Chapter 3
DC Circuits

eThinking in Circuits with PSpice

Mechanical Engineering
National Central University

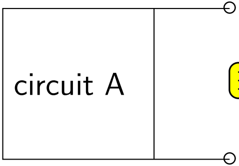
March 12, 2017

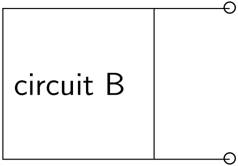
Outlines

1. Circuit Theorems
   - Equivalence
   - Source transformations
   - Circuit reductions
   - Thevenin/Norton Equivalents
2. Circuits Analysis
   - Loop-current/Mesh method
   - Node-voltage method
3. Superposition
   - Independent sources
   - Dependent sources

Equivalence

Two electrical circuits are equivalent if they have the same $v - i$ characteristics at the external terminals,
$\forall R$, including $R = 0$ and $R = \infty$.





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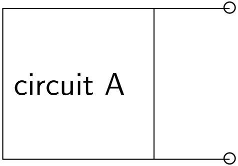

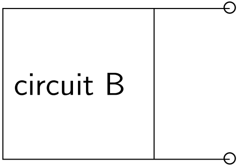

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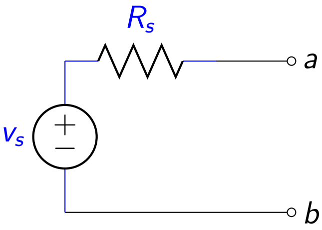
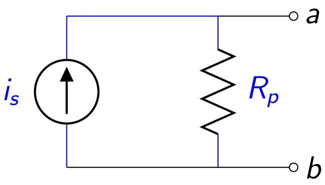
Simply put it,
Telling no difference seen from v-i, at the output terminal.

Thevenin & Norton





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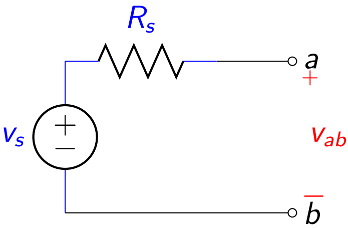
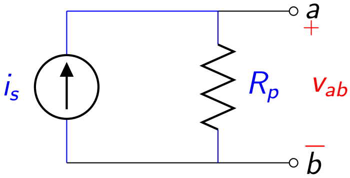
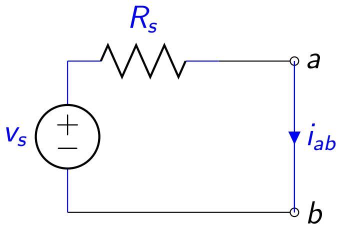
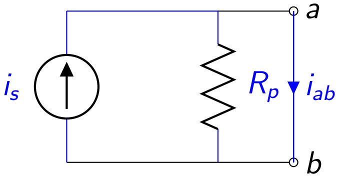




Open circuit $(R = \infty)$: $v_{ab} = v_s$ Open circuit : $v_{ab} = i_s R_p$

Thevenin & Norton





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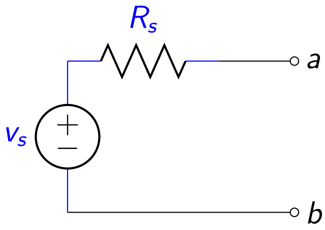

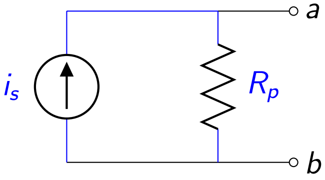

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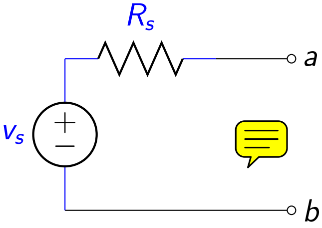

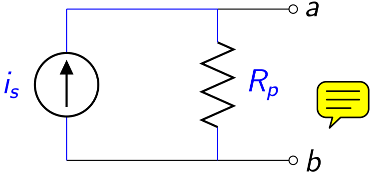

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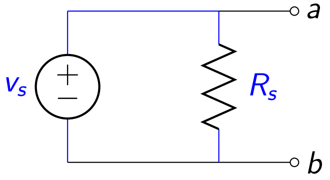
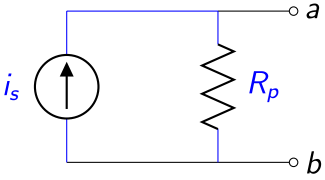
Can you now transform between these two sources?

