

NCTS-TCA Summer Student Program 2025
Mini-Workshop

Radiative Processes and Transport

Sheng-Yuan Liu (ASIAA)

Astronomy and Astrophysics an Observational Science



https://cdn.shopify.com/s/files/1/0742/7719/1954/files/famous_astronomers_1024x1024.jpg?v=1682805109

Astronomy and Astrophysics

an Observational Science

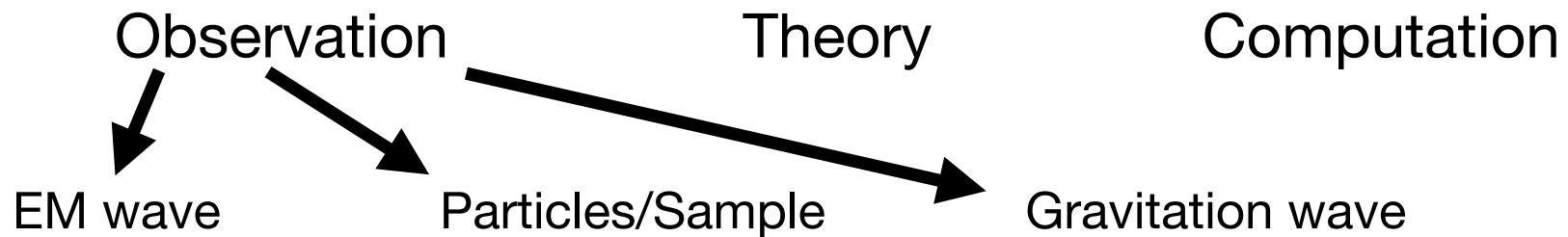
Observation

Theory

Computation

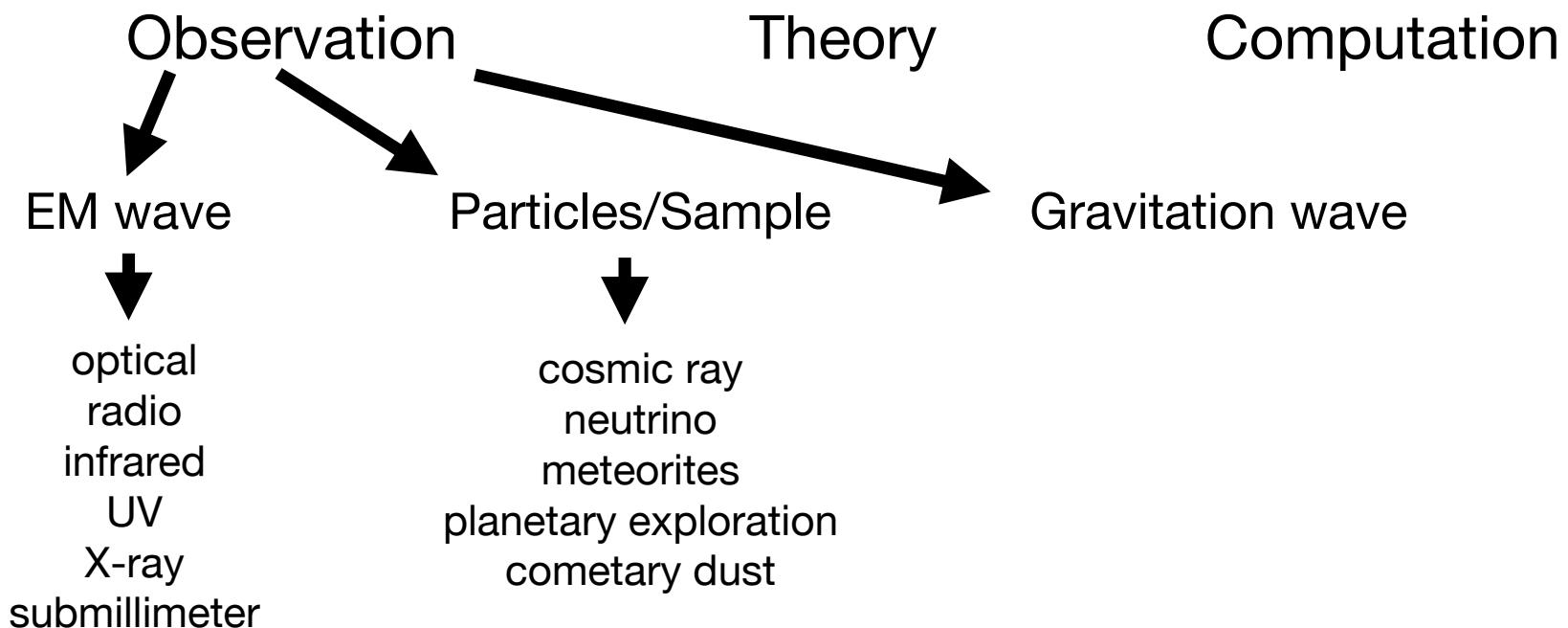
Astronomy and Astrophysics

an Observational Science



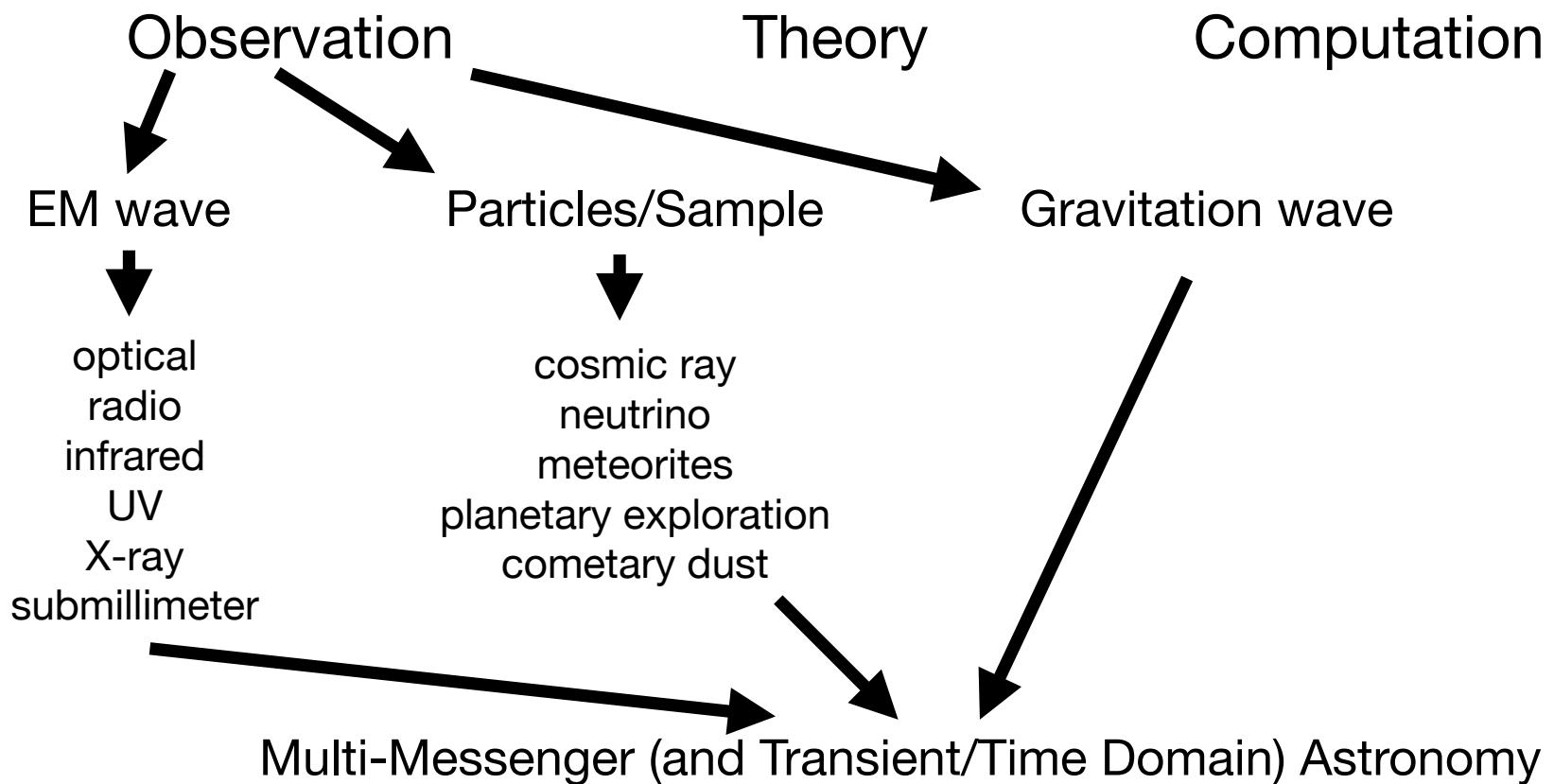
Astronomy and Astrophysics

an Observational Science

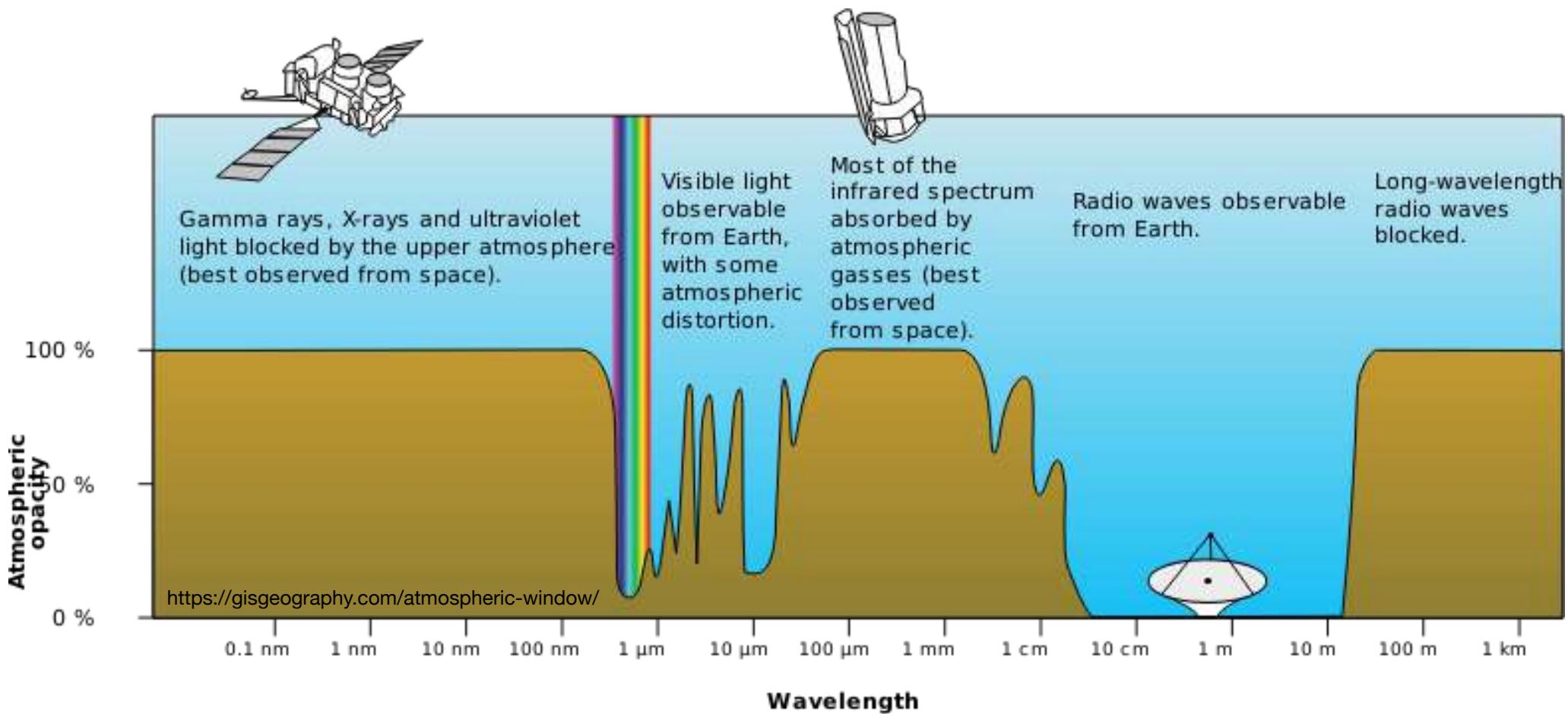


Astronomy and Astrophysics

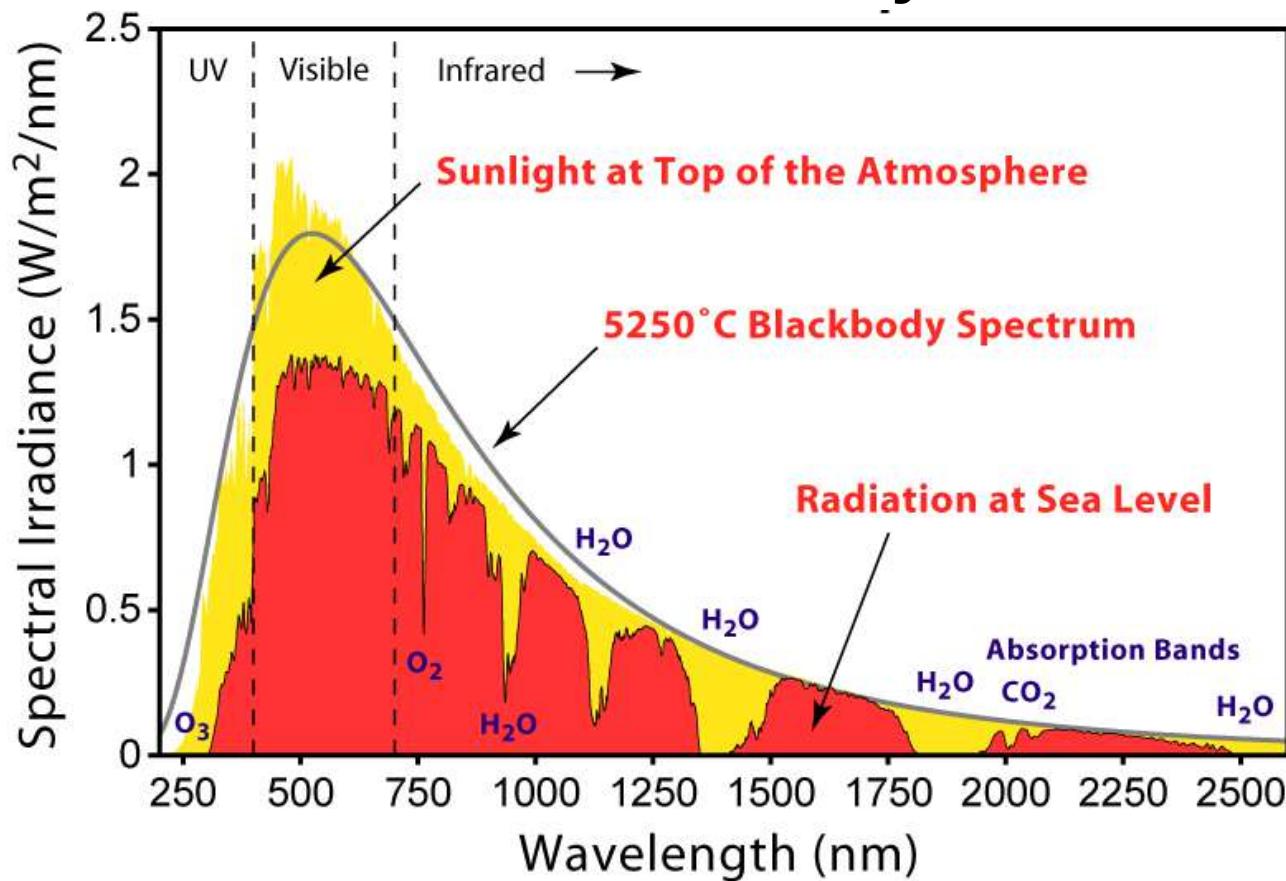
an Observational Science



Electromagnetic Wave and Atmospheric Window



Solar Radiation Spectrum as a Blackbody



<https://physics.stackexchange.com/questions/130209/how-can-it-be-that-the-sun-emits-more-than-a-black-body>

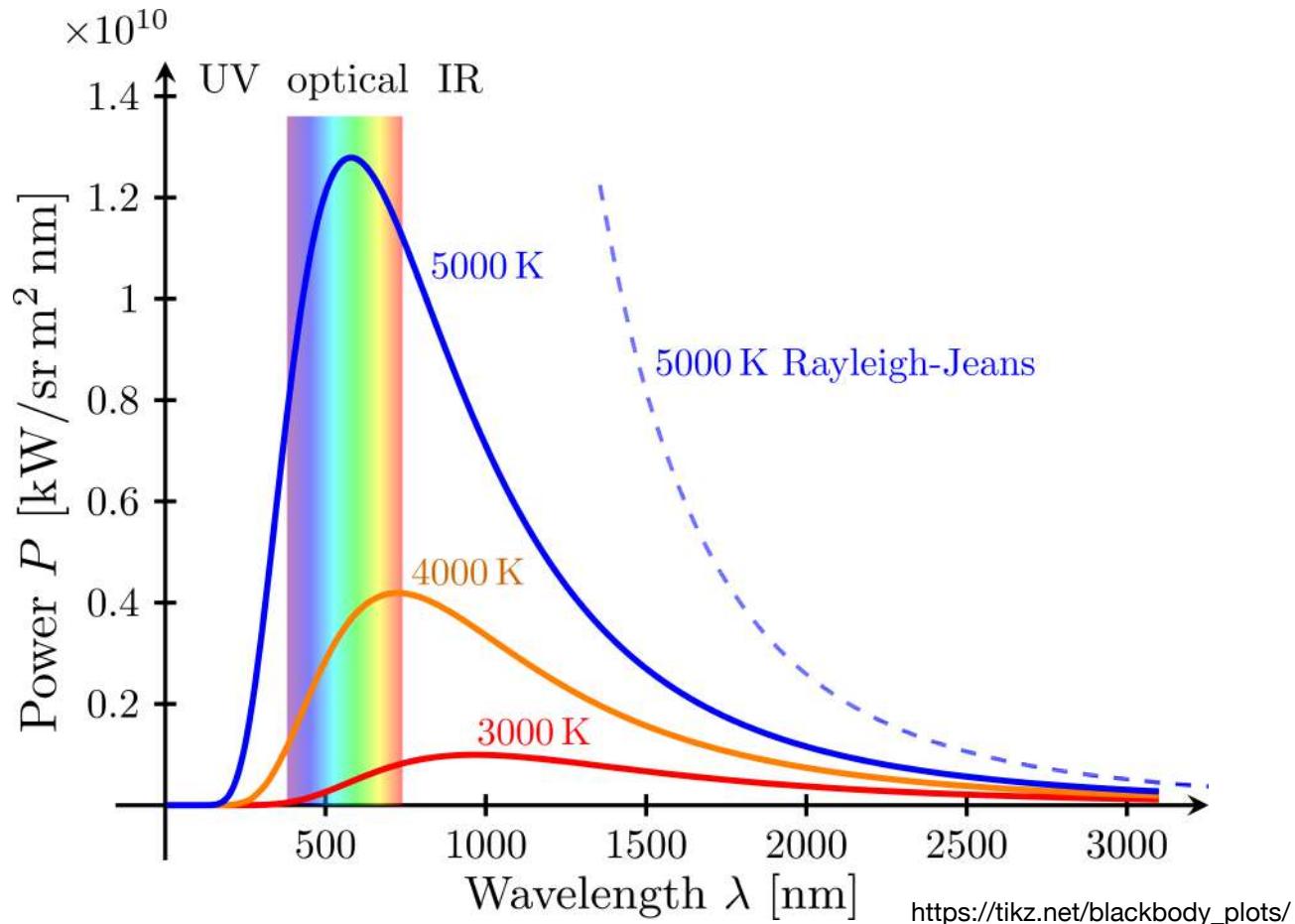
Radiation at multiple wavelengths - Blackbody Radiation

