



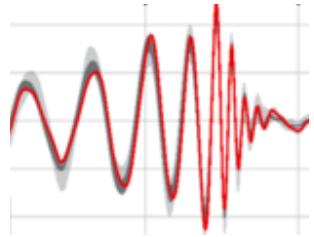
Rich Ormiston

MEASURING THE STOCHASTIC GRAVITATIONAL WAVE BACKGROUND WITH LIGO

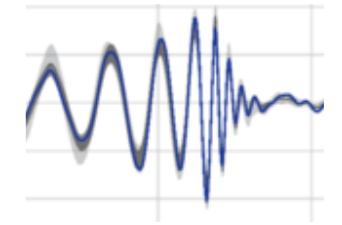
January 31, 2019

Overview

- Description of gravitational waves
- What is the stochastic gravitational wave background (SGWB) and why do we care?
- How can we test for the SGWB with LIGO and what are the current results?
- Can we enhance the sensitivity of the SGWB search through nonlinear data cleaning via machine learning?



Gravitational Waves



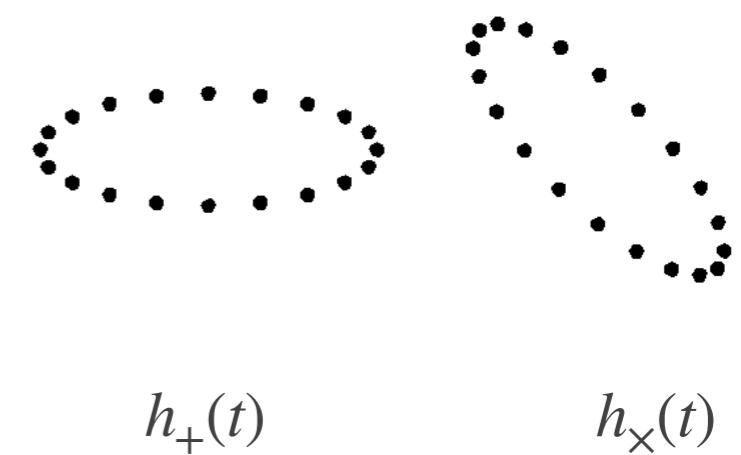
- Gravitational waves are first order perturbations to the spacetime metric.
- GWs were theorized by Poincare in 1905, formalized by Einstein in 1915, and only *indirectly* measured for the first time in 1974 ([Hulse-Taylor binary](#)).
- The first direct detection occurred on September 14, 2015 by the LIGO & Virgo Collaboration ([GW150914](#))

$$\frac{\Delta L}{L} = \frac{1}{2} h(t) = \frac{1}{2} (h_+ \hat{e}_{ab}^+ + h_\times \hat{e}_{ab}^\times) D^{ab}$$

Fractional change
in IFO arm length

GW Strain

Detector Tensor

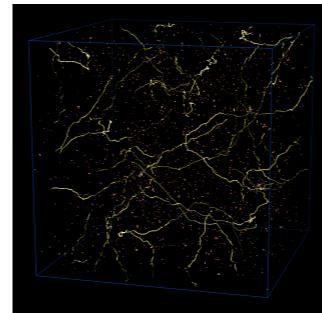


Effects of "plus" (left) and "cross" (right)
gravitational wave polarizations on
a ring of test masses

Sources of Gravitational Waves



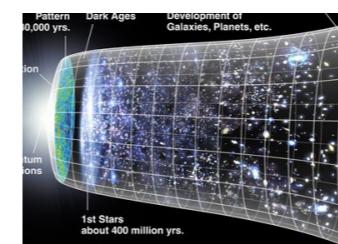
Binary Black Holes



Cosmic Strings



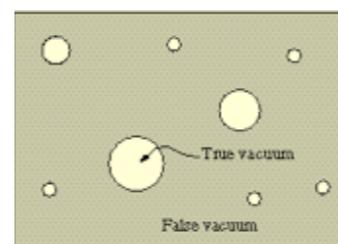
Binary Neutron Stars



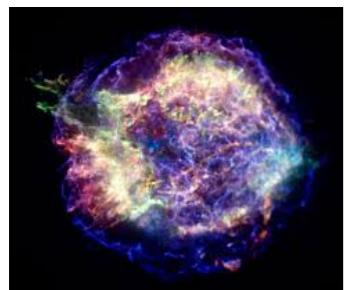
Inflation



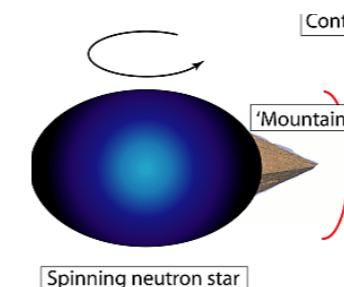
Black Hole +
Neutron Star



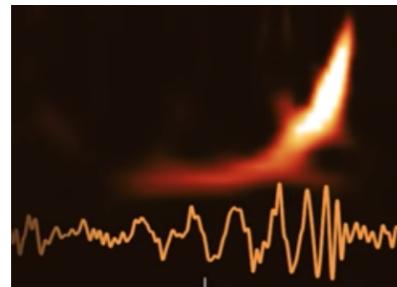
Phase Transitions



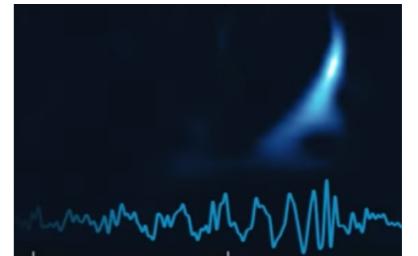
Supernovae



Non-symmetric
Neutron Stars



From GR to GW



We start by expanding Einstein's equation to first order in the metric. Assuming $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $|h_{\mu\nu}| \ll 1$, we have

$$\begin{aligned}\frac{8\pi G}{c^4} T_{\mu\nu} &= R_{\mu\nu}^{(1)} - \frac{1}{2} g_{\mu\nu} R^{(1)} \\ &= \frac{1}{2} \left(-\frac{\partial^2 h}{\partial x^\mu \partial x^\nu} + \frac{\partial^2 h_\nu^\alpha}{\partial x^\mu \partial x^\alpha} + \frac{\partial^2 h_\mu^\alpha}{\partial x^\nu \partial x^\alpha} - \eta^{\alpha\beta} \frac{\partial^2 h_{\mu\nu}}{\partial x^\alpha \partial x^\beta} \right) - \frac{1}{2} \left(\frac{\partial^2 h^{\mu\nu}}{\partial x^\mu \partial x^\nu} - \eta^{\mu\nu} \frac{\partial^2 h}{\partial x^\mu \partial x^\nu} \right)\end{aligned}$$

Adopting the transverse traceless gauge simplifies the expression greatly

$$-\frac{16\pi G}{c^4} T_{\mu\nu} = \square \bar{h}_{\mu\nu}$$

Gravitational Waves!

The Stochastic GW Background I

The SGWB is the superposition of many sources emitting gravitational waves which are too weak to be individually detected.

Normalized Energy Density from GWs $\longrightarrow \Omega_{GW}(f) = \frac{1}{\rho_{c,0}} \frac{d\rho_{GW}(f)}{d \log f} \longleftarrow$

\uparrow

Energy density from GW contained in interval $f \rightarrow f + \Delta f$

Energy Density Required to Close the Universe = $3H_0^2c^2/(8\pi G)$

$$\Omega_{GW}(f) = \frac{1}{\rho_{c,0}} \frac{d\rho_{GW}(f)}{d \log f}$$

The Stochastic GW Background I

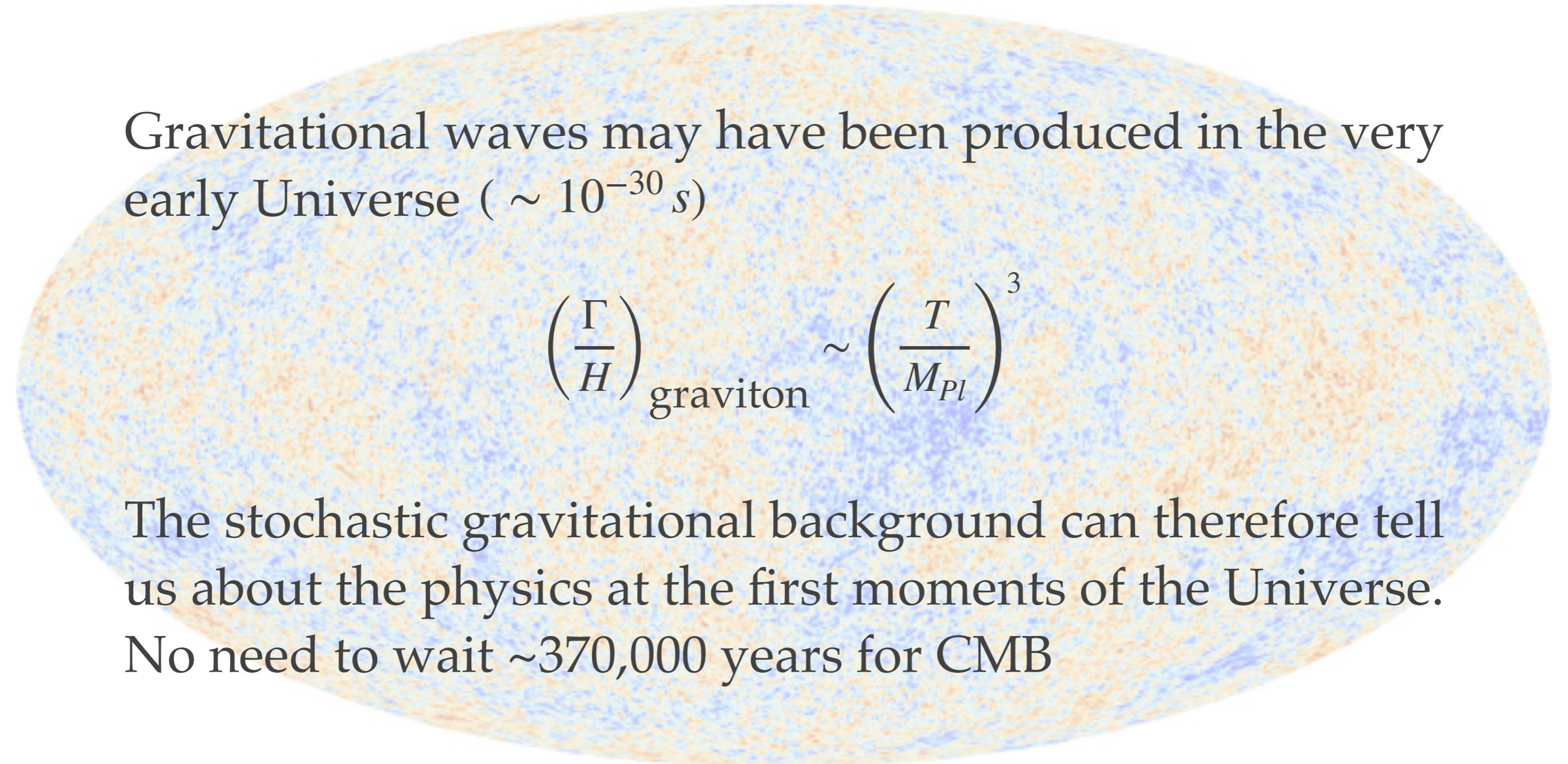
The SGWB is the superposition of many sources emitting gravitational waves which are too weak to be individually detected.

$$h_{100}^2 \Omega_{GW}(f) = \frac{h_{100}^2}{\rho_{c,0}} \frac{d\rho_{GW}(f)}{d \log f}$$

$$H_0 = h_{100} \times 100 \text{ km s}^{-1} \text{ Mpc}^{-1} = \mathbf{67.9} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Remove our ignorance of the Hubble constant from the analysis

The Stochastic GW Background II



Gravitational waves may have been produced in the very early Universe ($\sim 10^{-30} s$)

$$\left(\frac{\Gamma}{H}\right)_{\text{graviton}} \sim \left(\frac{T}{M_{Pl}}\right)^3$$

The stochastic gravitational background can therefore tell us about the physics at the first moments of the Universe. No need to wait $\sim 370,000$ years for CMB

The Stochastic GW Background III

Assumptions about SGWB:

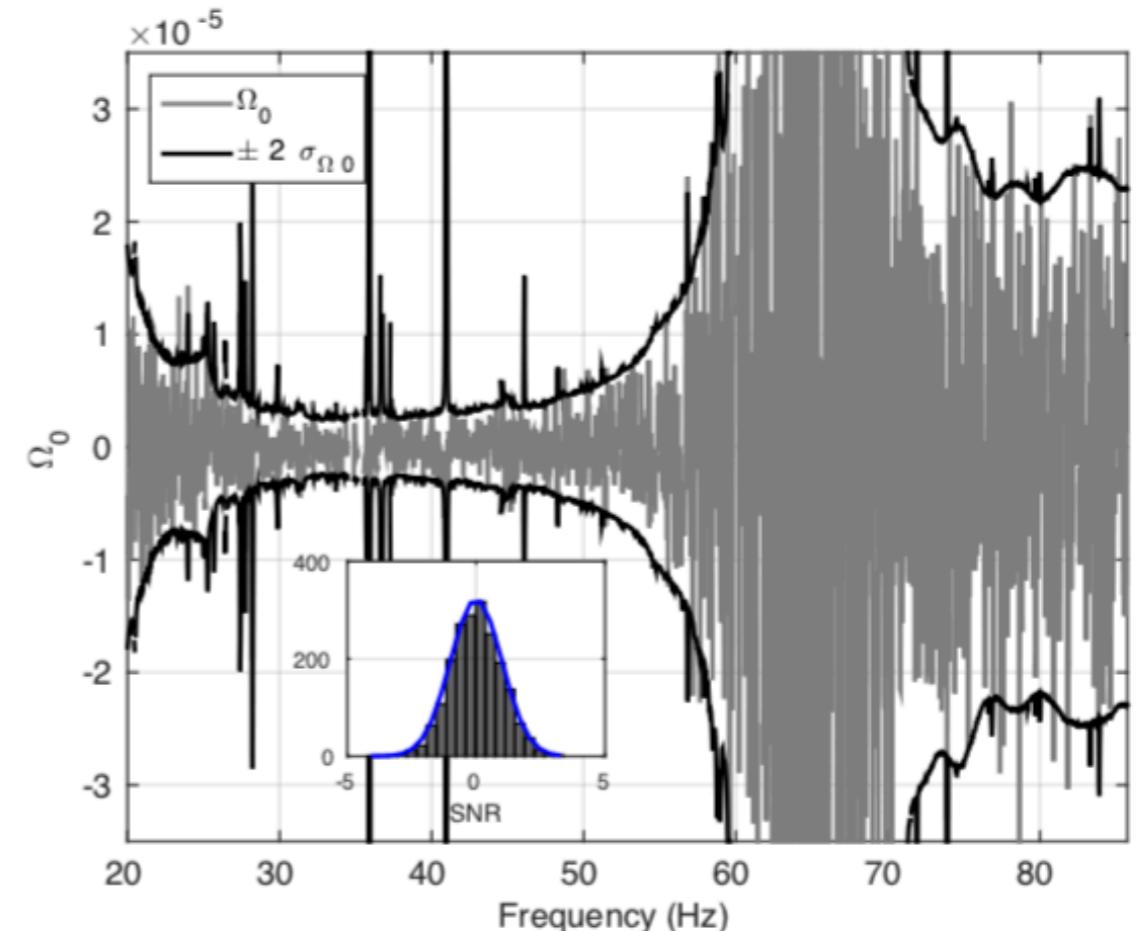
- Isotropic
- Gaussian
- Unpolarized
- Stationary

Because random noise is also gaussian (and loud), we cannot use a single detector to measure the background.

Instead, we must *cross-correlate* the signal S between two detectors:

$$S = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{s}_H^*(f) \tilde{s}_L(f') \tilde{Q}(f') \delta(f - f') df df'$$

$$\tilde{s}_i(f) = \tilde{n}_i(f) + \tilde{h}_i(f)$$



Bin-by-bin estimator for O1 isotropic search with uncertainty. Inset plot is SNR over frequency bins with gaussian fit

Calculating the SGWB Energy Density

To find $\rho_{GW}(f)$ we consider the T_{00} component of the stress energy tensor

$$\rho_{GW}(f) = T_{00} = \frac{c^2}{32\pi G} \left\langle \dot{h}_{TT}^{ij} \dot{h}_{ij}^{TT} \right\rangle$$

We then transform to frequency space (makes life easier) and can write the 2-point correlation

$$\left\langle h_A^*(f, \hat{\Omega}) h_{A'}(f', \hat{\Omega}') \right\rangle = \delta^2(\hat{\Omega}, \hat{\Omega}') \delta(f - f') \delta_{AA'} \frac{1}{16\pi} H(f)$$

Using the plane-wave expansion for the GWs and plugging it all in we get

$$\int df \frac{d\rho_{GW}}{df} = \frac{\pi c^2}{4G} \int_0^\infty df f^2 H(f) = \frac{\pi c^2}{4G} \int_0^\infty df f^2 \frac{3H_0^2}{2\pi^2} \frac{\Omega_{GW}(f)}{f^3}$$

Calculating the SGWB Energy Density

which implies...

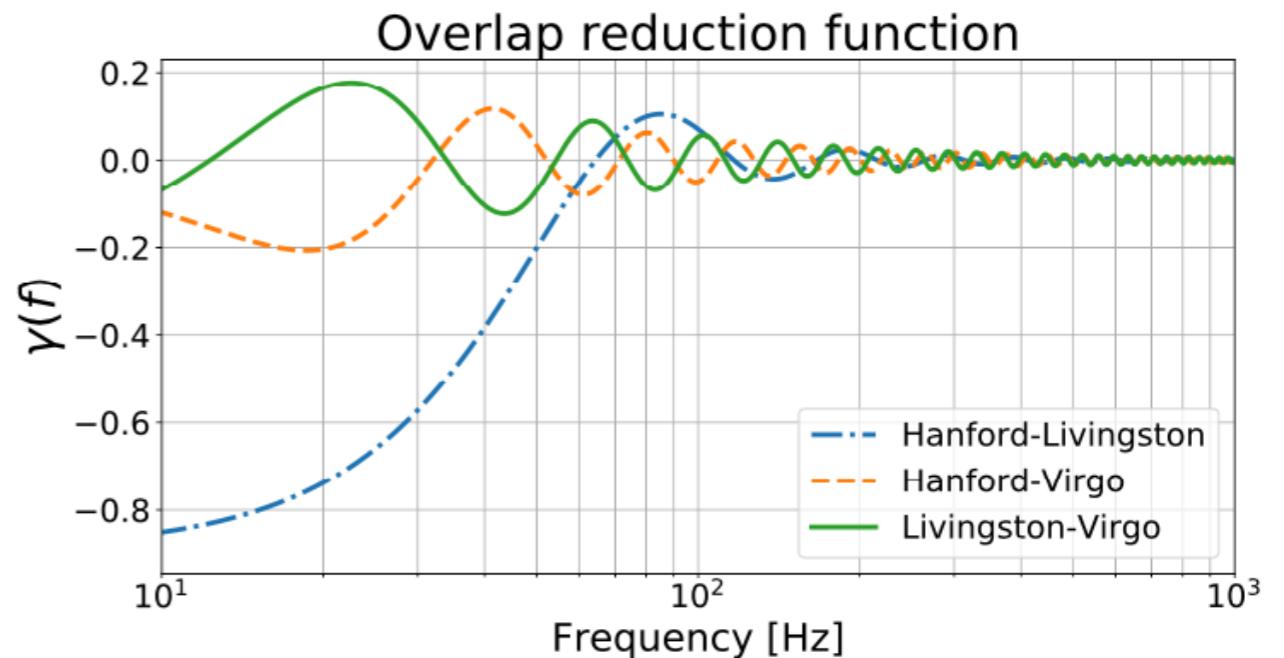
$$\left\langle \tilde{h}_H^*(f) \tilde{h}_L(f') \right\rangle = \frac{3H_0^2}{20\pi^2} \frac{\Omega_{GW}(f)}{f^3} \gamma(f) \delta(f - f')$$

This means that we have direct access to $\Omega_{GW}(f)$!

Calculating the SGWB SNR

The detector geometry and antenna pattern is encoded in the *overlap reduction function*

$$\gamma(f) \equiv \frac{5}{8\pi} \sum_A \int d\hat{\Omega} e^{2\pi i f \hat{\Omega} \cdot (\mathbf{x}_H - \mathbf{x}_L)/c} F_H^A(\hat{\Omega}) F_L^A(\hat{\Omega})$$



where $F(\hat{\Omega})$ is the projection of the GW vector onto the detector's response tensor. We can then find the SNR to be

$$SNR = \frac{\mu}{\sigma} = \frac{3H_0^2}{10\pi^2} \left[2T \int_0^\infty df \frac{\gamma^2(f) \Omega_{GW}^2(f)}{f^6 P_1(f) P_2(f)} \right]^{1/2}$$

Learning from the SNR

$$SNR = \frac{\mu}{\sigma} = \frac{3H_0^2}{10\pi^2} \left[2T \int_0^\infty df \frac{\gamma^2(f) \Omega_{GW}^2(f)}{f^6 P_1(f) P_2(f)} \right]^{1/2}$$

What this tells us about the isotropic SGWB search

1. If we observe for long enough, we will detect *something*
2. We are most sensitive at low frequencies
3. SNR depends on model we pick. How to choose?