Thevenin & Norton





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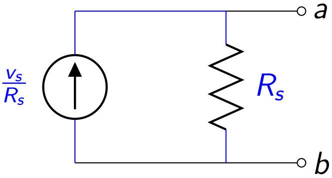

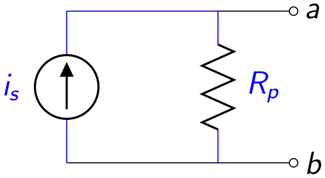

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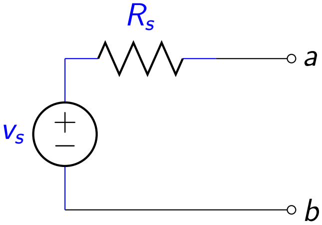

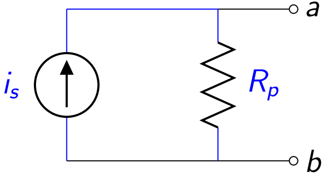

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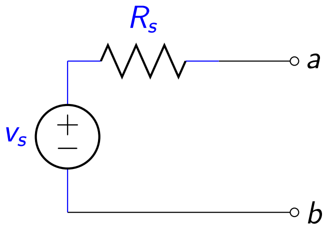

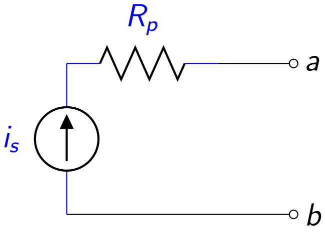

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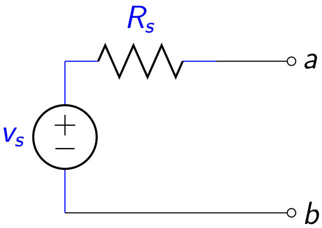

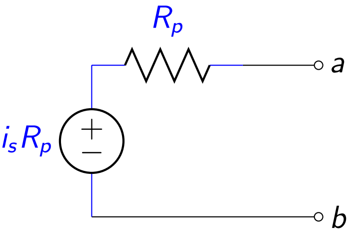

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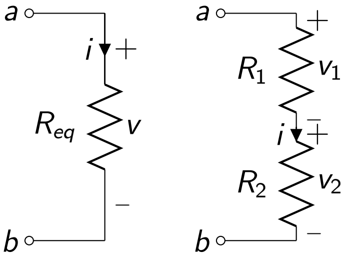
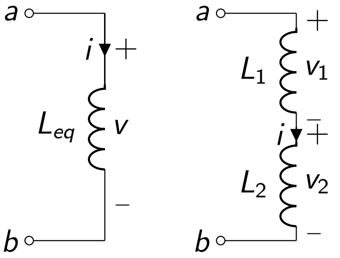
Series



By equivalence

$$R_{eq} = R_1 + R_2$$

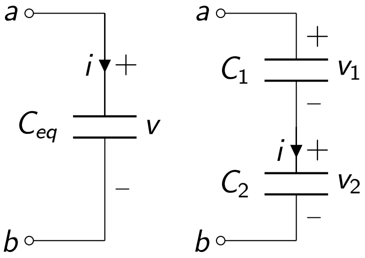
Series



By equivalence

$$L_{eq} = L_1 + L_2$$

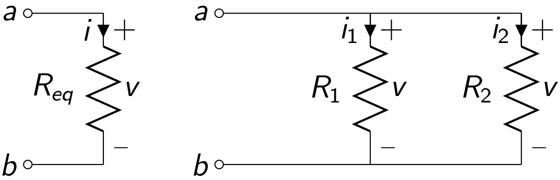
Series



By equivalence

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel



By equivalence

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

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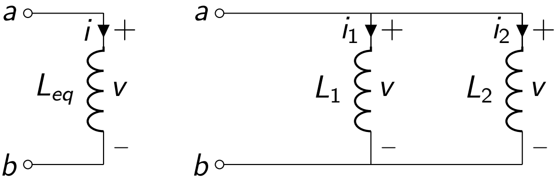

