



國立清華大學  
NATIONAL TSING HUA UNIVERSITY

NCTS

MOST 科技部  
Ministry of Science and Technology

NARLabs 國家實驗研究院

國家高速網路與計算中心  
National Center for High-performance Computing



# Radiative process (neutrino) radiation transport

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NCTS-TCA Summer Student Program 2021





# Outline

- Introduction
- The Boltzmann equation for radiation transport
- Numerical methods for radiative transfer
- Application: Core-Collapse Supernova



# Introduction



# Introduction

Observation



Image credit: NASA

Simulation

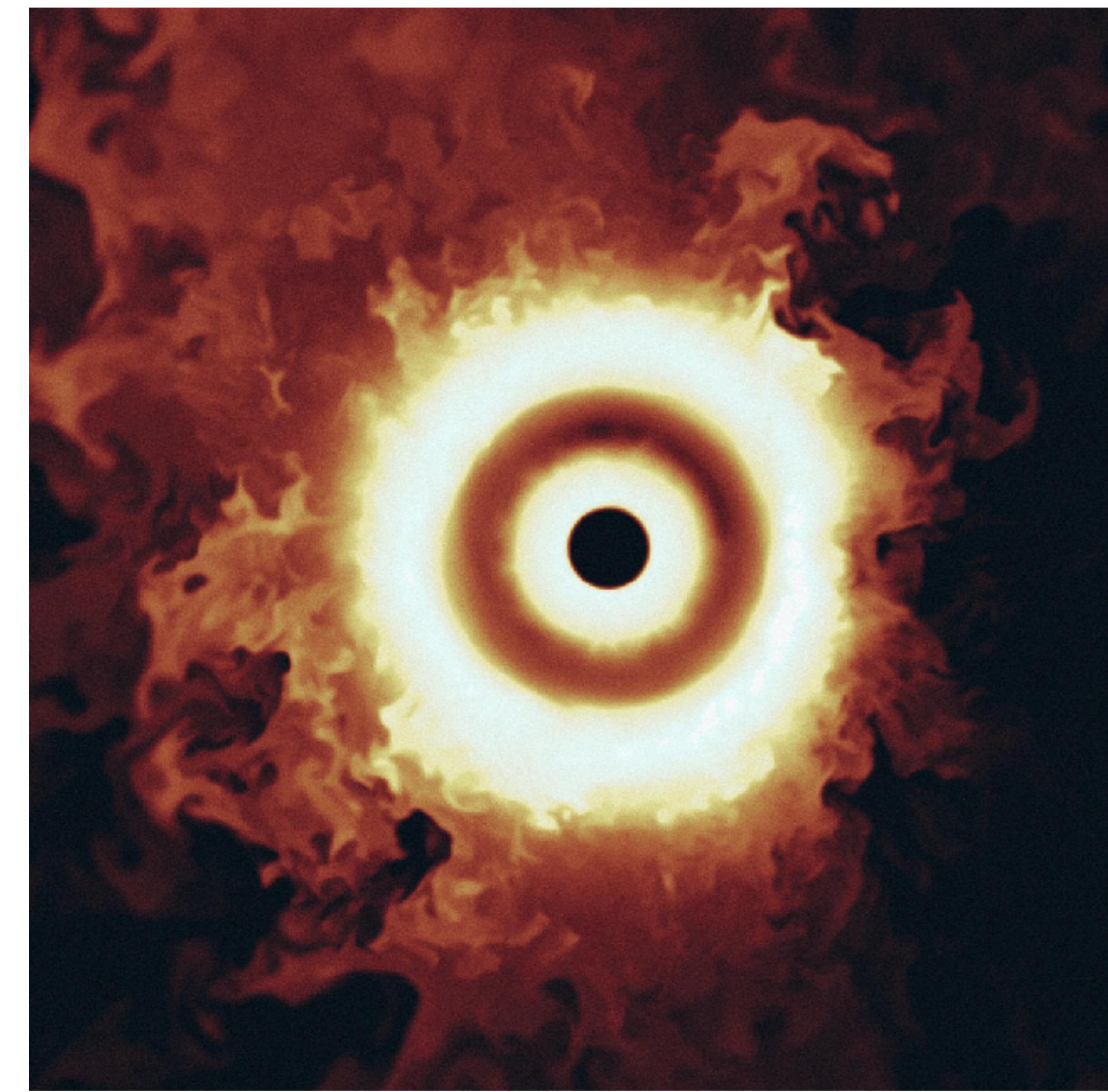


Image credit: K.-C. Pan



# Introduction (cont.)

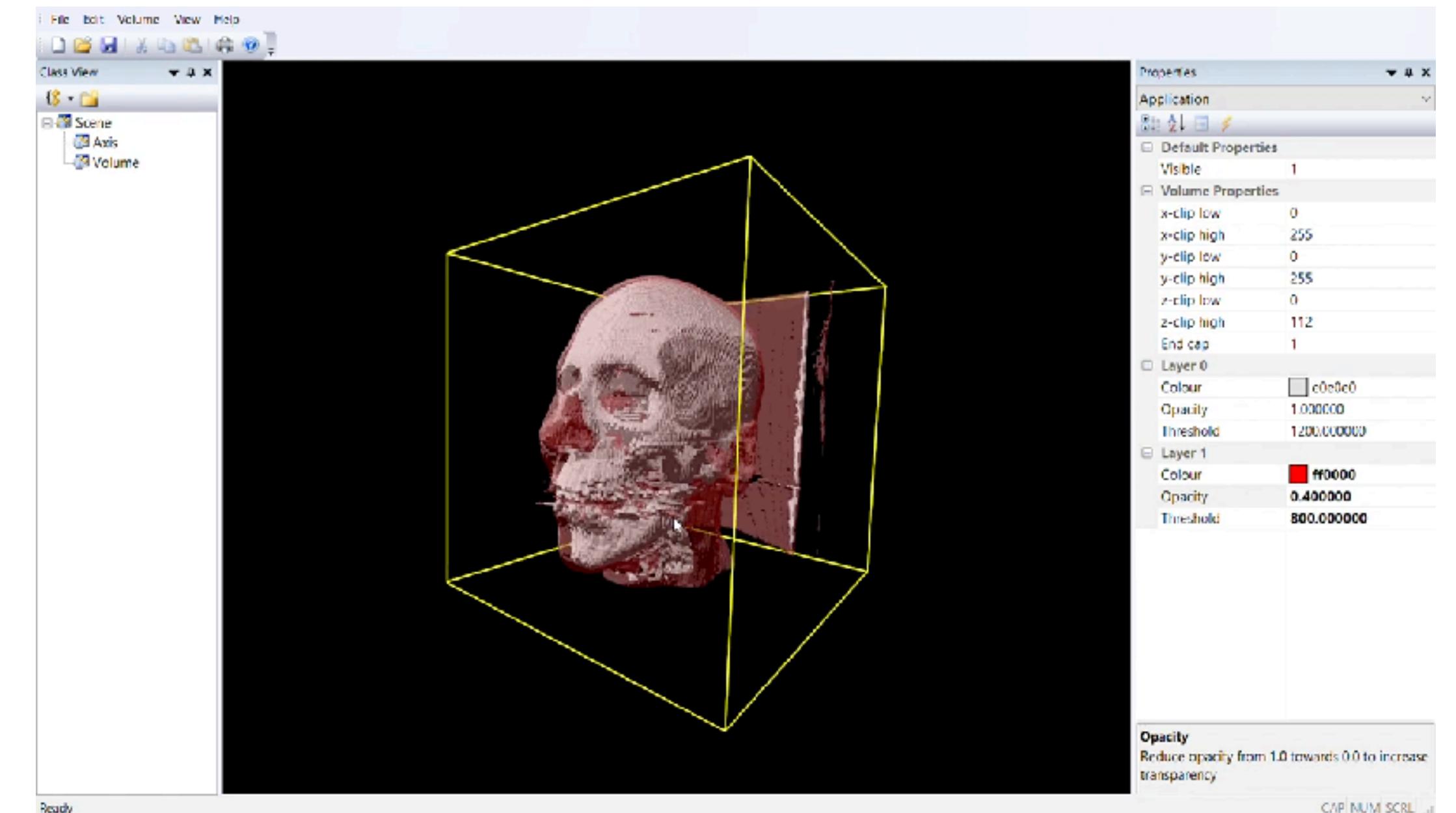
- Given an astrophysical system, how will it looks like?
- Given an observed astronomical object (i.e. an image or spectrum), what is the nature of the physical system?
- Radiative process link **astrophysical systems** with **astronomical observables**
- Light curves & spectra
- Chemistry, atomic/molecular lines, neutrino interactions, ...etc.
- Radiation feedback
- Radiation fields: EM waves (photons), neutrinos, ... (Multi-messengers)



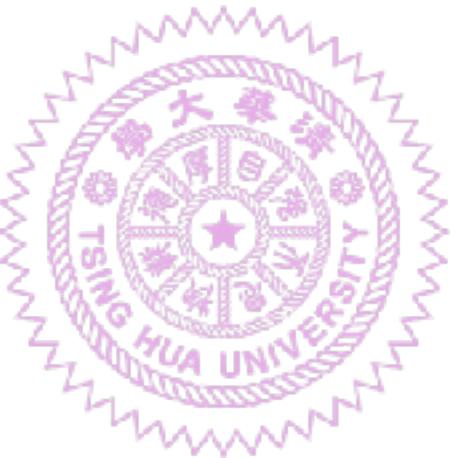
# Daily life examples



Video credit: [https://www.youtube.com/watch?v=CIMFsY\\_QKBM](https://www.youtube.com/watch?v=CIMFsY_QKBM)



Video credit: <https://www.youtube.com/watch?v=9dPlkHiJ6A4>



# Limb Darkening

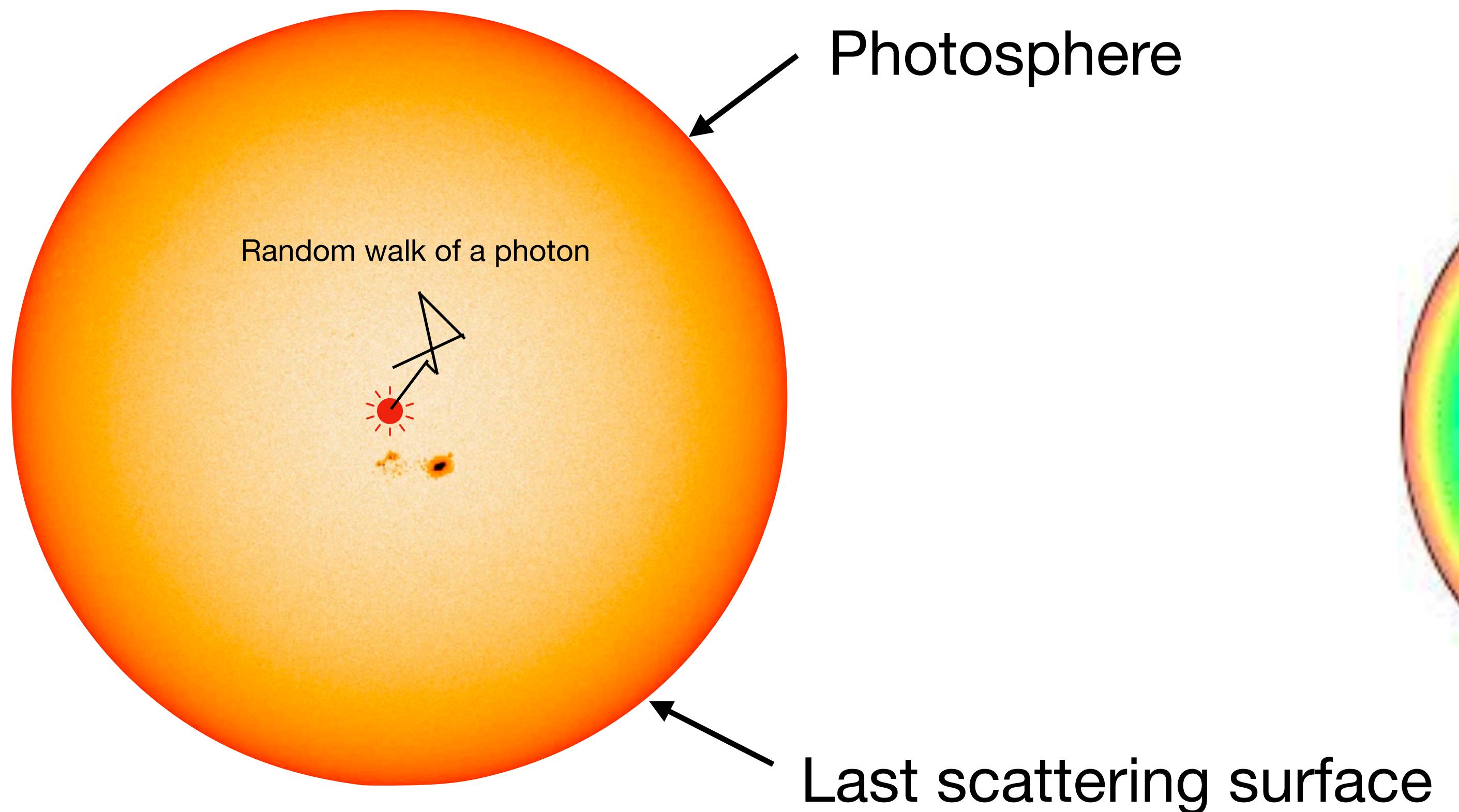
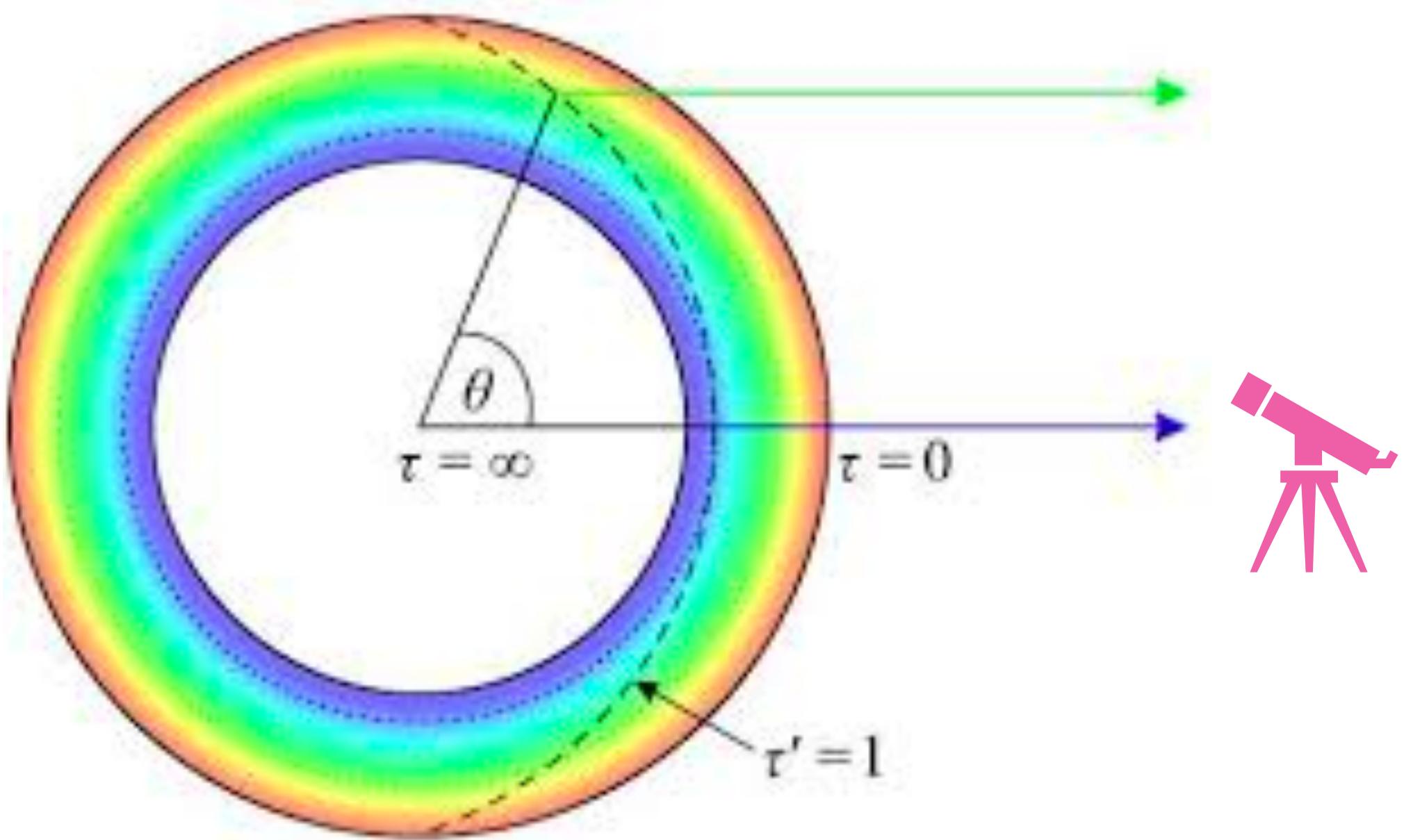