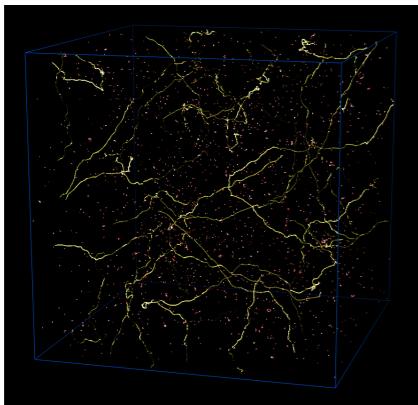
Choosing Models I

In general we assume a power law spectrum which allows us to write

$$\Omega_{GW}(f) = \Omega_0 \left(\frac{f}{f_{ref}} \right)^\beta$$

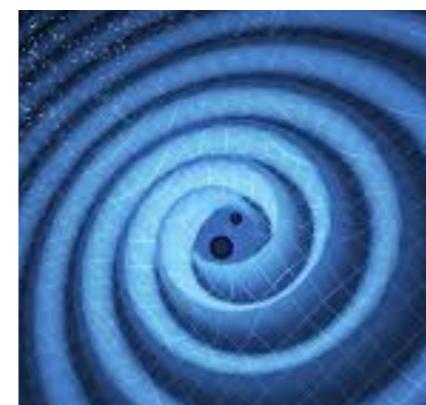
Currently Implemented Models

Cosmological Background



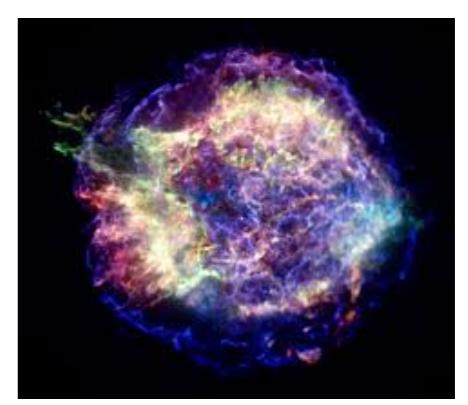
$$\beta = 0$$

Compact Binaries



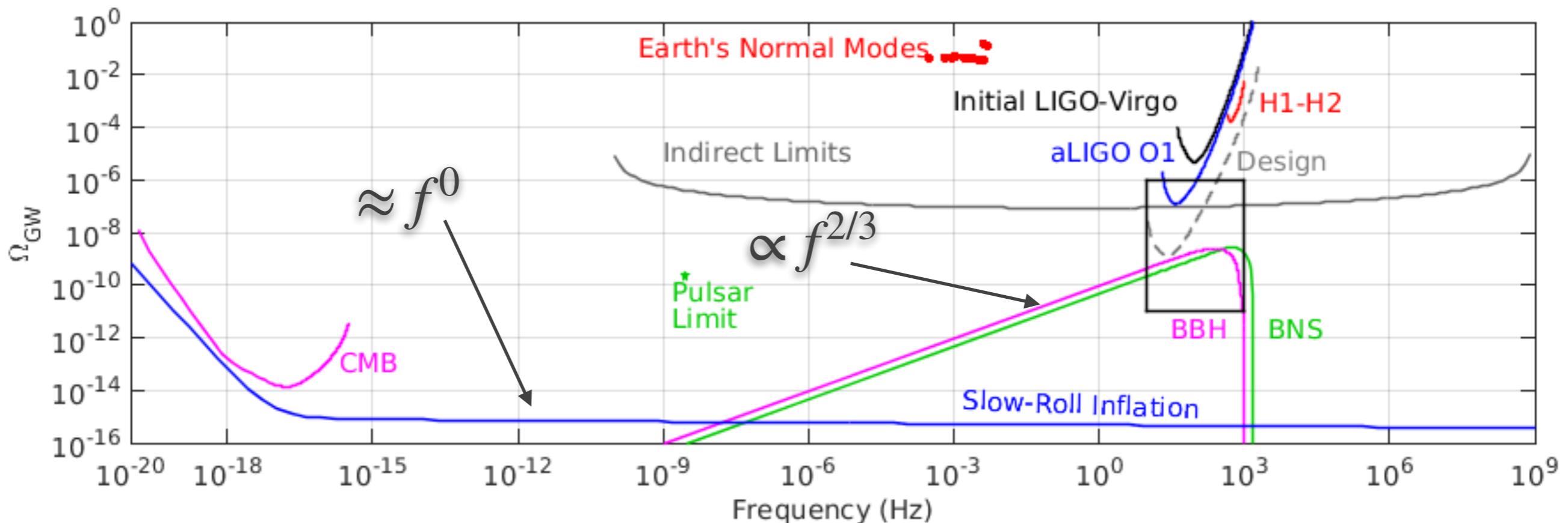
$$\beta = 2/3$$

Supernova



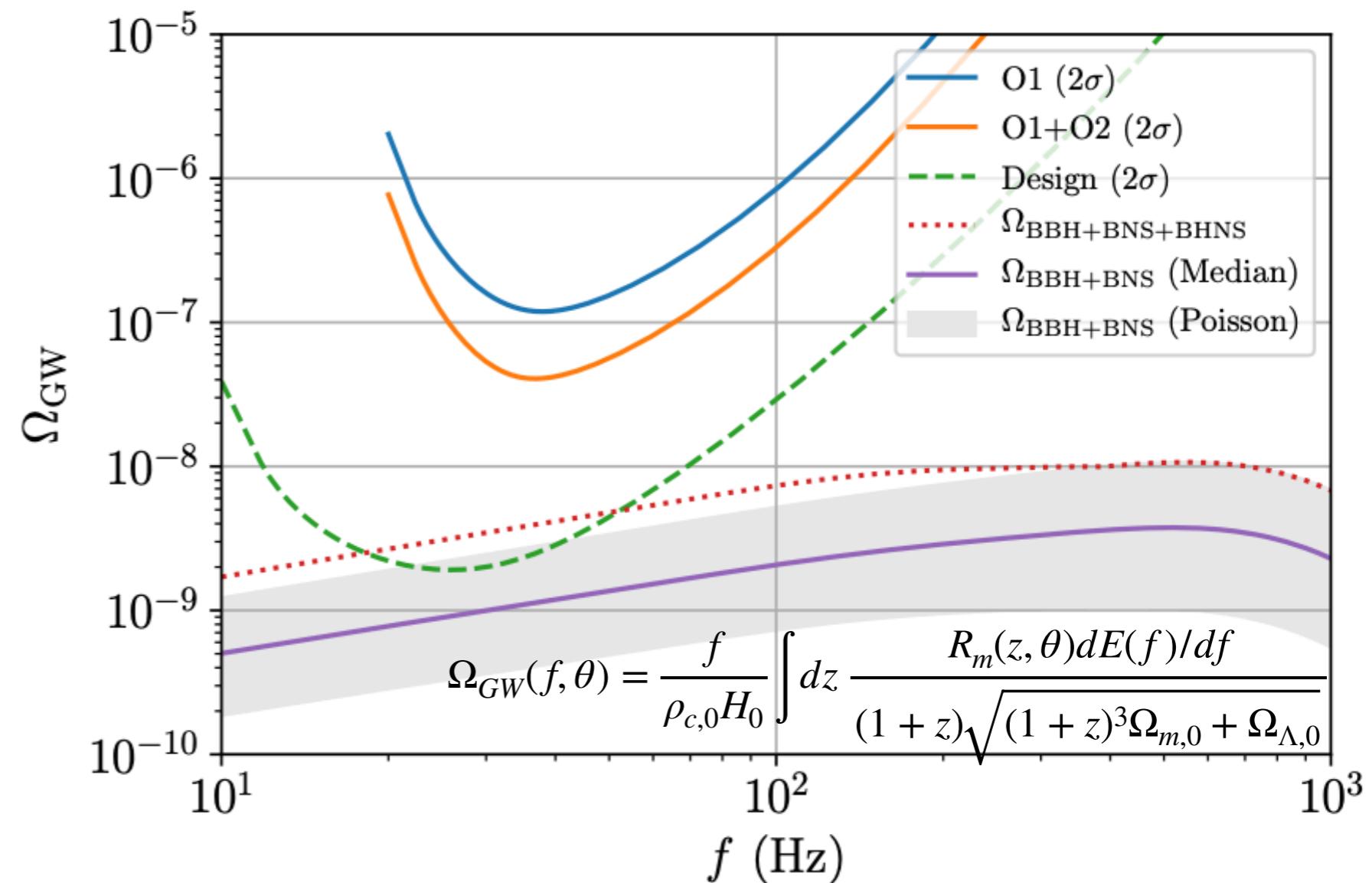
$$\beta = 3$$

Choosing Models II

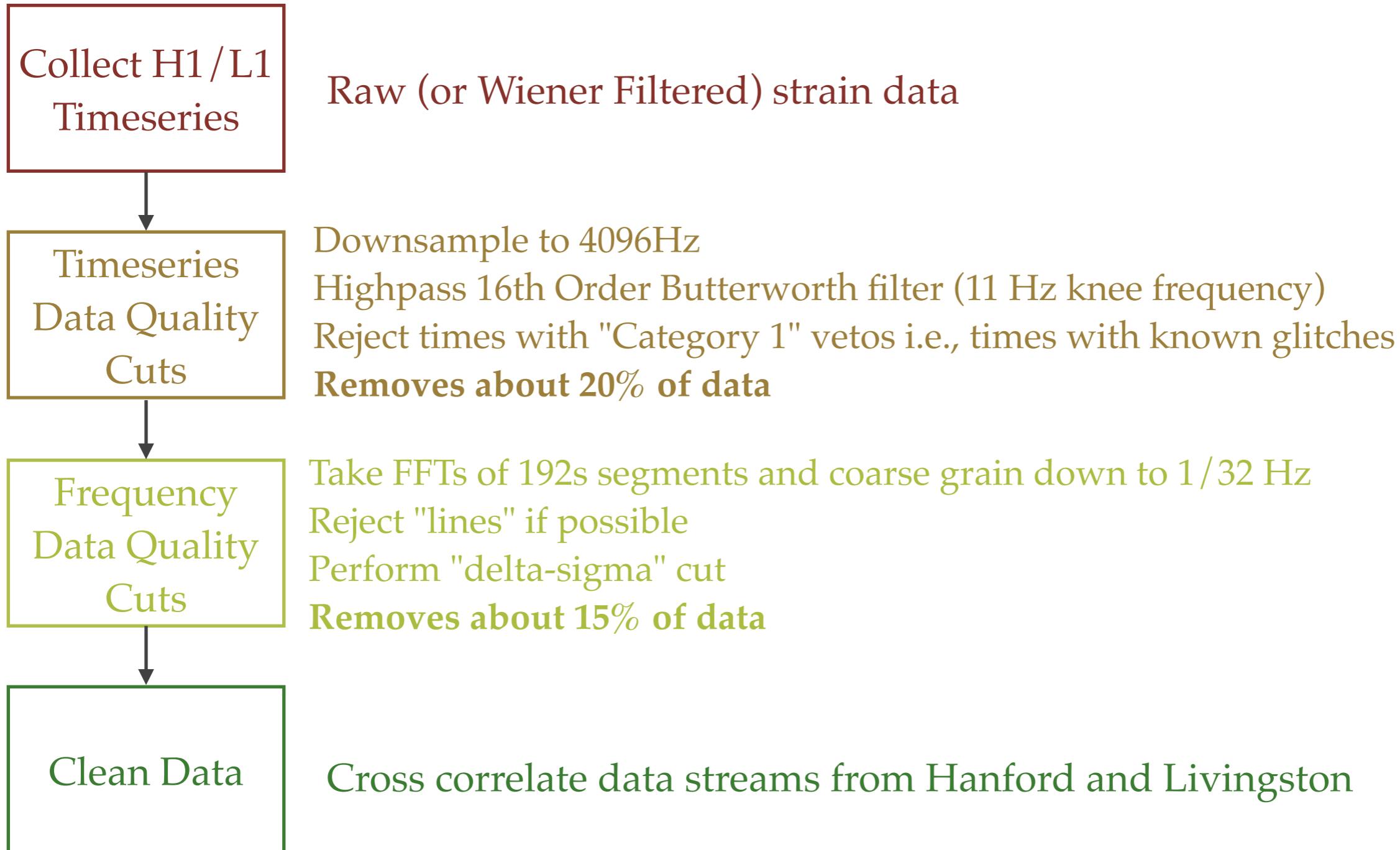


CBC Model

Sensitivity curves for O1, combined O1+O2, and design sensitivity. A power law stochastic background which lies tangent to one of these curves is detectable with 2σ significance. Design sensitivity assumes that the LIGO noise curve is determined by fundamental noise sources only.



Probing the SGWB I



Probing the SGWB II

Define cross-correlation statistic and variance for each frequency bin

$$\hat{C}(f) = \frac{20\pi^2 f^3}{3H_0^2 T} \frac{\text{Re}[\tilde{s}_L^*(f)\tilde{s}_H(f)]}{\gamma(f)}$$

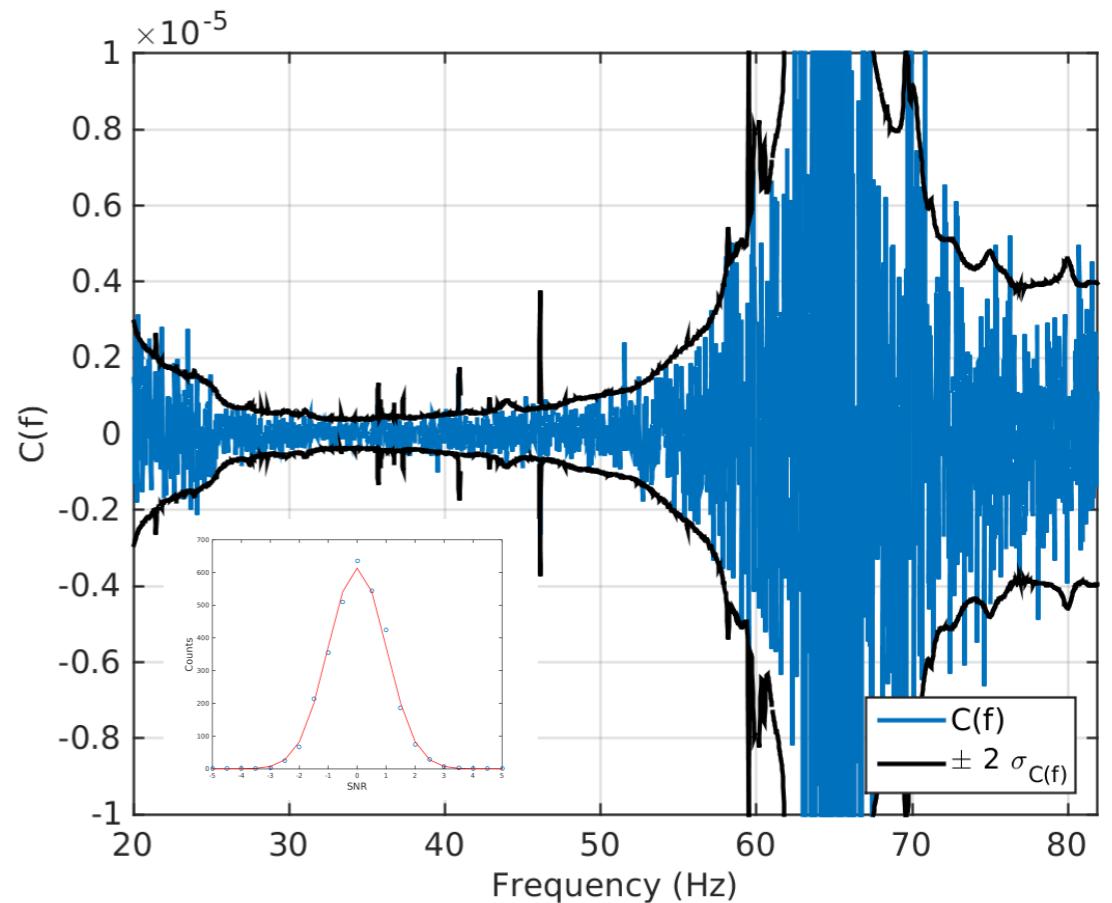
$$\sigma^2(f) = \left(\frac{10\pi^2}{3H_0^2} \right)^2 \frac{f^6}{2T\Delta f} \frac{P_1(f)P_2(f)}{\gamma^2(f)}$$

