Astrophysical Fluid Dynamics

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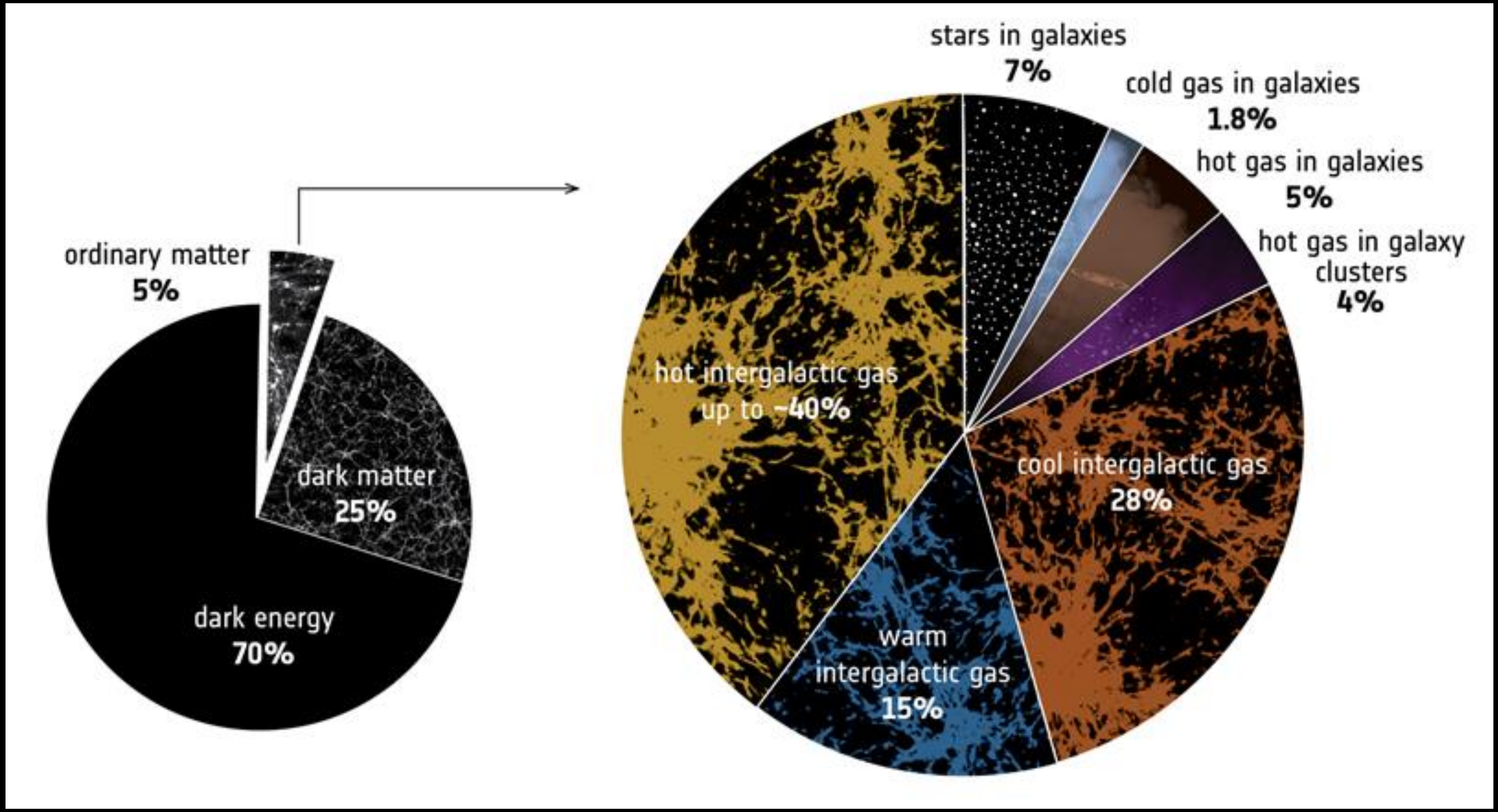
July 1st, 2025 | NCTS SSP mini-workshop

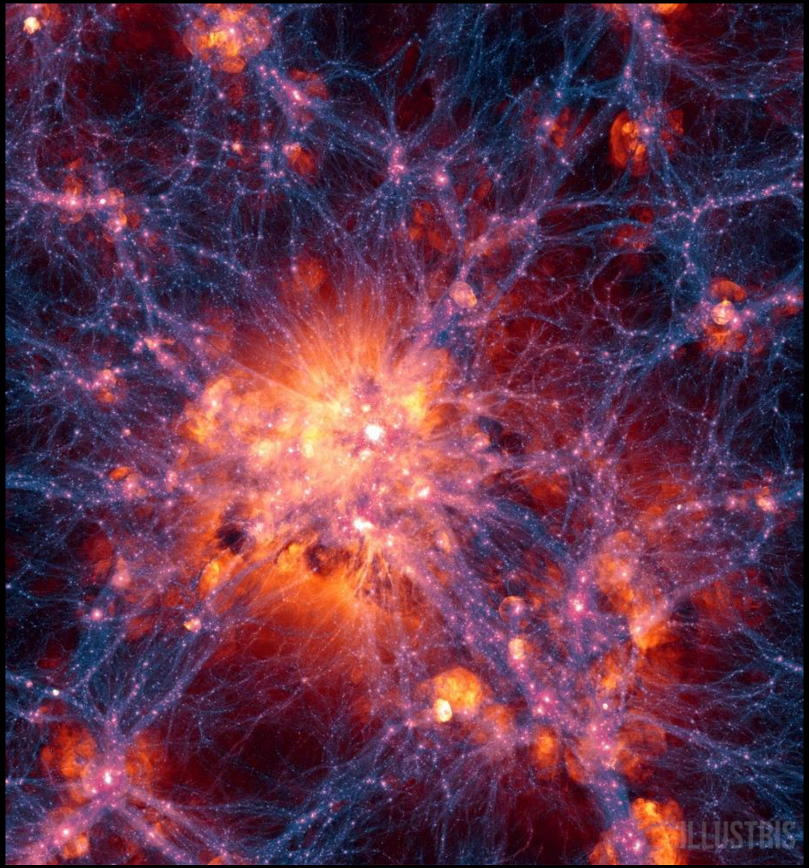
Outline

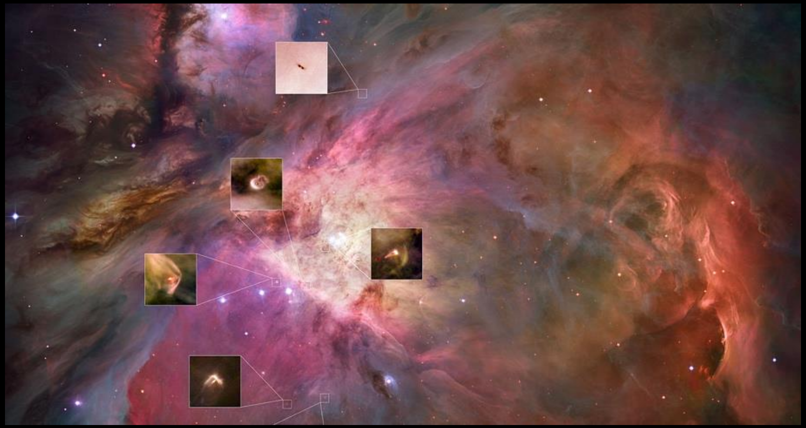
- Governing equations
- Linear waves
- Shocks
- Fluid instabilities
- Turbulence

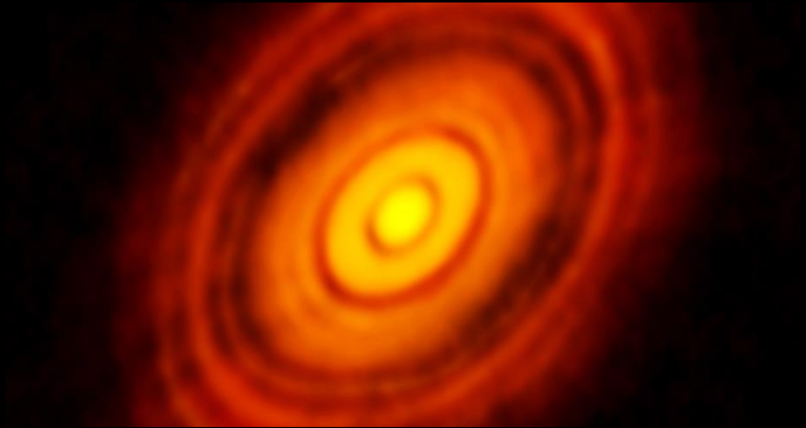
Why Fluid Dynamics?

- The energy budget of the Universe is dominated by dark matter and dark energy. Normal matter only amounts to ~5%.
- More than 90% of the normal matter is in the form of gas!
- The evolution of the cosmic gas is governed by fluid dynamics.









Gas is everywhere in the Universe!

Fluid Description

- A fluid is a continuous medium that

  - can easily deform (gas & liquid)

  - has locally well-defined macroscopic properties (density, temperature, etc.)

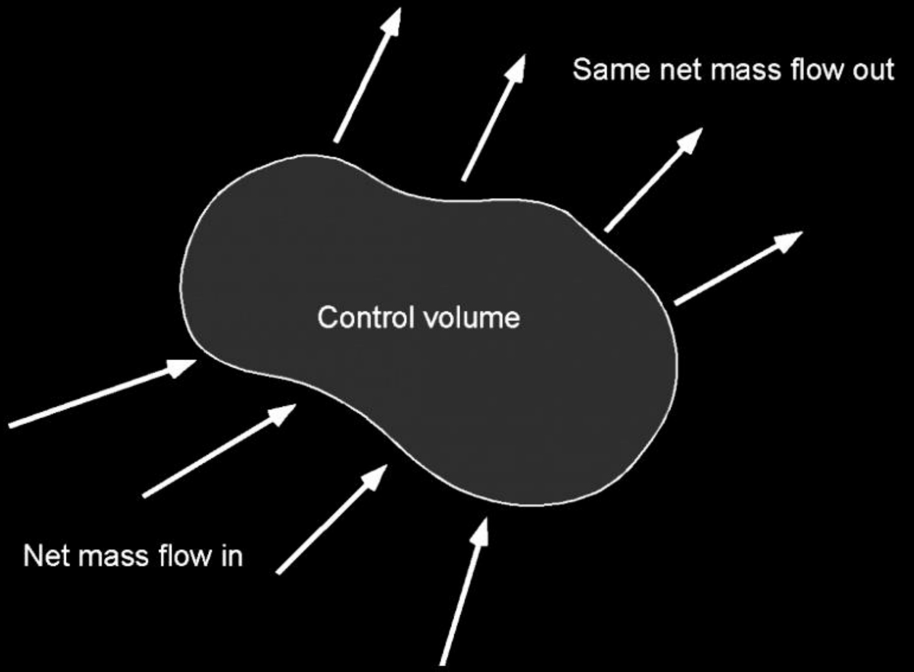
- For the fluid description to hold, we need the dimensionless Knudsen number

$$\boxed{\mathrm{Kn} \equiv \frac{l_{mfp}}{L} \ll 1}$$

collision mean free path

length scale
of interest

Ideal Fluids (Euler Equations)



- An ideal fluid is a fluid that has no dissipation such as viscosity and conduction.
- Governing equations are conservation laws.

Mass conservation:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0,$$

Momentum conservation:

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla(\rho \mathbf{v}\mathbf{v}^T + P) = 0,$$

Energy conservation:

$$\frac{\partial}{\partial t}(\rho e) + \nabla[(\rho e + P)\mathbf{v}] = 0,$$

$$e = \frac{1}{2}|\mathbf{v}|^2 + u$$

Equation of State

Mass conservation (continuity equation):

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0,$$

Momentum conservation:

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla(\rho \mathbf{v}\mathbf{v}^T + P) = 0,$$

Energy conservation:

$$\frac{\partial}{\partial t}(\rho e) + \nabla[(\rho e + P)\mathbf{v}] = 0,$$

- 6 unknowns, 5 equations
- We need an additional equation to close the system.
- Equation of state:

$$\boxed{P = (\gamma - 1)\rho u}$$

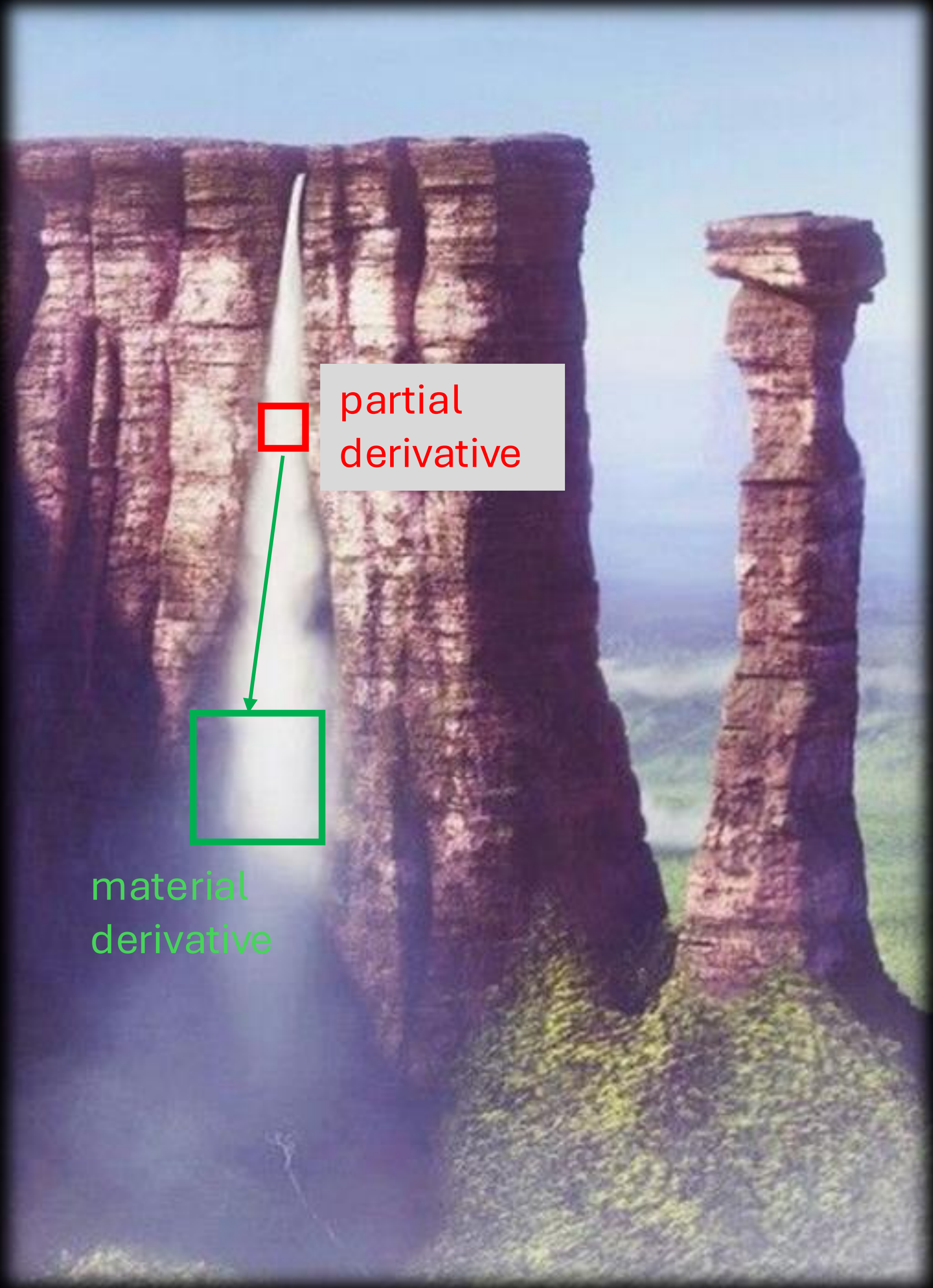
Material Derivative

- Material derivative: the rate of change of a property measured on a frame comoving with the fluid element.

$$\boxed{\mathrm{d}/\mathrm{d}t} = \boxed{\partial/\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}$$

material derivative   partial derivative

- Also known as:
  convective derivative,
  Lagrangian derivative,
  substantial derivative,
  total derivative, …



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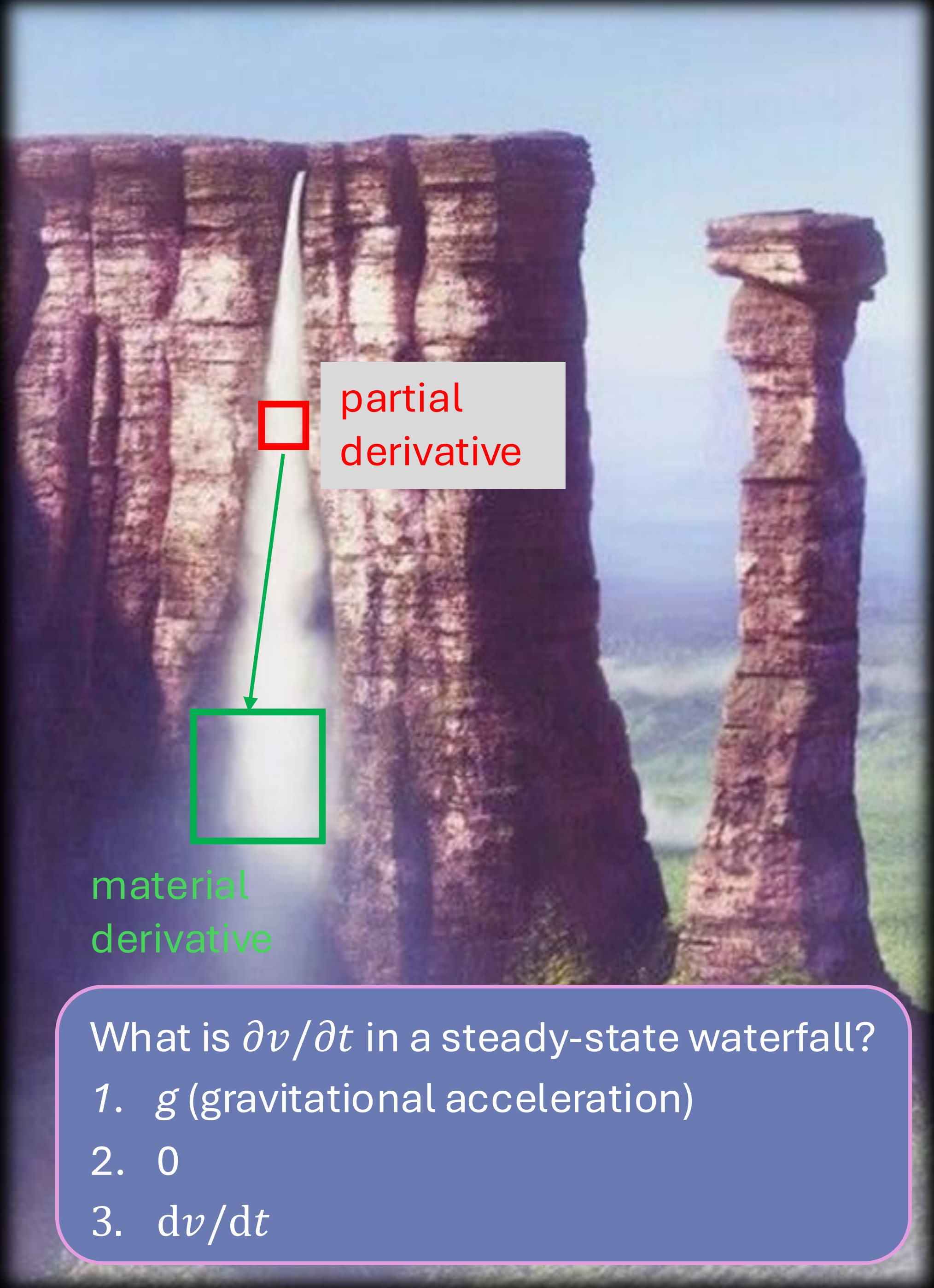
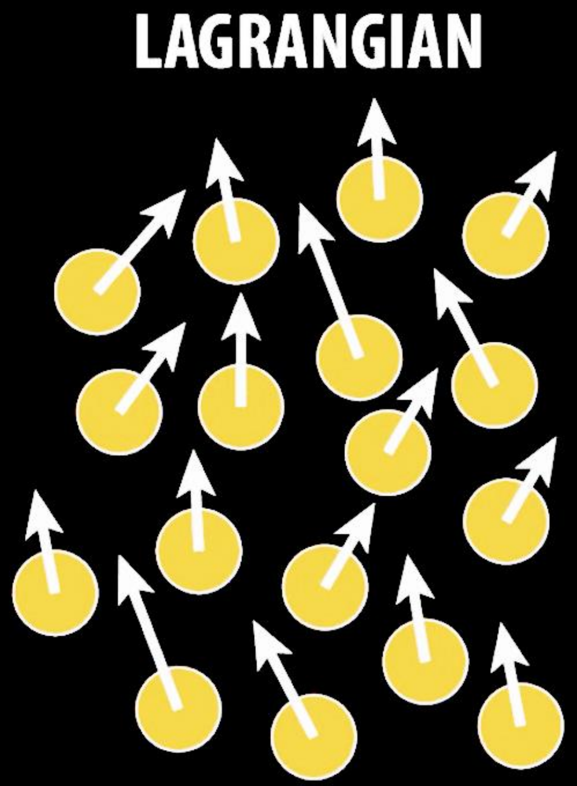
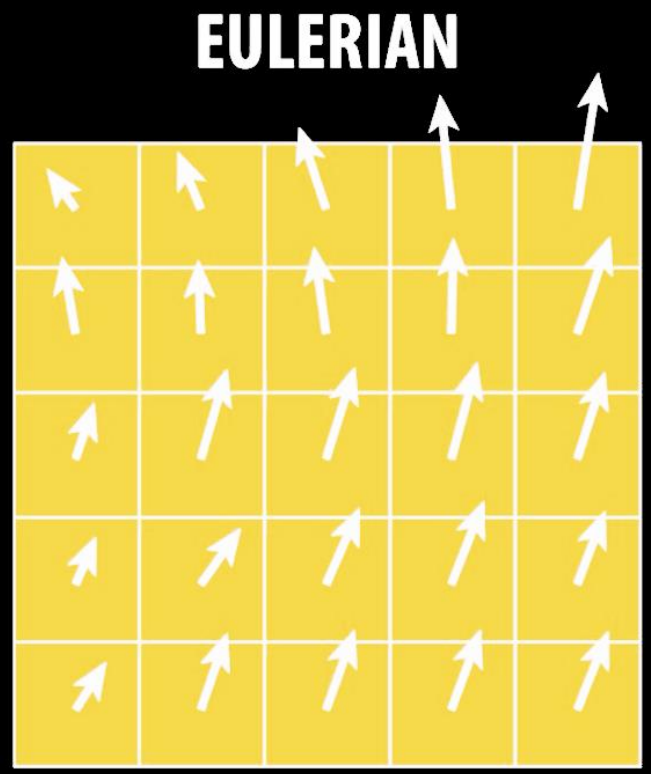


What is $\partial v/\partial t$ in a steady-state waterfall?
1. g (gravitational acceleration)
2. 0
3. $\mathrm{d}v/\mathrm{d}t$





Eulerian vs. Lagrangian viewpoints

- Two different ways to description the evolution of the fluid.

  - Eulerian: follow the evolution of a fluid at a fixed location

  - Lagrangian: follow the evolution of a fluid element

Euler equations: Lagrangian formulation

- Using material derivative, the Euler equations can be cast into the following form:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0,$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla(\rho \mathbf{v}\mathbf{v}^T + P) = 0,$$

$$\frac{\partial}{\partial t}(\rho e) + \nabla[(\rho e + P)\mathbf{v}] = 0,$$

Eulerian (stationary)

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v},$$

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho},$$

$$\frac{du}{dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v},$$

Lagrangian (comoving)

Euler equations: Lagrangian formulation

- The Lagrangian formulation has simple physical interpretations:

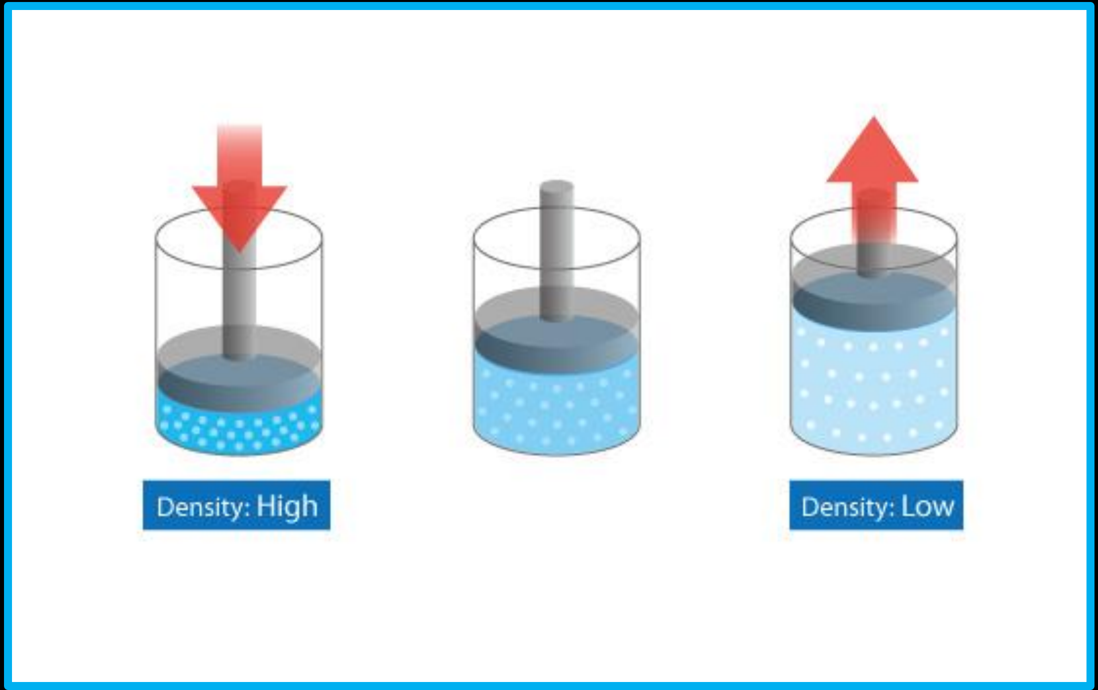
$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{v},$$ expansion rate of fluid

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho},$$ Newton's 2nd law (f=ma)

$$\frac{du}{dt} = -\frac{P}{\rho}\nabla\cdot\mathbf{v},$$ adiabatic heating/cooling

Incompressible flows

- Liquid (e.g. water) can be regarded as incompressible.
  - It is hard to change its density by squeezing it.