Image credit: NASA



Recall H. Hirashita's talk



# Sun's “Surface”s

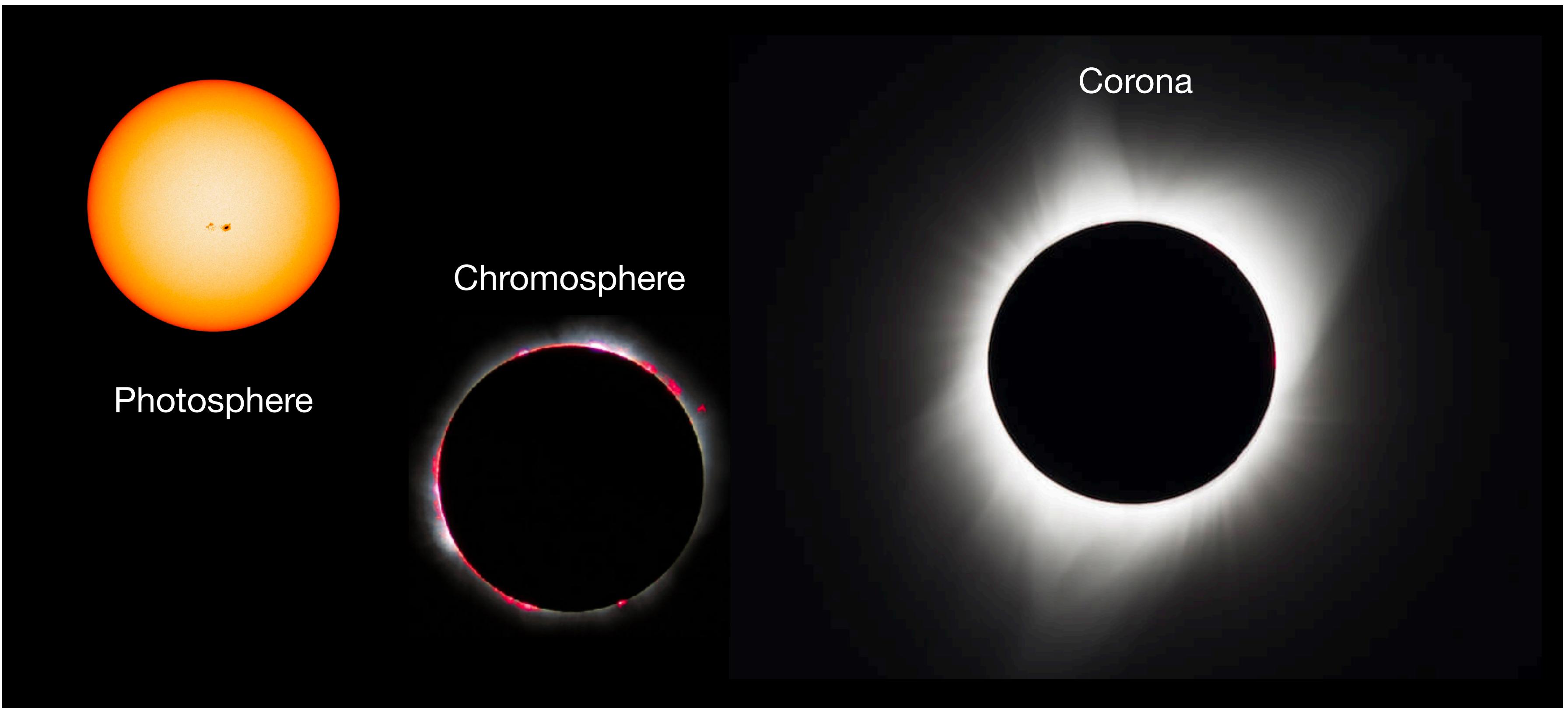


Image credit: NASA

Image credit: Luc Viatour

Image credit: NASA

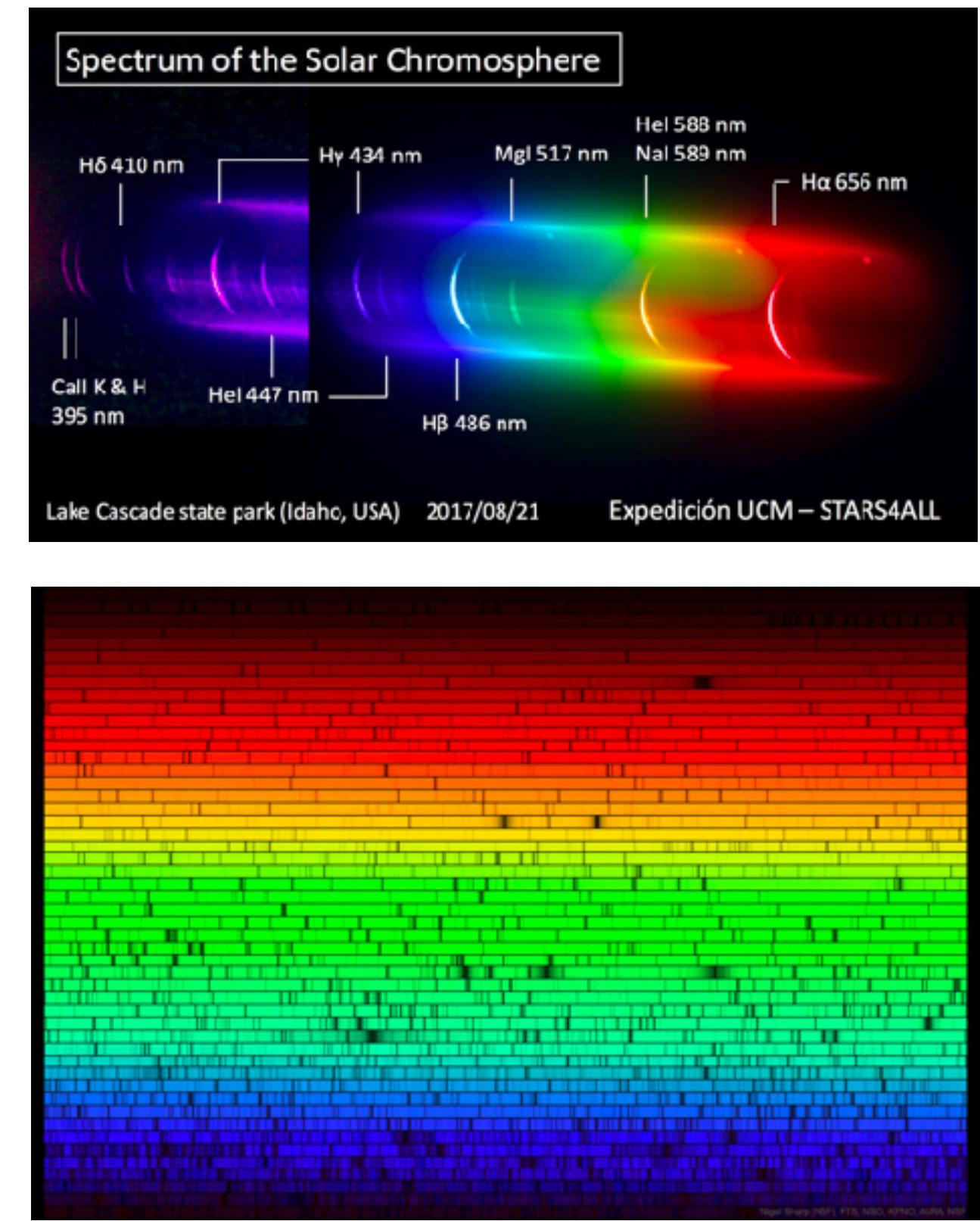
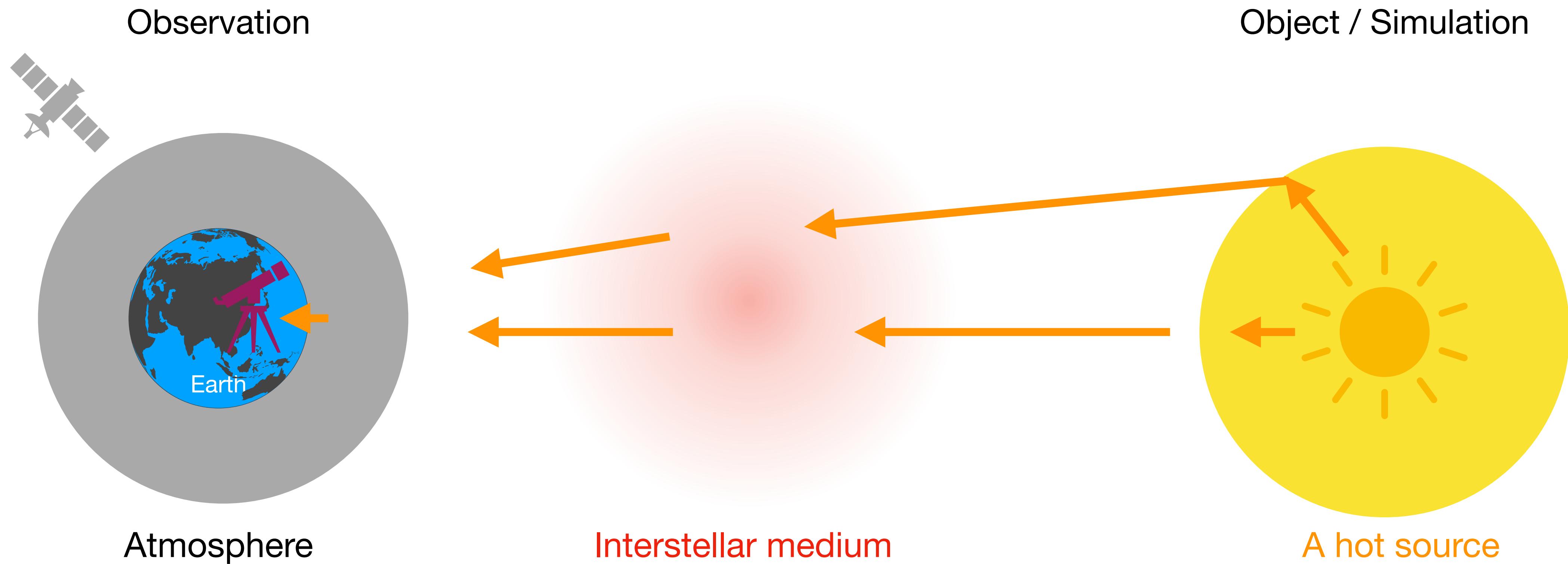


Image credit: Nigel Sharp



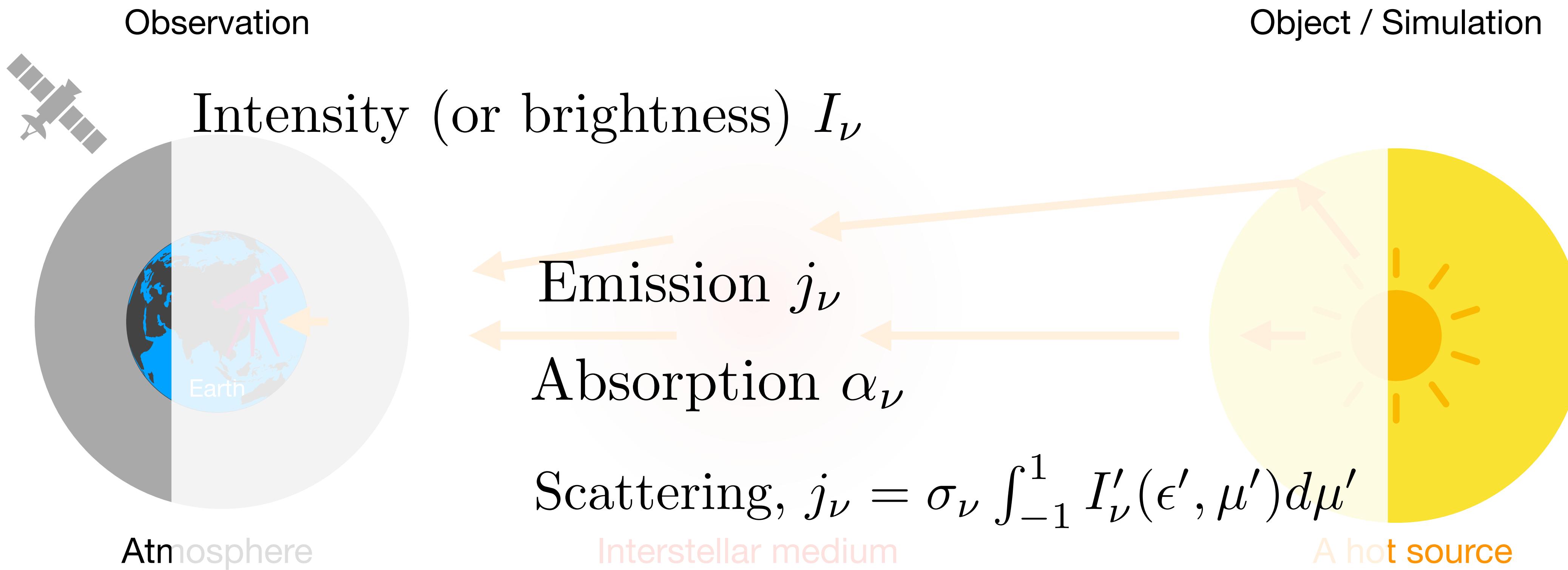
# Introduction



Recall: Hi. Hirashita's Talk

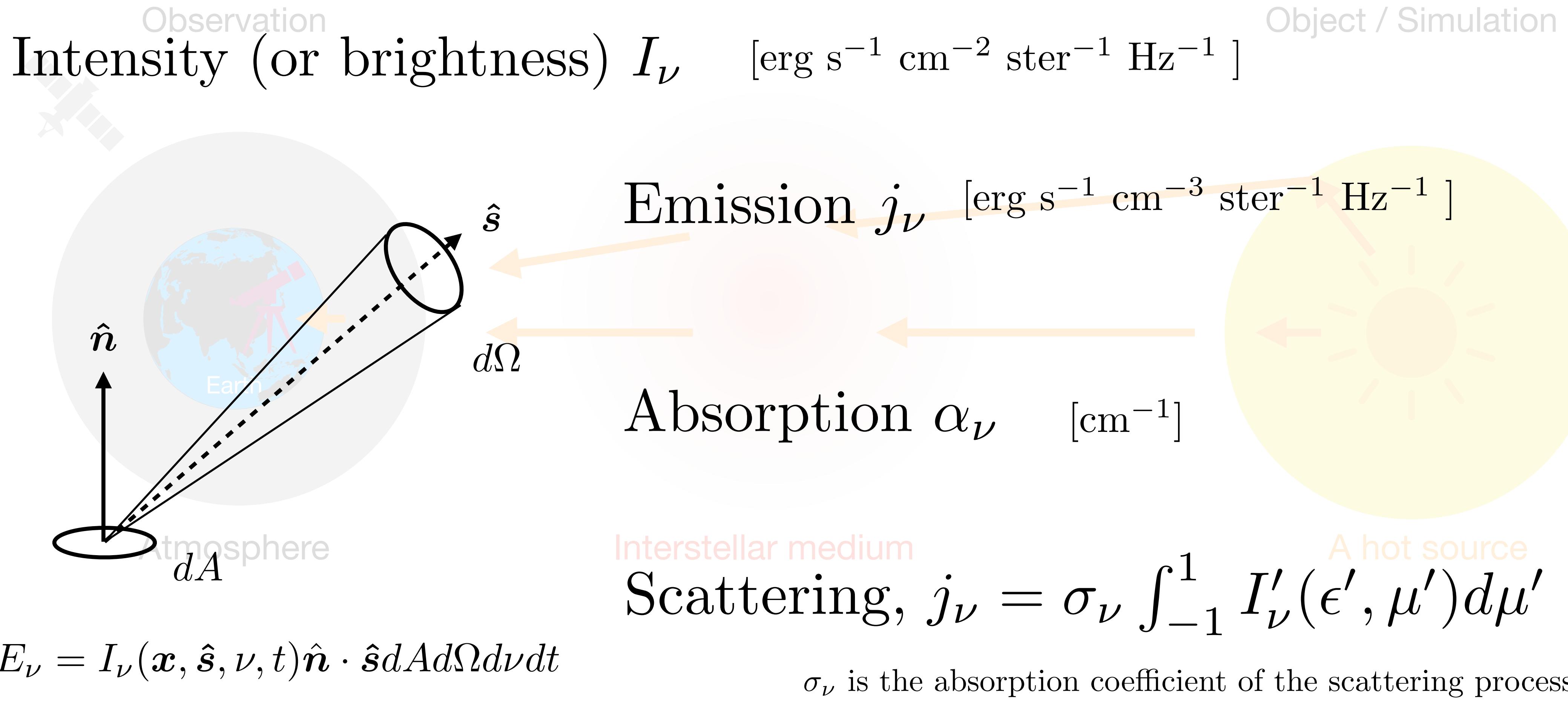


# Introduction





# Introduction



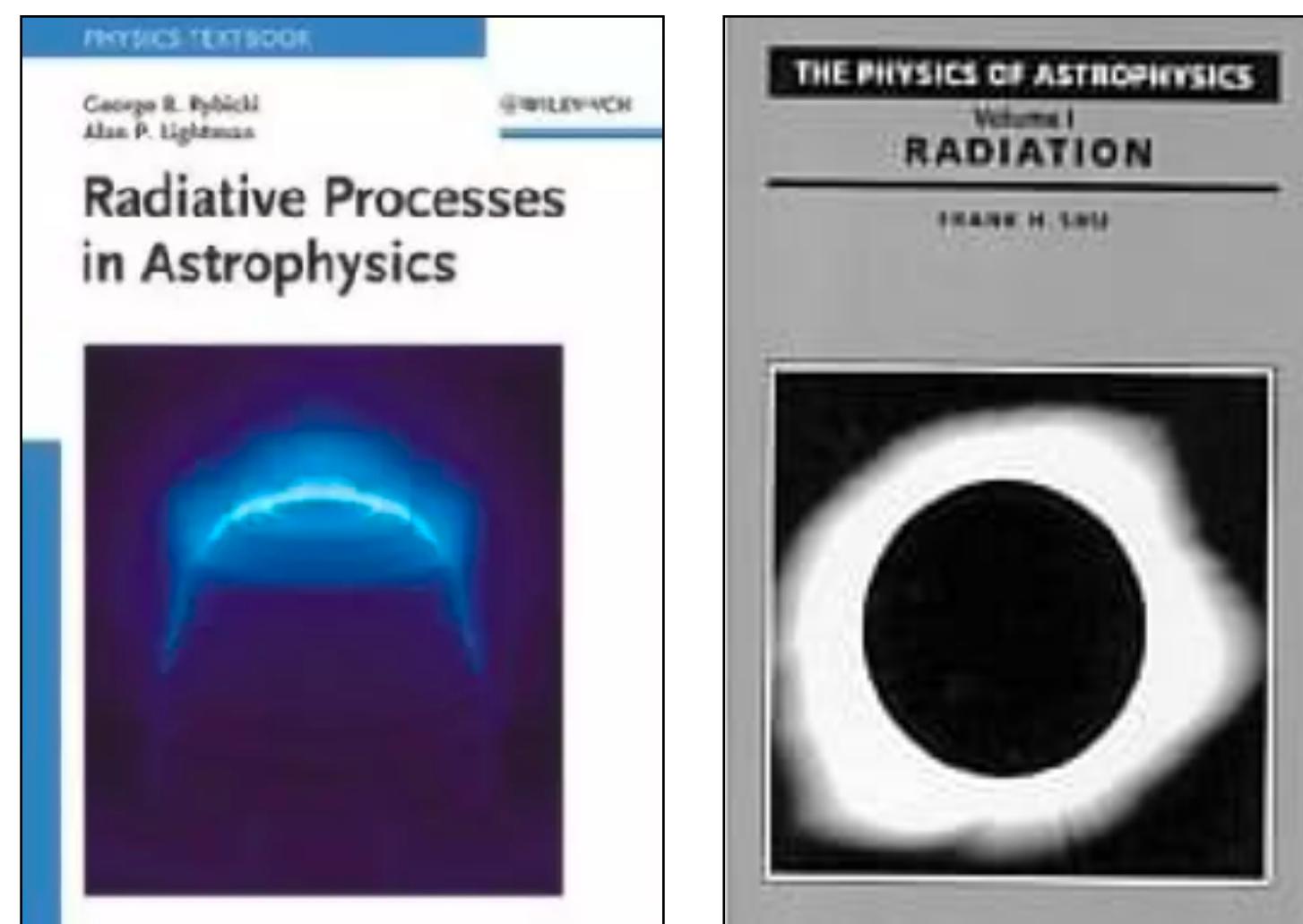


# The Transport Equations



# Suggested textbooks

- “Radiative Processes in Astrophysics”, Rybicki & Lightman
- “The Physics of Astrophysics”, Frank Shu





# Review: Basic Radiative Transfer Equation

assume no scattering process

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

Absorption      Emission

Recall H. Hirashita's talk

Or,

$$\frac{dI_\nu}{d\tau_\nu} = -(I_\nu + S_\nu)$$

Optical depth,  $d\tau_\nu = \alpha_\nu ds$

Source function,  $S_\nu = \frac{j_\nu}{\alpha_\nu}$

Opacity,  $\kappa_\nu = \frac{\alpha_\nu}{\rho}$  [cm<sup>2</sup> g<sup>-1</sup>]

- Optically **thick**:  $\tau_\nu > 1$
- Optically **thin**:  $\tau_\nu < 1$

Special cases: emission or absorption only



# Scattering

Scattering  $\sigma_\nu \int_{-1}^1 I'_\nu(\epsilon', \mu') d\mu'$

- Coherent, isotropic scattering

$$\frac{dI_\nu}{ds} = - (\alpha_\nu + \sigma_\nu) (I_\nu - S_\nu)$$

( Scattering coefficient,  $\sigma_\nu$   
Mean intensity,  $J_\nu$

$$S_\nu = \frac{\alpha_\nu j_\nu + \sigma_\nu J_\nu}{\alpha_\nu + \sigma_\nu}$$

Average of two source functions, weighted by  
their respective absorption coefficients



# Derived from Boltzmann equation

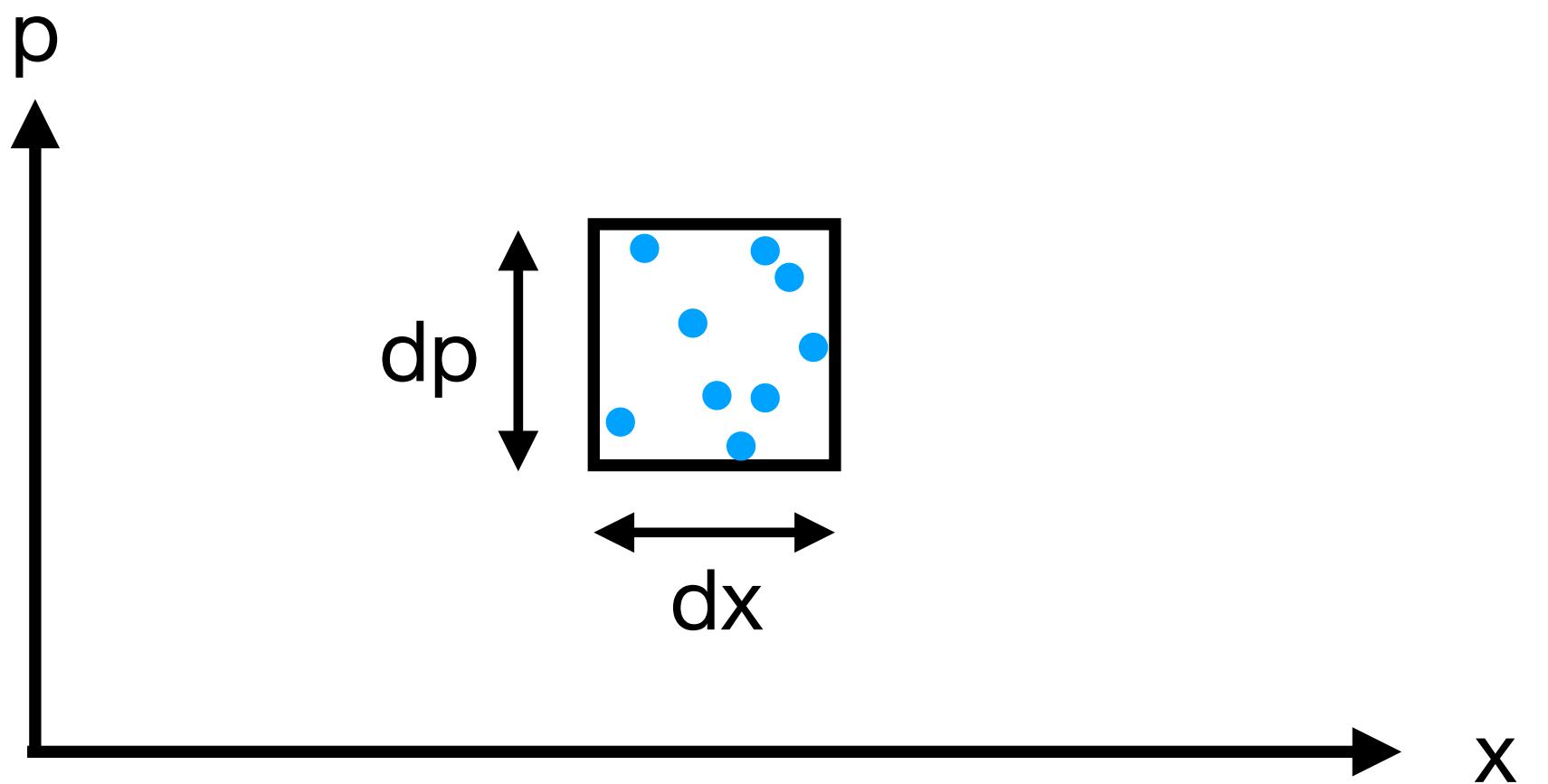
- Consider a cavity containing a gas of particles. The mean number of particles in this cavity is

$$N = \int f(x, p, t) d^3x d^3p,$$

where  $f$  is the distribution function.  $(x, p)$  is the phase spaces of momentum and position coordinates.



