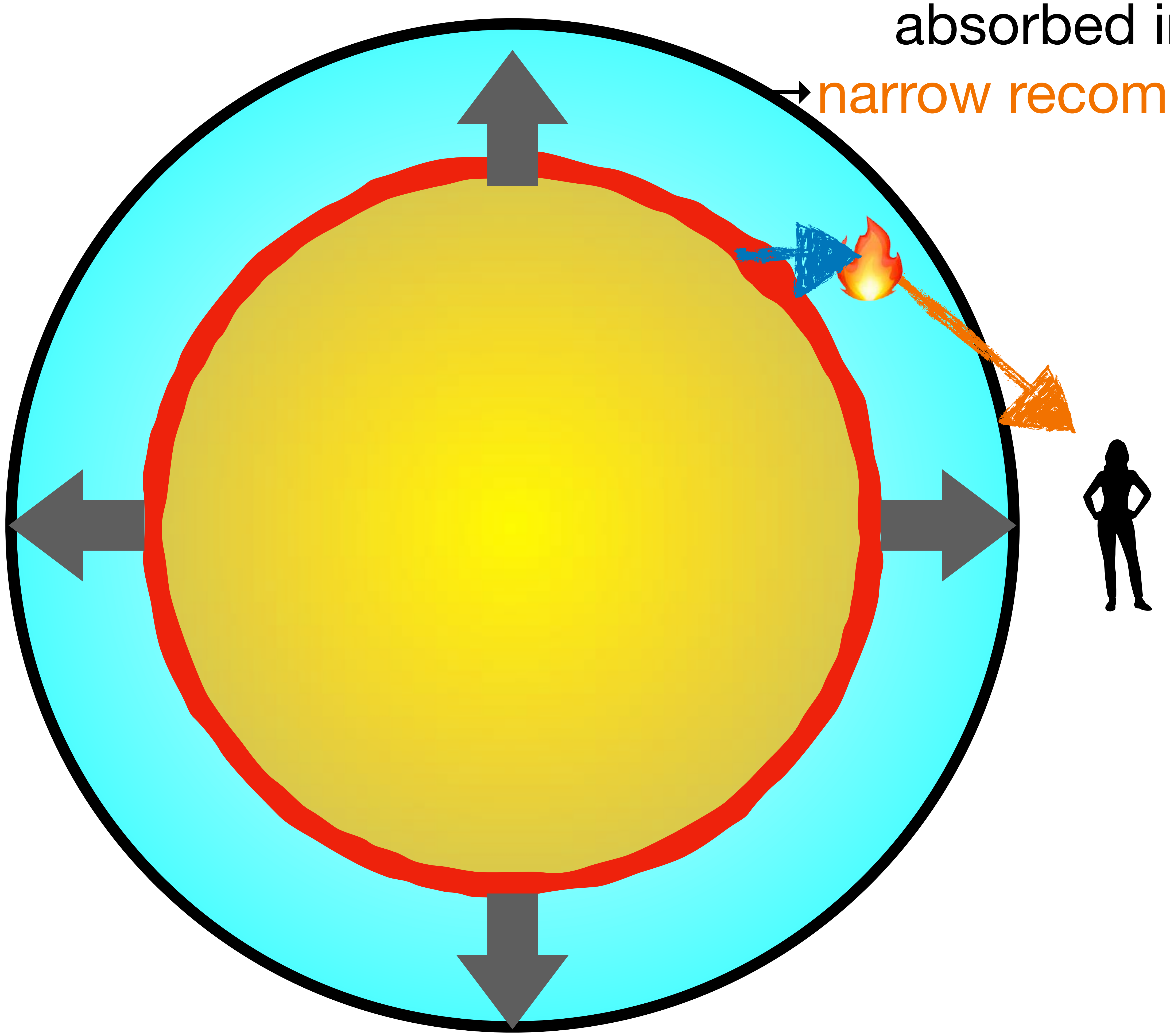
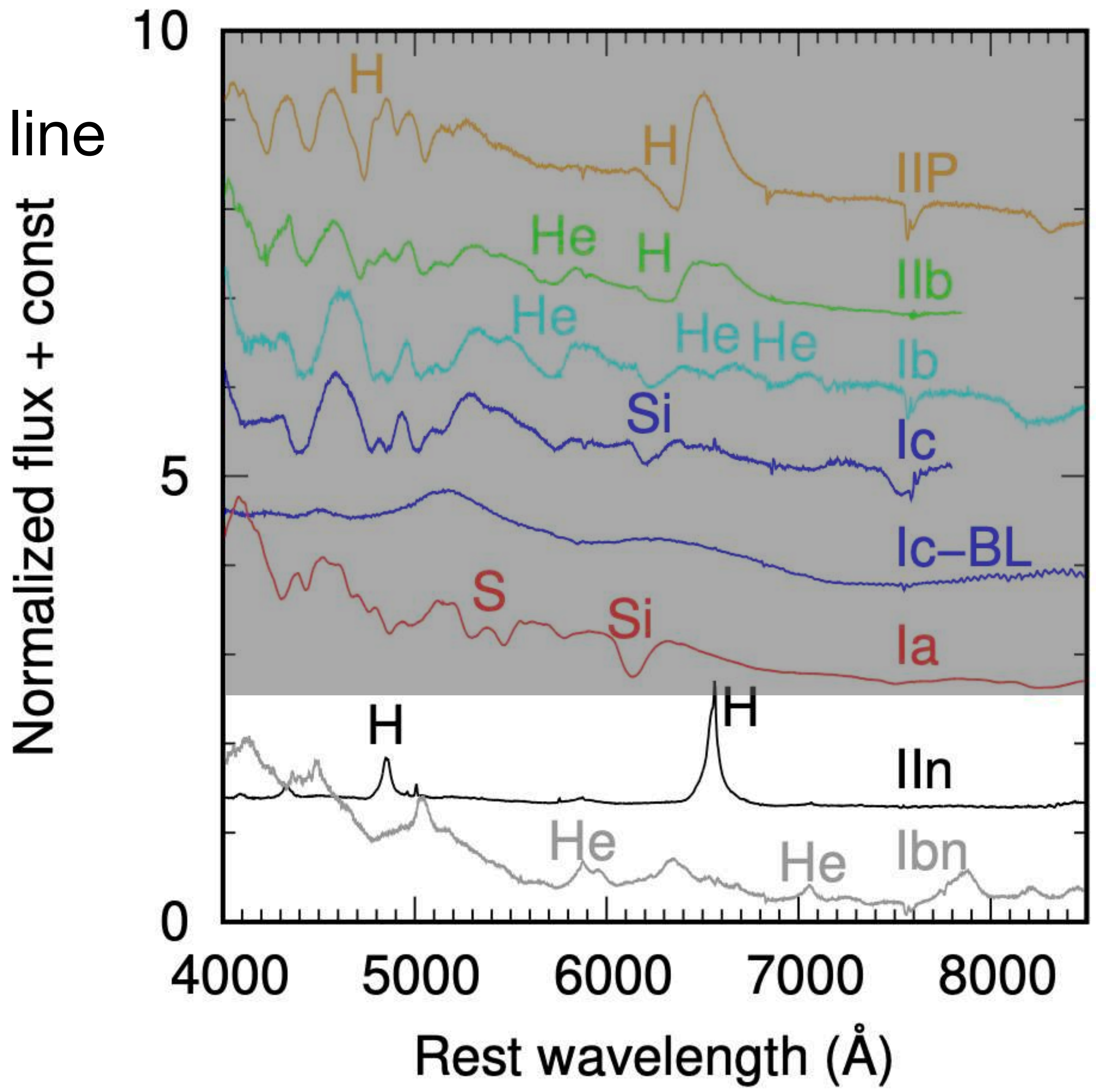
Spectra — with CSM

Photons would be absorbed in CSM
→narrow recombination line





Maeda22

Numerical modeling has demonstrated that SN-CSM interaction can yield narrow emission lines (e.g,. Dessart+15)

Advanced: Numerical codes for SN modelings

- Light curve modeling: STELLA (Blinnikov+98)

$$\frac{\partial r}{\partial t} = u \,, \tag{2.7}$$

$$\frac{\partial u}{\partial t} = -4\pi r^2 \frac{\partial (p+q)}{\partial m} - \frac{Gm}{r^2} + a_r + a_{\mathrm{mix}} \,, \tag{2.8}$$

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \,. \tag{2.9}$$

$$\mathscr{J}_\nu = \tfrac{1}{2}\int_{-1}^{1} d\mu f_\nu \,,$$

$$\mathscr{H}_\nu = \tfrac{1}{2}\int_{-1}^{1} d\mu\mu f_\nu \,,$$

$$\mathscr{K}_\nu = \tfrac{1}{2}\int_{-1}^{1} d\mu\mu^2 f_\nu \,, \tag{2.3}$$

then after multiplication by $(2h\nu^3/c^2)$ one obtains the usual angular moments of intensity I_ν,

$$J_\nu = \tfrac{1}{2}\int_{-1}^{1} d\mu I_\nu \,,$$

$$H_\nu = \tfrac{1}{2}\int_{-1}^{1} d\mu\mu I_\nu \,,$$

$$K_\nu = \tfrac{1}{2}\int_{-1}^{1} d\mu\mu^2 I_\nu \,. \tag{2.4}$$

- Spectral modeling: CMFGEN (Hiller+98)

$$\frac{1}{r^2}\frac{\partial(r^2 H)}{\partial r} - \left(\frac{\nu v}{rc}\right)\left[\frac{\partial(J-K)}{\partial \nu} + \left(\frac{d\ln v}{d\ln r}\right)\frac{\partial K}{\partial \nu}\right] = \eta - \chi J \quad (2)$$