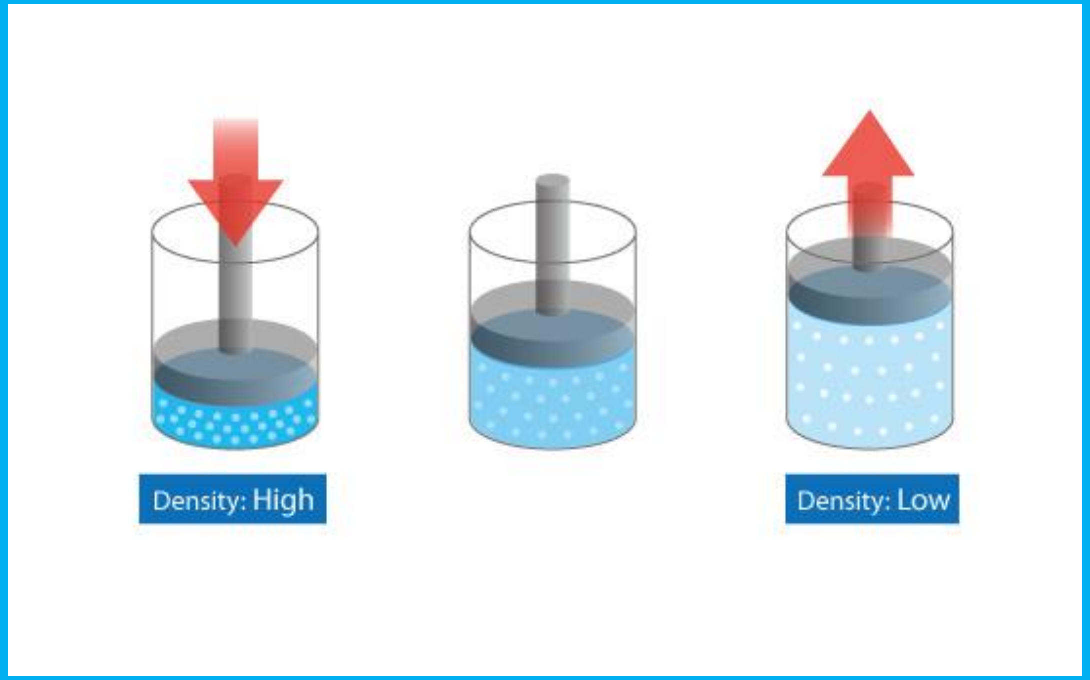


Incompressible flows

- Liquid (e.g. water) can be regarded as incompressible.
  - It is hard to change its density by squeezing it.
- What about gas?

Is gas incompressible?

1. Yes
2. No
3. It depends



Incompressible flows

Momentum conservation:

$$\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{v/T} + \underbrace{(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v}}_{v^2/L} = -\underbrace{\frac{\boldsymbol{\nabla}P}{\rho}}_{\delta P/\rho L} \quad \Rightarrow \quad \delta P/\rho \sim v^2$$

Variations of pressure and density are related via:

$$\delta P \sim C^2 \delta\rho$$

$$C = \sqrt{(\partial P/\partial \rho)_s}$$ (sound speed)

$$\boxed{\frac{\delta\rho}{\rho} \sim \frac{v^2}{C^2}}$$

Sub-sonic flows are incompressible!

Incompressible flows

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Sub-sonic flows are incompressible!

Continuity equation:

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{v}, \Rightarrow \boxed{\boldsymbol{\nabla}\cdot\mathbf{v} \simeq 0}$$

incompressible approximation

Viscous fluids

- In reality, fluids have non-zero viscosity.

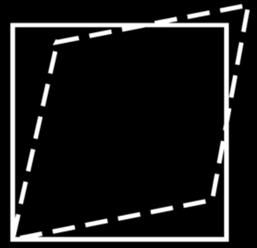
- Viscosity = stickiness
  - Water: low viscosity
  - Honey: high viscosity

Viscous stress tensor:

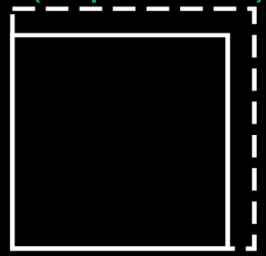
$$\mathbf{\Pi} = \eta \left[\boxed{\nabla \mathbf{v} + (\nabla \mathbf{v})^T - \frac{2}{3}(\nabla \cdot \mathbf{v})\mathbf{1}} \right] + \xi(\boxed{\nabla \cdot \mathbf{v}})\mathbf{1},$$

$$\boxed{\boldsymbol{\omega} \equiv \boldsymbol{\nabla} \times \mathbf{v}}$$

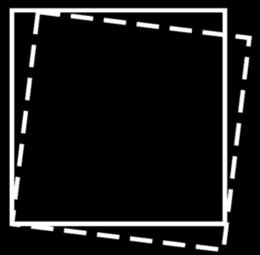
shear (distortion)



divergence (expansion)



vorticity (rotation)



Governing Equations for Viscous Fluids

Viscous stress tensor: $$\mathbf{\Pi} = \eta \left[\nabla \mathbf{v} + (\nabla \mathbf{v})^T - \frac{2}{3}(\nabla \cdot \mathbf{v})\mathbf{1} \right] + \xi (\nabla \cdot \mathbf{v})\mathbf{1},$$

- Microscopically, viscosity is caused by friction which transforms kinetic energy into heat.

  - a dissipative, irreversible process!

Mass conservation: $$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0,$$

Momentum conservation: $$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla(\rho \mathbf{v}\mathbf{v}^T + P) = \underline{\underline{\nabla\, \mathbf{\Pi}}},$$

Energy conservation: $$\frac{\partial}{\partial t}(\rho e) + \nabla[(\rho e + P)\mathbf{v}] = \underline{\underline{\nabla(\mathbf{\Pi}\mathbf{v})}}.$$

Navier–Stokes equation

Reynolds Number

- We can make the Navier-Stokes equation dimensionless using the characteristic length, velocity, and density:

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{V_0}, \qquad \hat{\mathbf{x}} = \frac{\mathbf{x}}{L_0}, \qquad \hat{P} = \frac{P}{\rho_0 V_0^2}.$$

$$\hat{t} = \frac{t}{L_0/V_0}, \qquad \hat{\rho} = \frac{\rho}{\rho_0}, \qquad \hat{\nabla} = L_0 \nabla.$$

$$\Rightarrow \quad \boxed{\frac{D\hat{\mathbf{v}}}{D\hat{t}} = -\frac{\hat{\nabla}\hat{P}}{\hat{\rho}} + \frac{\nu}{L_0 V_0}\hat{\nabla}^2\hat{\mathbf{v}}.}$$

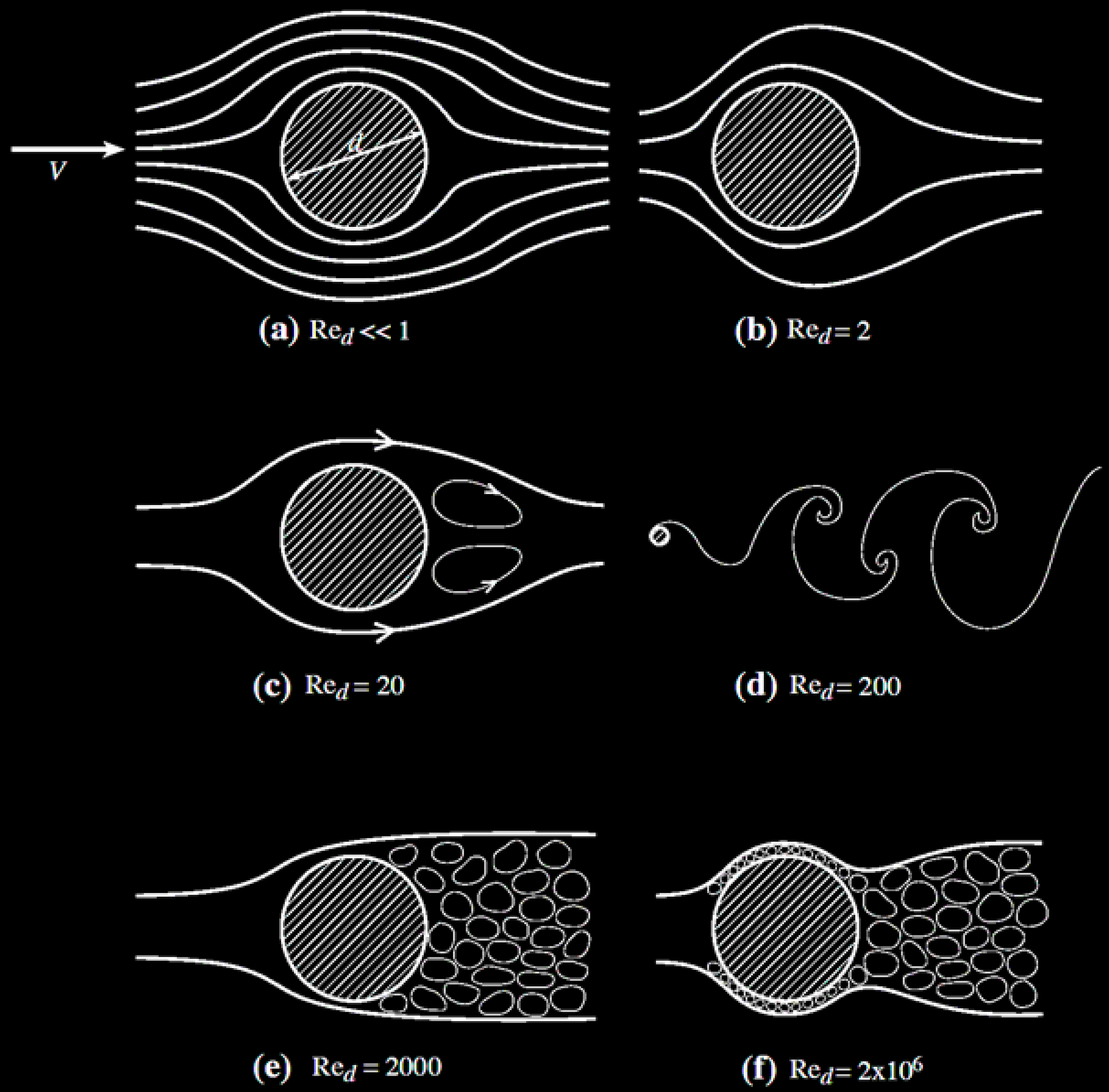
- The Reynolds number characterizes the relative importance of viscosity.

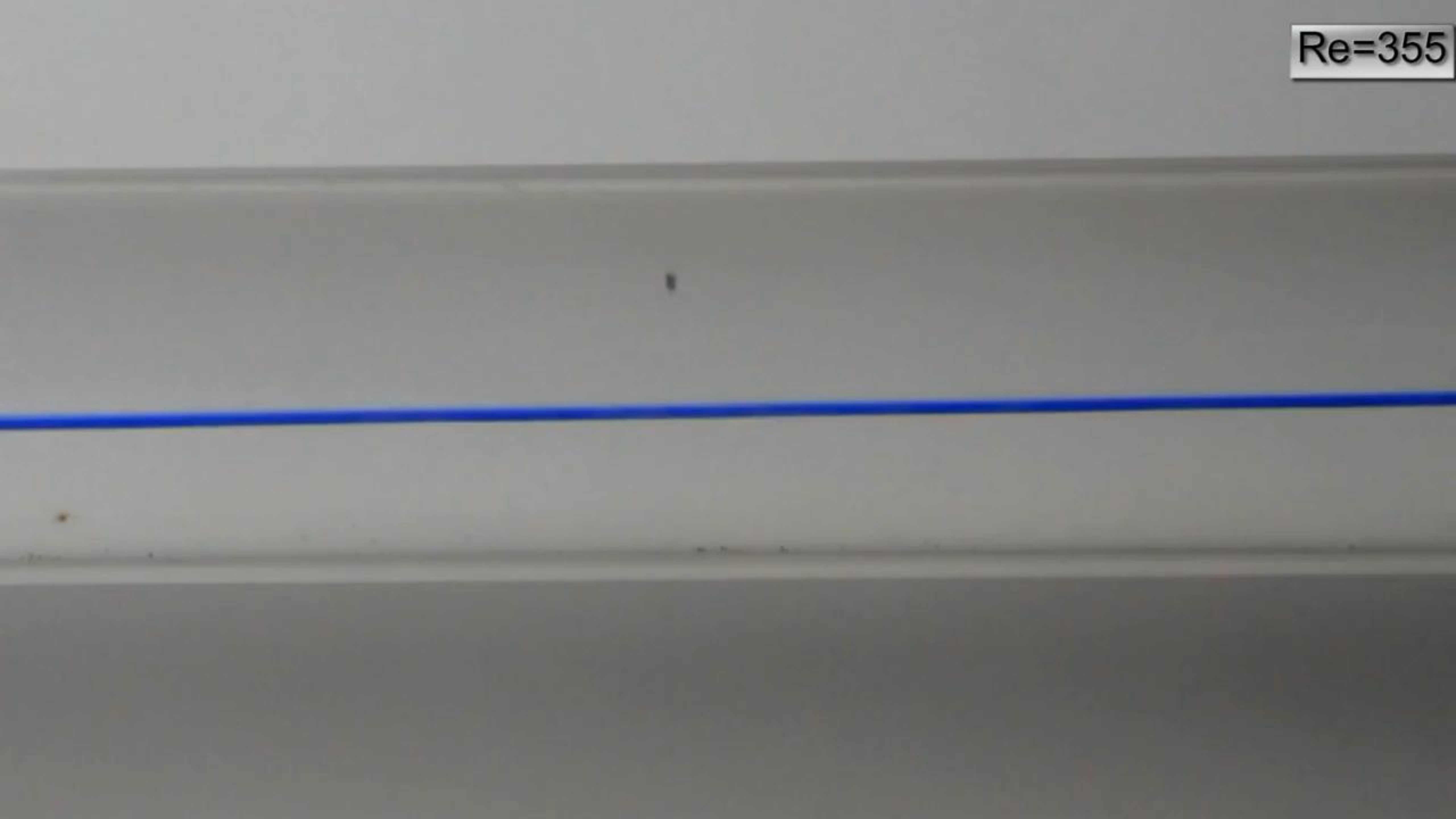
$$\mathrm{Re} \approx \frac{\text{inertial forces}}{\text{viscous forces}} \approx \frac{D\mathbf{v}/Dt}{\nu\nabla^2\mathbf{v}} \approx \frac{V_0/(L_0/V_0)}{\nu V_0/L_0^2} = \frac{L_0 V_0}{\nu}.$$

- In astrophysics, the spatial scales are so large that viscosity can usually be neglected ($\mathrm{Re} \gg 1$)!

Transition from Laminar to Turbulent Flows

The Reynolds number controls the characteristics of the flow.




Re=355

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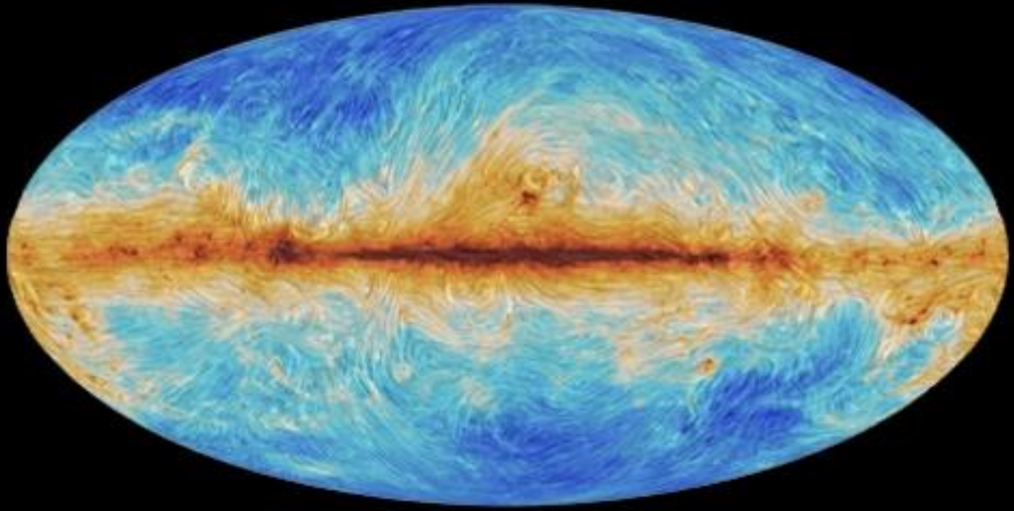
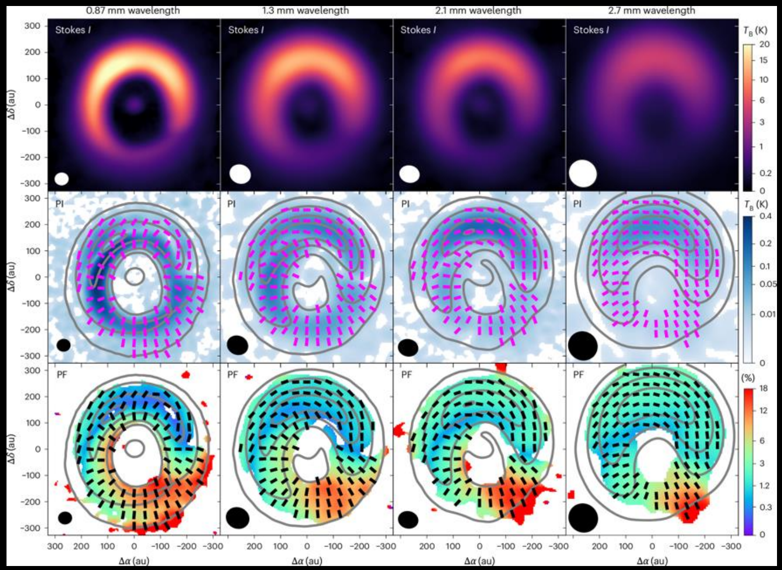
Turbulence is ubiquitous in astrophysics!





Magnetohydrodynamics (MHD)

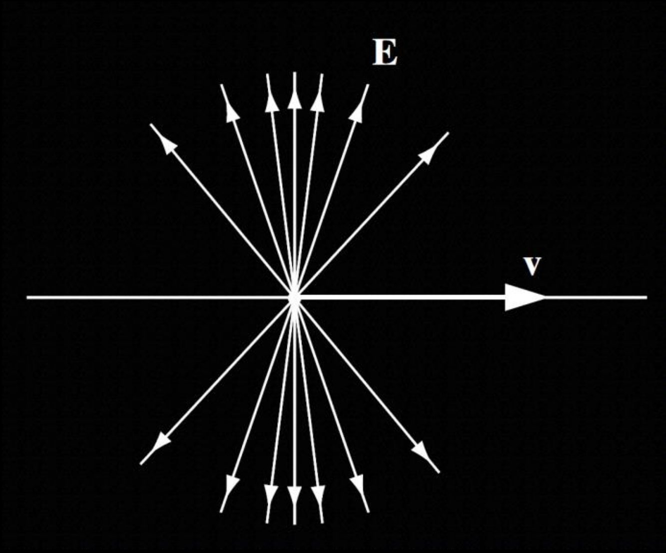
- In ionized gas (plasma), the gas is an electrically conducting fluid.
- Charged current leads to magnetic fields, which affect the dynamics of gas.

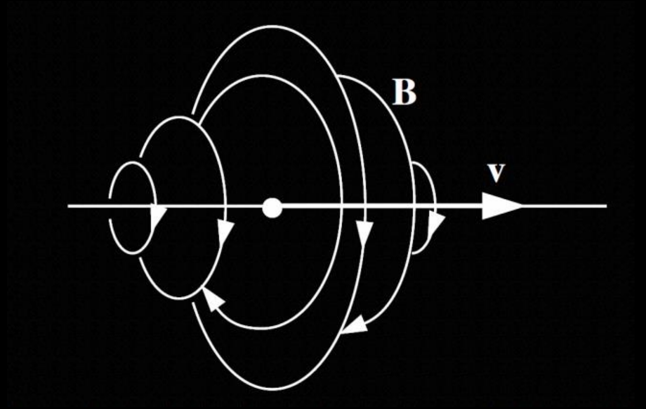
Magnetohydrodynamics (MHD)

- In E&M, the E- and B-fields can be transformed from one coordinate to another:

$$
\begin{aligned}
E'_{\parallel} &= E_{\parallel}, \\
\mathbf{E}'_{\perp} &= \gamma(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}/c), \\
B'_{\parallel} &= B_{\parallel}, \\
\mathbf{B}'_{\perp} &= \gamma(\mathbf{B}_{\perp} - \mathbf{v} \times \mathbf{E}/c),
\end{aligned}
$$

$$\gamma = (1 - v^2/c^2)^{-1/2}$$





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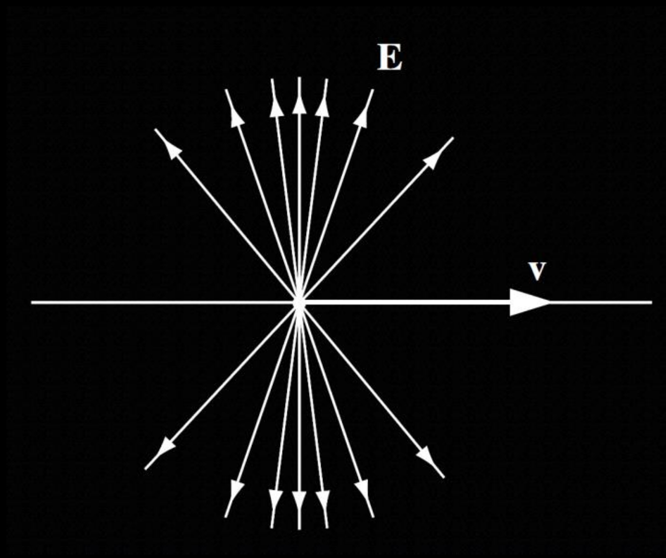


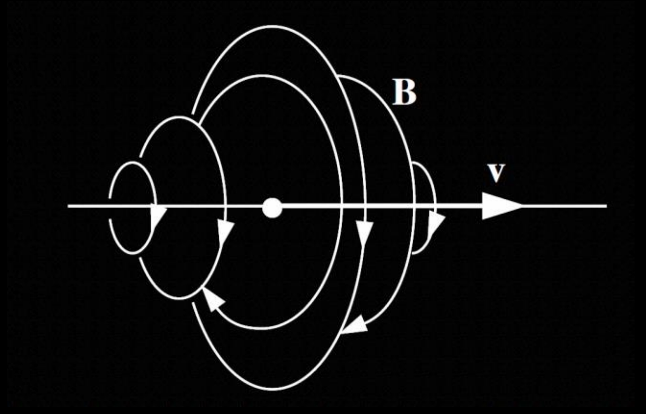
- For nonrelativistic gas, we can drop the v^2/c^2 term.
- In the comoving frame, assuming gas is a perfect conductor:

$$\mathbf{E}' = 0$$

which leads to:

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}/c$$ in Lab frame.

$$\mathbf{B}' = \mathbf{B}[1 + \mathcal{O}(v^2/c^2)],$$



- In (nonrelativistic) MHD, $\mathbf{E} \ll \mathbf{B}$, and $\mathbf{B}$ is frame-independent!