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp(hc/k\lambda T) - 1}$$

Wien's Displacement Law

$$\lambda_{peak} = 0.002898 \left(\frac{T}{K}\right) m$$

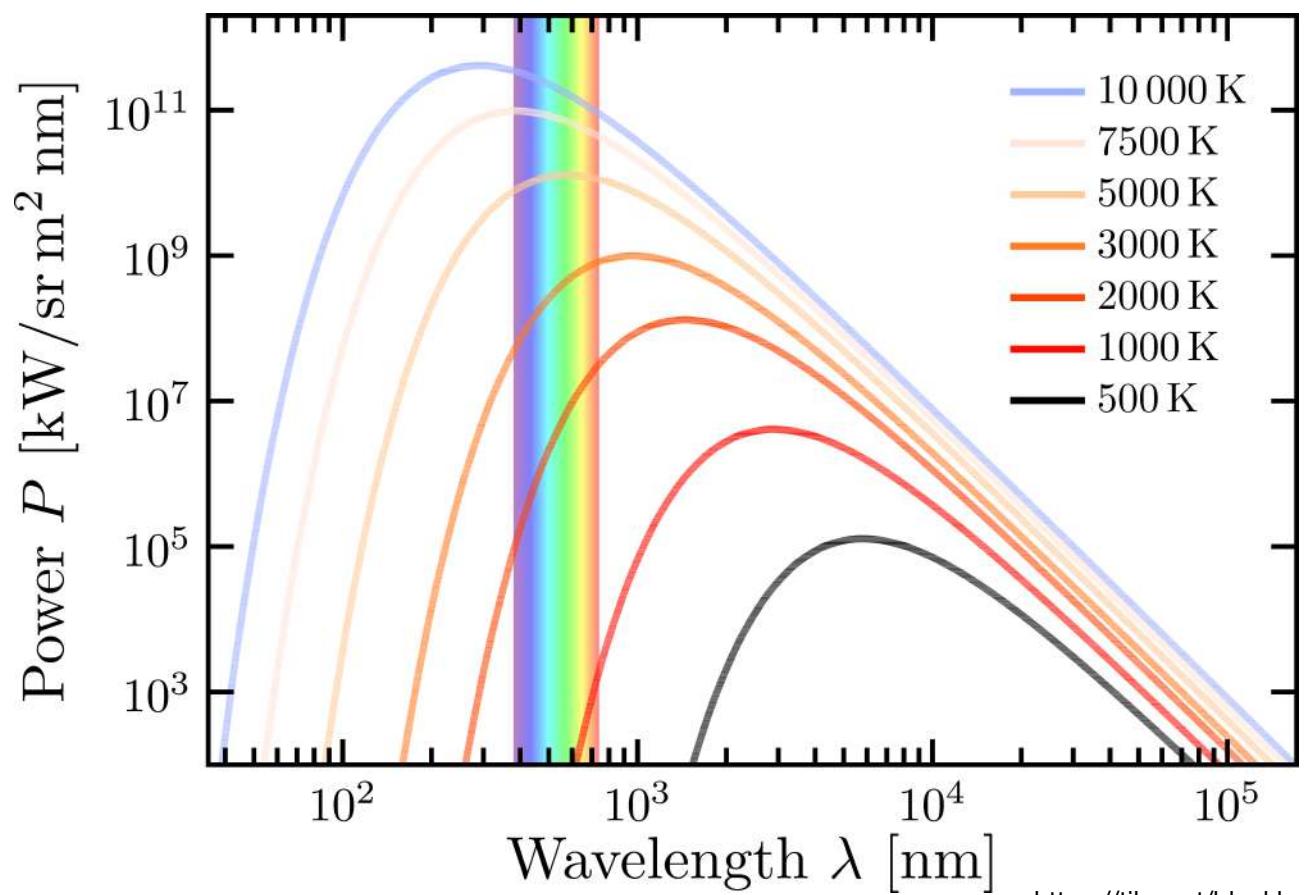


https://tikz.net/blackbody_plots/

Blackbody Radiation

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Blackbody Radiation

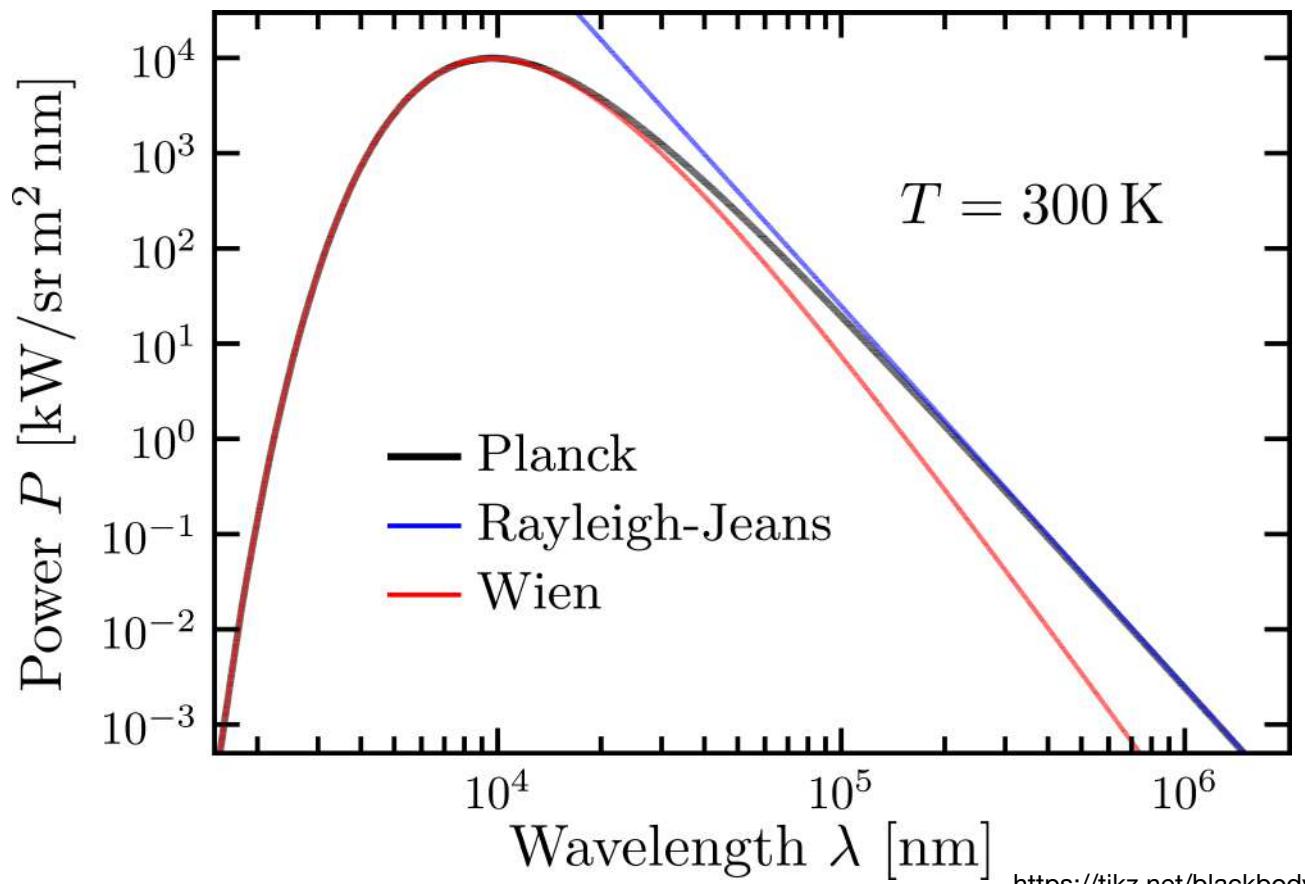
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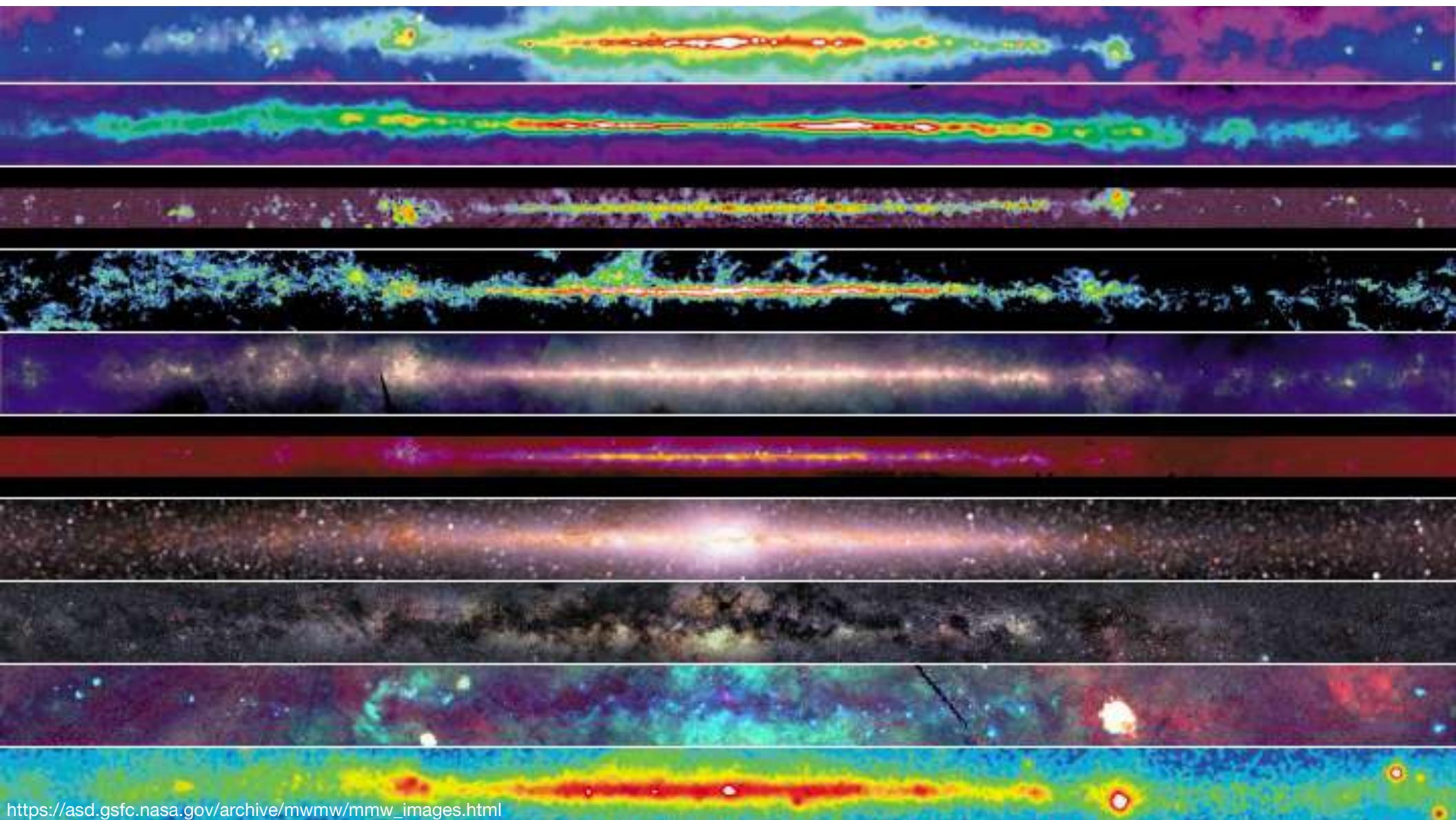
Approximations

$$B_\nu(T) = \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{k_B T}} \quad \text{Wien} \quad (\nu \gg 1)$$

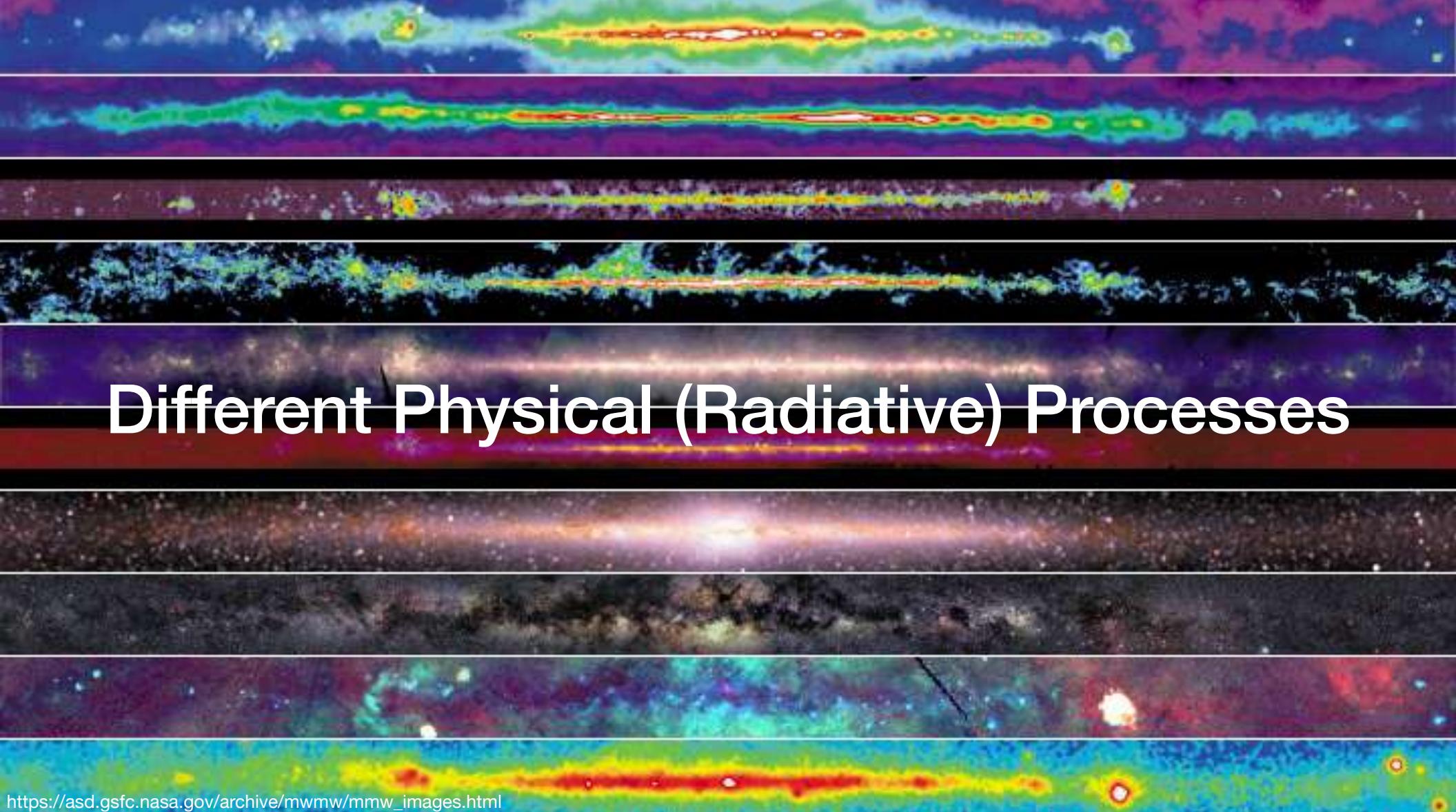
$$B_\nu(T) = \frac{2\nu^2 k_B T}{c^2} \quad \text{Rayleigh-Jeans} \quad (\nu \ll 1)$$



https://tikz.net/blackbody_plots/



https://asd.gsfc.nasa.gov/archive/mwmw/mmw_images.html



Different Physical (Radiative) Processes

**Transport (of EM Waves) =
Radiative Transfer**

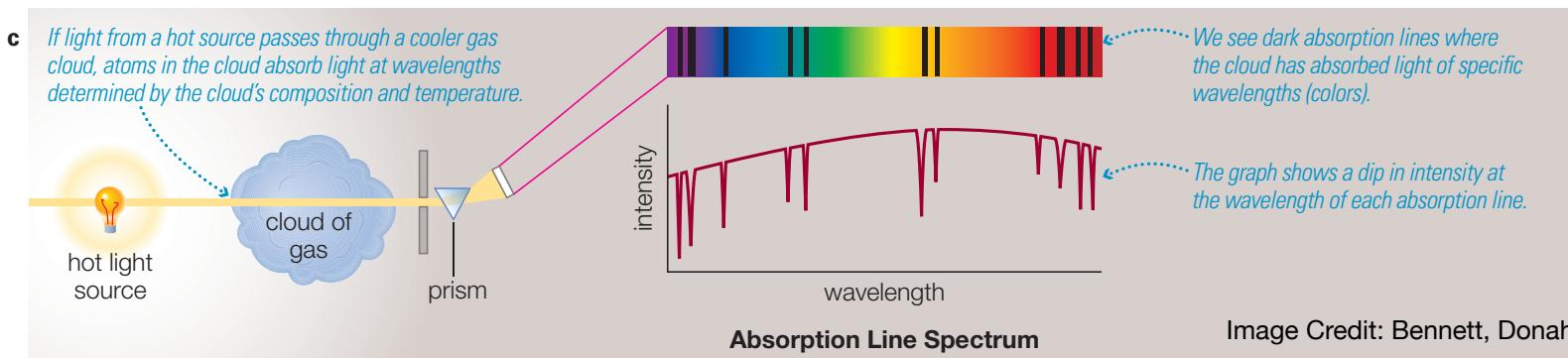
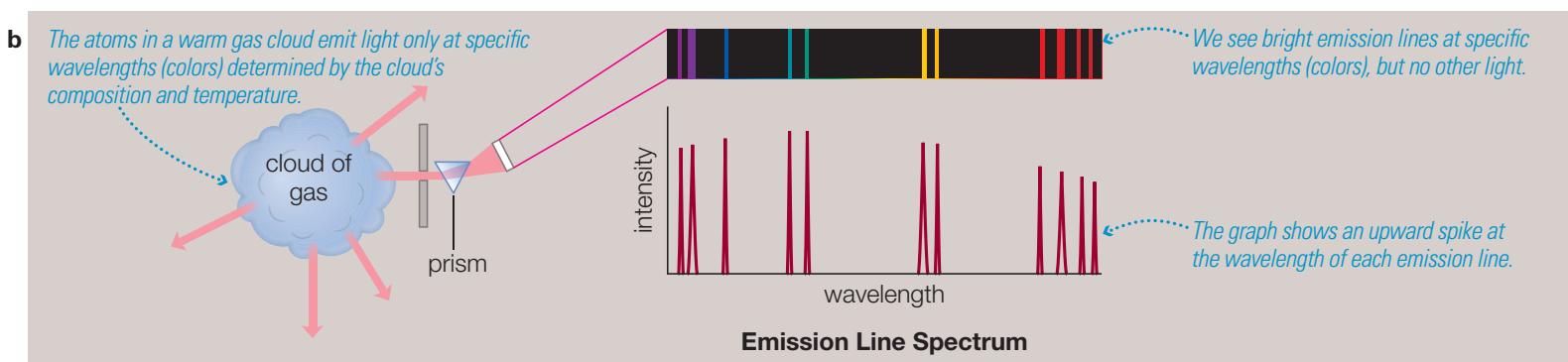
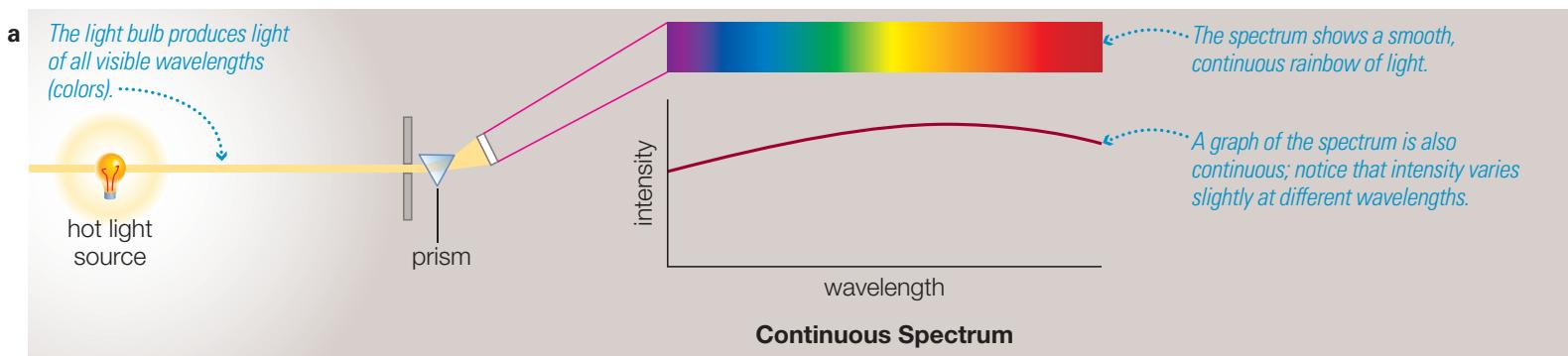
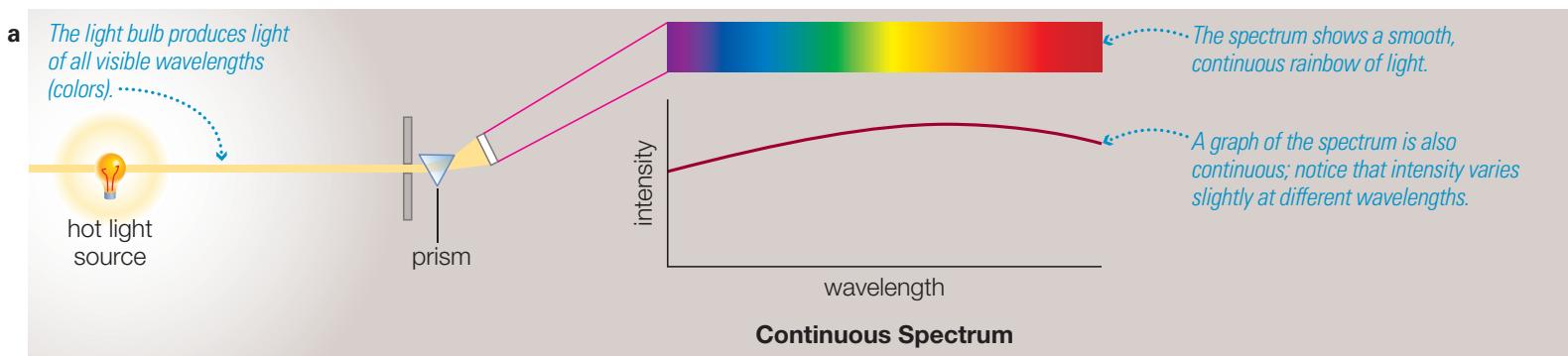
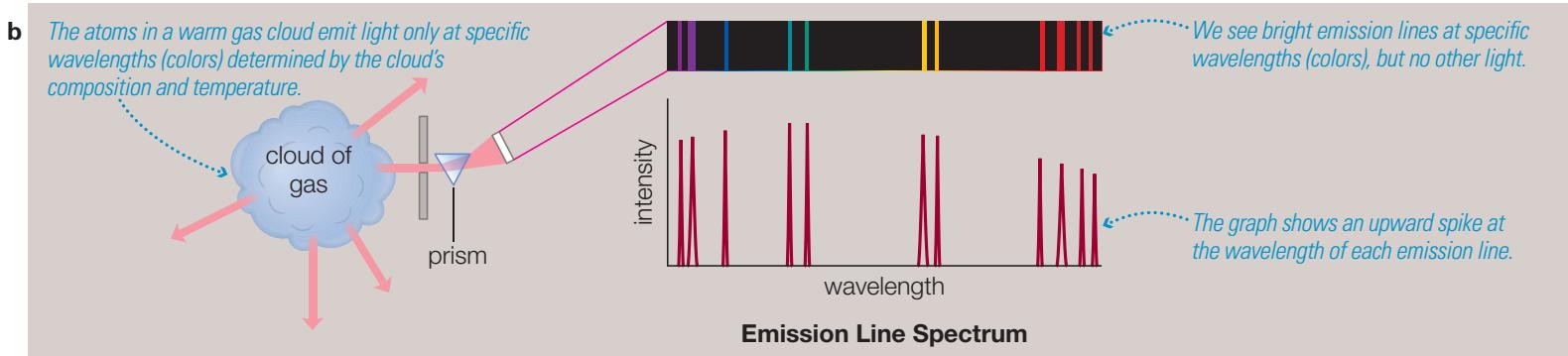


