



Chapter 2

Circuit Laws

eThinking in Circuits with PSpice

Mechanical Engineering
National Central University

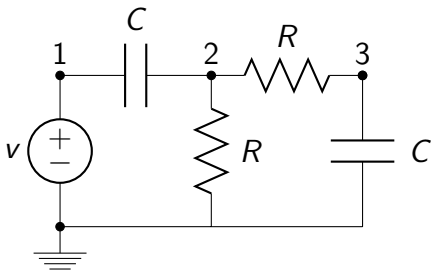
February 28, 2013–March 10, 2020

Outlines

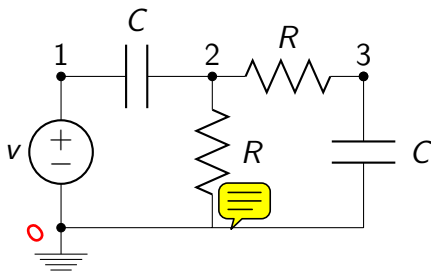
1 Definitions

2 Circuit Laws

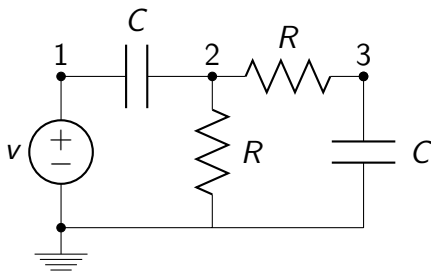
- Kirchhoff's **voltage** law
- Kirchhoff's **current** law
- Examples
- Voltage divider
- Current divider



❶ An electrical circuit — elements connected by conductors.

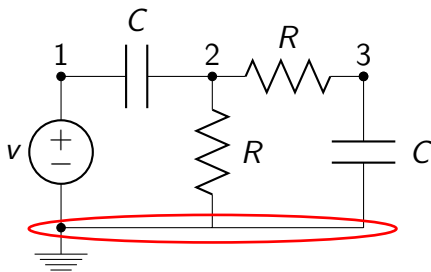


- 1 An electrical circuit — elements connected by conductors.
- 2 A node — a point at which elements are joined together.

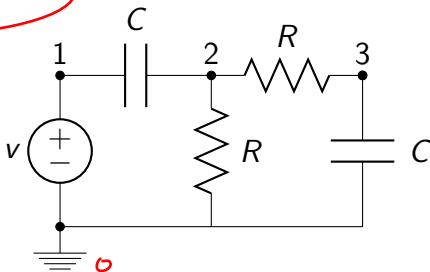



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- ② A node — a point at which elements are jointed together.
- ③ A loop — a closed path.



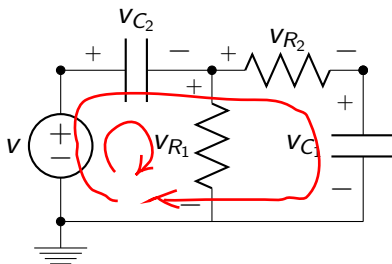


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- ① An electrical circuit — elements connected by conductors.
- ② A node — a point at which elements are jointed together.
- ③ A loop — a closed path.
- ④ A branch — a conductor. 
- ⑤ A branch without any element — a node.

KVL



$$\sum V_i \text{ (up)} = 0, \quad \sum V_i \text{ (down)} = 0$$

$$-v + v_{C_2} + v_{R_1} = 0$$

$$-v_{R_1} + v_{R_2} + v_{C_1} = 0$$

$$-v + v_{C_2} + v_{R_2} + v_{C_1} = 0$$

In a specified direction,

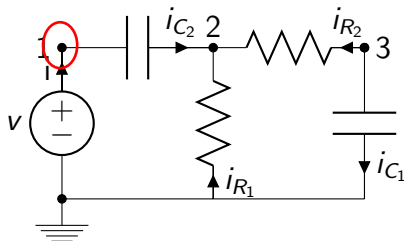
$$\sum V_i \uparrow = \sum V_i \downarrow$$