This gives the *optimal detection statistic*

$$\hat{\Omega}_{ref} = \frac{\sum_k w(f_k)^{-1} \hat{C}(f_k) \sigma^{-2}(f_k)}{\sum_k w(f_k)^{-2} \sigma^{-2}(f_k)}$$

where

$$\sigma_{\Omega}^{-2} = \sum_k w(f_k)^{-2} \sigma^{-2}(f_k) \quad \text{and} \quad w(f) = \frac{\Omega_{GW}(f_{ref})}{\Omega_{GW}(f)}$$



Cross-correlation statistic for LIGO during the second observing run. Loss of sensitivity at 64Hz is from a zero in the overlap reduction function

Isotropic SGWB Search Results

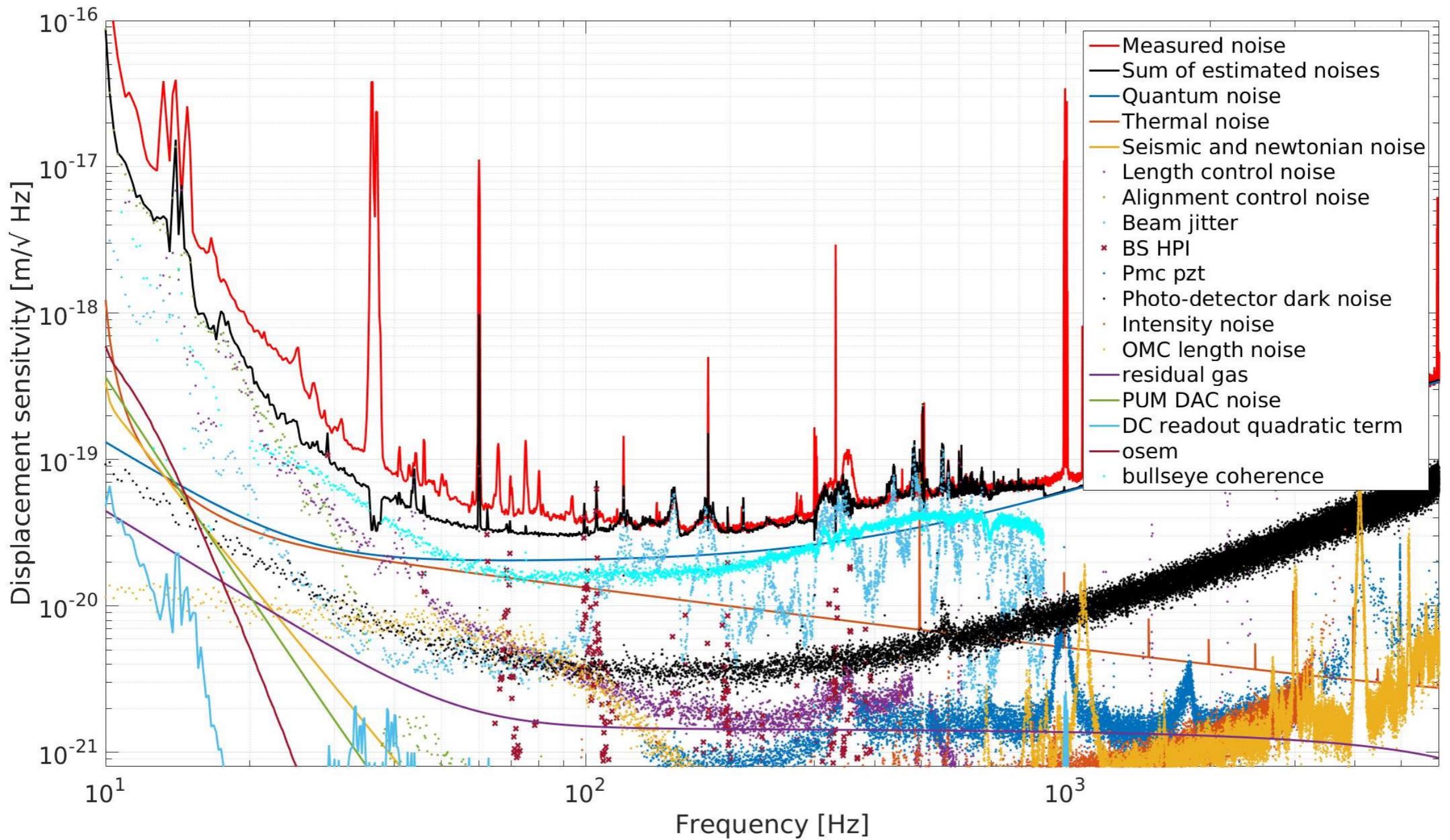
α	$\hat{\Omega}_{\text{ref}} \text{ (O2)}$	$\hat{\Omega}_{\text{ref}} \text{ (O1)}$	O2 Sensitive band
0	$(2.2 \pm 2.2) \times 10^{-8}$	$(4.4 \pm 6.0) \times 10^{-8}$	20-85.8 Hz
$2/3$	$(2.0 \pm 1.6) \times 10^{-8}$	$(3.5 \pm 4.4) \times 10^{-8}$	20-95.2 Hz
3	$(3.5 \pm 2.8) \times 10^{-9}$	$(3.7 \pm 6.6) \times 10^{-9}$	20-305 Hz

Point estimate with 1σ uncertainties for the optimal estimator for O1 and O2. The "sensitive band" contains 99% of the sensitivity for the shown spectral index.

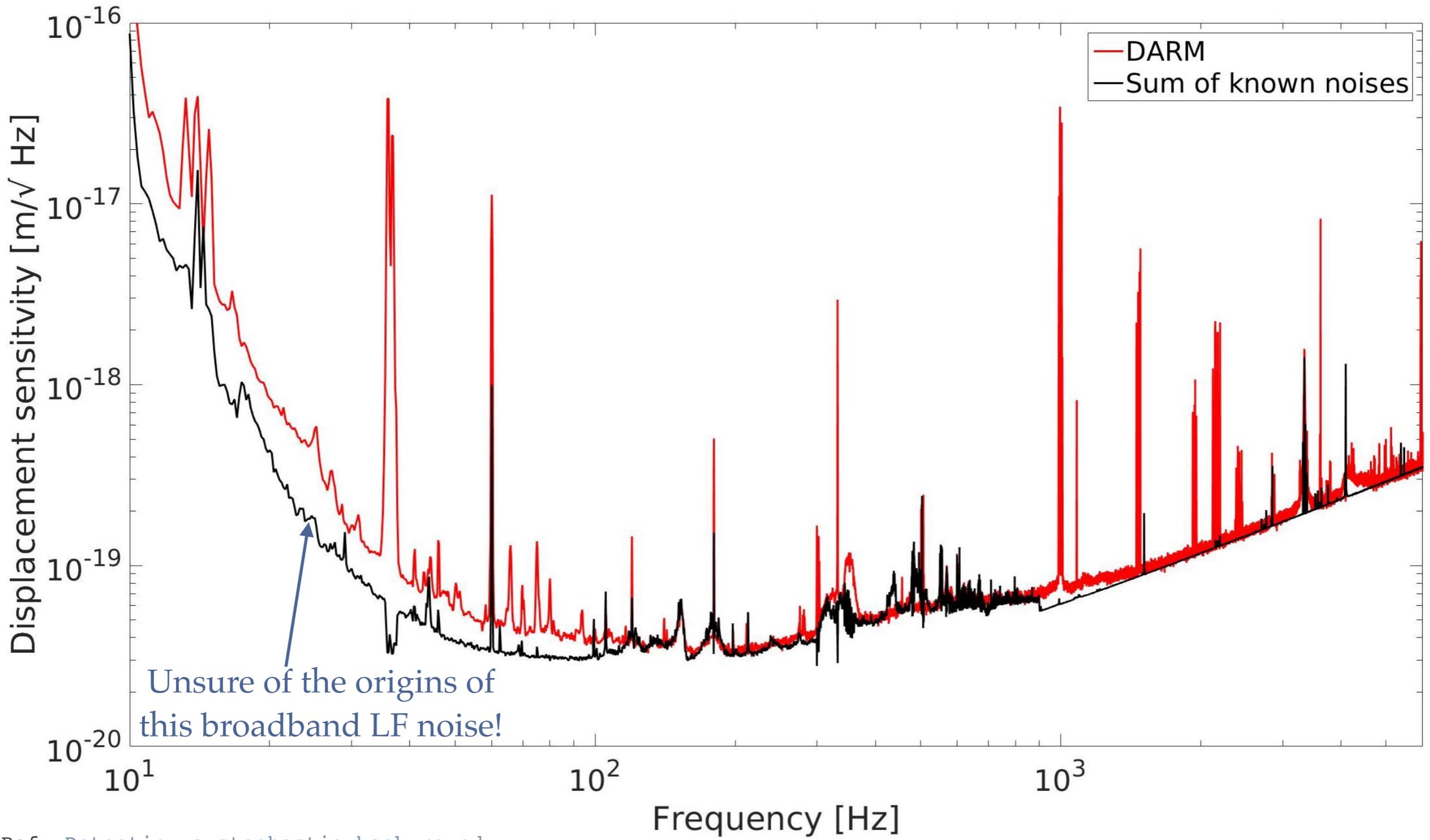
Still ~ 1 order of magnitude above the CBC PI curve at ~ 30 Hz