Magnetohydrodynamics (MHD)

Maxwell's equations:

$$\boxed{\begin{aligned} 4\pi\mathbf{j} + \partial\mathbf{E}/\partial t &= c\boldsymbol{\nabla}\times\mathbf{B}, \\ \partial\mathbf{B}/\partial t &= -c\boldsymbol{\nabla}\times\mathbf{E}, \\ \boldsymbol{\nabla}\cdot\mathbf{E} &= 4\pi\sigma, \\ \boldsymbol{\nabla}\cdot\mathbf{B} &= 0, \end{aligned}}$$

Perfect conductor:

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$$4\pi\mathbf{j} - \cancel{\frac{\partial}{\partial t}(\mathbf{v}\times\mathbf{B})/c} = c\nabla\times\mathbf{B}.$$

displacement current
is 2nd order (negligible) in MHD

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Lorentz force: $\mathbf{F}_\mathrm{L} = \frac{1}{c}\mathbf{j}\times\mathbf{B} = \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}.$

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Lorentz force: $\mathbf{F}_{\mathrm{L}} = \frac{1}{c}\mathbf{j}\times\mathbf{B} = \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}.$

Momentum equation in MHD: $\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla p + \boxed{\frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}} + \rho\mathbf{g},$

Magnetic fields are tied to the gas

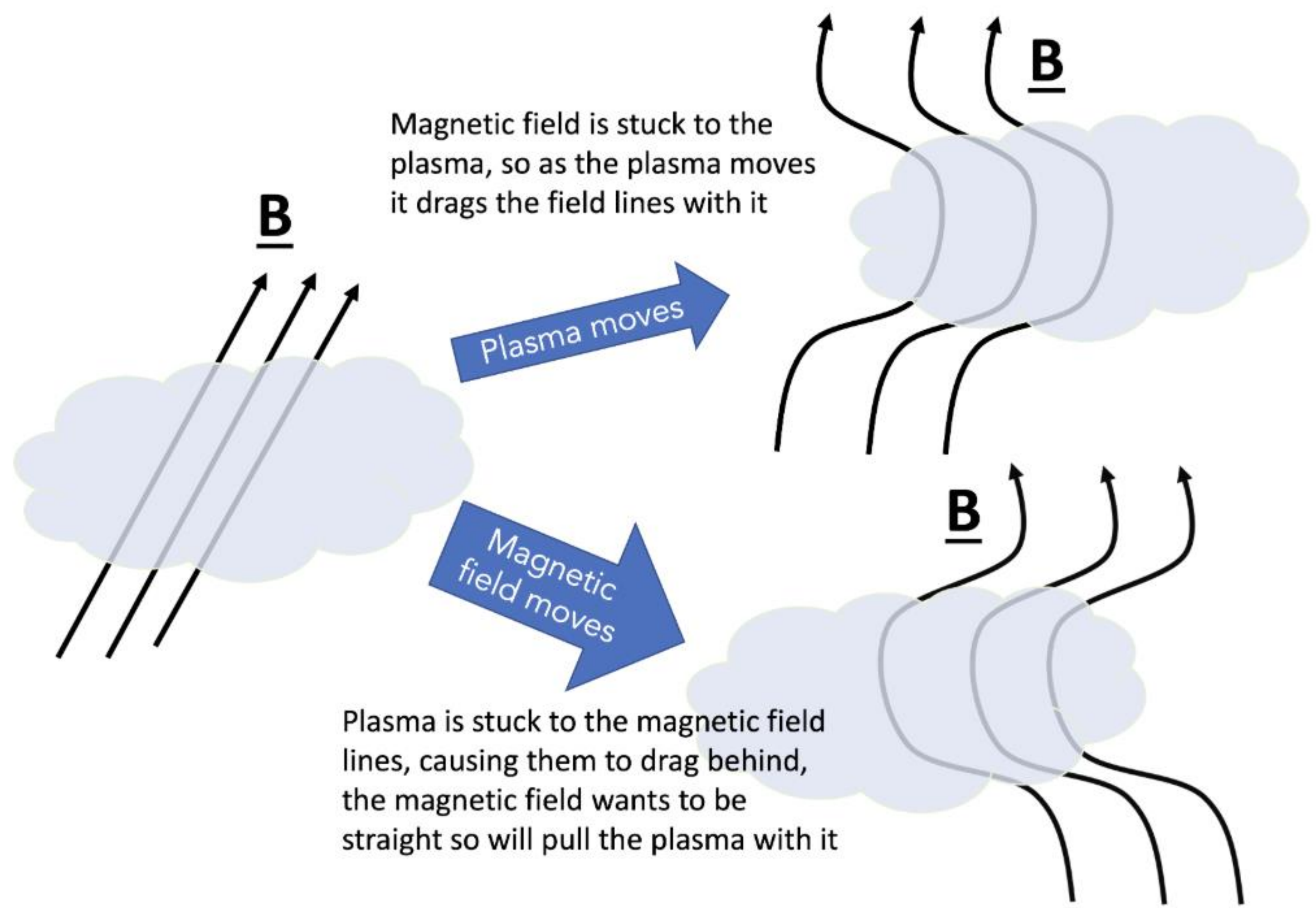
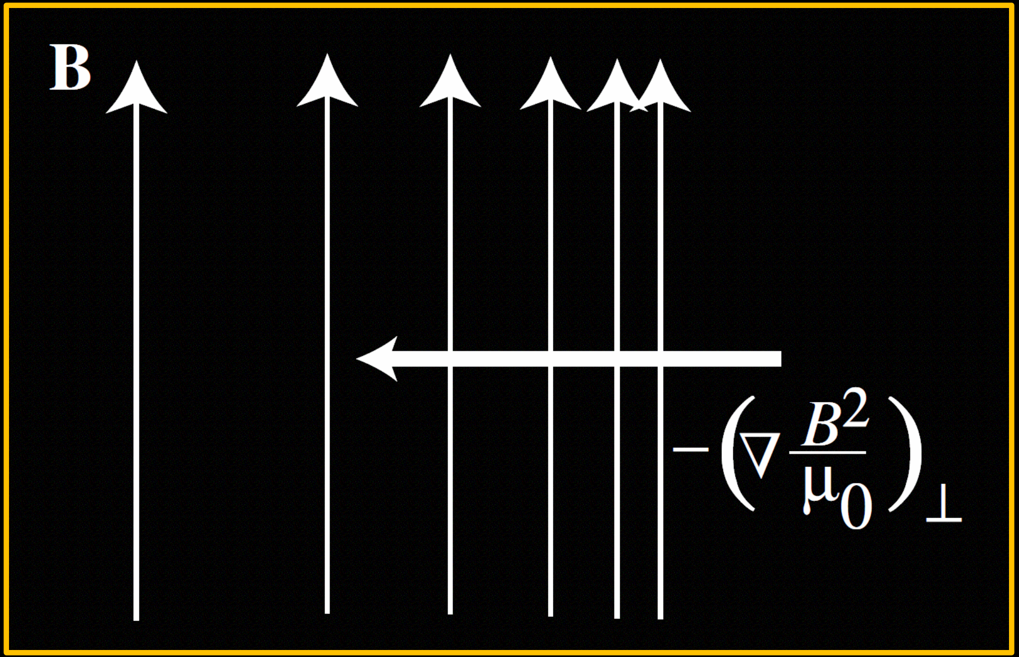
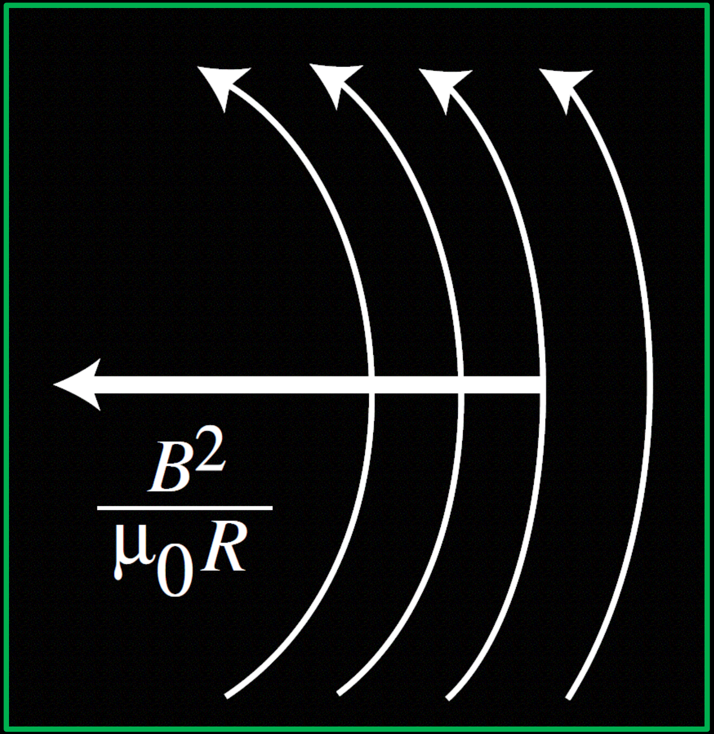


Diagram of plasma and magnetic field interacting due to the frozen in theorem.

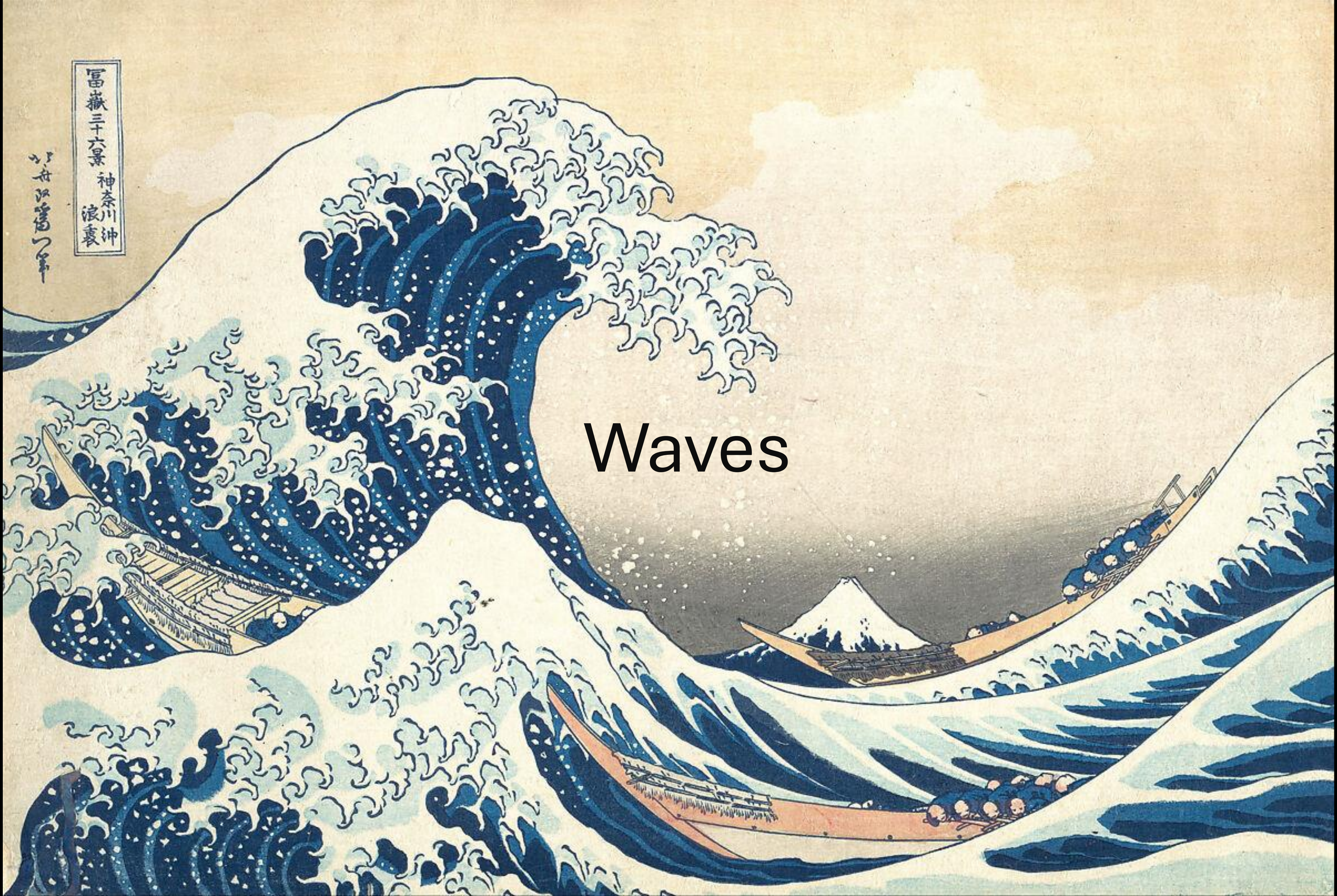
Magnetic Pressure vs. Magnetic Tension

- The Lorentz force can be decomposed into two parts:
  - Magnetic pressure
  - Magnetic tension





Waves



Sound Waves

Introduce perturbations in a static, uniform medium:

$$\begin{cases} P = P_0 + \delta P \\ T = T_0 + \delta T \\ \rho = \rho_0 + \delta\rho \\ \mathbf{v} = \mathbf{0} + \delta\mathbf{v} \end{cases}$$

$$\delta P = \left(\frac{\partial P}{\partial \rho}\right)_s \delta\rho$$

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Euler equations

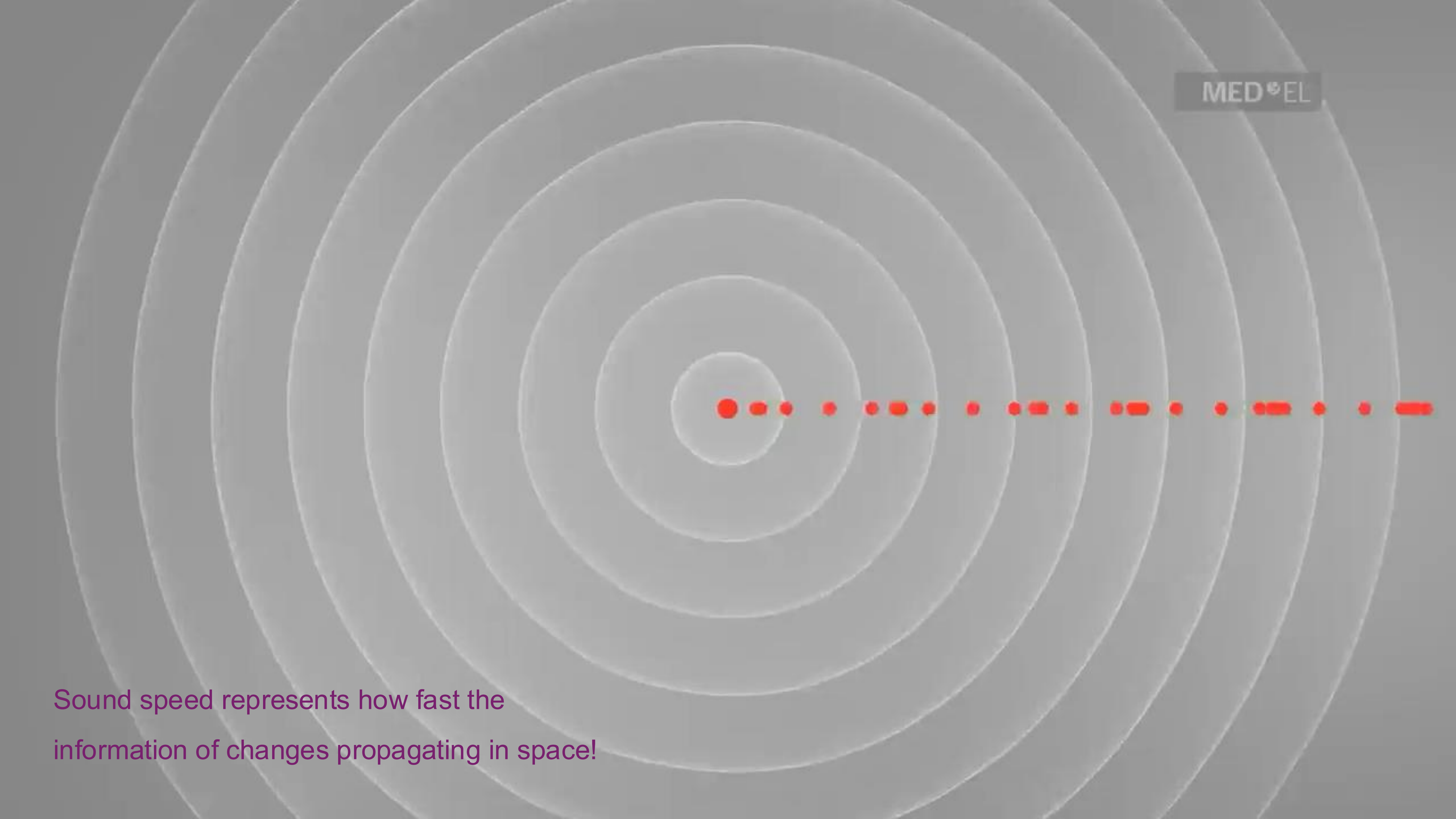
$$\boxed{\begin{aligned} &\frac{\partial \rho}{\partial t} + \nabla(\rho\mathbf{v}) = 0, \\ &\frac{\partial}{\partial t}(\rho\mathbf{v}) + \nabla(\rho\mathbf{v}\mathbf{v}^T + P) = 0, \end{aligned}}$$

Substitute them into the Euler equations, we get

$$\begin{cases} \dfrac{\partial \delta\rho}{\partial t} + \rho_0 \nabla \cdot \delta\mathbf{v} = 0 \\ \rho_0 \dfrac{\partial \delta\mathbf{v}}{\partial t} = -\nabla \delta P \end{cases} \quad \Rightarrow \quad \boxed{\Delta\delta\rho - \frac{1}{c_s^2}\frac{\partial^2 \delta\rho}{\partial t^2} = 0 \quad \text{with} \quad c_s^2 = \left(\frac{\partial P}{\partial \rho}\right)_s}$$

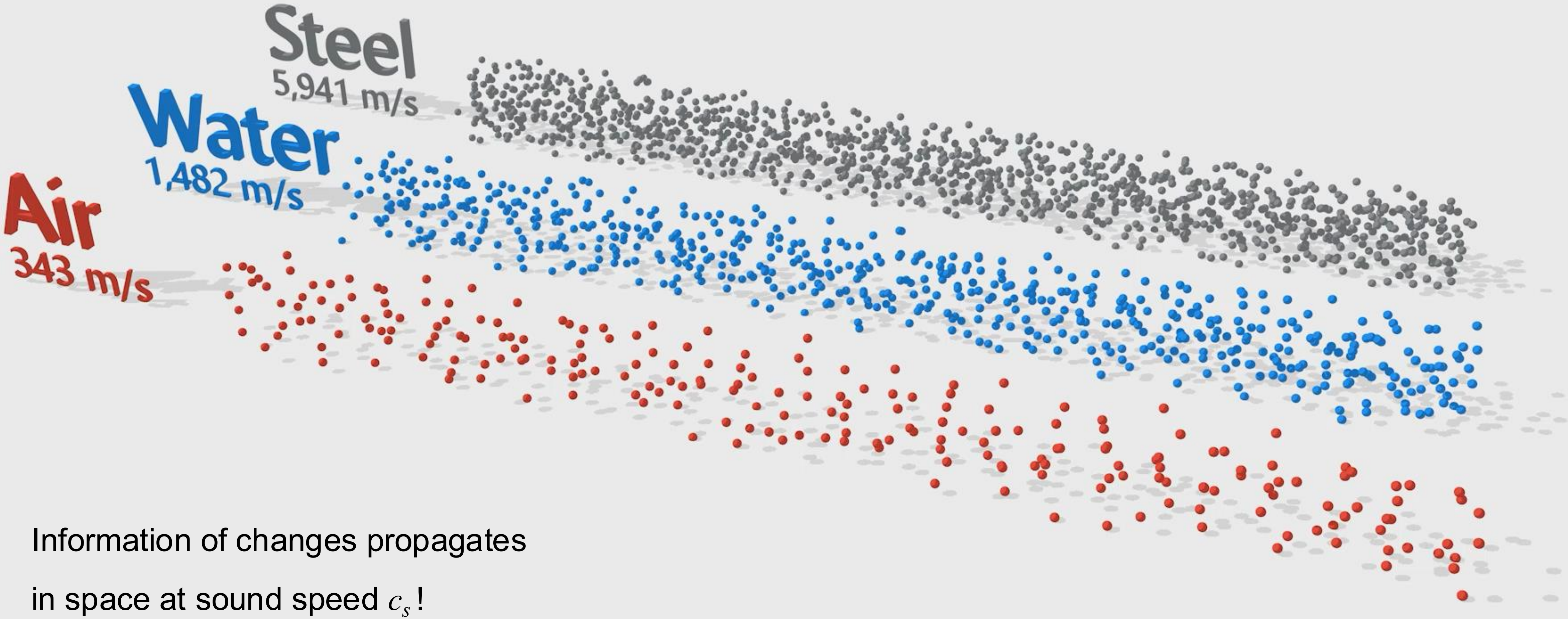
Density perturbation follows a wave equation,

with a wave speed of c_s



Sound speed represents how fast the information of changes propagating in space!

Sound Speed is the Signal Speed of Fluids



Information of changes propagates

in space at sound speed c_s!

credit: Higgsino physics

Wave Steepening

- For sound waves, we have dropped the nonlinear term

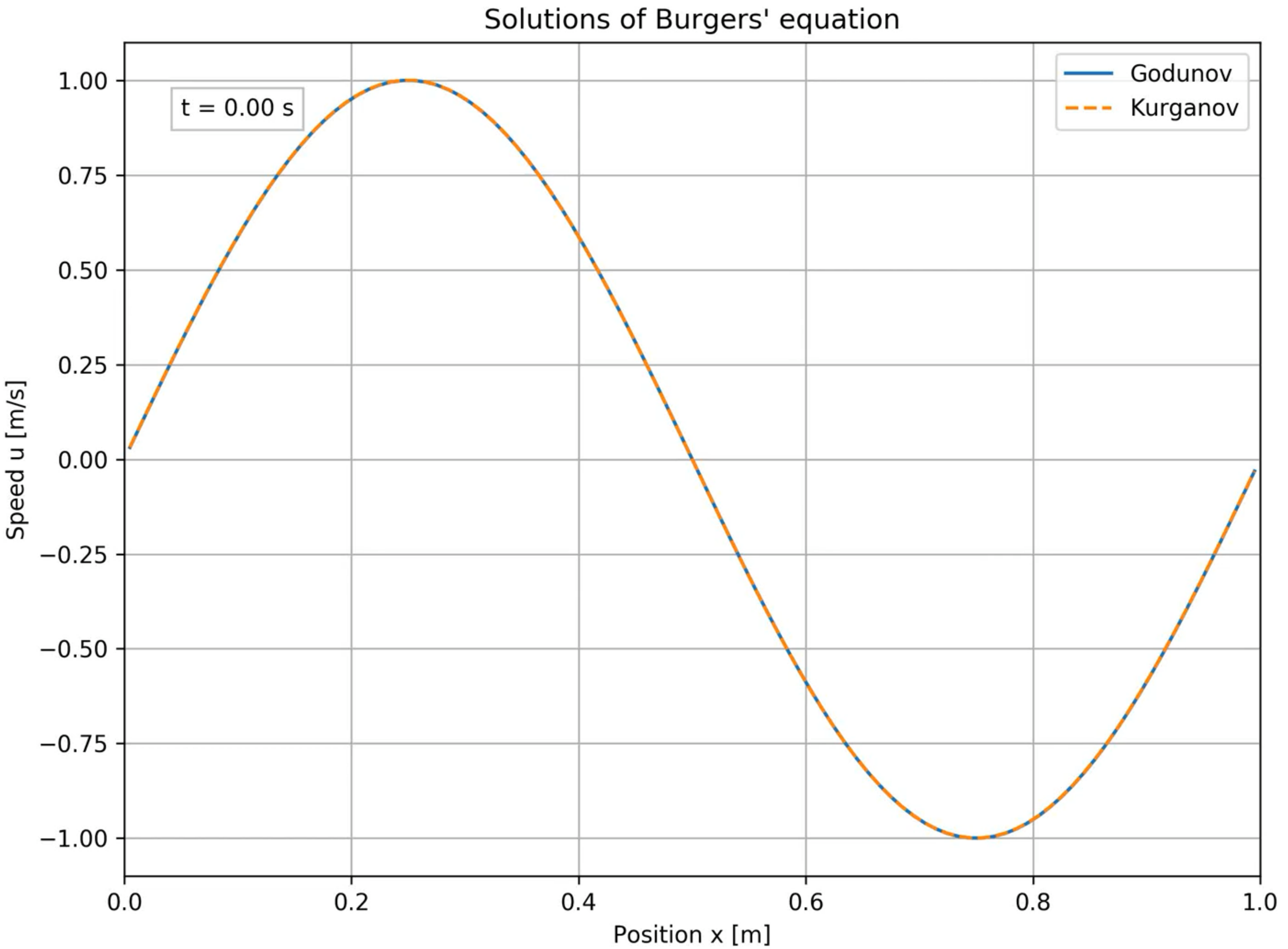
- In reality, the nonlinear term leads to wave steepening!

- Consider the 1D Burger's equation (pressureless fluid):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

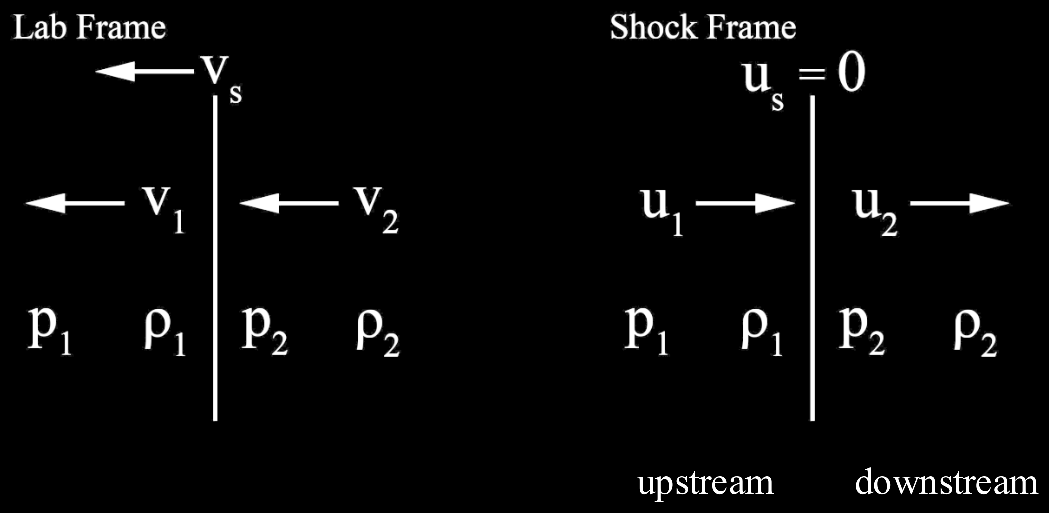
One could also add a viscosity term:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \nu\frac{\partial^2 u}{\partial x^2} = 0, \qquad \nu > 0$$


Solutions of Burgers' equation
t = 0.00 s
Godunov
Kurganov
Speed u [m/s]
Position x [m]
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0

Shocks

- Shock waves are discontinuities in fluid properties (e.g., density, temperature, velocity) propagating supersonically.
- Shock waves are dissipative, irreversible processes where entropy is generated post-shock, converting kinetic energy into thermal energy.