Image Credit: Bennett, Donahue, Schneider, Voit



hot vs. cold

continuum vs. line



foreground vs. background

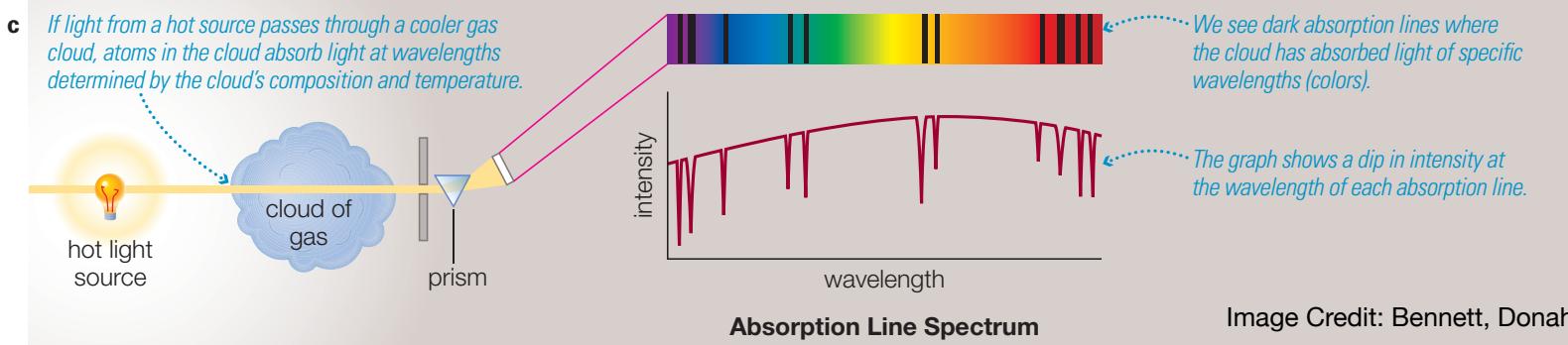
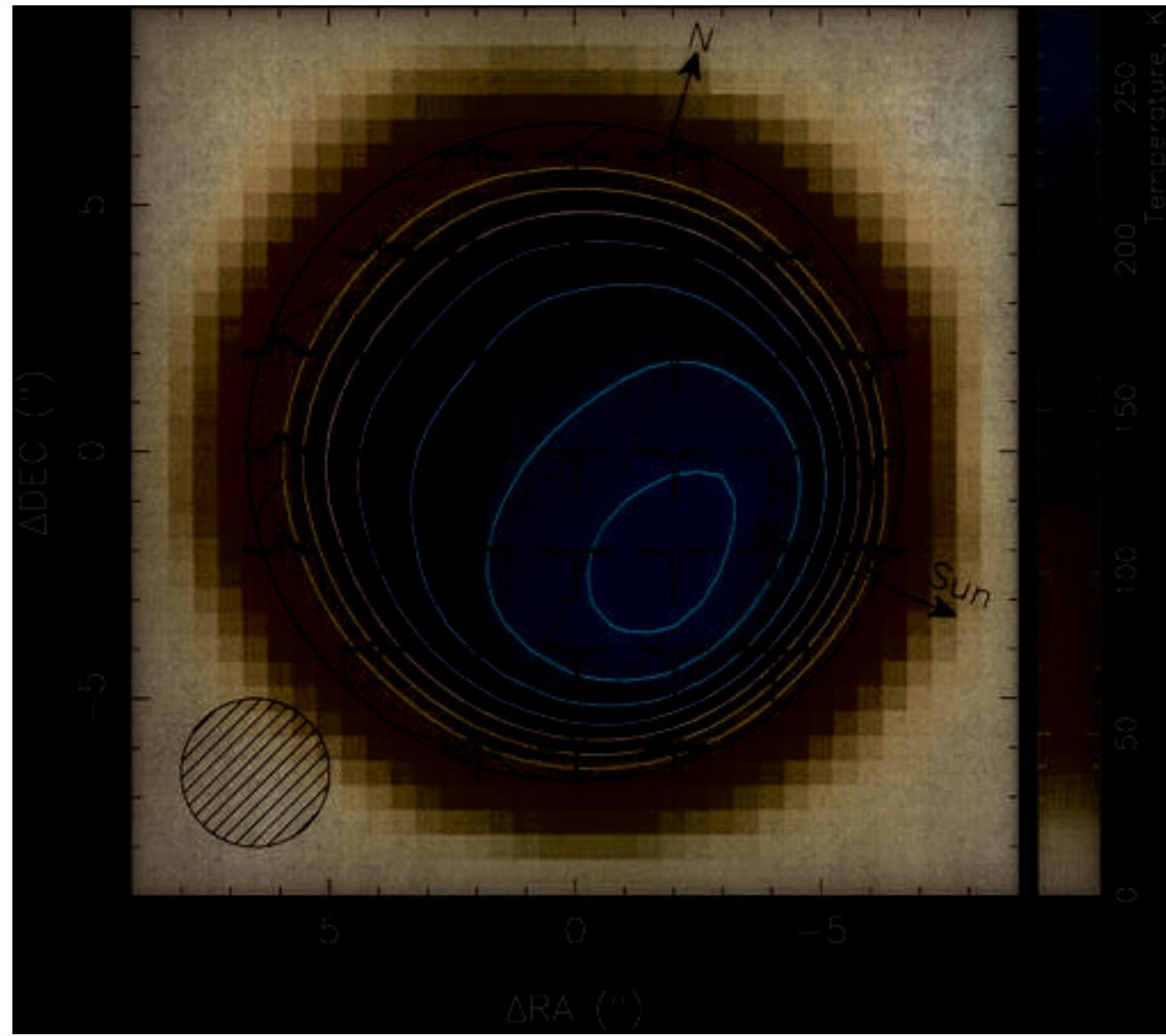


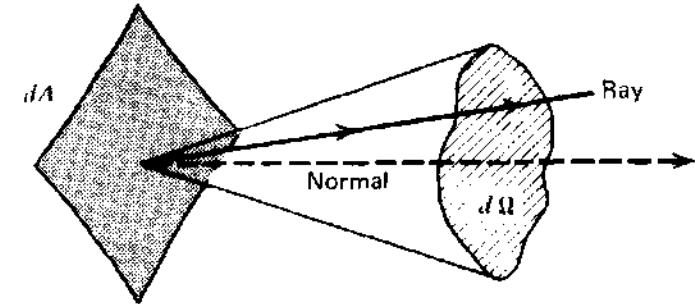
Image Credit: Bennett, Donahue, Schneider, Voit



How bright is it?

Radiation Fundamentals

- Specific Intensity I_ν : $dE = I_\nu d\sigma \cos\theta d\omega d\nu dt$
 - or Intensity, or Brightness, or Brightness Temperature
 - rate of energy transport, along a particular direction, per unit area, per unit solid angle, and per unit frequency
 - conserved along a ray through free space
 - unit: e.g. $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{Sr}^{-1}$
- Flux Density S_ν (or F_ν): $\int I_\nu(\omega \cos(\theta)) d\omega$
- Flux F : $\int S_\nu d\nu$
 - energy per time per area, depends on the relative position of source and the observer
- Power P : $\int F d\sigma$
 - or Luminosity
 - energy per time given off by a source



Geometry for normally incident rays.

Image Credit: Rybicki and Lightman

How bright is it?

Radiation Fundamentals

- Specific Intensity I_ν : $dE = I_\nu d\sigma \cos\theta d\omega d\nu dt$
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Recall: Approximations

$$B_\nu(T) = \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{k_B T}} \quad (\nu \gg 1) \quad \text{Wien}$$

$$B_\nu(T) = \frac{2\nu^2 k_B T}{c^2} \quad (\nu \ll 1) \quad \text{Rayleigh-Jeans}$$

Radiative Transfer (of EM wave)

- Basic equation of (radiative) transfer
 - $dI_\nu = j_\nu \, ds - \kappa_\nu \, I_\nu \, ds$
 - j_ν : emission coefficient
 - α_ν : absorption or extinction coefficient (cm^{-1}) = $\kappa_\nu \rho$ where κ_ν is the mass absorption coefficient
 - Both j_ν and α_ν are a function of intrinsic properties and physical condition of the medium
 - $\frac{dI_\nu}{\alpha_\nu ds} = - I_\nu + \frac{j_\nu}{\alpha_\nu}$
 - $S_\nu \equiv \frac{j_\nu}{\alpha_\nu}$; $d\tau_\nu \equiv \alpha_\nu \, ds$ where τ_ν is the optical depth
- $$\frac{dI_\nu}{d\tau} = - I_\nu + S_\nu$$

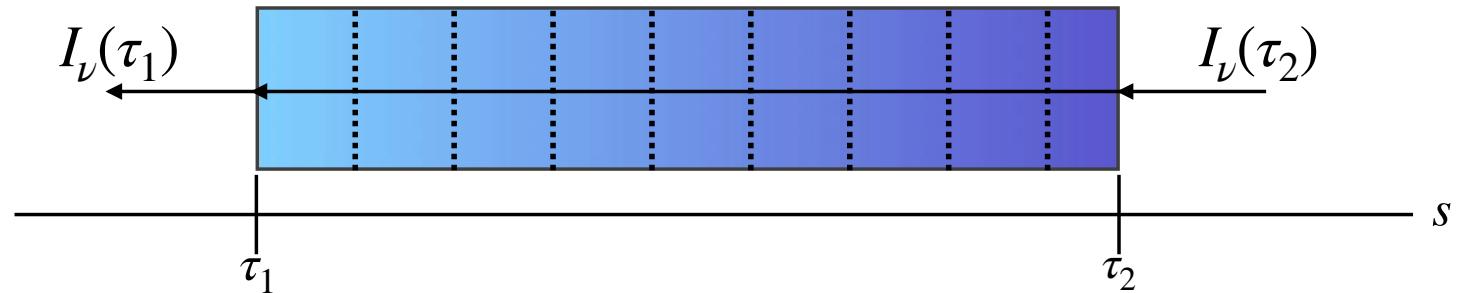
Radiative Transfer (of EM wave)

- Differential form

- $$\frac{dI_\nu}{d\tau} = -I_\nu + S_\nu$$

- A general solution in integral form (considering a one dimension plane parallel slab geometry)

- $$I_\nu(\tau_1) = I_\nu(\tau_2) e^{-(\tau_2-\tau_1)} + \int_{\tau_1}^{\tau_2} S_\nu(t) e^{-(t-\tau_1)} dt$$



Radiative Transfer (of EM wave)

- Differential Form

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- The above equation can be fully solved if S_ν is known. However, S_ν generally depends on the (local) radiation field. Namely, the emission and absorption properties of the material may depend on I_ν . Hence, given the physical condition, one needs to solve I_ν and S_ν numerically and iteratively.

Radiative Transfer (of EM wave)

- Differential Form

- $$\frac{dI_\nu}{d\tau} = -I_\nu + S_\nu$$

- A general solution in integral form (considering a one dimension plane parallel slab geometry)

- $$I_\nu(\tau_1) = I_\nu(\tau_2) e^{-(\tau_2-\tau_1)} + \int_{\tau_1}^{\tau_2} S_\nu(t) e^{-(t-\tau_1)} dt$$

- Think about special cases

- No foreground emission term, namely, first term only
- No background emission term, namely, second term only
- constant source function in optically thick case vs. optically thin case

**Mechanisms (of EM Waves) =
Radiative Processes**

Radiative Processes (of EM wave)

Continuum
(SED: spectral Energy Distribution)

Line
(Intensity, Frequency Center and
Width, or Profile)

Thermal
(characterized by a single
(kinetic) temperature)

Free-Free
Dust
Inverse Compton Scattering

Atomic (Recombination)
Molecular/Ionic Line

Non-Thermal
(characterized NOT by a single
(kinetic) temperature) but
energy distribution, e.g.)

Synchrotron

Maser

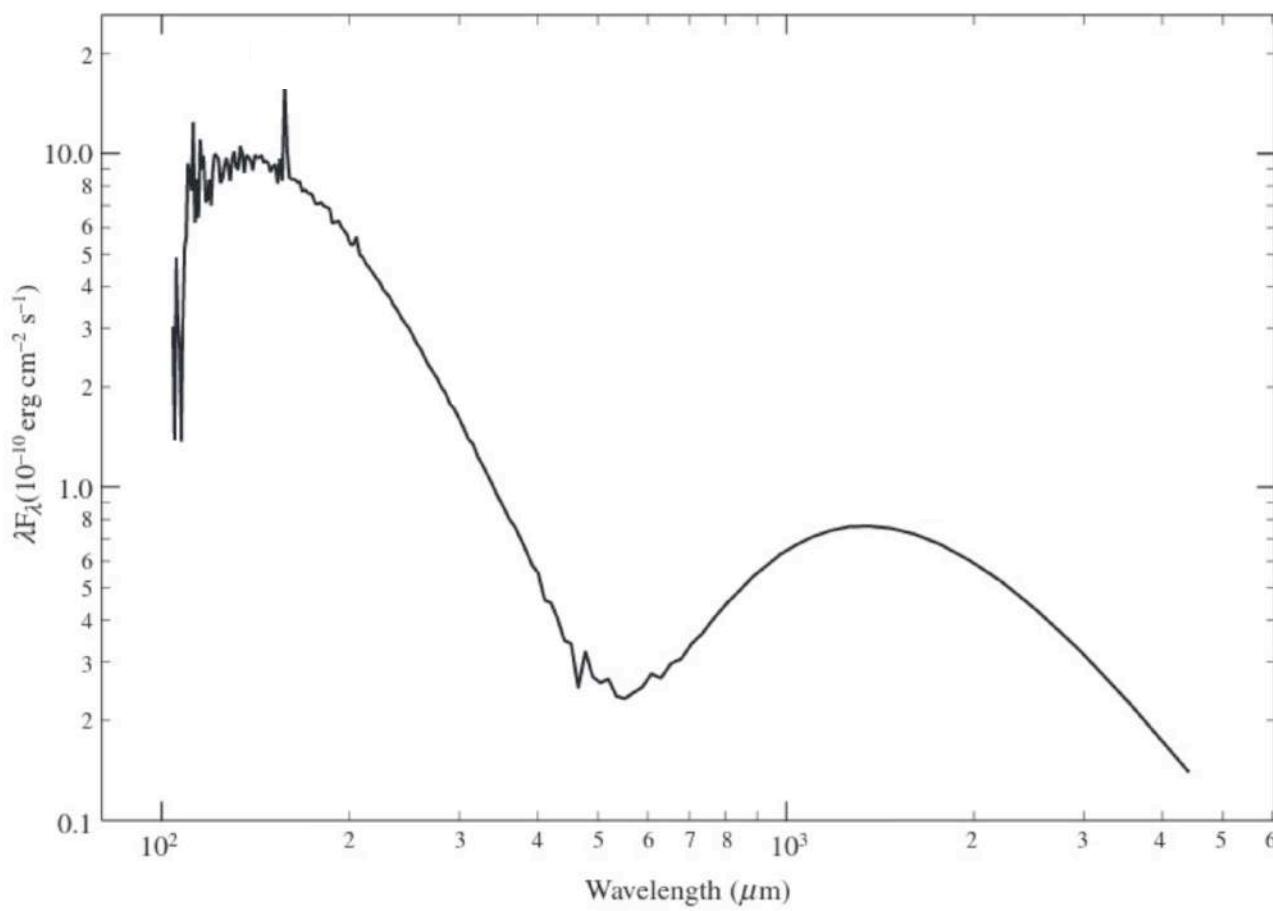


Figure 1.3

The integrated spectrum of the galactic plane as observed by the *Cosmic Background Explorer* (*COBE*) satellite.

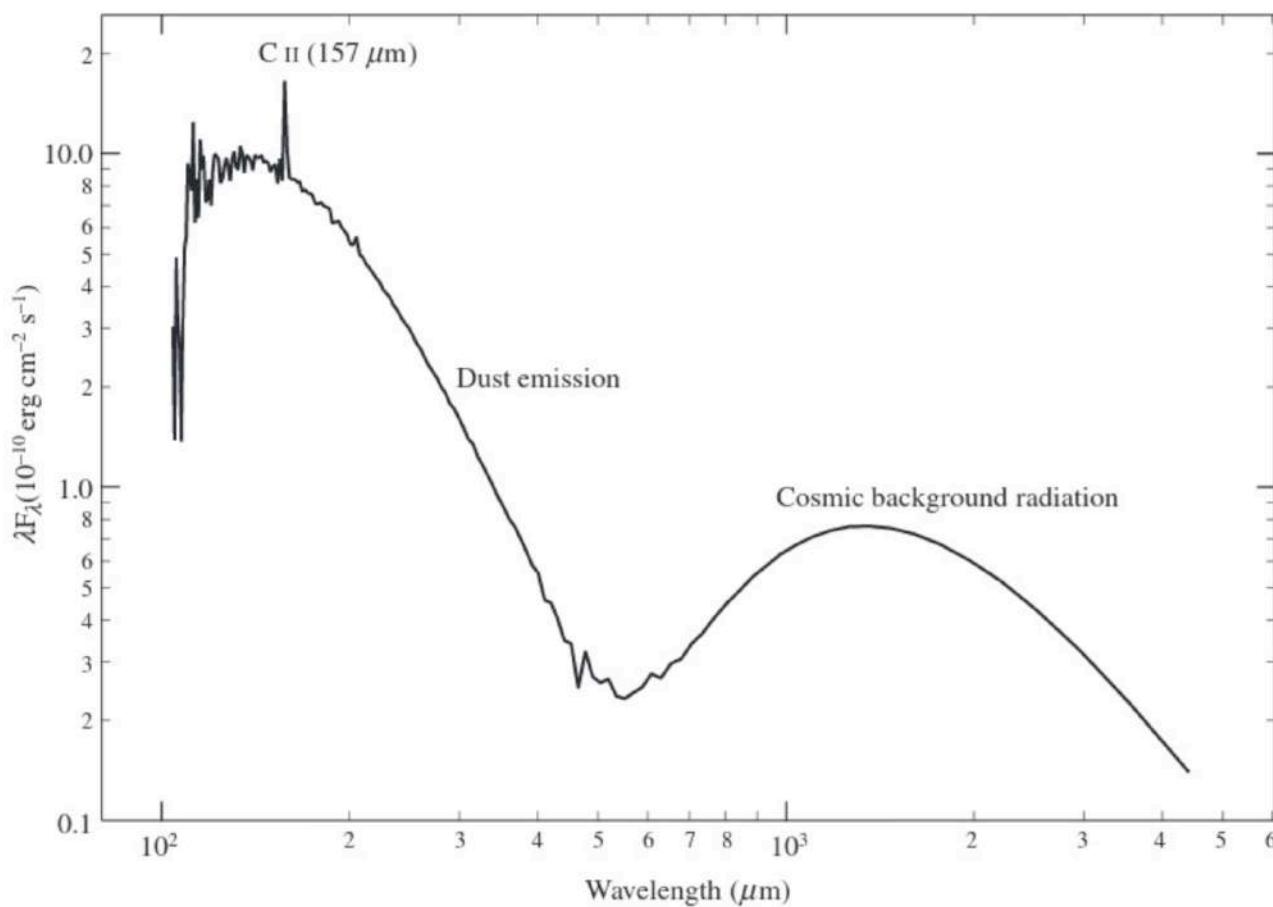
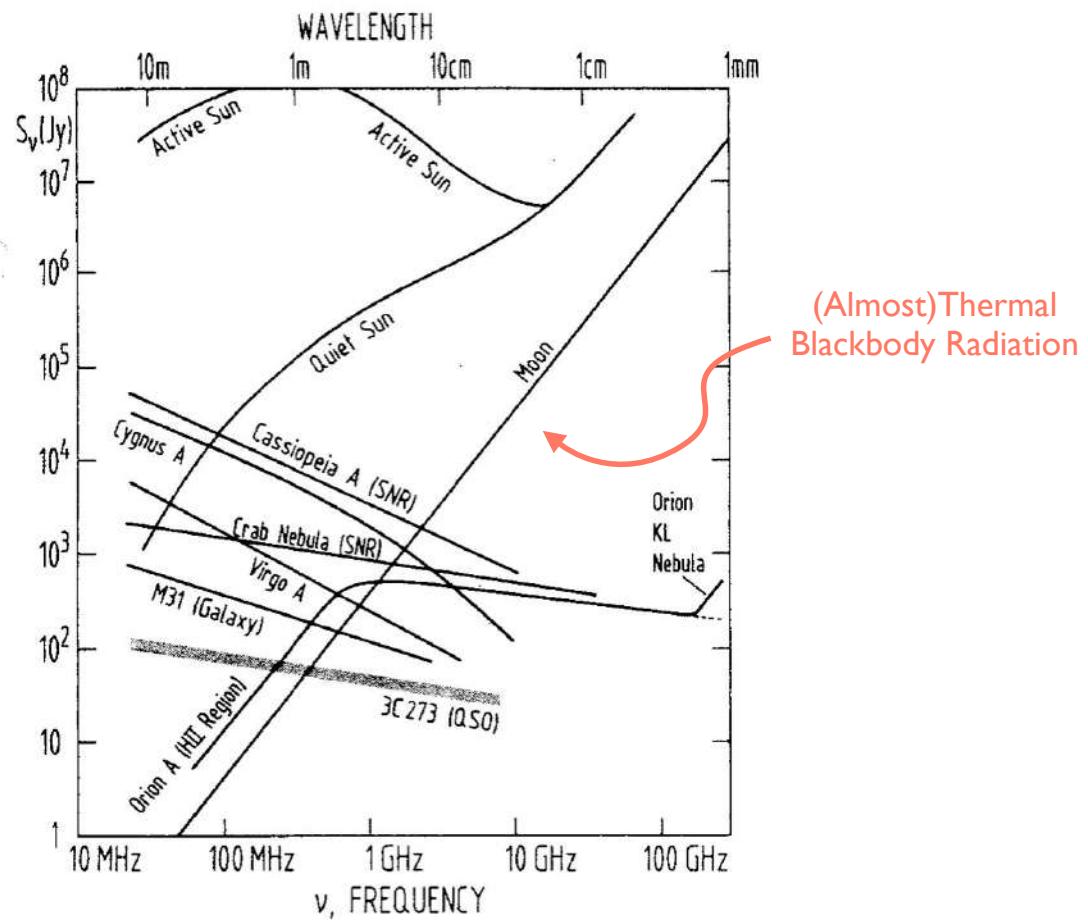