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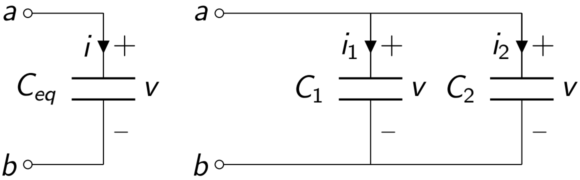

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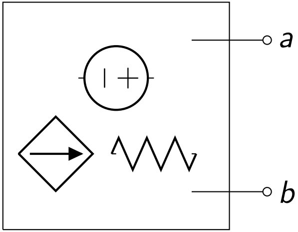
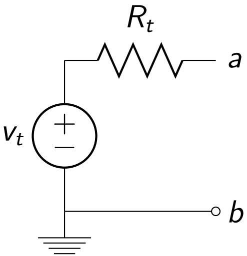
Thevenin equivalent





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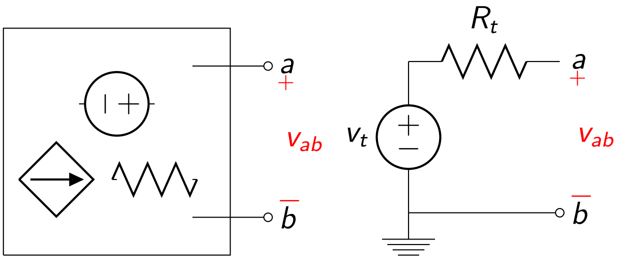
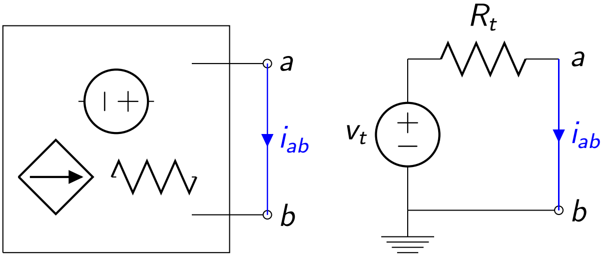


Open circuit $(R = \infty)$: $v_{ab} = v_t = v_{oc}$

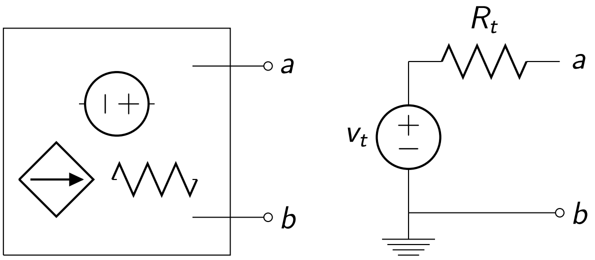
Thevenin equivalent



$$\text{Open circuit } (R=\infty) \quad : \quad v_{ab} = v_t = v_{oc}$$

$$\text{Short circuit } (R=0) \quad : \quad i_{ab} = \frac{v_t}{R_t} = i_{sc}$$

Thevenin equivalent



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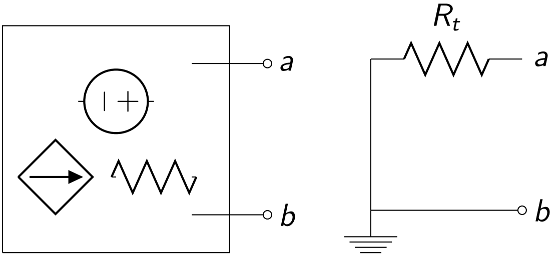
We need to solve the circuit on the left or use measurement.

Thevenin equivalent



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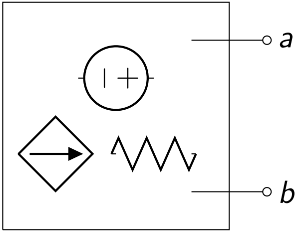
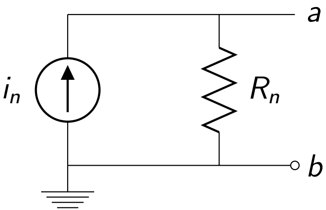
We need to solve the circuit on the left or use measurement.
A quick way to find R_t is setting v_t to zero.