and

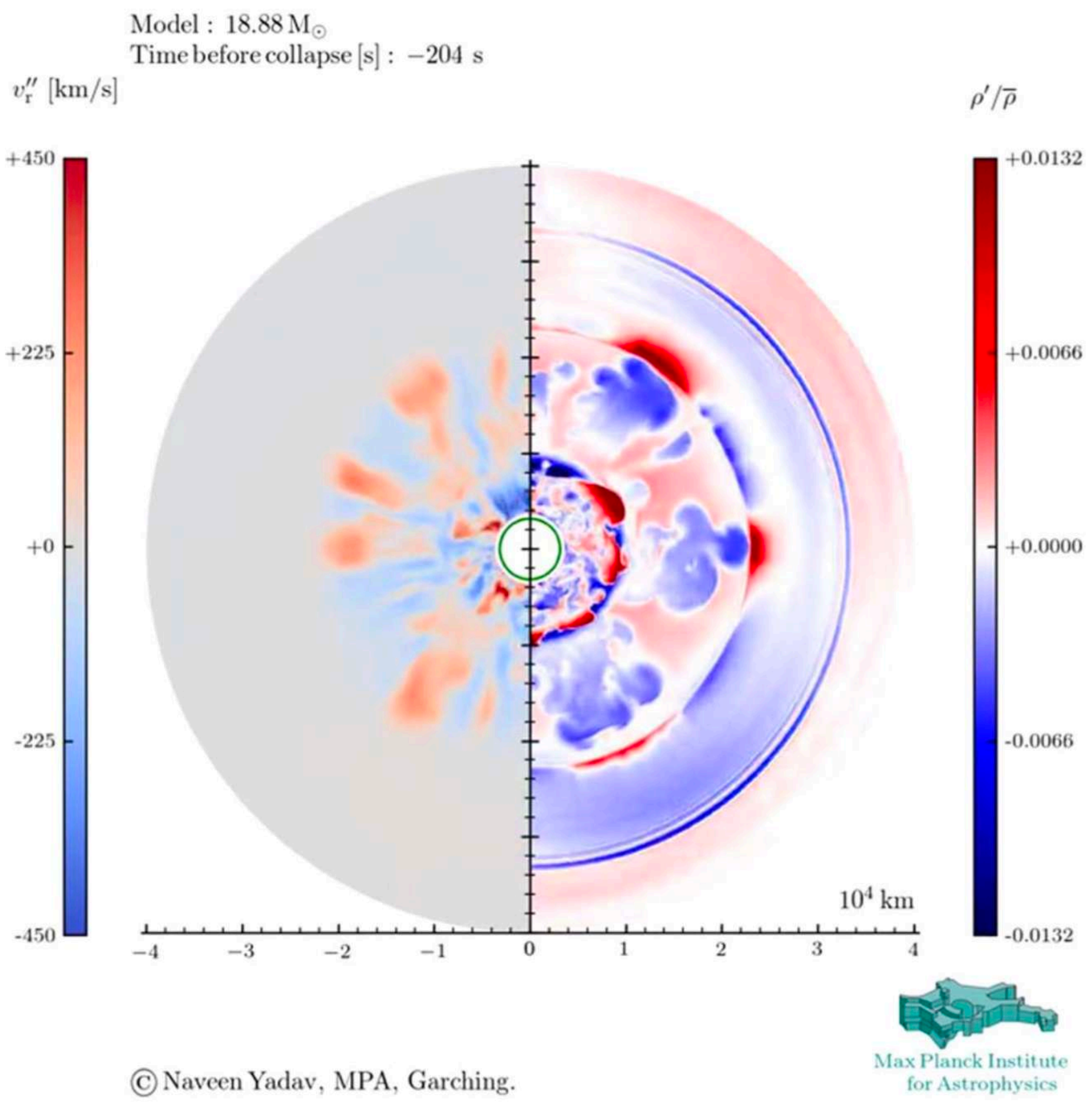
$$\frac{\partial K}{\partial r} + \frac{(3K-J)}{r} - \left(\frac{\nu v}{rc}\right)\left[\frac{\partial(H-N)}{\partial \nu} + \left(\frac{d\ln v}{d\ln r}\right)\frac{\partial N}{\partial \nu}\right] = -\chi H \,, \quad (3)$$

where

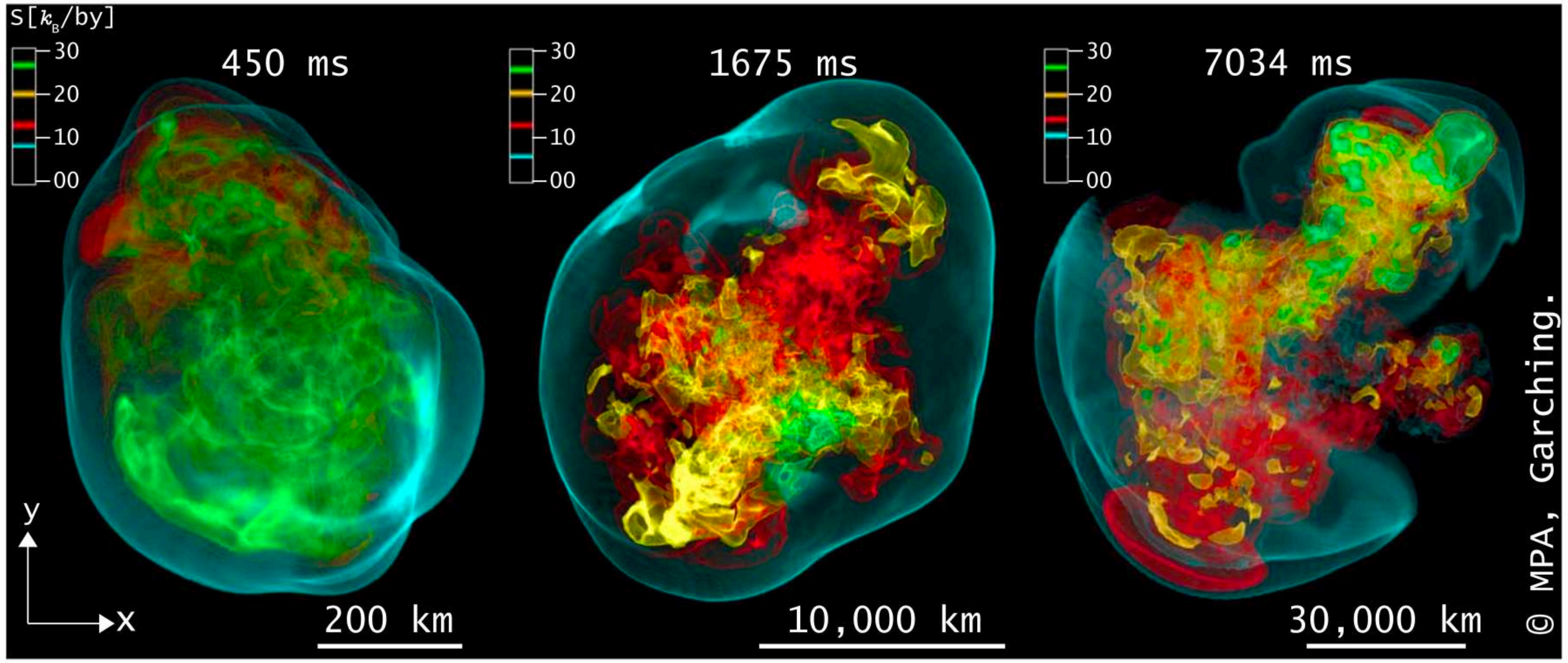
$$[J, H, K, N] = \tfrac{1}{2}\int_{-1}^{1} I(\mu, r)[1, \mu, \mu^2, \mu^3] d\mu \quad (4)$$

Contents

Successful CCSN? Invoking progenitor disturbance



Yadav+20

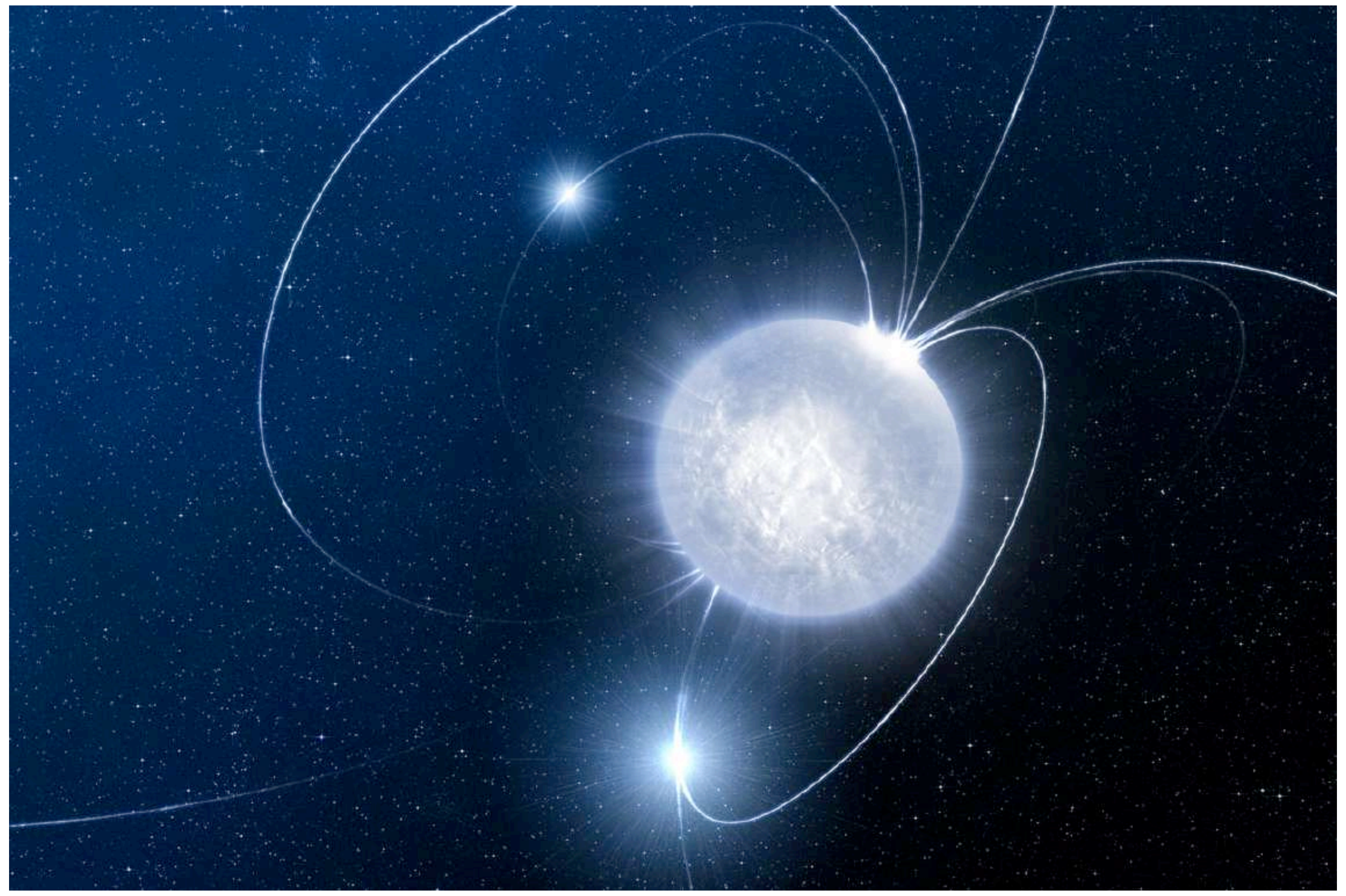


Bollig+21

From simulations they found that progenitor asphericities might help the explosion. Further investigation will be expected to discover physical explanations