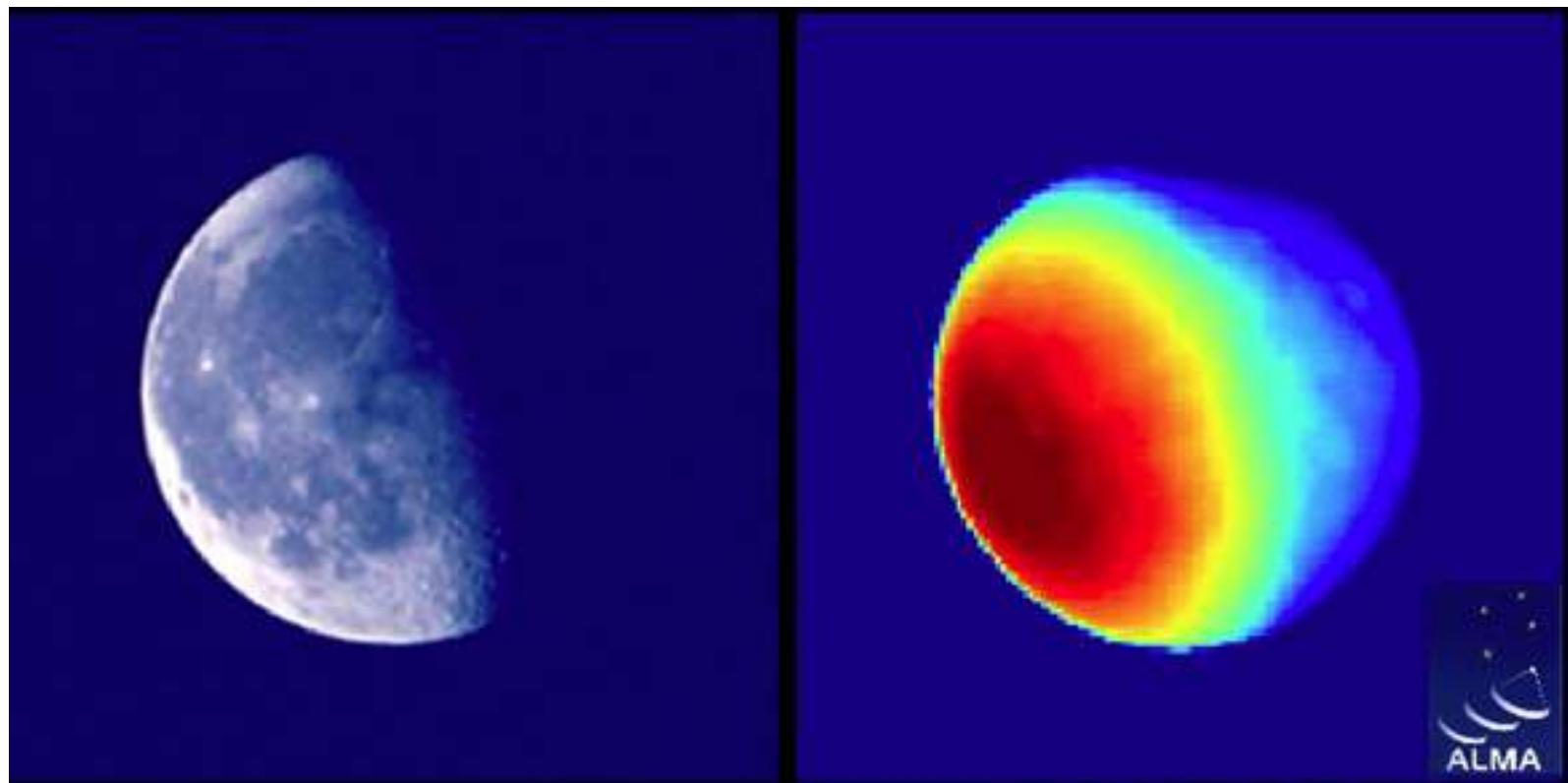
Figure 1.3

The integrated spectrum of the galactic plane as observed by the *Cosmic Background Explorer (COBE)* satellite. Other than the cosmic background radiation, which is seen over the whole sky, the ISM shows strong continuum emission from cool dust of a temperature of ~ 20 K. A line due to ionized carbon is also prevalent on the galactic plane.

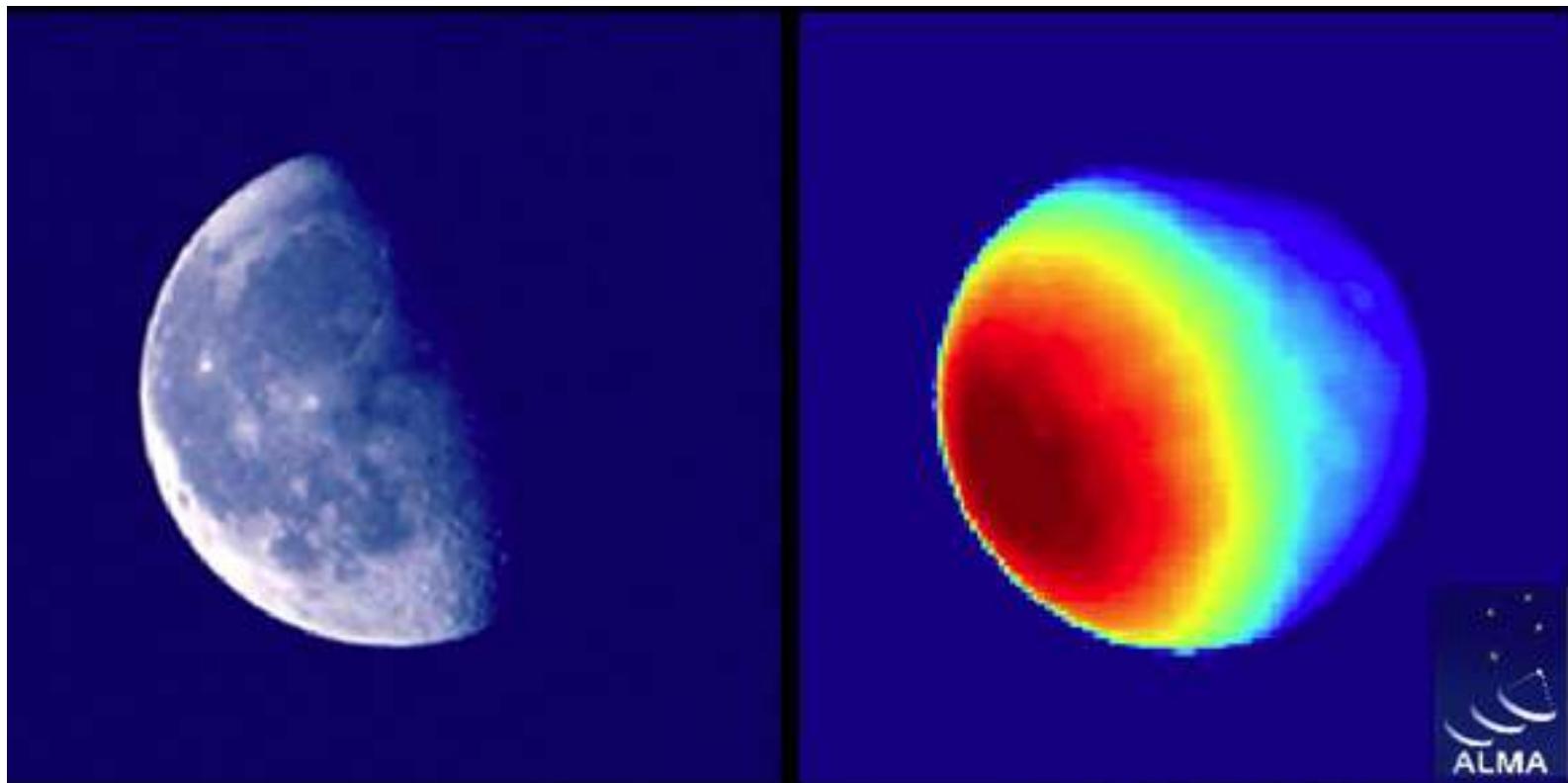
Spectral Energy Distribution (in radio regime)



The Power Spectrum (in radio regime)



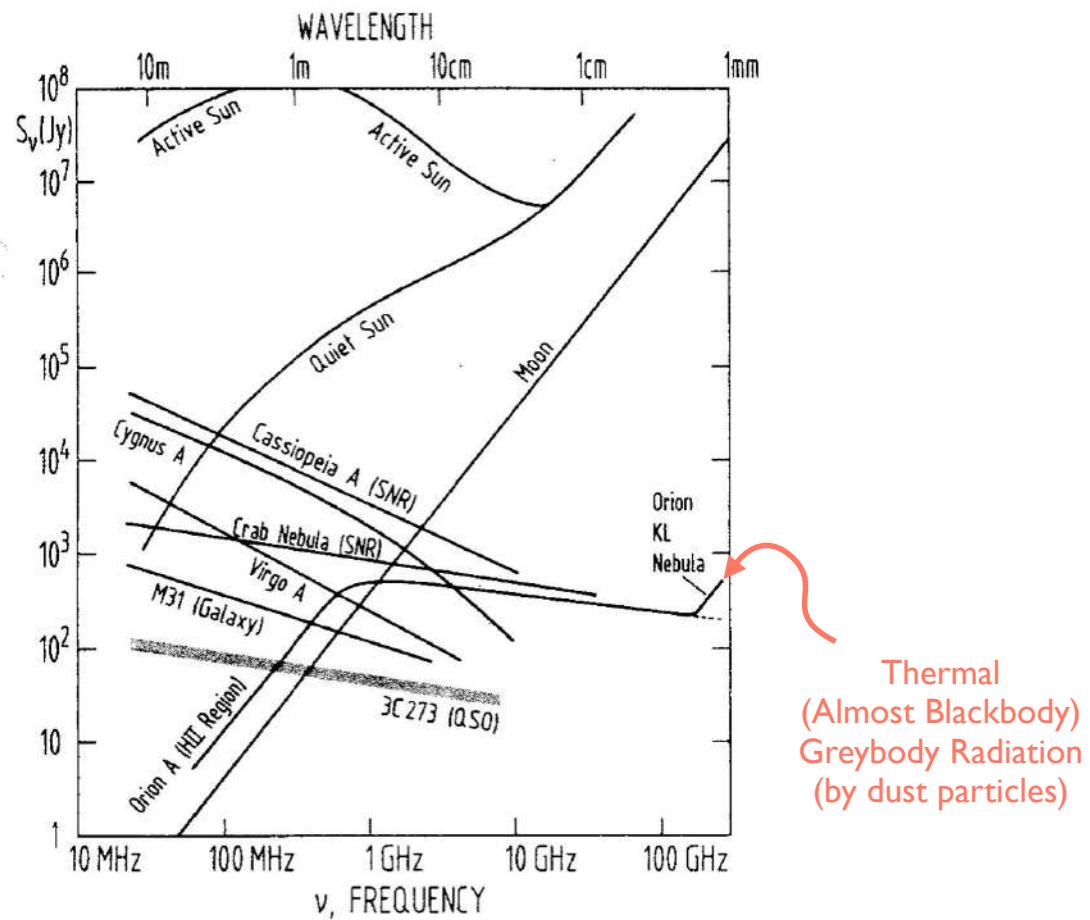
The Power Spectrum (in radio regime)



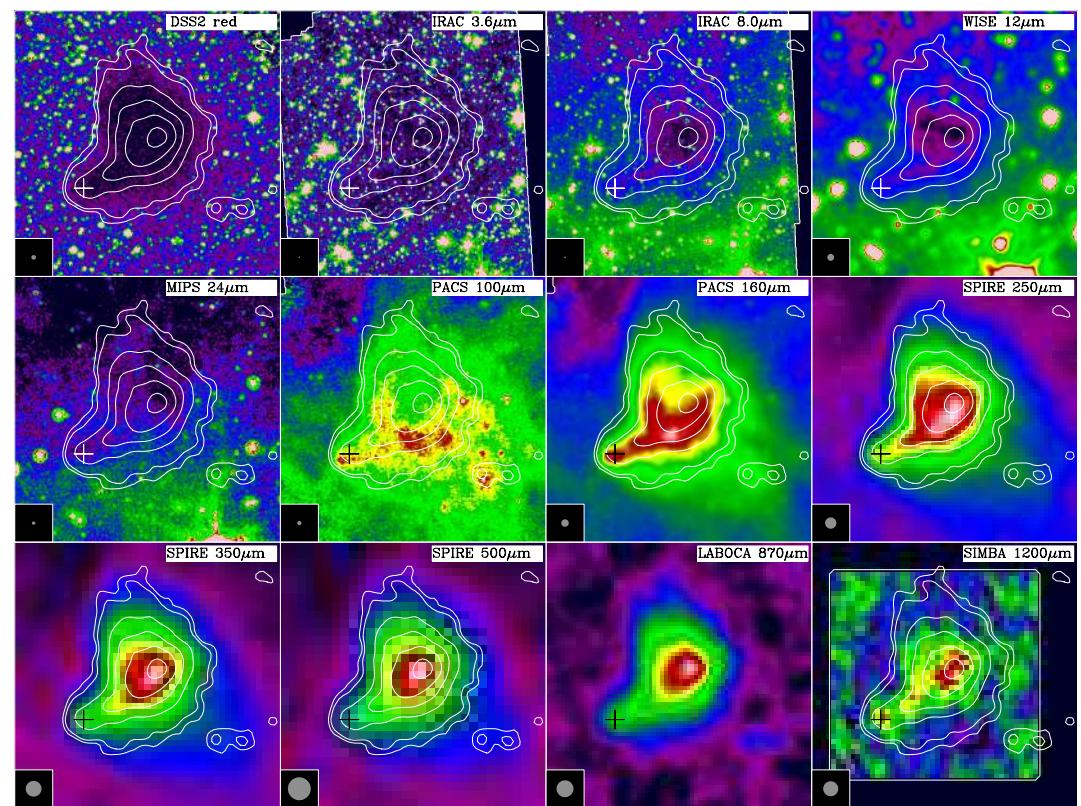
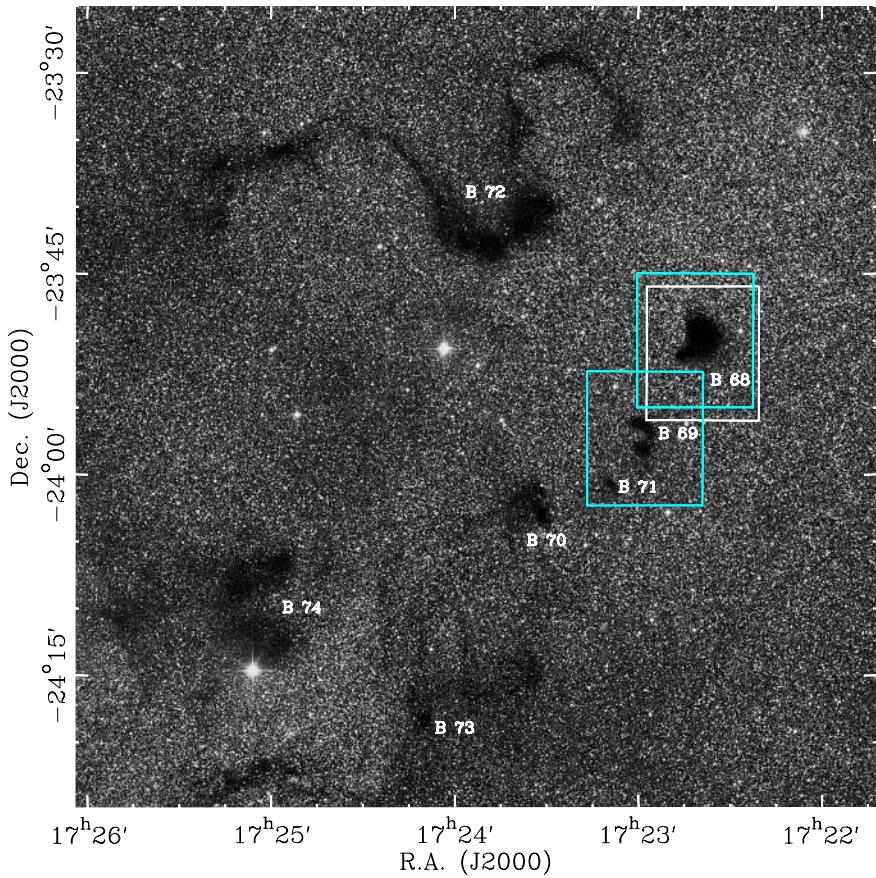
First radio image of the Moon taken with a 12-meter ALMA antenna. The optical image at left shows the sunlight reflected by the Moon's surface, whereas the radio image shows the physical temperature distribution of the Moon. The ALMA image clearly shows the temperature distribution of the Moon, including weak radio emission from the dark side of the Moon.

(Source: <https://public.nrao.edu/gallery/moon-as-seen-by-alma/>)

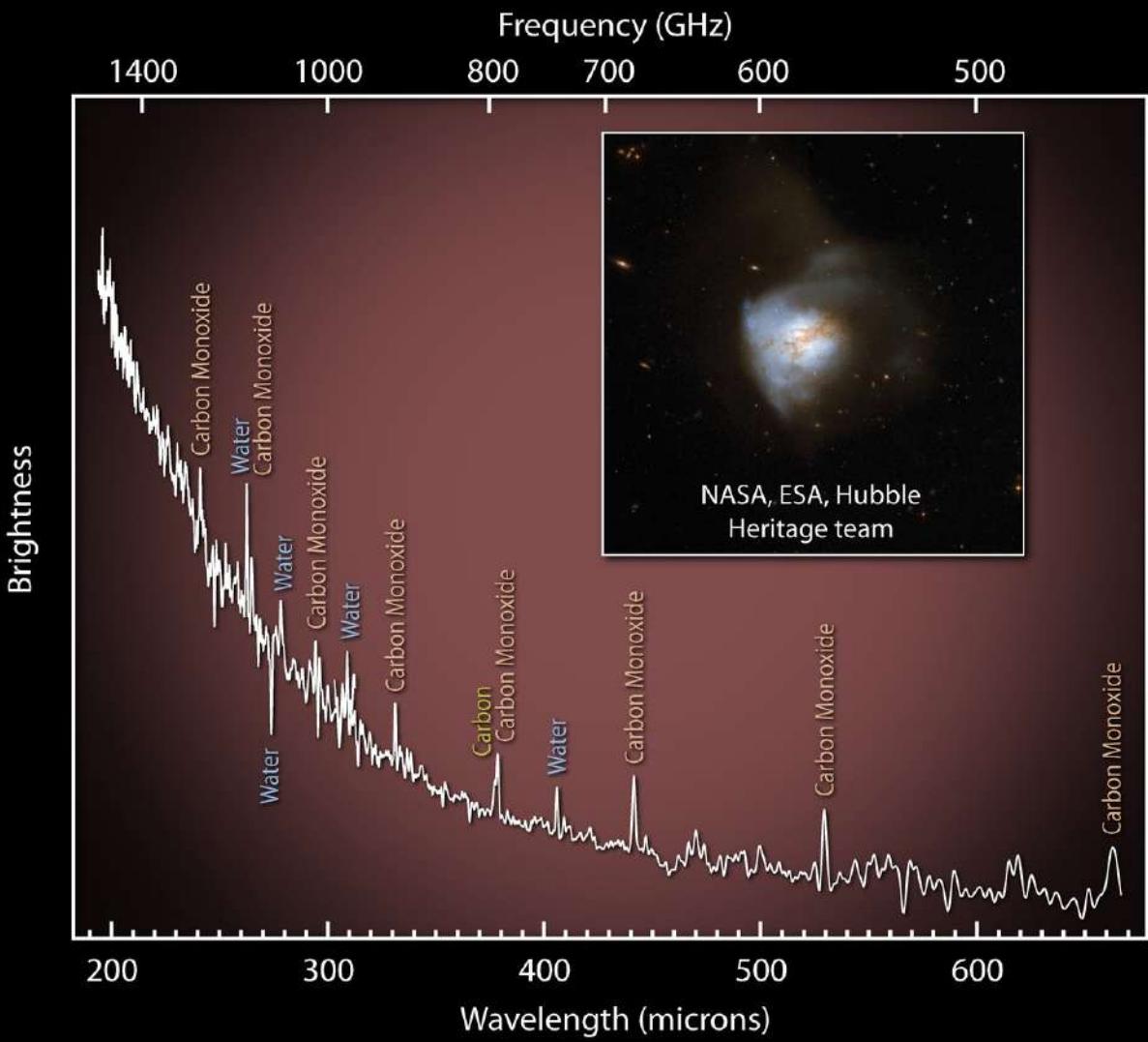
Spectral Energy Distribution (in radio regime)



Bok Globule B68



Nielbock et al. (2012) A&A



Arp 220

© ESA and the SPIRE consortium

Radiative Process (Thermal Dust)

- dust extinction (absorption + scattering) and (nearly BB) emission

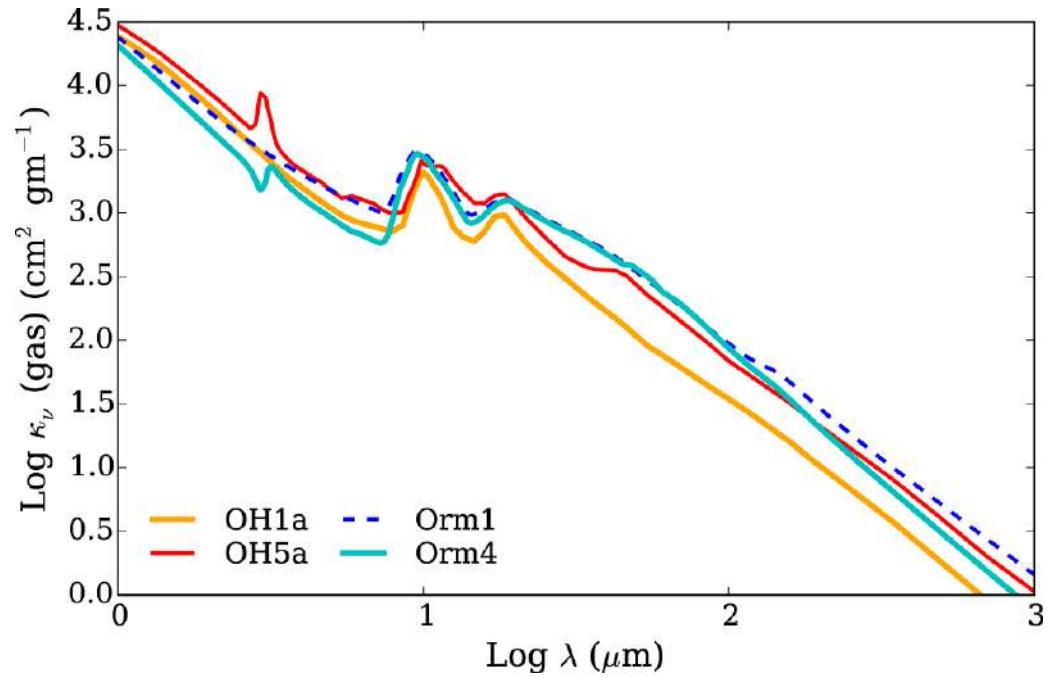
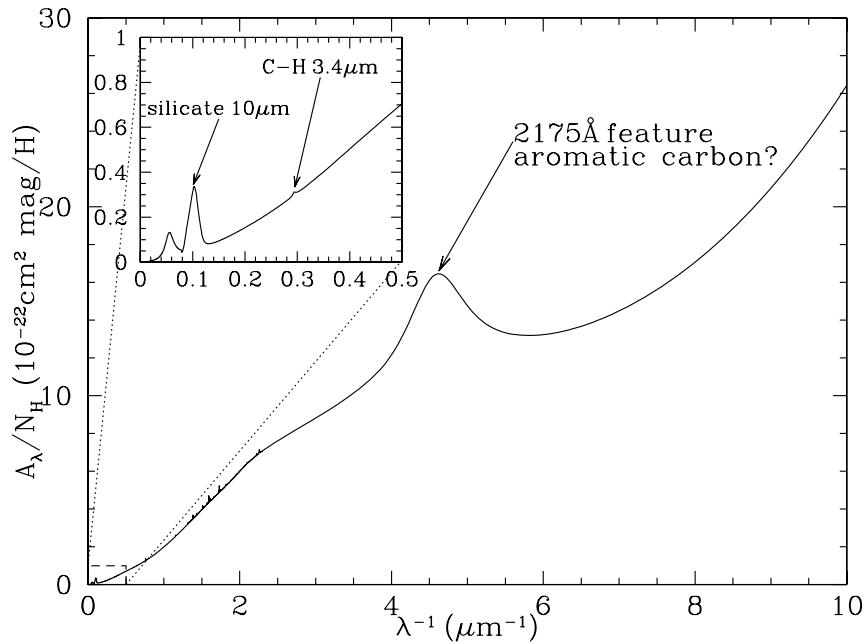


Figure 21.1 Extinction versus inverse wavelength λ^{-1} on a typical sightline in the local diffuse ISM. The inset shows the extinction at $\lambda > 2 \mu\text{m}$.