Enhancing Sensitivity Via Machine Learning

LIGO O2 Noise Budget



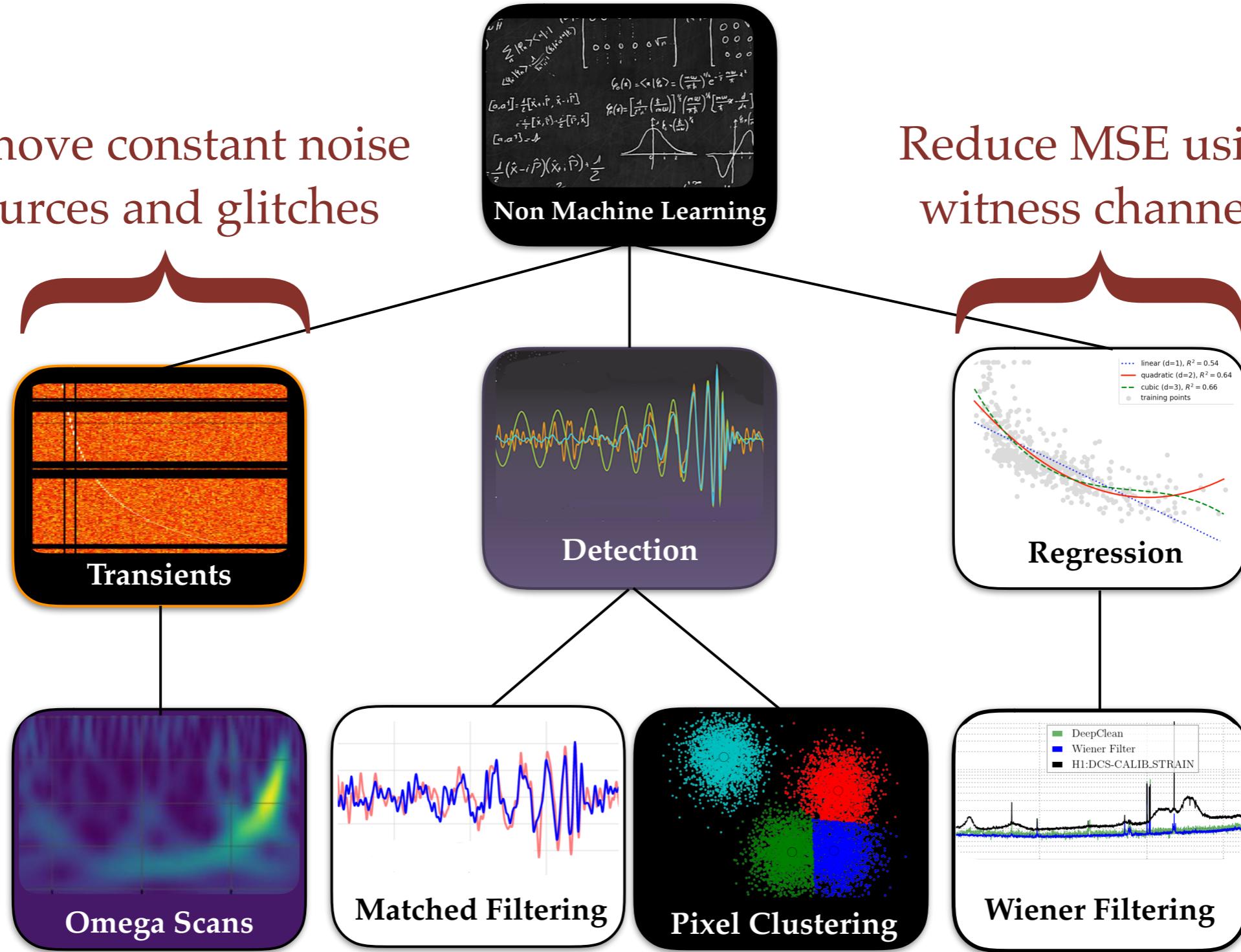
O2 Noise Budget - A Closer Look



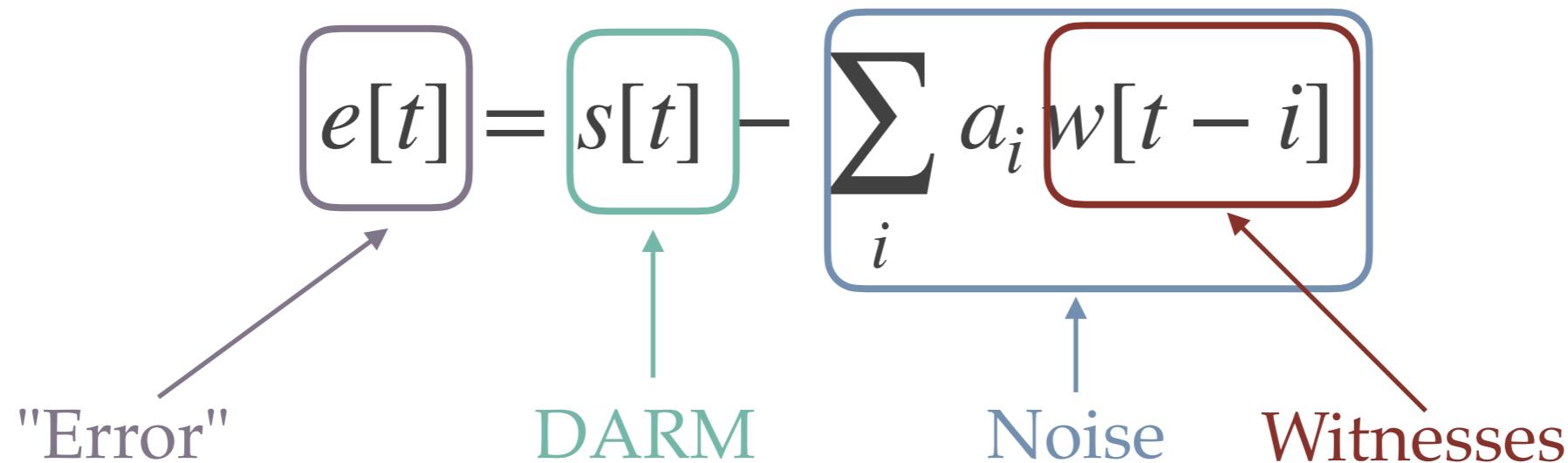
Data Quality

Remove constant noise
sources and glitches

Reduce MSE using
witness channels



MISO - Wiener Filtering

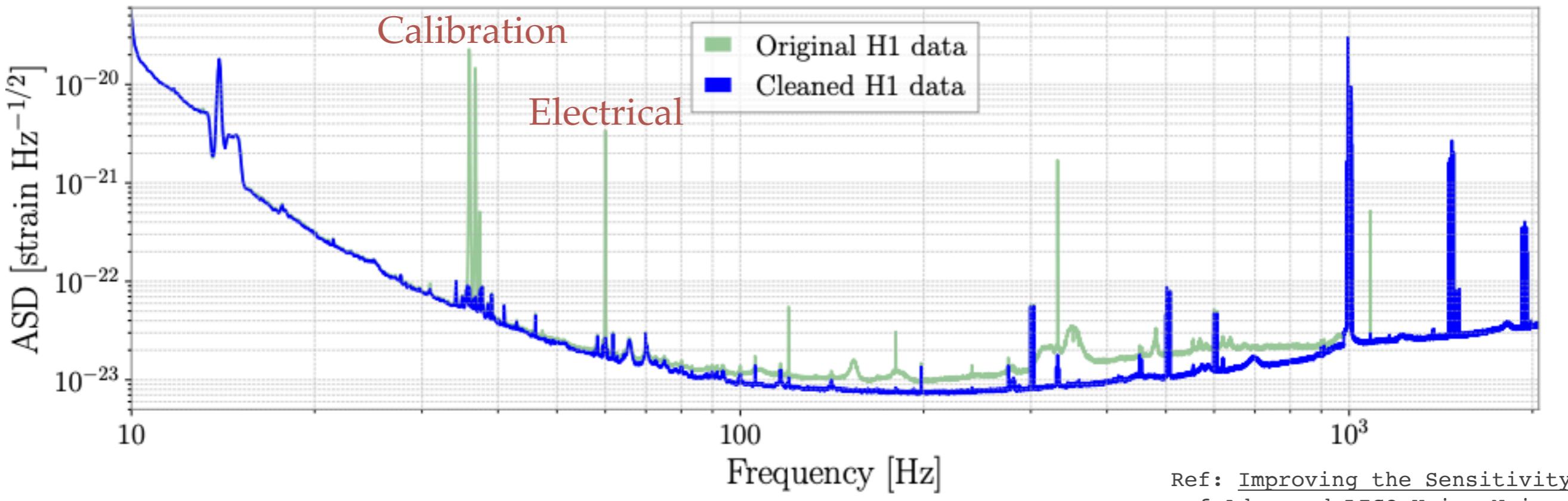


Estimate the transfer function by minimizing the MSE:

$$\frac{\partial}{\partial a_j} \langle e[t]^2 \rangle = 0$$

$$= 2 \sum_j \langle w[t - j]w[t - i] \rangle a_j - 2 \langle w[t - i]s[t] \rangle$$

Wiener Filtering Results

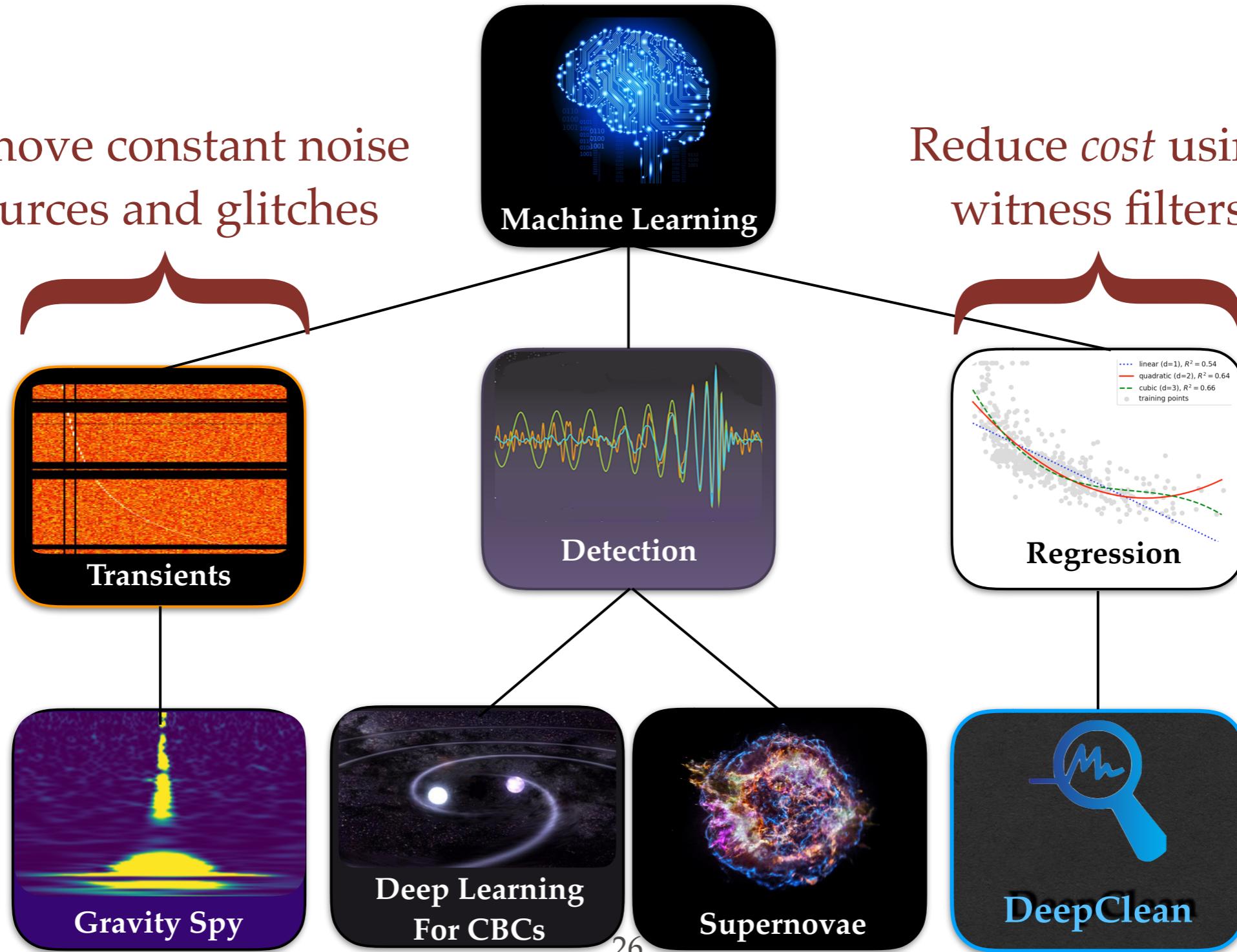


- Filtering was run from 10 - 1024 Hz
- Essentially no subtraction below 80 Hz
- Subtracted noise from ~100 - 1000 Hz is primarily beam jitter

