Compositionality and Expressiveness (with Rely/Guarantee notation)

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Compositionality

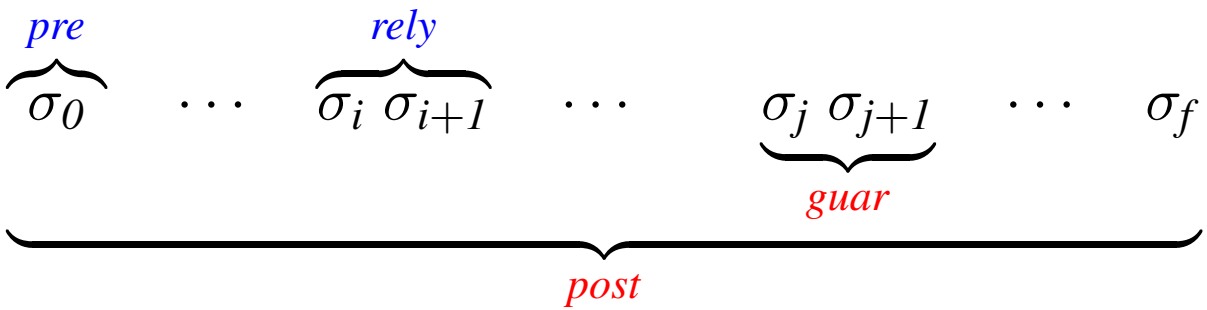
- when we write:
  $\{\textbf{true}\}SORT\{s\text{-}ordered(s') \wedge is\text{-}permutation(s', s)\}$
  we accept any (proven) implementation
- we have chosen what to express! (relation between states)
- post conditions take a position on what is observable
- notice (engineering):
  $\{\textbf{true}\}SIEVE\{s' = s - Composites\}$ is more useful than
  $\{s = \{1..n\}\}SIEVE\{s' = s - Composites\}$
  because the former is less reliant on the environment
  $INIT: s \leftarrow \{1..n\}$ being only one possibility
- compositionality is **crucial** for concurrency
- as with $SIEVE$ above, it's about separation
  - moot with symmetric processes (e.g. $SIEVE =||_i REM(i)$)
  - clearer with asymmetric processes: different R/Gs
  - e.g. $QREL$ (aka "union/find")

Document interference with Rely/Guarantee (R/G)

- using R/G also takes a position on what can be expressed
- *rely* relations abstract the interference to be tolerated
- R/G is expressively weak
  - only relations
  - (same as for post conditions)
  - reflexive (for no interference); transitive (for multiple steps)
- maybe the weakness is their strength?
  - simple(r) POs
  - clear separation
- the other extreme
  - the code of the environment
  - makes it possible to reason about all interleavings
  - unattractive: non-compositional and brittle

Rely/Guarantee (R/G) idea is simple

. . . but requires adopting an R/G mindset!



- assumptions $pre/rely$
- commitments $guar/post$
- reduce to non-concurrent by rely: $x' = x$; guar: **true**

Prejudices

- post conditions should be relations [Jon80, Jon86]
  - well-founded loop relation vs. “variant function” arguments
- data abstraction/reification
  - is at least as important as control structures
  - this is reinforced in concurrency [Jon07, JY15]

Undermining compositionality

- minimal information about the (concurrent) environment
  - R/G facilitates this
  - clearest in examples of asymmetric concurrent processes
  - Owicki/Gries reasons about the final code!
- "ghost" variables dent/destroy compositionality
  - in the extreme, they can dictate the entire environment code
    e.g. $(PC = 1 \Rightarrow x = 5) \wedge (PC = 2 \Rightarrow x = 6) \wedge \cdots$
  - in many cases, such "auxiliary" variables can be avoided
  - if I have to use them, I want a test
    (cf. homomorphic rule vs. "biased" specifications)

Possible values

- in the presence of interference
  $\{P\}x \leftarrow y\{x' = y \vee x' = y'\}$
  is not enough — write:
  $\{P\}x \leftarrow y\{x' \in \widehat{y}\}$
- need identified in [JP11]
  - development of Simpson's "4-slot" algorithm for ACMs
  - (asymmetric $WRITE \mid\mid READ$)
- [JH16] has a beautifully clear specification of ACMs
- posvals extends expressive power
- . . . avoids some apparent "needs" for ghost variables
- have to decide if this limits implementations too much!

Example: GC

it is the underlying points, not the example, that matter

- concurrent Garbage Collection: Ben Ari [BA84]
- an “Owicki/Gries” proof by Jan van de Snepscheut [vdS87]
- illustrates:
  - asymmetric processes (and therefore R/G): $Collector \,||\, Mutator$
  - three $Mutator$ options: $Redirect/Malloc/Zap$
- highlights a limitation(!) of R/G
  - let’s us explore options

(Concurrent) garbage collection: abstract specification

Σ_0 :: $busy$: $Addr$-**set**
$free$: $Addr$-**set**

where

$inv\text{-}\Sigma_0(mk\text{-}\Sigma_0(busy, free)) \triangleq busy \cap free = \{\}$