Norton equivalent





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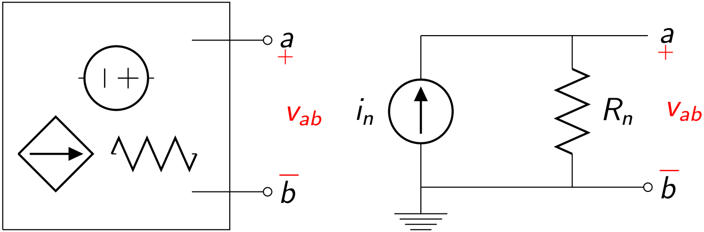

Open circuit $(R = \infty)$: $v_{ab} = i_n R_n = i_s R_n = v_{oc}$

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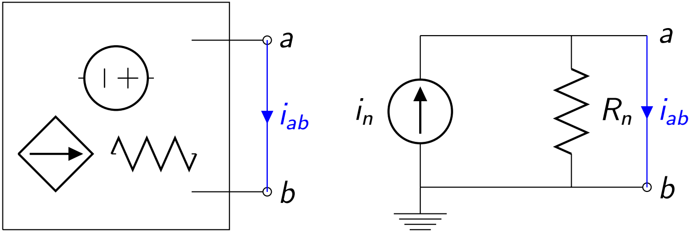

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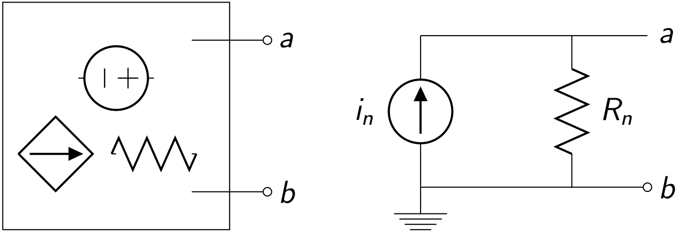
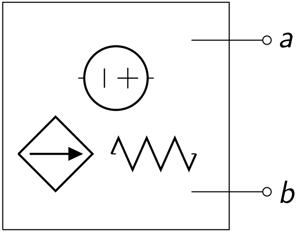


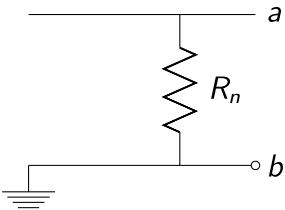
Open circuit $(R = \infty)$ $v_{ab} = i_n R_n = i_s R_n = v_{oc}$

Short circuit $(R = 0)$ $i_{ab} = i_n = i_{sc}$

We need to solve the circuit on the left or use measurement

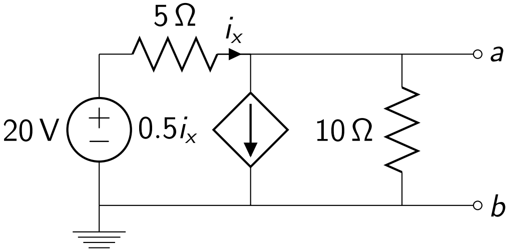
Norton equivalent





$$\begin{aligned}\text{Open circuit } (R=\infty) \quad &: \quad {\color{blue} v_{ab} = i_n R_n = i_s R_n = v_{oc}} \\ \text{Short circuit } (R=0) \quad &: \quad {\color{red} i_{ab} = i_n = i_{sc}}\end{aligned}$$

We need to solve the circuit on the left or use measurement
A quick way to find R_t is setting i_n to zero.



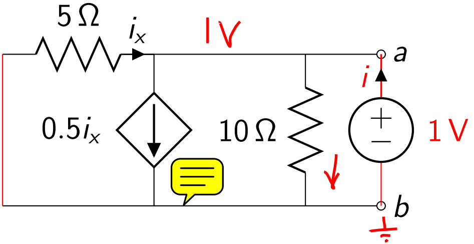
(a) $i = 0.5i + \frac{1}{10}$, $\quad i = 0.2A$, $\quad R_t = \frac{1}{0.2} = 5\Omega$.