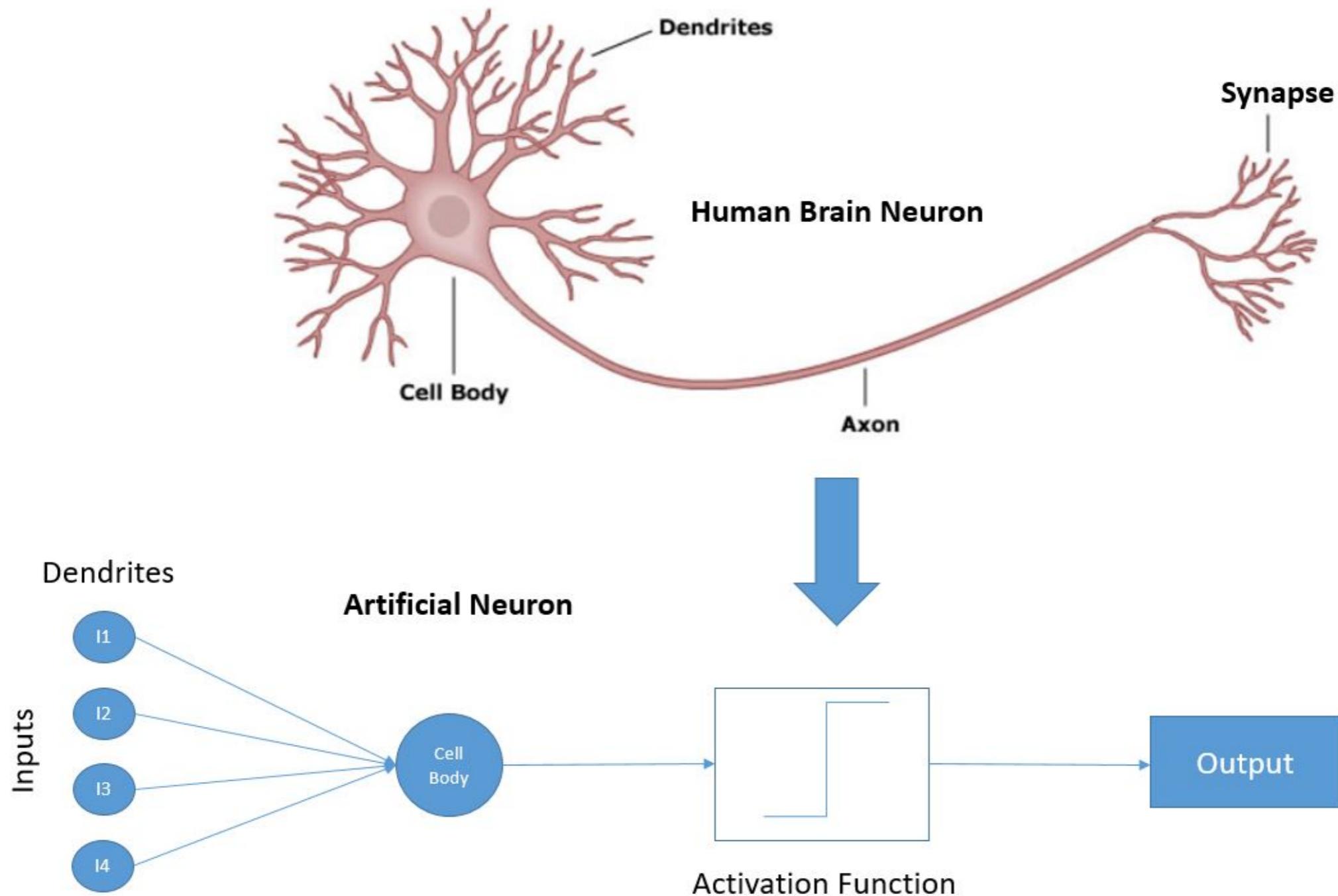
Machine Learning

Remove constant noise
sources and glitches

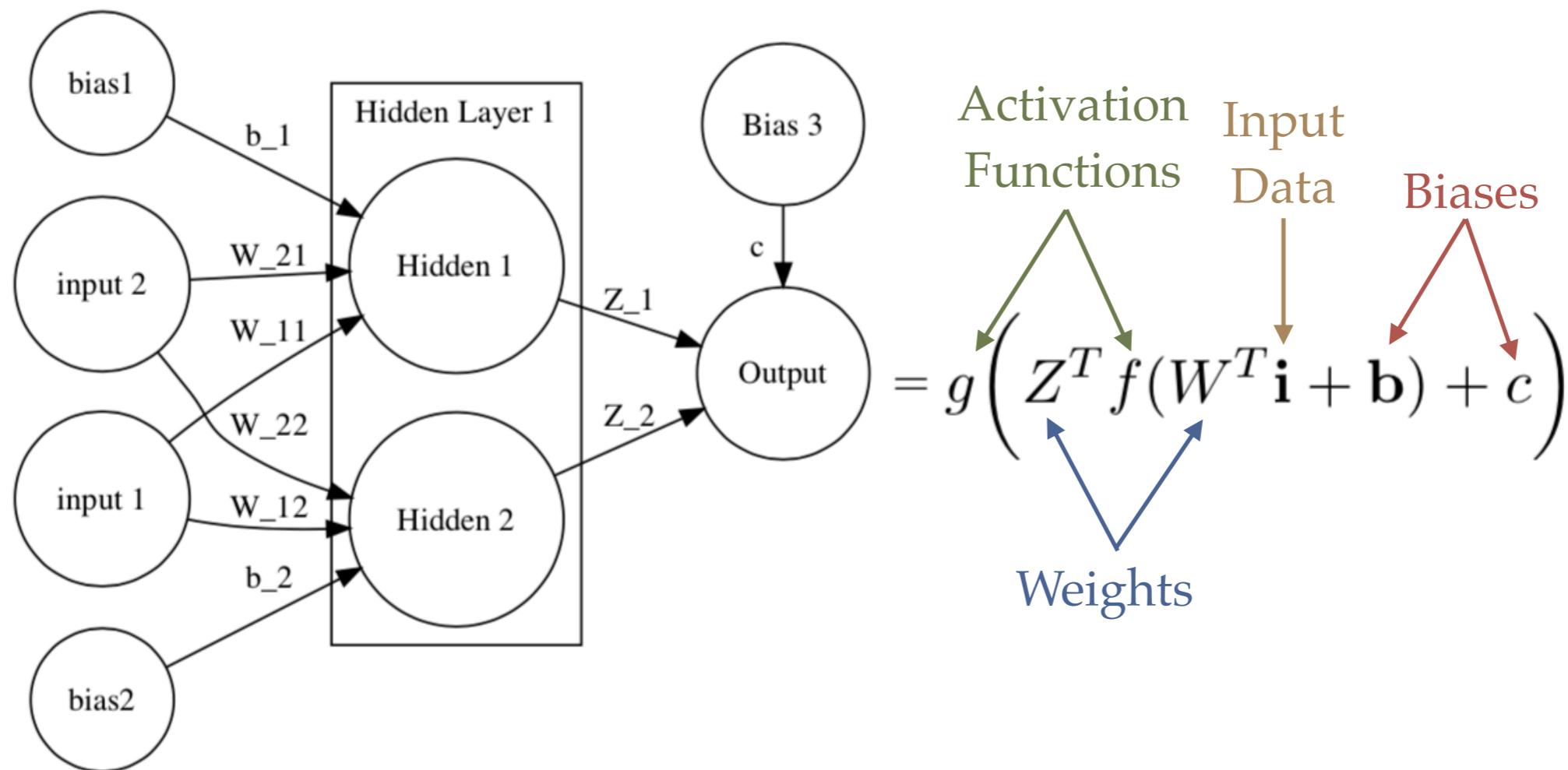
Reduce *cost* using
witness filters



How MLAs Work - I

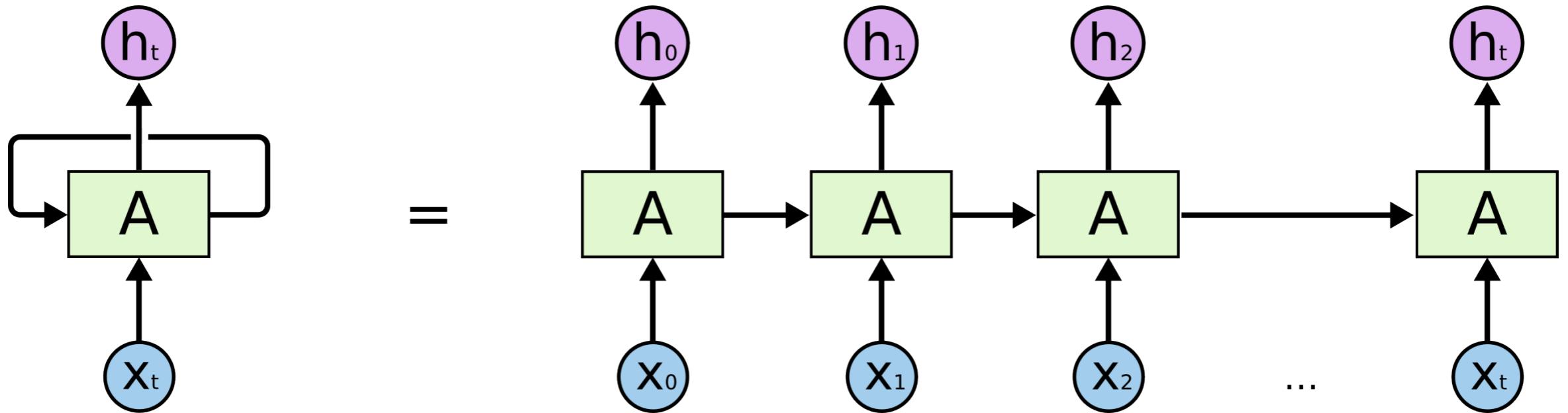


How MLAs Work - 2



- Vanilla feed-forward network. The network is comprised of the hidden layer, activation functions, weights, and biases.
- **We** construct the architecture of the hidden layer and activation functions.
- **The network's** goal is to learn the values of the weights and biases.

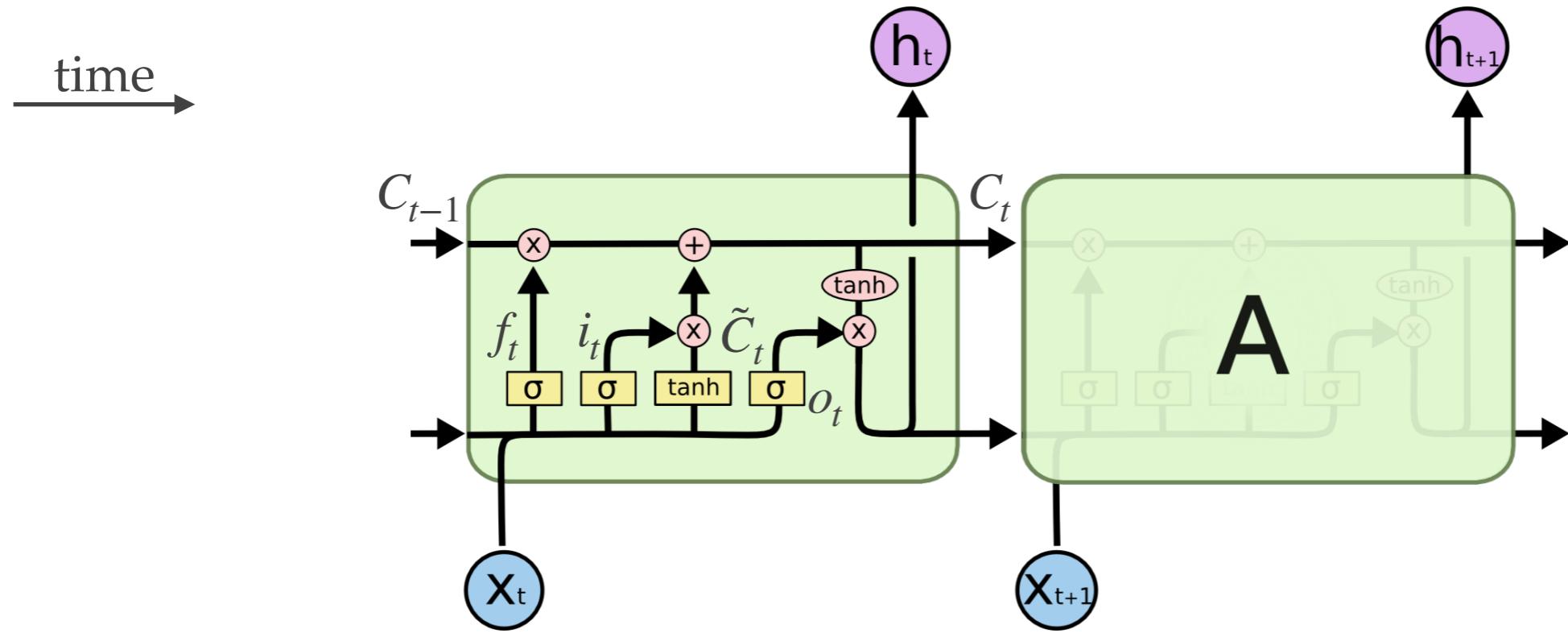
Recurrent Neural Networks I



Each "A" block is a feed-forward network which receives previous output and current data as input. Therefore, past and present data together informs the predictions. In this sense, the network has memory.

Expanding on this leads to Long Short-Term Memory (LSTM) Networks.