Compact objects

Successful explosion
→neutron star

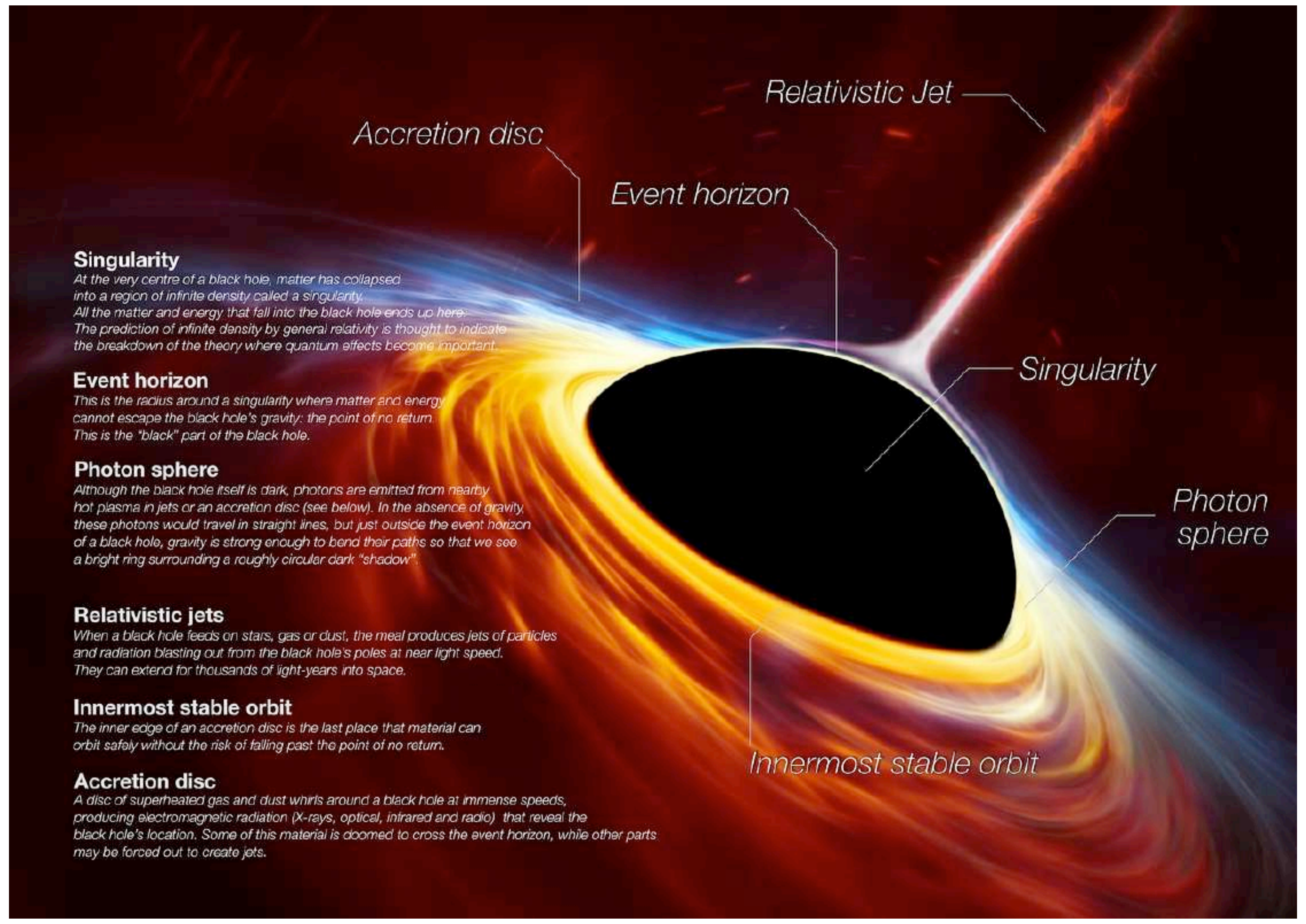


credit: ESO

Pulsar, Magnetar, and probably fast radio bursts…etc

$$M \sim M_\odot$$

Unsuccessful explosion
→ fallback of material
→ black hole (see Ishika's talk)



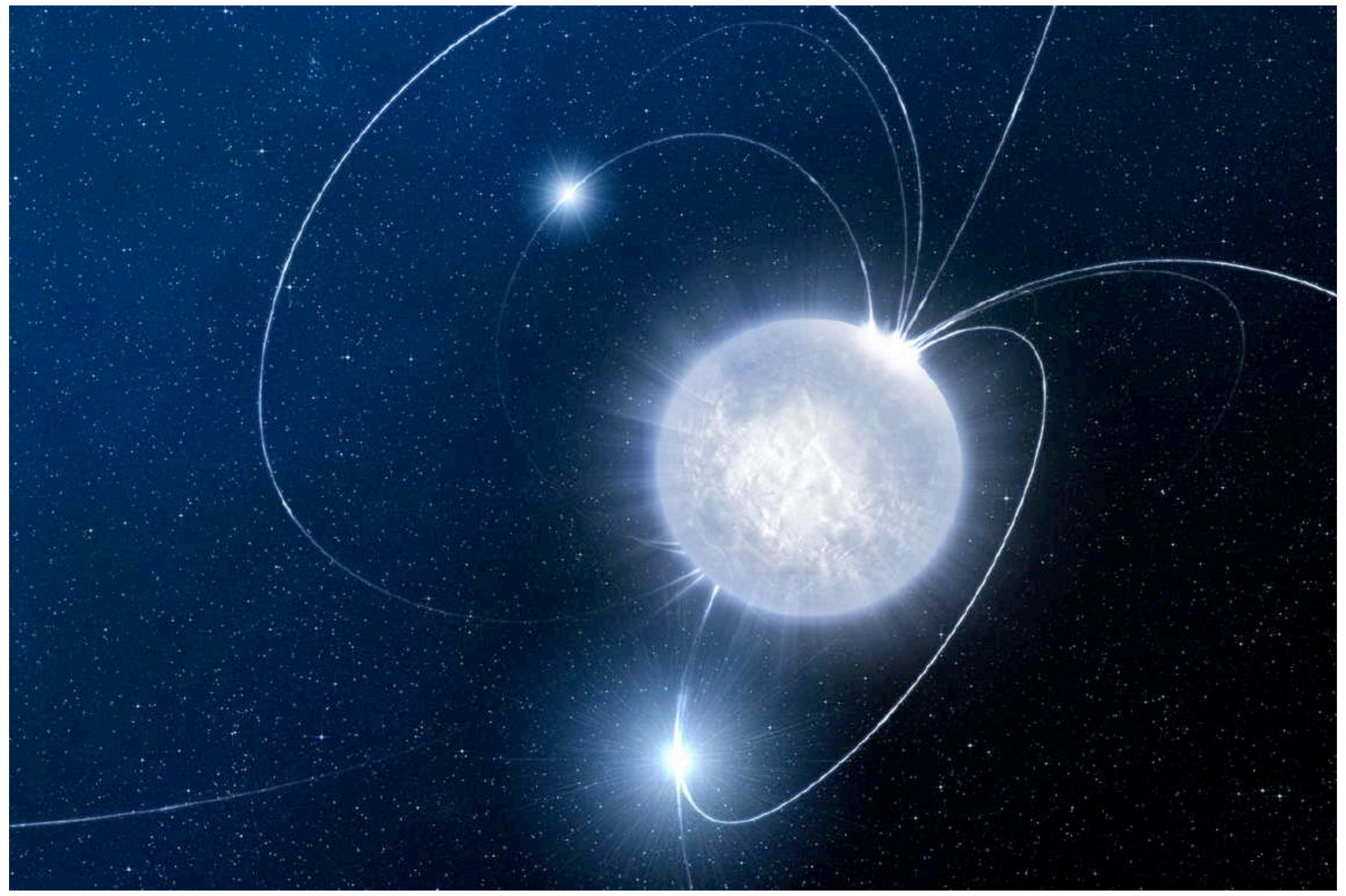
credit: ESO

Active galactic nuclei, Supermassive black holes in galaxies…etc

$$M \gtrsim 10 M_\odot$$

Compact objects

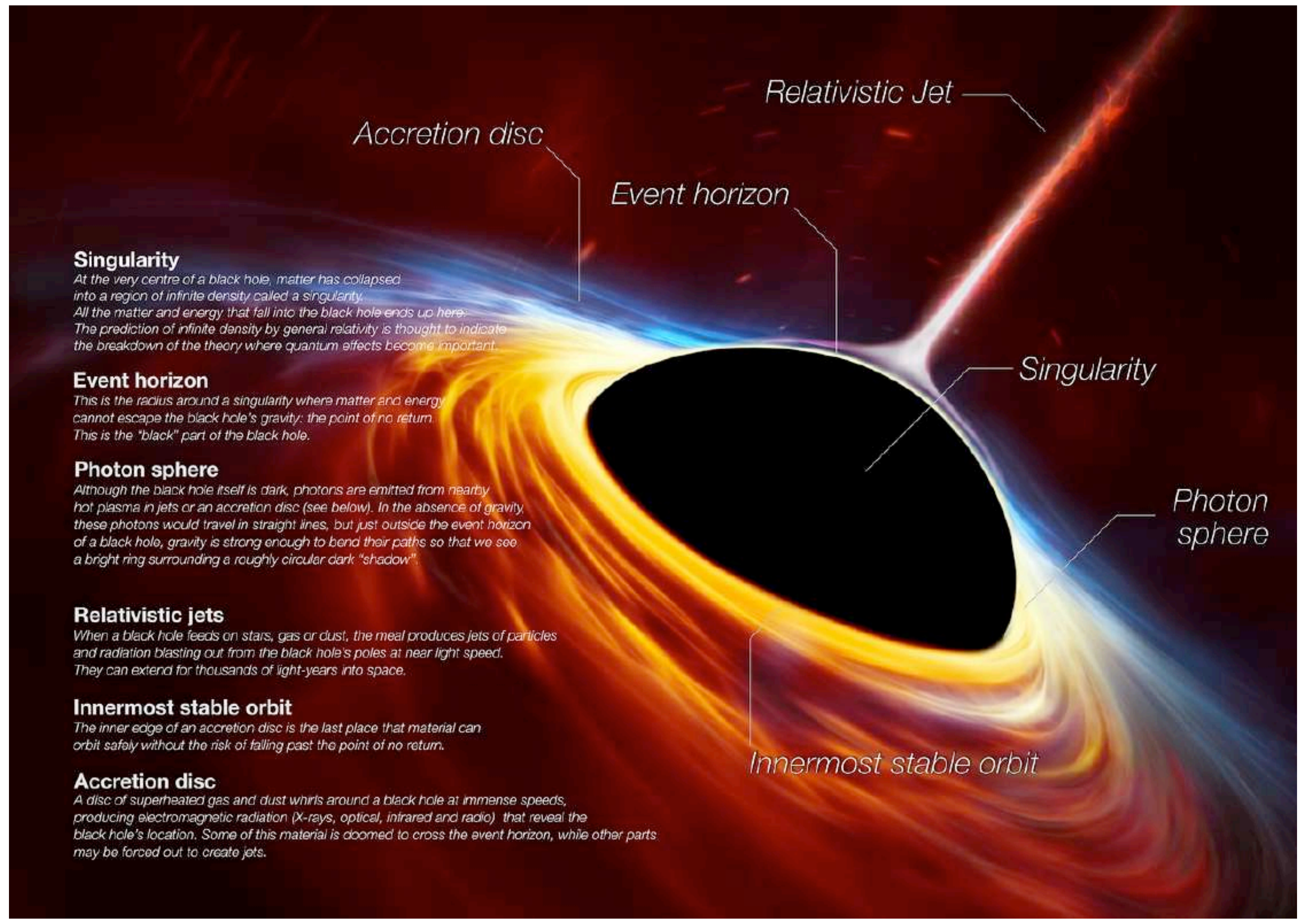
Successful explosion
→neutron star



credit: ESO

?