# The Boltzmann equation



- Particles are subject to an external force field “F”
- The Boltzmann equation

$$f(\mathbf{x} + \mathbf{u}dt, m\mathbf{u} + \mathbf{F}dt, t + dt) - f(\mathbf{x}, \mathbf{p}, t) = [\Delta f]_{\text{coll.}}$$



# The Boltzmann equation

$$f(\mathbf{x} + \mathbf{u}dt, m\mathbf{u} + \mathbf{F}dt, t + dt) - f(\mathbf{x}, \mathbf{p}, t) = [\Delta f]_{\text{coll.}} \quad \text{Recall H.-Y. Pu's talk}$$

$$\frac{df}{dt} = \left[ \frac{\partial f}{\partial t} \right]_{\text{coll.}}$$

or

$$\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} + F_i \frac{\partial f}{\partial u_i} = \left[ \frac{\partial f}{\partial t} \right]_{\text{coll.}}$$

- The evolution of the distribution function in the six dimensional space

Number density

$$n(x, t) = \int f(x, p, t) dp$$

Mass density

$$\rho(x, t) = \int m f(x, p, t) dp$$

Bulk velocity

$$v(x, t) = \int m u f(x, p, t) dp$$



# The Boltzmann equation

- When collisions are “elastic” and the density of the medium is low enough for collisions involving more than two particles to be neglected
- Then, in absence of external force,  $f$  is obtained from statistical mechanism and is given by the **Maxwellian velocity distribution**

$$f(x, \mathbf{u}, t) d\mathbf{u} = n(x, t) \left[ \frac{m}{2\pi k T(x, t)} \right]^{\frac{3}{2}} \exp \left[ -\frac{m(u - v)^2}{2k T(x, t)} \right]$$

\* Using the same concept, we could derive the hydrodynamics equations as well



# The Boltzmann equation for photons

$$f(\mathbf{x} + \mathbf{u}dt, m\mathbf{u} + \mathbf{F}dt, t + dt) - f(\mathbf{x}, \mathbf{p}, t) = [\Delta f]_{\text{coll.}} \quad \text{Recall H.-Y. Pu's talk}$$

$$\frac{df}{dt} = \left[ \frac{\partial f}{\partial t} \right]_{\text{coll.}}$$

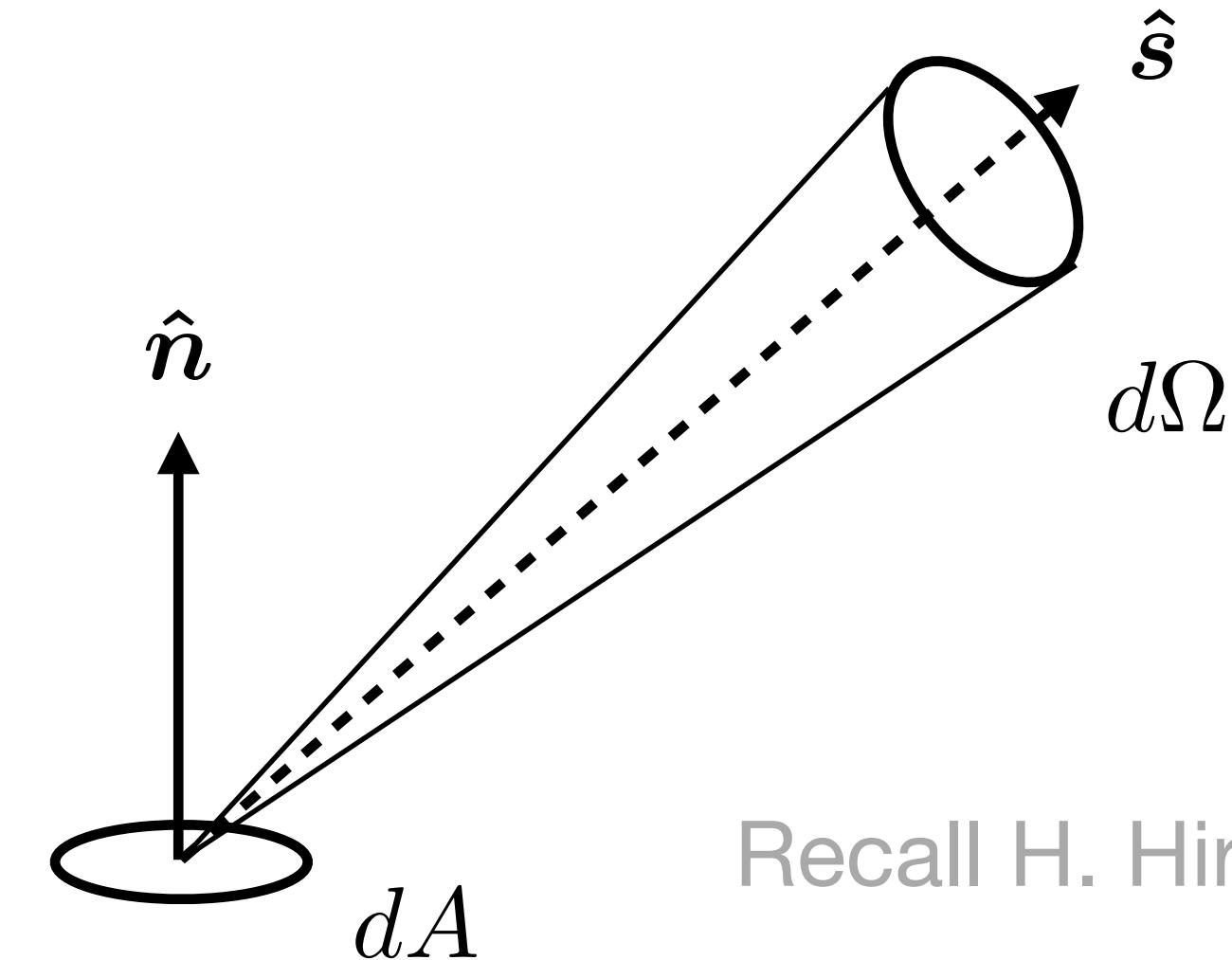
or

$$\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} + F_i \frac{\partial f}{\partial u_i} = \left[ \frac{\partial f}{\partial t} \right]_{\text{coll.}}$$

- Photon transport:  $f = f_\gamma$

$$( \begin{aligned} dE_\nu &= h\nu f_\gamma(\mathbf{x}, \mathbf{p}, t) dx dp \\ dE_\nu &= I_\nu(\mathbf{x}, \hat{\mathbf{s}}, \nu, t) \hat{\mathbf{n}} \cdot \hat{\mathbf{s}} dA d\Omega d\nu dt \end{aligned} )$$

$$I_\nu = (h\nu/c)(h^2\nu)f_\gamma = \frac{h^4\nu^3}{c^2}f_\gamma = C_1 f_\gamma$$



Recall H. Hirashita's talk



# Photon Transport Equation

$$\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} + F_i \frac{\partial f}{\partial u_i} = \left[ \frac{\partial f}{\partial t} \right]_{\text{coll.}}$$

- Photon transport:  $f = f_\gamma$

$$\frac{1}{C_1} \left[ \frac{\partial I_\nu}{\partial t} + c(\hat{s} \cdot \nabla) I_\nu \right] = \frac{1}{C_1} \left[ \frac{\partial f}{\partial t} \right]_{\text{coll.}} \quad \mathbf{F} = 0 \text{ for Newtonian photons}$$

$$\boxed{\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{s} \cdot \nabla I_\nu = -\epsilon_\nu I_\nu + j_\nu + [\text{Other scattering terms}]}$$

Extinction,  $\epsilon_\nu = \sigma_\nu + \alpha_\nu$



# Moments of the Boltzmann equation for Photons

- The moments of the Boltzmann equation define the dynamical equations for the radiation field.

0 th moment: mean Intensity

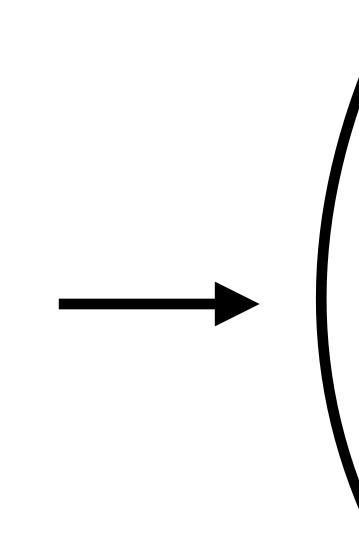
$$J_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu d\Omega,$$

1st moment: radiation flux

$$H_\nu^i = \frac{1}{4\pi} \int_{4\pi} I_\nu s_i d\Omega,$$

2nd moment: tensor

$$K_\nu^{ij} = \frac{1}{4\pi} \int_{4\pi} I_\nu s_i s_j d\Omega,$$



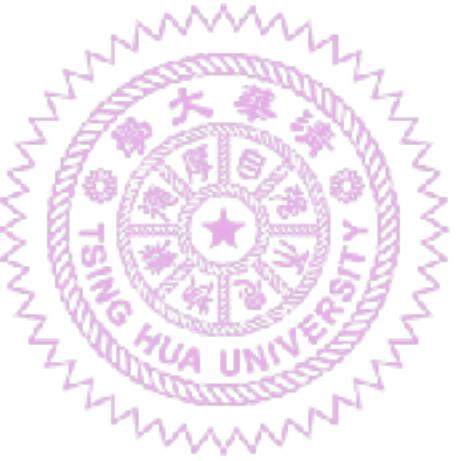
$$\frac{1}{c} \frac{\partial J_\nu}{\partial t} + \nabla \cdot \mathbf{H}_\nu + \alpha_\nu \rho (J_\nu - B_\nu) = 0$$

Radiation energy equation

$$\frac{1}{c} \frac{\partial H_\nu^i}{\partial t} + \sum_j \frac{\partial K_\nu^{ij}}{\partial x_j} + \epsilon_\nu \rho H_\nu^i = 0$$

Radiation momentum equation

Additional closure relations are necessary for "K<sup>ij</sup>"



# Example: Optically thick limit

- e.g. the interior of a star, mean free path is much less than the radius of the star → local thermodynamics equilibrium (LTE)

$$\left( \begin{array}{l} \frac{1}{c} \frac{\partial J_\nu}{\partial t} + \nabla \cdot \mathbf{H}_\nu + \alpha_\nu \rho (J_\nu - B_\nu) = 0 \\ \frac{1}{c} \frac{\partial H_\nu^i}{\partial t} + \sum_j \frac{K_\nu^{ij}}{\partial x_j} + \epsilon_\nu \rho H_\nu^i = 0 \end{array} \right) \xrightarrow{\text{Equilibrium (thermal timescale is long)}}$$

$$I_\nu \sim B_\nu(T) \text{ and } S_\nu = B_\nu(T)$$

almost isotropic  $K_\nu \sim \frac{1}{3} J_\nu$

$$H_\nu = -\frac{1}{\rho \alpha_\nu} \nabla \cdot \mathbf{K}_\nu = -\frac{1}{3\rho \alpha_\nu} \frac{\partial B_\nu}{\partial T} \nabla T$$

$$F_\nu = 4\pi H_\nu \text{ and } F = \int_0^\infty F_\nu d\nu$$

$$F = -\frac{4\pi}{3\rho \alpha_\nu} \nabla T \int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu = -\frac{4\pi}{3\rho \alpha_\nu} \nabla(aT^4)$$

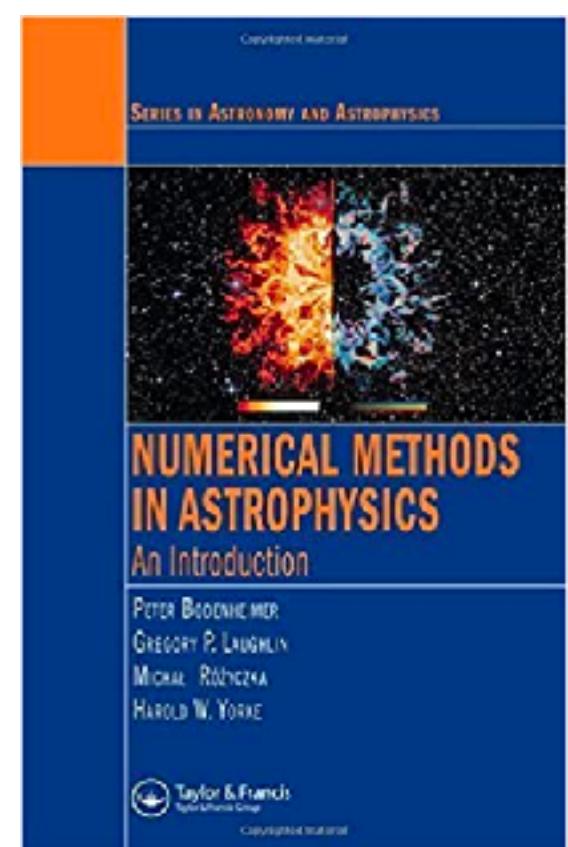


# Numerical methods for solving transport equations



# Codes and Methods

- “Numerical methods in Astrophysics”, Bodenheimer et al.
- Astrophysics Source Code Library (<http://ascl.net/>)
- Odssey.edu (<https://odysseyedu.wordpress.com/>) by H.-Y. Pu
- More ...



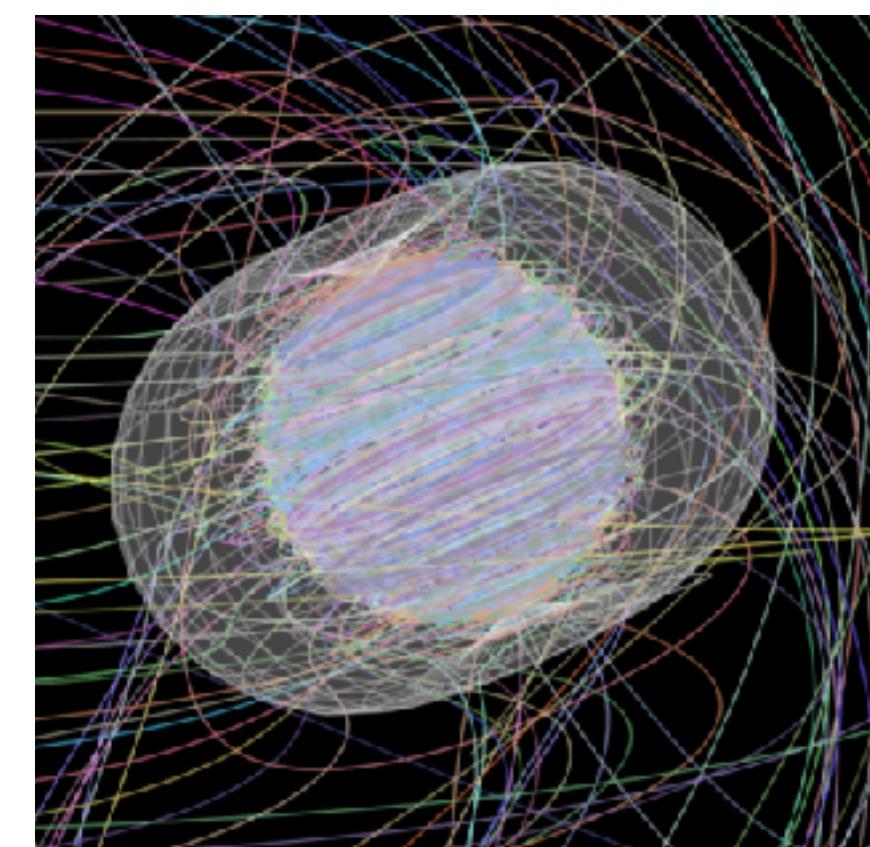
Bodenheimer et al.



## Welcome to the ASCL

The Astrophysics Source Code Library (ASCL) system astronomers, and lists codes that have is indexed by the [SAO/NASA Astrophysics Data System](#). ascl ID can be used to link to the code entry by

<http://ascl.net>

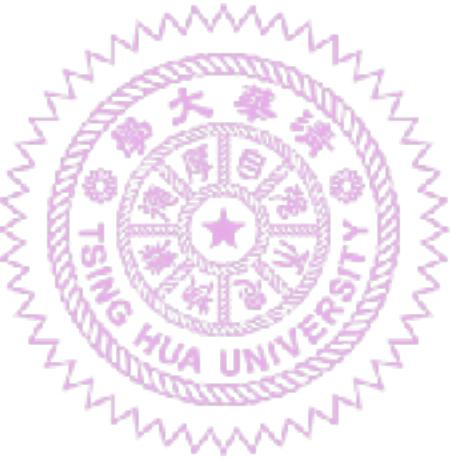


Pu and Yun (2016)



# Numerical Challenging

- Can be time-dependent or time independent
- Can be coupled with gas (hydro/mhd) or via post-processing
- (magneto-) hydrodynamics: 4D (t, x, y, z)
- Radiation transport: 7D (t, x, y, z, theta, phi, e/f) —> slow to compute
- If 100 grid points in each dimension —>  $10^{12}$  points per time step (~ 8TB)
- Could have a wide opacity range (from optically thin to thick)
- Additional complexity from multi-dimensional fluids
- Approximations are usually necessary



# Numerical approaches - Outline

Non- Transport

- Efficient / Ad-hoc / Poor: polytropic EoS
- Tabulated heating /cooling
- Photon/Neutrino leakage

Rad. Transport

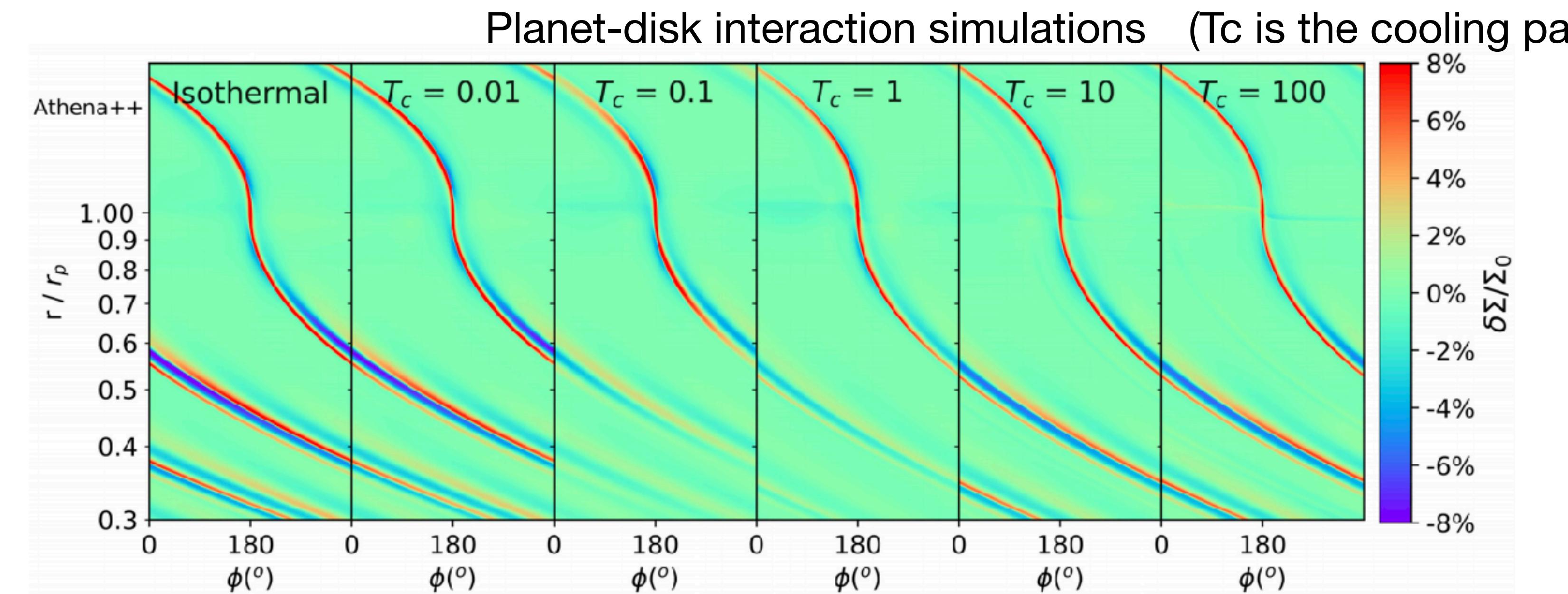
- Flux-limited diffusion
- Ray-tracing
- Moment schemes
- Boltzmann transport

\* Each method could have several variants



# Non-transport: Isothermal EoS

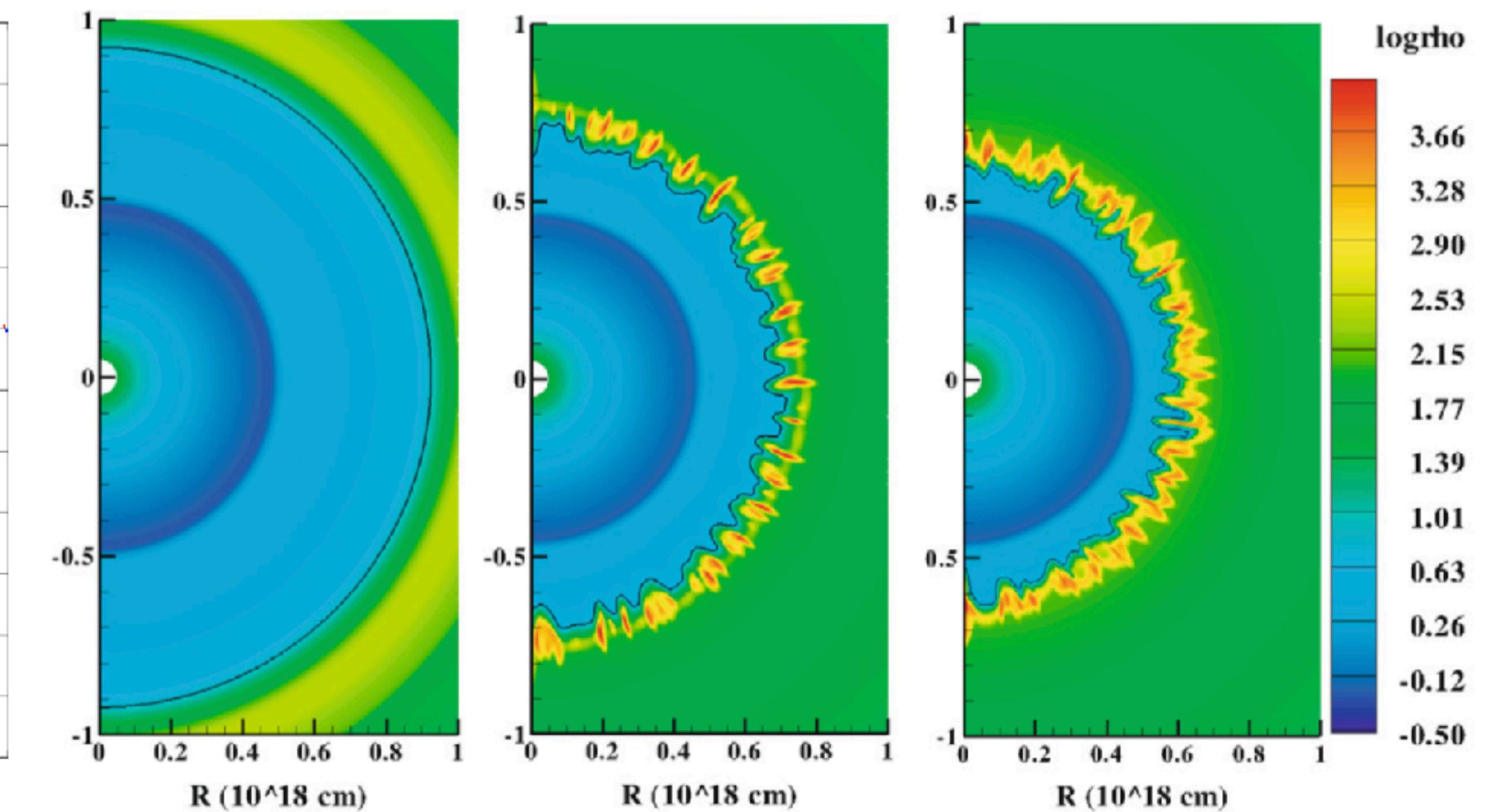
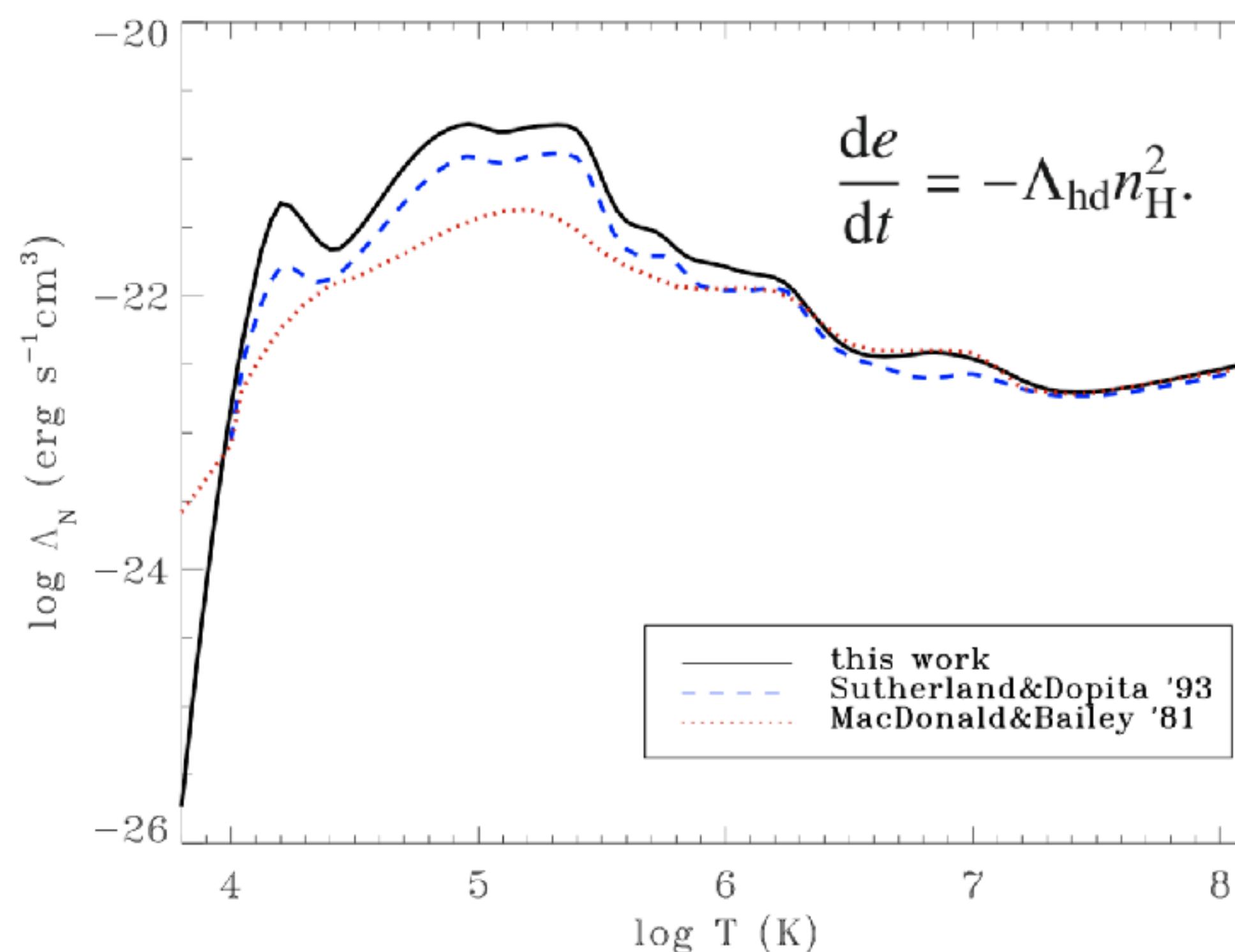
- If radiative cooling is so efficient that the isothermal assumption is applicable
- Simply use the isothermal EoS ( $\gamma=1$ ) or polytropic EoS ( $\gamma > \sim 1$ )





# Non-transport: Tabulated heating/cooling

- Add the cooling curve (from a table or a formula) in the energy equation.

