BRADLEY UNIVERSITY
Electrical and Computer Engineering
ECE443/543 — HW 2
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Problem 1: Code up gradient descent for a cost function with scalar input. Suppose that $g(w) = \log(1 + e^{w^2})$, whose first derivative is given as $\frac{dg(w)}{dw} = \frac{2e^{w^2}w}{1+e^{w^2}}$. Select the initial point $w(1) = 1$ and a step length fixed at $\alpha = 10^{-1}$. Plot the corresponding curves for $w(k)$ and $g(w(k))$.

Problem 2: Code up newton's method for problem 1. The second derivative is given as

$$\frac{d^2g(w)}{dw^2} = \frac{2e^{w^2}(2w^2 + e^{w^2} + 1)}{(1 + e^{w^2})^2}$$

$$w(k) = w(k-1) - \frac{\dot{g}(w(k-1))}{\ddot{g}(w(k-1))}$$

Plot $g(w(k))$. Discuss and compare the convergence with gradient descent algorithm.

Problem 3: Code up gradient descent for a cost function with vector input.

$$g(\mathbf{w}) = -\cos(2\pi\mathbf{w}^T\mathbf{w}) + 2\mathbf{w}^T\mathbf{w}$$

where $\mathbf{w}$ is a 2-d vector. The gradient is

$$\bigtriangledown g(\mathbf{w}) = 4\pi \sin(2\pi\mathbf{w}^T\mathbf{w})\mathbf{w} + 4\mathbf{w}$$

Let the starting point be $w(1) = [-0.7, 0]^T$ and $\alpha = 10^{-3}$. Plot the result.