Unsuccessful explosion
→ fallback of material
→ black hole (see Ishika's talk)



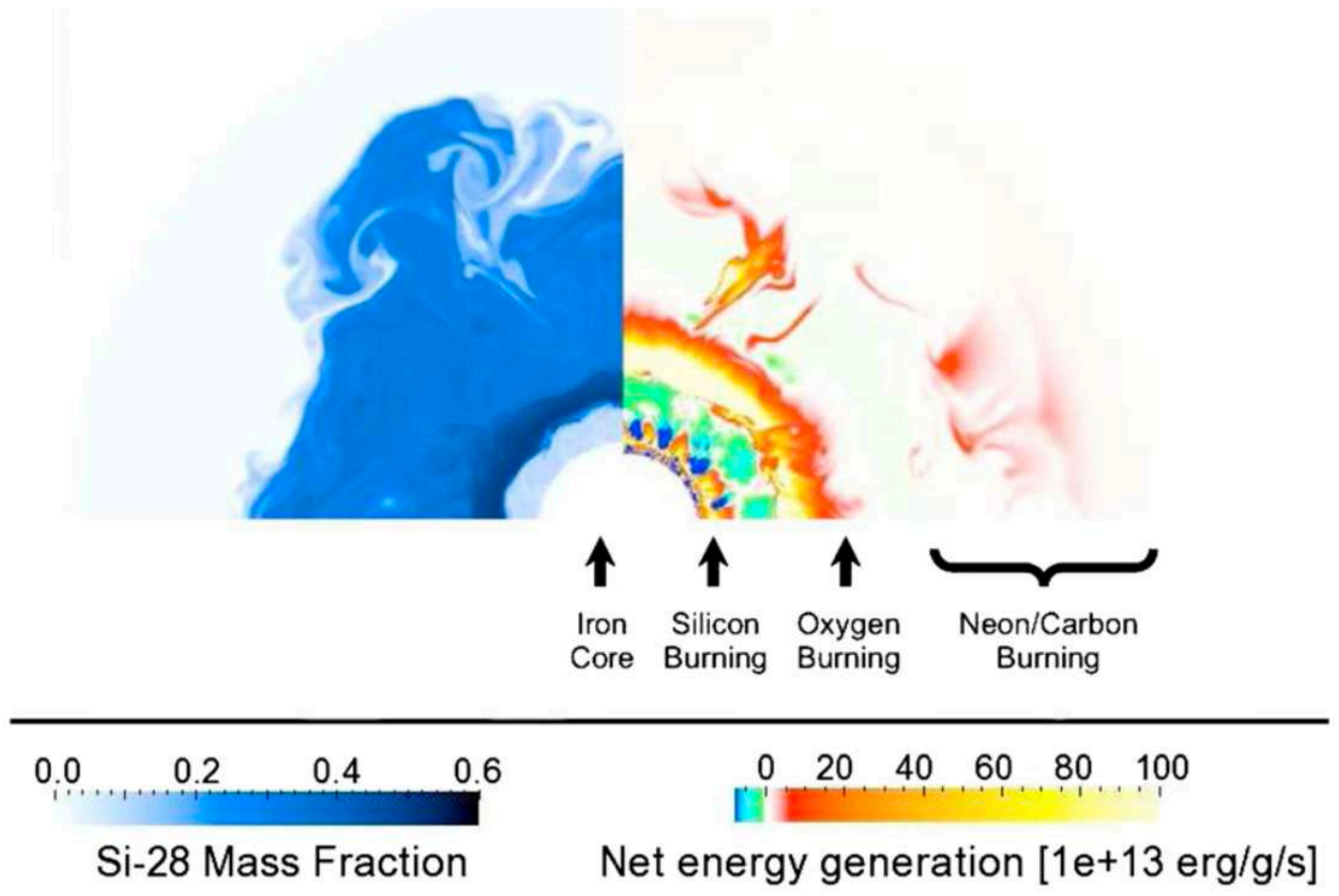
credit: ESO

Black holes with $2\,M_\odot \lesssim M \lesssim 5M_\odot$ have not so much been discovered. Why? Potential hints for explosion mechanism…etc

Origin of CSM

SN observations indicate the (universal) presence of massive CSM, implying mass-loss activity stronger than stellar wind, but how?