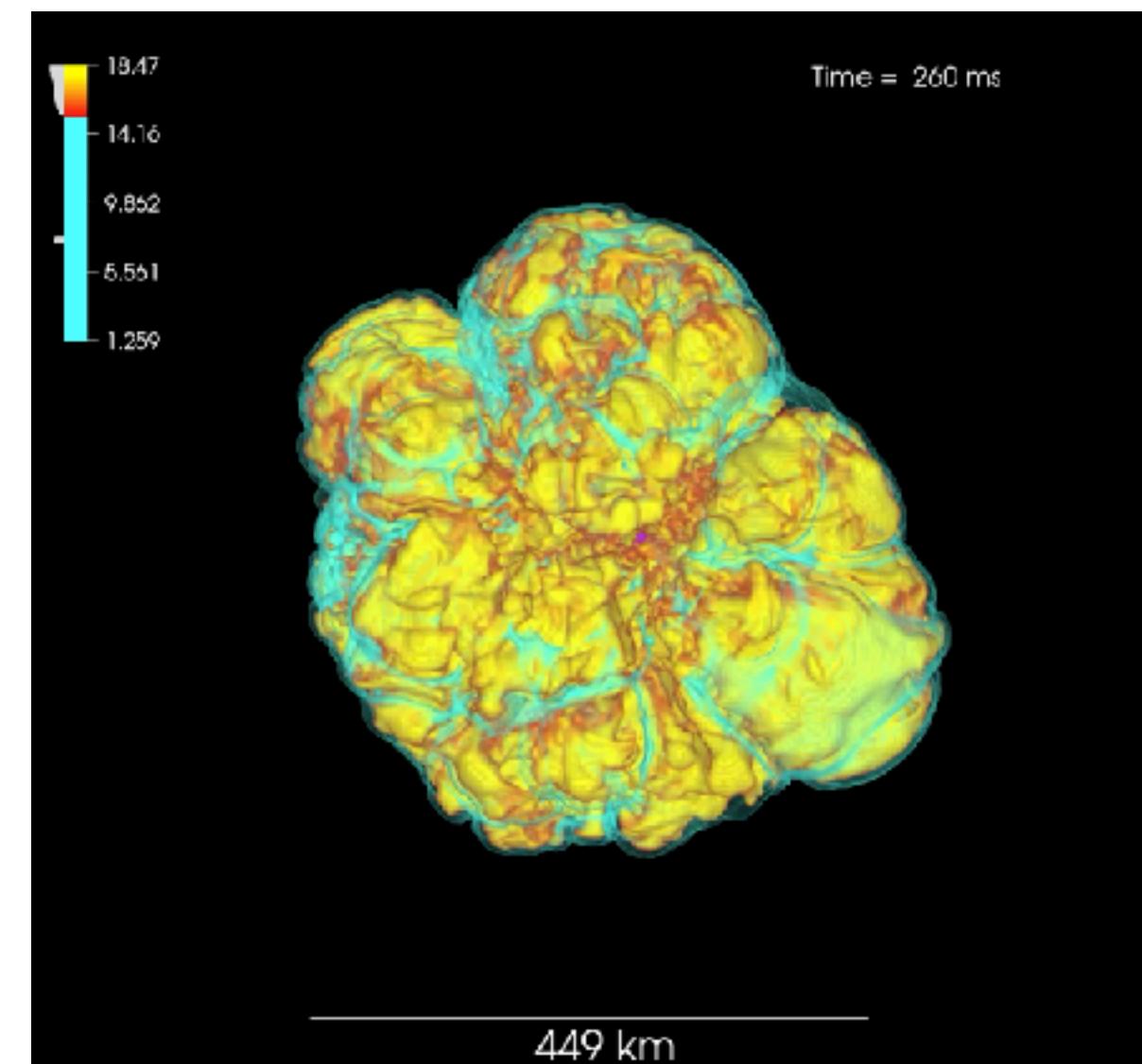


Schure et al. (2009)



# Non-transport: Leakage scheme

- The leakage scheme provides approximate energy and number emission/absorption rates based on local thermodynamics and the optical depth.
- The rate of energy emission can be determined by the interpolation between two limiting regimes
- The optical depth requires a non-local calculation



Couch & O'Connor (2013)



# Rad.-transport: FLD

- Radiation-hydrodynamics equations:

$$\frac{d\rho}{dt} + \rho \frac{\partial v_j}{\partial x_j} = 0$$

$$\rho \frac{dv_i}{dt} = - \frac{\partial P_g}{\partial x_i} + \frac{1}{c} \epsilon_F \rho F_{\text{rad},i}$$

$$\rho \frac{de}{dt} + (e + P_g) \frac{\partial v_j}{\partial x_j} = -4\pi \kappa_P \rho B + c \kappa_E \rho u$$

\* ignore gravity, magnetic fields, and viscosity

$F_{\text{rad}}$  (or  $F$ ), radiation flux integrated over frequency  
 $B$ , the Planck function integrated over frequency  
 $u$ , energy density in the radiation field

$$\kappa_E = \frac{1}{u} \int_0^\infty \kappa_\nu u_\nu d\nu,$$

$$\kappa_P = \frac{1}{B} \int_0^\infty \kappa_\nu B_\nu(T) d\nu,$$

$$\epsilon_F = \frac{1}{F} \int_0^\infty \epsilon_\nu F_\nu d\nu,$$



# Rad.-transport: FLD (cont.)

- Diffusion Approximation (optically thick limits)
- Frequency integrated (gray) or with different frequency bins (multi-groups)

$$F = -\frac{c}{3\kappa_R \rho} \nabla u, \quad \text{or} \quad F = -\frac{c\lambda}{\kappa_R \rho} \nabla u,$$

- In the optical thin limit, flux becomes unphysical large
- We need to adjust lambda in optically thin limit (need a flux limiter)



# Rad.-transport: FLD (conti.)

- Flux-limited diffusion
- Define a dimensionless quantity R

$$R = \frac{|\nabla u|}{\kappa_R \rho u},$$

Is the ratio of mean free path to the energy scale height

$$\lambda = \frac{2 + R}{6 + 3R + R^2} \quad \begin{cases} R - > 0 \text{ optically thick } \lambda = \frac{1}{3} \\ R - > \infty \text{ optically thin } \lambda = \frac{1}{R} \end{cases}$$

Levermore & Pomraney (1981)



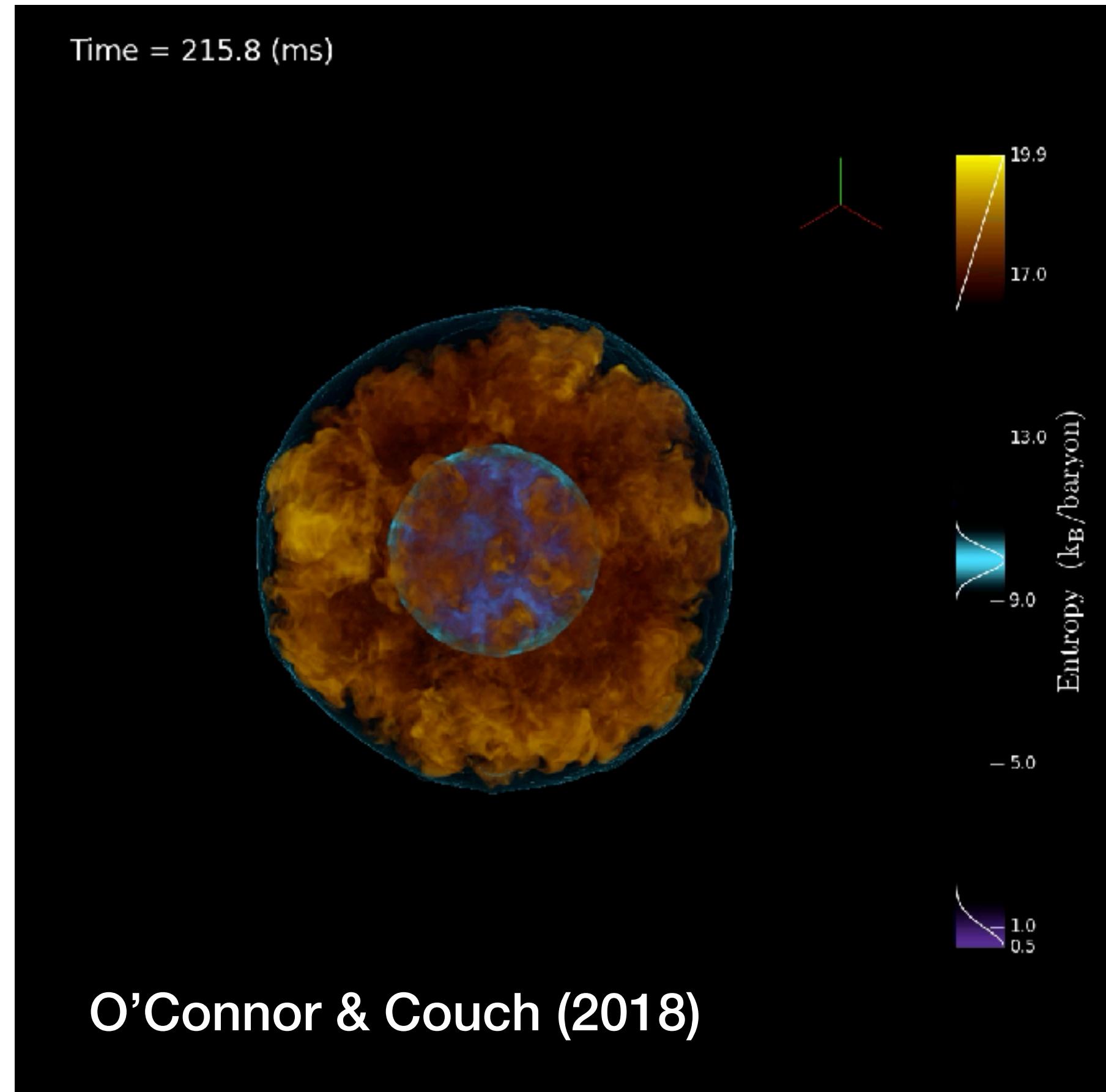
# Rad.-transport: Moments methods

$$\left( \begin{array}{l} \frac{1}{c} \frac{\partial J_\nu}{\partial t} + \nabla \cdot \mathbf{H}_\nu + \alpha_\nu \rho (J_\nu - B_\nu) = 0 \\ \frac{1}{c} \frac{\partial H_\nu^i}{\partial t} + \sum_j \frac{K_\nu^{ij}}{\partial x_j} + \epsilon_\nu \rho H_\nu^i = 0 \end{array} \right)$$

- The moment equations describes the radiation fields, which are related to the energy, energy flux, and radiation pressure
- Consider up to the 1st moment (or M0)  $\rightarrow$  FLD
- Consider up to the 2nd moment with assumptions of closure (M1, variable Eddington tensor)
- Multi-energy M1 scheme could be very expensive !!!



# Rad.-transport: Moments methods



- Three neutrino species
- M1 scheme with 12 energy bins
- $3 \text{ (species)} \times 4 \text{ (1 energy + 3D flux)} \times (12 \text{ energy bins}) = 144$  radiation variables
- Takes  $> 10M$  core-hours



# Rad.-transport: Ray-tracing methods

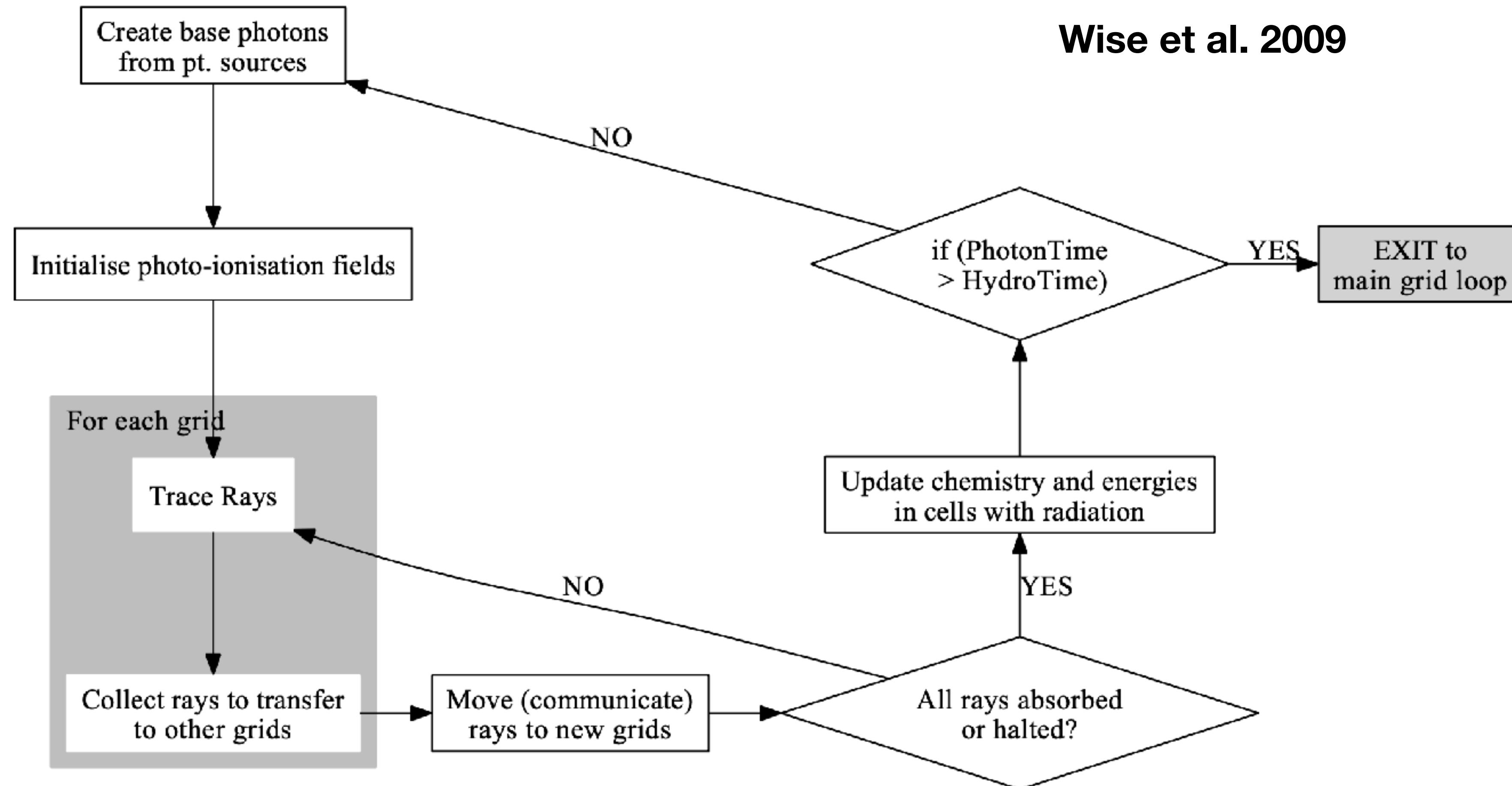
- Usually via post-processing
- Write the transfer equation in Lagrangian coordinates

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

- Shoot a discrete set of light rays from a point source, solving the transfer equation along the path of the light ray.
- The difficulty is then to choose the appropriate number of rays (adaptive ray-tracing method)
- Use Monte-Carlo approach (short or long characteristic)



# Rad.-transport: Ray-tracing methods





# Microphysics

- How about opacity, emissivity, and mean free path?
- These depend on complex microphysics which itself depends on the transport of the radiation field
- Usually stored in a table (atomic data, chemical network, eos)  $\sim$ GB
- Could assume LTE (Saha eq.) or non-LTE (solve reaction network)