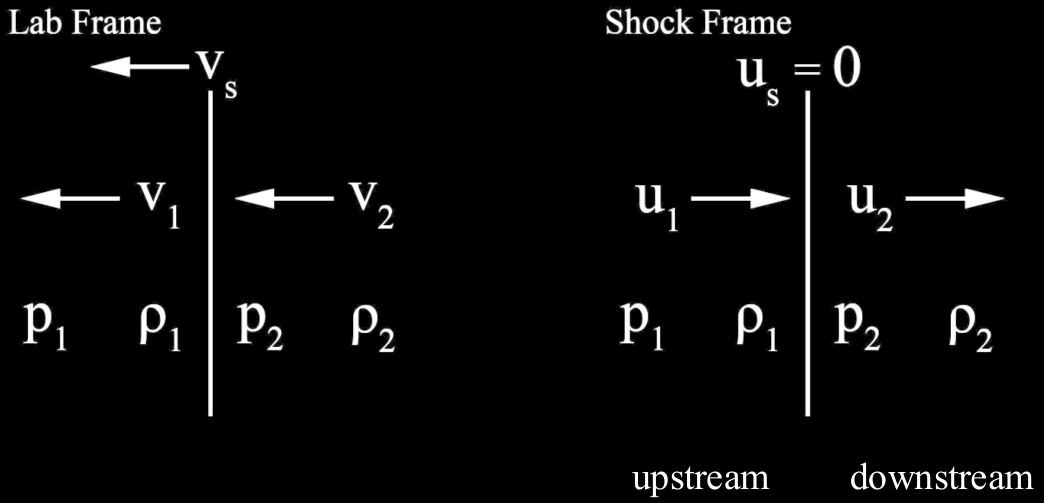
Jump conditions

- Consider a 1D, steady-state shock front, the fluxes of the conserved quantities must vanish, and the conservation laws in the "shock frame" become:

$$\rho_1 v_2 = \rho_2 v_2,$$

$$\rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2,$$

$$(\rho_1 e_1 + P_1) v_1 = (\rho_2 e_2 + P_2) v_2.$$



Jump conditions

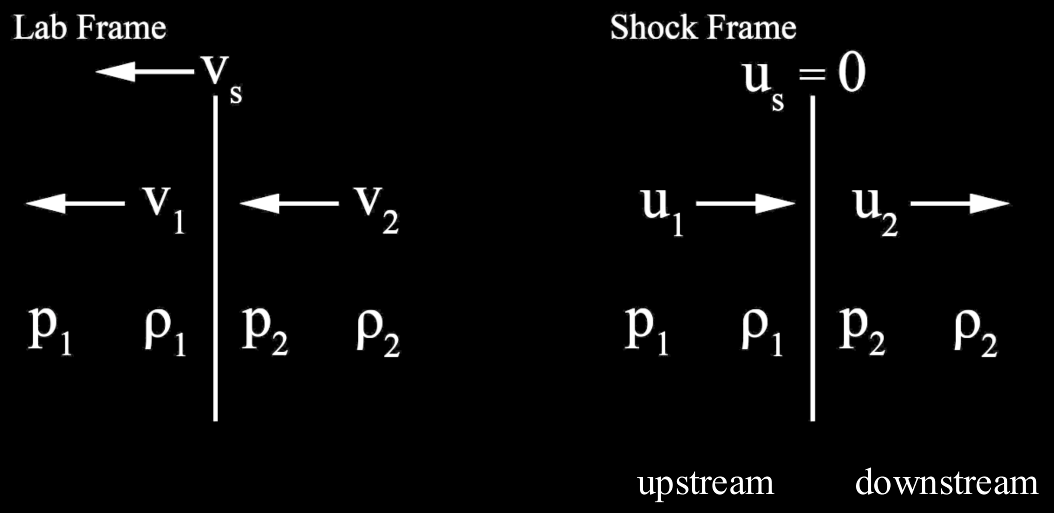
- This leads to the Rankine-Hugoniot jump conditions:

$$\boxed{\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{v_2}{v_1} = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma + 1)M^2}},$$

$$\boxed{\frac{P_2}{P_1} = \frac{2\gamma M^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1}},$$

where $M \equiv M_1 = v_1/C_1 = \sqrt{v_1^2/\gamma P_1 V_1}$

is the upstream *Mach number*.



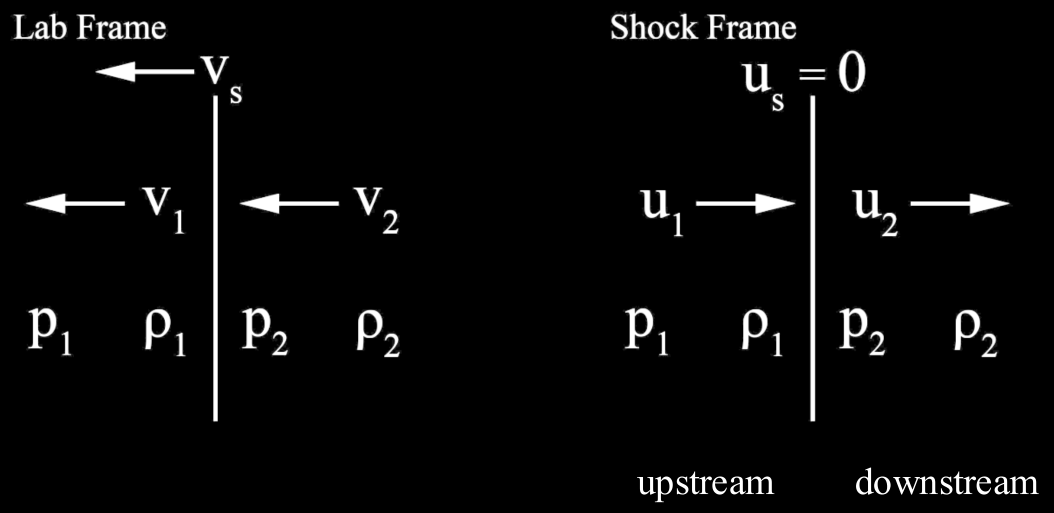
Jump conditions

- For strong shocks, $M \gg 1$ the jump conditions become

$$\boxed{\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{v_2}{v_1} \simeq \frac{\gamma - 1}{\gamma + 1}},$$

$$\boxed{\frac{P_2}{P_1} \simeq \frac{2\gamma M^2}{\gamma + 1}}.$$

- The density compression is at most 4x for $\gamma = 5/3$, while pressure can jump unbound.



Sedov-Taylor Blast Wave

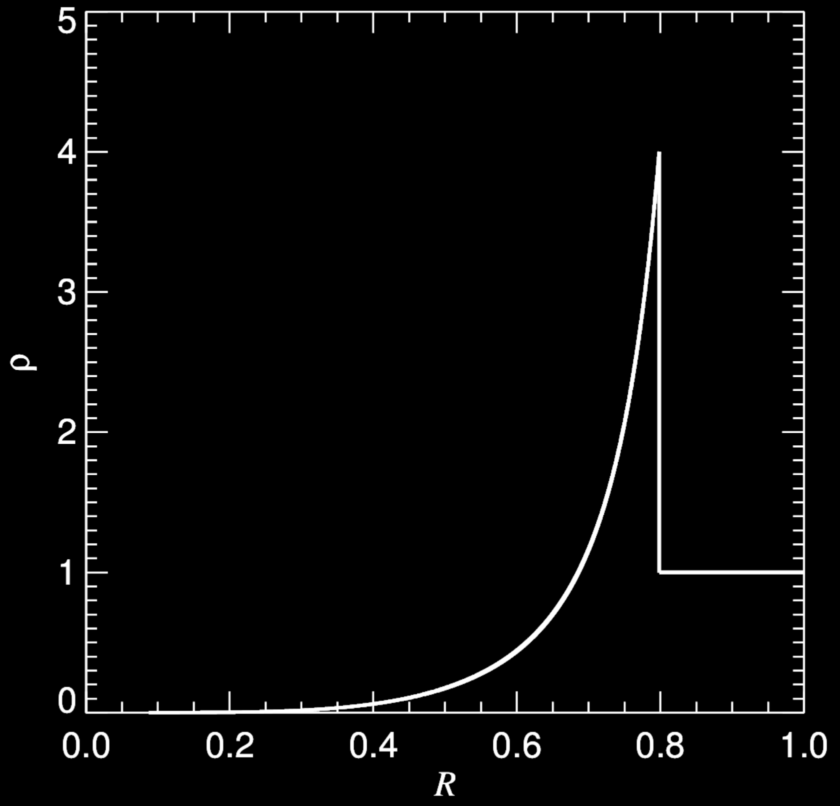
- A strong shock driven by a point explosion of energy E occurring in a uniform medium of density ρ_1 .

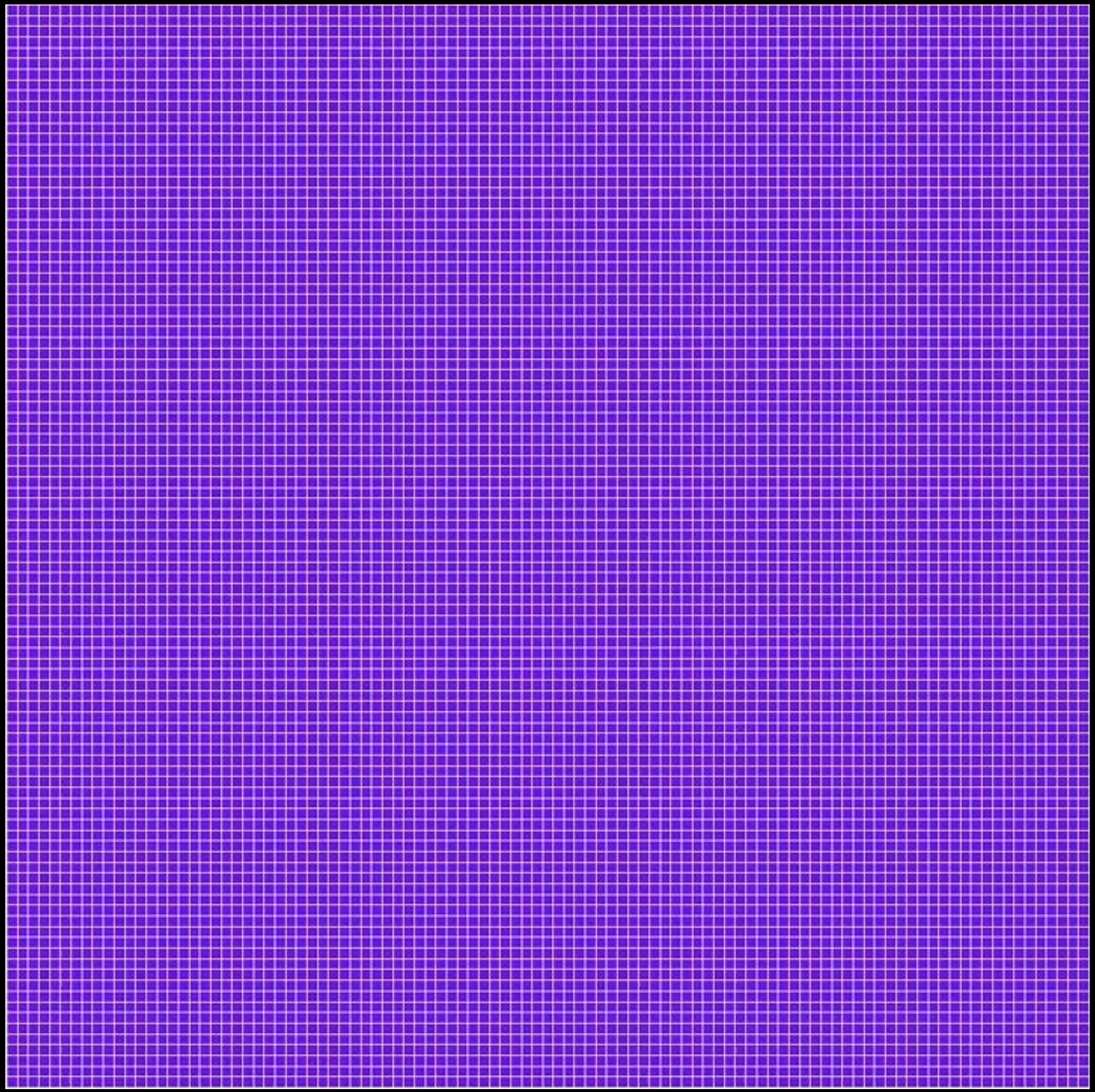
- Dimensional analysis:

$$
\begin{aligned}
[\rho_1] &= \frac{\mathrm{M}}{\mathrm{L}^3} \\
[E] &= \mathrm{M}\frac{\mathrm{L}^2}{\mathrm{T}^2} \\
[t] &= \mathrm{T}
\end{aligned}
\quad \Rightarrow \quad \left[\left(\frac{Et^2}{\rho_1}\right)^{\frac{1}{5}}\right] = \mathrm{L} \quad \Rightarrow \quad \boxed{R(t) = \eta_s \left(\frac{Et^2}{\rho_1}\right)^{1/5}}
$$

Sedov-Taylor Blast Wave

$$\boxed{R(t) = \eta_s \left(\frac{E t^2}{\rho_1} \right)^{1/5}}$$





Credit: Kevin Schaal

Sedov-Taylor Blast Wave

The formation of a blast wave by a very intense explosion. II. The atomic explosion of 1945

By Sir Geoffrey Taylor, F.R.S.

(*Received* 10 *November* 1949)

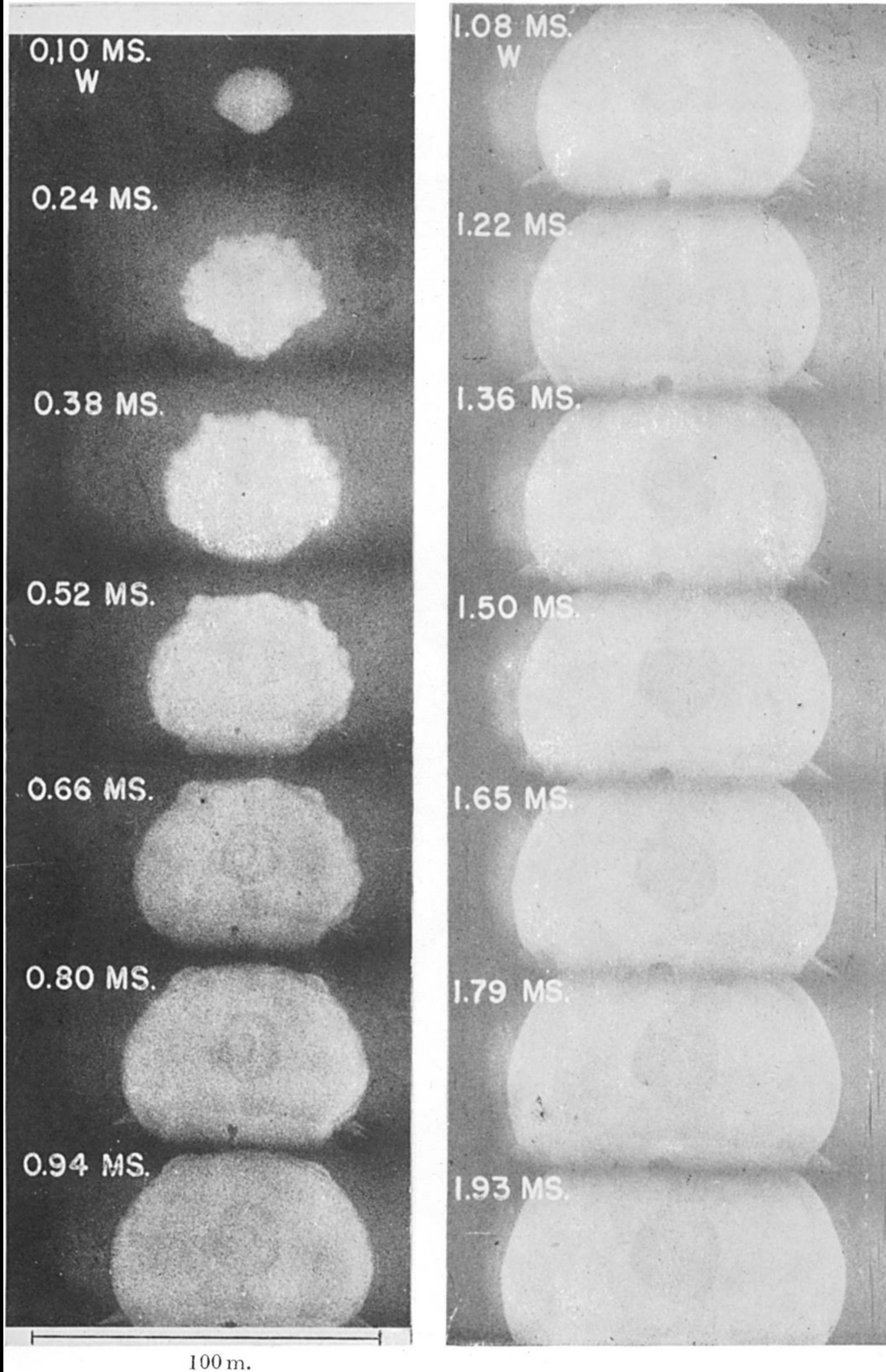
[Plates 7 to 9]

Photographs by J. E. Mack of the first atomic explosion in New Mexico were measured, and the radius, R, of the luminous globe or 'ball of fire' which spread out from the centre was determined for a large range of values of t, the time measured from the start of the explosion. The relationship predicted in part I, namely, that $R^{\frac{5}{2}}$ would be proportional to t, is surprisingly accurately verified over a range from $R = 20$ to 185 m. The value of $R^{\frac{5}{2}} t^{-1}$ so found was used in conjunction with the formulae of part I to estimate the energy E which was generated in the explosion. The amount of this estimate depends on what value is assumed for γ, the ratio of the specific heats of air.

Two estimates are given in terms of the number of tons of the chemical explosive T.N.T. which would release the same energy. The first is probably the more accurate and is 16,800 tons. The second, which is 23,700 tons, probably overestimates the energy, but is included to show the amount of error which might be expected if the effect of radiation were neglected and that of high temperature on the specific heat of air were taken into account. Reasons are given for believing that these two effects neutralize one another.

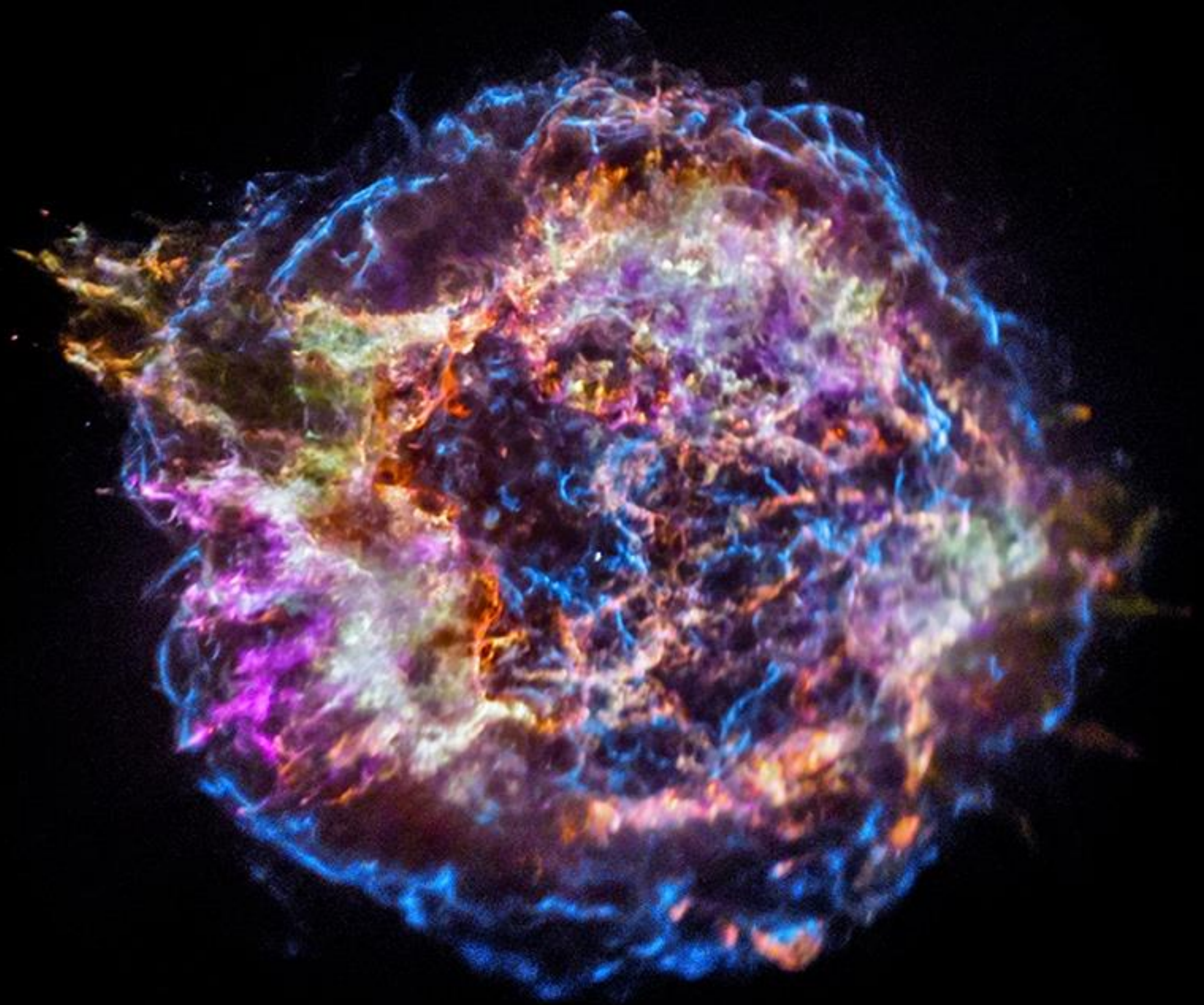
After the explosion a hemispherical volume of very hot gas is left behind and Mack's photographs were used to measure the velocity of rise of the glowing centre of the heated volume. This velocity was found to be 35 m./sec.

Until the hot air suffers turbulent mixing with the surrounding cold air it may be expected to rise like a large bubble in water. The radius of the 'equivalent bubble' is calculated and found to be 293 m. The vertical velocity of a bubble of this radius is $\frac{2}{3}\sqrt{(g\,29300)}$ or 35·7 m./sec. The agreement with the measured value, 35 m./sec., is better than the nature of the measurements permits one to expect.

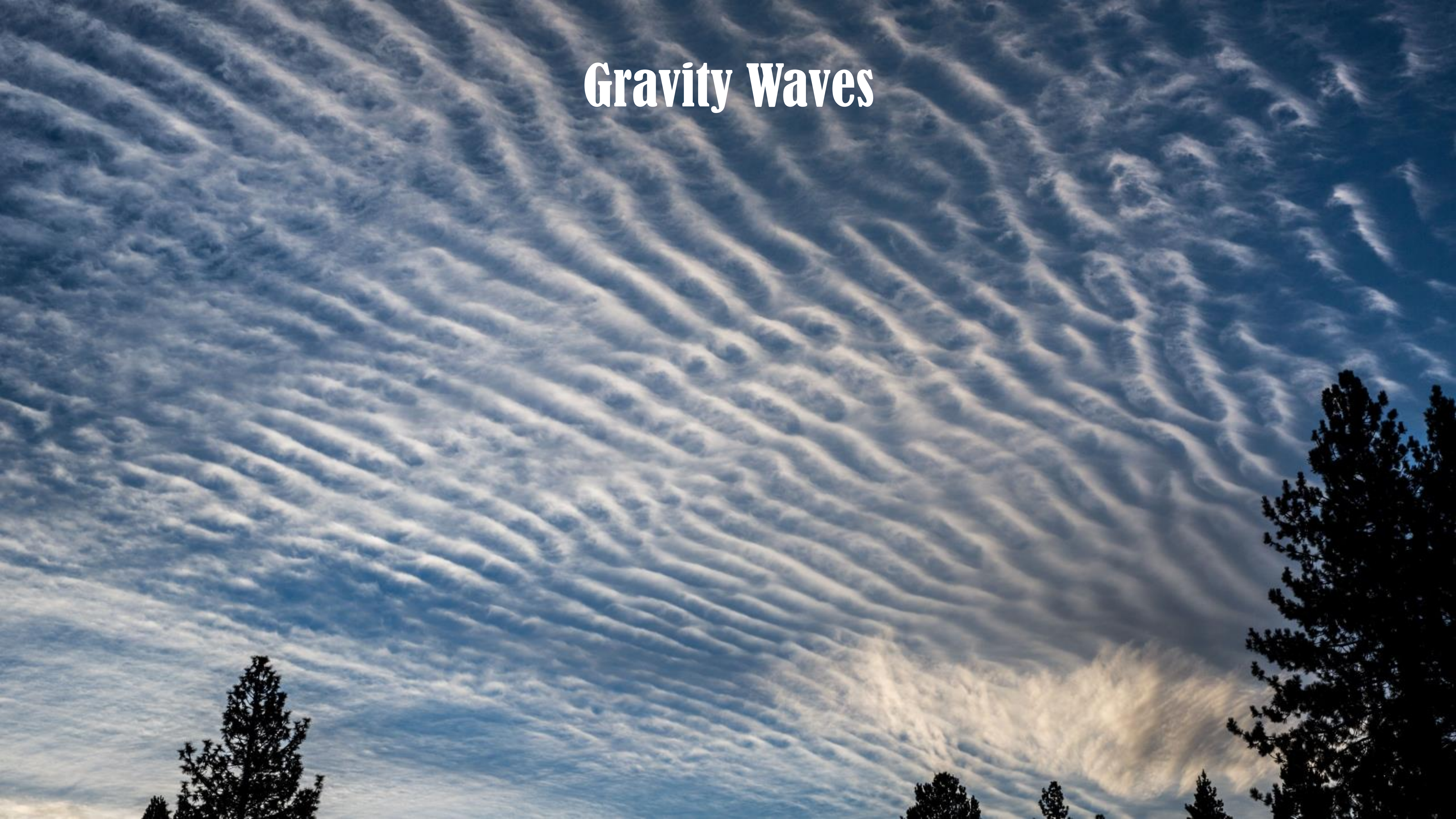


Astrophysical Blast Waves: Supernova Remnants

- Cassiopeia A in X-ray: a supernova remnant exploded ~ 300 years ago.