(b) Open circuit

$$\frac{20 - V_a}{5} = 0.5i_x + \frac{V_a}{10}$$
$$\frac{V_a}{10} = i_x - 0.5i_x.$$

Solving, we have $i_x = 2A$. Thus $V_a = 10V = V_{oc}$.

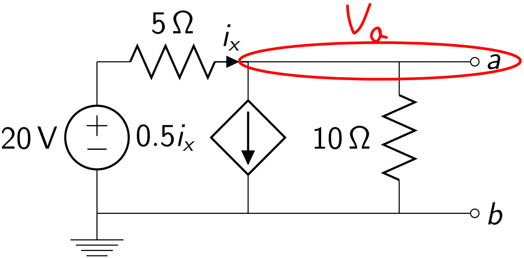
(c) Short Circuit: $i_x = \frac{20}{5} = 4A$, $\quad i_{sc} = 2A$.



(a) $i_x = 0.5 i_x + \frac{1}{10}$, $\quad i_x = 0.2A$, $\quad R_t = \frac{1}{0.2} = 5\Omega$.
(b) Open circuit

$$\frac{20 - V_a}{5} = 0.5 i_x + \frac{V_a}{10}$$
$$\frac{V_a}{10} = i_x - 0.5 i_x.$$

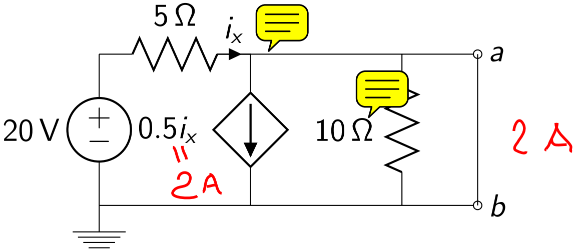
Solving, we have $i_x = 2A$. Thus $V_a = 10V = V_{oc}$.
(c) Short Circuit: $i_x = \frac{20}{5} = 4A$, $\quad i_{sc} = 2A$.



(a) $i = 0.5i + \frac{1}{10}, \quad i = 0.2A, \quad R_t = \frac{1}{0.2} = 5\Omega.$
(b) Open circuit

$$\frac{20 - V_a}{5} = 0.5 i_x + \frac{V_a}{10}$$
$$\frac{V_a}{10} = i_x - 0.5 i_x.$$

Solving, we have $i_x = 2A$. Thus $V_a = 10V = V_{oc}$.
(c) Short Circuit: $i_x = \frac{20}{5} = 4A, \quad i_{sc} = 2A.$



(a) $i = 0.5i + \frac{1}{10}, \quad i = 0.2A, \quad R_t = \frac{1}{0.2} = 5\Omega.$

(b) Open circuit

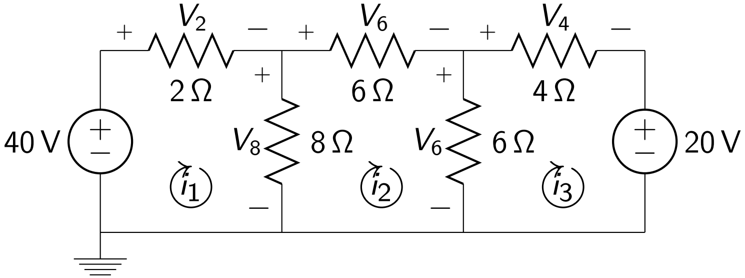
$$\begin{aligned} \frac{20 - V_a}{5} &= 0.5i_x + \frac{V_a}{10} \\ \frac{V_a}{10} &= i_x - 0.5i_x. \end{aligned}$$

Solving, we have $i_x = 2A$. Thus $V_a = 10V = V_{oc}$.