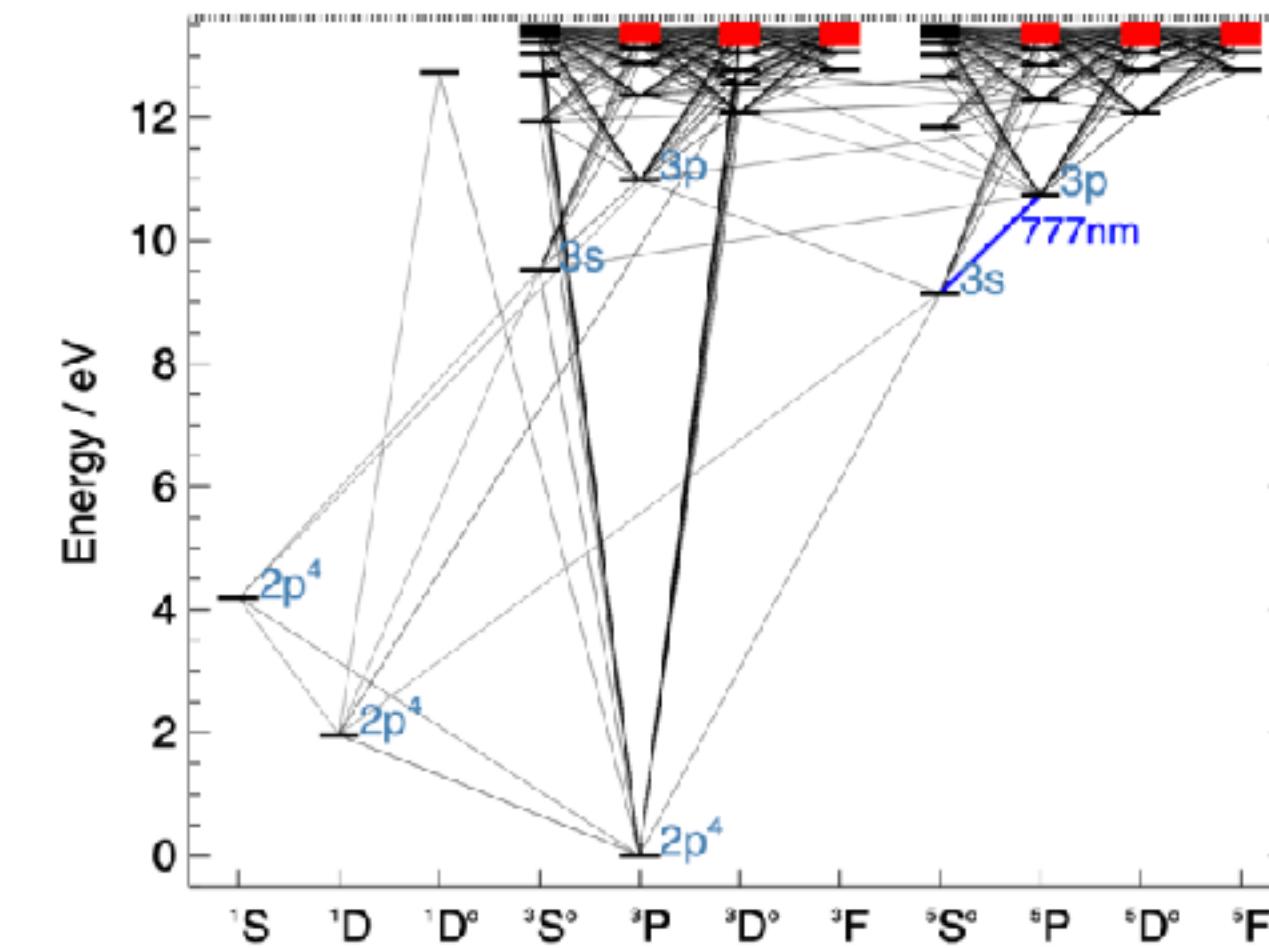


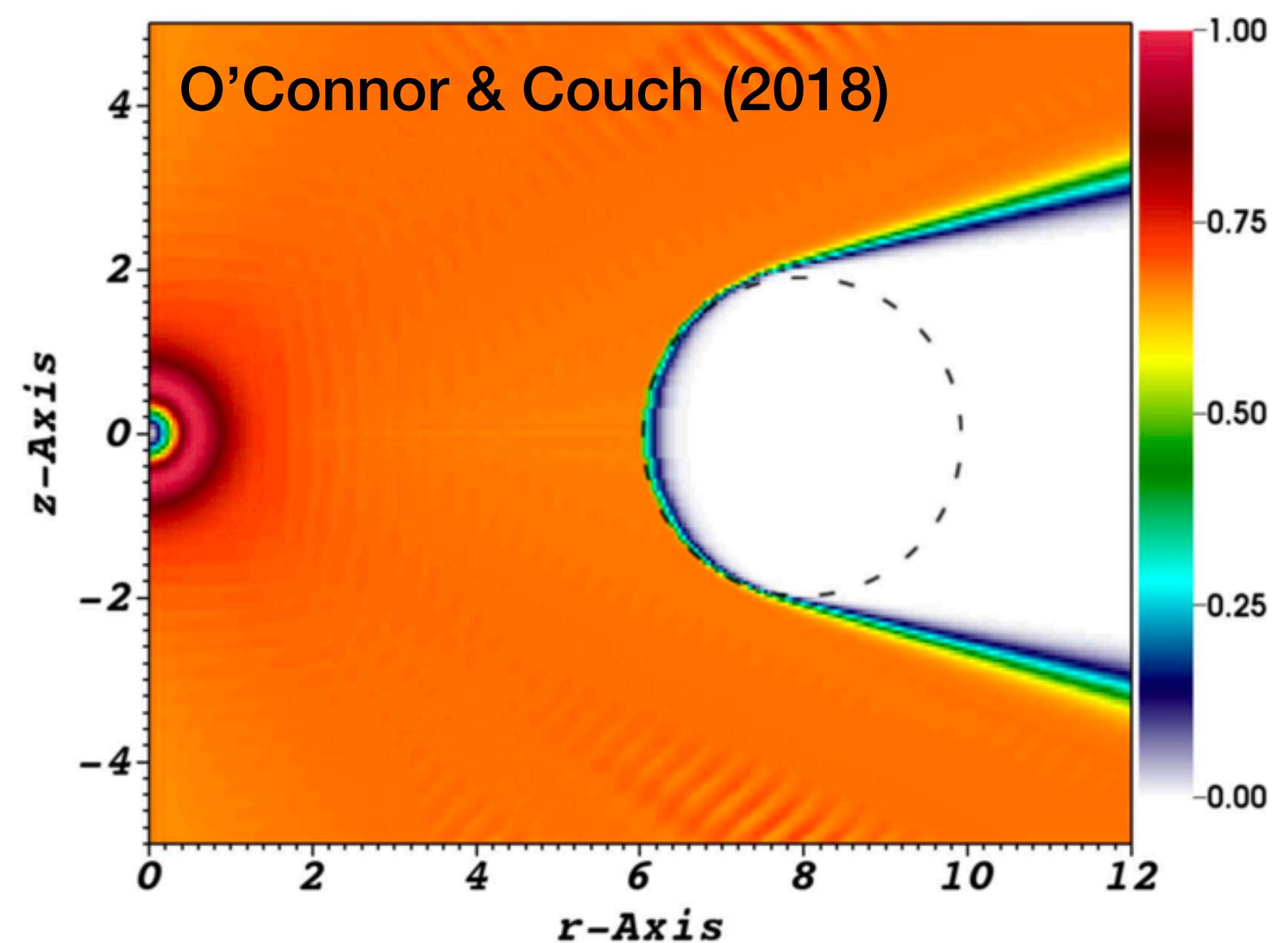
Illustration of a comprehensive "model atom" for neutral oxygen, that describes the structure and radiative and collisional transitions, and is used when calculating the departures from Saha-Boltzmann equilibrium in stellar atmospheres (Amarsi et al., 2015, A&A, 516, 89).



# Benchmarks

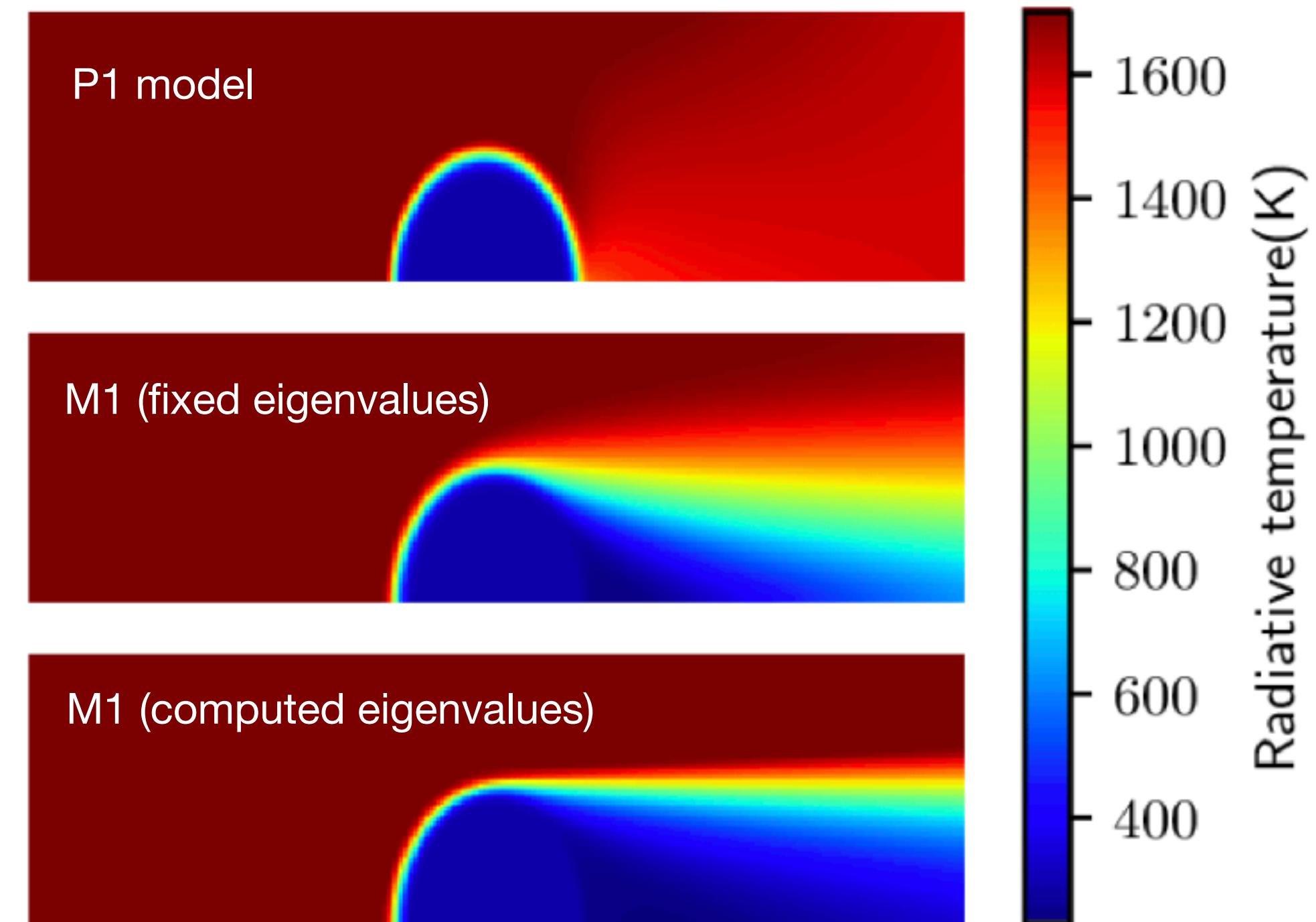
- Free streaming shadow test

THE ASTROPHYSICAL JOURNAL, 854:63 (19pp), 2018 February 10



**Figure 13.** Neutrino energy density multiplied by  $r^2$  in our M1 shadow test in 2D cylindrical coordinates. There is a spherical emission source located at the origin and a perfectly absorbing region (marked by the dashed circle) at  $r = 8$  with a radius of 2. This test closely follows the setup of Just et al. (2015).

Bloch et al. (2020)



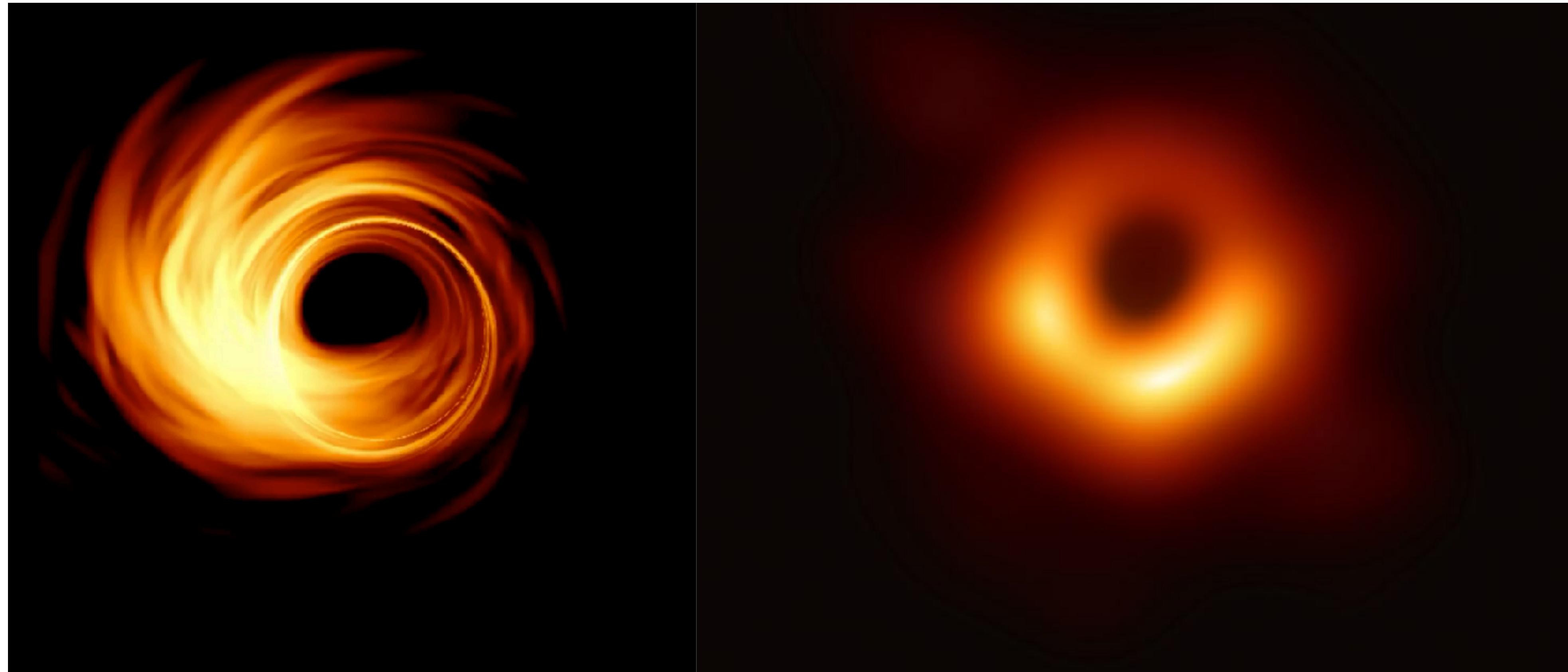
Shadow simulation, showing snapshots of the radiative temperature at time  $t_f = 10 - 10$  s with different closure relations: P1 model (upper panel), M1 model with fixed eigenvalues (middle panel), and M1 model with computed eigenvalues (lower panel).



# GRMHD + post-precessing ray tracing

Supermassive BH in M87

Recall H.-Y. K. Yang's talk



Shiokawa et al. (2017)

Alberdi et al. (2019)



# Numerical approaches - Outline

## Non- Transport

- Polytropic EoS
- Tabulated heating /cooling
- Photon/Neutrino leakage

## Radiation Transport

Optically thick

Optically thin

Simple

Gray-FLD

Multi-groups-FLD

IDSA

Ray-Tracing

Hard

M1

Boltzmann solver



|                        | Diffuse Regime            | Semi-transparent          | Transparent Regime    |
|------------------------|---------------------------|---------------------------|-----------------------|
| Boltzmann solver       | Truncation errors in flux |                           | Inefficient ang. res. |
| Flux-limited diffusion |                           | Flux factor estimated     | Flux factor unknown   |
| Ray-tracing            | Short mean free path      | Limited by reaction rates |                       |

The ideal algorithm combines the three green regions.  
However, it might be too complicated.  
Alternatives: variable Eddington factor method; M1, and the [IDSA](#)

[Adjusted from M. Liebendörfer]



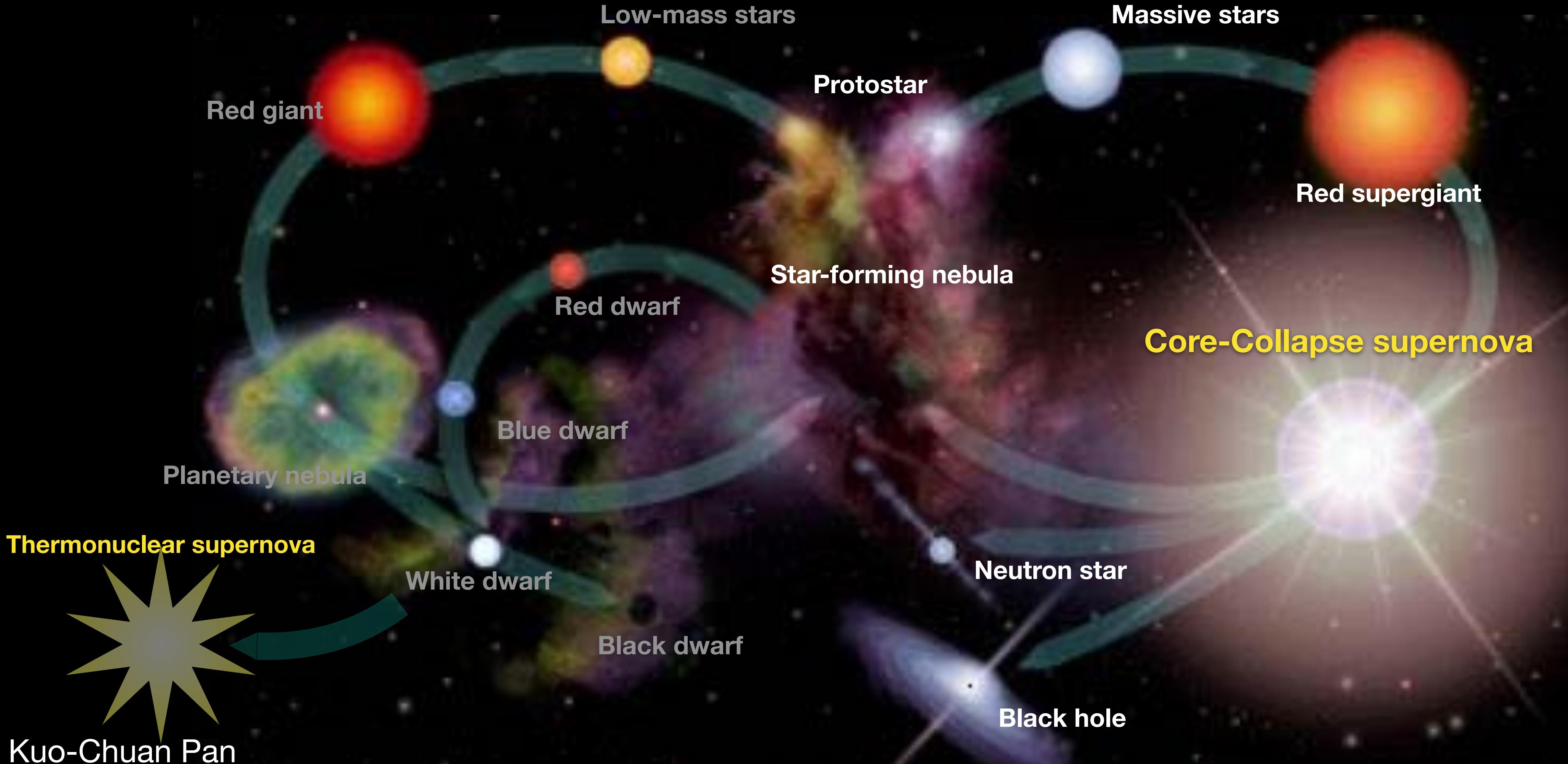
# Application: Supernova with Neutrino Transport



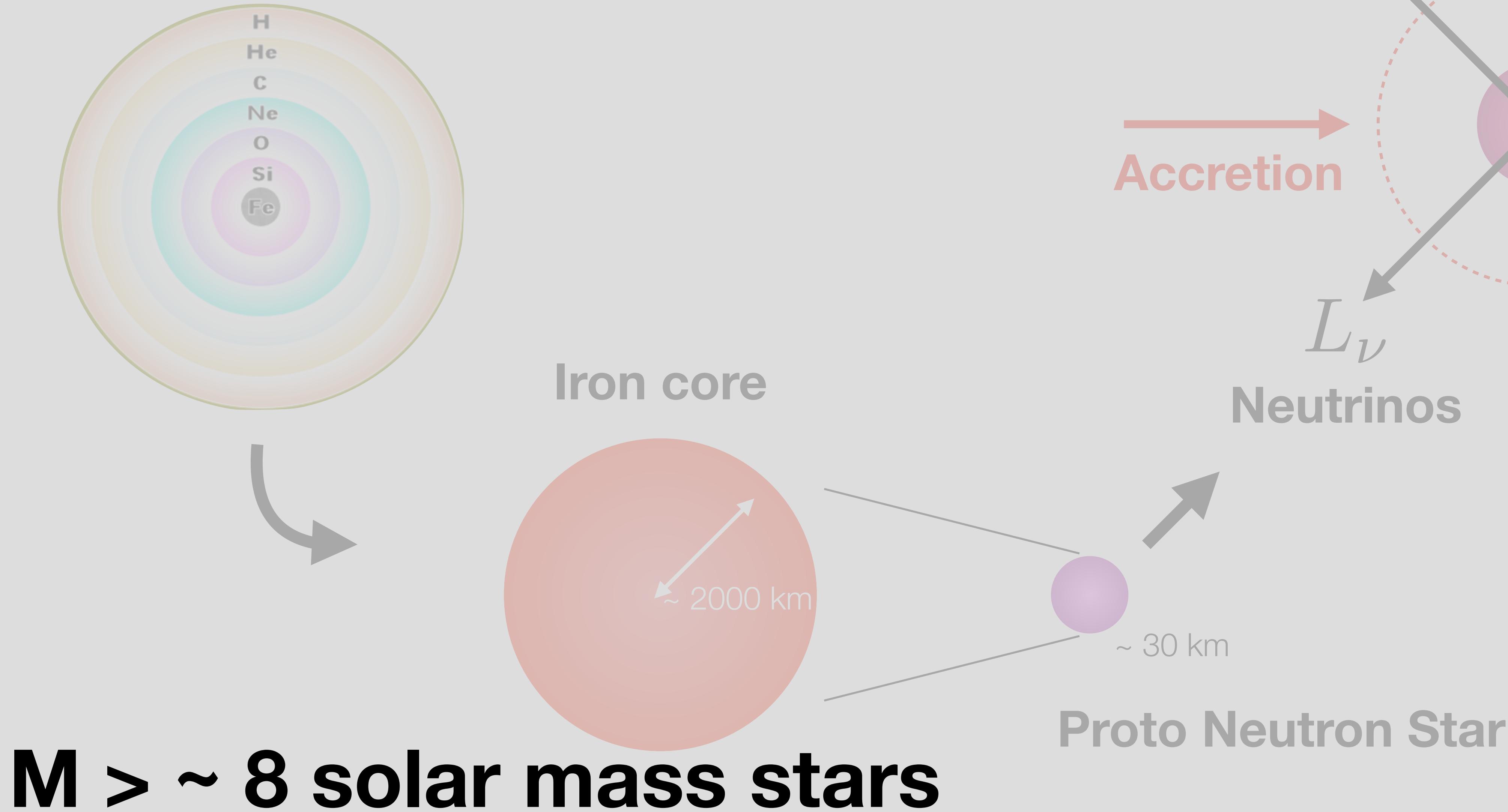
# Application: Core-Collapse supernovae

- Neutrino Transport (not photons)                      Recall H-Y Karen Yang's talk
- Not only supernovae, but also neutron star mergers, ... etc.
- Neutrinos are fermions (photons are bosons)
- Neutrinos have difference flavors (and anti-neutrinos)
- Relativistic effects can not be ignored
- Complicated neutrino interactions (oscillations?)
- Cover both optically thick and thin