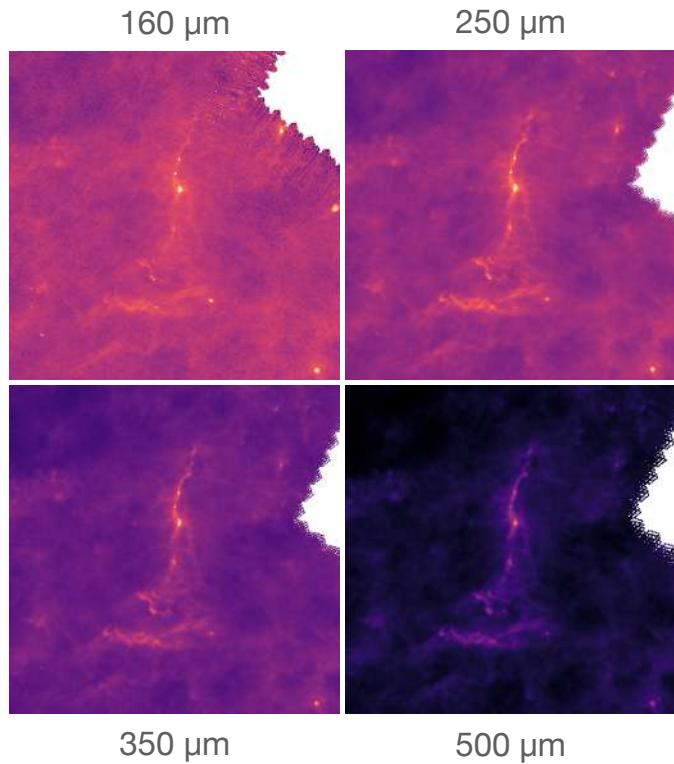
Webb et al. (2017)

Draine (Physics of Interstellar and Intergalactic Medium)

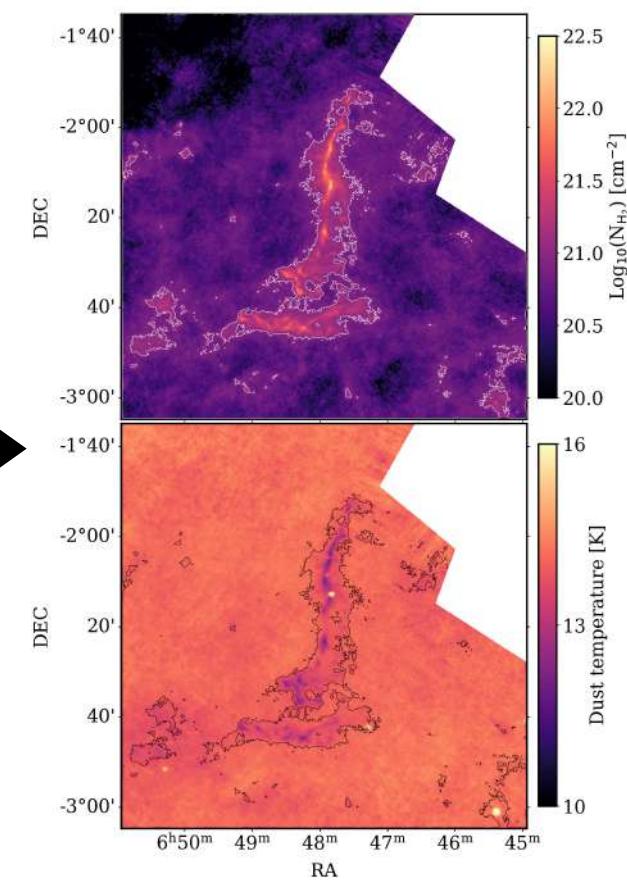
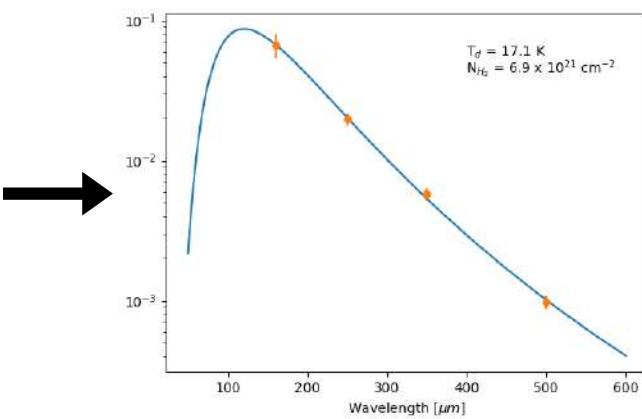
Slide Credit: Seamus Clarke

Column density estimation

Fitting spectral energy distributions



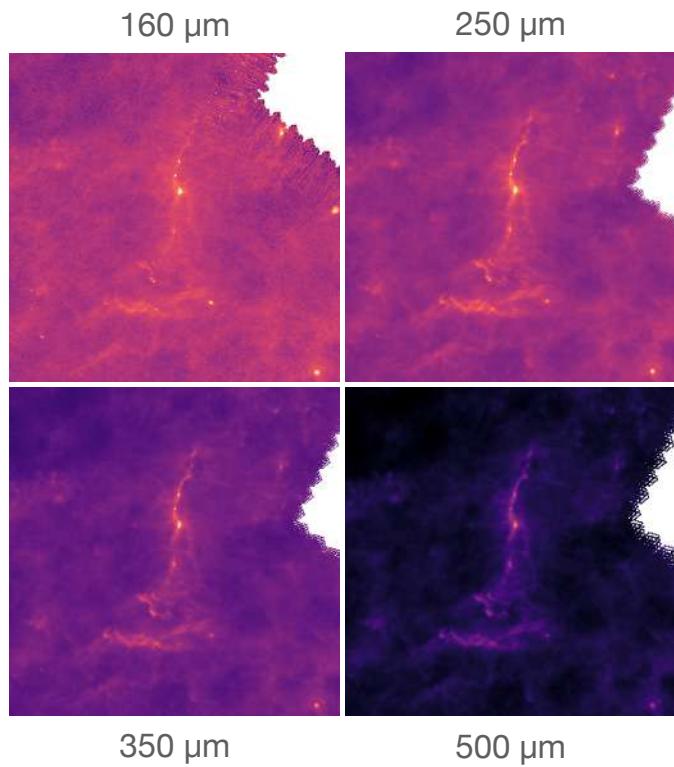
$$I_{\nu, \text{obs}} = \tau_{\nu} B_{\nu}(T_d) = \kappa_{\nu} N_d B_{\nu}(T_d)$$



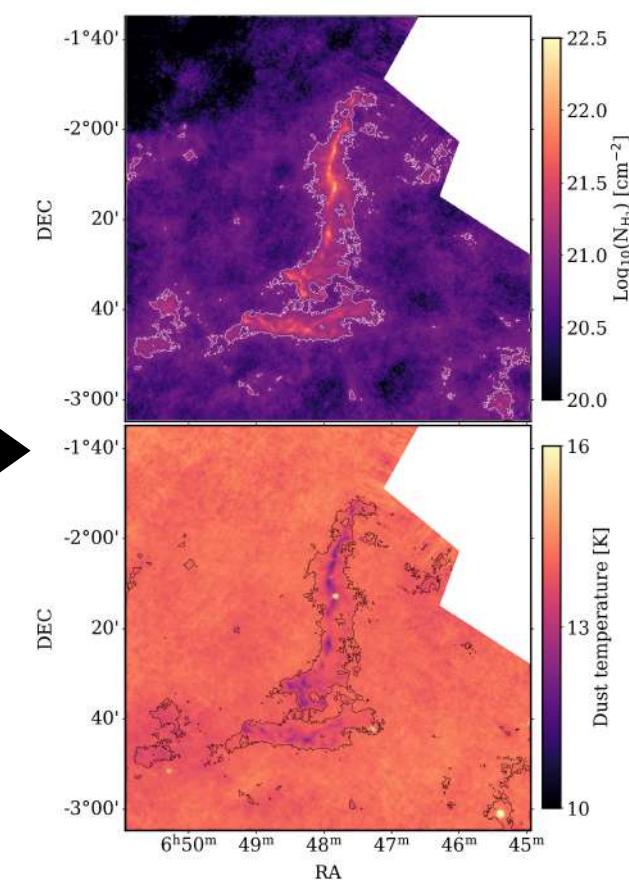
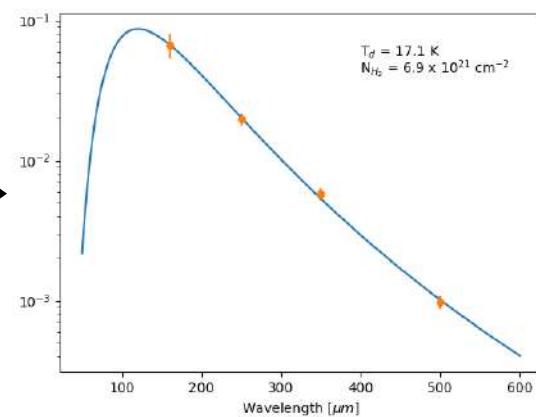
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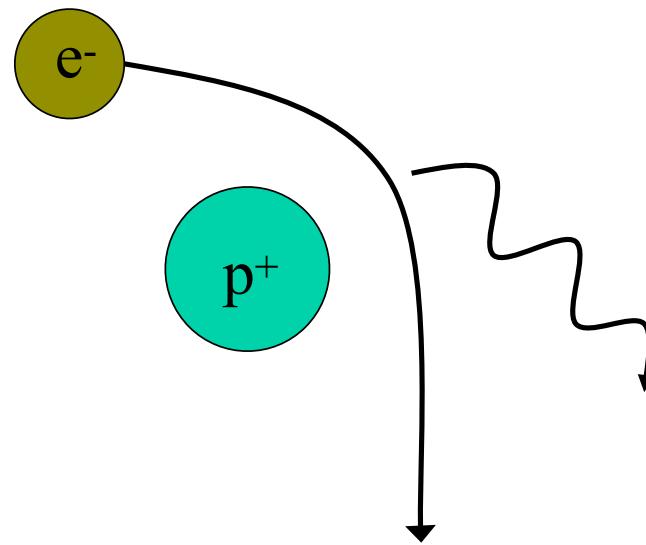


$$I_{\nu, \text{obs}} = \tau_{\nu} B_{\nu}(T_d) = \kappa_{\nu} N_d B_{\nu}(T_d)$$



Radiative Process (Free-Free)

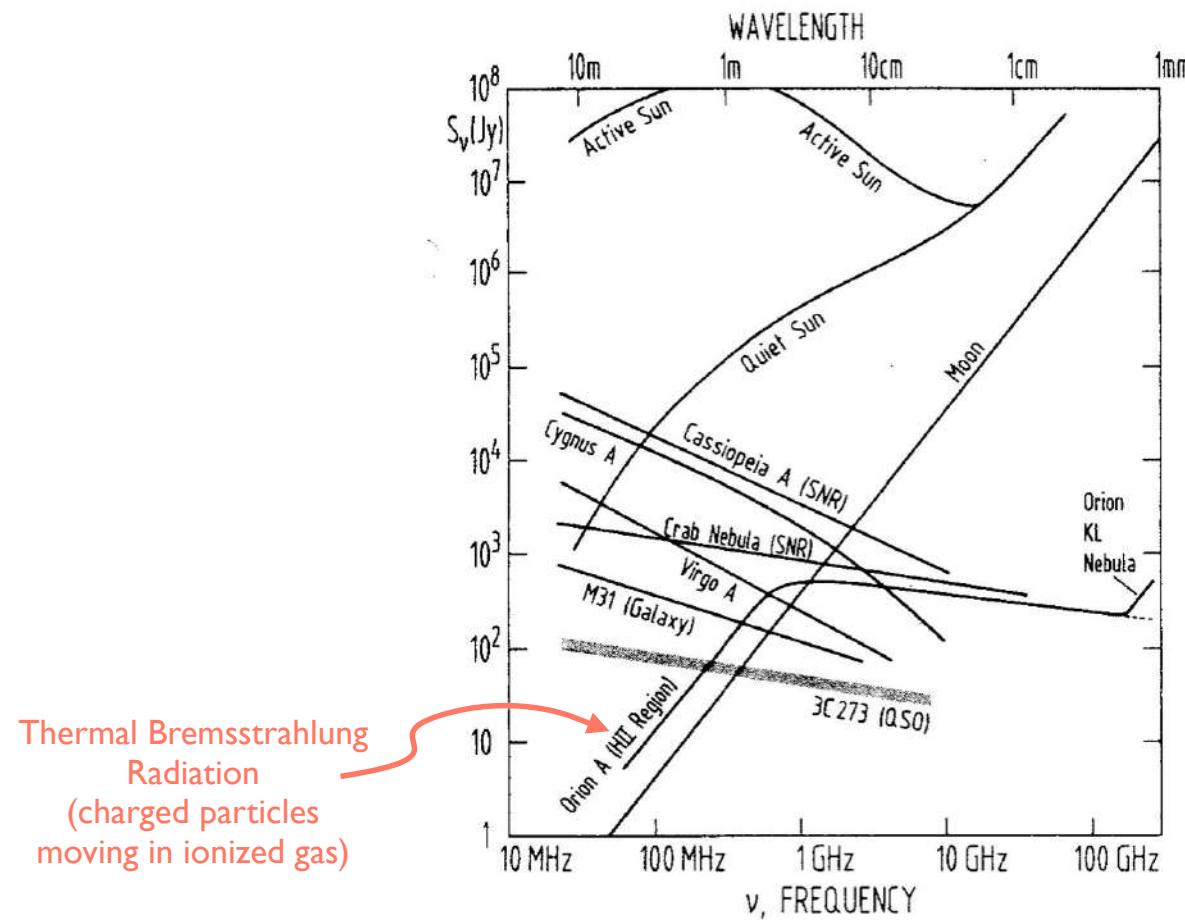
Bremsstrahlung (free-free)



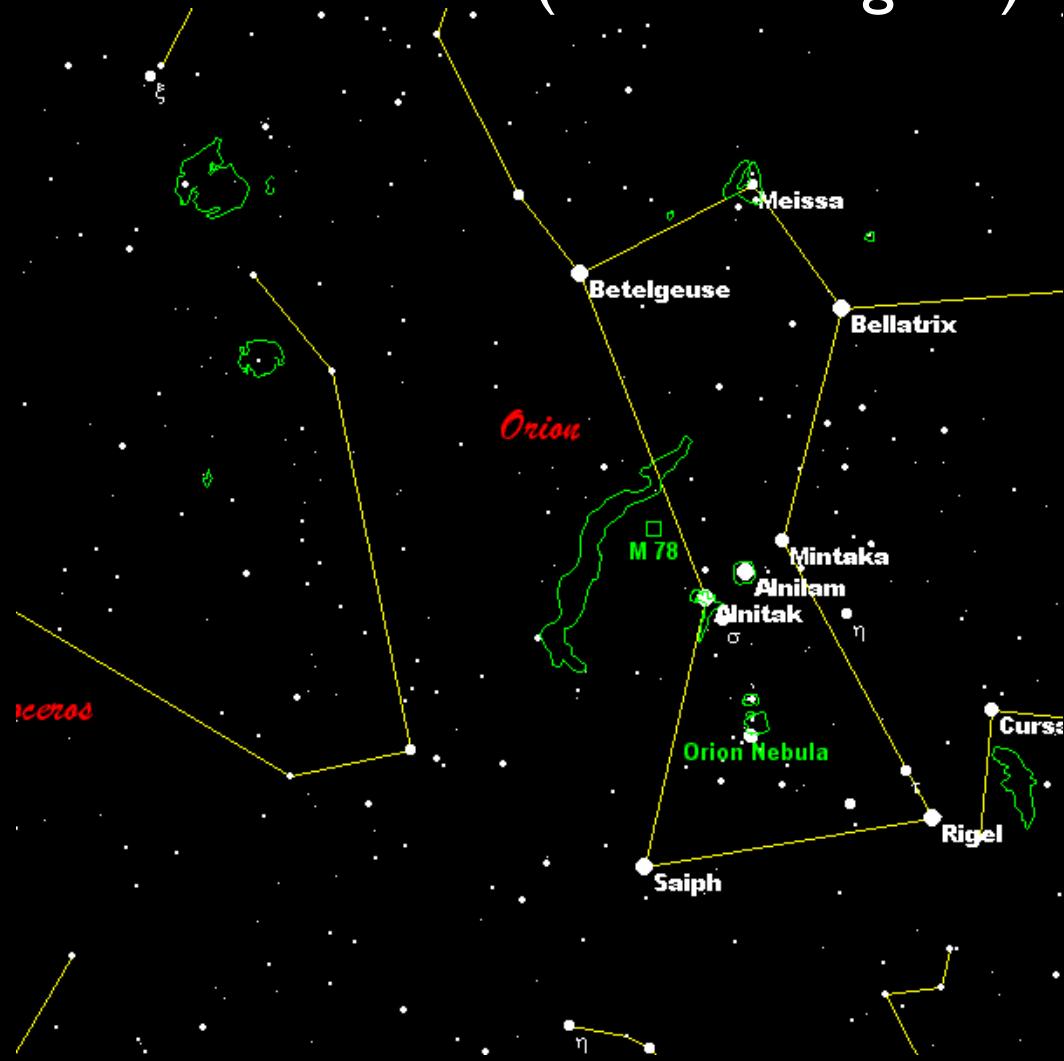
Ionized gas:
e.g., H II region
Intracluster medium

Image Credit: Hiro Hirashita

Spectral Energy Distribution (in radio regime)



Orion Nebula (in radio regime)



Orion Nebula (in radio regime)

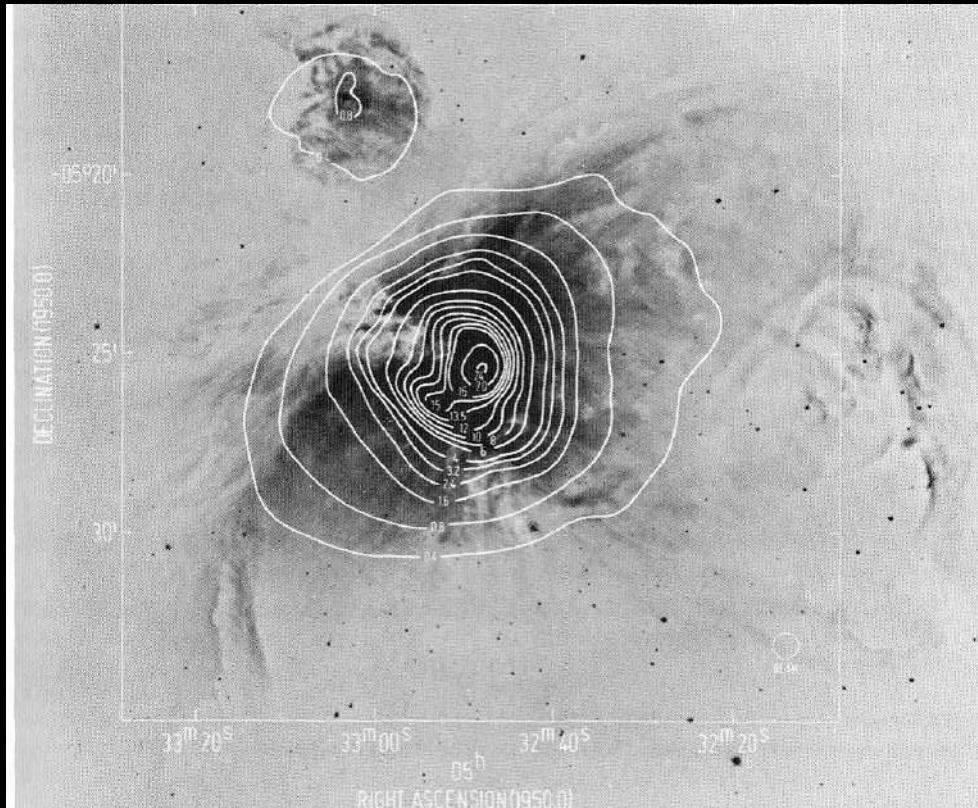


Fig. 2.3. The 23-GHz radio continuum contours, in units of main-beam brightness temperature, on an optical photo in H α and [NII] of NGC 1976 (Orion A, M42), below, and NGC 1982 (M43), above. The angular resolution is 42'', which at the distance of Orion A, corresponds to a linear resolution of 0.10 pc (Westerhout & de Bruyn 1984).