(c) Short Circuit: $i_x = \frac{20}{5} = 4A, \quad i_{sc} = 2A.$

1. Loop-current method: Adding voltage
   - Only voltage sources
   - Involve current sources
2. Node-voltage method: Adding current
   - Only current sources
   - Involve voltage sources

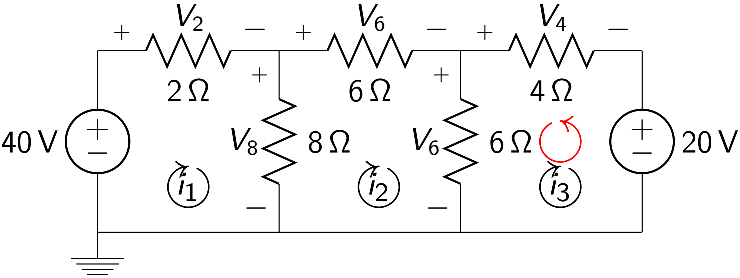
Example 3.8



$$-40 + 2i_1 + 8(i_1 - i_2) = 0$$
$$-[-8(i_2 - i_1)] + 6i_2 + 6(i_2 - i_3) = 0$$

$$-[-6(i_3 - i_2)] + 4i_3 + 20 = 0$$

Example 3.8



$$-40 + 2i_1 + 8(i_1 - i_2) = 0$$

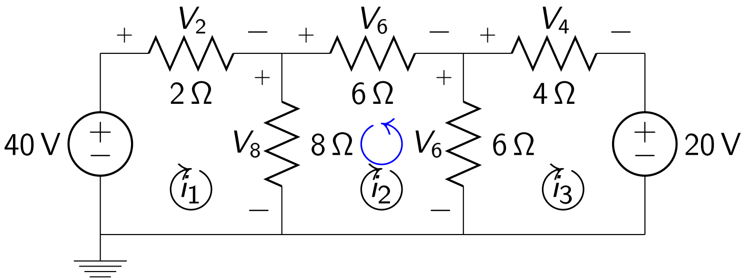
$$-[-8(i_2 - i_1)] + 6i_2 + 6(i_2 - i_3) = 0$$

$$-[-8(i_2 - i_1)] + 6i_2 + 6(i_2 + i_3) = 0$$

$$-[-6(i_3 - i_2)] + 4i_3 + 20 = 0$$

$$6(i_3 + i_2) - [-4i_3] - 20 = 0$$

Example 3.8

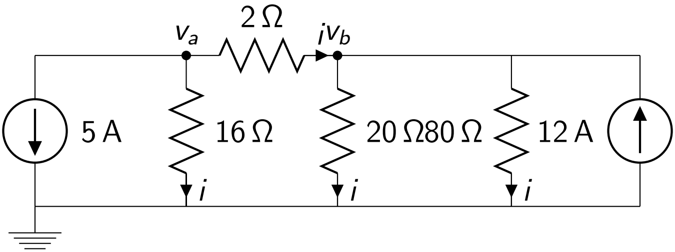


$$-40 + 2i_1 + 8(i_1 - i_2) = 0$$
$$-[-8(i_2 - i_1)] + 6i_2 + 6(i_2 - i_3) = 0$$

$$8(i_2 + i_1) - [-6i_2] - [-6(i_2 + i_3)] = 0$$
$$-[-6(i_3 - i_2)] + 4i_3 + 20 = 0$$

$$-[-6(i_3 + i_2)] + 4i_3 + 20 = 0$$

Example 3.16



Assign leaving current to be positive

$$5 + \frac{v_a}{16} + \frac{v_a - v_b}{2} = 0$$

$$\frac{v_b}{20} + \frac{v_b}{80} - \frac{v_a - v_b}{2} - 12 = 0$$

Solving, we have $v_a = 48V$, $v_b = 64V$.

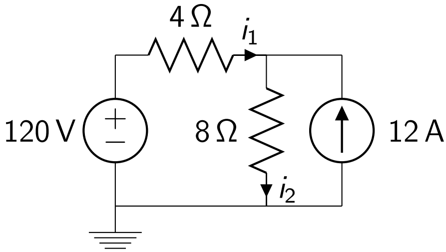
Linear system

$f(ax_1 + bx_2) = af(x_1) + bf(x_2), a, b \in R.$

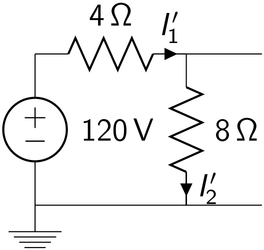
Total response=sum of individual responses

