Introduction to Bode Plot

- 2 plots – both have logarithm of frequency on x-axis
  - y-axis magnitude of transfer function, H(s), in dB
  - y-axis phase angle

The plot can be used to interpret how the input affects the output in both magnitude and phase over frequency.

<u>Where do the Bode diagram lines comes from?</u>

1) Determine the Transfer Function of the system:

$$H(s) = \frac{K(s+z_1)}{s(s+p_1)}$$

2) Rewrite it by factoring both the numerator and denominator into the **<u>standard</u>** form

$$H(s) = \frac{Kz_1(s/z_1 + 1)}{sp_1(s/p_1 + 1)}$$

where the ***z*** s are called zeros and the ***p*** s are called poles.

3) Replace ***s*** with ***j?*** . Then find the **<u>Magnitude</u>** of the Transfer Function.

$$H(jw) = \frac{Kz_1(jw/z_1 + 1)}{jwp_1(jw/p_1 + 1)}$$

If we take the $\log_{10}$ of this magnitude and multiply it by 20 it takes on the form of

$$20\log_{10}(H(jw)) = 20\log_{10}\left(\frac{Kz_1(jw/z_1 + 1)}{jwp_1(jw/p_1 + 1)}\right) =$$

$$20\log_{10}|K| + 20\log_{10}|z_1| + 20\log_{10}\left|(jw/z_1 + 1)\right| - 20\log_{10}|p_1| - 20\log_{10}|jw| - 20\log_{10}\left|(jw/z_1 + 1)\right|$$

Each of these individual terms is very easy to show on a logarithmic plot. The entire Bode log magnitude pl the result of the superposition of all the straight line terms. This means with a little practice, we can quickly the effect of each term and quickly find the overall effect. To do this we have to understand the effect of the different types of terms.

These include: 1) Constant terms K

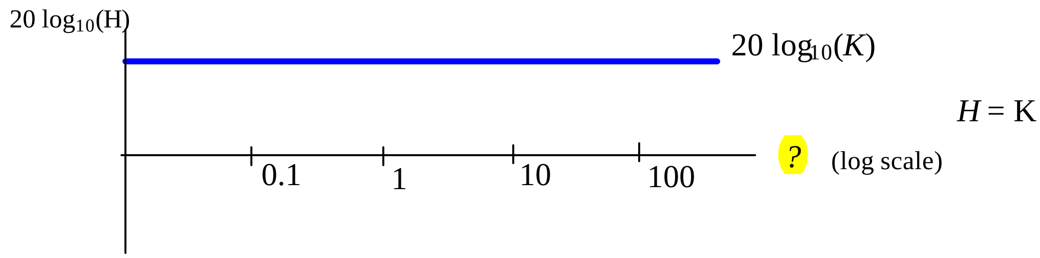
2) Poles and Zeros at the origin $|j?|$

3) Poles and Zeros not at the origin $\left|1 + \frac{jw}{p_1}\right|$ or $\left|1 + \frac{jw}{z_1}\right|$

4) Complex Poles and Zeros (addressed later)

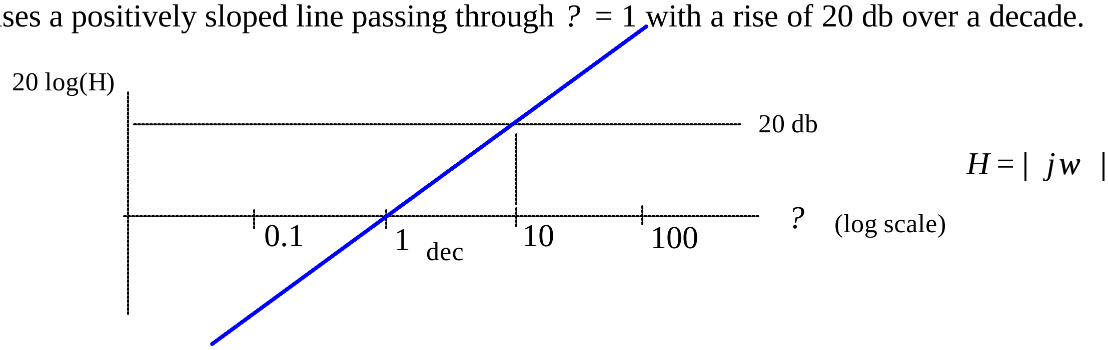
Effect of Constant Terms:

Constant terms such as ***K*** contribute a straight horizontal line of magnitude 20 $\log_{10}(K)$

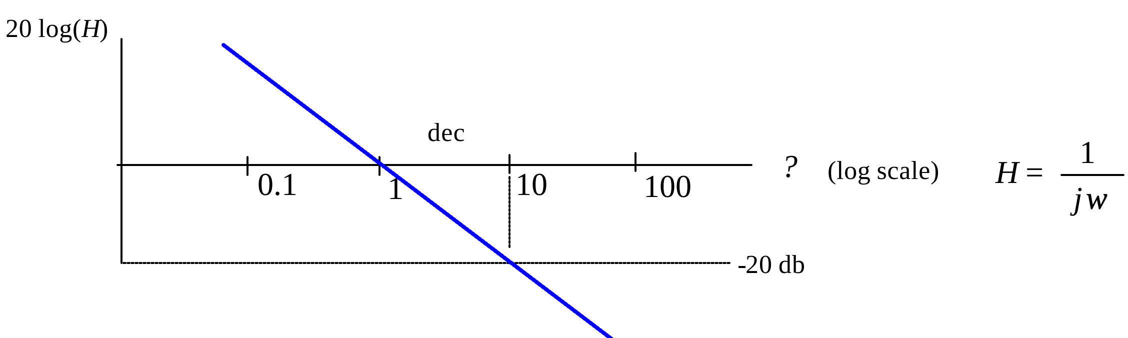


Effect of Individual Zeros and Poles at the origin:

A **zero** at the origin occurs when there is an ***s*** or ***j?*** multiplying the numerator. Each occurrence of this causes a positively sloped line passing through ? = 1 with a rise of 20 db over a decade.

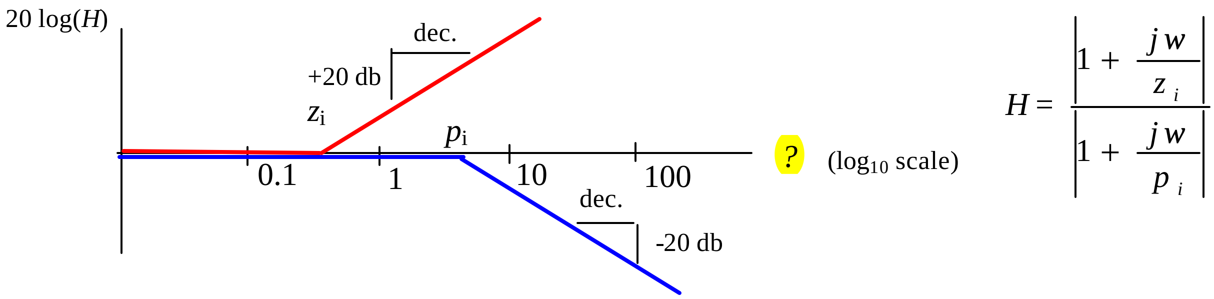


A pole at the origin occurs when there are ***s*** or ***j?*** multiplying the denominator. Each occurrence of this causes a negatively sloped line passing through ? = 1 with a drop of 20 db over a decade.



Effect of Individual Zeros and Poles Not at the Origin

Zeros and Poles **not at the origin** are indicated by the **(1+*j?* /z_i)** and **(1+*j?* /p_i)**. The values z_i and p_i in each of these expression is called a **critical frequency** (or break frequency). Below their critical frequency these terms do not contribute to the log magnitude of the overall plot. Above the critical frequency, they represent a ramp function of 20 db per decade. Zeros give a positive slope. Poles produce a negative slope.



- To complete the log magnitude vs. frequency plot of a Bode diagram, we superposition all the lines of the different terms on the same plot.