Recurrent Neural Networks II

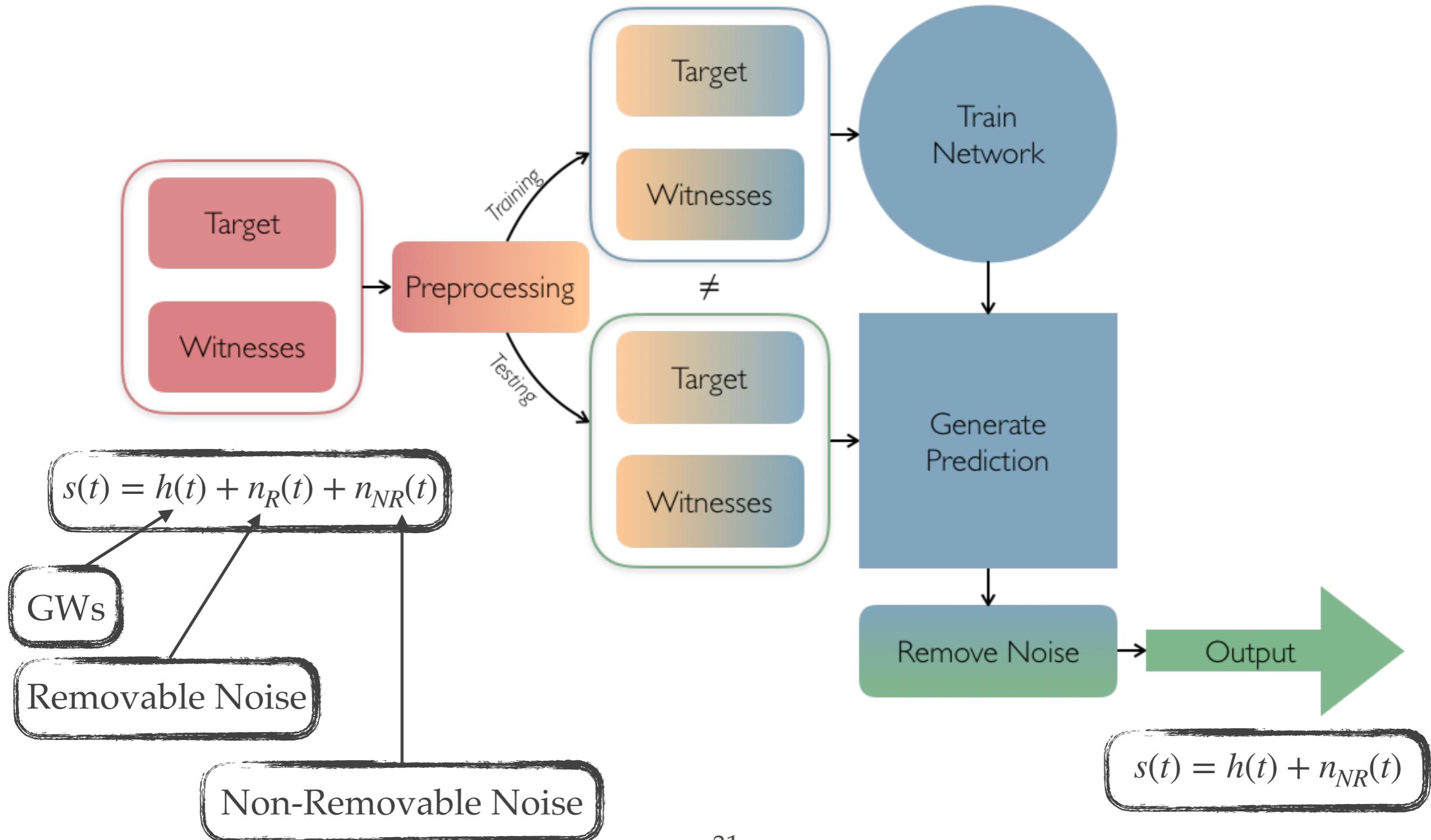


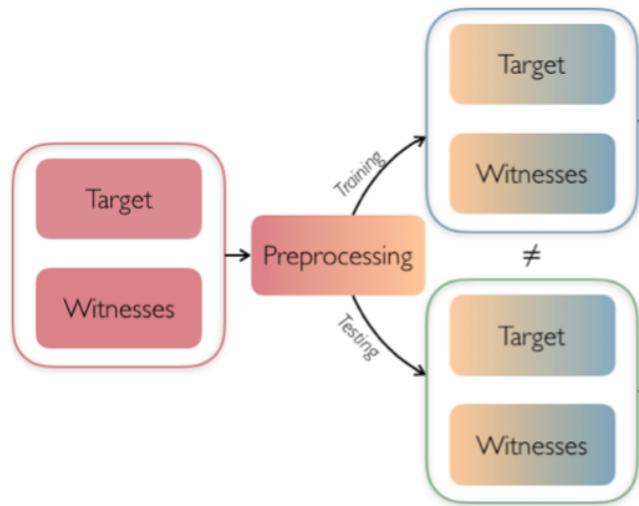
Each input time step is analyzed through a series of gates (forget, input, output) and this information is fed into a "cell state."

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$
$$C_t = f_t \cdot C_{t-1} + i_t \cdot \tilde{C}_t$$
$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$
$$h_t = o_t \cdot \tanh(C_t)$$



DEEP CLEAN Workflow I





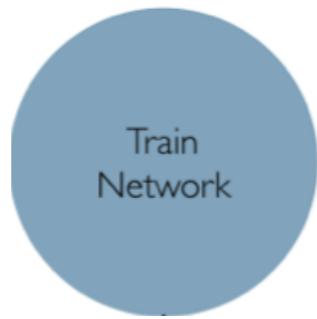
Workflow II

- Separate out the target from the reference data.
- Preprocessing can involve many steps. Generally, we normalize (or standardize):

$$Data \rightarrow \frac{Data - \mu}{\sigma}$$

To reduce training time and increase performance, we also bandpass the data into several bands and "loop" over the network performing subtraction in each band separately

- For supervised learning, split the data into training and testing samples. Network never "sees" testing data



Workflow III

Network training consists (roughly) of three parts:

- Given the witnesses $(\bar{w}_t, \dots, \bar{w}_{t-L})$ and lookback L, calculate a prediction \tilde{n}_{R_t} (repeat for each time step)

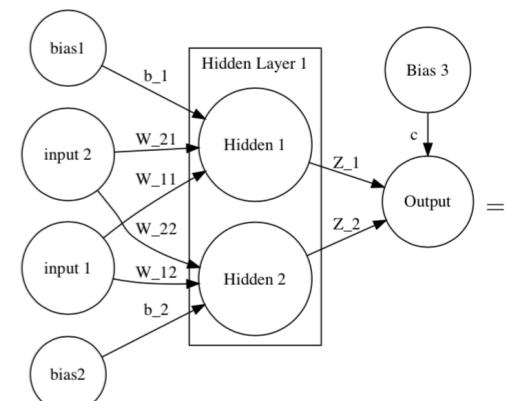
$$Network[\bar{w}_t, \bar{w}_{t-1}, \dots, \bar{w}_{t-L}] = \tilde{n}_{R_t} \quad (s(t) = h(t) + n_R(t) + n_{NR}(t))$$

- Calculate the error of the prediction with the target value through a cost function $J(s_t, \tilde{n}_{R_t})$. Typically use MSE

$$J = \mathcal{N} \sum_t J(s_t, \tilde{n}_{R_t}) = \mathcal{N} \sum_t (s_t - \tilde{n}_{R_t})^2$$

- Update the weights to minimize the cost function

$$\bar{Z} \rightarrow \bar{Z} - \eta \nabla J(s_t, \tilde{n}_{R_t})$$



Training Without a Formal Target I

Question:

How does the network "know" to only subtract noise and not to remove any of the true signal?

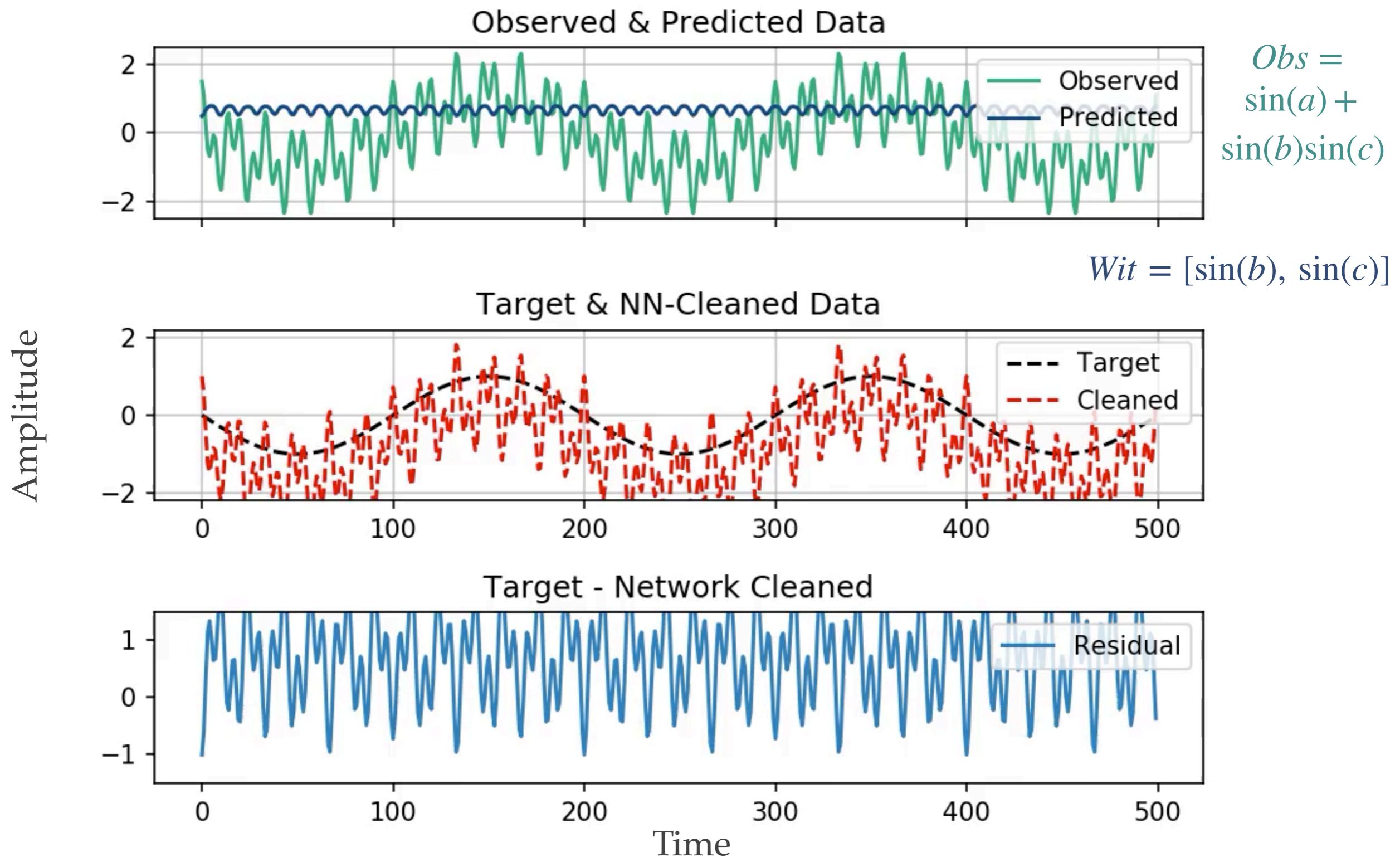
Answer:

The witness channels will not have GWs imprinted upon them (no way to measure GWs with a seismometer & magnetometer)

The neural network learns on (nearly) stationary noise sources; short transients such as glitches and GWs will not be learned

However, continuous wave sources which match the frequency of a witness channel will be removed!