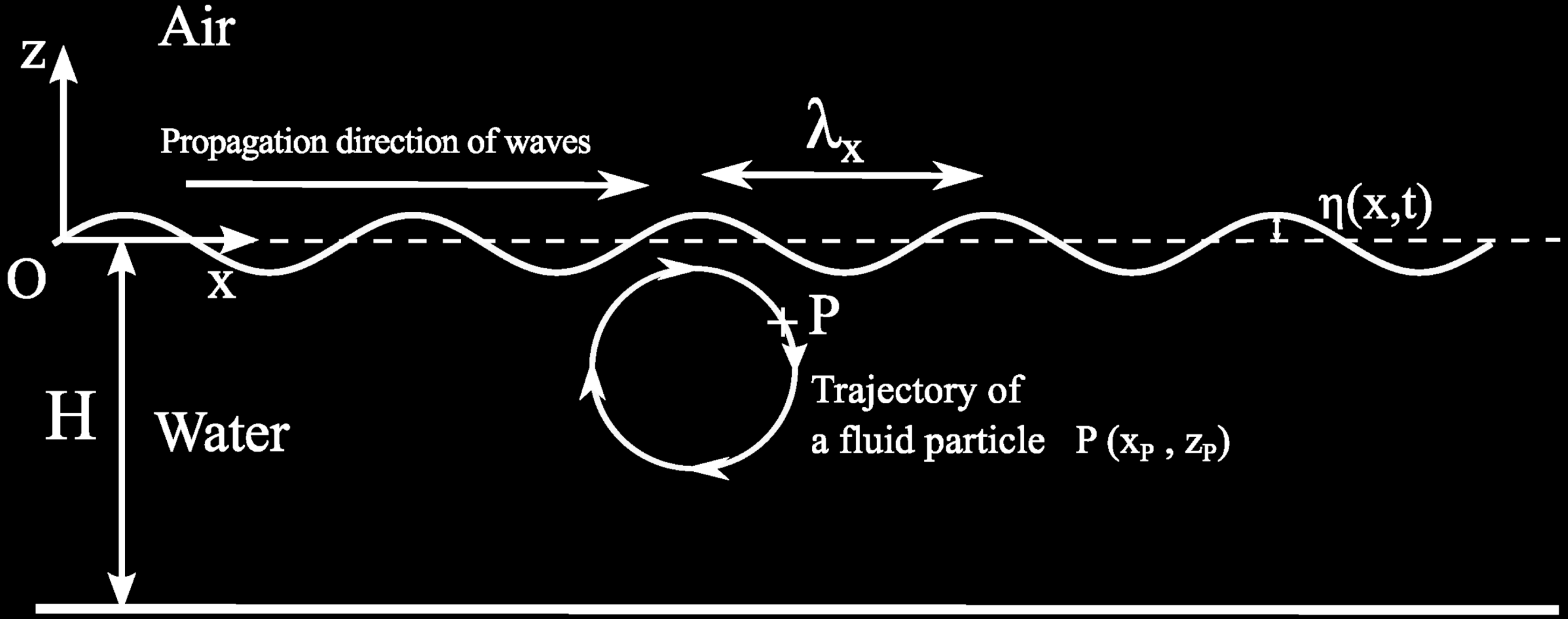
Gravity Waves



Gravity Waves

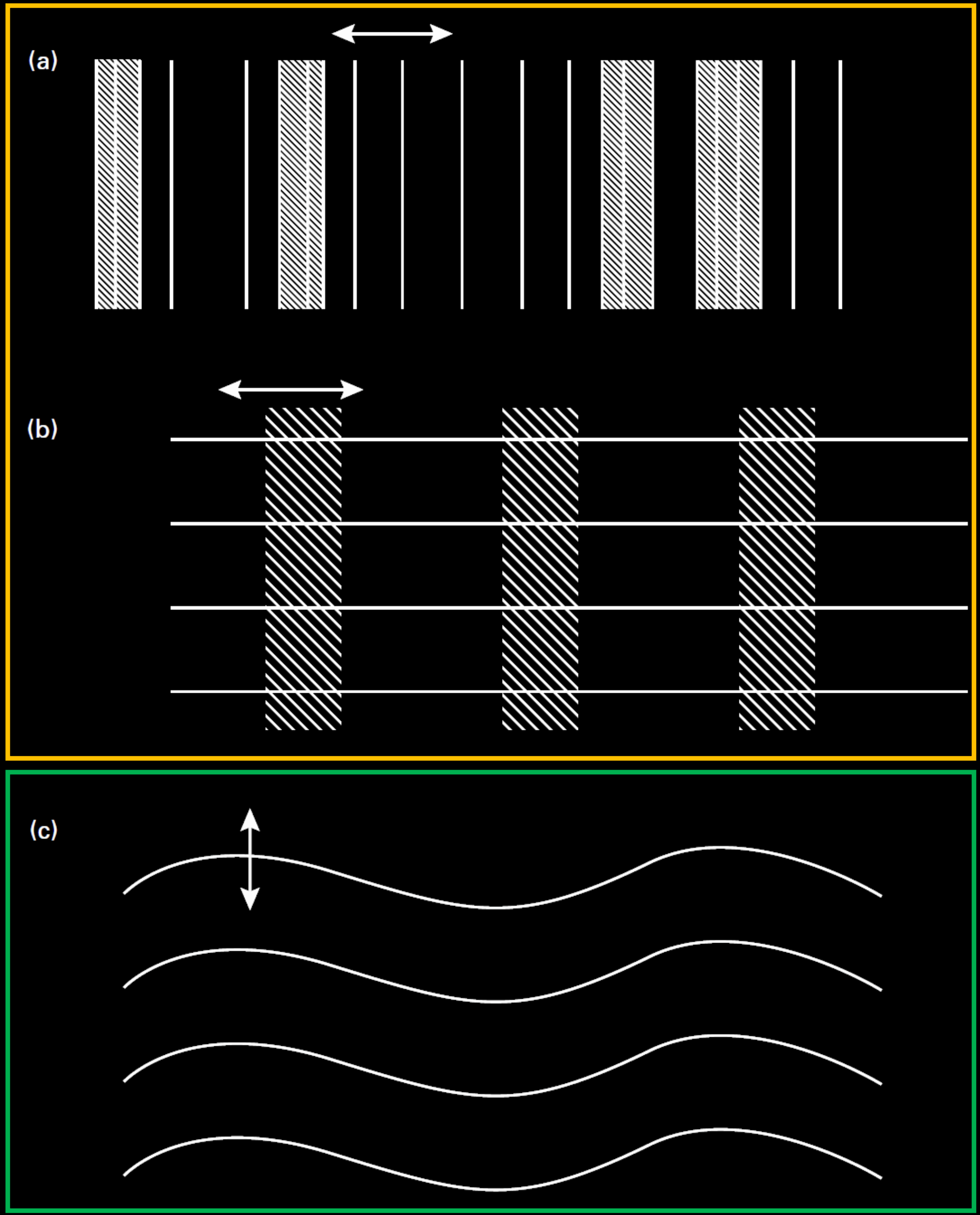
not to be confused with gravitational waves…

- Restoring forces: gravity (drop) and buoyancy (float)



MHD Waves

- Magnetic fields provide additional restoring force via magnetic pressure or magnetic tension.



fast magnetosonic wave
(magnetic pressure)

slow magnetosonic wave
(magnetic pressure)

Alfven wave
(magnetic tension)

MHD Waves

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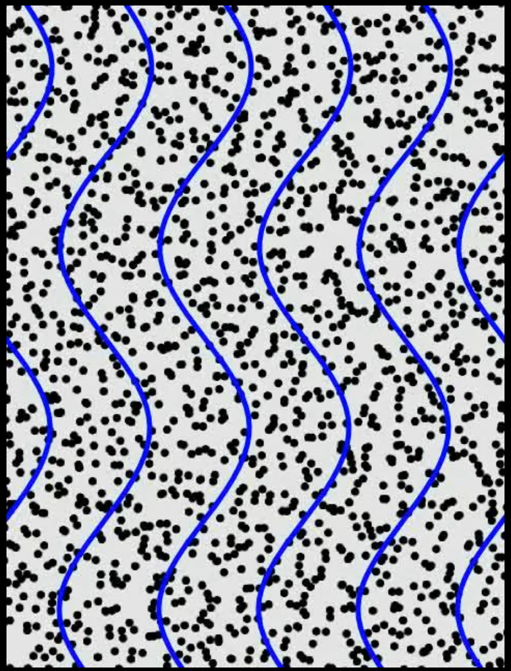

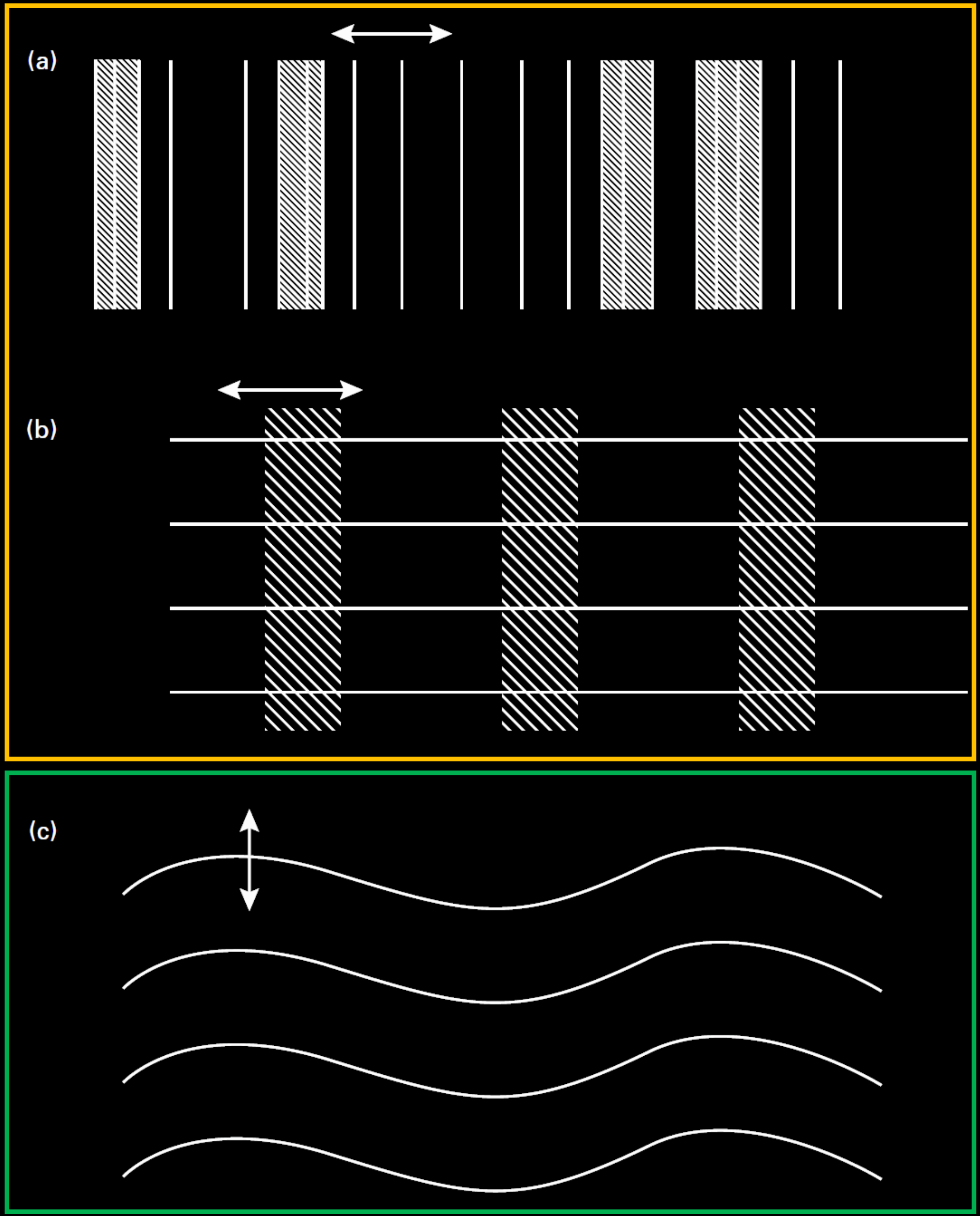

fast magnetosonic wave
(magnetic pressure)

slow magnetosonic wave
(magnetic pressure)

Alfven wave
(magnetic tension)


Instability

Gravitational instability

Similar to our linear sound waves, let's consider a static, uniform medium, but this time we include the self-gravity of the fluid.

$$
\begin{cases}
\dfrac{\partial \delta\rho}{\partial t} + \rho_0 \nabla \cdot \mathbf{v} = 0 \\[2ex]
\rho_0 \dfrac{\partial \mathbf{v}}{\partial t} = -\nabla \delta P \boxed{- \rho_0 \nabla \delta \Phi} \\[2ex]
\delta P = c_s^2 \delta\rho \\[2ex]
\boxed{\Delta \delta \Phi = 4\pi G \delta\rho} \quad \text{(Poisson's eqn.)}
\end{cases}
$$

$$c_s = \sqrt{\gamma P_0 / \rho_0}$$

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\boxed{\Delta \delta \Phi = 4\pi G \delta\rho} \quad \text{(Poisson's eqn.)}
\end{cases}
$$

$$c_s = \sqrt{\gamma P_0 / \rho_0}$$

Assume a plane-wave solution:

$$\delta\rho = \delta\rho_0 e^{i(\omega t + \mathbf{k}\cdot\mathbf{r})}, \quad \mathbf{v} = \mathbf{v}_0 e^{i(\omega t + \mathbf{k}\cdot\mathbf{r})}, \text{ etc.}$$

$$\omega^2 = c_s^2 k^2 \boxed{- 4\pi G \rho_0}$$

< 0 unstable (too much gravity)

> 0 stable

Gravitational instability

Similar to our linear sound waves, let's consider a static, uniform medium, but this time we include the self-gravity of the fluid.

$$\begin{cases} \dfrac{\partial \delta\rho}{\partial t} + \rho_0 \nabla \cdot \mathbf{v} = 0 \\ \rho_0 \dfrac{\partial \mathbf{v}}{\partial t} = -\nabla \delta P \boxed{- \rho_0 \nabla \delta \Phi} \\ \delta P = c_s^2 \delta\rho \\ \boxed{\Delta \delta\Phi = 4\pi G \delta\rho} \end{cases}$$

(Poisson's eqn.)

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$$\omega^2 = c_s^2 k^2 \boxed{- 4\pi G \rho_0}$$

< 0 unstable (too much gravity)

> 0 stable

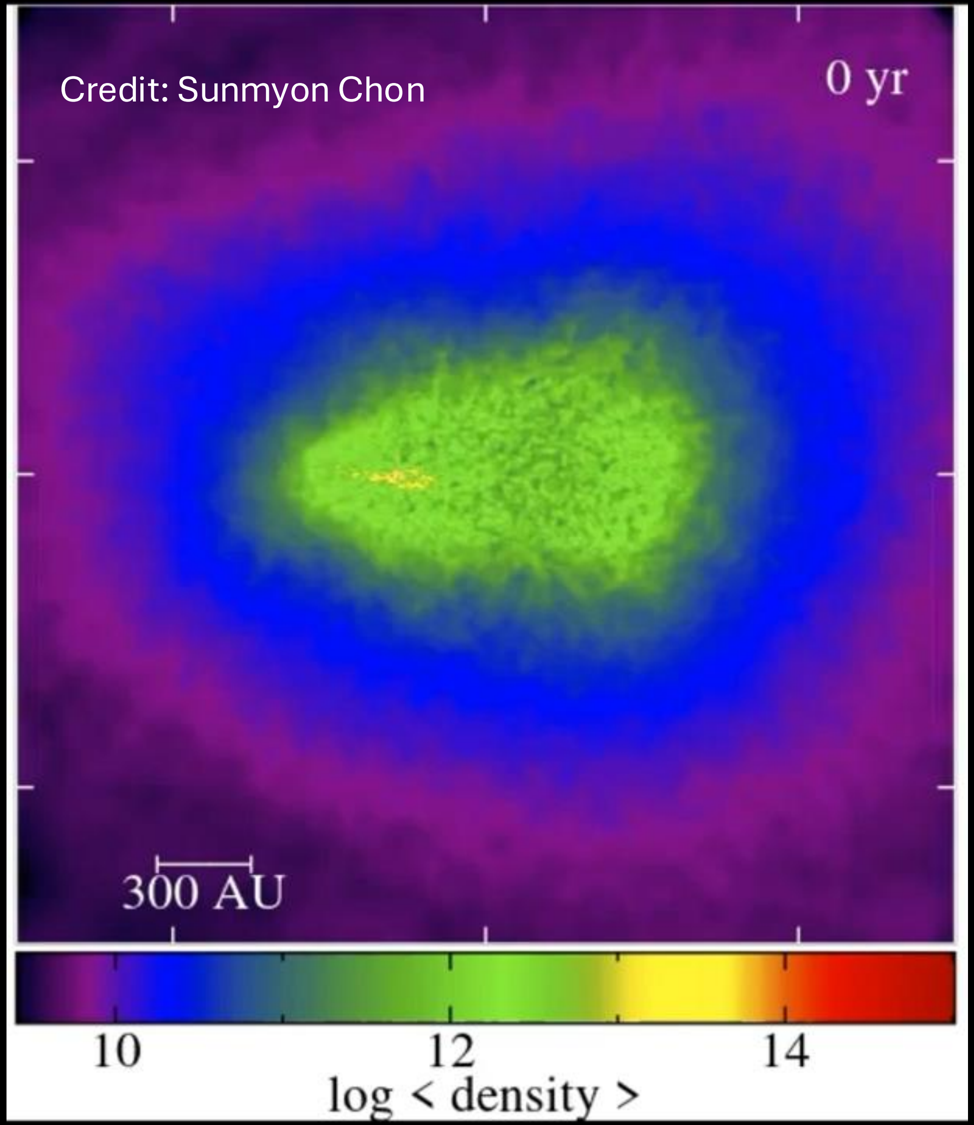
Critical wavenumber and wavelength:

$$\boxed{k_J = \sqrt{\frac{4\pi G \rho_0}{c_s^2}}} \qquad \lambda_J = 2\pi / k_J$$

Jeans length!

Gravitational instability

- Gravitational instability is responsible for the formation of galaxies, stars, planets, black holes, …



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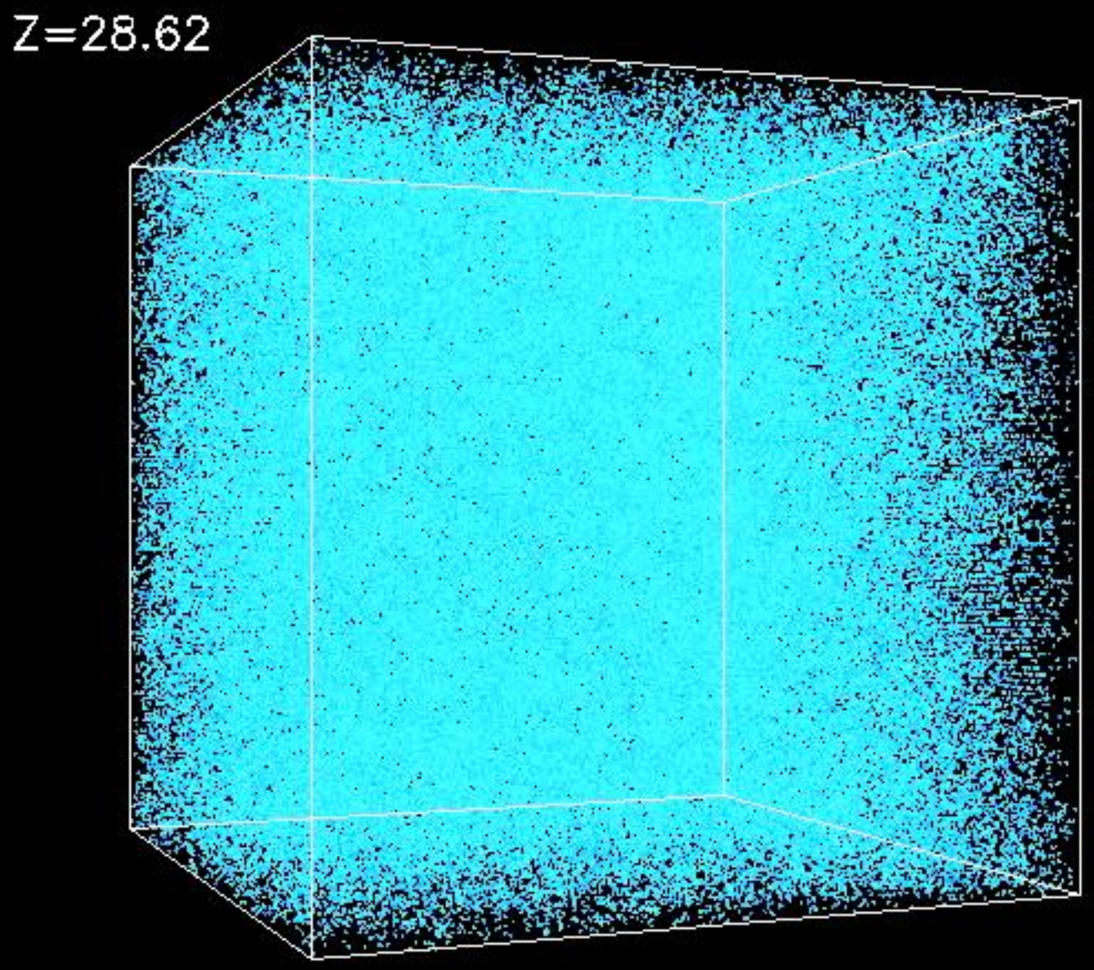
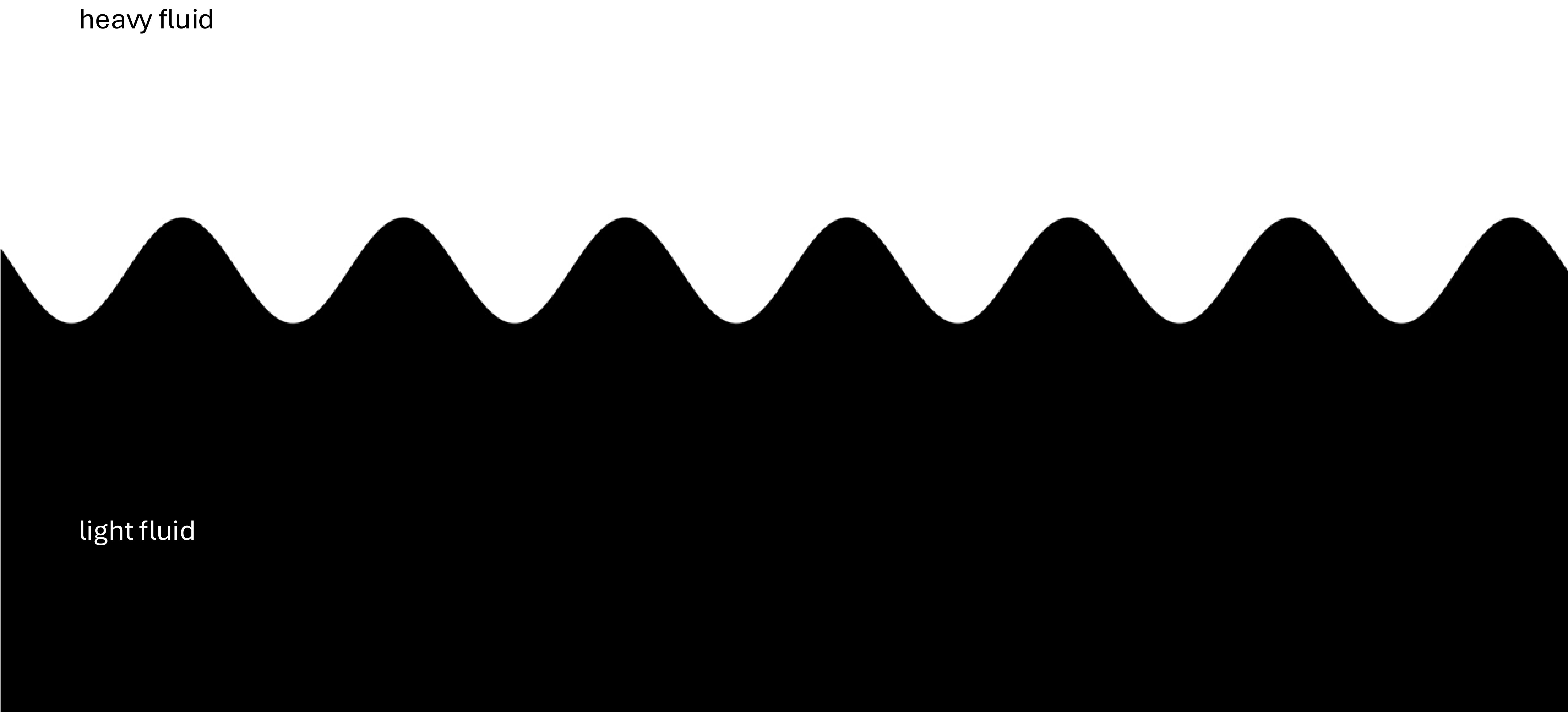