<u>Example 1:</u>
For the transfer function given, sketch the Bode log magnitude diagram which shows how the log magnitude of the system is affected by changing input frequency. (*TF=transfer function)*

$$TF = \frac{1}{2s+100}$$

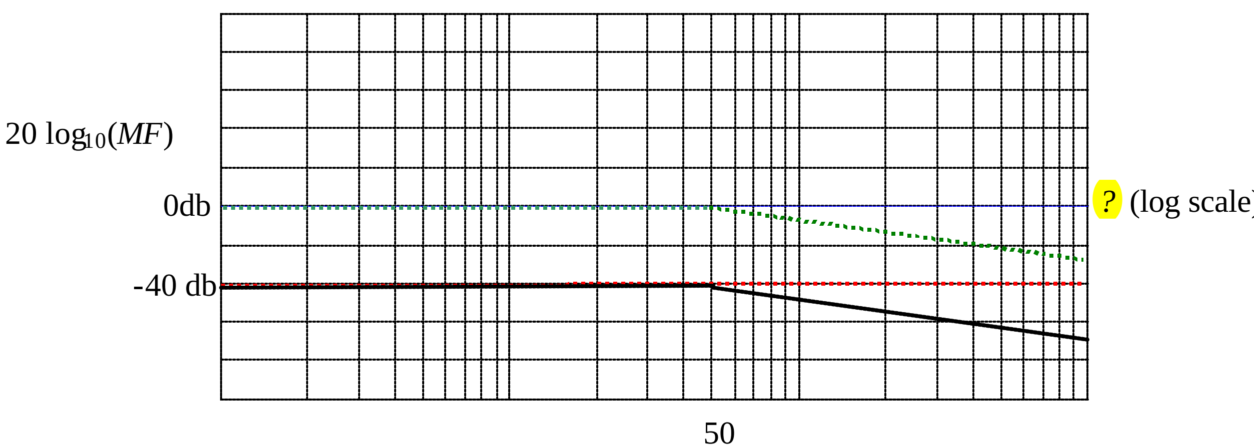
Step 1: Repose the equation in Bode plot form:

$$TF = \frac{\left(\frac{1}{100}\right)}{\frac{s}{50}+1} \qquad \text{recognized as} \qquad TF = \frac{K}{\frac{1}{p_1}s+1}$$

with $K = 0.01$ and $p_1 = 50$

For the constant, K: $20\log_{10}(0.01) = -40$

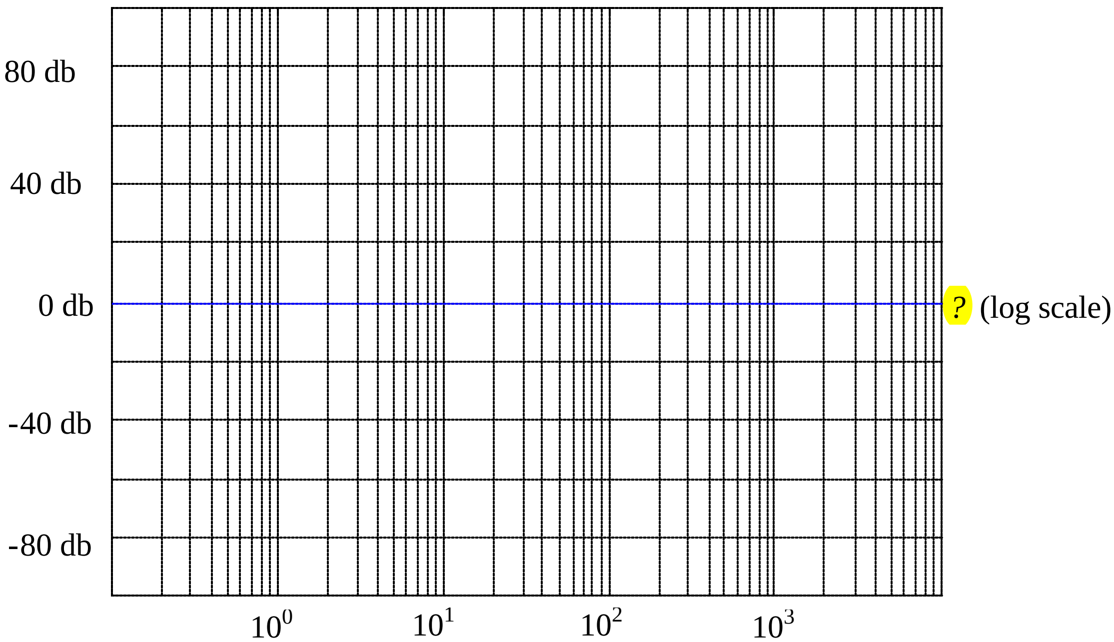
For the pole, with critical frequency, p_1:



Example 2:
Your turn. Find the Bode log magnitude plot for the transfer function,

$$TF = \frac{5x10^4 s}{s^2 + 505s + 2500}$$

Start by simplifying the transfer function form:



Example 2 Solution:
Your turn. Find the Bode log magnitude plot for the transfer function,

$$TF = \frac{5x10^4 s}{s^2 + 505s + 2500}$$

Simplify transfer function form:

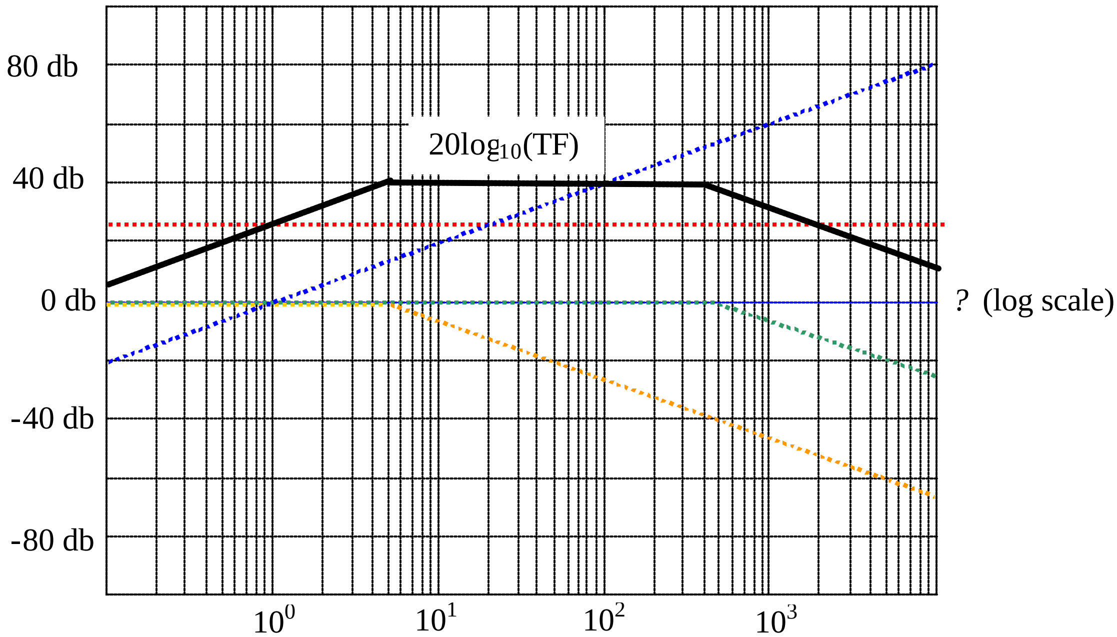
$$TF = \frac{5x10^4 \quad s}{(s+5)(s+500)} = \frac{\frac{5x10^4}{5*500} \quad s}{(\frac{s}{5}+1)(\frac{s}{500}+1)} = \frac{20 \quad s}{(\frac{s}{5}+1)(\frac{s}{500}+1)}$$

Recognize: K = 20 → 20 log10(20) = 26.02

1 zero at the origin

2 poles: at p_1 = 5 and p_2=500

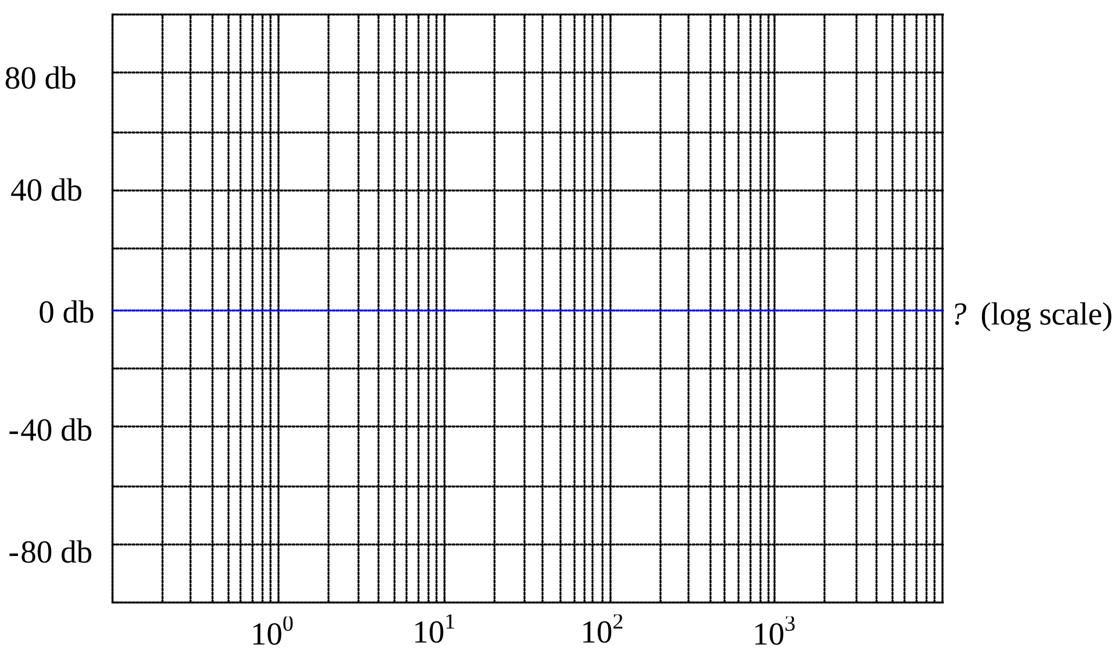
Technique to get started:
1) Draw the line of each individual term on the graph
2) Follow the combined **pole-zero at the origin** line back to the left side of the graph.
3) Add the constant offset, **20 log$_{10}$(*K*),** to the value where the **pole/zero at the origin** line intersects the left side of the graph.
4) Apply the effect of the **poles/zeros not at the origin**. working from left (low values) to right (higher values) of the poles/zeros.