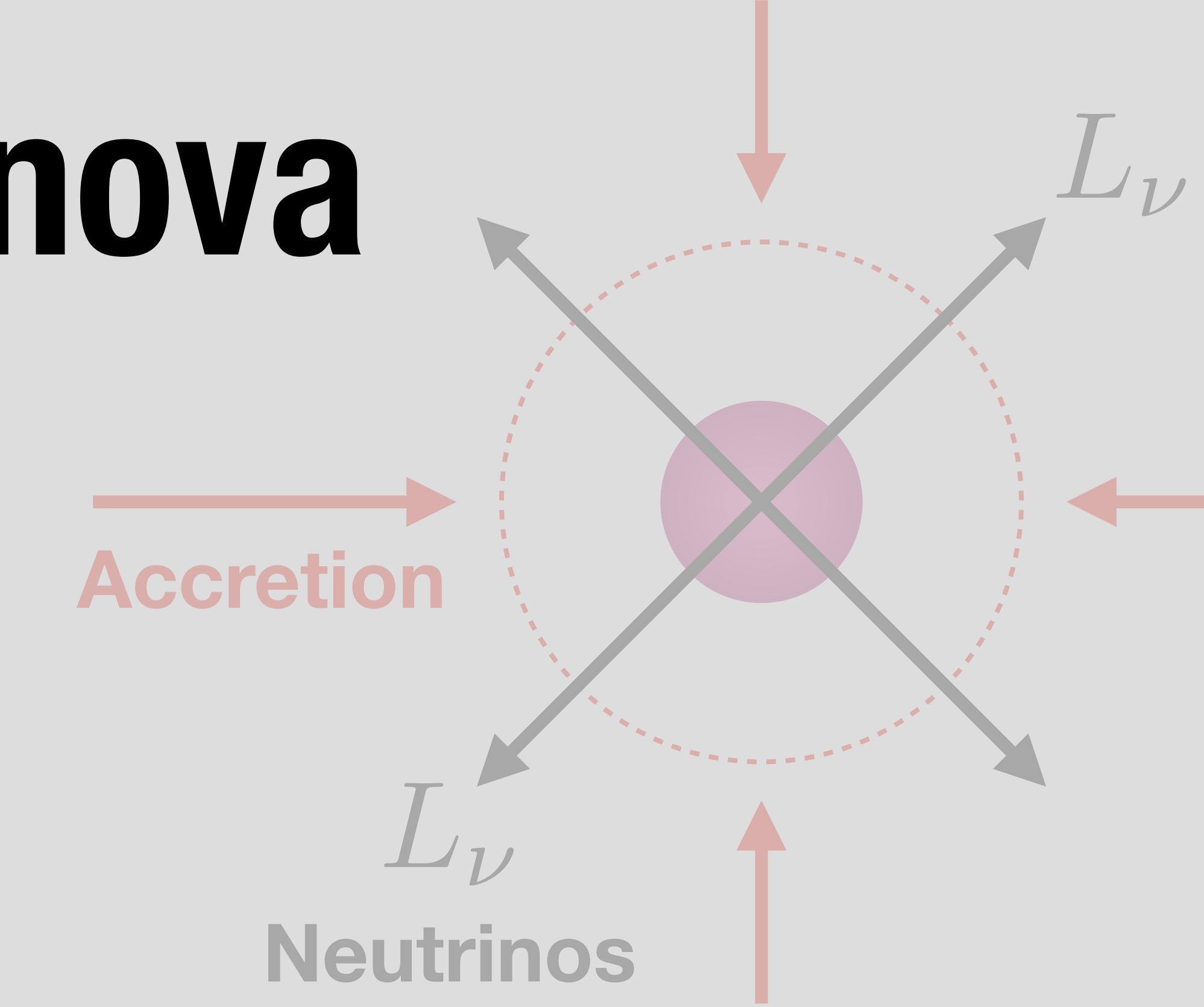
# Stellar evolution 100



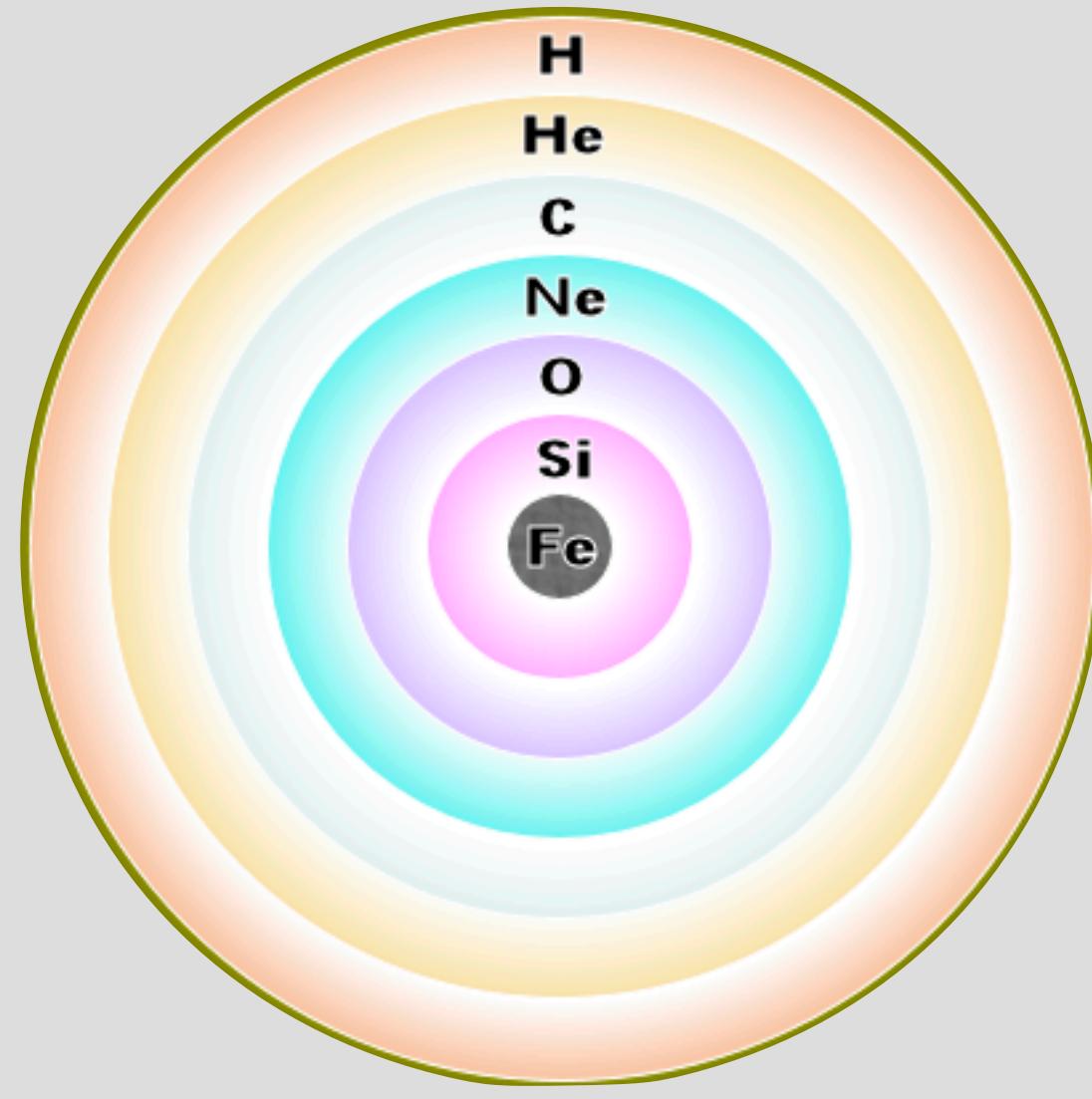
# Core-Collapse Supernova



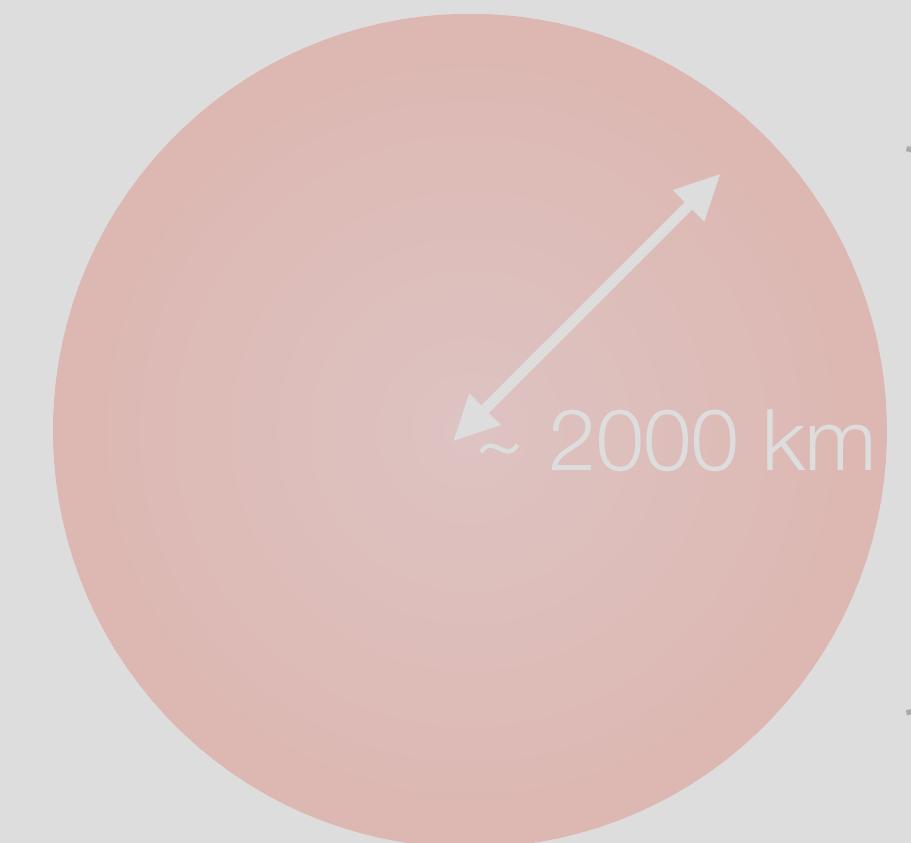
$M > \sim 8$  solar mass stars



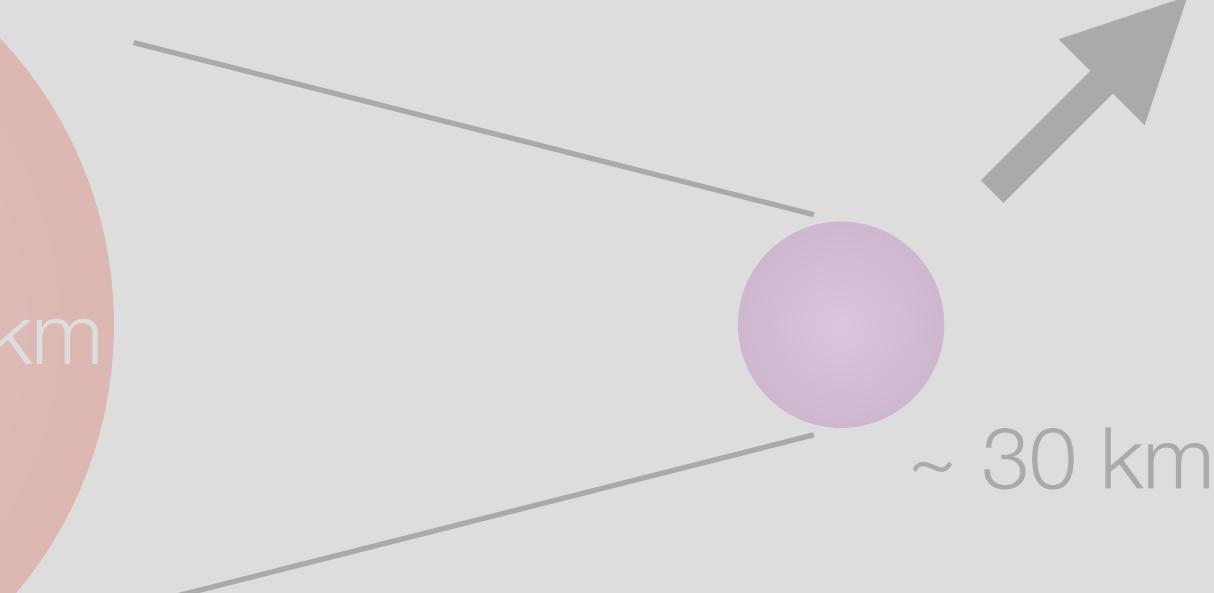
# Core-Collapse Supernova



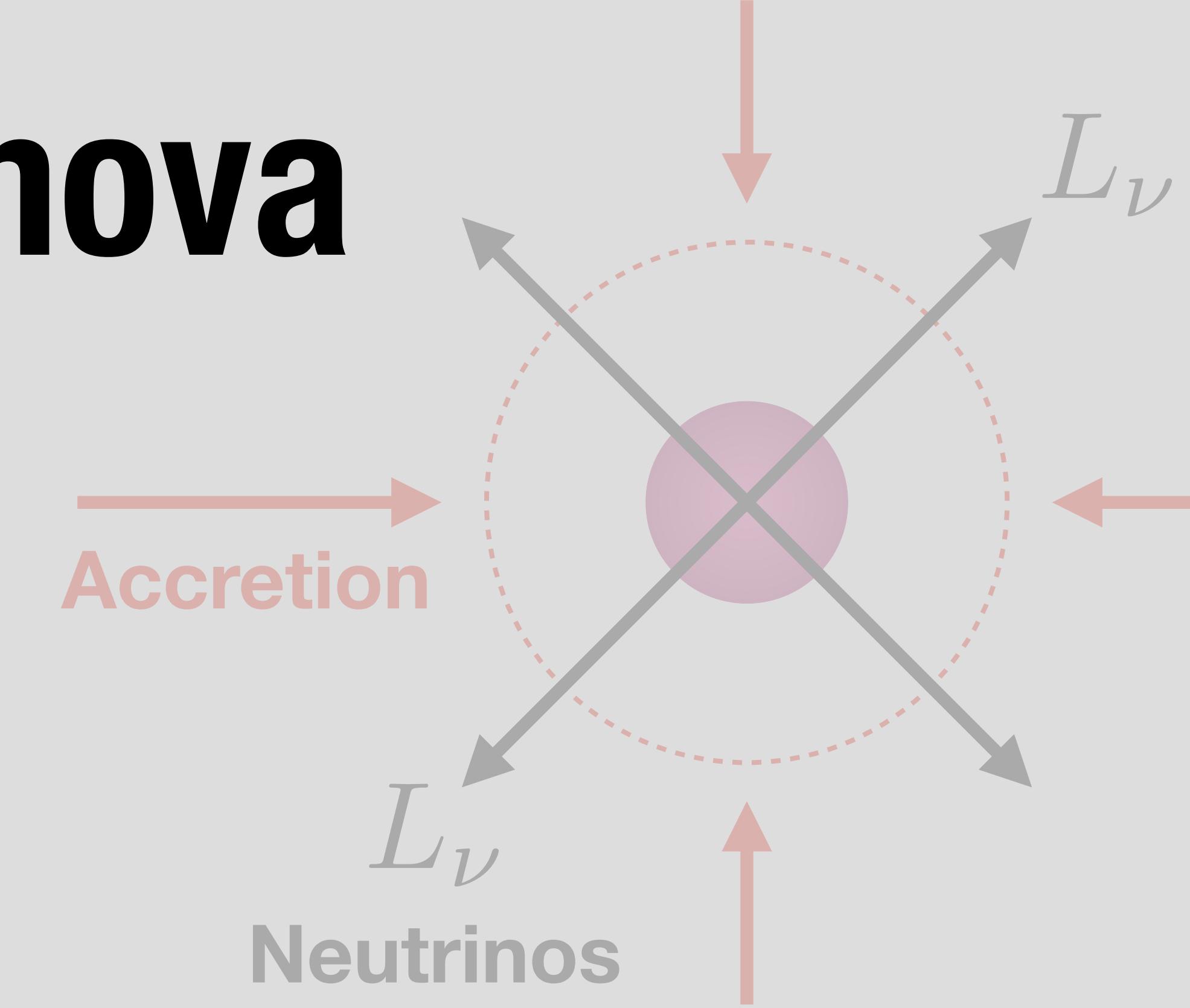
Iron core



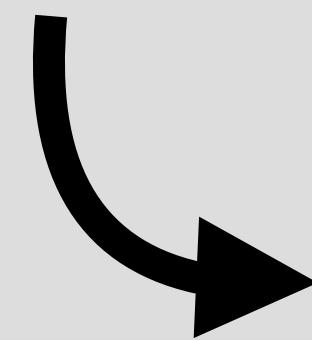
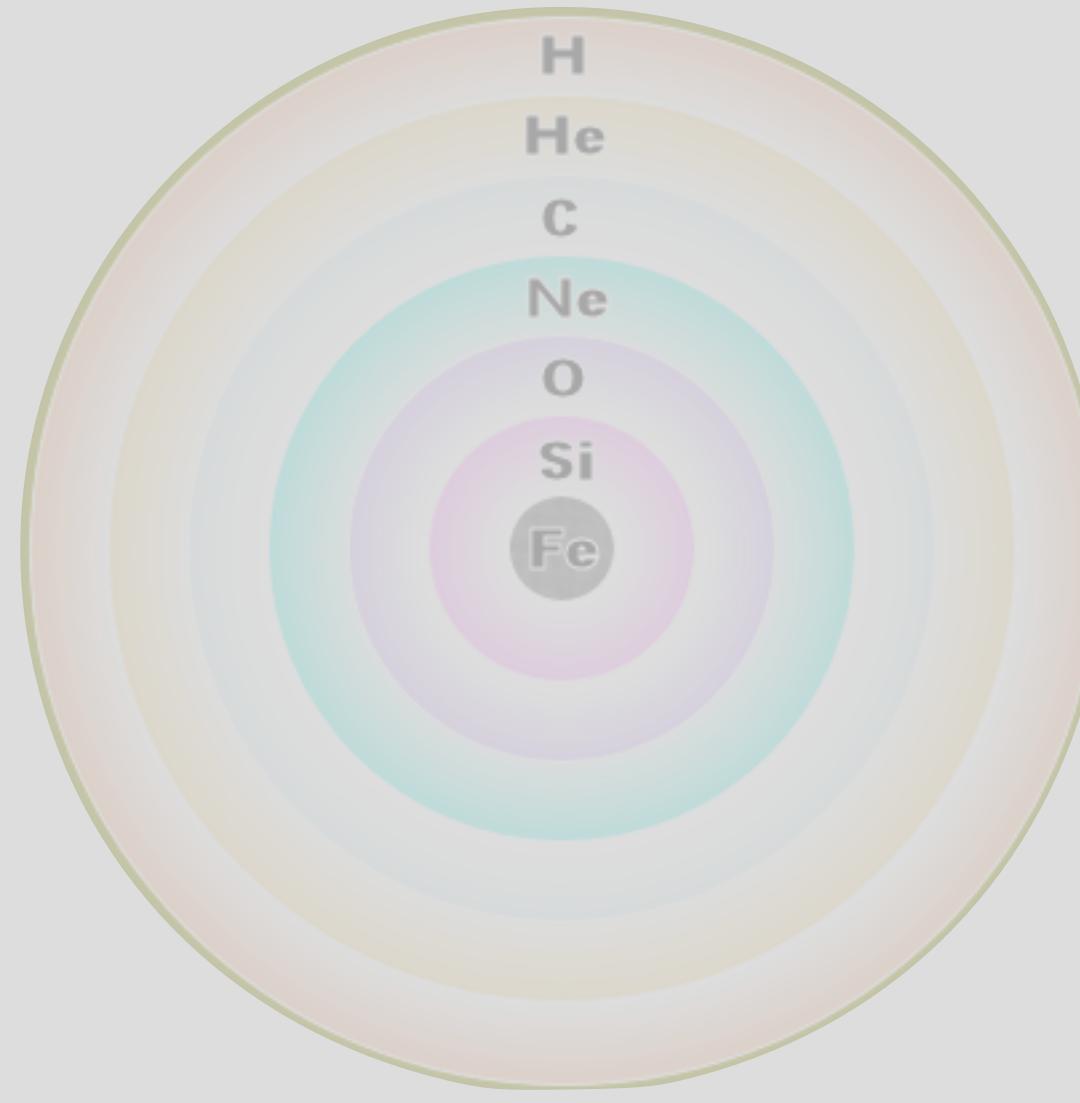
~ 2000 km