$$v = v_{C_2} + v_{R_1}$$

$$v_{R_1} = v_{R_2} + v_{C_1}$$

$$v = v_{C_2} + v_{R_2} + v_{C_1}$$

KCL



At a node, $\sum I_i = 0$

1 $i - i_{C_2} = 0$

2 $i_{C_2} + i_{R_1} + i_{R_2} = 0$

3 $-i_{R_2} - i_{C_1} = 0$

0 $-i - i_{R_1} + i_{C_1} = 0$



At a node, $\sum I_i \rightarrow = \sum I_i \rightarrow$

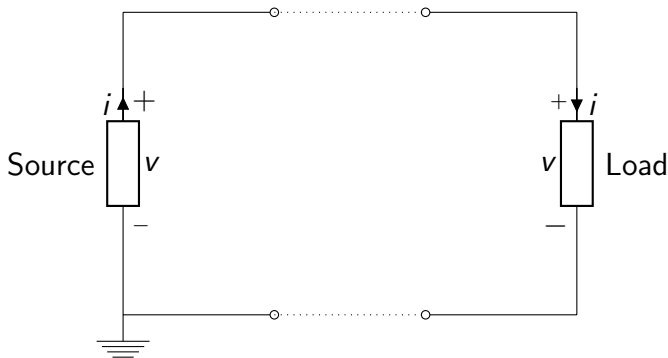
$i = i_{C_2}$

$i_{C_2} + i_{R_1} + i_{R_2} = 0$

$+i_{R_2} + i_{C_1} = 0$

$i + i_{R_1} = i_{C_1}$

Overall Closed-loop Circuit



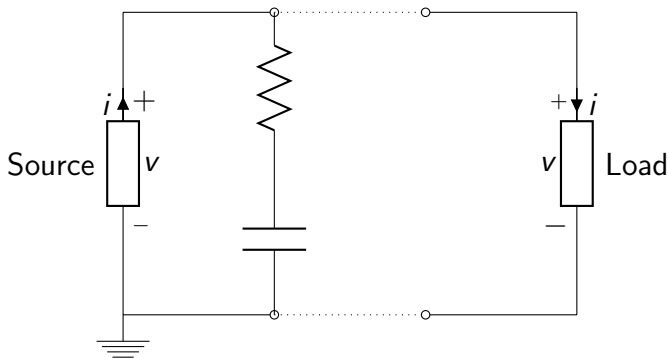
Sources: provide, deliver, send, generate

Load: absorb, consume, receive, draw

Current has direction.

Voltage has polarities.

Overall Closed-loop Circuit



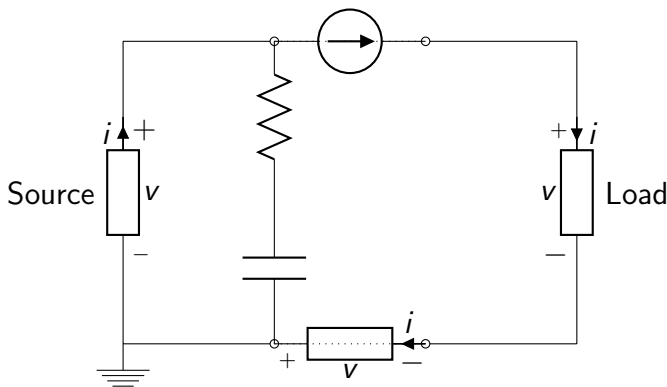
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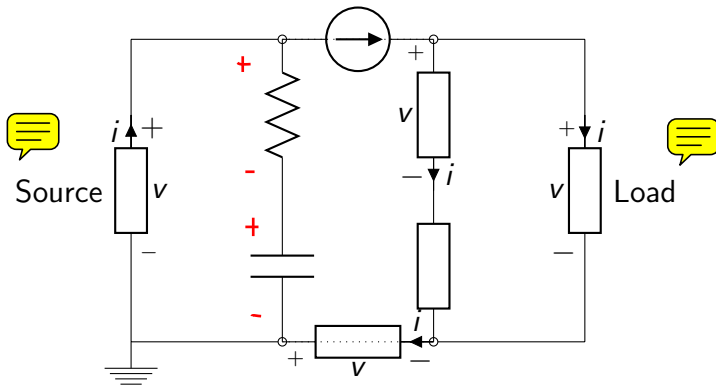
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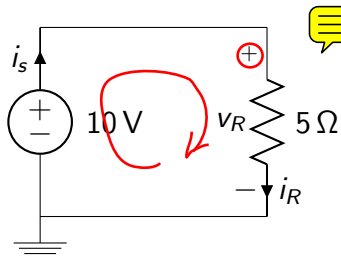
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Example 2.1, Arbitrary direction