GC: $Collector^*$

$Collector$
wr $free$
rd $busy$
pre true
...
post $(Addr - busy) \subseteq \bigcup \overset{\frown}{free}$ lower bound for GC

data reification gets us to:

$$\Sigma_1 :: roots: Addr\text{-}\mathbf{set}$$
$$hp: Heap$$
$$free: Addr\text{-}\mathbf{set}$$

where

$$inv\text{-}\Sigma_1(mk\text{-}\Sigma_1(roots, hp, free)) \triangleq$$
$$\mathbf{dom}\, hp = Addr \wedge$$
$$free \cap reach(roots, hp) = \{\}$$

upper bound for GC

$$Heap = Addr \xrightarrow{m} Node$$

$$Node = Addr^*$$

Simplification: here **nil** is ignored

Mark any *Addr* reachable from *roots*

$Collector \triangleq (Unmark; Mark; Sweep)$

$\Sigma_2 :: roots: Addr\text{-}\mathbf{set}$
$hp: Heap$
$free: Addr\text{-}\mathbf{set}$
$marked: Addr\text{-}\mathbf{set}$

where

$inv\text{-}\Sigma_2(mk\text{-}\Sigma_2(roots, hp, free, marked)) \triangleq$
$\mathbf{dom}\, hp = Addr \wedge$
$free \cap reach(roots, hp) = \{\} \wedge$
$(roots \cup free) \subseteq marked$

Stage 1: non-interfering case

(transition from sequential to interfering shows an R/G "pattern")

$Mark$

wr $marked$

rd $hp, roots, free$

pre true

rely $marked' = marked \wedge free' = free \wedge hp' = hp$

guar true

post $marked' = (free \cup reach(roots, hp))$

$Mark \triangleq$

repeat

$mc \leftarrow$ **card** $marked$;

$Propagate$

until card $marked = mc$

$Propagate$:

$consid \leftarrow \{\}$;

do while $consid \neq Addr$

let $x \in (Addr - consid)$ **in**

if $x \in marked$ **then** $Mark\text{-}kids(x)$ **else skip**;

$consid \leftarrow consid \cup \{x\}$

od

proofs of (sequential) loops are straightforward

Stage 2: accept concurrent changes to *heap*

have to enlist *Mutator* to help with marking process!

Unmark
wr *marked*
rd *free*
pre true
rely $marked \subseteq marked' \wedge free' \subseteq free$
guar $marked' \subseteq marked$
post $\forall a \in (Addr - (roots \cup free)) \cdot \exists m \in \overbrace{marked} \cdot a \notin m$

simplification: upper bound of marking (lower of GC) ignored for now

Mark: (simplified)

Pretend $Mutator(Redirect)$ can make change and mark atomically

$$< hp(a), marked \leftarrow hp(a) \dagger \{i \mapsto b\}, marked \cup \{b\} >$$

Mark
wr $marked$
rd $hp, roots, free$
pre true
rely $marked \subseteq marked' \wedge$
$\forall (a,i) \in \mathbf{dom}\, hp \cdot hp'(a,i) \neq hp(a,i) \Rightarrow hp'(a,i) \in marked'$
guar $marked \subseteq marked'$
post $reach(roots, hp') \subseteq marked'$

slightly more complicated but previous proof *strategy* works
(with exactly the same code — just different R/G)

Stage 3: concurrent case

. . . the *Mutator* cannot redirect/mark atomicaly

- I have worked on "fiction of atomicity/atomicity refinement" but I don't think that's the right approach here
- the essence of the correctness is a "three state" argument:
  Mutator can change $hp(a, i))$ to point to b
  but then go to sleep
  if the $Collector/Proagate$ has already passed a, its (obvious) post condition fails
  but we will get to another pointer to b
  unless the *Mutator* destroys that link

  but in that case, the *Mutator* must have marked b to get to that point
- such a three state argument is not expressible as relations

Concurrent case: alternatives

- reject the restrictions of R/G and use Temporal Logic
  - I worry that RGITL is too expressive!
  - slippery slope to arguing about the combined code
- concede a ghost variable!

$$< hp(a), tbm \leftarrow hp(a) \dagger \{i \mapsto b\}, b >;$$
$$< marked, tbm \leftarrow marked \cup \{tbm\}, \textbf{nil} >$$

but we would have a test!

- either of these alternatives damage compositionality!
- I think that one could specify that the *Mutator* preserves *can-be-completed*
- destroys compositionality! the designer of *Mutator* . . .

(My) current preferred alternative

Consider:

$mr\text{-}1: < hp(a) \leftarrow hp(a) \dagger \{i \mapsto b\} >;$
$mr\text{-}2: < marked \leftarrow marked \cup \{b\} >$

With:
$Collector \mid\mid (< mr\text{-}1 >; < mr\text{-}2 >)^*$
the $Collector$ has to tolerate interference in the following cases:
$(< mr\text{-}1 >; < mr\text{-}2 >)^*$ (which is covered above)
$(< mr\text{-}1 >; < mr\text{-}2 >)^*; < mr\text{-}1 > (guar\text{-}mr\text{-}2 \Leftrightarrow hp' = hp)$
$(< mr\text{-}2 >)$ (OK here this is like the counter-example if the $Collector$ marked first)

Clearly this also limits compositionality — but . . .
Comment (most) welcome

Conclusions

- the layered R/Gs are rather nice!
- compositionality = good engineering
- compositionality wrt a notation
- R/G can't specify GC fully compositionally
  - the 3-state argument looks like a clear test
- the "jury is still out" on the least damaging solution
- actually, I think all candidate workarounds tell us something
- (for me) examples are essential!

Background R/G literature

- [Jon81, Jon83a, Jon83b] used keywords (VDM style)
- elsewhere presented as $\{P, R\}\ S\ \{G, Q\}$
- . . . especially for the proof rules
- useful intro [Jon96]
- many uses — including Bornat's 4-slot [BA10, BA11]
- more recently, with Ian Hayes [HJC14, JHC15]:
  - R/G completely recast in refinement calculus style
  - Ian's newer semantics gets more out of algebraic properties
- links to Separation Logic [JY15]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