Spectral Energy Distribution (in radio regime)

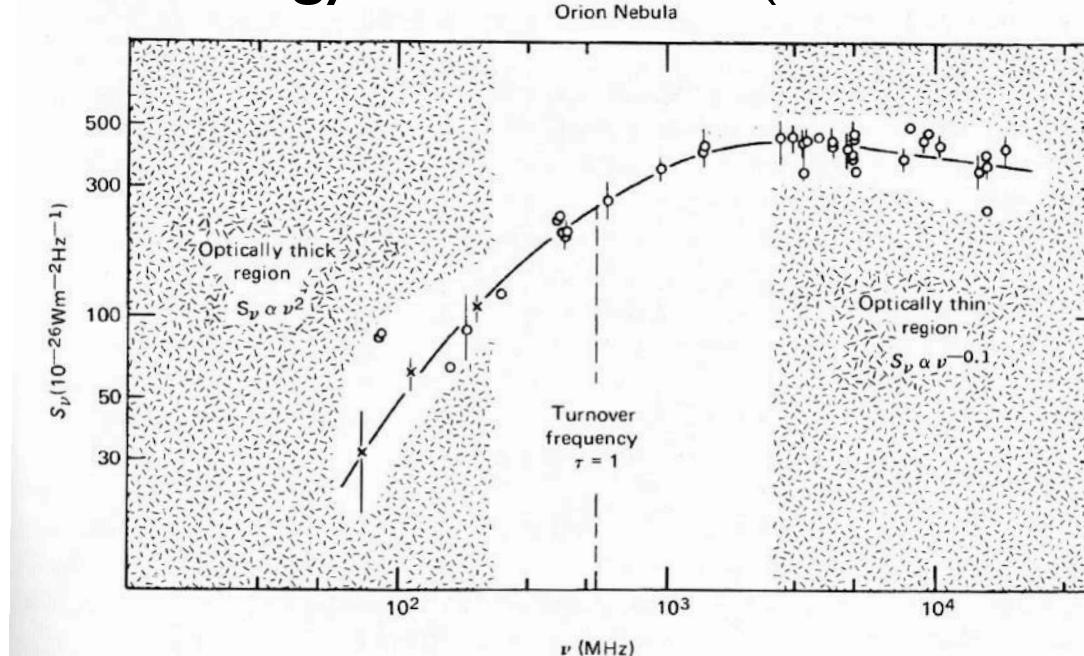


Fig. 2.2. Spectral flux density of the Orion Nebula plotted against frequency. The shaded regions mark the optically thick and thin regions of the spectrum. (Reprinted with permission by Gordon and Breach Science Publishers from: Terzian, Y. and Parrish A., *Astrophysical Letters*. Vol. 5(1970), pp. 261.

$$\tau_\nu \approx 8.235 \times 10^{-2} \left(\frac{T_e}{K}\right)^{-1.35} \left(\frac{\nu}{\text{GHz}}\right)^{-2.1} \left(\frac{EM}{pc \text{ cm}^{-6}}\right) \alpha(\nu, T) \quad \frac{EM}{pc \text{ cm}^{-6}} = \int \left(\frac{n_e}{cm^{-3}}\right)^2 d\left(\frac{s}{pc}\right)$$

$$\alpha(\nu, T) \sim 1$$

Spectral Energy Distribution (in radio regime)

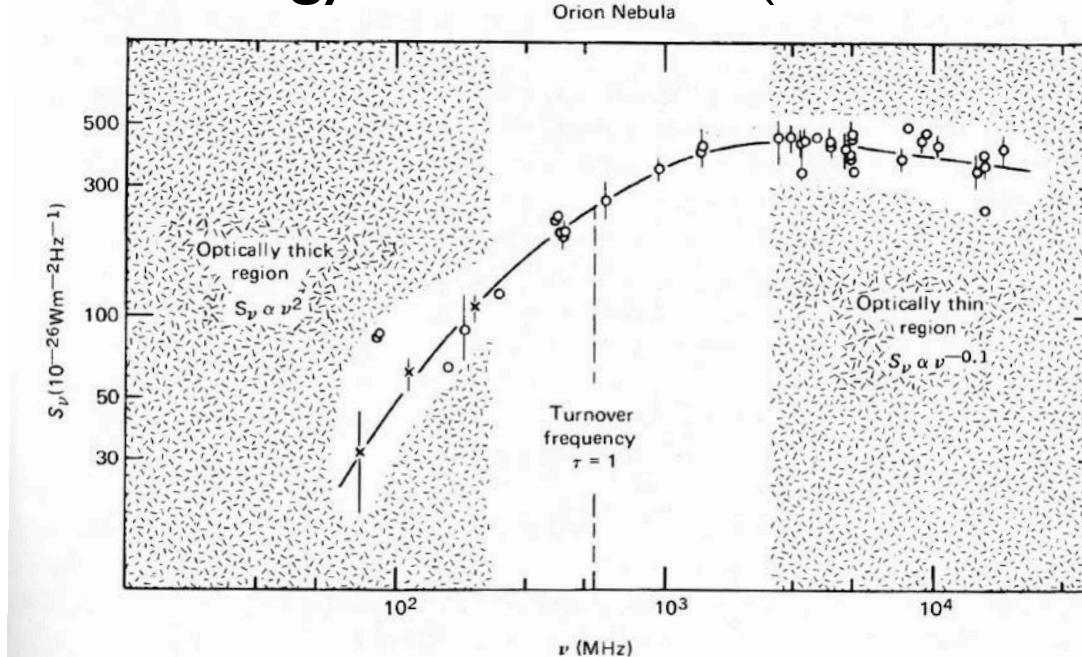


Fig. 2.2. Spectral flux density of the Orion Nebula plotted against frequency. The shaded regions mark the optically thick and thin regions of the spectrum. (Reprinted with permission by Gordon and Breach Science Publishers from: Terzian, Y. and Parrish A., *Astrophysical Letters*. Vol. 5(1970), pp. 261.

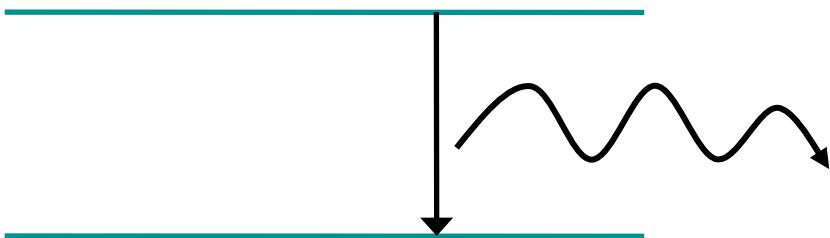
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Emission Measure

$$\frac{EM}{pc \text{ cm}^{-6}} = \int \left(\frac{n_e}{cm^{-3}}\right)^2 d\left(\frac{s}{pc}\right)$$

Radiative Process (Spectral Line)

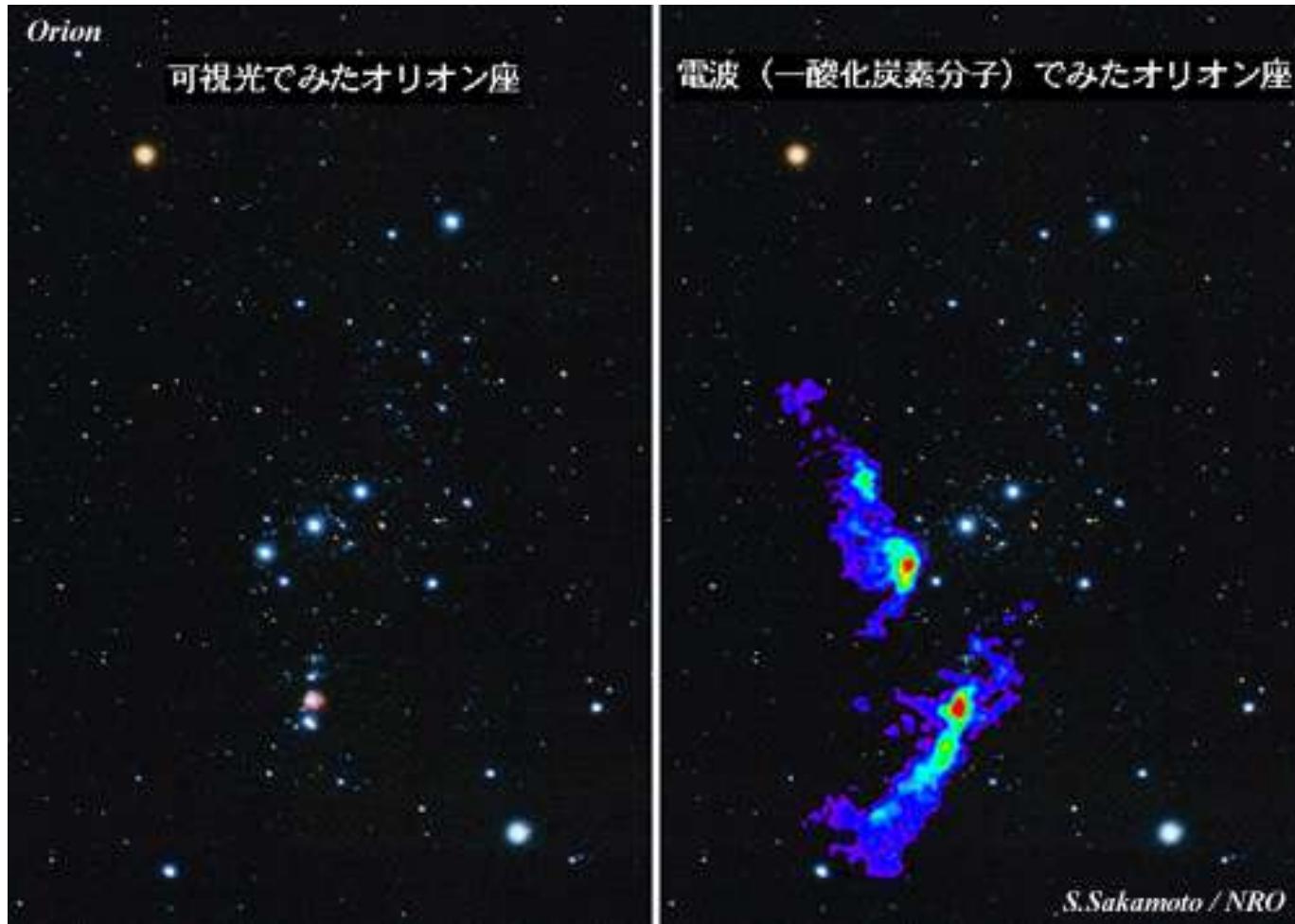
Radiative transition



electric ~ UV, optical
molecular vibrational
~ near-IR
molecular rotational
~ radio

Image Credit: Hiro Hirashita

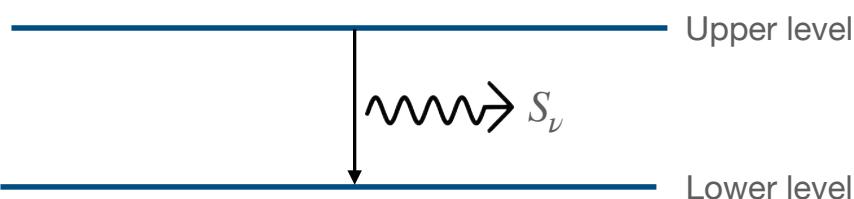
Radiative Process (Spectral Line)



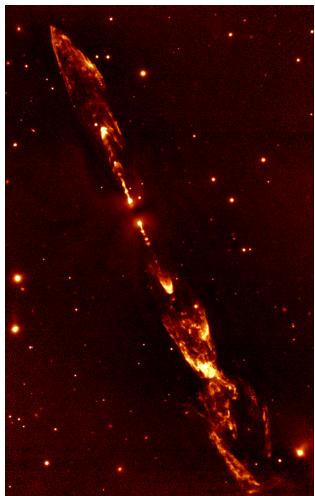
Slide Credit: Seamus Clarke

Radiation sources

Line emission



HH212



Molecular vibrational transitions:

Near-IR wavelengths

e.g. H₂ 2.12 μm

Simeis 147

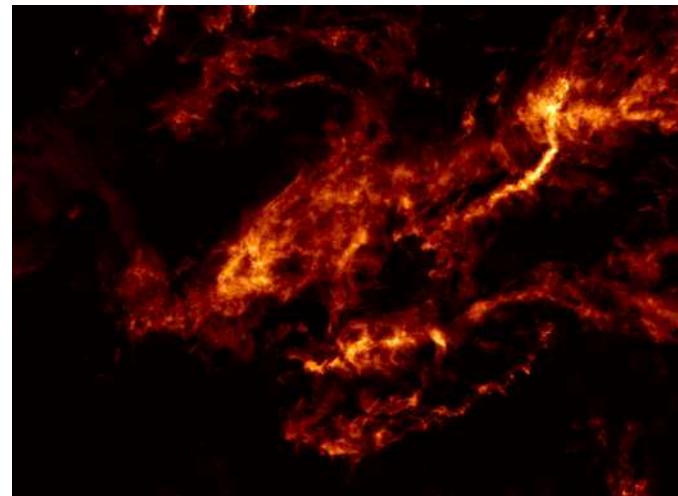


Electronic transitions:

Optical - UV wavelengths

e.g. Hα, [OIII], [SII]

Taurus molecular cloud



Molecular rotational transitions:

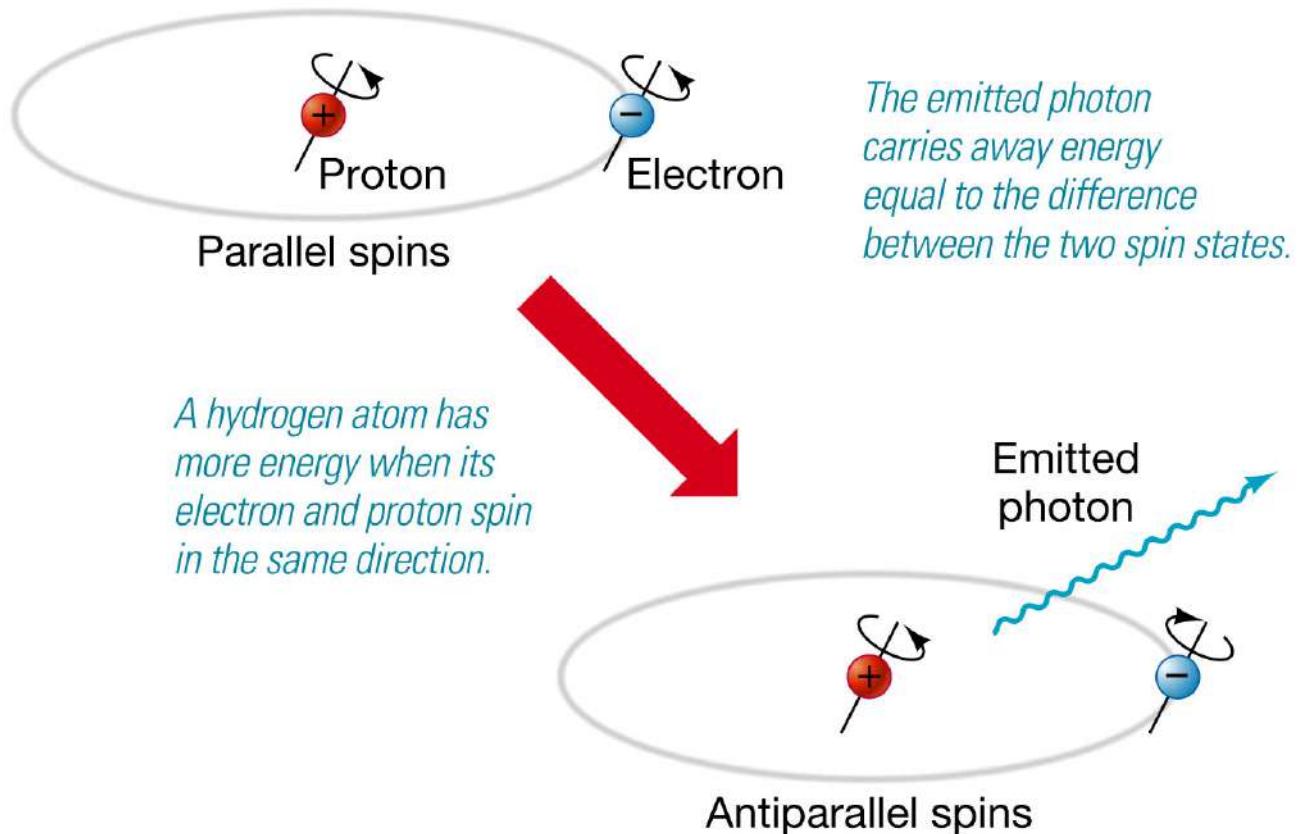
Sub-mm - radio wavelengths

e.g. CO, NH₃, CH₃OH

Image credit: Nicolas Kizilian, Mark McCaughrean, Paul Goldsmith

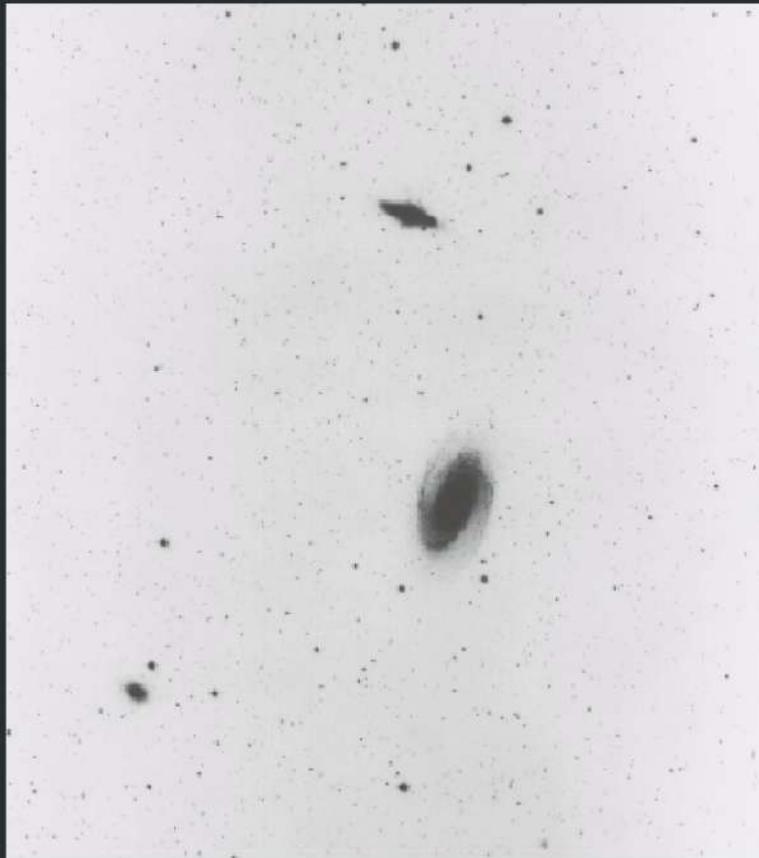
18.4 21-Centimeter Radiation

Interstellar gas emits low-energy radiation, due to a transition in the hydrogen atom

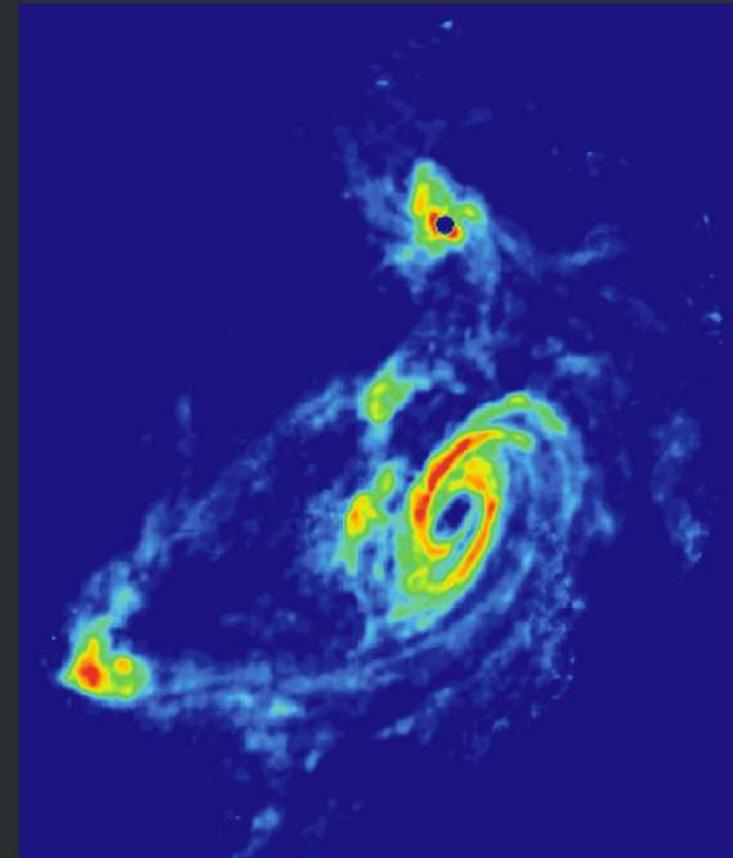


TIDAL INTERACTIONS IN M81 GROUP

Stellar Light Distribution



21 cm HI Distribution



(NRAO Archive)

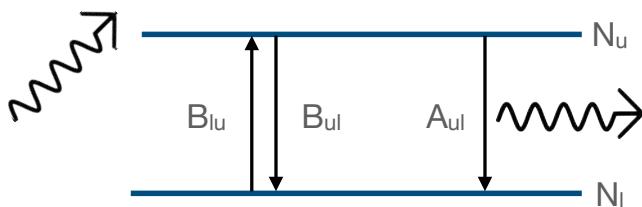
Line Radiation

- Spectral Line Radiation
 - Peak Frequency : velocity
 - Peak Intensity : opacity (hence column density, if with known excitation)
 - Line-width : thermal and non-thermal (turbulence and pressure) broadening
 - Polarization : splitting (due to magnetic fields)

Slide Credit: Seamus Clarke

Population levels and excitation

Level occupation



We wish to determine what would the absorption and emission terms be for this system.