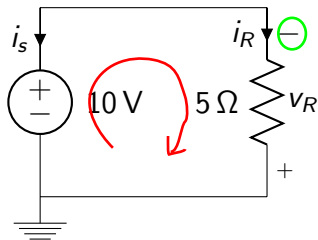


$$-10 + 5i_R = 0, \quad i_R = 2A$$

$$i_s = i_R$$

$$p_s = v_s i_s = 10 \times 2 = 20W, \text{ delivering.}$$

$$p_R = v_R i_R = 10 \times 2 = 20W, \text{ absorbing.}$$



$$-10 - v_R = 0, \quad v_R = -10V.$$

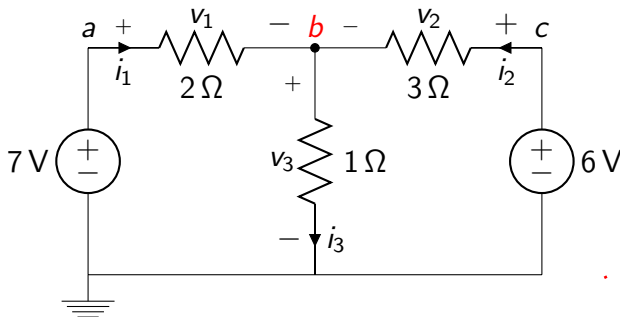
$$-10 - (-5i_R) = 0, \quad i_R = 2A.$$

$$i_s + i_R = 0, \quad i_s = -2A.$$

$$p_s = v_s i_s = 10 \times (-2) = -20W, \text{ absorbing.}$$

$$p_R = v_R i_R = -10 \times 2 = -20W, \text{ delivering.}$$

Example 2.5: Mix of KCL and KVL, Passive convention



Solution: KCL at node b is

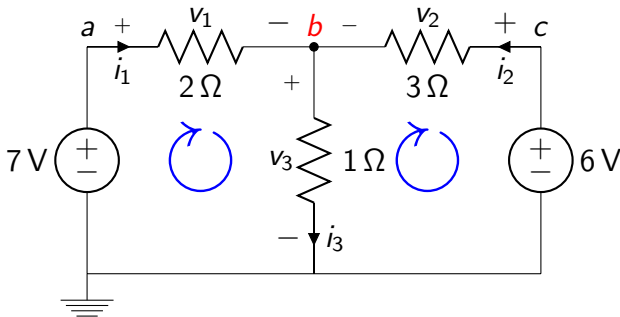
$$i_1 + i_2 - i_3 = 0 \quad \text{adding currents}$$

KVLs for the two clockwise loops yield

$$-7 + 2i_1 + i_3 = 0 \quad \text{adding voltages}$$

$$-i_3 - 3i_2 + 6 = 0 \quad \text{adding voltages}$$

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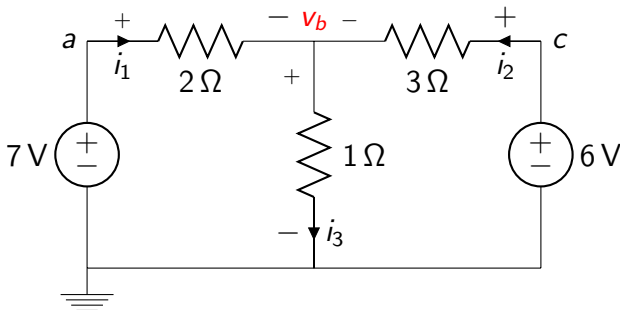
$$-7 + 2i_1 + i_3 = 0 \quad \text{adding voltages}$$

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Example 2.6: Node-Voltage Method, Passive convention

Label each node with a voltage variable v_a, v_b, v_c .



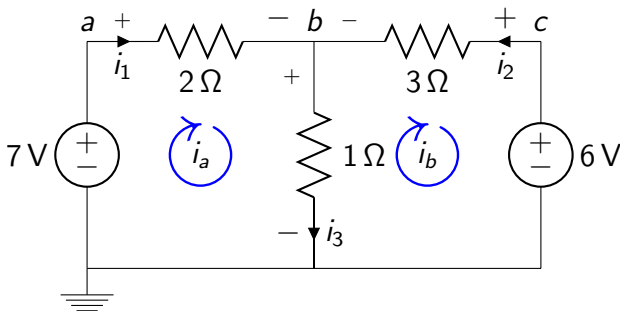
Solution: *KCL* at node *b* leads to

$$i_1 + i_2 - i_3 = \frac{7 - v_b}{2} + \frac{6 - v_b}{3} - \frac{v_b}{1} = 0$$

Solving, we have $v_b = 3\text{V}$, $i_1 = 2\text{A}$, $i_2 = 1\text{A}$, $i_3 = 3\text{A}$.