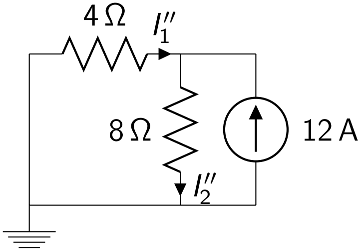
Independent voltage source $\rightarrow$ a short circuit

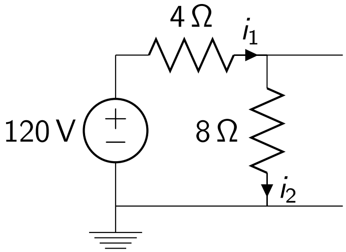
Independent current source $\rightarrow$ an open circuit



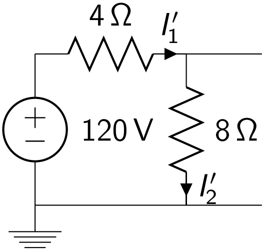
$i_1 = i_1' + i_1''$ $i_2 = i_2' + i_2''$

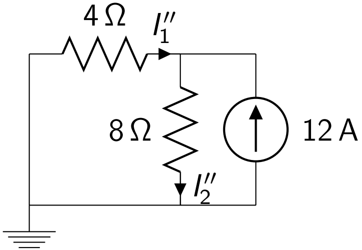


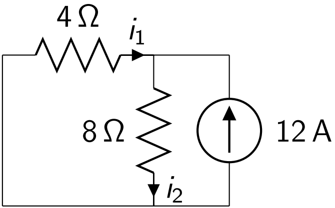




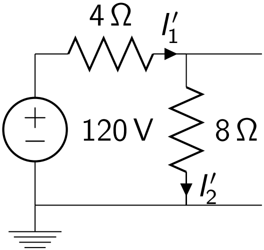
$i_1 = i_1' + i_1''$ $\quad i_2 = i_2' + i_2''$

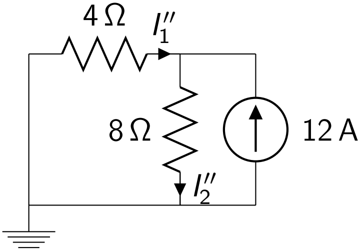


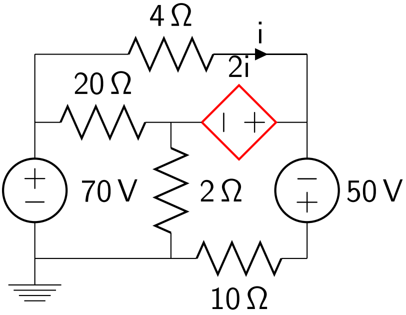




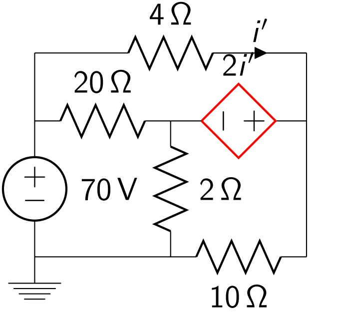
$i_1 = i_1' + i_1'' \quad i_2 = i_2' + i_2''$

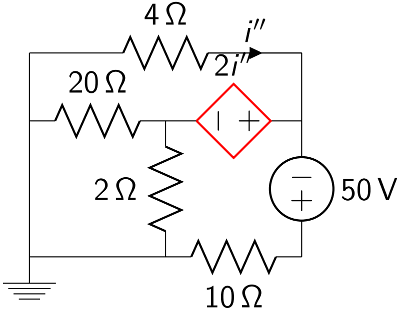


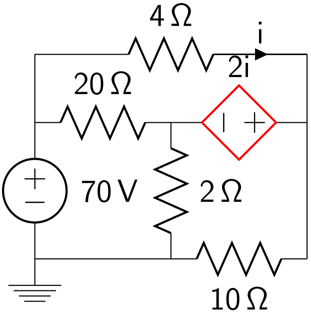




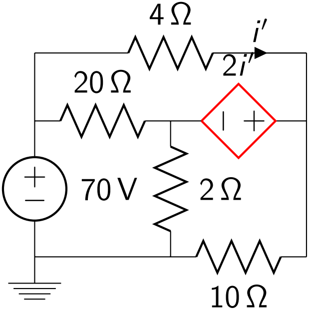
$$i_1 = i_1' + i_1'' \quad i_2 = i_2' + i_2''$$