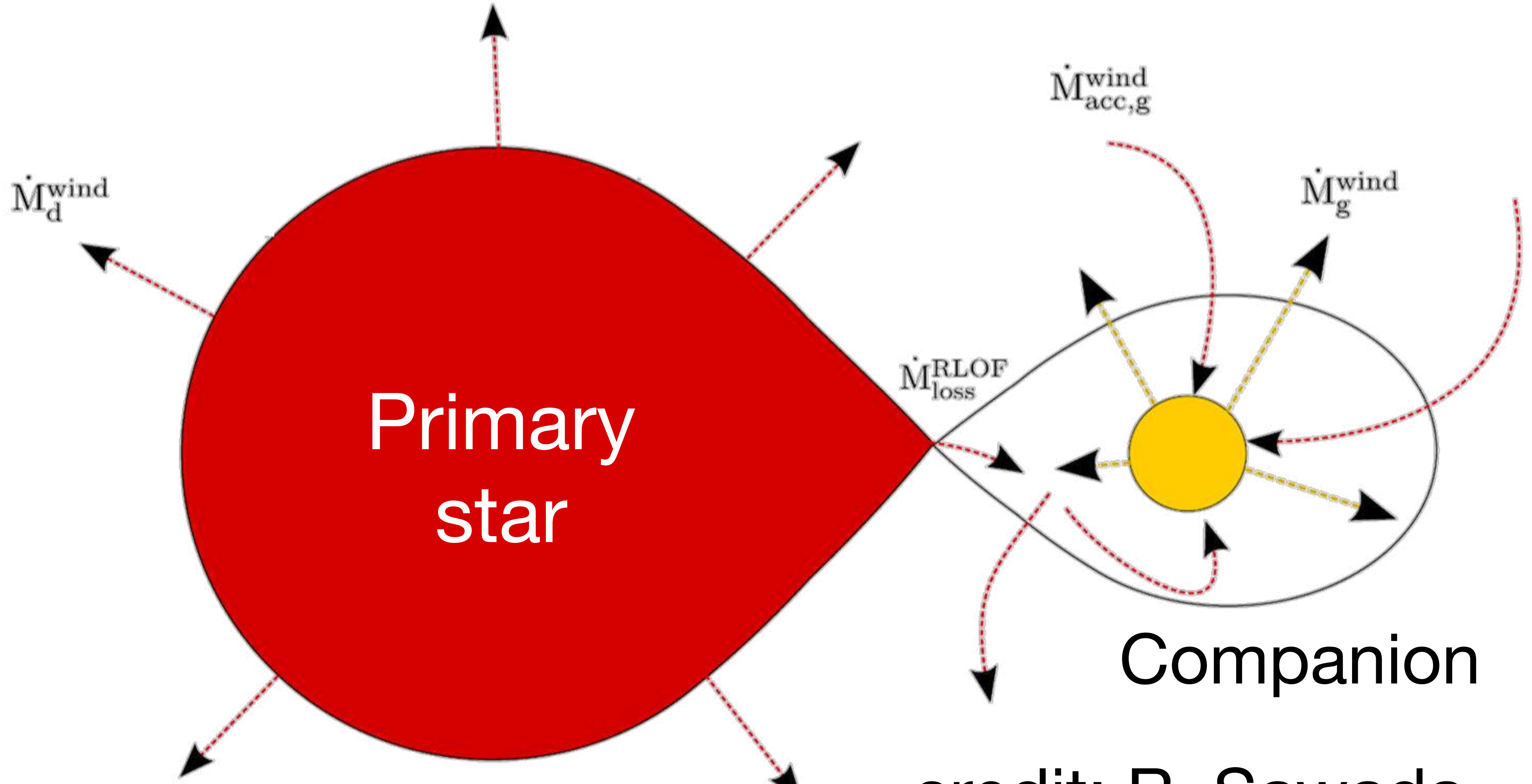
- Single star's activity



credit:Arnett+11

e.g., Arnett+11, Quataert+12

- Binary interaction



credit: R. Sawada

e.g., TM+24, Ercolino+24

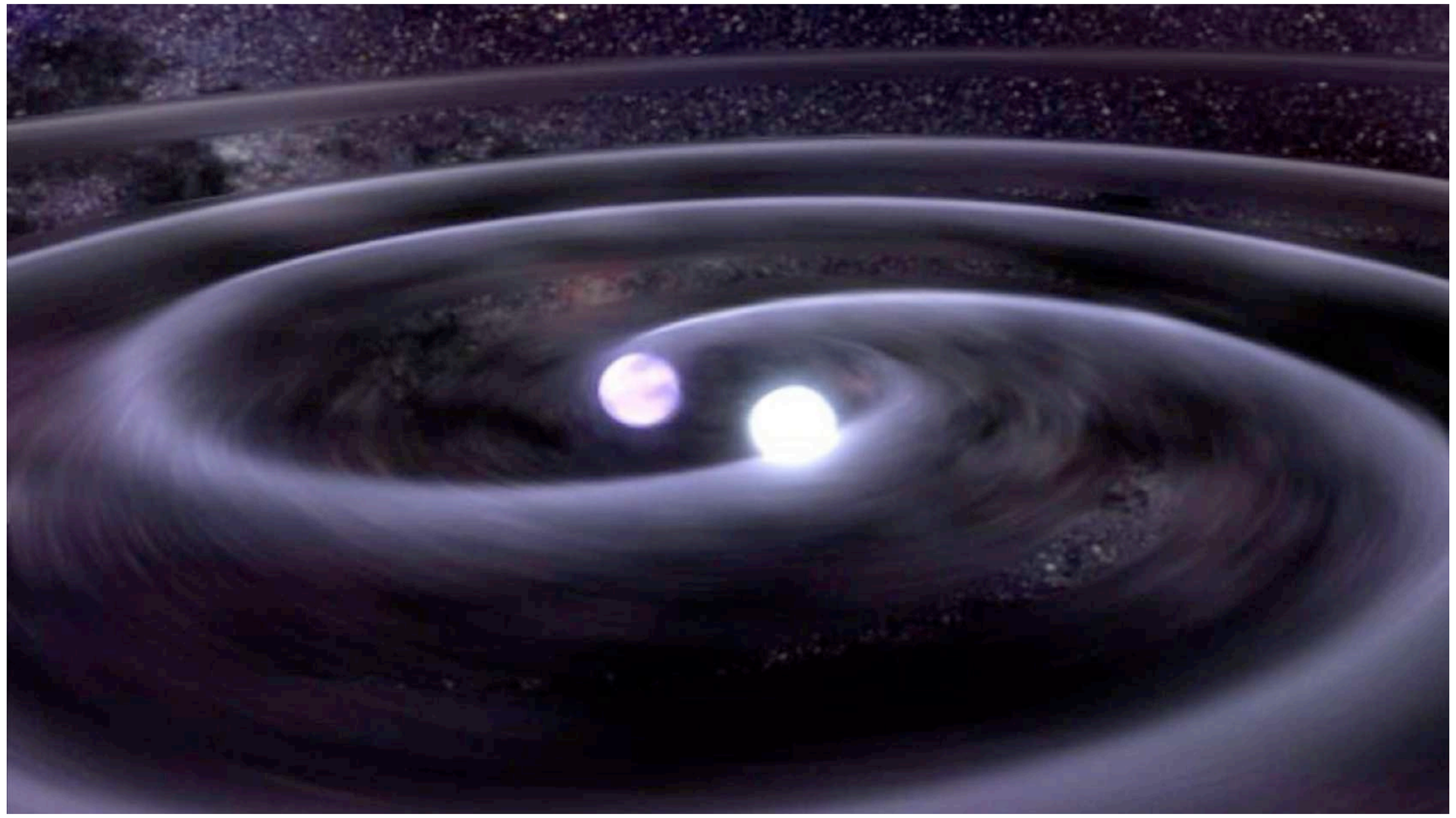
Who explodes as Type Ia SN?

In a binary, white dwarf can achieve weighing up to Chandrasekhar mass.
But, how?

Single degenerated
Main sequence/Red giant — White dwarf

Mass transfer from the donor?

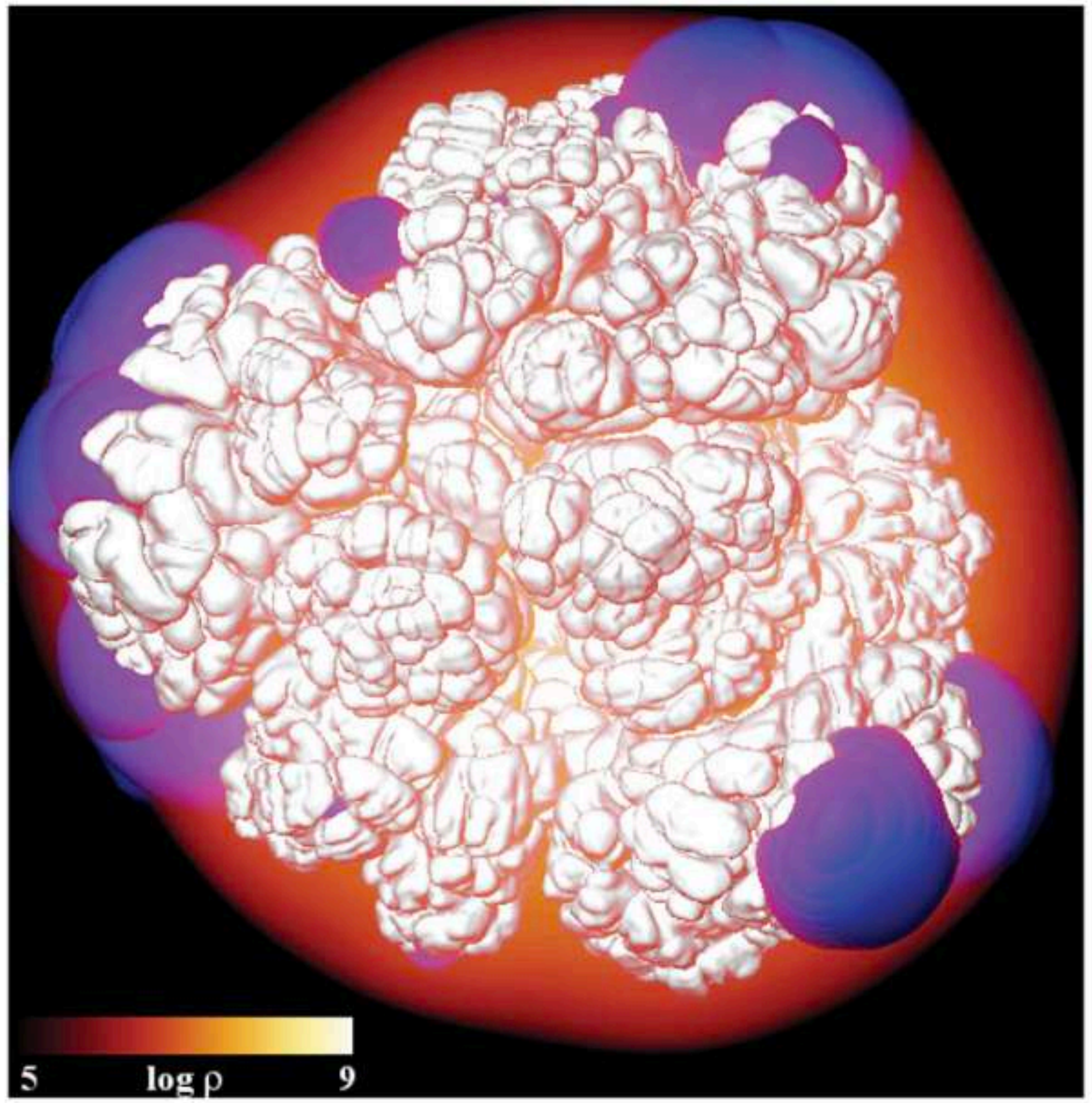
Double degenerated
White dwarf — White dwarf



Merger?

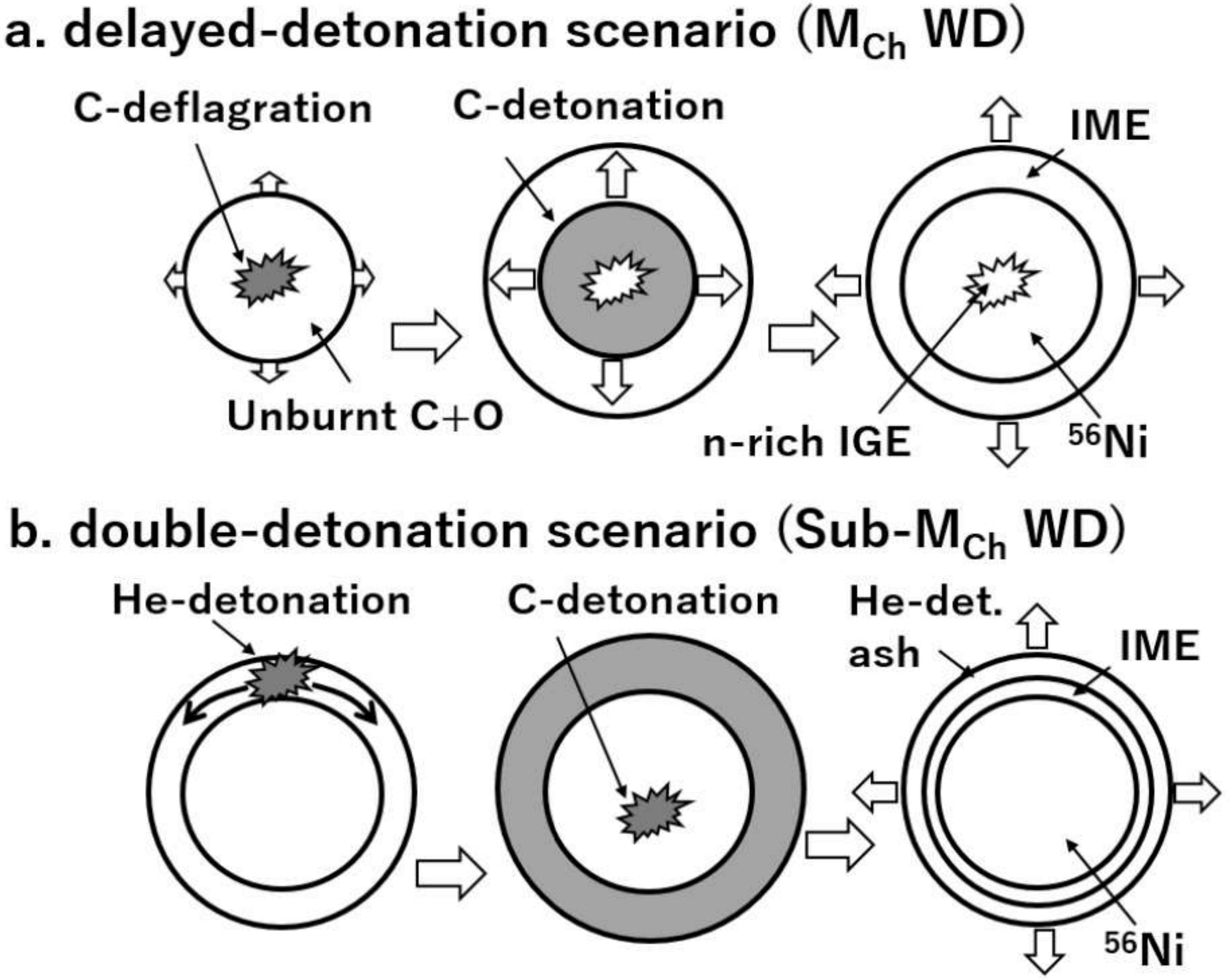
Credit: AAS nova

How does Type Ia SN explode?