Credit: Andrey Kravtsov

Rayleigh-Taylor Instability



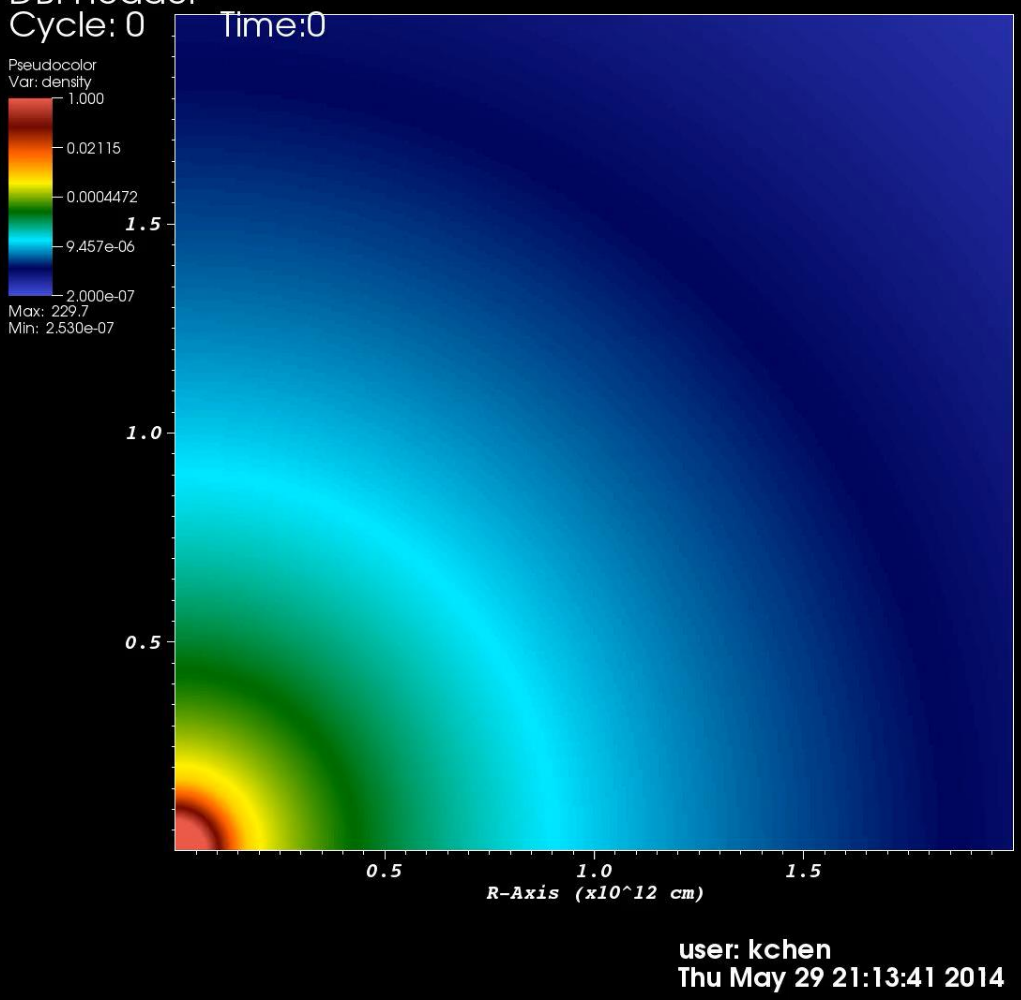
Rayleigh-Taylor Instability

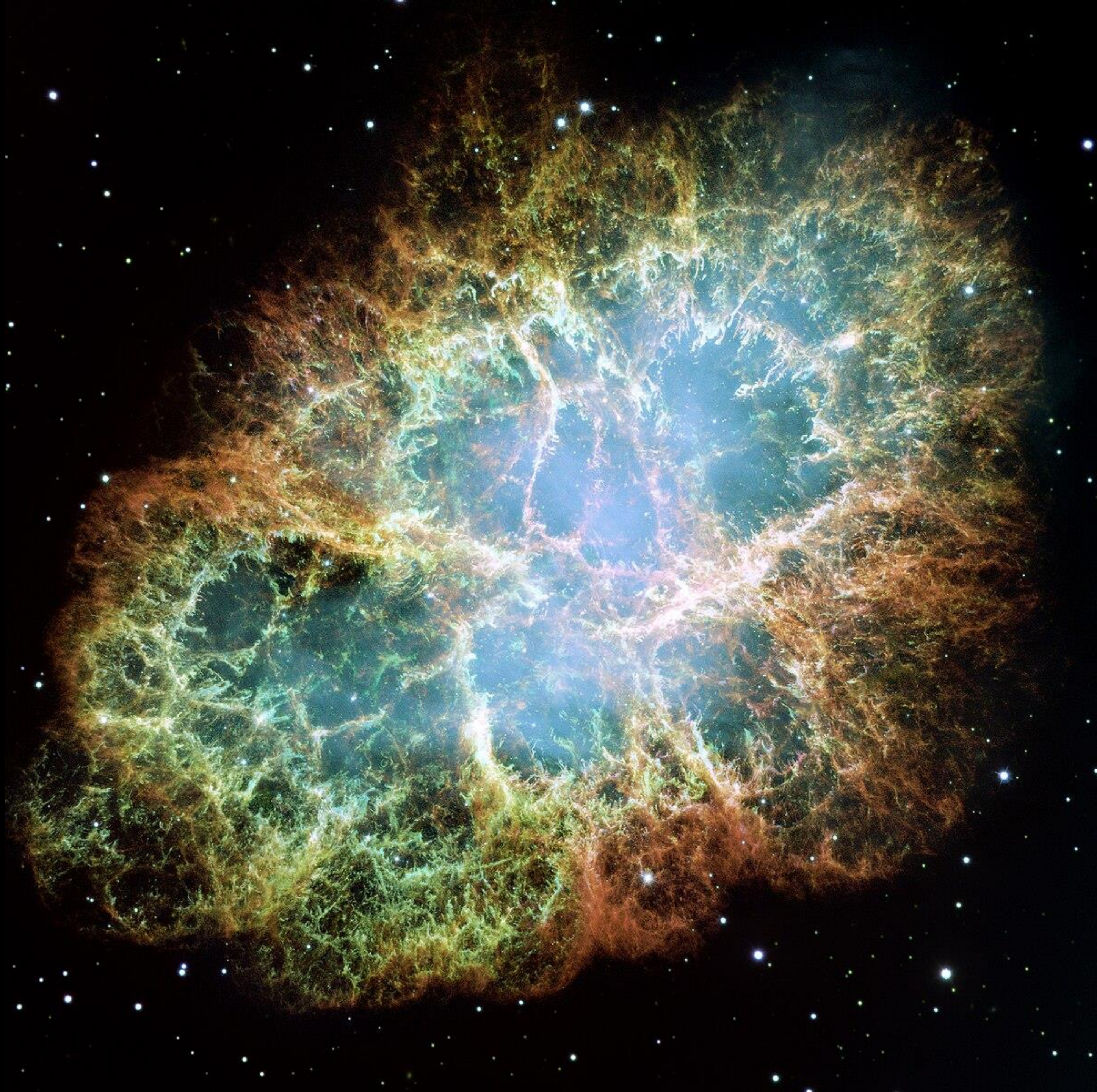
What happens if the two fluids are in free fall?

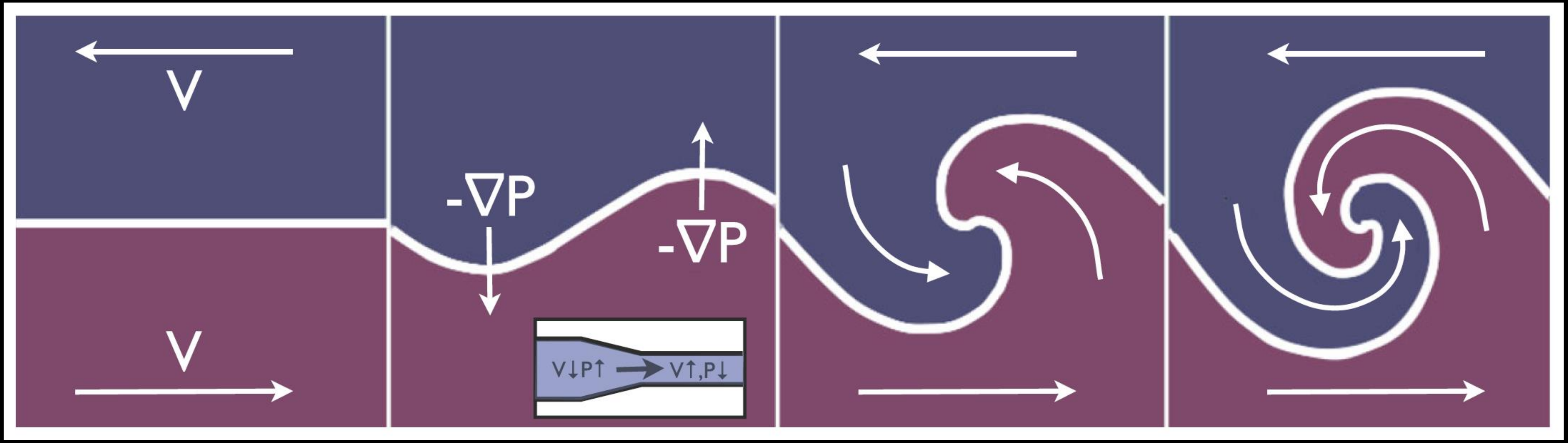
1. No instability
2. Enhanced instability
3. It depends

Rayleigh-Taylor Instability

Credit: Ken Chen (ASIAA)



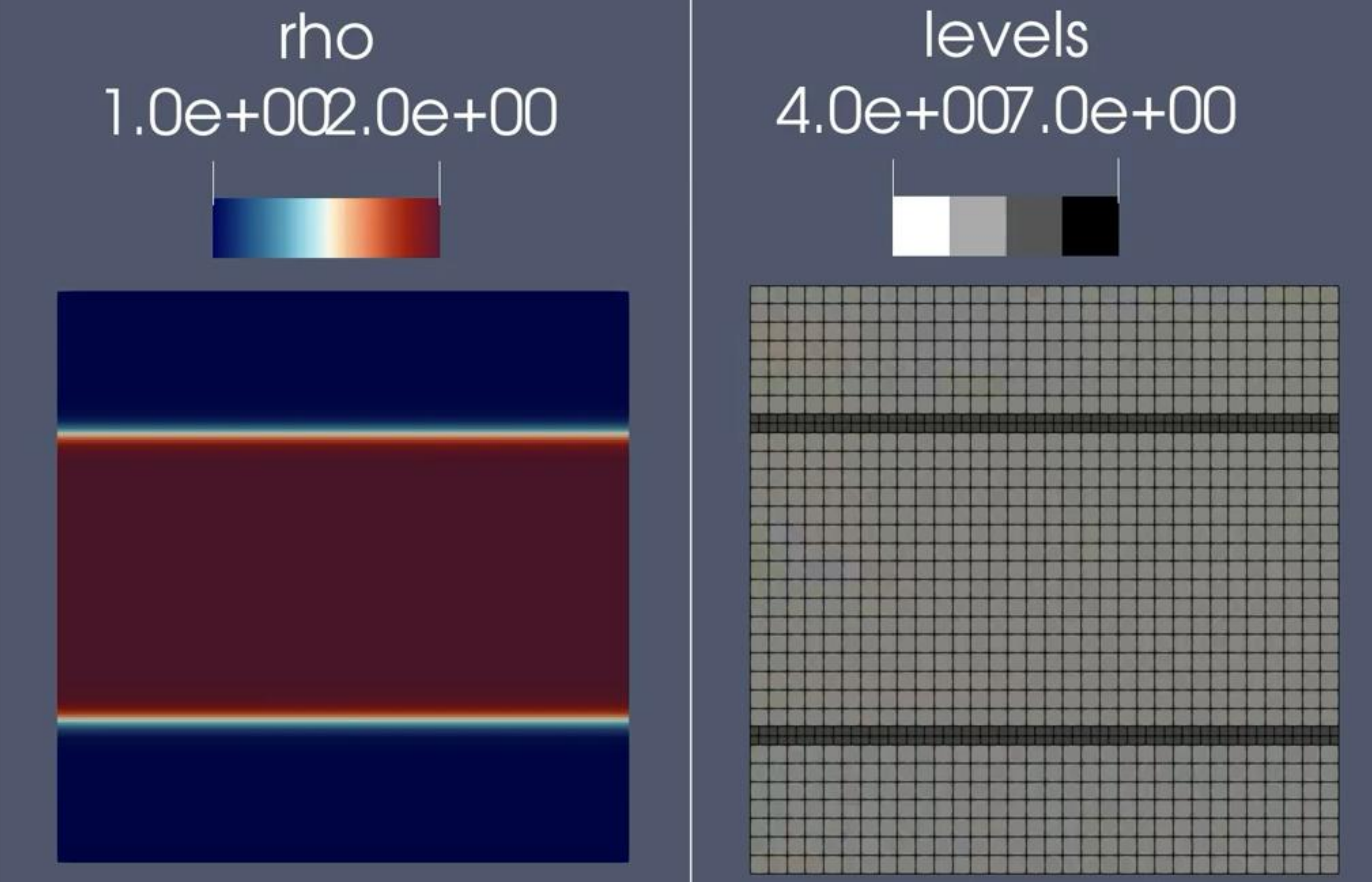




- The Kelvin–Helmholtz instability occurs at the interface between two fluids with velocity shear.

Kelvin-Helmholtz Instability

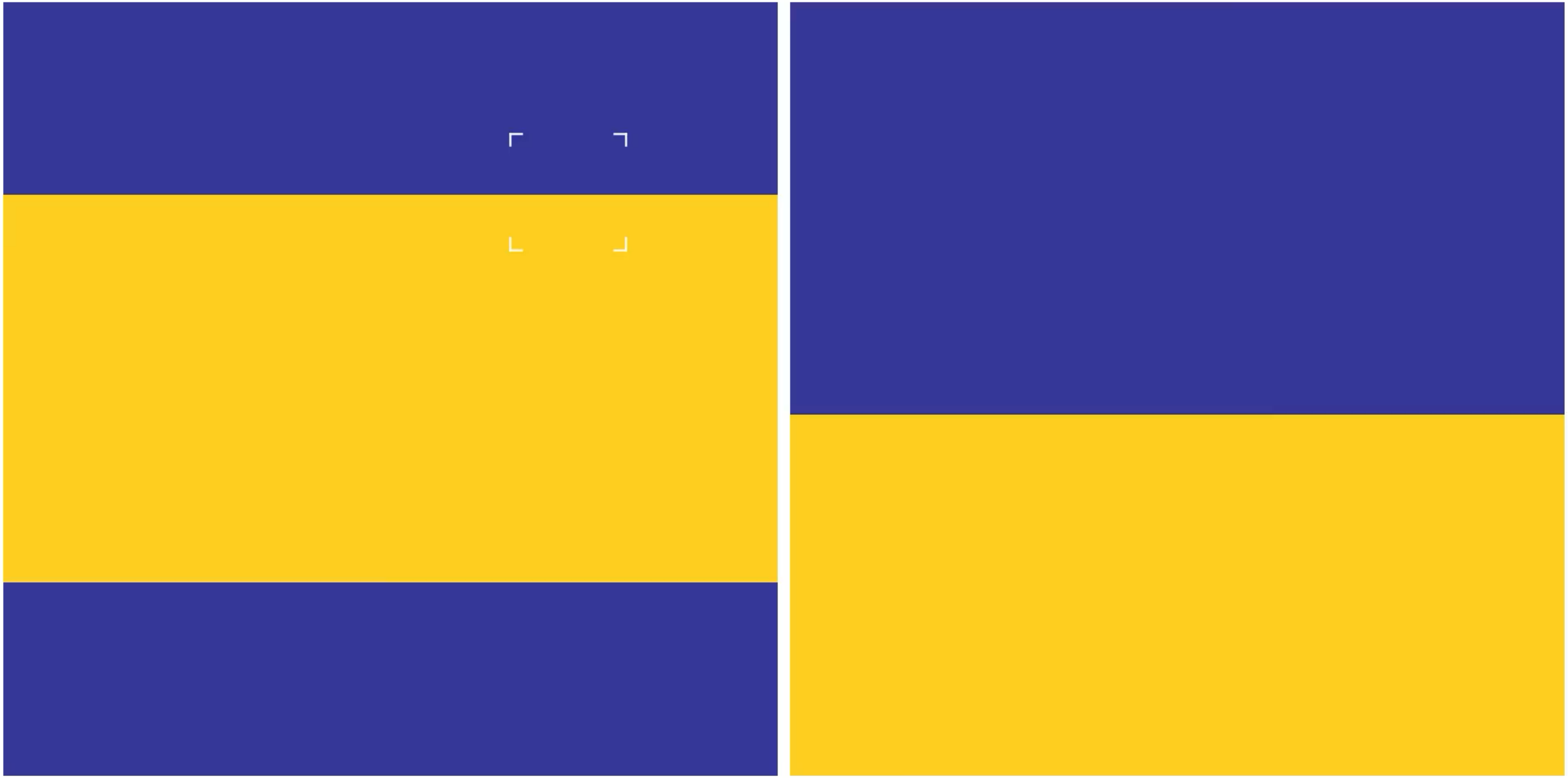
Kelvin-Helmholtz instability



Credit: Gregor Gassner

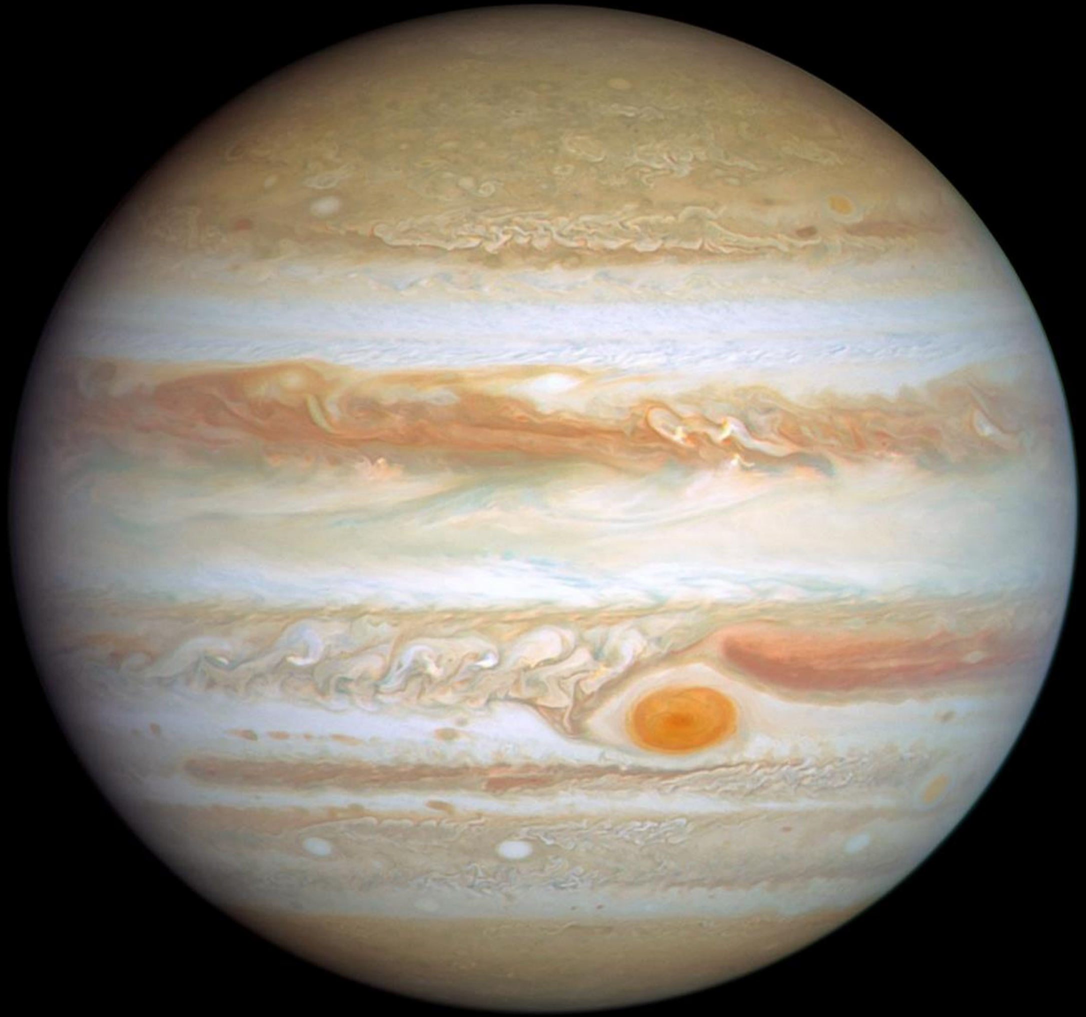
Kelvin-Helmholtz Instability

Credit: Kevin Schaal



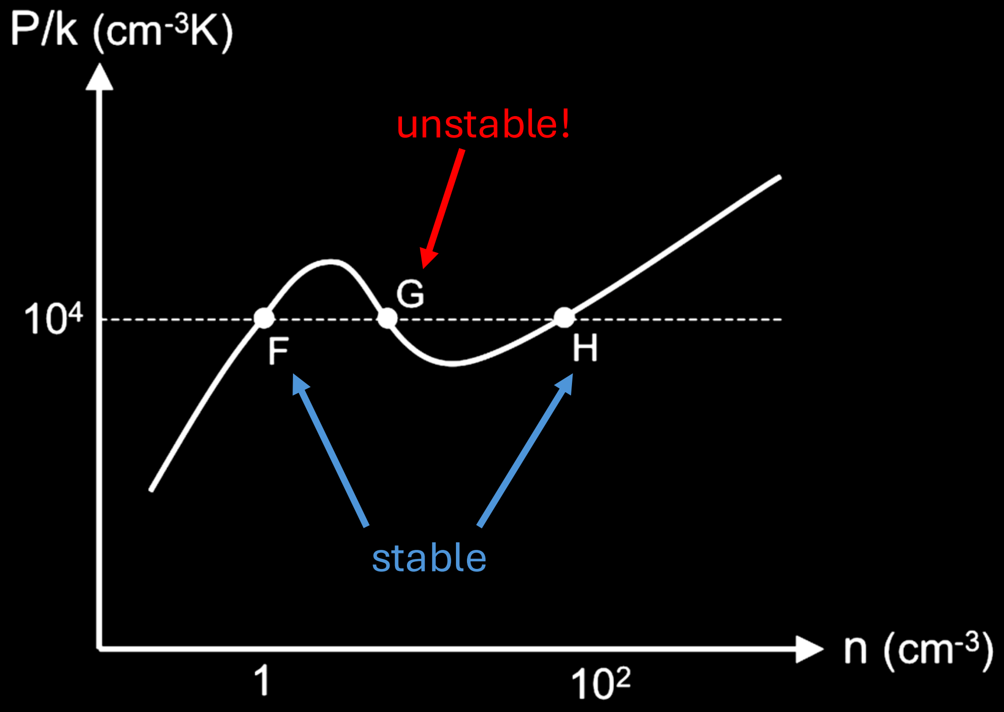

Kelvin-Helmholtz instability in real life

Kelvin-Helmholtz Instability in Planetary Atmospheres



Thermal Instability

- For a given ISM pressure, thermal equilibrium allows 3 possible solutions (F, G, H). F & H are stable; G suffers from thermal instability.
- Thermal instability: density decreases => net heating => density decreases even more ($P \sim n\,T$) => even stronger heating => ...
- Thermal instability naturally leads to 2 stable phases (WNM & CNM)!



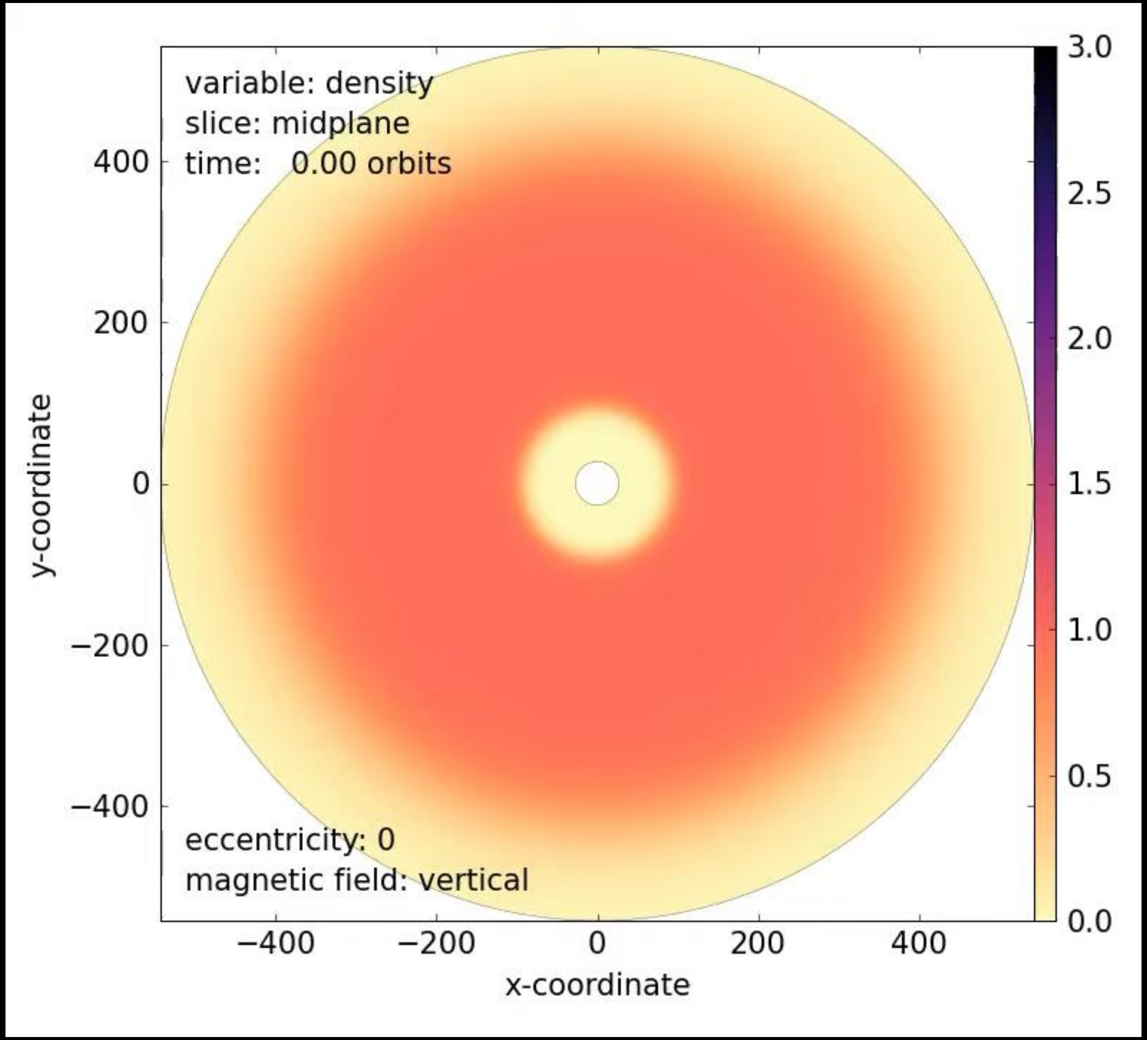
Thermal Instability

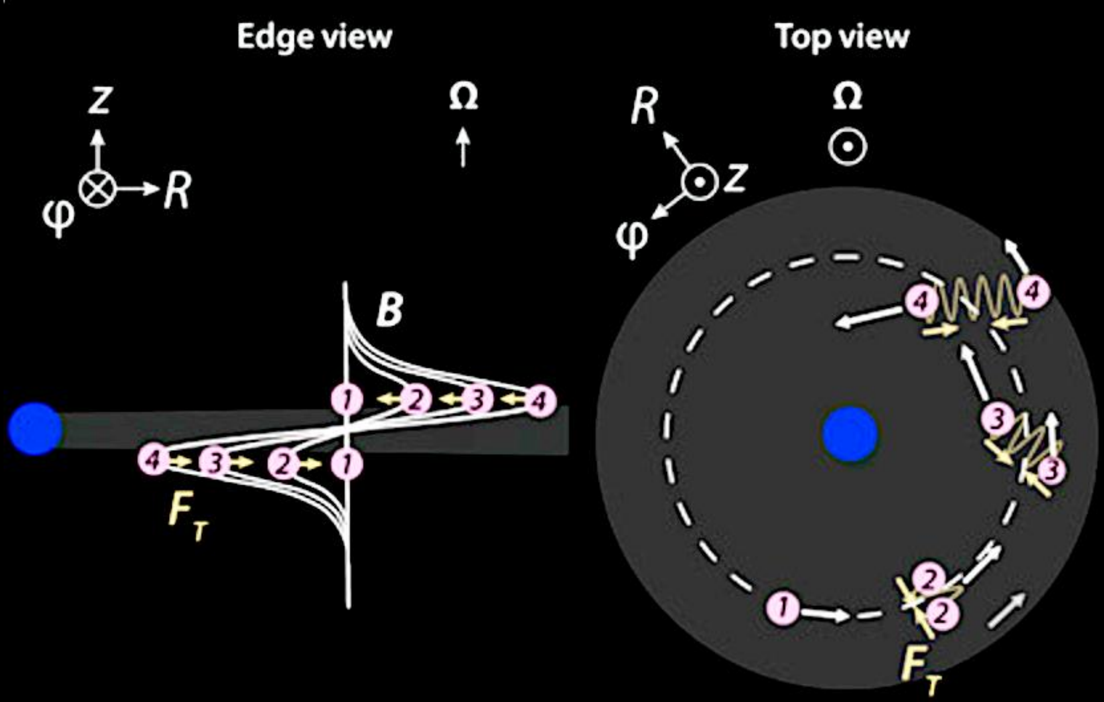
- Thermal instability naturally leads to 2 stable phases (WNM & CNM)!