Example 3: One more time. This one is harder. Find the Bode log magnitude plot for the transfer function,

$$TF = \frac{200(s+20)}{s(2s+1)(s+40)}$$

Simplify transfer function form:



Technique to get started:

1) Draw the line of each individual term on the graph
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4) Apply the effect of the **poles/zeros not at the origin**. working from left (low values) to right (higher values) of the poles/zeros.

Example 3 Solution: Find the Bode log magnitude plot for the transfer function,

$$TF = \frac{200(s+20)}{s(2s+1)(s+40)}$$

Simplify transfer function form:

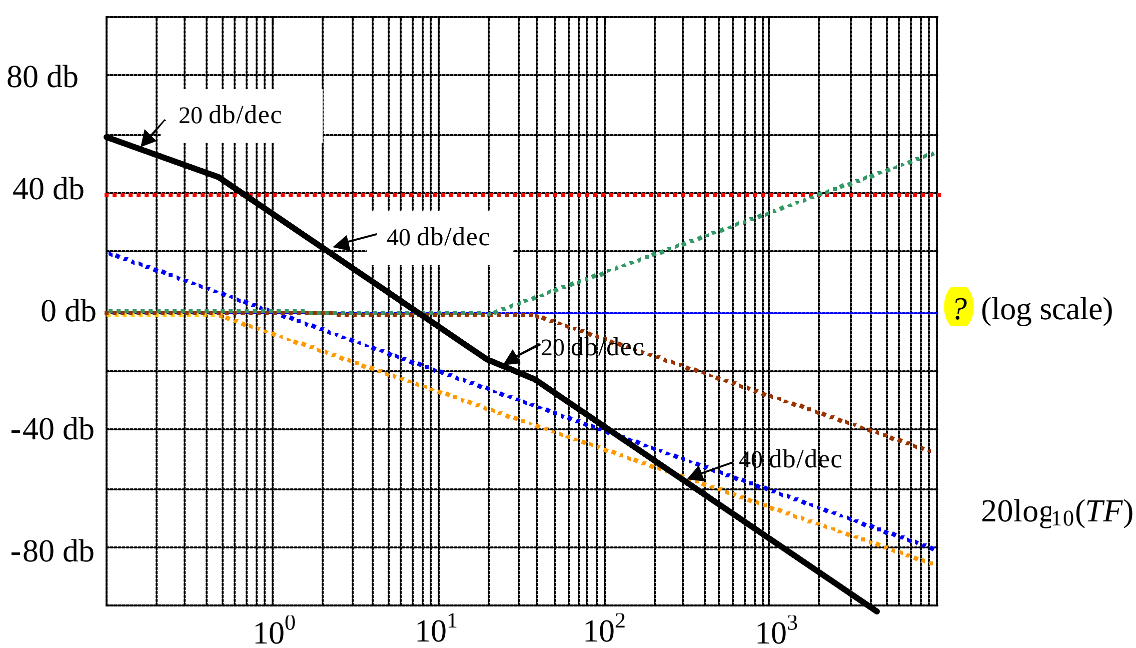
$$TF = \frac{200(s+20)}{s(2s+1)(s+40)} = \frac{\frac{200*20}{40}\quad(\frac{s}{20}+1)}{s(\frac{s}{0.5}+1)(\frac{s}{40}+1)} = \frac{100\quad(\frac{s}{20}+1)}{s(\frac{s}{0.5}+1)(\frac{s}{40}+1)}$$

Recognize: $K = 100$ → 20 log10(100) = 40

1 pole at the origin

1 zero at $z_1 = 20$

2 poles: at $p_1 = 0.5$ and p_2=40



Technique to get started:

1) Draw the line of each individual term on the graph
2) Follow the combined **pole-zero at the origin** line back to the left side of the graph.
3) Add the constant offset, **20 $\log_{10}(K)$,** to the value where the **pole/zero at the origin** line intersects the left side of the graph.
4) Apply the effect of the **poles/zeros not at the origin**. working from left (low values) to right (higher values) of the poles/zeros.

The plot of the **log magnitude** vs**. input frequency** is only half of the story.
We also need to be able to plot the **phase angle** vs. **input frequency** on a log scale as well to complete the full Bode diagram..

For our original transfer function,

$$H(jw) = \frac{Kz_1(\frac{jw}{z_1}+1)}{jwp_1(\frac{jw}{p_1}+1)}$$

the cumulative phase angle associated with this function are given by

$$\angle H(jw) = \frac{\angle K \angle z_1 \angle(\frac{jw}{z_1}+1)}{\angle jw \angle p_1 \angle(\frac{jw}{p_1}+1)}$$

Then the cumulative phase angle as a function of the input frequency may be written as

$$\angle H(jw) = \angle\left[K + z_1 + (\frac{jw}{z_1}+1) - (jw) - p_1 - (\frac{jw}{p_1}+1)\right]$$

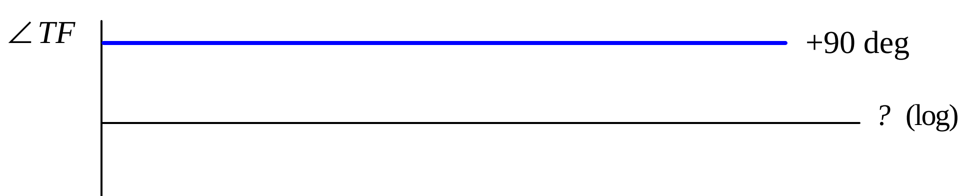
Once again, to show the phase plot of the Bode diagram, lines can be drawn for each of the different terms. Then the total effect may be found by superposition.

<u>Effect of Constants on Phase:</u>

A **positive** constant, ***K*>0,** has no effect on phase. A **negative** constant, ***K*<0,** will set up a phase shift of $\pm180^o$. (Remember real vs imaginary plots – a negative real number is at $\pm180^o$ relative to the origin)

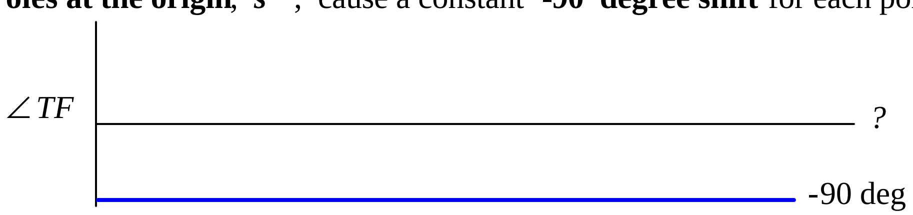
<u>Effect of Zeros at the origin on Phase Angle:</u>

Zeros at the origin, *s*, cause a constant **+90 degree shift** for each zero.



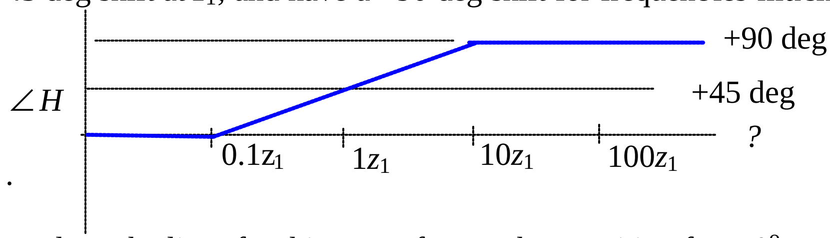
<u>Effect of Poles at the origin on Phase Angle:</u>

Poles at the origin, s^{-1}, cause a constant **-90 degree shift** for each pole.



Effect of Zeros not at the origin on Phase Angle:

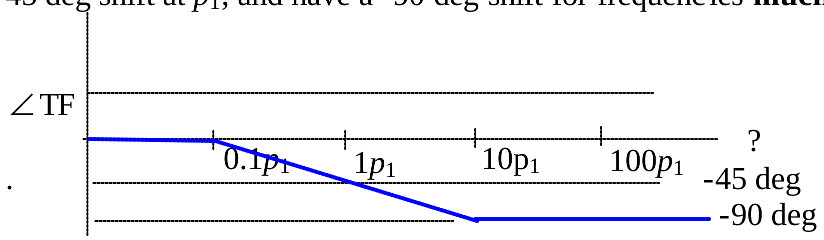
Zeros not at the origin, like $\left|1 + \frac{jw}{z_1}\right|$, have no phase shift for frequencies **much lower** than z_i, have a +45 deg shift at z_1, and have a +90 deg shift for frequencies **much higher** than z_1.