(f) N100; t = 1.00 s

Seitenzahl+13

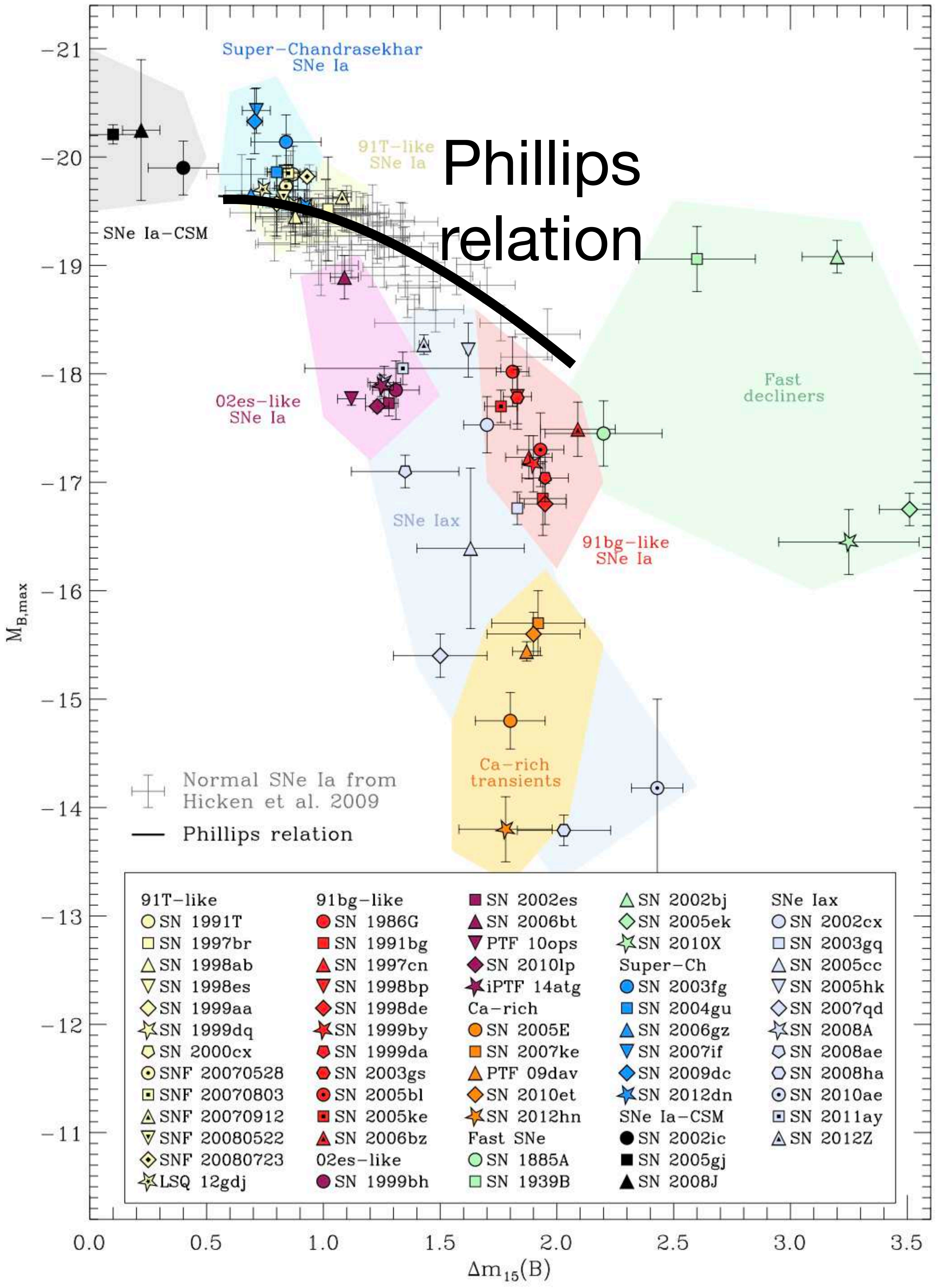


Maeda22

Homogeneity & Inhomogeneity of Type Ia SNe

- Since $M \sim M_{\rm ch}$ is expected, the explosive way of Type Ia SNe are often similar
  — Phillips relation between peak magnitude and decay rate
  — Used as a standard candle to measure distance in Cosmology (See Tomomi's talk)

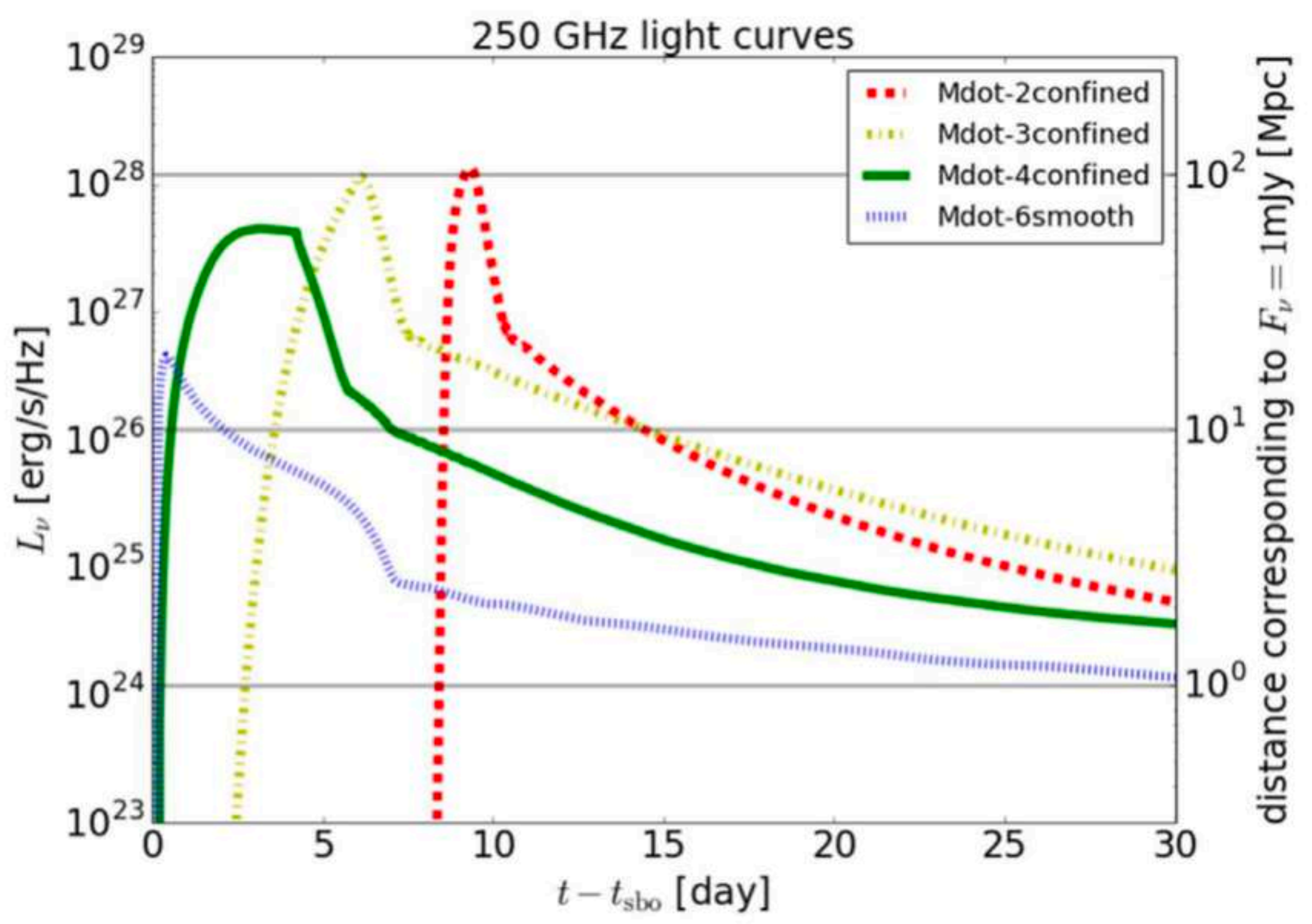
- That said, there are peculiar Type Ia SNe reported, which deviates from Phillips relationship.
  — Different progenitor?
  — Different explosion mechanism?



