BRADLEY UNIVERSITY
Electrical and Computer Engineering
ECE443/543 — March 7, 2024

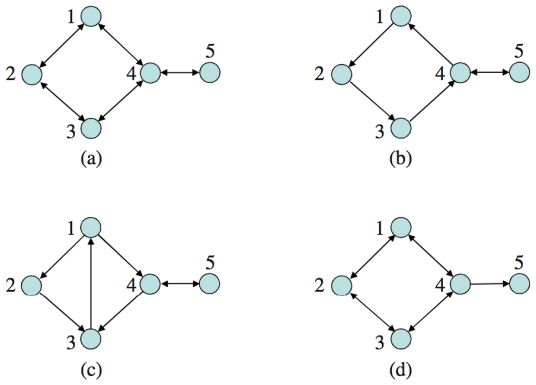
Test 1

Instructions

1. Put on your name first.
2. Write only on the front side of the paper.
3. Assemble your work for each problem in logical order.
4. Justify all your conclusions.
5. You must work on it independently. Open-book and open-notes.

Full Name: _____

Problem 1. (10 points) Consider the following different types of graphs of five agents.



1) Identify the undirected graph, the balanced graph, the weakly connected graph, and the strongly connected graph.

2) For graphs (a) and (c), find the corresponding adjacency matrix $\mathcal{A}$, degree matrix $\mathcal{D}$, and Laplacian matrix $\mathcal{L}$. (Hint: Assume A is a binary matrix, that is, $a_{ij} = 1$ if there is a link.)

3) Is there a spanning tree in graph (c)? If so, point it out.

Problem 2. (10 points) Consider the following graph. Find the Laplacian matrix L (Assume the adjacency matrix is a binary matrix). Draw a picture to show how its eigenvalues are localized in the complex plane.

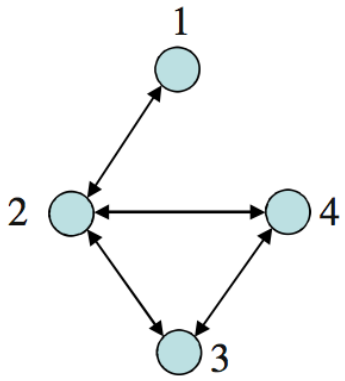


Figure 1: A sensing/communication graph of four agents

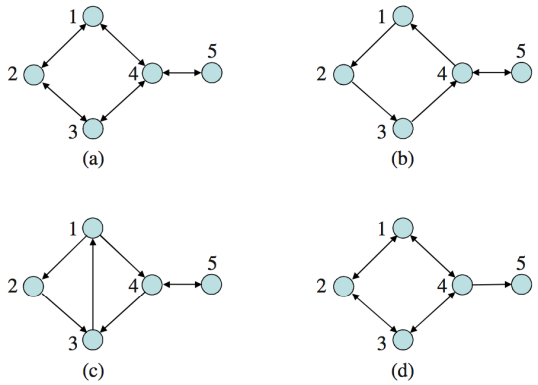
Problem 3. (10 points) Consider the cooperative control of four agents

$$\dot{x}_i = u_i, \quad i = 1, 2, 3, 4, 5 \tag{1}$$

where $x_i \in \Re$, and $u_i \in \Re$. Initial conditions are $x_1(0) = 3; x_2(0) = 2; x_3(0) = 6; x_4(0) = -2, x_5(0) = -5$. Let the cooperative control for agent i be

$$u_i = \sum_{j=1}^{5} a_{ij}(x_j - x_i) \tag{2}$$

where a_{ij} are nonnegative constants depending on sensing/communication topologies. (You are free to choose the values for a_{ij} accordingly.)



- Consider four different sensing/communication topologies given in the above figure, respectively. For each of them, show whether under control (2), the system consensus will be reached? If so, what is the consensus value?
- Write a Matlab code to simulate the system under the communication topologies given in the above figure, respectively. For each sensing/communication topology, plot the system responses for all agents $x_i(t)$ into one figure.

Problem 4: (10 points) Given the following Laplacian matrix

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Write a Matlab program to use the distributed algorithm to estimate its left eigenvector corresponding to eigenvalue 0.