Credit: Jeong-Gyu Kim

Magnetorotational Instability (MRI)

MRI is an efficient way to transport angular momentum in accretion disks



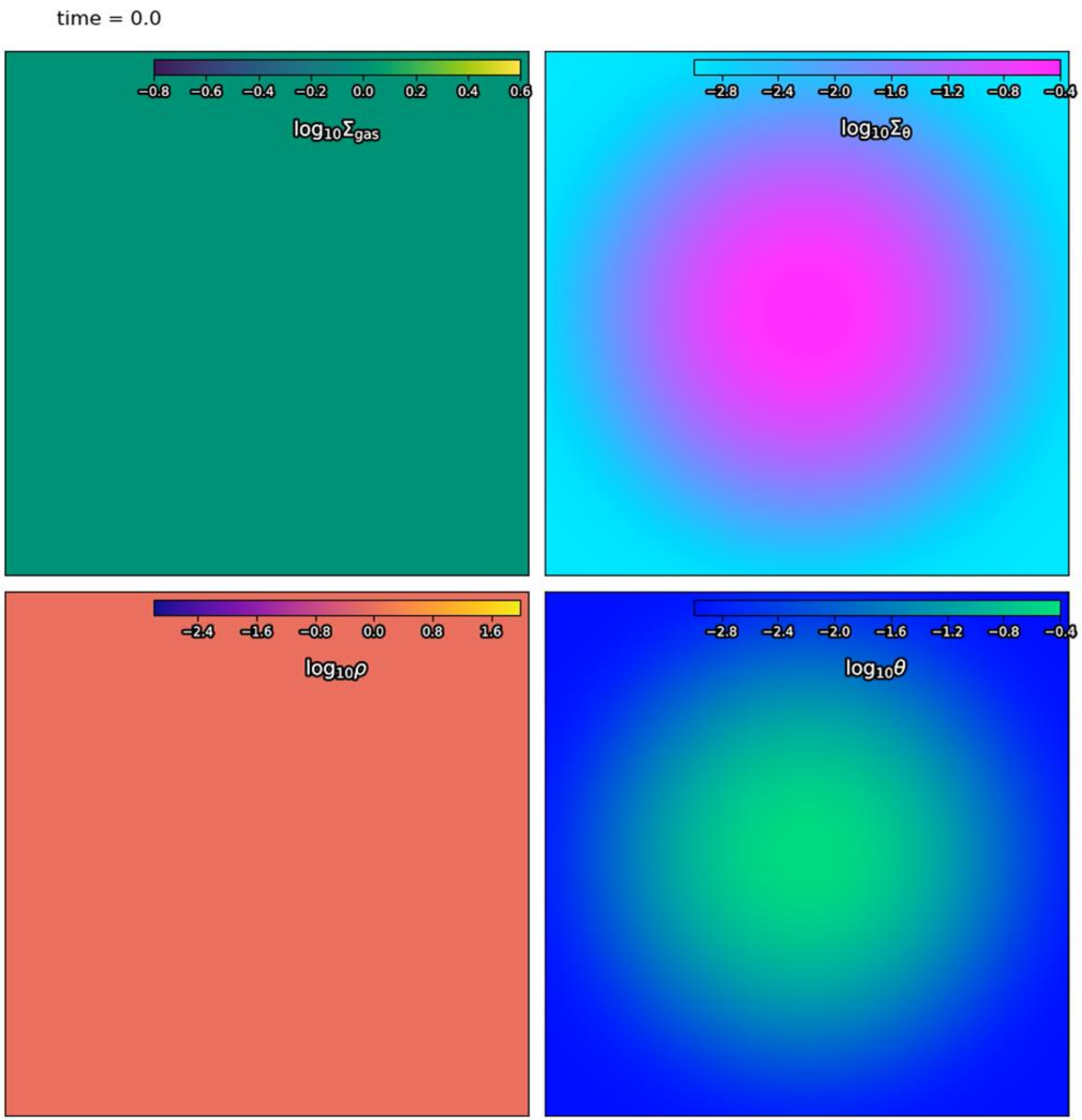


Weiss+ 2021

Turbulence

Turbulence

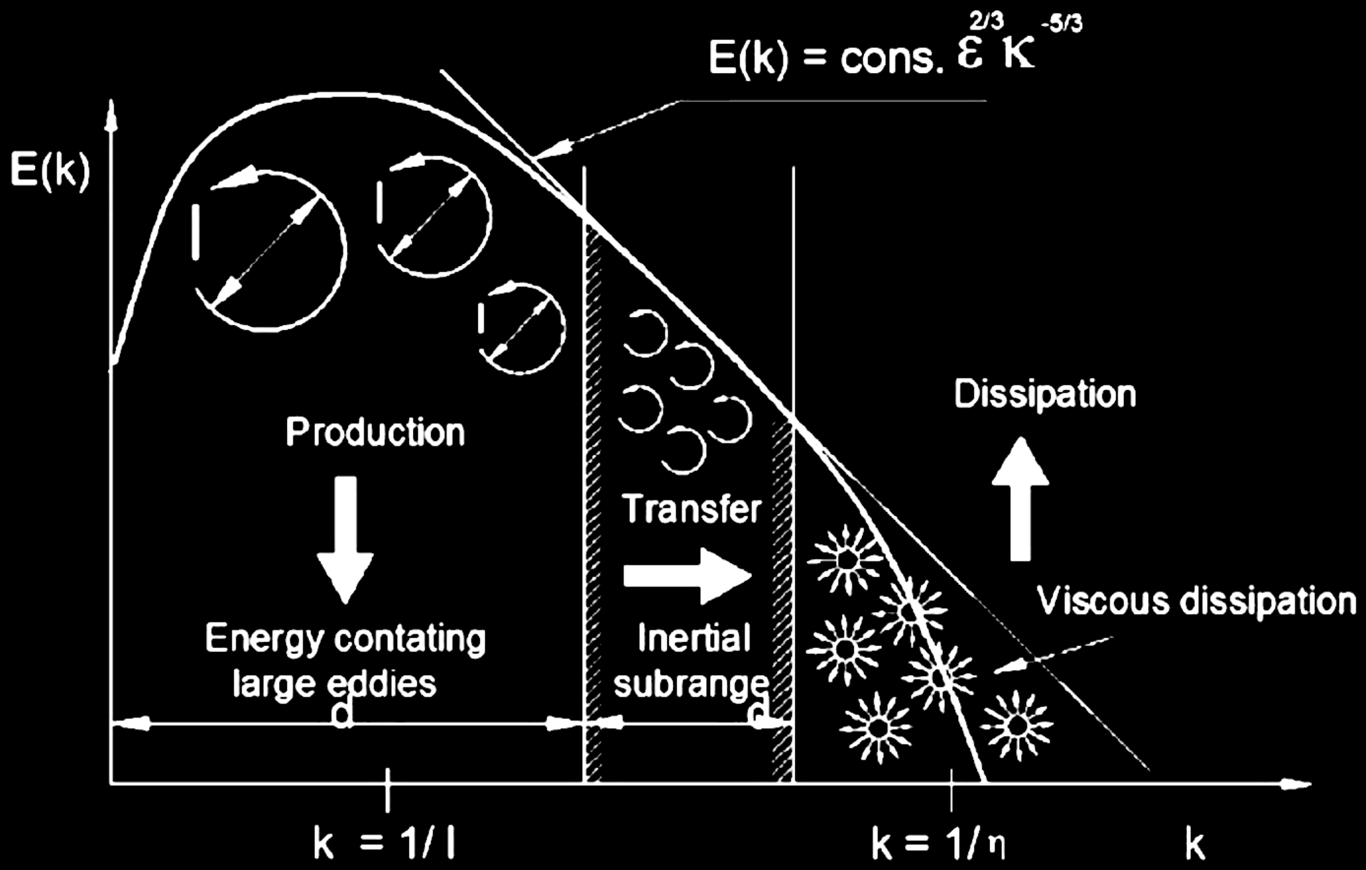
- Chaotic, unpredictable motion.
- Vortices (or eddies)
- Mixing
- Dissipation



Kolmogorov’s energy spectrum

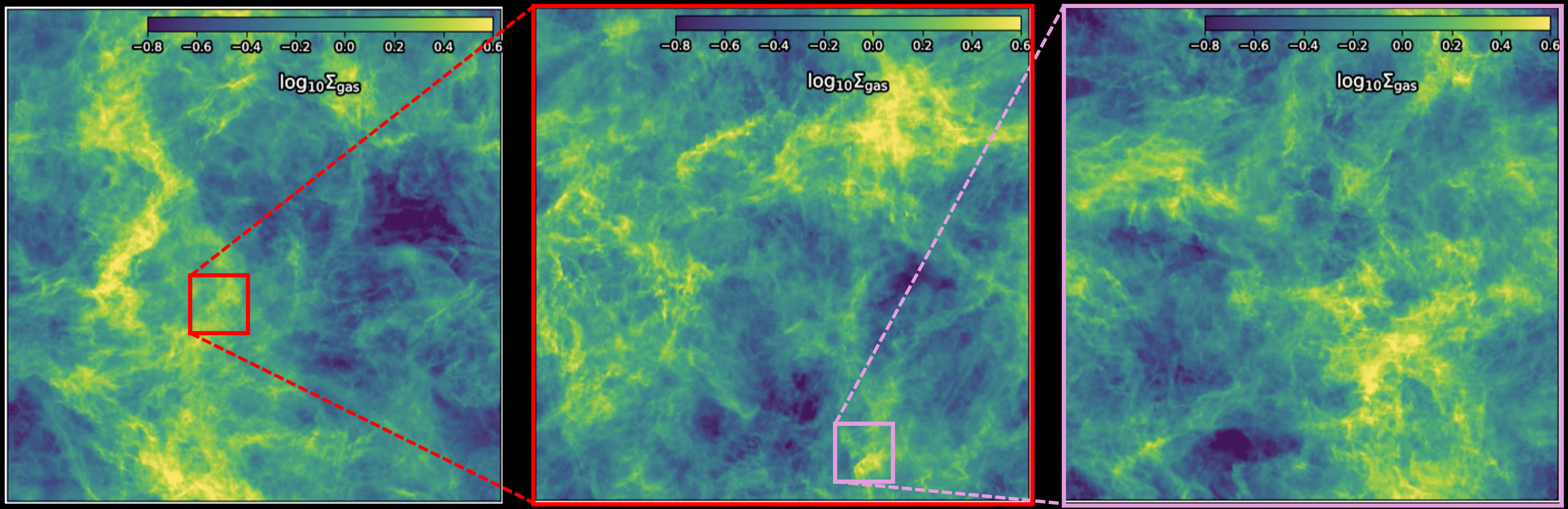
- Kinetic energy of turbulent eddies is quickly transferred from large scales to smaller scales via the energy cascade and is eventually dissipated into heat via viscosity.

- The energy spectrum follows a power law of ~ $k^{-5/3}$ (Kolmogorov’s 5/3 law)!



Turbulence is self-similar!

- In the inertial range, turbulence is scale free (i.e., the statistical properties are independent of the spatial scale of interest).



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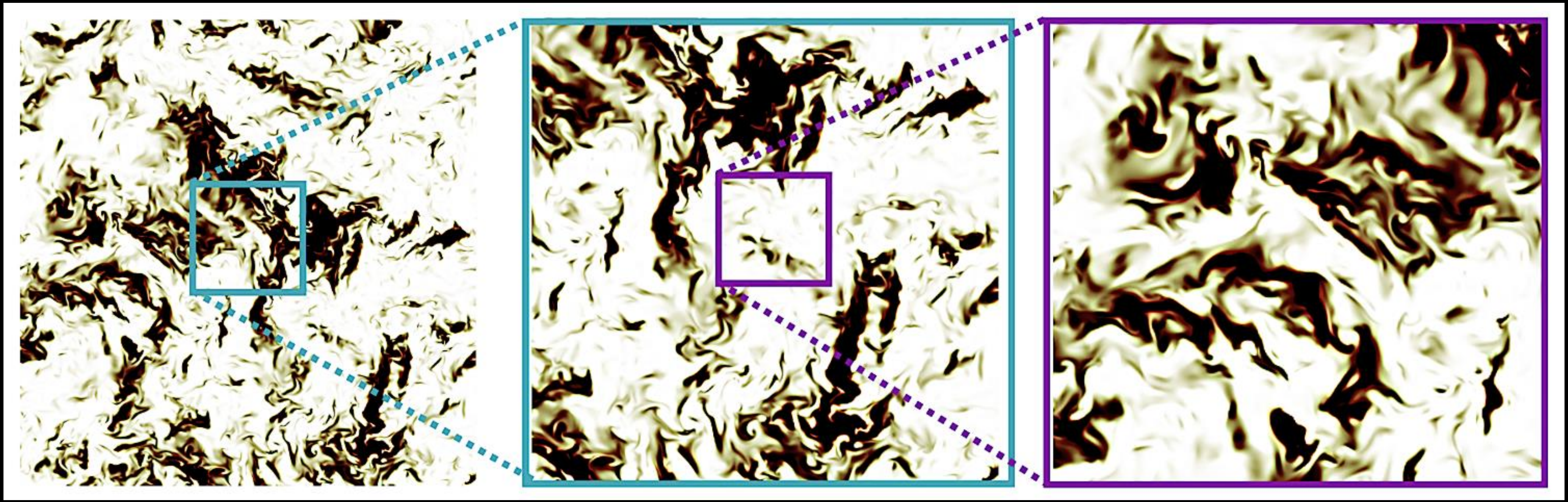
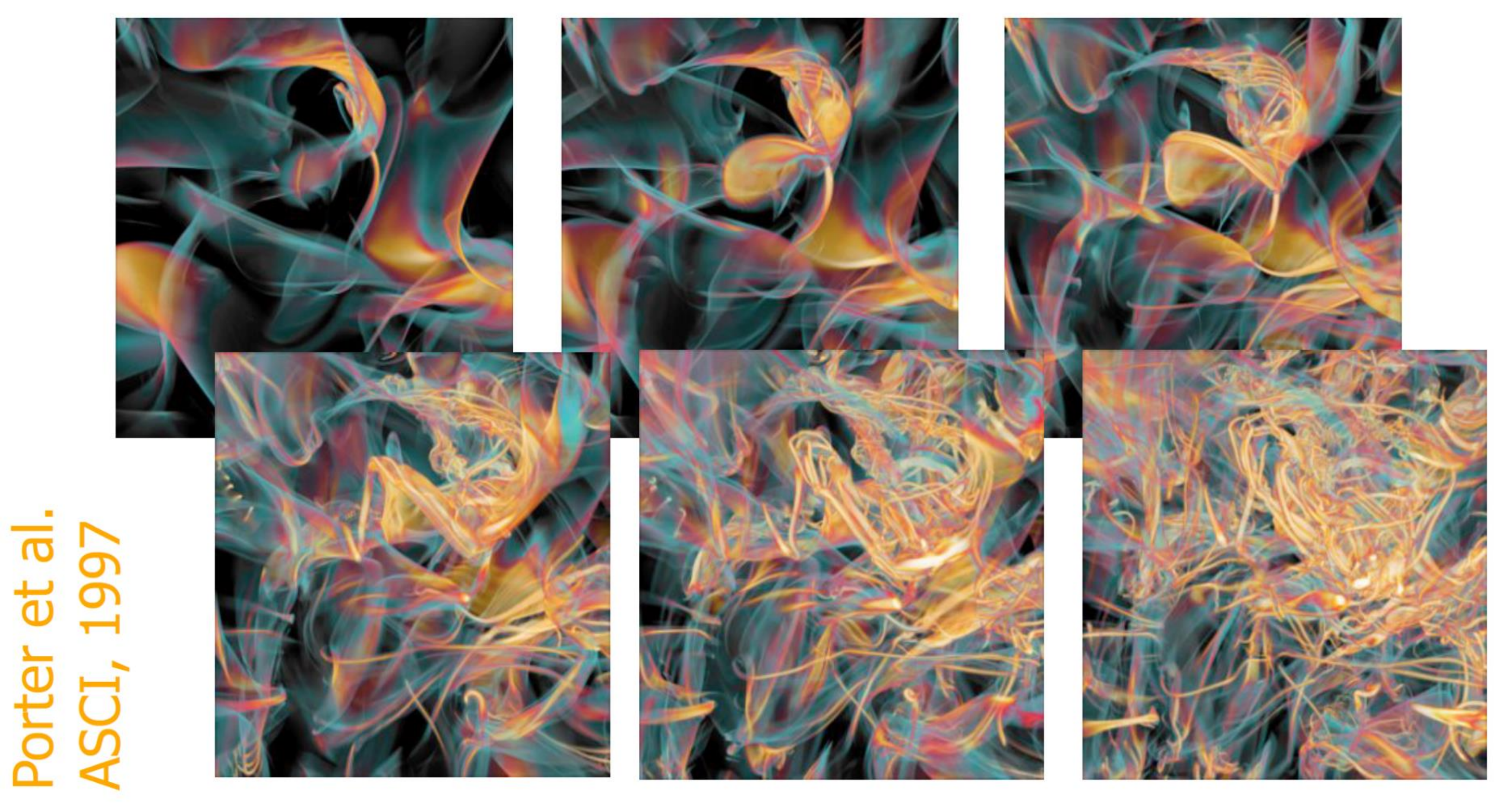


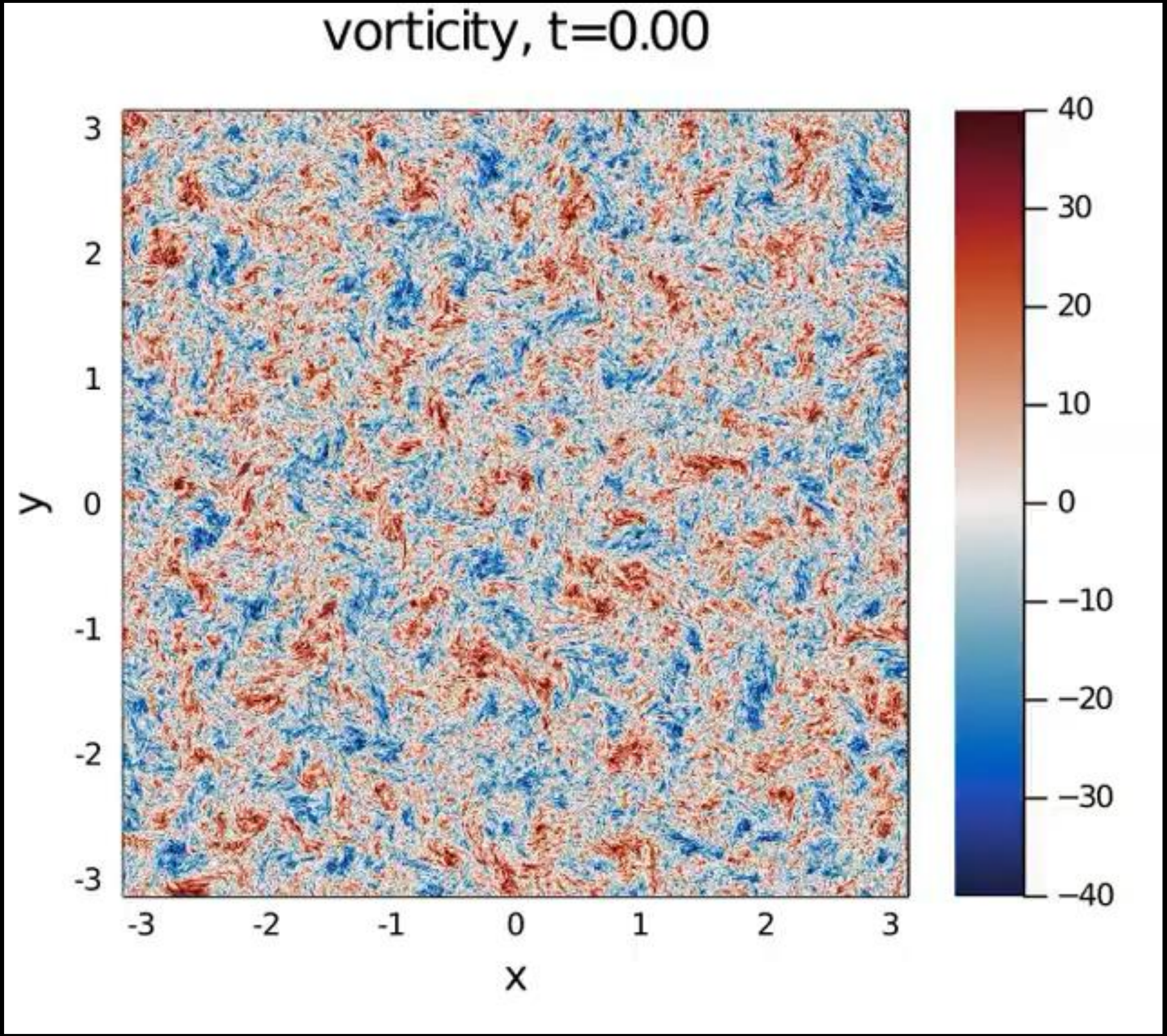
Turbulence is a fundamentally 3D phenomenon



- Energy is transferred from large scales to small scales as vortices are stretched and folded, which can only occur in 3D!

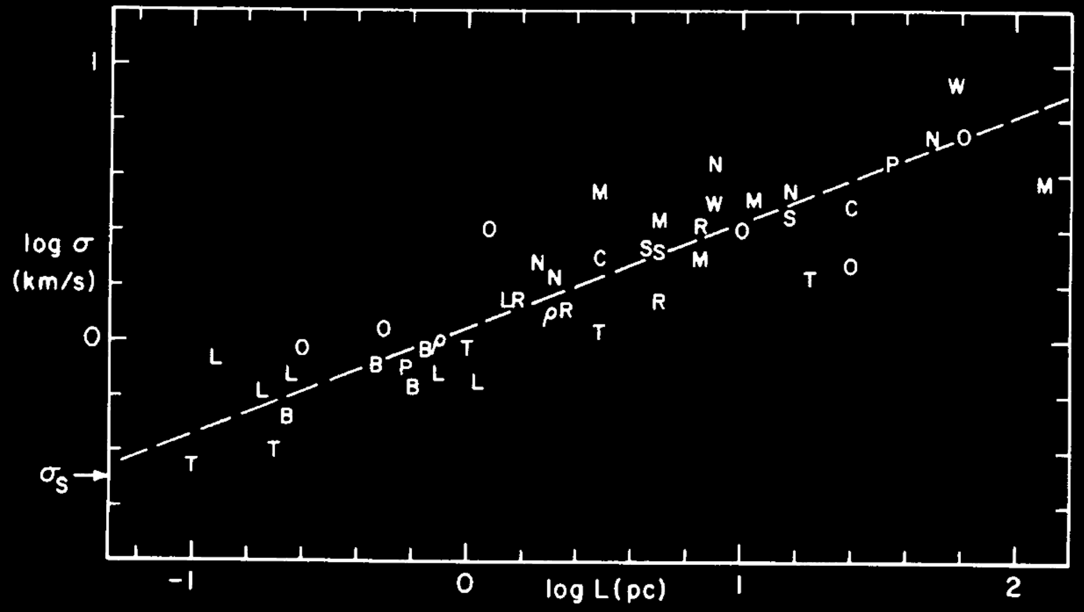
“Turbulence” in 2D behaves very differently

Inverse cascade: energy transferred from small scales to large scales!



Larson's Relation

- Molecular clouds are supersonically turbulent!