First we need some additional definitions.

Fractional occupation number

$$n_i = \frac{N_i}{N}$$

$$\sum_i^{all} n_i = 1$$

Thermally distributed states (LTE)

$$\frac{n_i}{n_j} = \frac{N_i}{N_j} = \frac{g_i}{g_j} e^{-(E_i - E_j)/k_B T}$$

The partition function

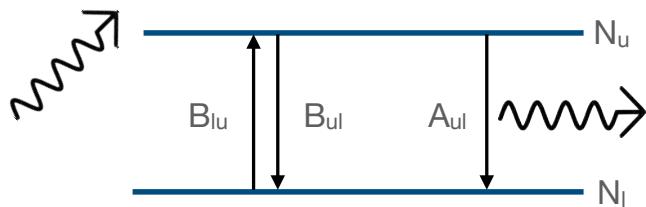
$$Z(T) = \sum_i^{all} g_i e^{-E_i/k_B T}$$

$$n_i = \frac{1}{Z(T)} g_i e^{-E_i/k_B T}$$

Slide Credit: Seamus Clarke

Population levels and excitation

Emissivity and absorption



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

So how do we go about determining these two quantities?

Energy per photon

$$j_\nu = \frac{h\nu_{ul}}{4\pi} A_{ul} N_u \phi(\nu)$$

Per steradian

Rate of photon release per unit volume

Line profile

Rate of photon absorption per unit volume per unit intensity

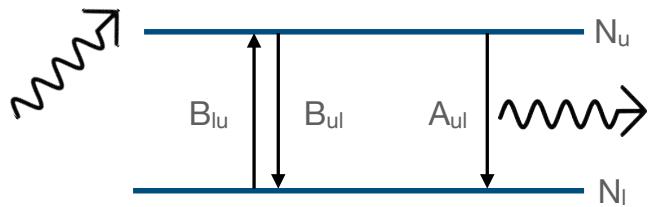
$$\alpha_\nu = \frac{h\nu_{ul}}{4\pi} (N_l B_{lu} - N_u B_{ul}) \phi(\nu)$$

$N_l B_{lu} I$ = Rate of stimulated absorption

Slide Credit: Seamus Clarke

Population levels and excitation

Einstein's relations



Einstein's relations

$$A_{ul} = \frac{2h\nu_{ul}^3}{c^2} B_{ul}$$
$$g_u B_{ul} = g_l B_{lu}$$

Assume that LTE is valid

$$\frac{N_u}{N_l} = \frac{g_u}{g_l} e^{-(E_u - E_l)/k_B T} = \frac{g_u}{g_l} e^{-h\nu_{ul}/k_B T_{ex}}$$

$$j_\nu = \frac{h\nu_{ul}}{4\pi} A_{ul} N_u \phi(\nu)$$



$$\alpha_\nu = \frac{h\nu_{ul}}{4\pi} (N_l B_{lu} - N_u B_{ul}) \phi(\nu)$$

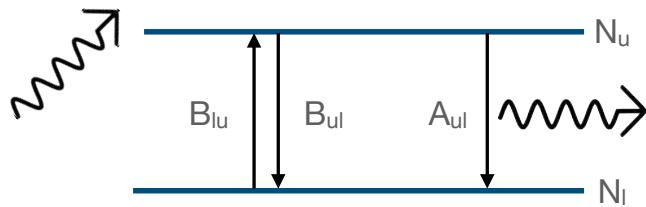
$$\alpha_\nu = \frac{c^2}{8\pi\nu_{ul}^2} (e^{h\nu_{ul}/k_B T_{ex}} - 1) A_{ul} N_u \phi(\nu)$$

$$S_\nu = \frac{2h\nu_{ul}^3}{c^2} \frac{1}{(e^{h\nu_{ul}/k_B T_{ex}} - 1)} = B_\nu(T_{ex})$$

Slide Credit: Seamus Clarke

Population levels and excitation

Einstein's relations



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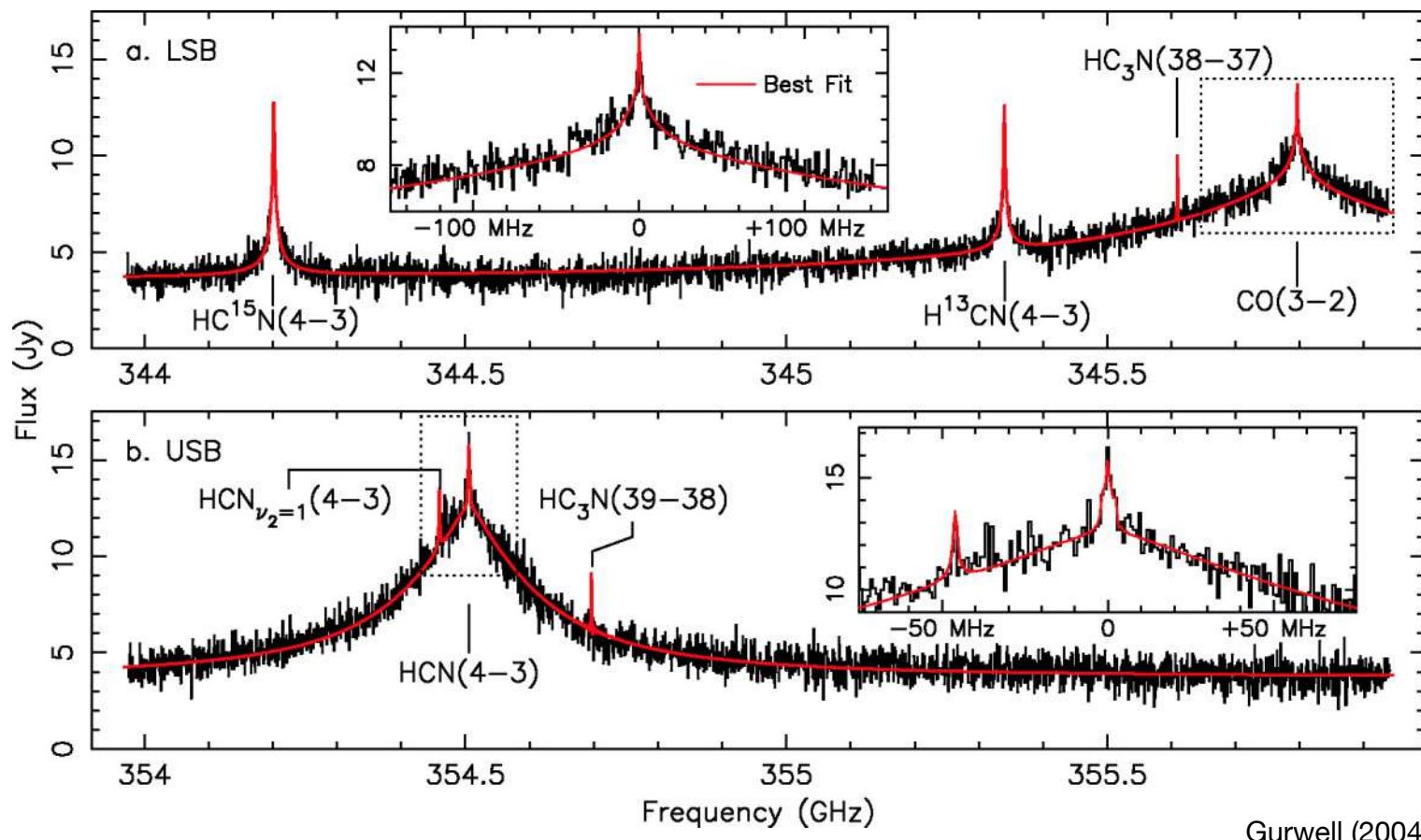
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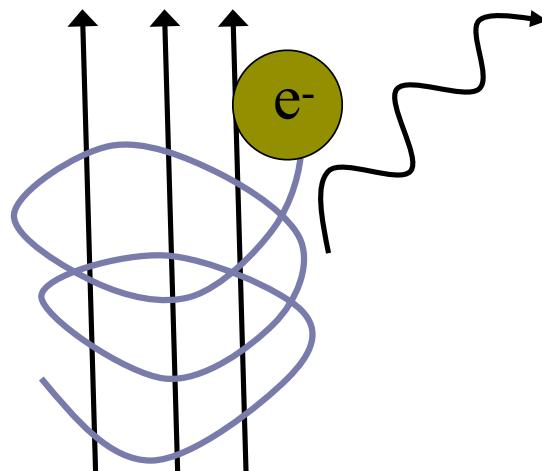
Line Radiation [broadening]

Submillimeter Observations of Titan: Global Measures of Stratospheric Temperature, CO, HCN, HC₃N, and the Isotopic Ratios ¹²C/¹³C and ¹⁴N/¹⁵N



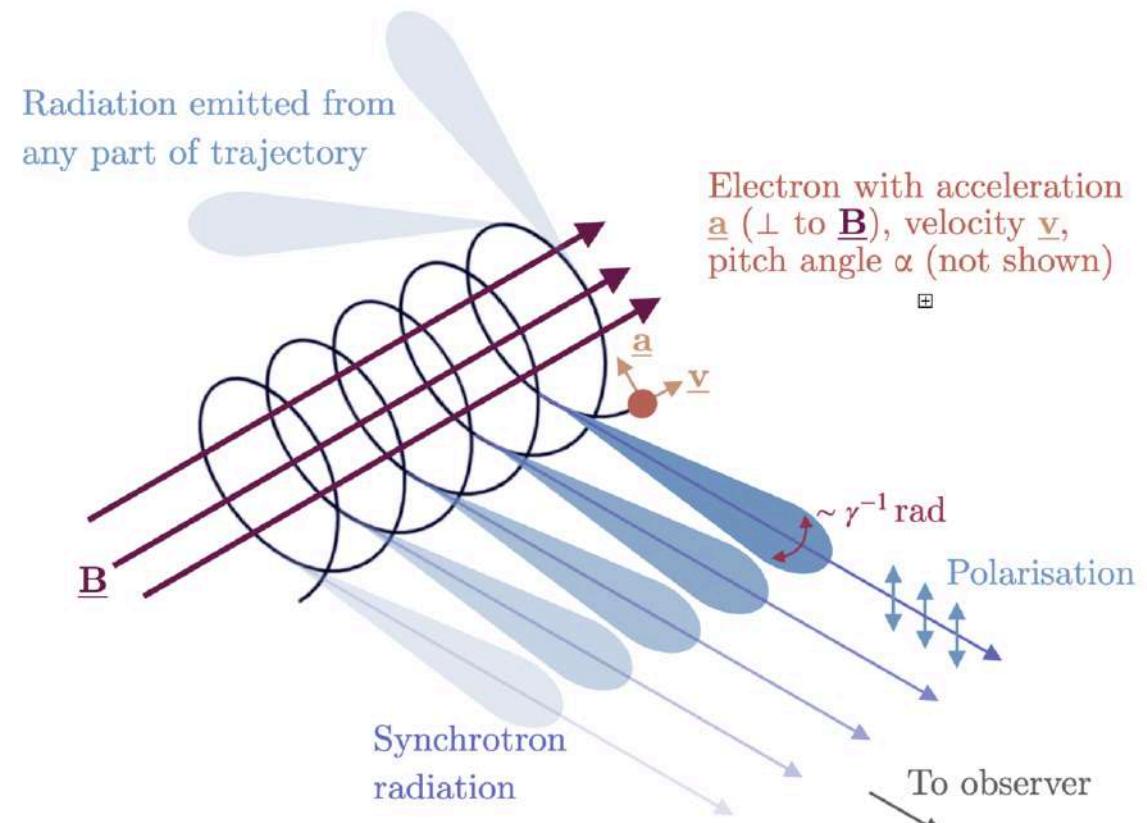
Radiative Process (Synchrotron)

Synchrotron (magnetic
bremsstrahlung)



B Supernovae
AGN jet

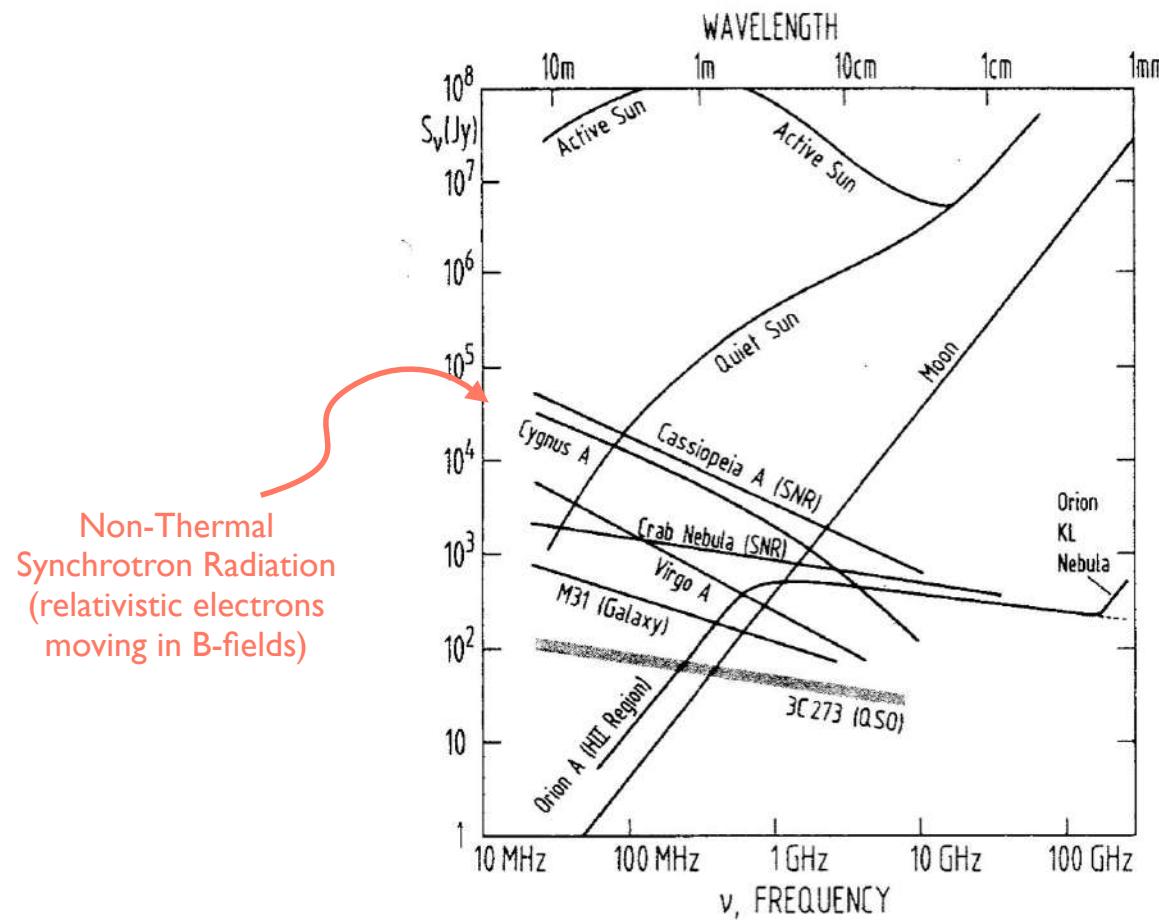
Image Credit: Hiro Hirashita



Emma Alexander

https://en.wikipedia.org/wiki/Synchrotron_radiation

Spectral Energy Distribution (in radio regime)



Spectral Energy Distribution (in radio regime)

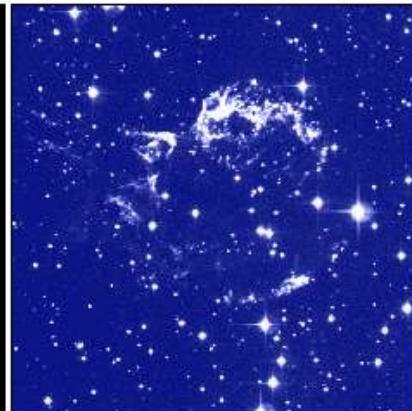
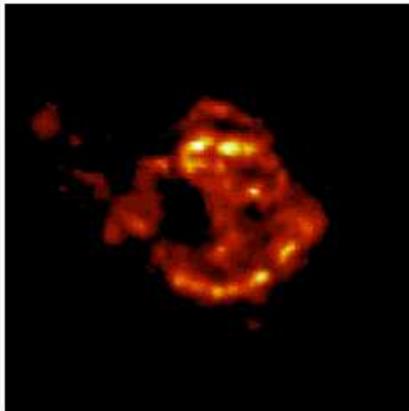
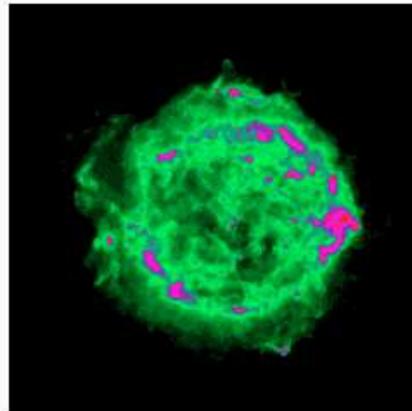
Cassiopeia A: a star that died in \sim 1700

RADIO

INFRARED

OPTICAL

XRAY

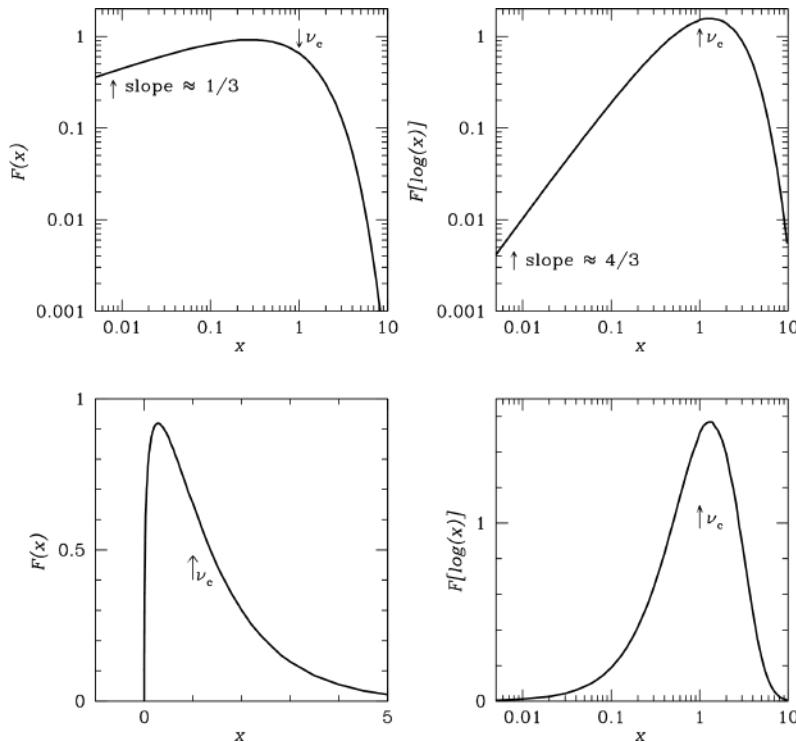


Synchrotron Radiation (single particle)

- synchrotron radiation

- spectrum

$$P(\nu) = \frac{\sqrt{3}e^3 B \sin\alpha}{m_e c^2} \left(\frac{\nu}{\nu_c}\right) \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta) d(\eta) \quad \text{with} \quad \nu_c = \frac{3}{2} \gamma^2 \nu_G \sin\alpha$$

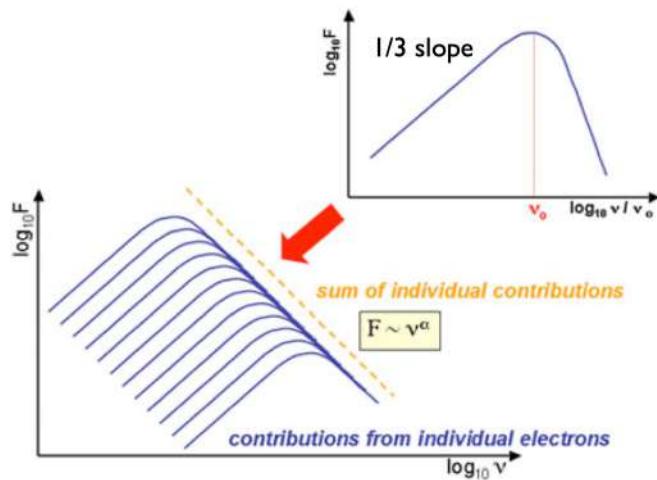


relatively flat spectrum at frequencies lower than the critical frequency and tailing off roughly at the maximum frequency

$$\nu_c = \left(\frac{3}{2} \sin\alpha\right) \left(\frac{E}{mc^2}\right)^2 \frac{eB}{2\pi m_e c} \propto E^2 B_\perp$$

$$\nu_{max} \approx \pi \gamma^2 \nu_G \sin\alpha \propto \gamma^2 B_\perp$$

Synchrotron Spectrum



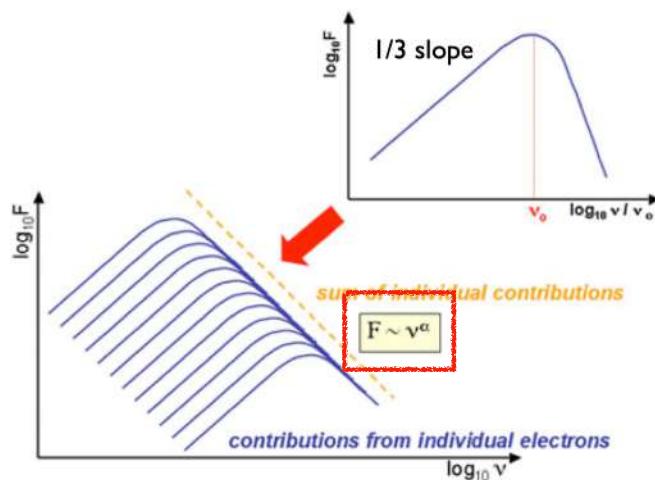
- Powerlaw.
- Observed spectrum is a superposition of the individual electron spectra.