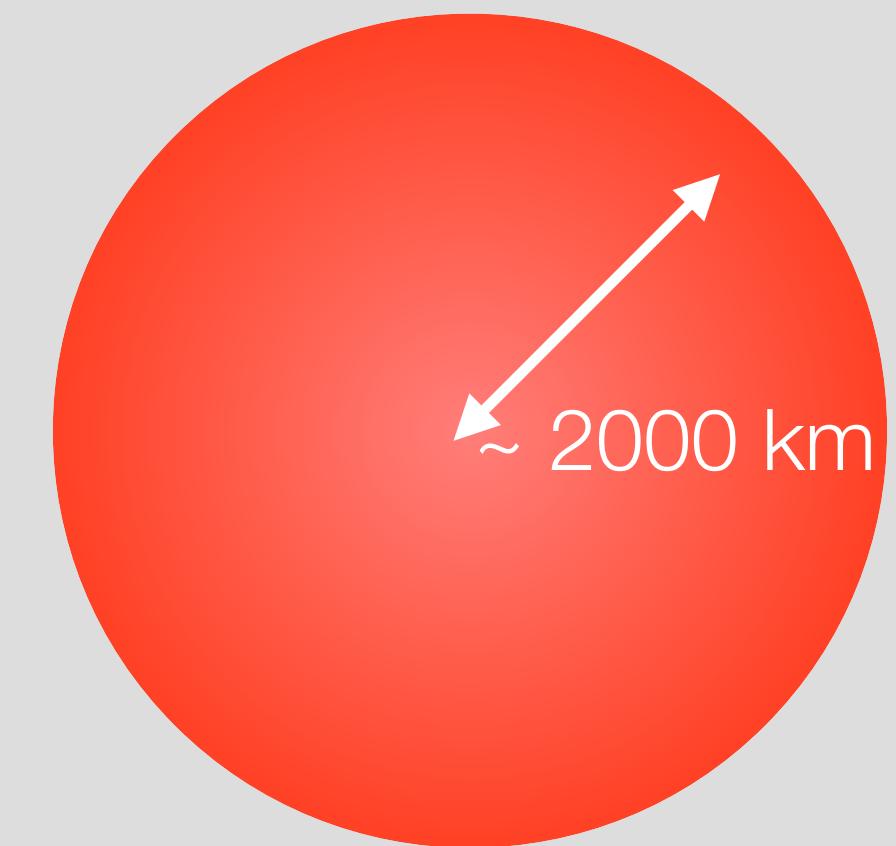
Proto Neutron Star



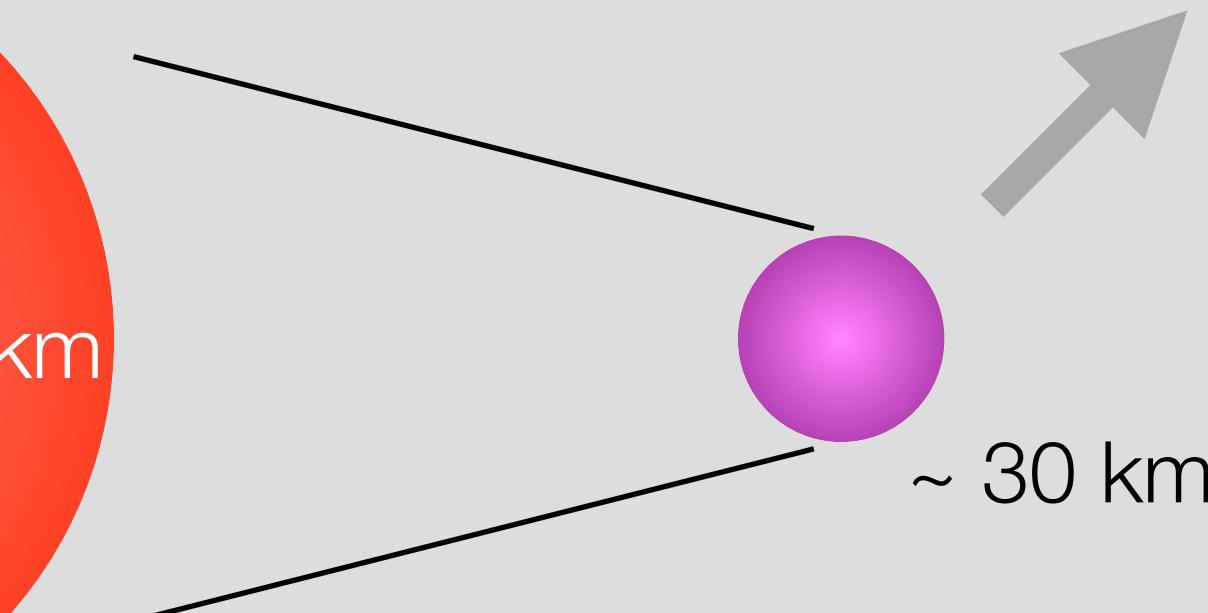
# Core-Collapse Supernova



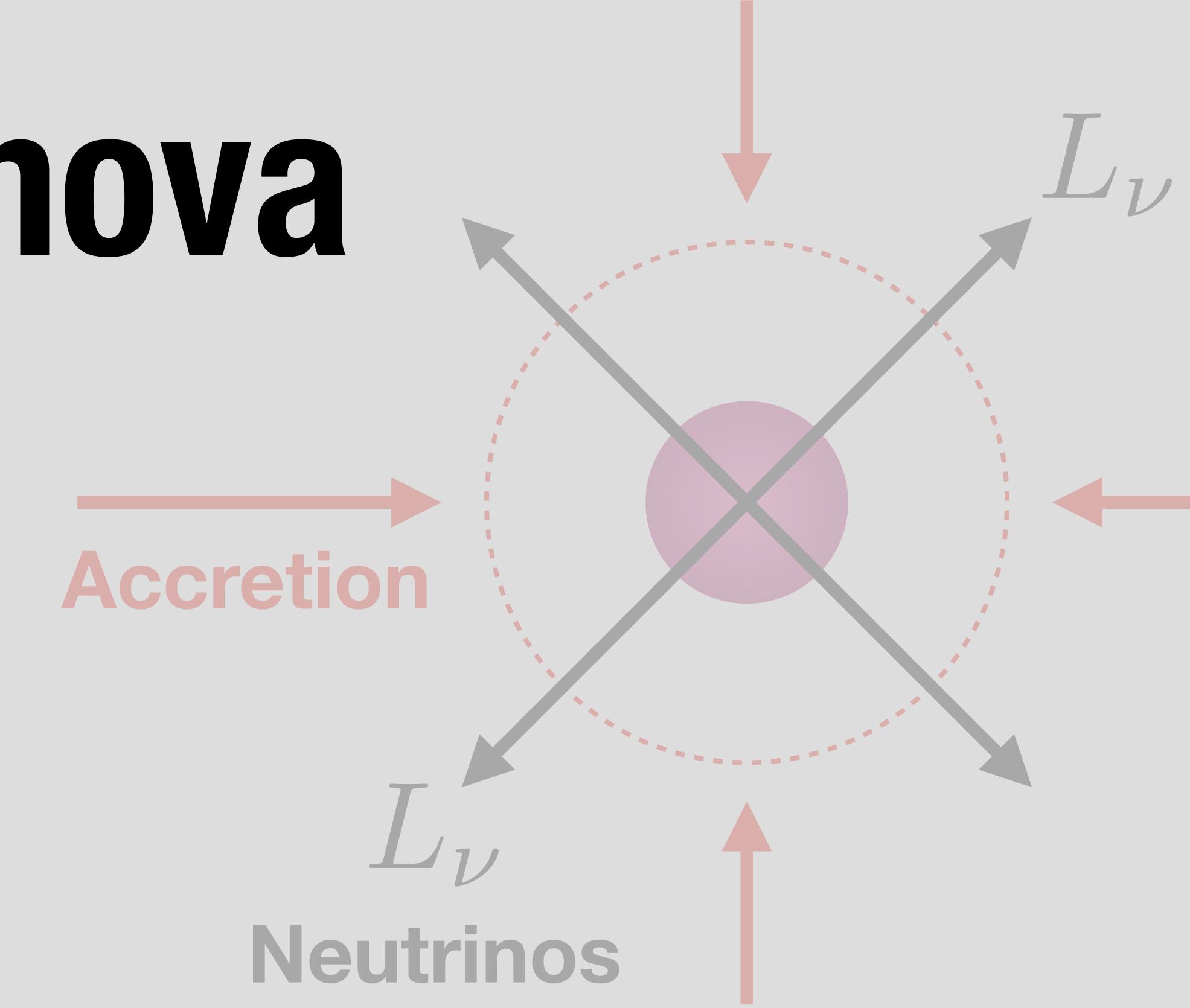
Iron core



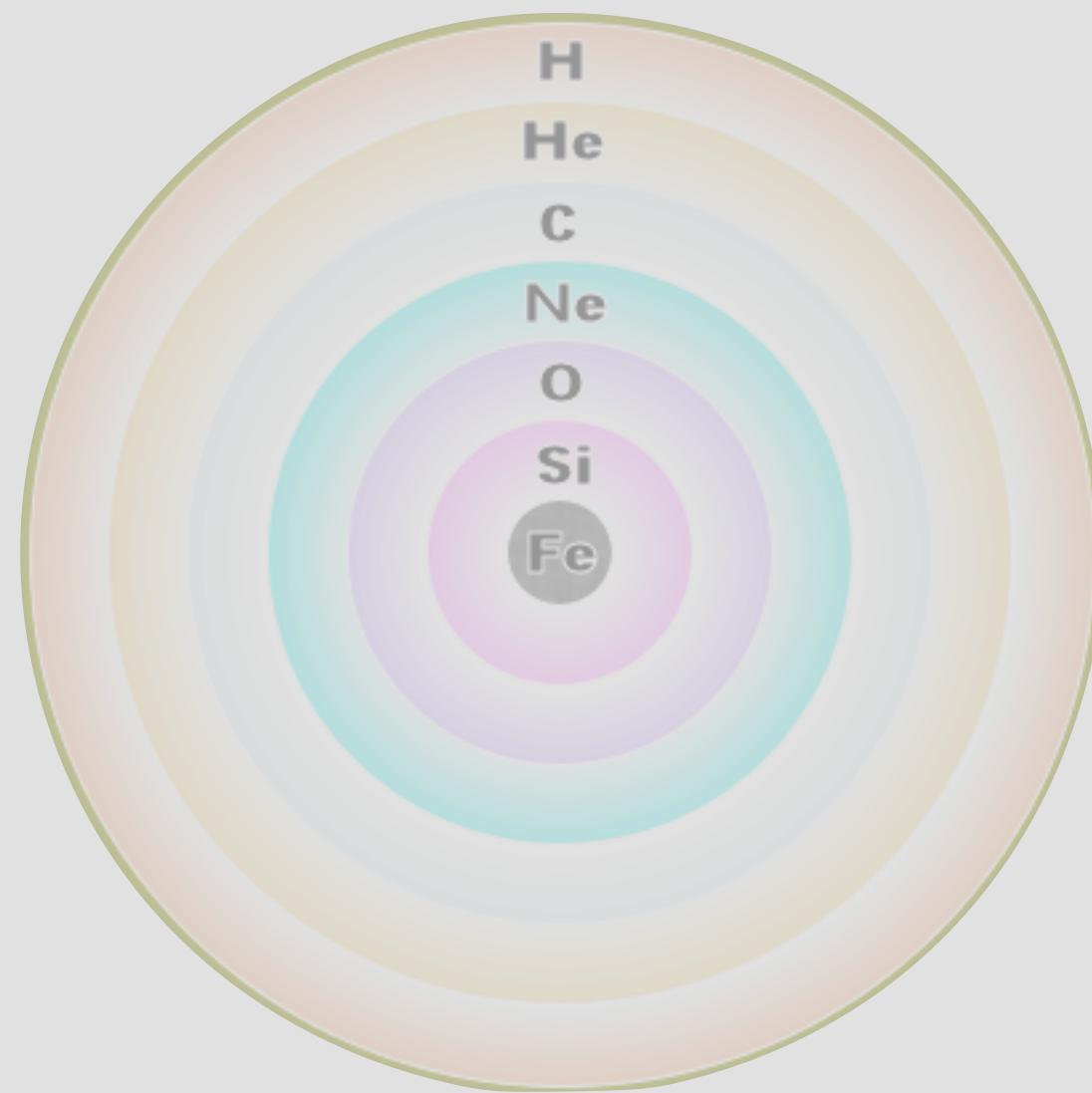
~ 2000 km



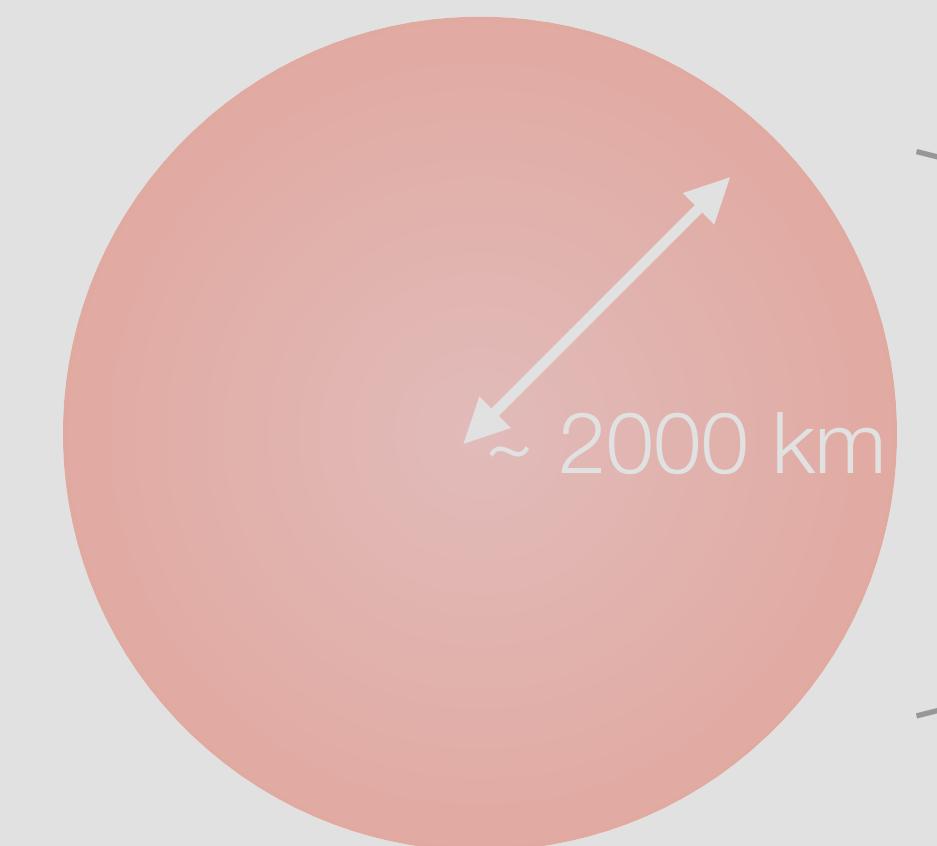
Proto Neutron Star



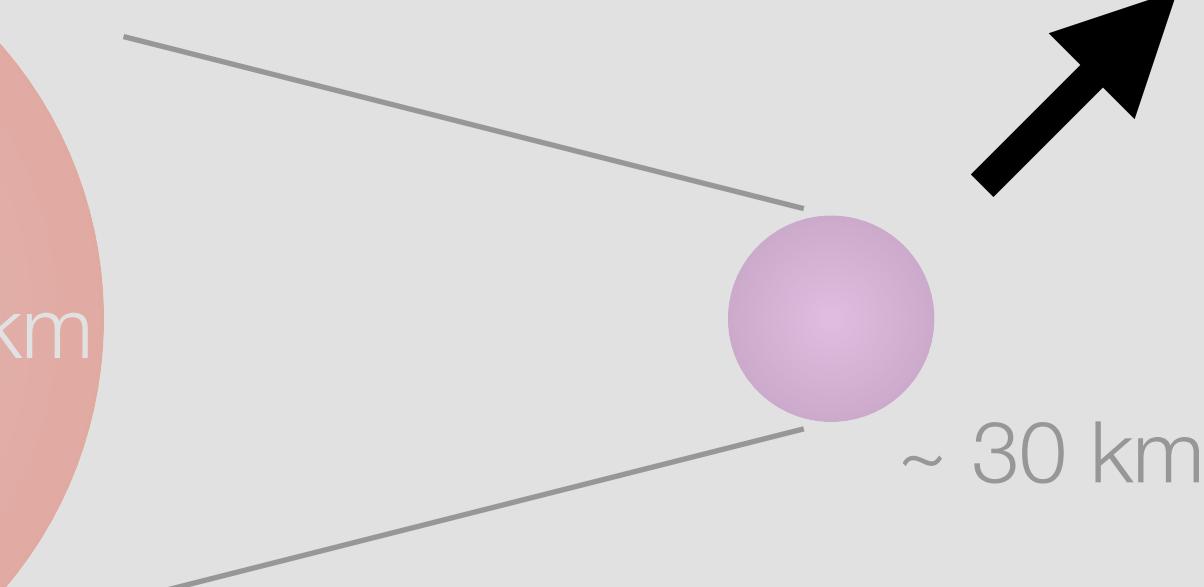
# Core-Collapse Supernova



Iron core



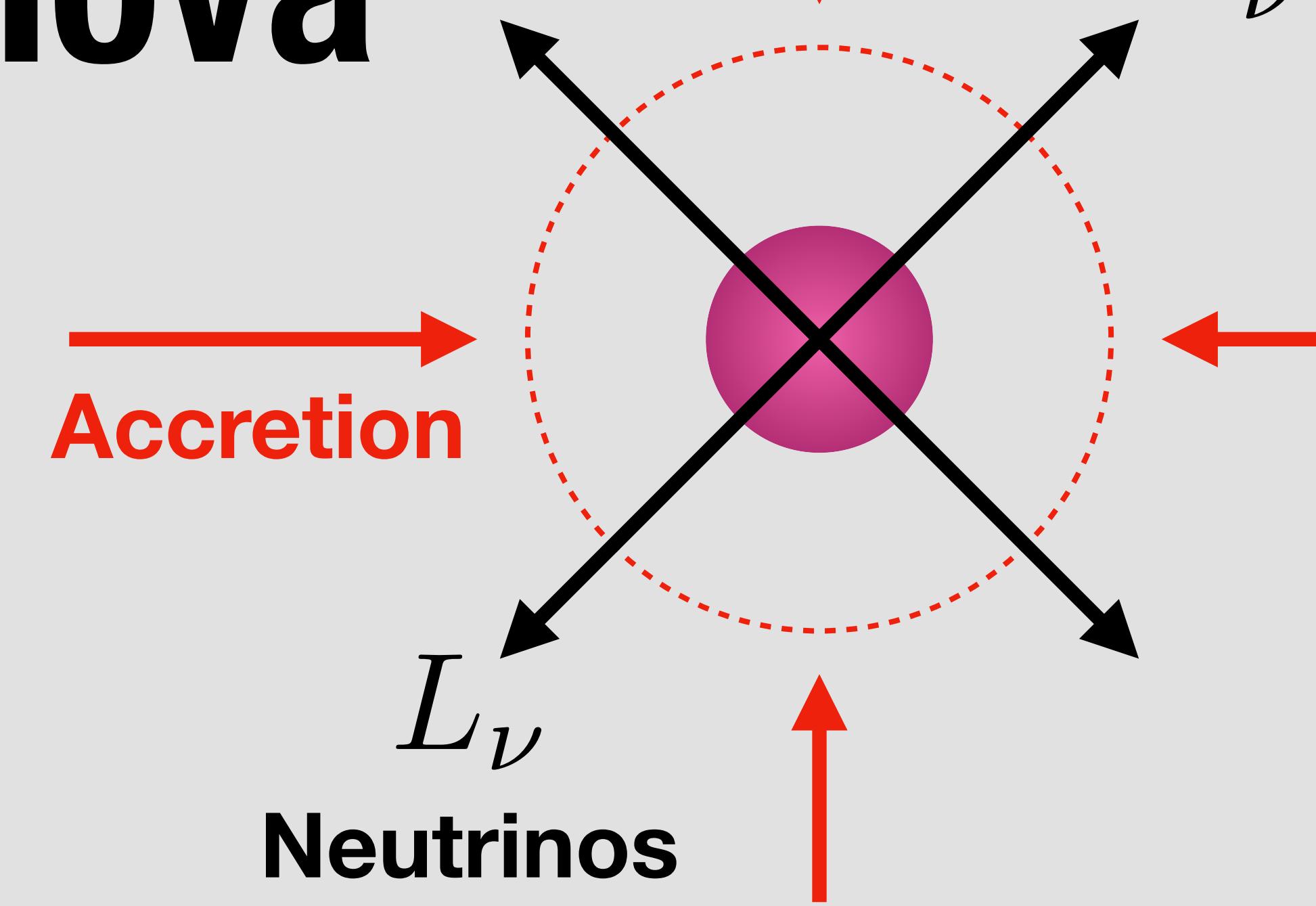
~ 2000 km



Proto Neutron Star

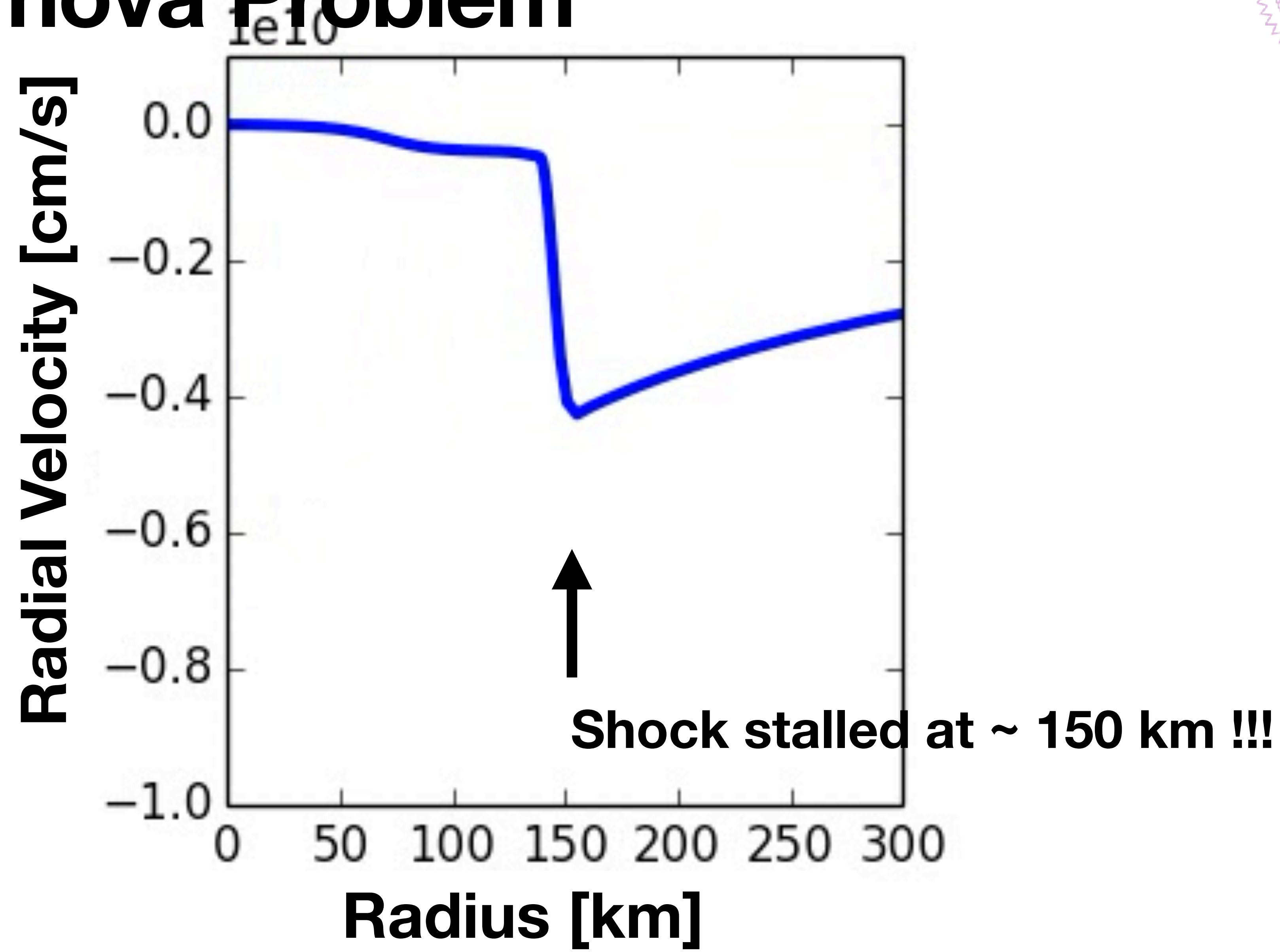
Accretion

$L_\nu$   
Neutrinos



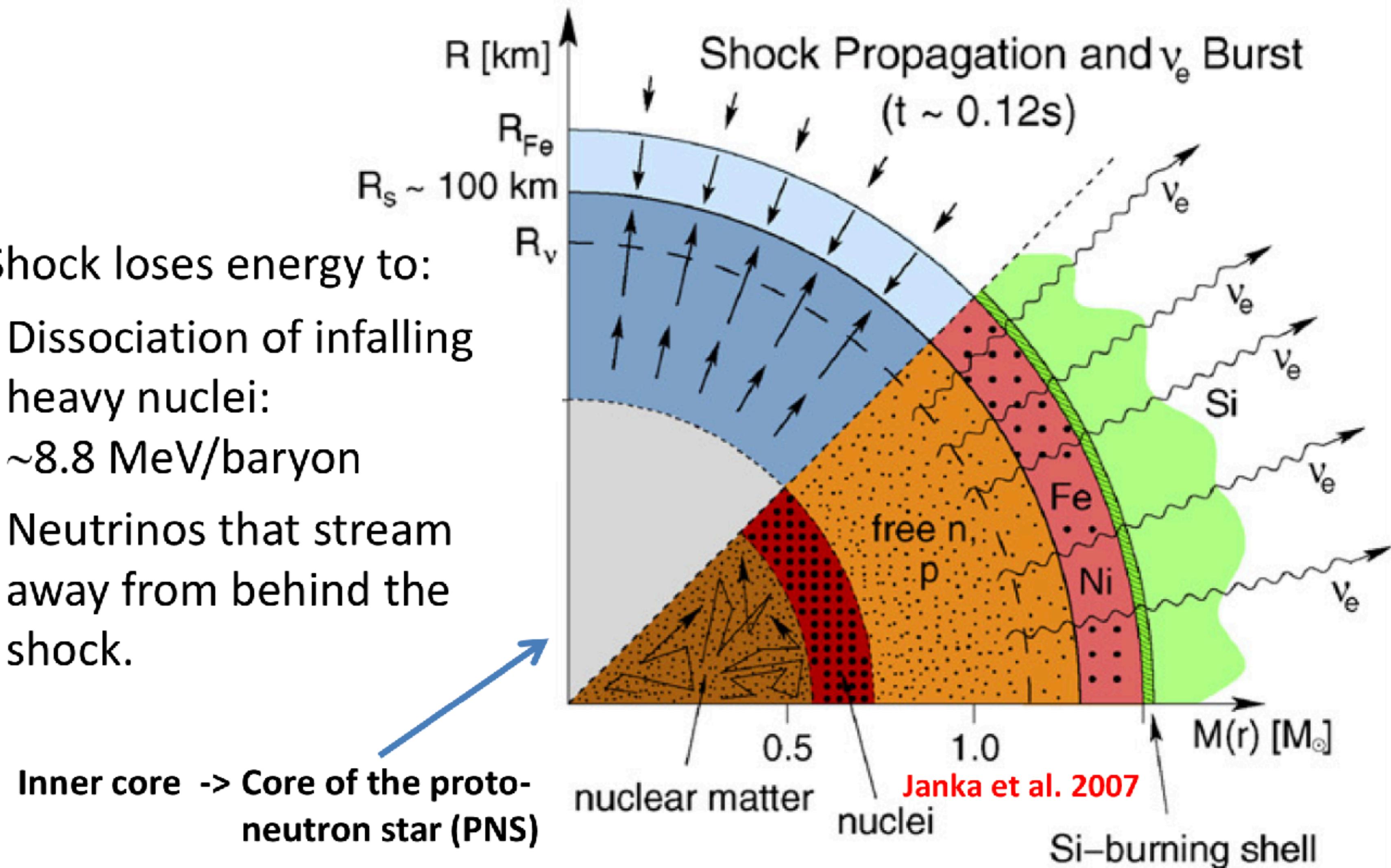


# The Supernova Problem

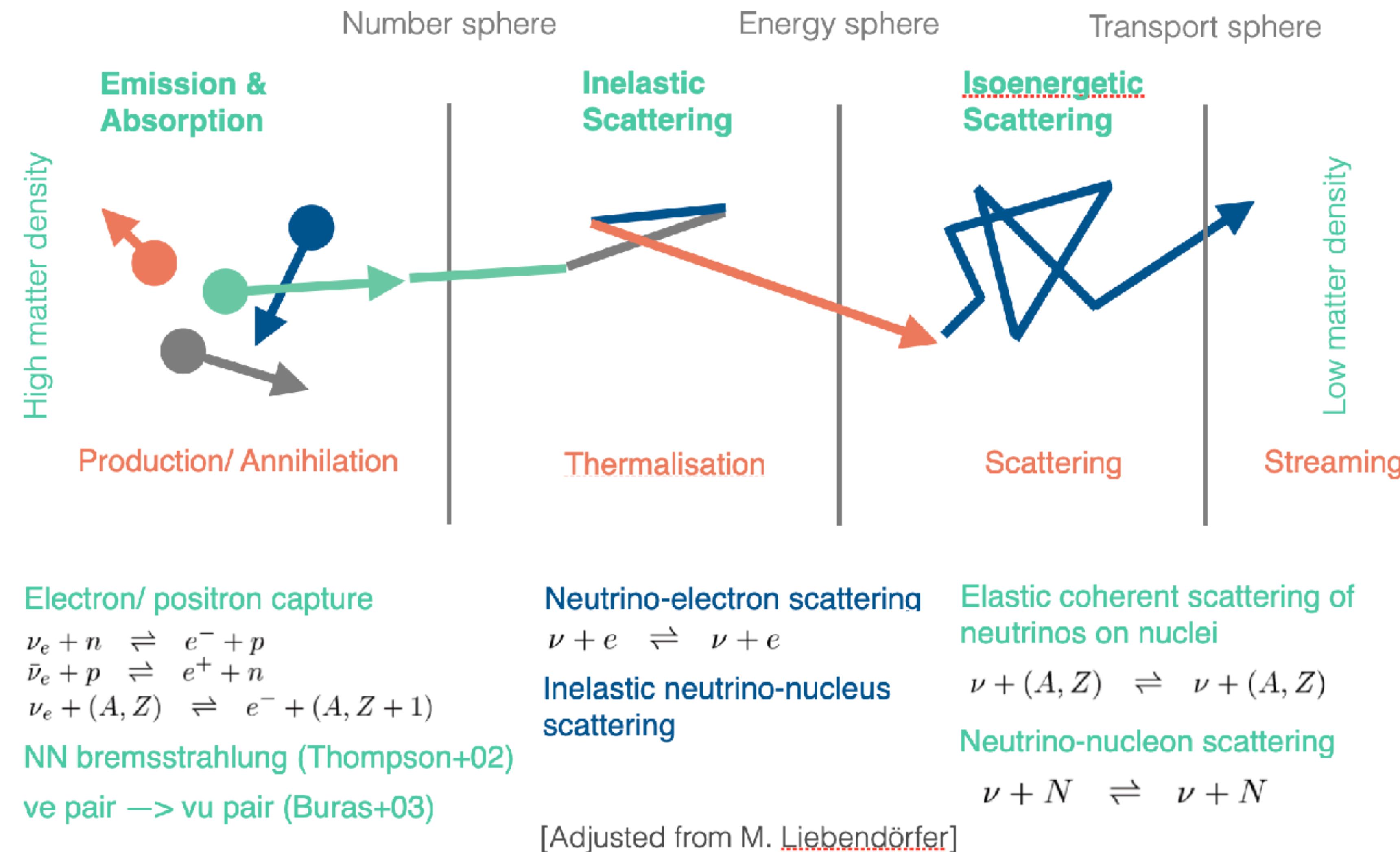


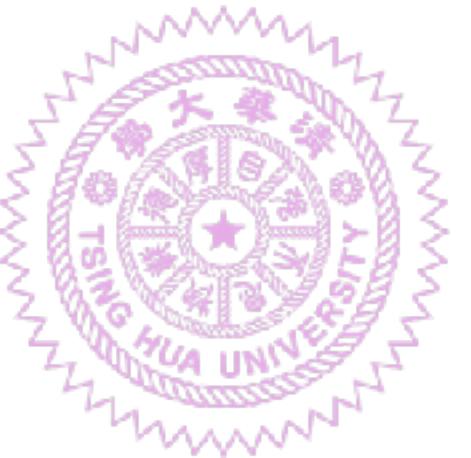
# Why does the shock stall?

- Shock loses energy to:
  - Dissociation of infalling heavy nuclei:  
 $\sim 8.8 \text{ MeV/baryon}$
  - Neutrinos that stream away from behind the shock.



# Neutrino Matter Interactions





# Neutrino Luminosity

- Optical depth

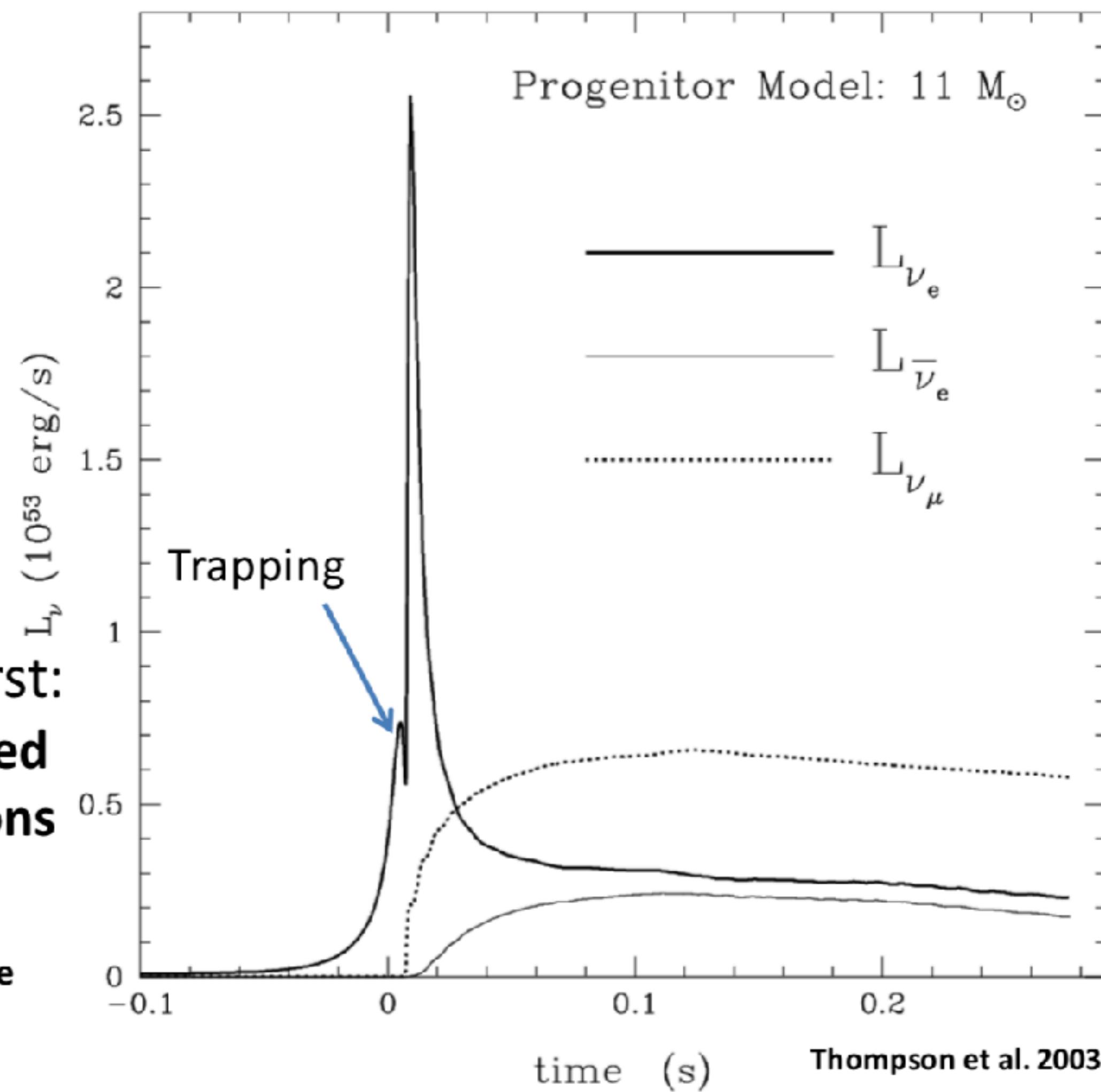
$$\tau_\nu(r) = \int_{\infty}^r \frac{1}{\lambda_\nu} dr'$$

- Neutrinosphere:

$$R_\nu = R \left( \tau_\nu = \frac{2}{3} \right)$$

Depends on  $(\epsilon_\nu)^2$

- Postbounce neutrino burst:  
**Release of neutrinos created by  $e^-$  capture on free protons in shocked region when shock ‘breaks out’ of the  $\nu_e$  neutrinospheres.**





# Supernova mechanism

- Collapse to neutron star  $\sim 300$  B