Example 2.7: Loop-Current Method, Passive convention

Label each loop with a current variable i_a, i_b, i_c .



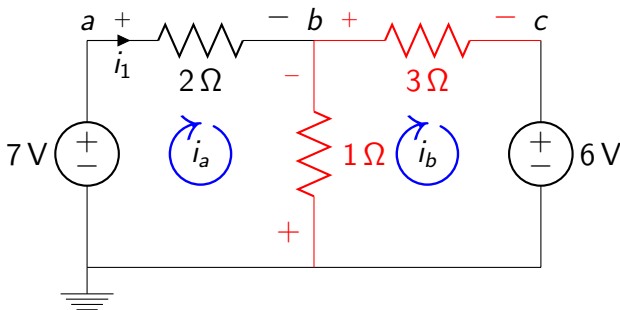
Solution: write *KVL* for loop *oabo*

$$-7 + 2i_a + 1(i_a - i_b) = 0$$

$$\begin{aligned} 0 &= -[-(i_b - i_a)] - [-3i_b] + 6 \\ &= (i_b - i_a) + 3i_b + 6 \end{aligned}$$

Example 2.7: Loop-Current Method, Passive convention

Label each loop with a current variable i_a, i_b, i_c .



Solution: write *KVL* for loop *oabo*

$$-7 + 2i_a + 1(i_a - i_b) = 0$$

$$\begin{aligned} 0 &= -[-(i_b - i_a)] - [-3i_b] + 6 \\ &= (i_b - i_a) + 3i_b + 6 \end{aligned}$$

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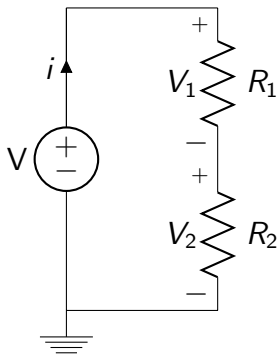
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- Simultaneous equations. 

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- Combined method: KVL+KCL.
- Simultaneous equations.
- Interpretation of decision variables — Use your logics.



$$I = \frac{V}{R_1 + R_2}$$

Ohm's law finds ...

$$V_2 = V\left(\frac{R_2}{R_1 + R_2}\right), \quad V_1 = V\left(\frac{R_1}{R_1 + R_2}\right)$$

Generalized, $V_{R_i} = V \left(\frac{R_i}{\sum_{i=1}^r R_i} \right)$

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V(G_1 + G_2)$$

Rearranging,

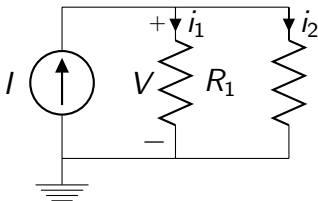
$$V = \frac{R_1 R_2}{R_1 + R_2} I = \frac{I}{G_1 + G_2}$$

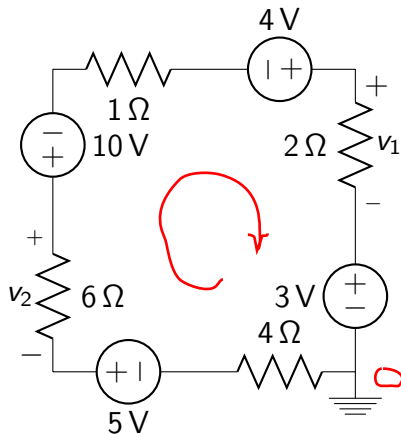
Ohm's law finds ...

$$I_1 = I \left(\frac{R_2}{R_1 + R_2} \right) = I \left(\frac{G_1}{G_1 + G_2} \right)$$

$$I_2 = I \left(\frac{R_1}{R_1 + R_2} \right) = I \left(\frac{G_2}{G_1 + G_2} \right)$$

Generalized, $I_{R_i} = I \left(\frac{G_i}{\sum_{i=1}^n G_i} \right)$





$$5 - 10 + 4 - 3 = -4$$