Strong dependence on the Lorentz factor

$$v_{max} = v_e \gamma^2 \quad P_e \sim 2 \gamma^2 U_{mag}$$

- Since ($\gamma \gg 1$) this boosts the emission frequency of synchrotron emission into the radio domain.

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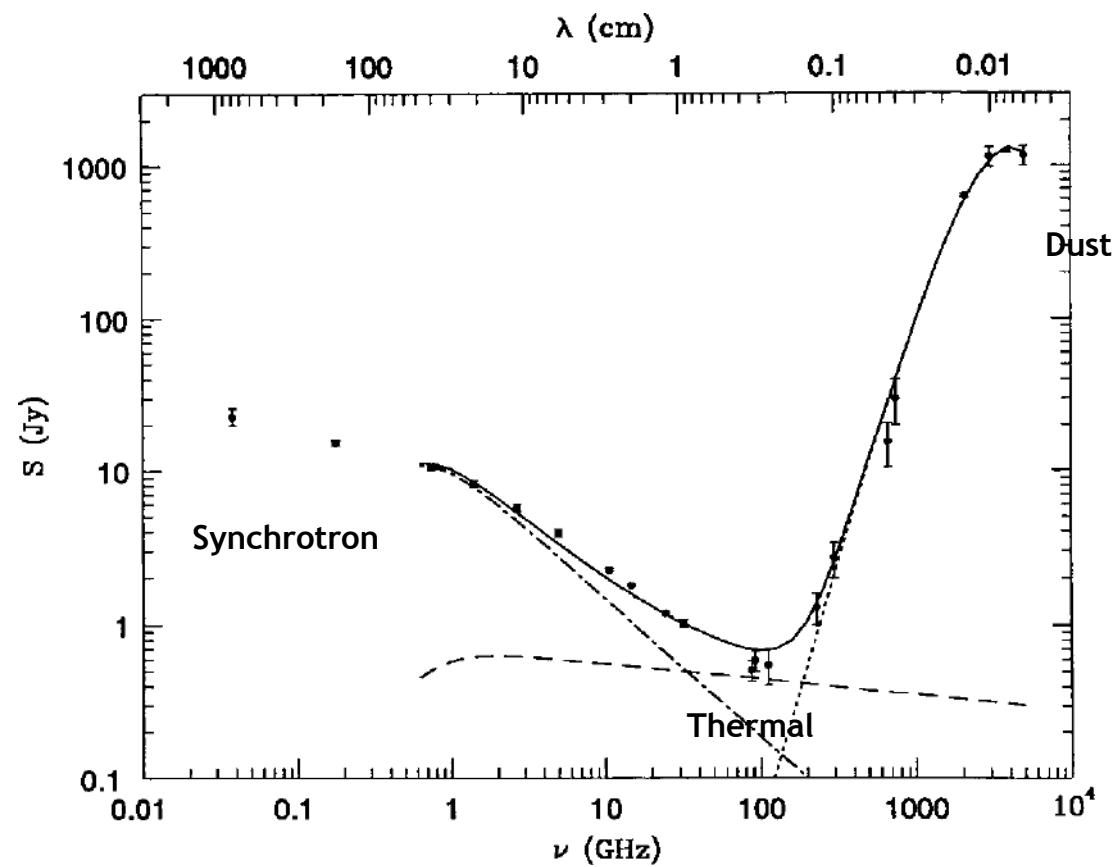
Galactic Radio Astronomy - SNR

- SNR in extragalactic sources : e.g. M82



Credit: Josh Marvil (NM Tech/NRAO), Bill Saxton (NRAO/AUI/NSF), NASA

“Nonthermal emission” Synchrotron emission: electrons spiraling around a magnetic field at relativistic velocities.



Radiative Process (Inverse Compton)

(Inverse) Compton Radiation

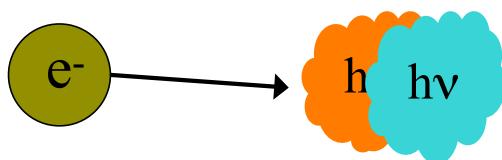
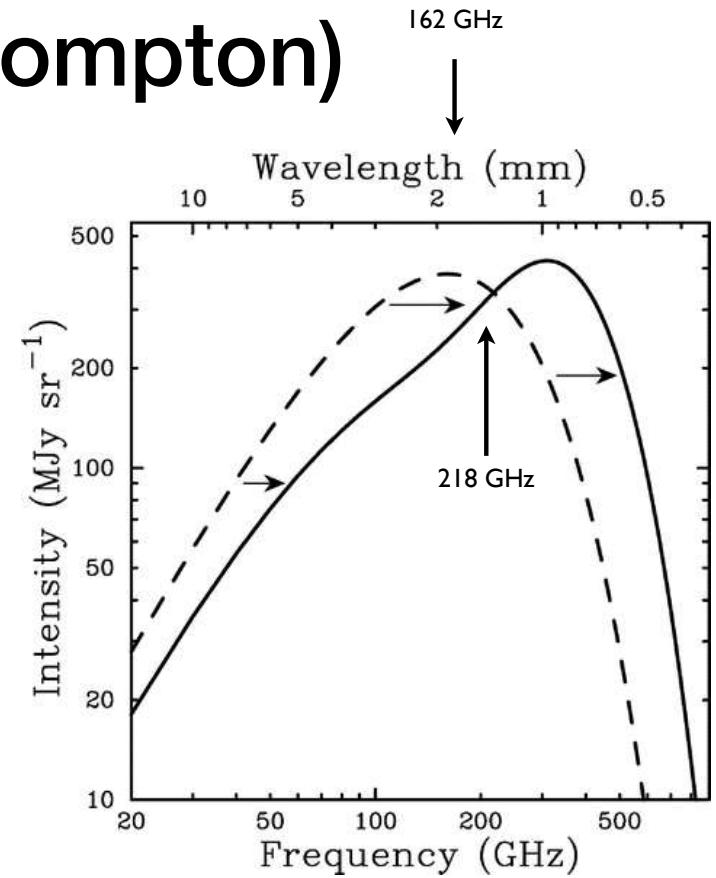


Image Credit: Hiro Hirashita

Intracluster medium
(Sunyaev-Zeldovich effect)
AGN jet, accretion disk

- Sunyaev-Zel'dovich effect
- Sunyaev & Zel'dovich (1970)
- CMB photons interact with 10^8 K plasma in clusters, typically extend on the Mpc scale (angular size of several arcmins)
- no confirmed results until late 1990's



$$\frac{\Delta T}{T} = \frac{2kT_e}{m_e c^2} \sigma_T N_e L$$
$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 6.65 \cdot 10^{-25} \text{ cm}^{-2}$$
$$= 2.24 \cdot 10^{-34} T_e N_e L$$

Radiative Process (Inverse Compton)

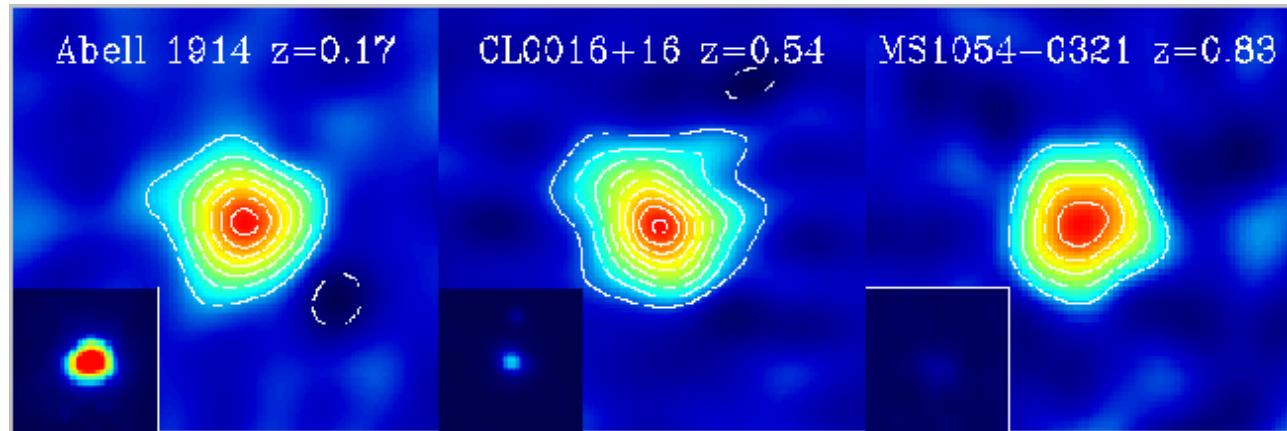
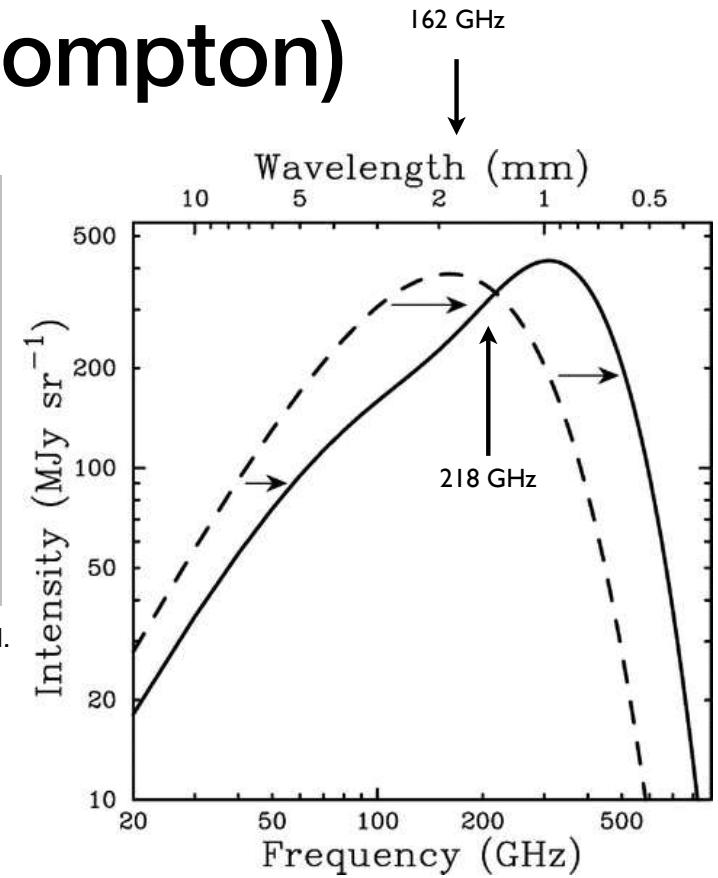


Image Credit: Carlstrom et al.

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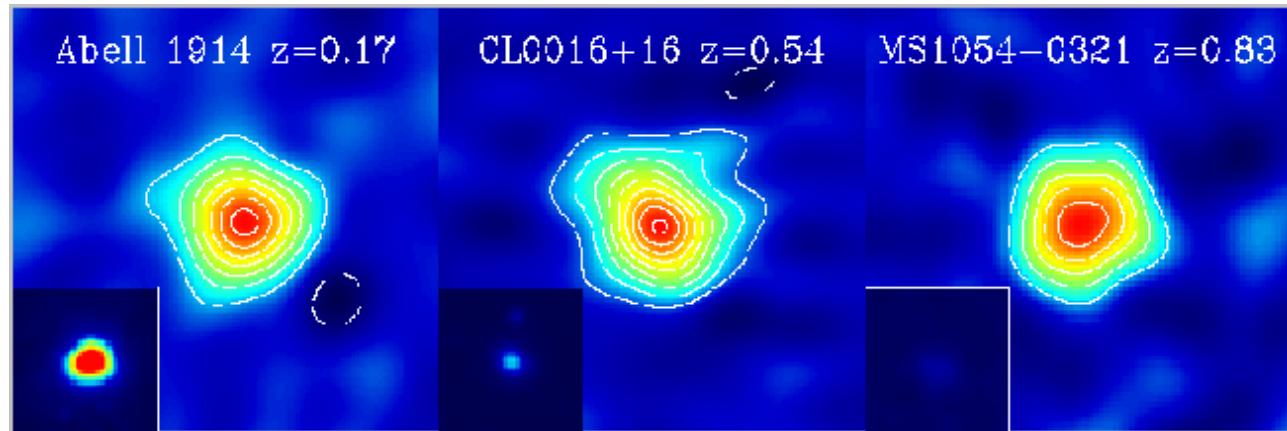
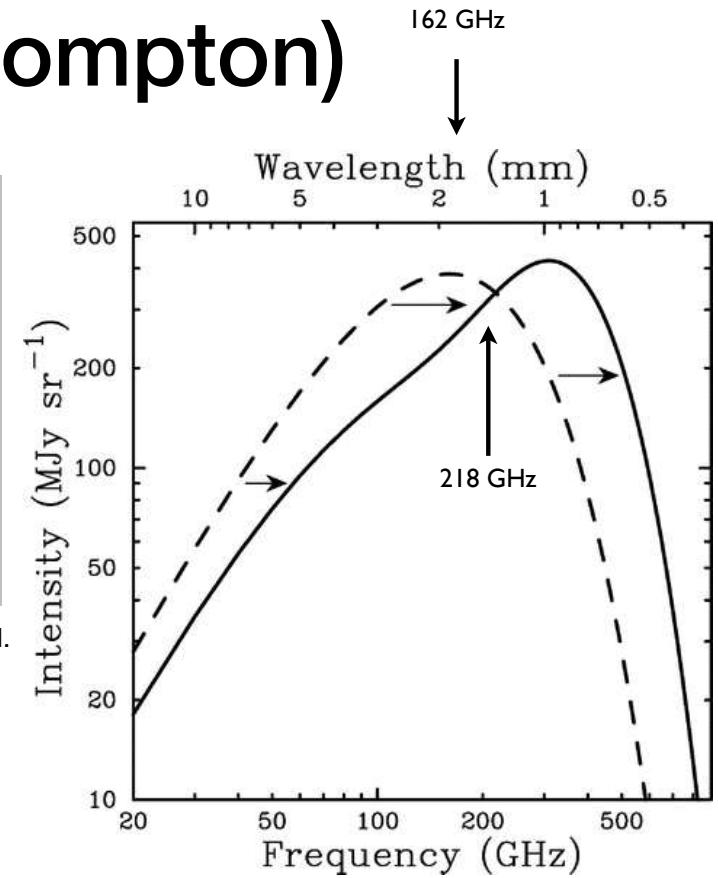


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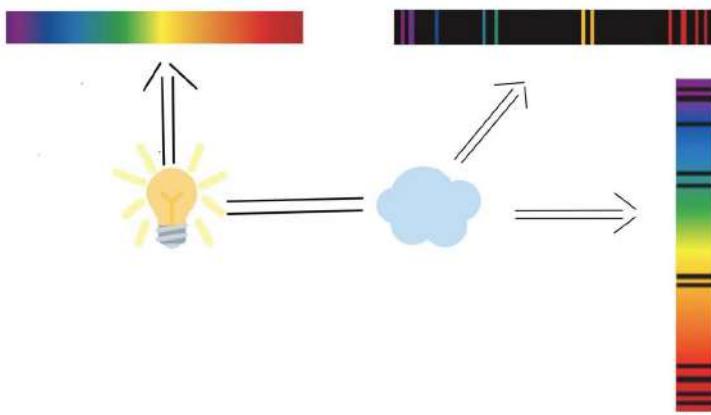


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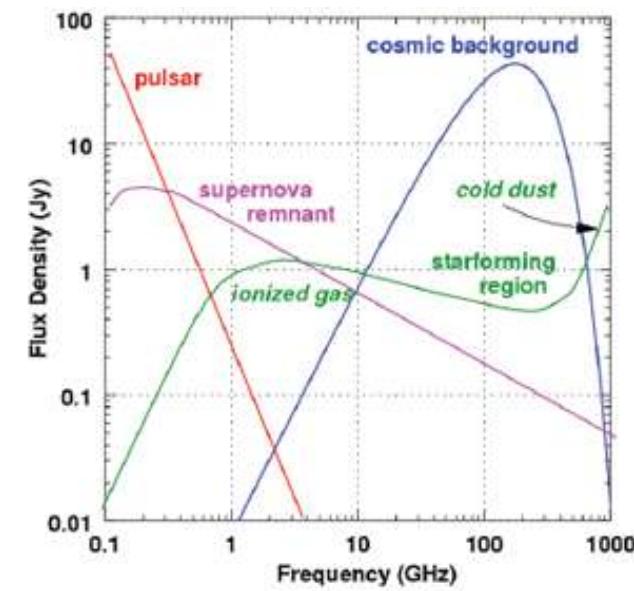
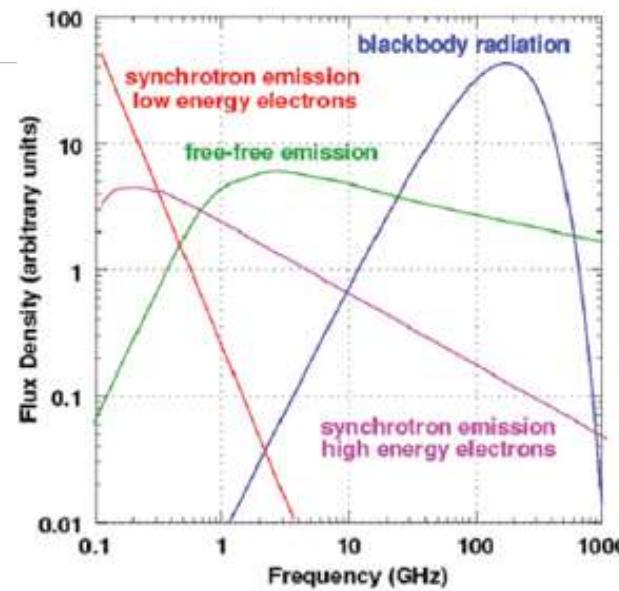
Take Home Message -

Think about what physical parameters can we extract?

Radiative Transfer



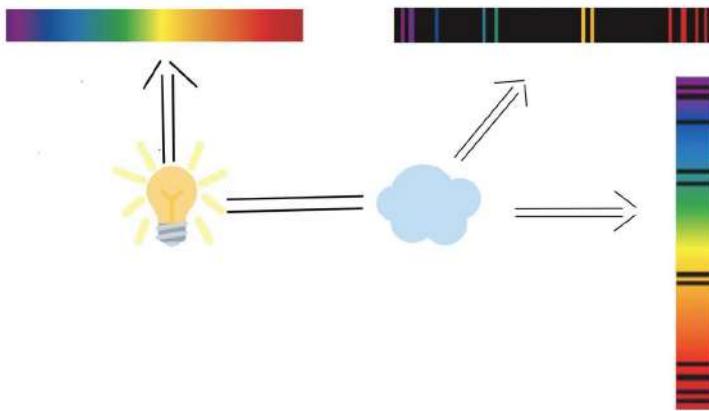
Radiative Processes



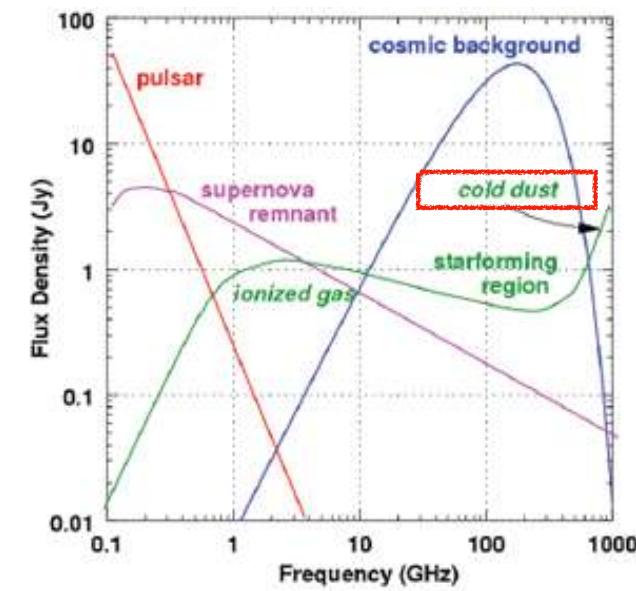
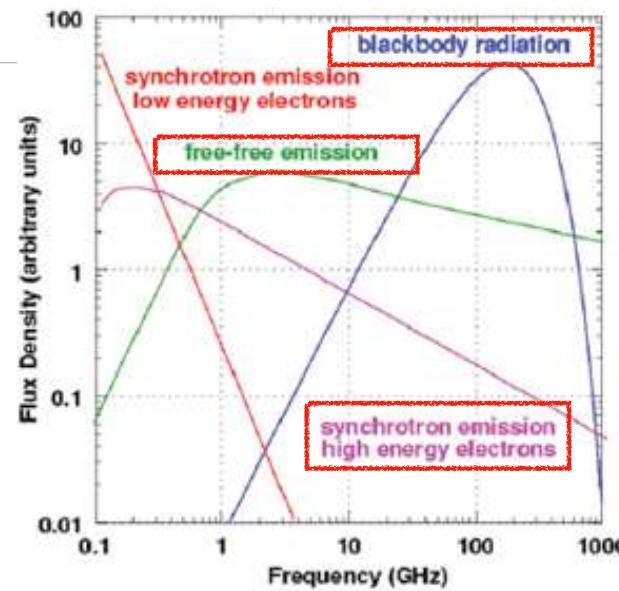
Take Home Message -

Think about what physical parameters can we extract?

Radiative Transfer



Radiative Processes



References

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- Draine, “Physics of the Interstellar and Intergalactic Medium”
- Seamus Clarke, ASIAA 2022 SSP Lecture, “Radiative Transfer”
- Hiroyuki Hirashita, ASIAA 2020 SSP Lecture, “Radiative Transfer”