- 1B kinetic and internal energy of the ejecta (or  $\sim 10$ B for hypernova)
- 99% of the energy is radiated as neutrinos over hundreds of seconds as the protoneutron star cools
- Explosion mechanism must tap the gravitational energy reservoir and convert the necessary fraction into energy of the explosion.



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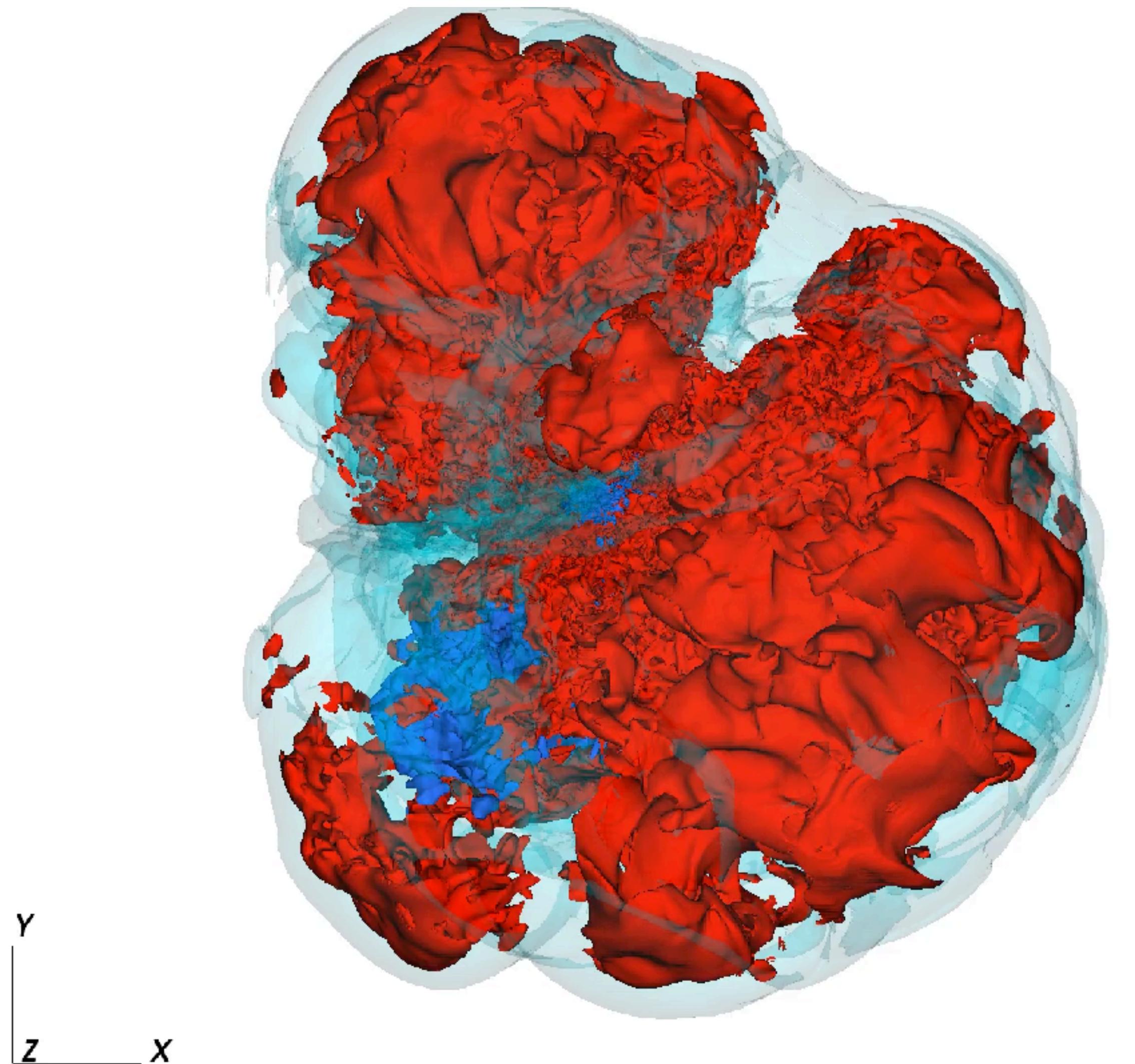
**NARLabs** 財團法人國家實驗研究院  
國家高速網路與計算中心  
National Center for High-performance Computing

# Core-Collapse Supernova Simulation

Visualization: Kuo-Chuan Pan (潘國全)  
Department of Physics  
Institute of Astronomy  
National Tsing Hua University, Taiwan

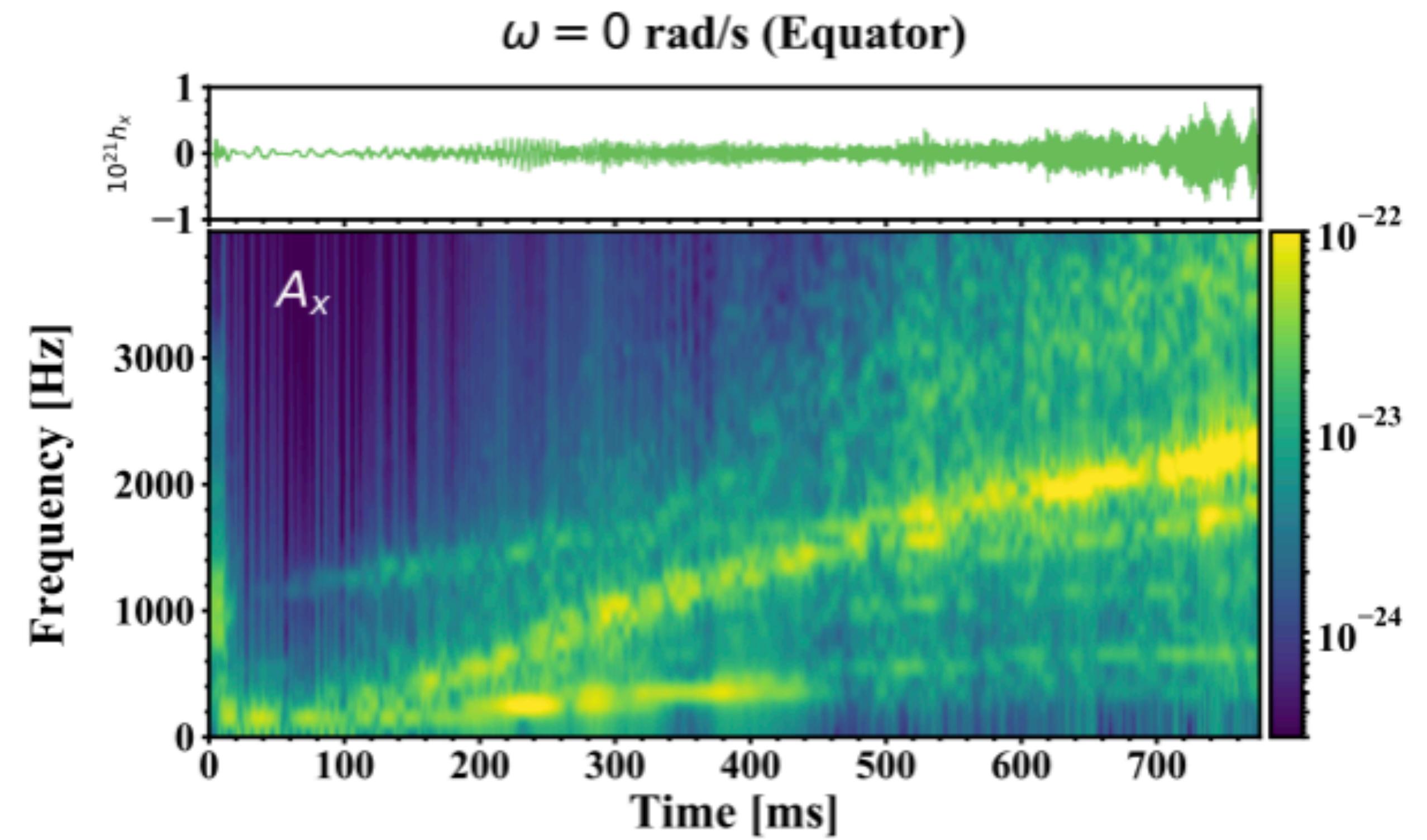
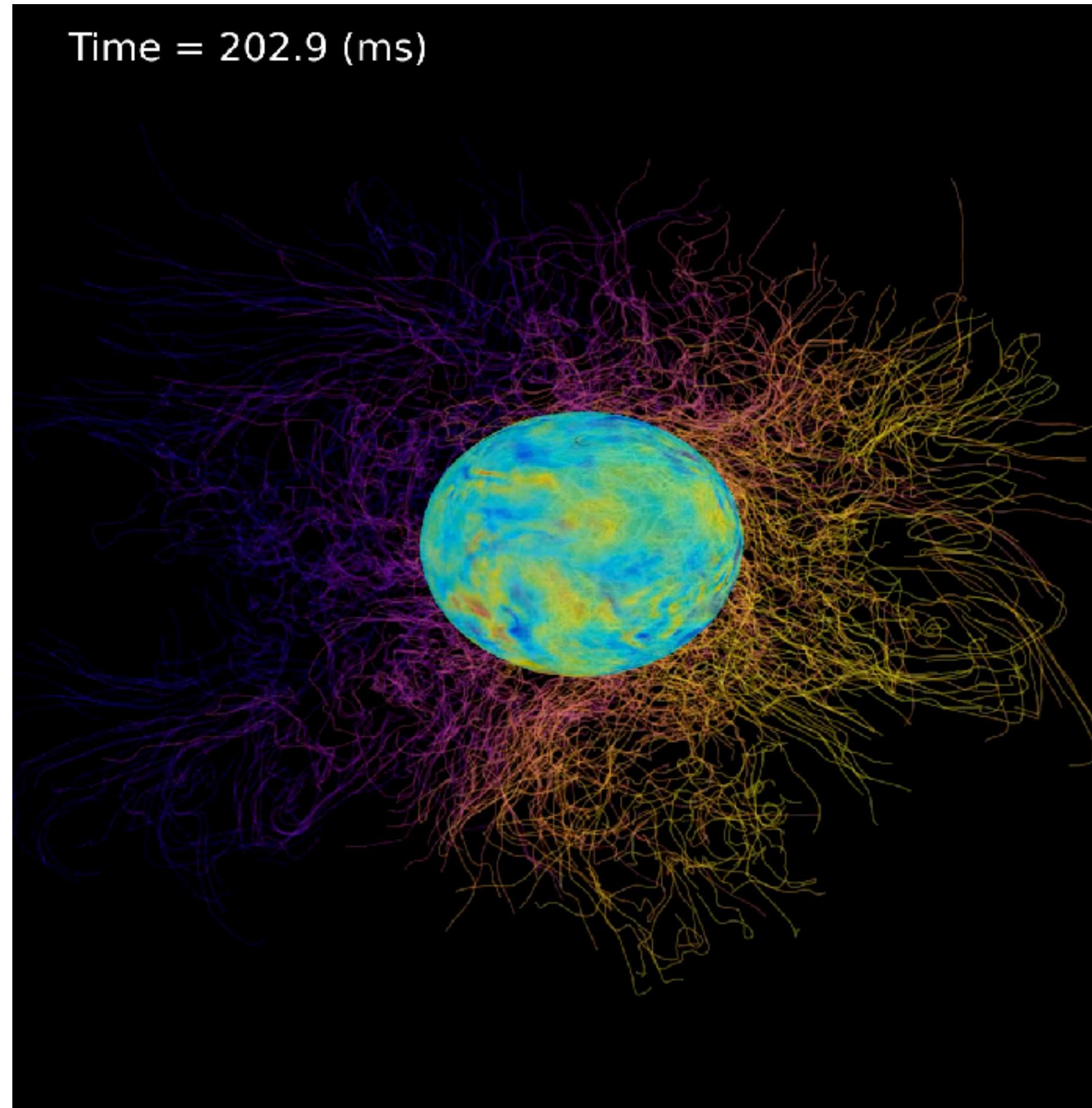


# Compositions





# Gravitational wave from CCSNe





# Summary

ray-tracing

Spontaneous Emission

Black hole shadow

flux-limited diffusion

Star formation atomic data

LTE Scattering process Monte Carlo

Boltzmann equation

# Radiation transport

IDSA Closure Hyperfine Splitting  
microphysics Stimulated emission

neutrino radiation Variable Eddington Tensor

Supernovae Zeeman splitting Absorption coefficient

Multi-group flux limited diffusion

moment methods

neutrino interactions