To draw the lines for this type of term, the transition from 0^o to $+90^o$ is drawn over 2 decades, starting at $0.1z_1$ and ending at $10z_1$.

Effect of Poles not at the origin on Phase Angle:

Poles not at the origin, like $\frac{1}{\left|1 + \frac{jw}{p_1}\right|}$, have no phase shift for frequencies **much lower** than p_i, have a -45 deg shift at p_1, and have a -90 deg shift for frequencies **much higher** than p_1.



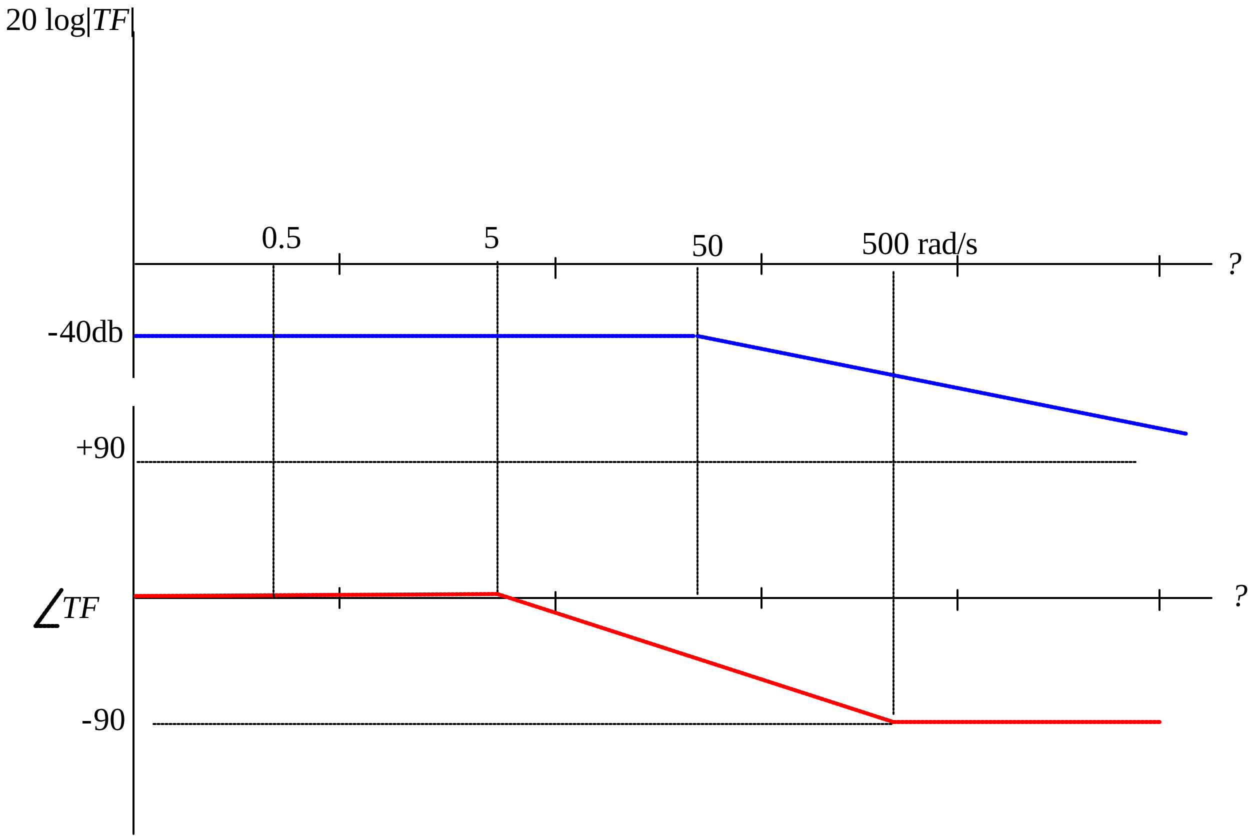
To draw the lines for this type of term, the transition from 0^o to -90^o is drawn over 2 decades, starting at $0.1p_1$ and ending at $10p_1$.

When drawing the phase angle shift for **not-at-the-origin zeros and poles**, first locate the critical frequency of the zero or pole. Then start the transition 1 decade before, following a slope of $\pm 45^o$ /decade. Continue the transition until reaching the frequency one decade past the critical frequency.

Now let's complete the Bode Phase diagrams for the previous examples:

Example 1:
For the Transfer Function given, sketch the Bode diagram which shows how the phase of the system is affected by changing input frequency.

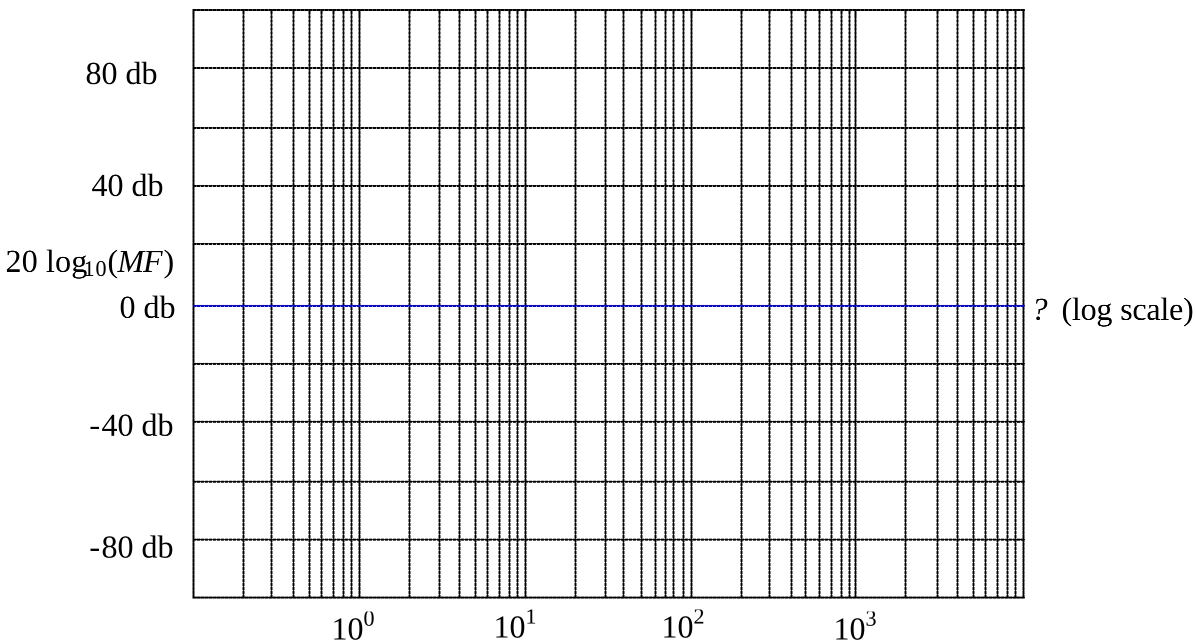
$$TF = \frac{1}{2s + 100} = \frac{(1/100)}{(\frac{s}{50} + 1)}$$

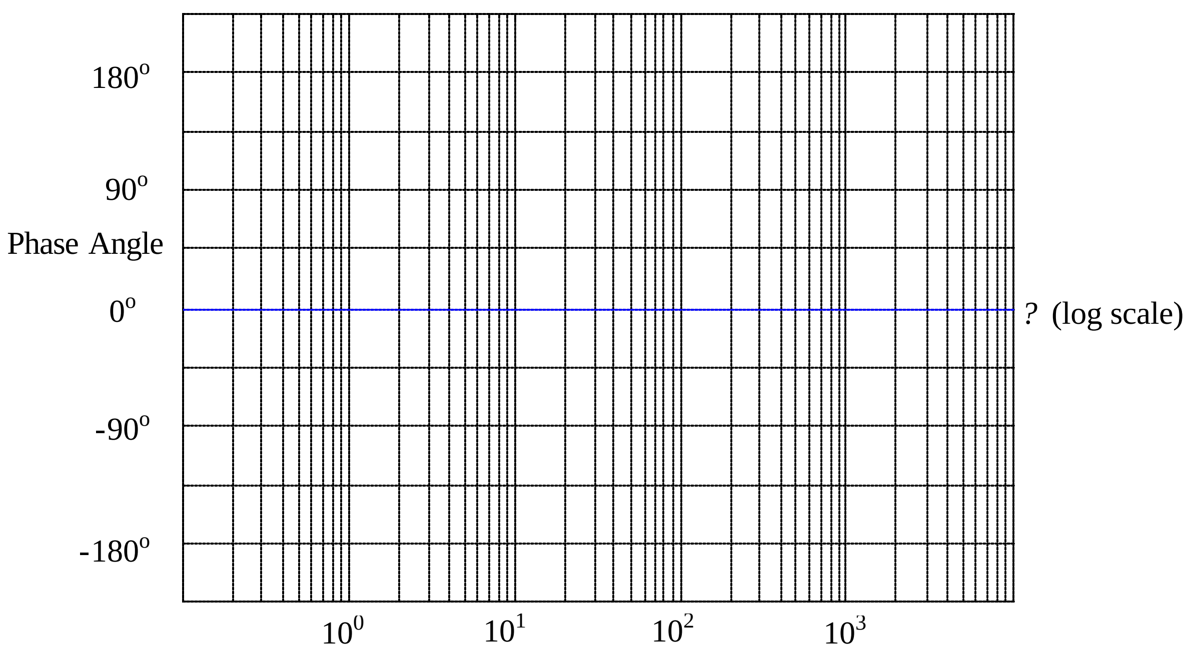


Example 2:
Repeat for the transfer function,

20log|TF|

$$TF = \frac{5x10^4 s}{s^2 + 505s + 2500} = \frac{20 \quad s}{(\frac{s}{5}+1)(\frac{s}{500}+1)}$$