Taubenberger17

Multi-wavelength astronomy in transients

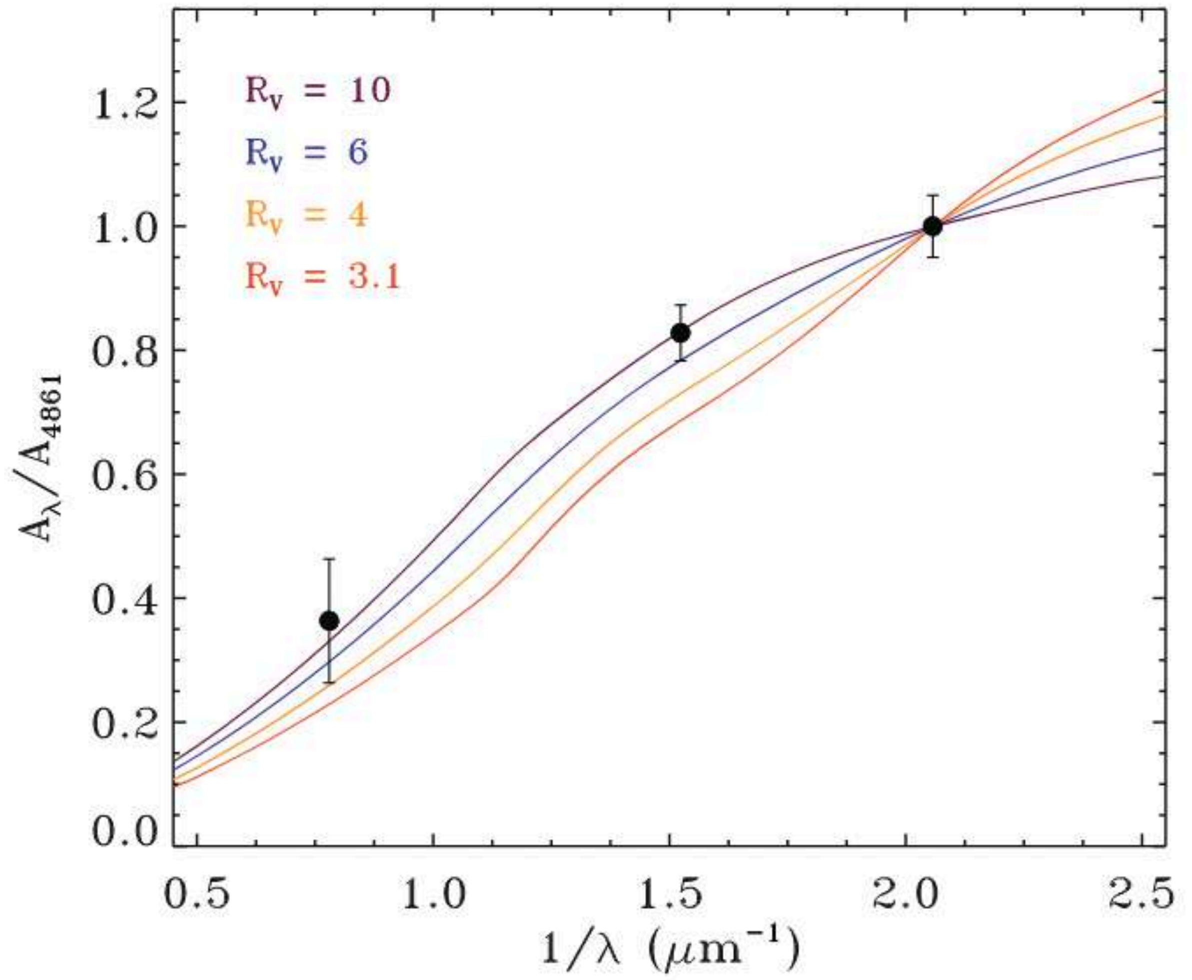
Radio



TM+19

mass-loss activity

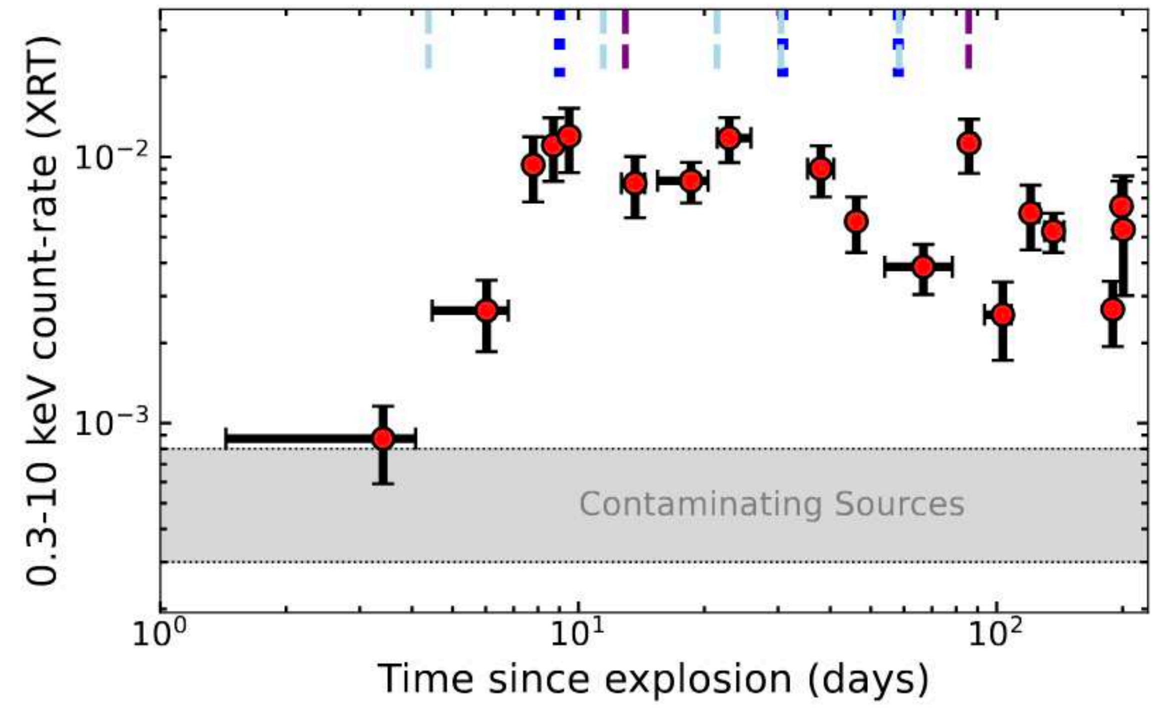
Infrared



Smith+20

Dust property, distribution, yield

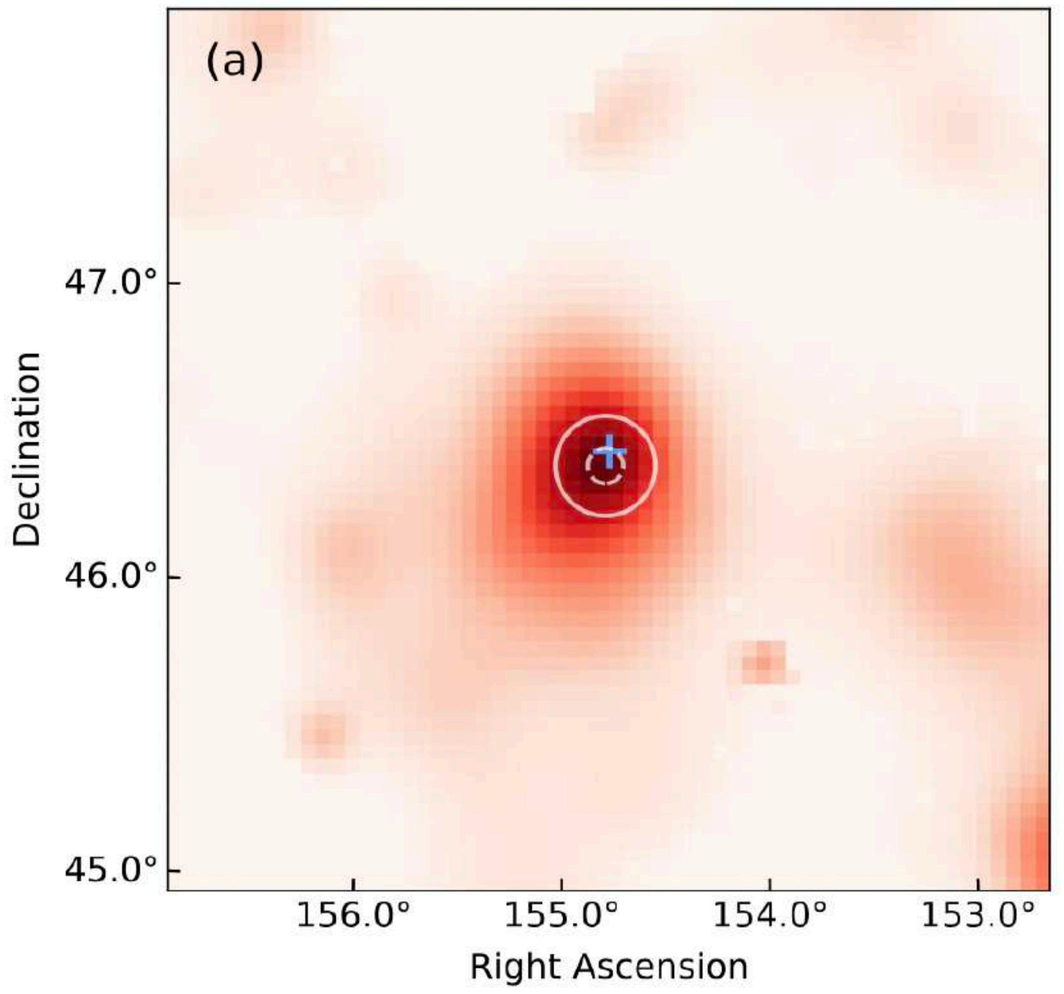
X-ray



Nayana+25

mass-loss activity & ejecta

Gamma-ray



Li+24

Particle acceleration

Multi-messenger astronomy in transients

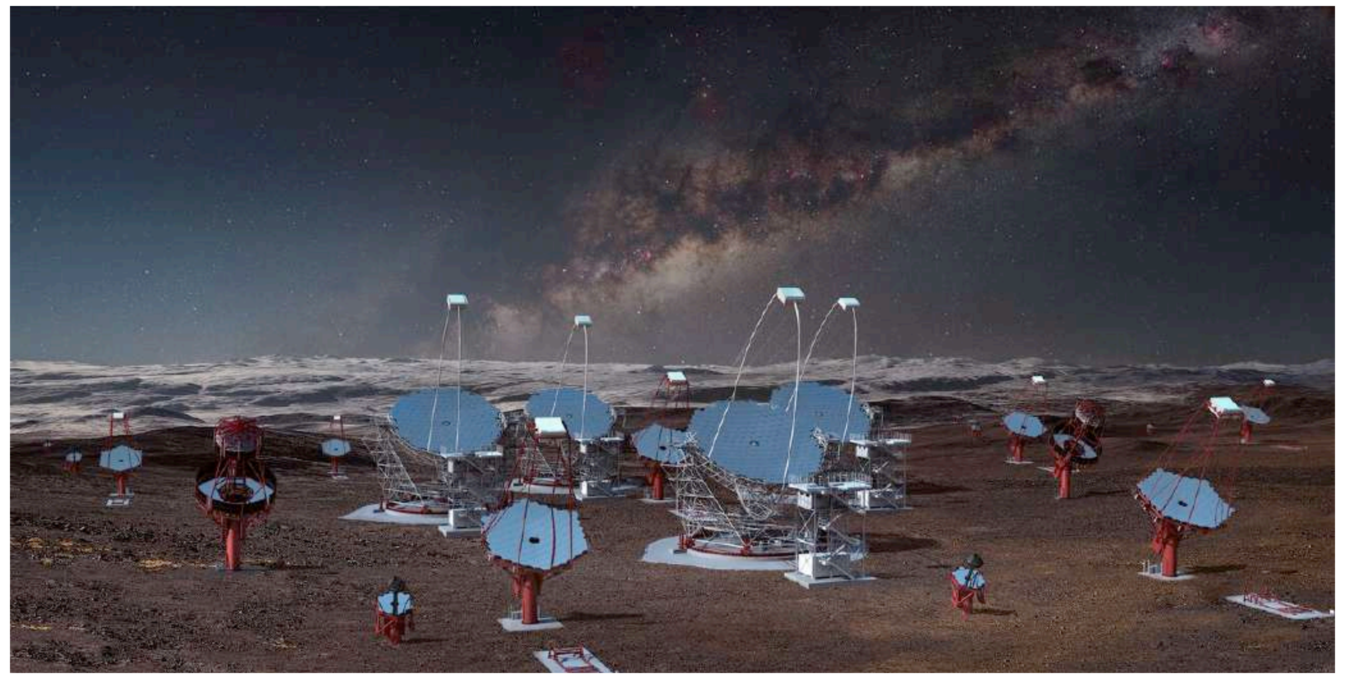
Electromagnetic wave

Cosmic-ray

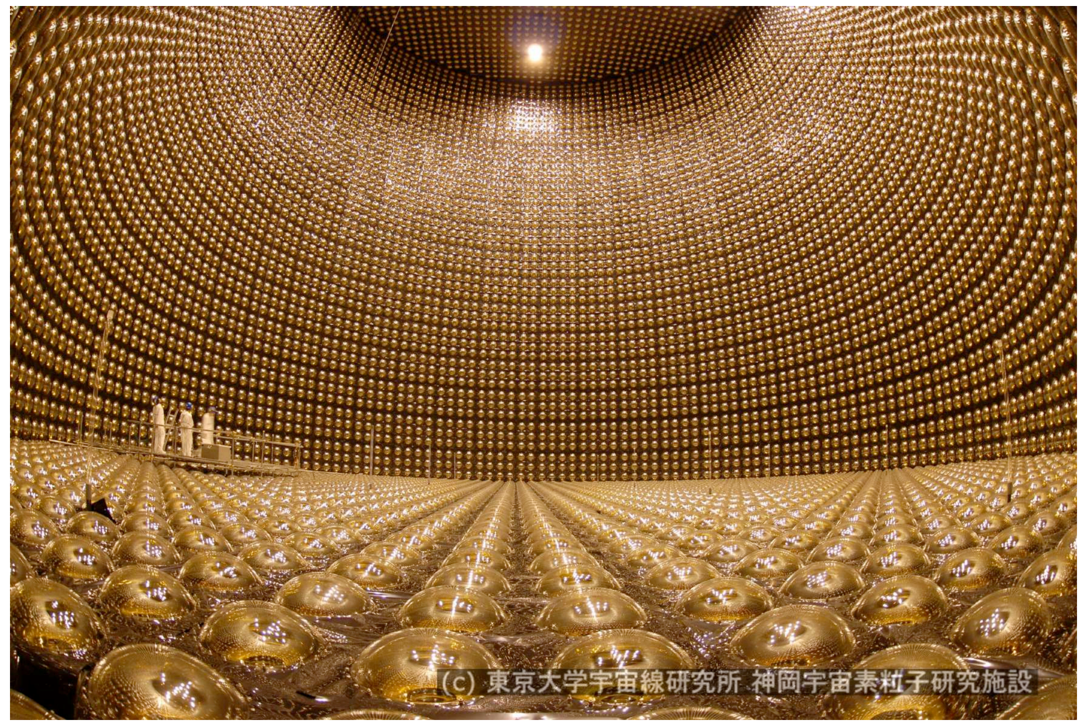
Neutrino

Gravitational wave

ZTF/IPAC



CTA/ESO



ICRR/U.Tokyo

LVG collaboration

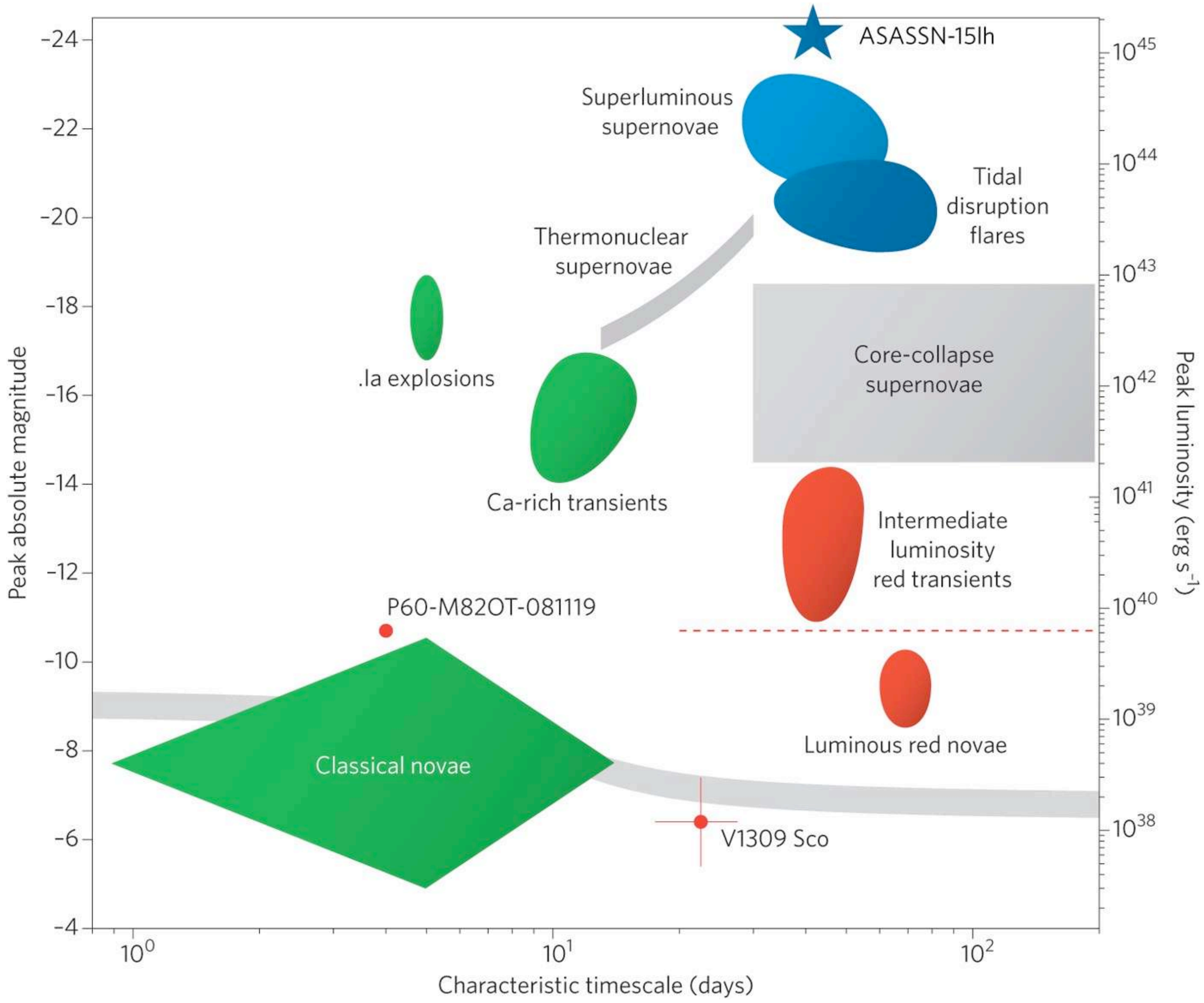
Astrophysics

Particle acceleration physics

Weak-interaction physics

General relativity

Do we only have supernovae? No.



Cenko17

Transient Zoo

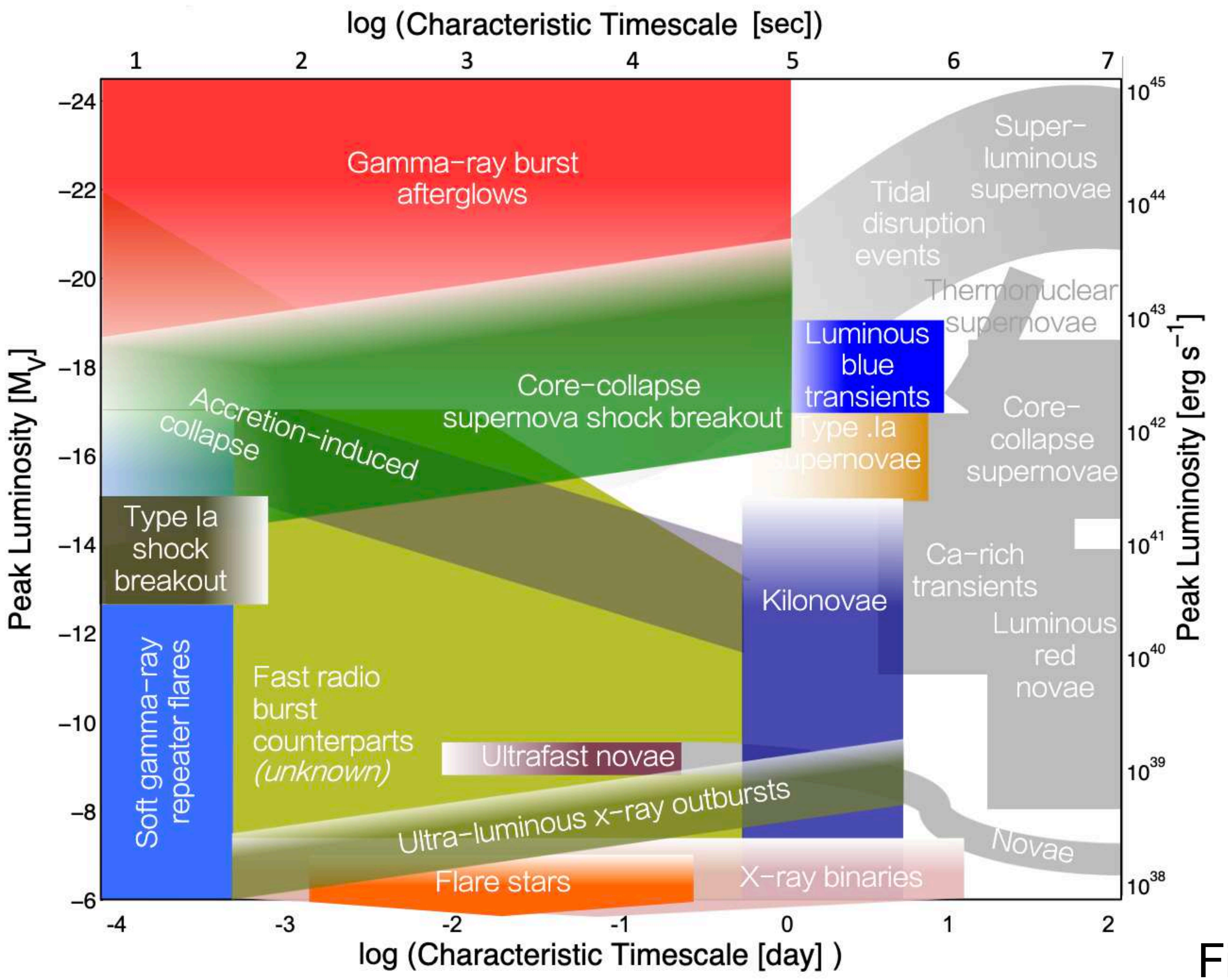


Figure 1 in Buckley+21
(arXiv:2011.02892)

Summary of supernovae

- A supernova is an explosion of a star
- Stars are basically stable because of the balance between pressure and gravity
- Stars are evolvoing slowly in a timescale of nuclear burning
- However, in the specific cases stars can be unstable ($\gamma = 4/3$ is important)
    - White dwarf: Exceeding Chandrasekhar mass → C+O combustion → Thermonuclear supernovae (Type Ia)
    - Fe core in massive stars: Fe photodissociation + electron capture → lost of pressure → core collapse SNe
- Interpreting observational data allows us to infer the nature of explosions and progenitor stars

Summary of my message

- ***Try to remember rough values of physical constants.***
- ***Put an emphasis not only on performing your simulations, but also on interpreting results of your simulations.***
    - Simulation is a "numerical experiment". Any experiments need interpretation.
    - I define interpretation as "giving an explanation based only on order estimate and simple algebra."
    - Simulation is one of the methods to understand astrophysical phenomena.
        - demonstration of the situation beyond analytical description
        - detailed reproduction of observational signatures to identify the nature
    - Do not stop or give up understanding physics.