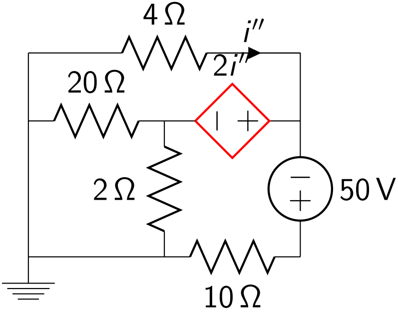


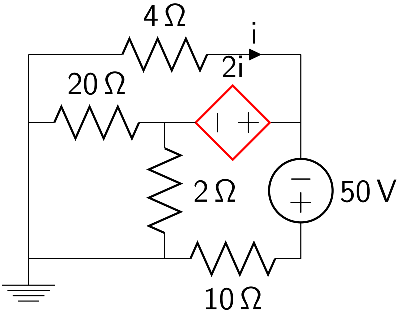




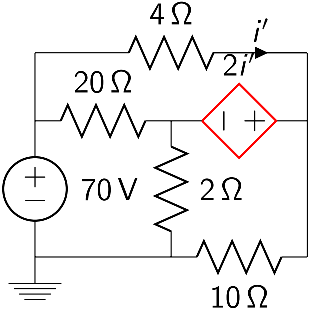
$$i_1 = i_1' + i_1'' \quad i_2 = i_2' + i_2''$$

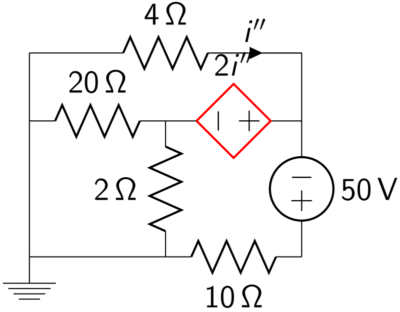


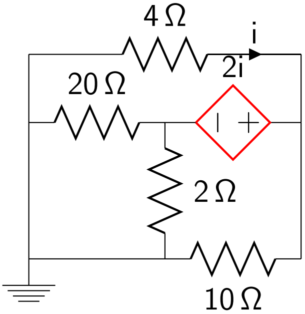




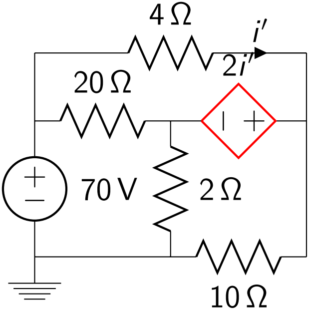
$$i_1 = i_1' + i_1'' \quad i_2 = i_2' + i_2''$$

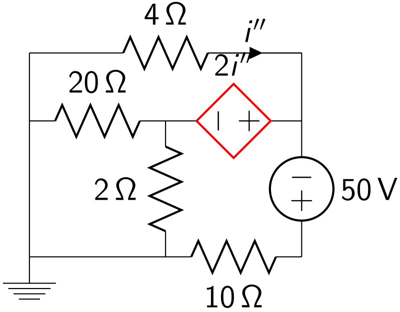


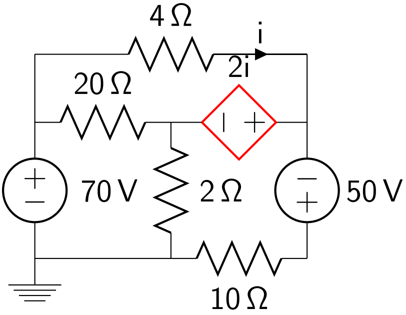




$$i_1 = i_1' + i_1'' \quad i_2 = i_2' + i_2''$$







No need to disable the dependent source.

$i_1 = i'_1 + i''_1 \quad i_2 = i'_2 + i''_2$