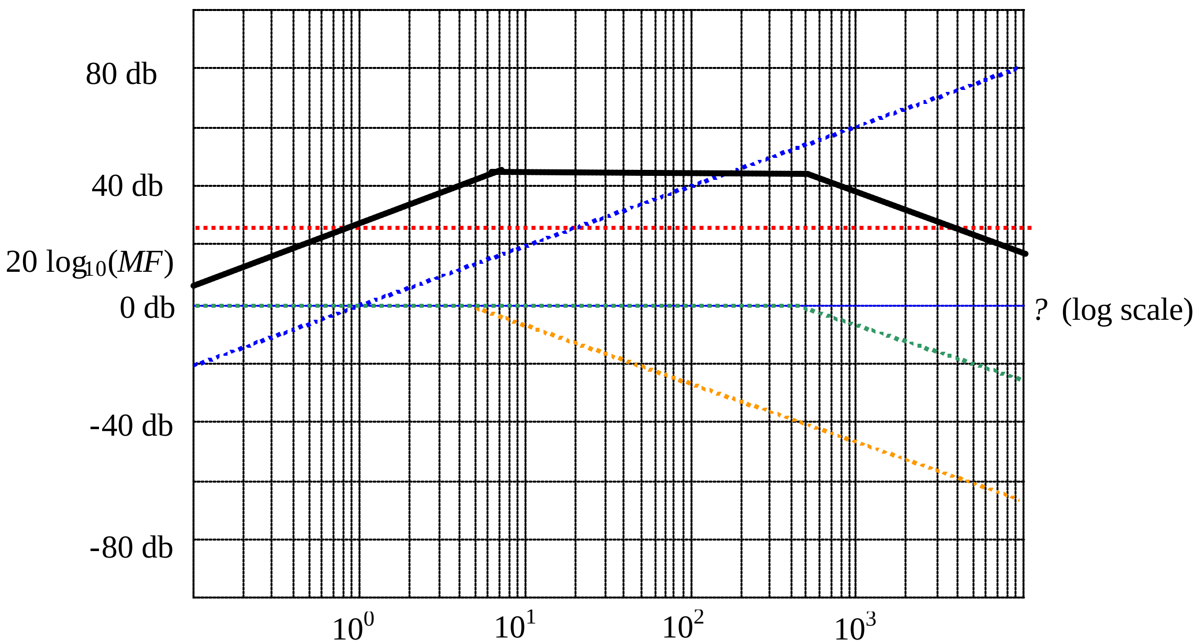


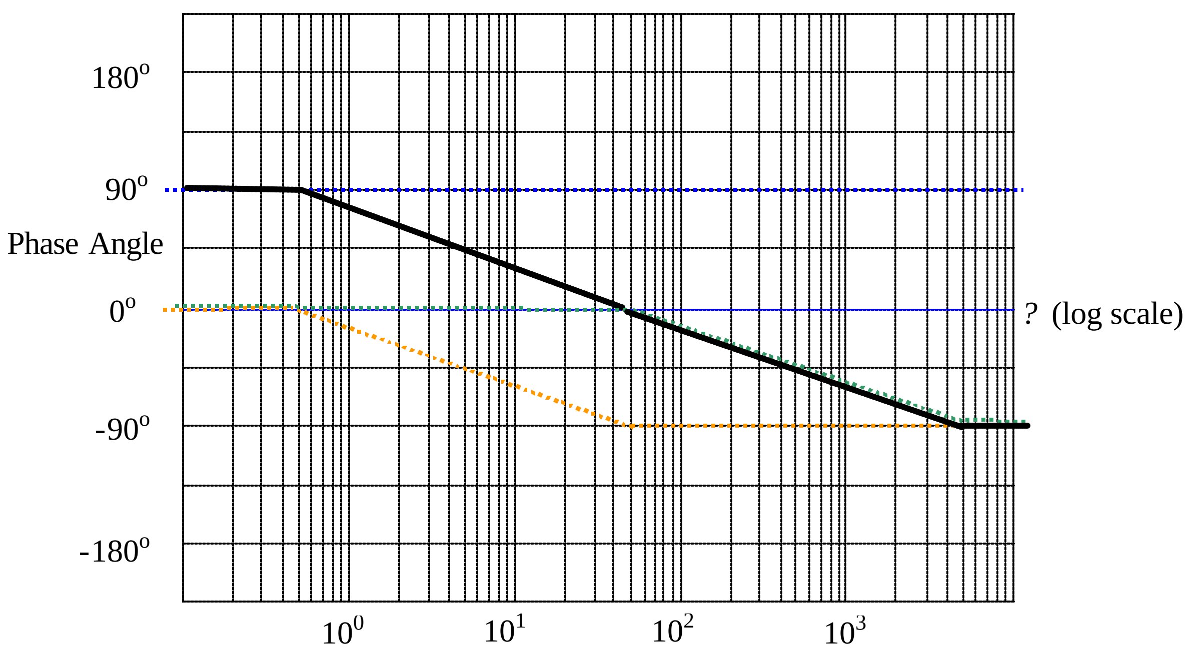


Example 2 Solution:
Repeat for the transfer function,

20log|*TF*|

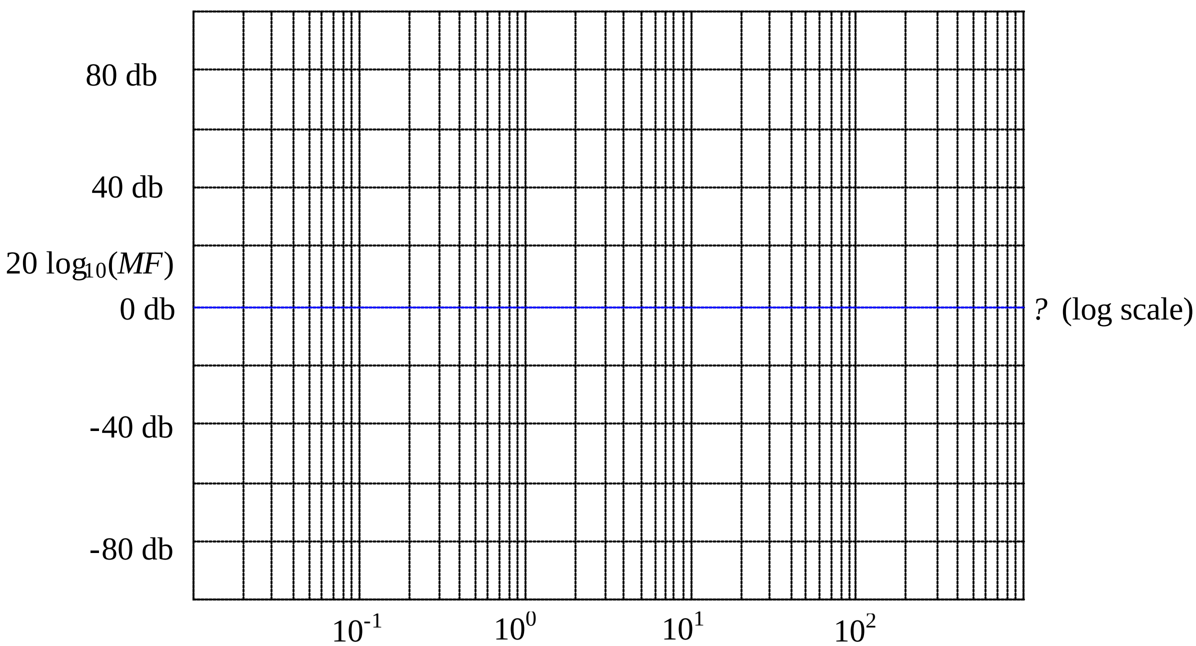
$$TF = \frac{5x10^4 s}{s^2 + 505s + 2500} = \frac{20 \quad s}{(\frac{s}{5}+1)(\frac{s}{500}+1)}$$