Larson 1981

Larson's Relation

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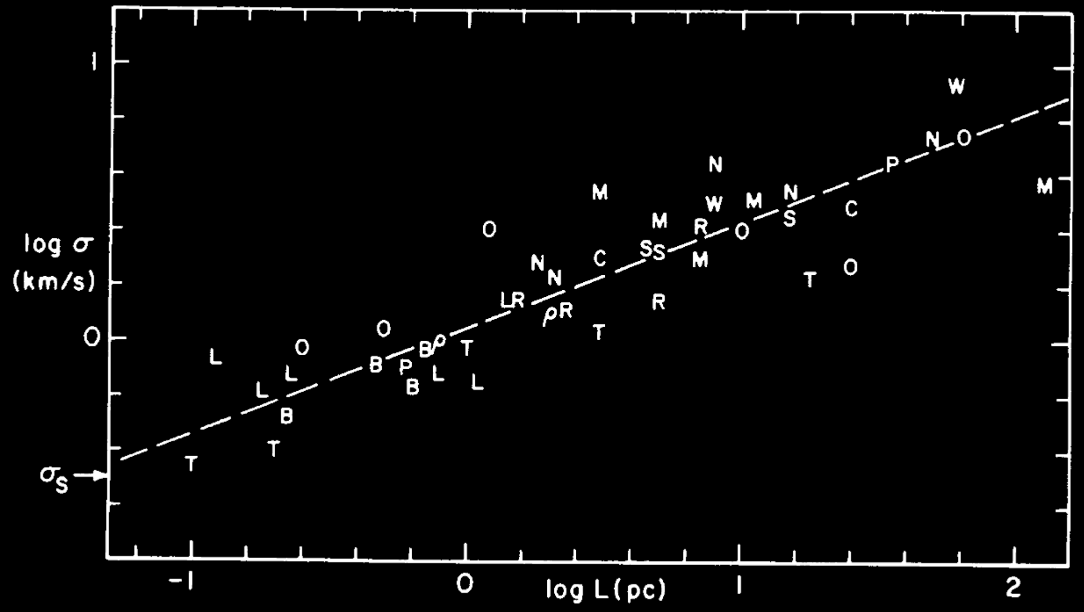
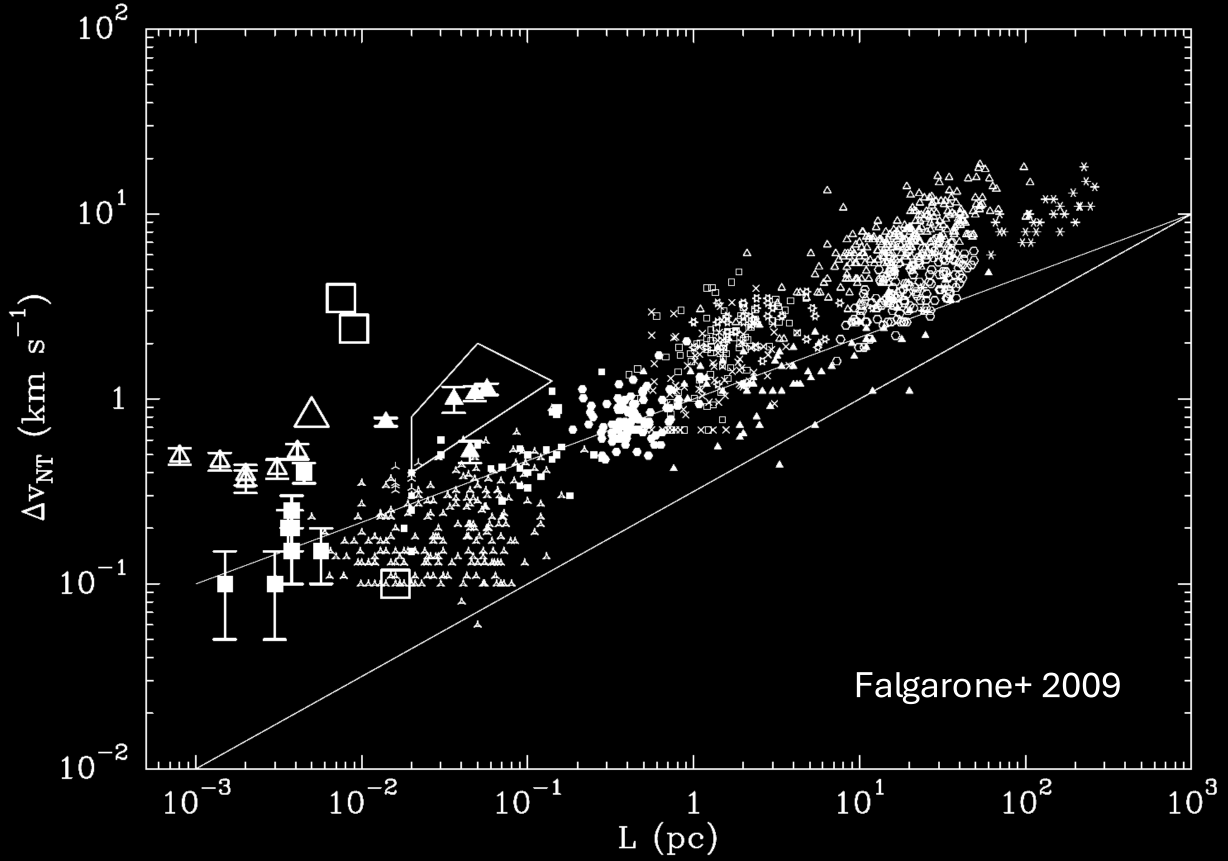
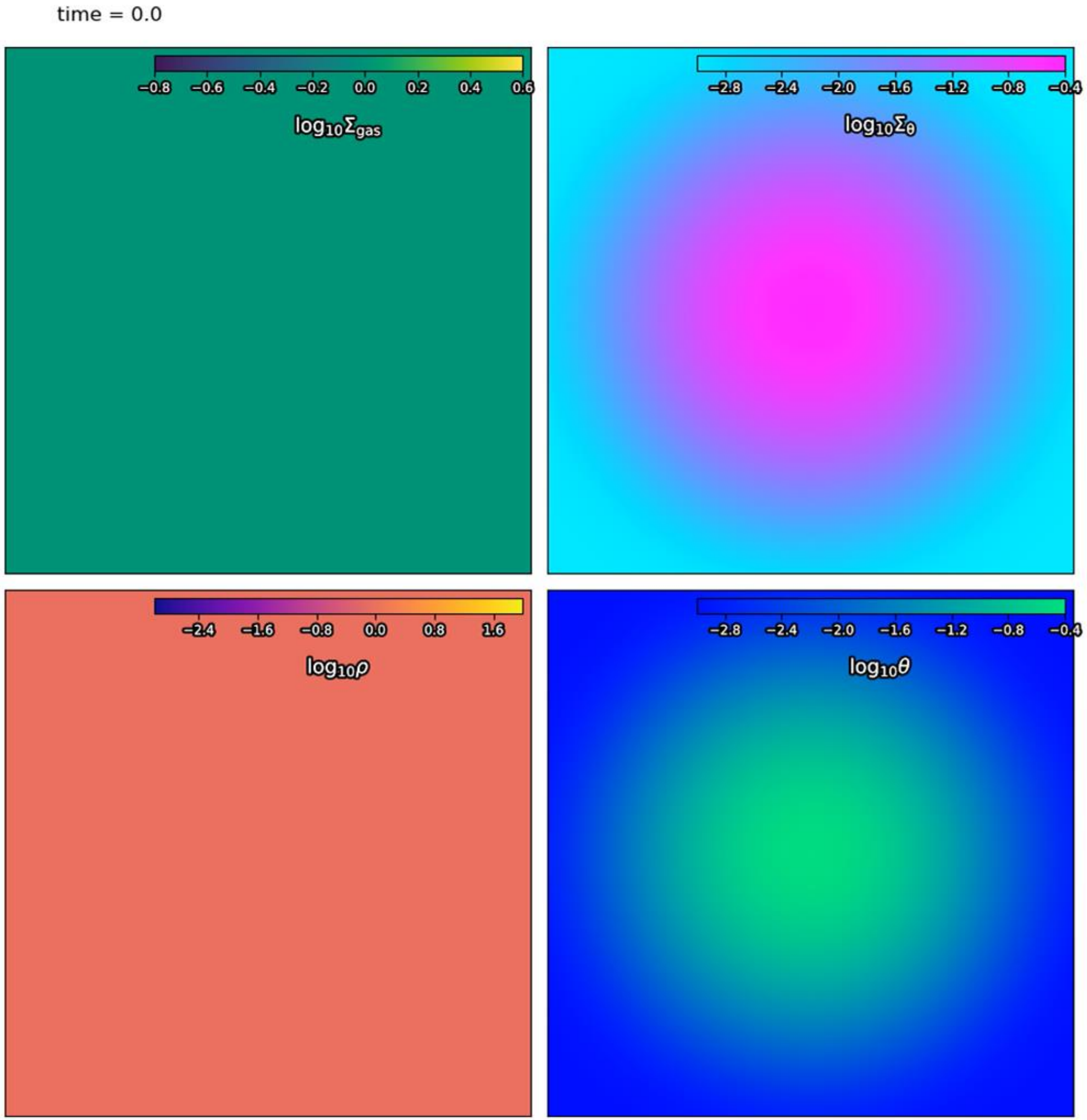


Larson 1981

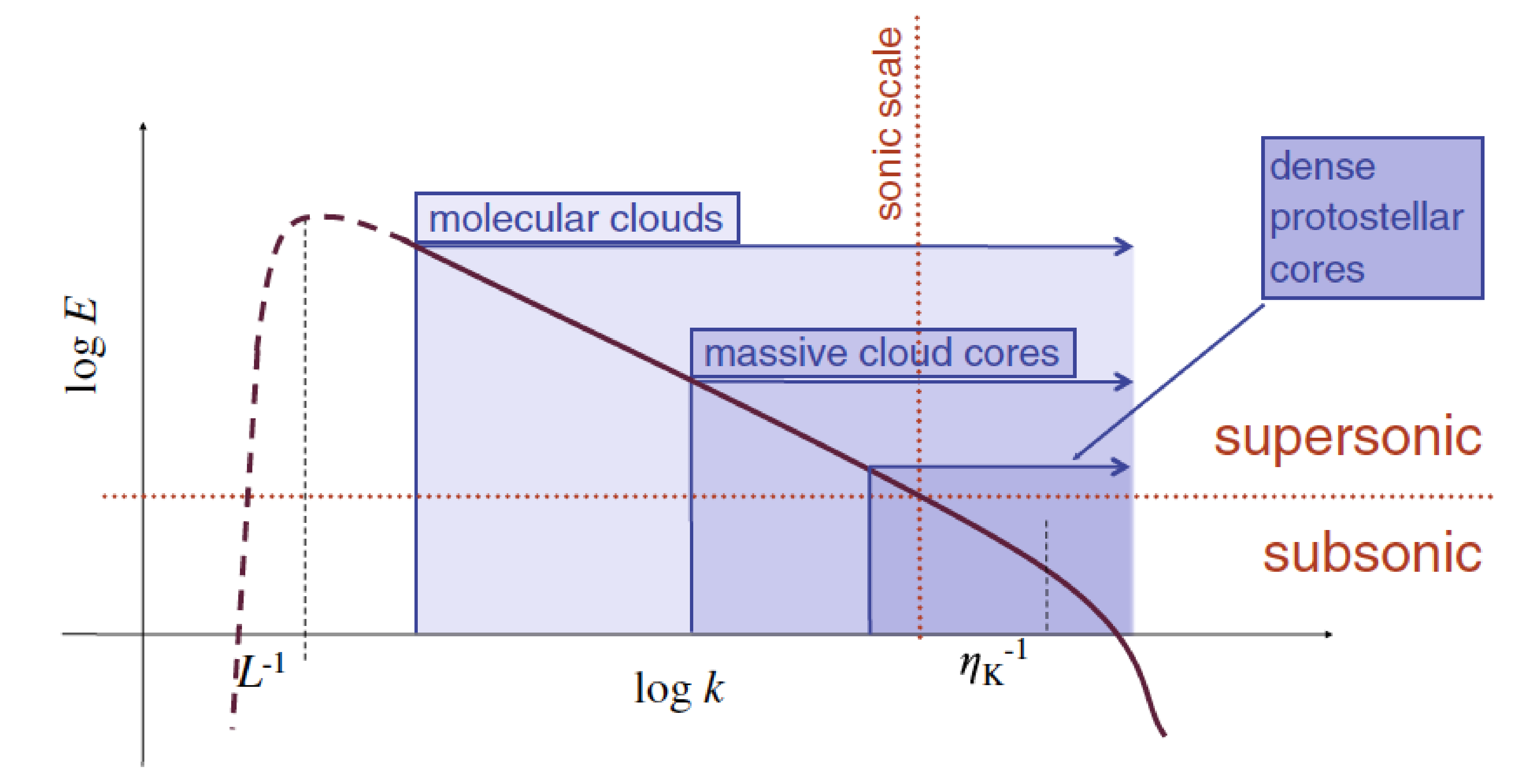


Supersonic Turbulence

- Compressible flows
- Shocks
- Density variations

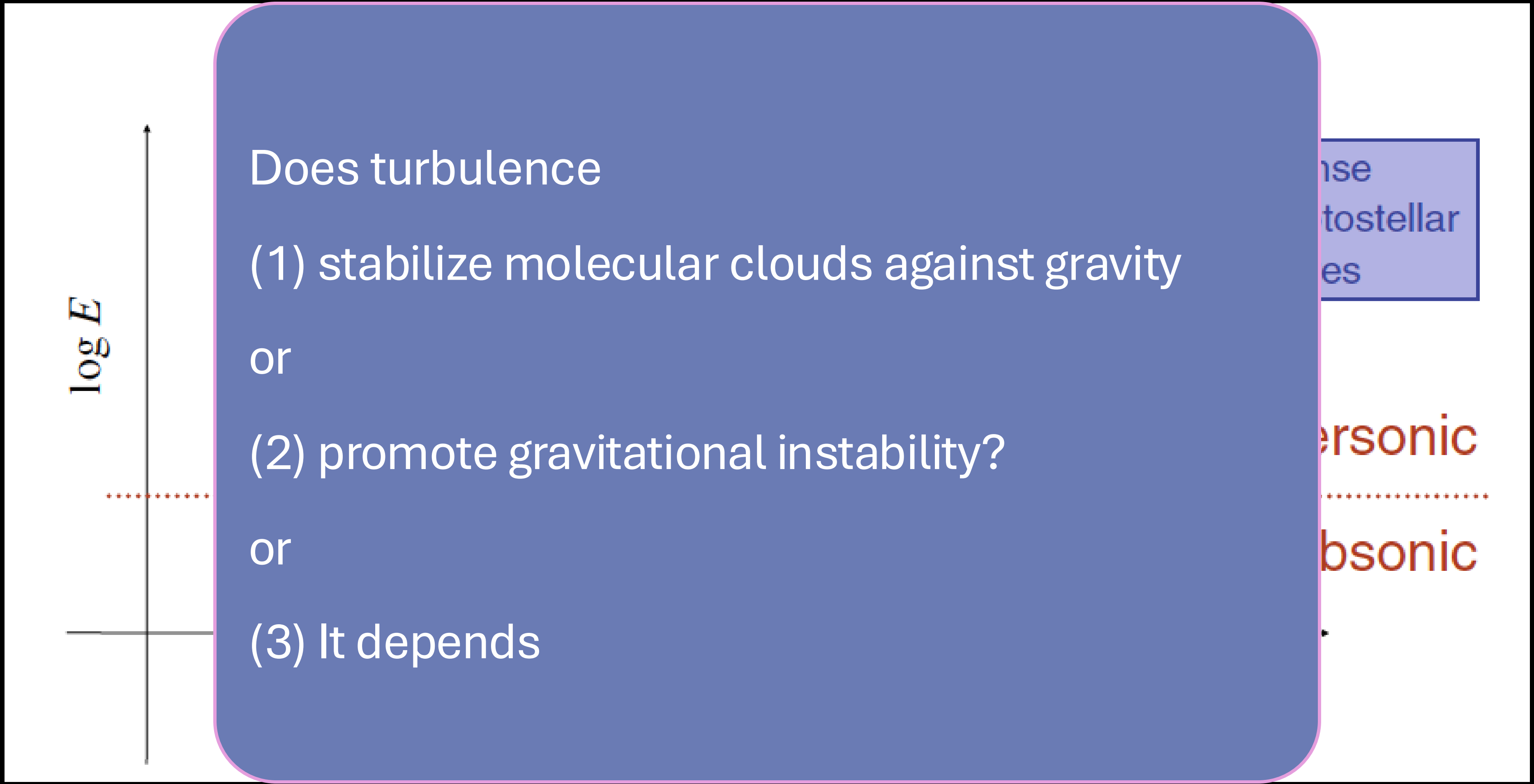


Supersonic Turbulence in Molecular Clouds



- Molecular clouds are structured from the cloud scale down to the sonic scale!

Supersonic Turbulence in Molecular Clouds



Does turbulence

(1) stabilize molecular clouds against gravity

or

(2) promote gravitational instability?

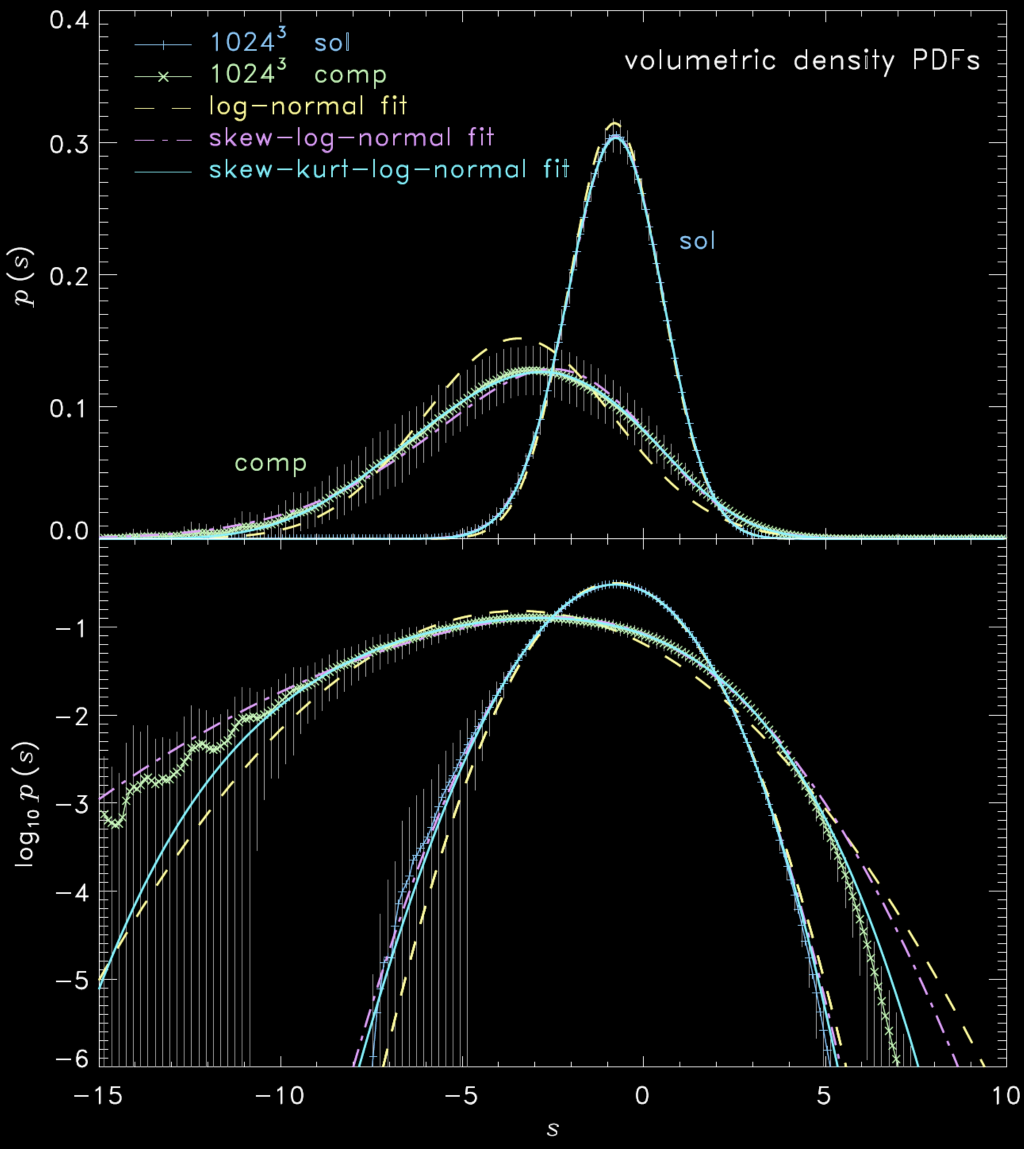
or

(3) It depends

- Molecular clouds are structured from the cloud scale down to the sonic scale!

Supersonic turbulence regulates star formation

- On large scales, it provides an additional nonthermal pressure support against gravity.
- On small scales, it creates over-densities where gas can collapse locally.



Federrath+ 2010

Summary

- Governing equations of fluid dynamics: conservation of mass, momentum, energy.
- **Waves**: perturbations oscillate around equilibrium

  - sound waves, gravity waves, MHD waves
- **Shocks**: inevitable due to nonlinear wave steepening

  - supernova blastwaves
- **Instabilities**: perturbations driven away from equilibrium

  - gravitational instability, Rayleigh-Taylor and Kelvin-Helmholz instability, thermal instability, MRI
- **Turbulence**: ubiquitous in astrophysics ($\mathrm{Re} \gg 1$)

  - chaotic energy cascade from large scales to small scales

Backup Slides

Magnetohydrodynamics (MHD)

$$\frac{\partial}{\partial t}\boldsymbol{\nabla}\cdot\mathbf{B}=0$$

$$\boldsymbol{\nabla}\cdot\mathbf{B}\;=\;0$$

$$\frac{\partial\mathbf{B}}{\partial t}=\boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{B})$$

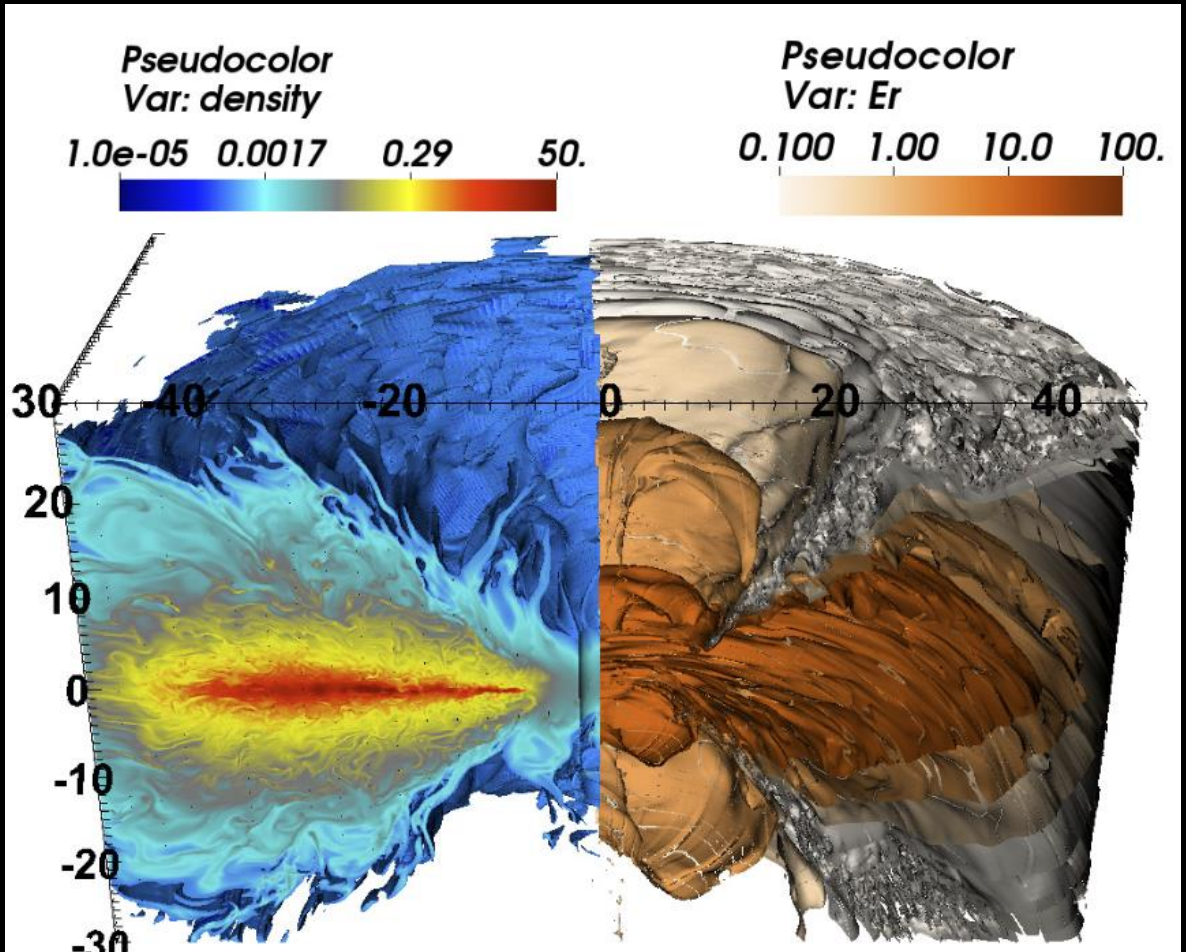
$$\mathbf{j}=\frac{c}{4\pi}\boldsymbol{\nabla}\times\mathbf{B}$$

$$\boldsymbol{\nabla}\cdot\mathbf{j}=0$$

$$\mathbf{B}'=\mathbf{B}[1+\mathcal{O}(v^2/c^2)],\qquad 4\pi\mathbf{j}-\frac{\partial}{\partial t}(\mathbf{v}\times\mathbf{B})/c=c\boldsymbol{\nabla}\times\mathbf{B}.$$

Magnetorotational Instability (MRI)

accretion disk near a black hole



Credit: Yan-Fei Jiang

NTU logo background removed