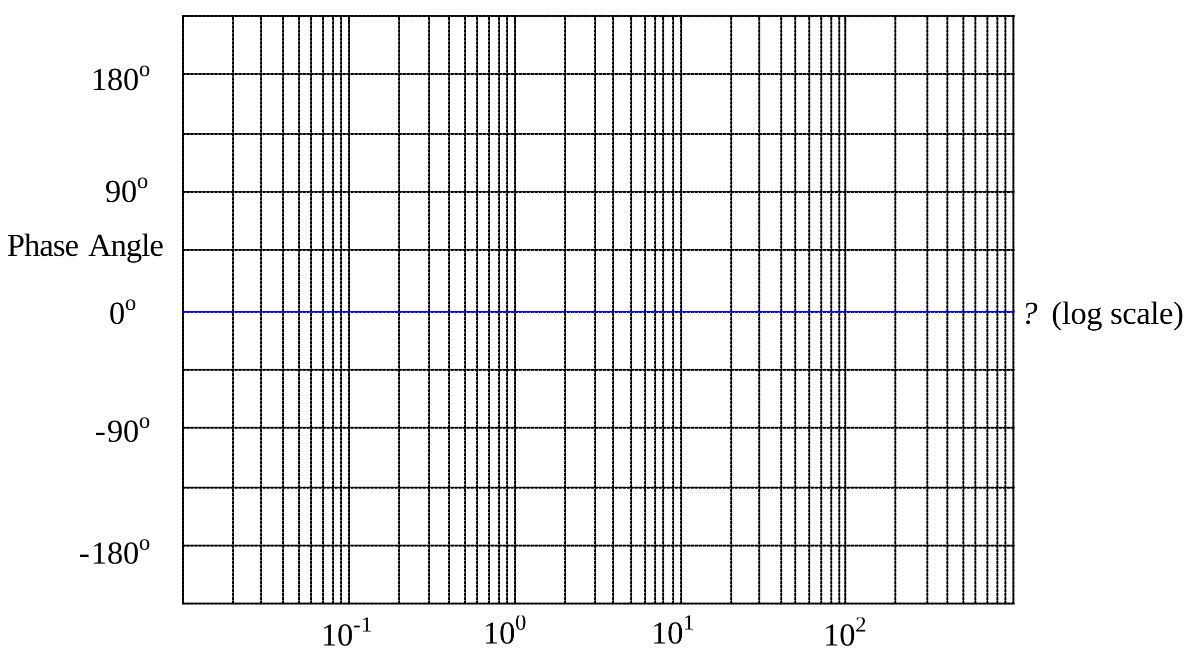




Example 3: Find the Bode log magnitude and phase angle plot for the transfer function,

$$TF = \frac{200(s+20)}{s(2s+1)(s+40)} = \frac{100 \quad (\frac{s}{20}+1)}{s(\frac{s}{0.5}+1)(\frac{s}{40}+1)}$$





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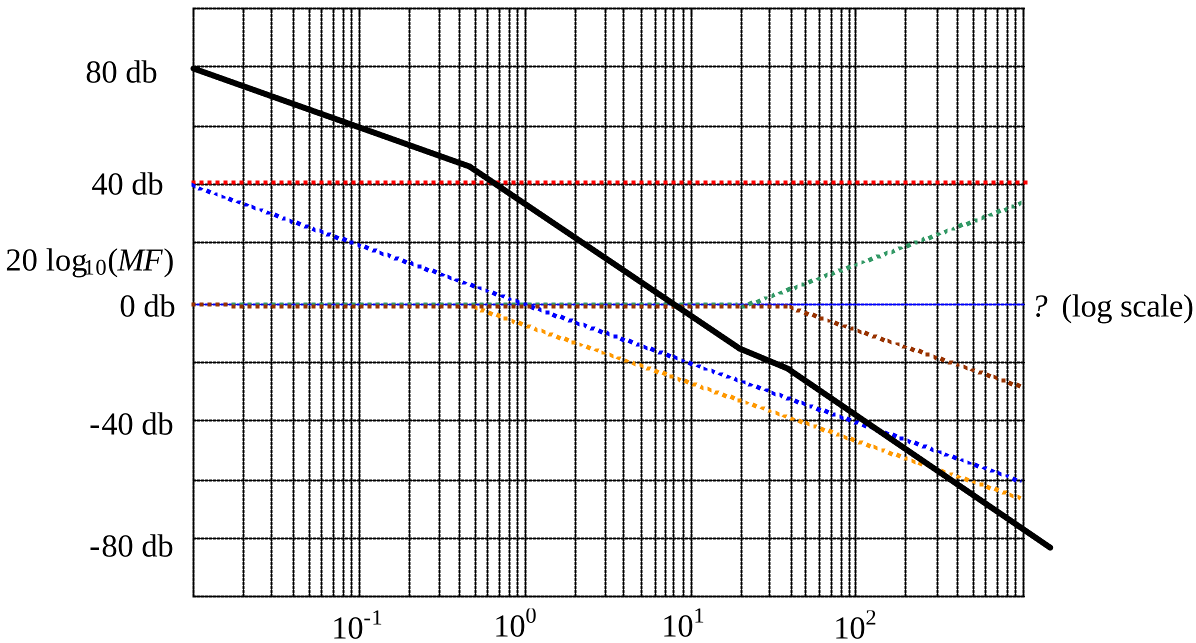
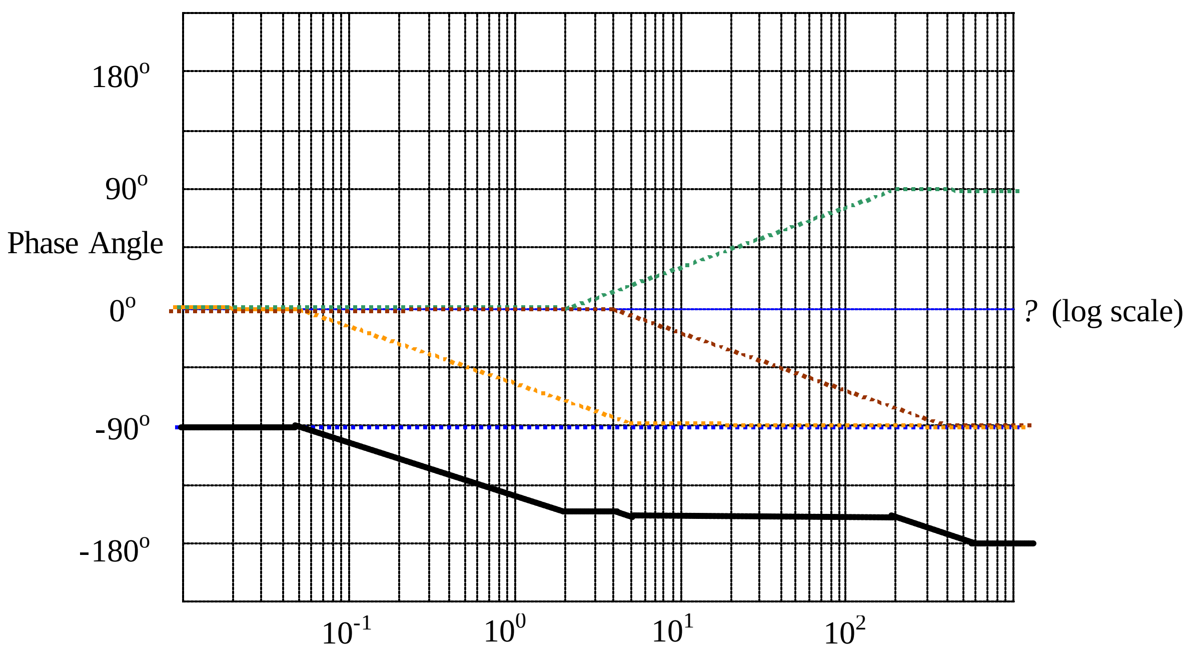


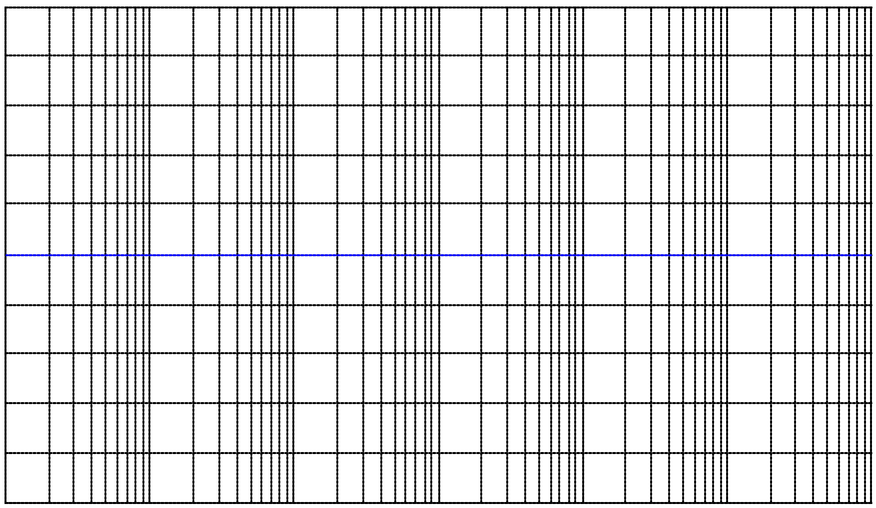


Example 4:

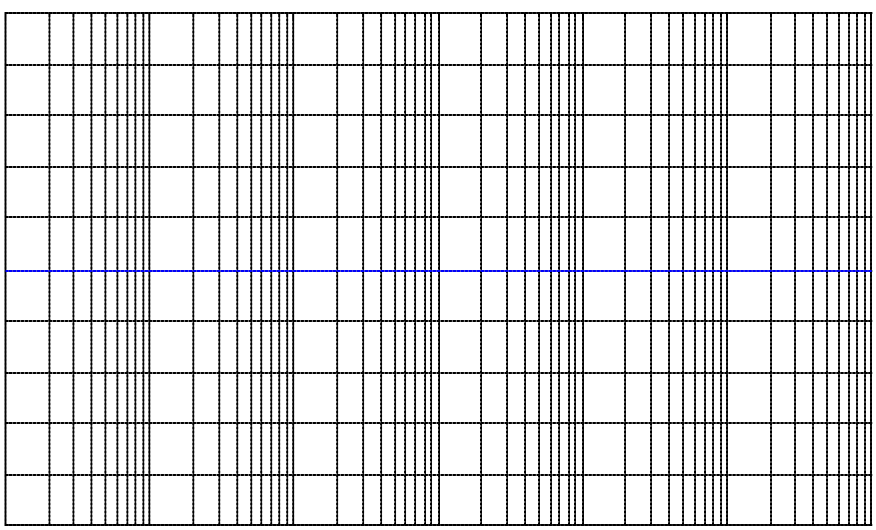
Sketch the Bode plot (Magnitude and Phase Angle) for

$$TF = \frac{100\times 10^{3}(s+1)}{(s+10)(s+1000)} =$$

$20\log_{10}|TF|$



Angle of TF



Example 4:

Sketch the Bode plot (Magnitude and Phase Angle) for

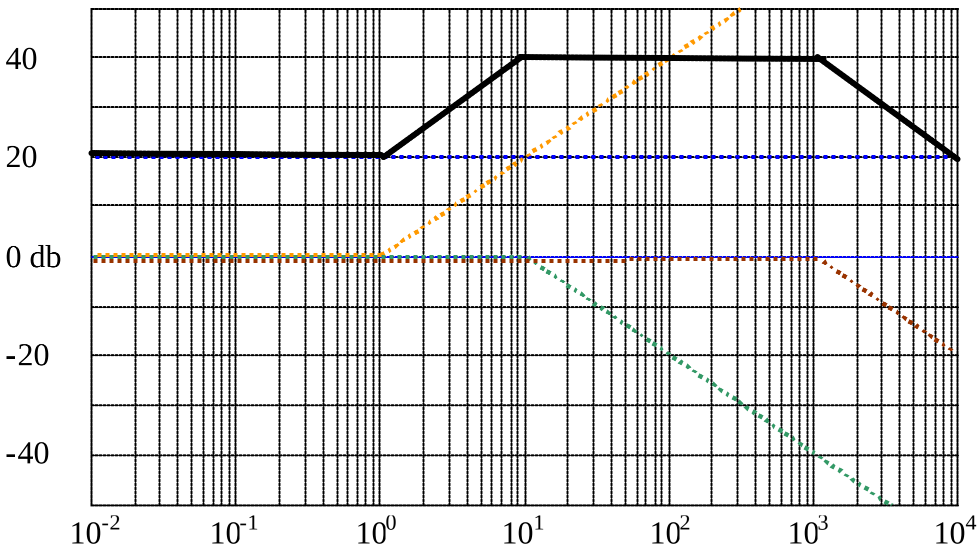
$$TF = \frac{100\times 10^{3}(s+1)}{(s+10)(s+1000)} \quad = \quad \frac{10\left(\frac{s}{1}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)}$$

Therefore: $K = 10$ so $20\log_{10}(10) = 20$ db

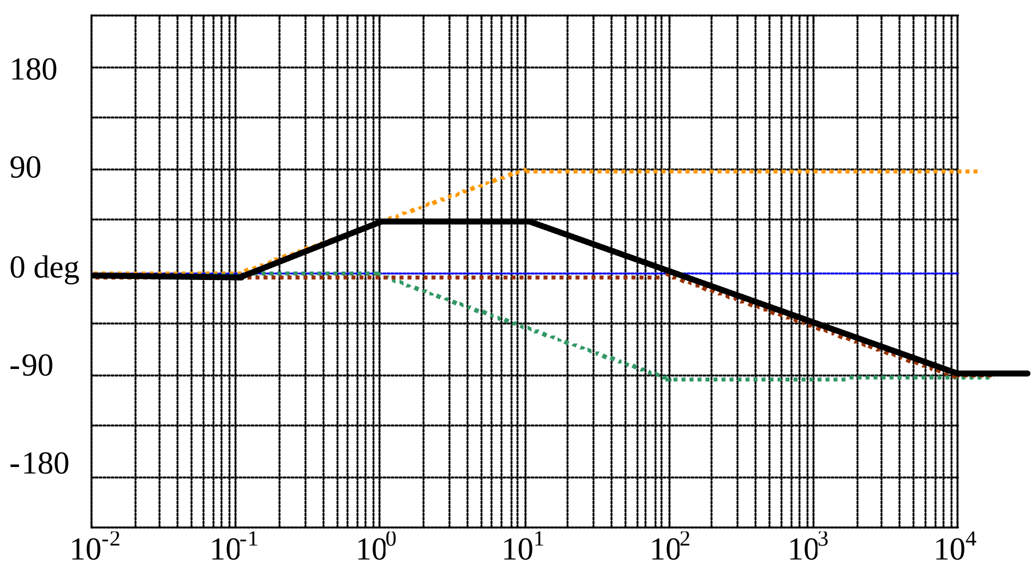
One zero: $z_1 = 1$

Two poles: $p_1 = 10$ and $p_2 = 1000$

$20\log_{10}|TF|$



Angle of TF

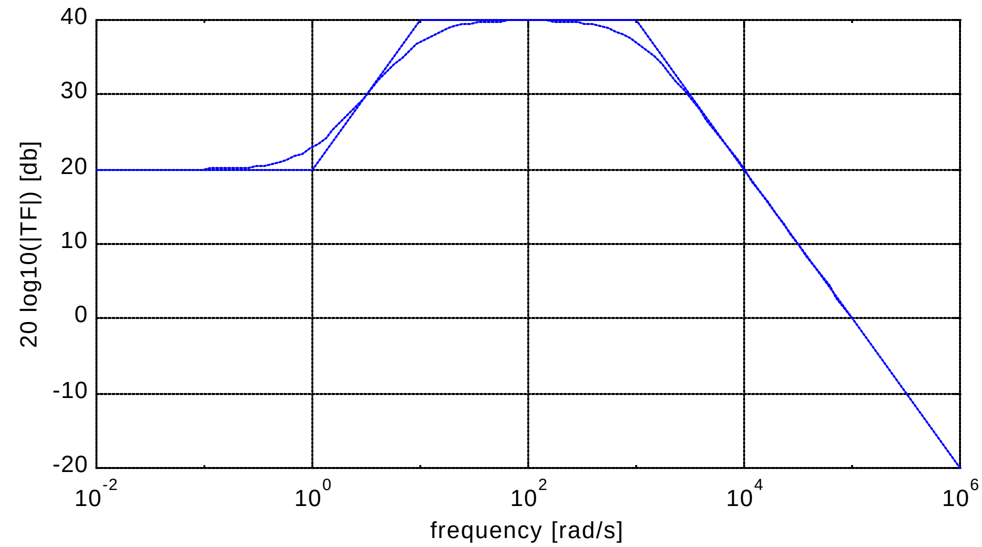


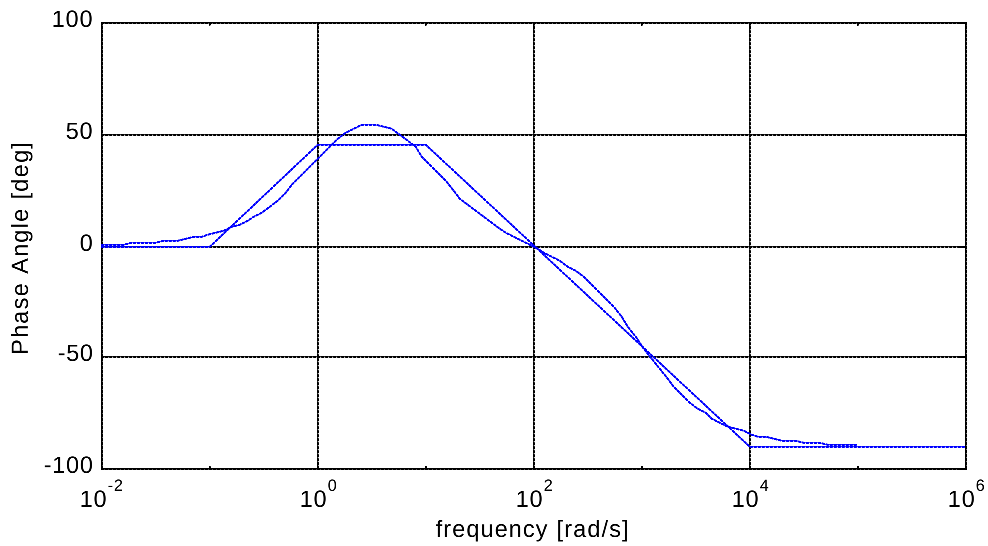
Matlab can also be used to draw Bode plots:

<u>Matlab</u> (with the sketched Bode Plot superimposed on the actual plot)

$$TF = \frac{100\times10^3(s+1)}{(s+10)(s+1000)}$$

```
w=logspace(-1,5,100);                                      %setup for x-axis
MagH=100000*sqrt(w.^2+1^2)./(sqrt(w.^2+10^2).*sqrt(w.^2+1000^2));
                                                           %transfer function
MagHdb=20*log10(MagH);                                     %transfer function converted to dB
PhaseHRad=atan(w/1)-atan(w/10)-atan(w/1000);               %phase done in radians
PhaseHDeg=PhaseHRad*180/pi;                             %phase done in degrees
subplot(2,1,1)
semilogx(w,MagHdb,':b',x,y,'-b')                        %semilog plot
xlabel('frequency [rad/s]'),ylabel('20 log10(|TF|) [db]'),grid  %xaxis label
subplot(2,1,2)
semilogx(w,PhaseHDeg,':b',xAng,yAngDeg,'-b')
xlabel('frequency [rad/s]'),ylabel('Phase Angle [deg]'),grid
```

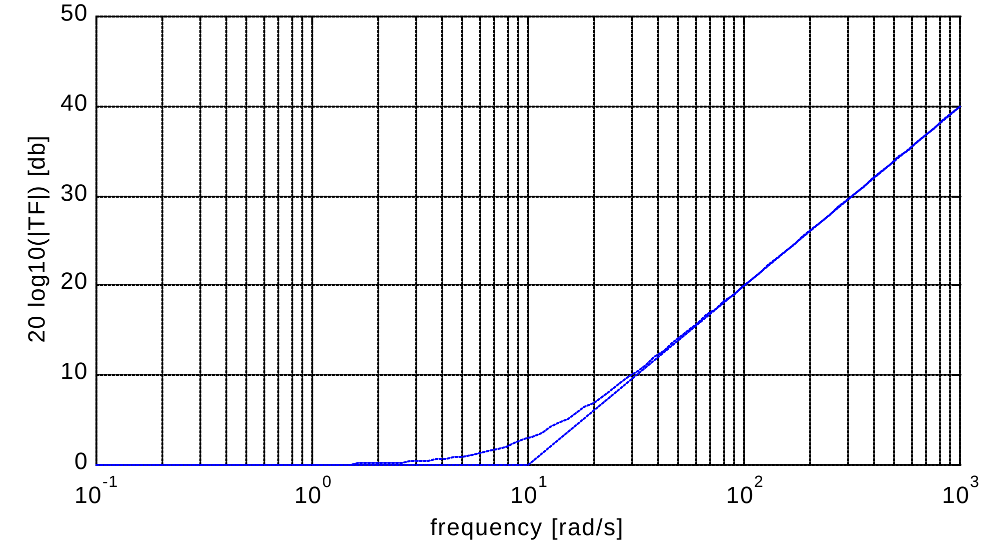


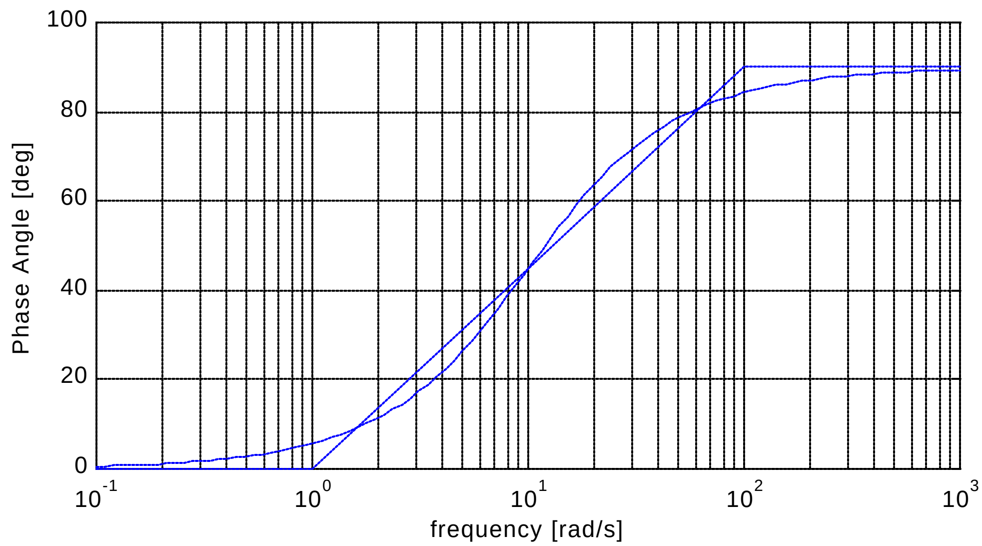


Notice that the actual plot does not follow the sketched plot exactly. There is error between our sketched method and the actual Bode plot. How much error is expected?

Let's look at an example of a zero, $TF = (1 + \frac{s}{10})$.   Note, ? $_{\text{critical}}$ = 10 rad/s

The largest error that occurs on the Magnitude plot is right at the critical frequency. It is on the order of 3





db.

The largest error that is shown on the Phase plot occurs at 0.1? $_{critical}$ and 10? $_{critical}$ (one decade above and below the critical frequency). Error at these points is about 6 degrees.

It's understood that sketching the Bode diagrams will contain some error but this is generally considered acceptable practice.

To quickly sketch the graphs:

1. Determine the starting value: |H(0)|
2. Determine all critical frequencies (break frequencies). Start from the lowest value and draw the graphs as follows:

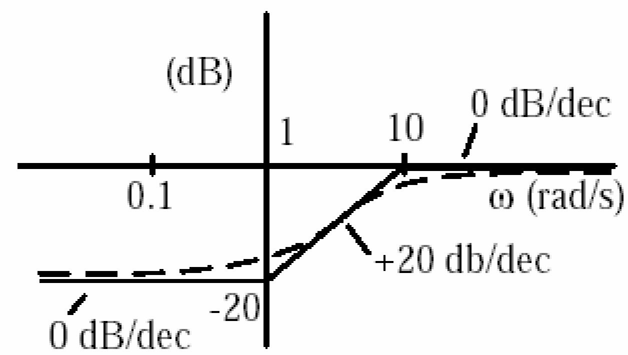
	Magnitude	Phase (create slope 1 decade below to 1 decade above $\omega_{critical}$)
Pole is negative	-20dB/dec	-45^o
Pole is positive	-20dB/dec	$+45^o$
Zero is negative	+20dB/dec	$+45^o$
Zero is positive	+20dB/dec	-45^o

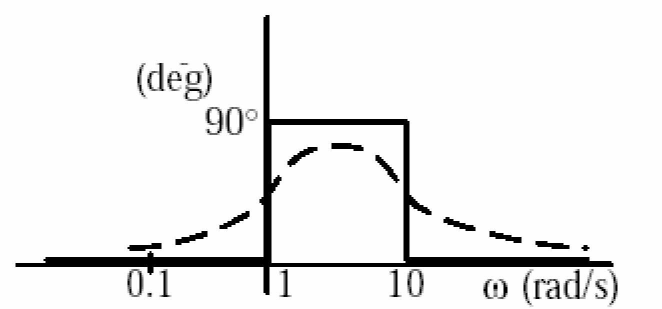
Add each value to the previous value.

Examples:

1. H(s) = $\frac{s+1}{s+10}$ |H(0)| = |0+1/0+10 = 1/10 |= |0.1| => -20dB

Critical frequencies: zero@ -1 and pole @ -10





Magnitude Plot Phase Plot

The dotted line is a more accurate representation.

2. H(s) = $10\frac{(s-1)}{(s-3)(s-10)}$ |H(0)| = |10*(-1)/(-3)(-10) |= |-1/3| = 1/3 => -10dB

Note that the angle of (-1/3 real value) is 180^o

critical frequencies: zero @ 1, pole@3 and 10