Training Without a Formal Target II

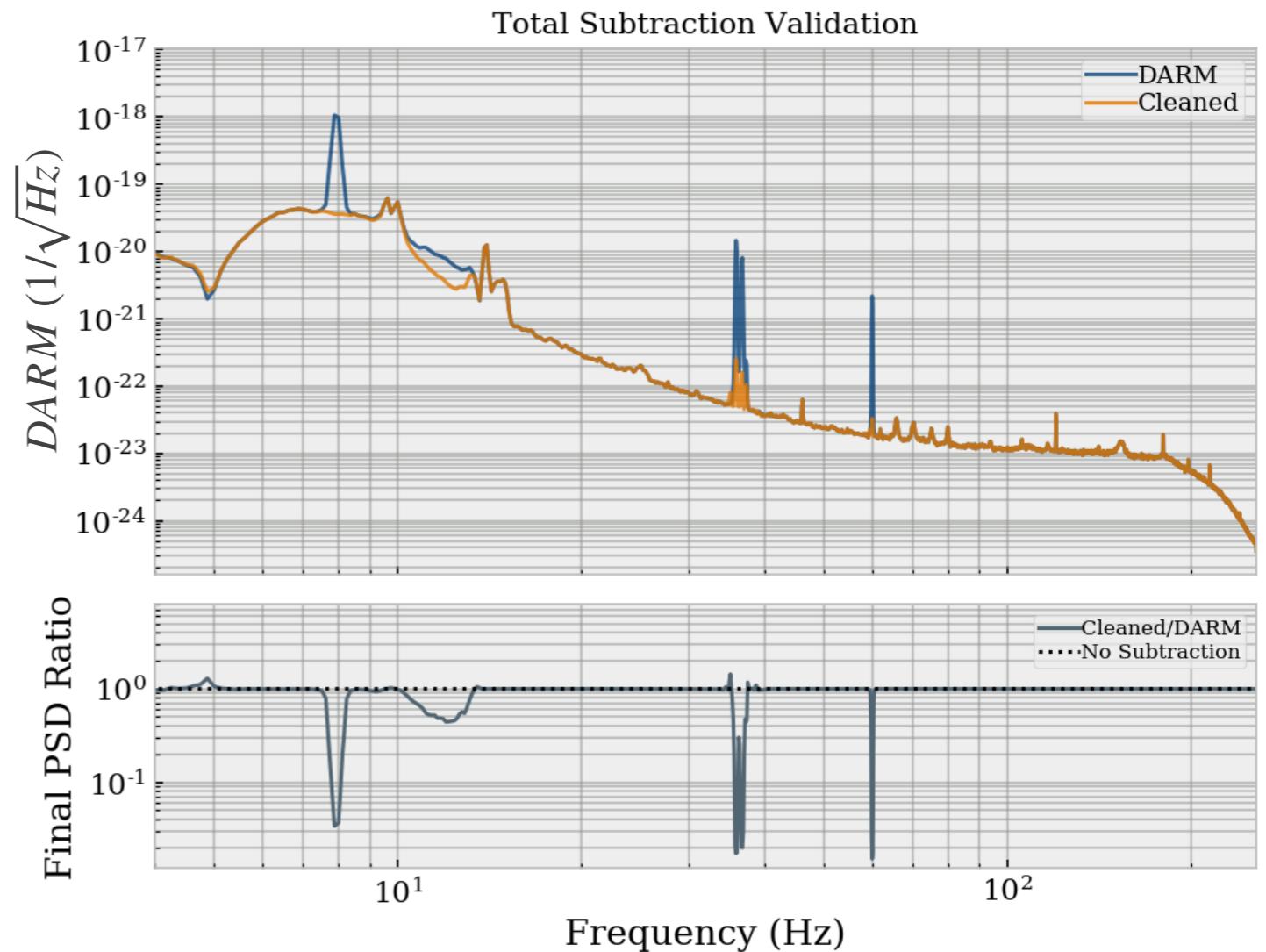


LHO O2 Calibration Lines

O2 Channel List

GDS-CALIB_STRAIN

PSL-DIAG_BULLSEYE_PIT_OUT_DQ
PSL-DIAG_BULLSEYE_YAW_OUT_DQ
PSL-DIAG_BULLSEYE_WID_OUT_DQ
MC-WFS_A_DC_PIT_OUT_DQ
IMC-WFS_B_DC_PIT_OUT_DQ
IMC-WFS_A_DC_YAW_OUT_DQ
IMC-WFS_B_DC_YAW_OUT_DQ
ASC-DHARD_P_OUT_DQ
ASC-DHARD_Y_OUT_DQ
ASC-CHARD_P_OUT_DQ
ASC-CHARD_Y_OUT_DQ
LSC-CAL_LINE_SUM_DQ
LSC-SRCL_IN1_DQ
LSC-MICH_IN1_DQ
LSC-PRCL_IN1_DQ
PEM-EY_MAINSMON_EBAY_1_DQ
PEM-EY_MAINSMON_EBAY_2_DQ
PEM-EY_MAINSMON_EBAY_3_DQ
CAL-CS_LINE_SUM_DQ
CAL-PCALY_TX_PD_OUT_DQ
CAL-PCALY_EXC_SUM_DQ
SUS-ETMY_L3_CAL_LINE_OUT_DQ

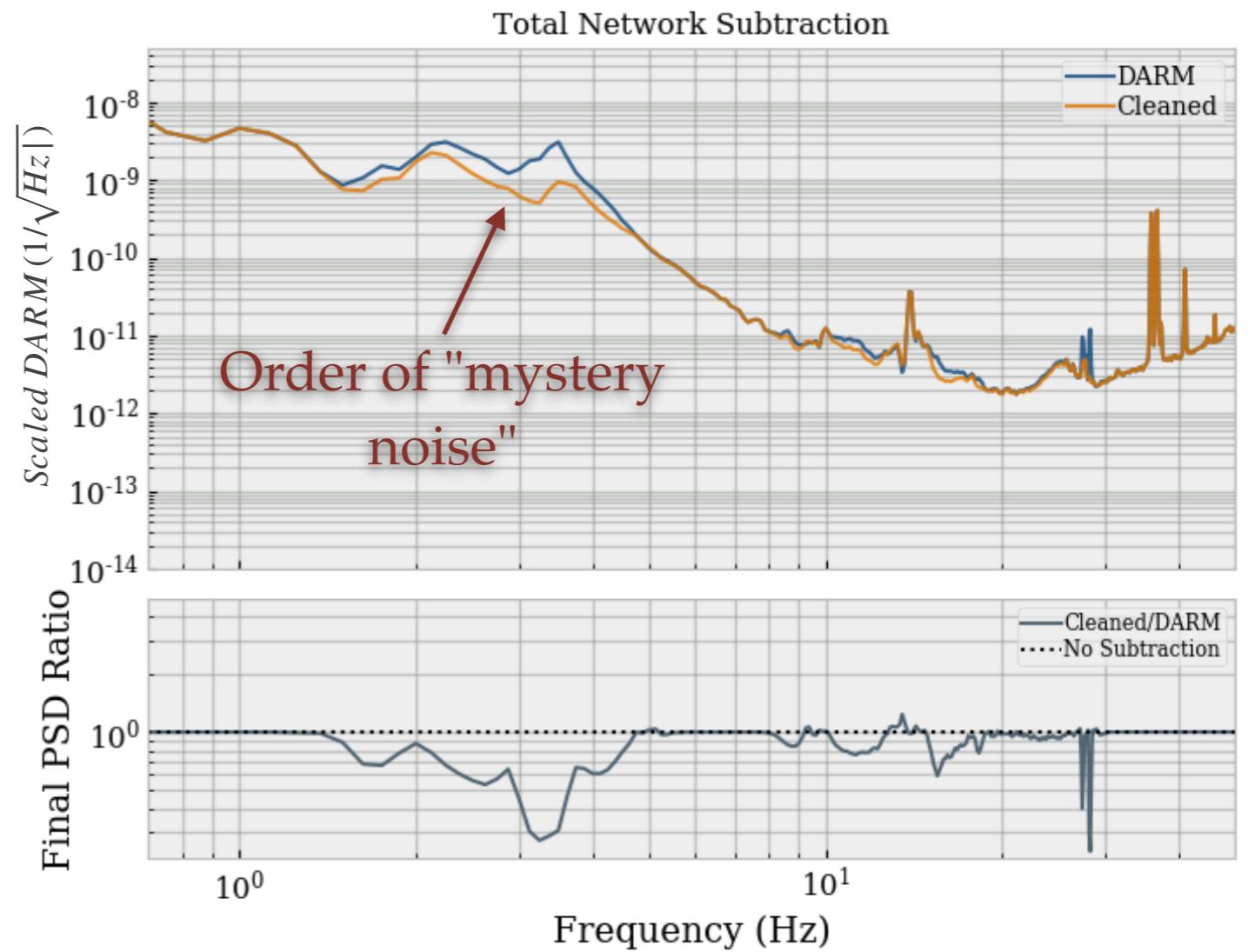


Removing calibration lines and 60 Hz mains from LHO during O2 (+ more?)

LHO 01 GW150914

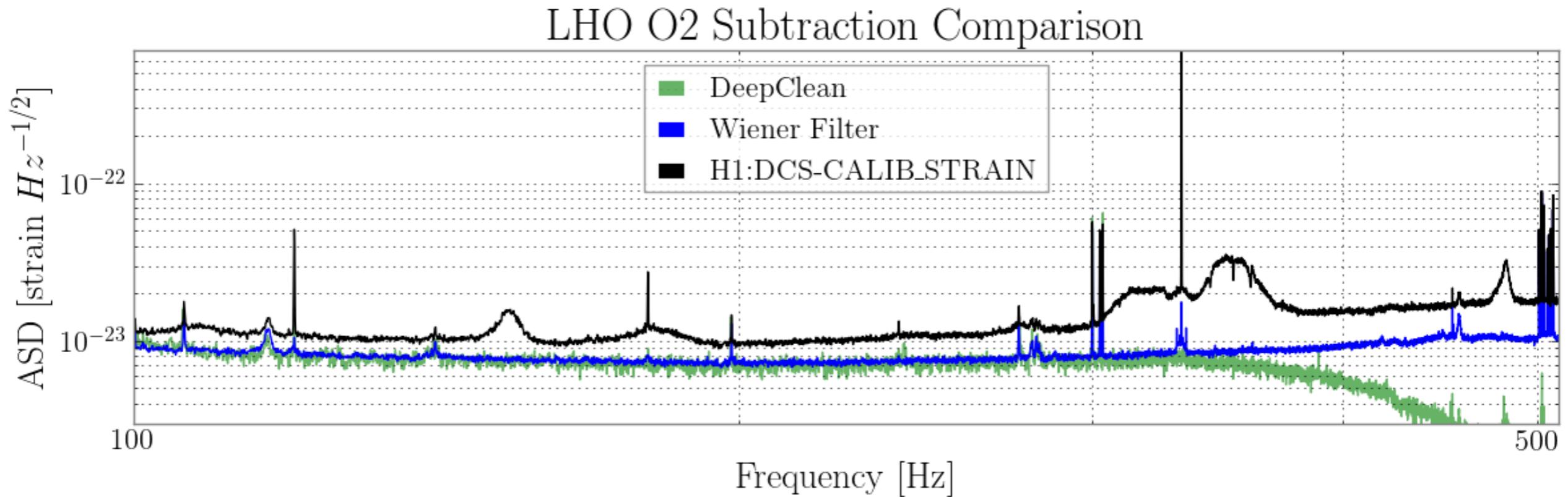
O1 Channel List

CAL-DELTAL_EXTERNAL_DQ
ASC-CHARD_P_OUT_DQ
ASC-CHARD_Y_OUT_DQ
ASC-DHARD_P_OUT_DQ
ASC-DHARD_Y_OUT_DQ
LSC-MICH_OUT_DQ
LSC-PRCL_OUT_DQ
LSC-SRCL_OUT_DQ
PEM-CS_ACC_HAM4_SR2_X_DQ
PEM-CS_ACC_HAM6_OMC_X_DQ
PEM-CS_ACC_HAM2_PRM_Y_DQ
PEM-CS_MIC_LVEA_INPUTOPTICS_DQ
PEM-CS_MIC_LVEA_OUTPUTOPTICS_DQ
ASC-X_TR_A_PIT_OUT_DQ
ASC-X_TR_A_YAW_OUT_DQ
ASC-X_TR_B_PIT_OUT_DQ
ASC-X_TR_B_YAW_OUT_DQ
ASC-Y_TR_A_PIT_OUT_DQ
ASC-Y_TR_A_YAW_OUT_DQ
ASC-Y_TR_B_PIT_OUT_DQ
ASC-Y_TR_B_YAW_OUT_DQ



Jitter subtraction at LHO using 1024s
of data surrounding GW150914

Comparison to Wiener Filter

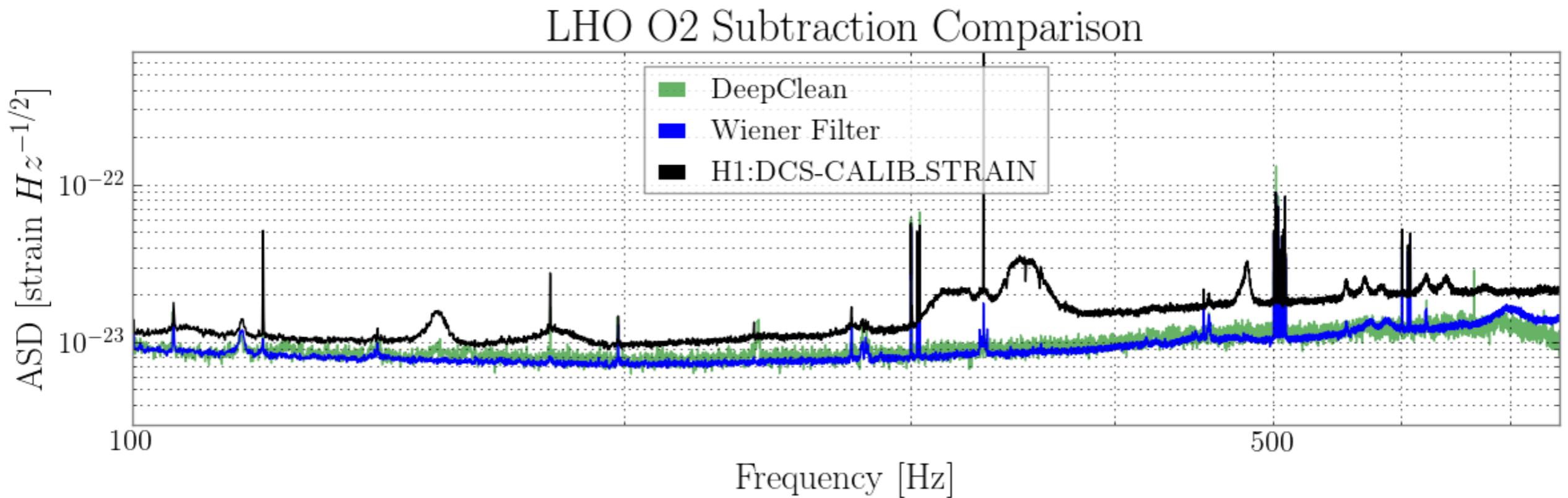


The broadband performance of DeepClean against the O2 linear subtraction is essentially identical (linear activation functions)

Validates the WF method and DeepClean's network.

Due to $\text{sample_rate} = 1024 \text{ Hz}$, the amplitude falls off as we approach the Nyquist frequency.

Comparison to Wiener Filter



Same network as previous slide, but with `sample_rate = 2048`.

The network isn't fully converged yet (twice as much data would need to run a little longer) but the performance is still roughly the same.

Next Steps for DeepClean

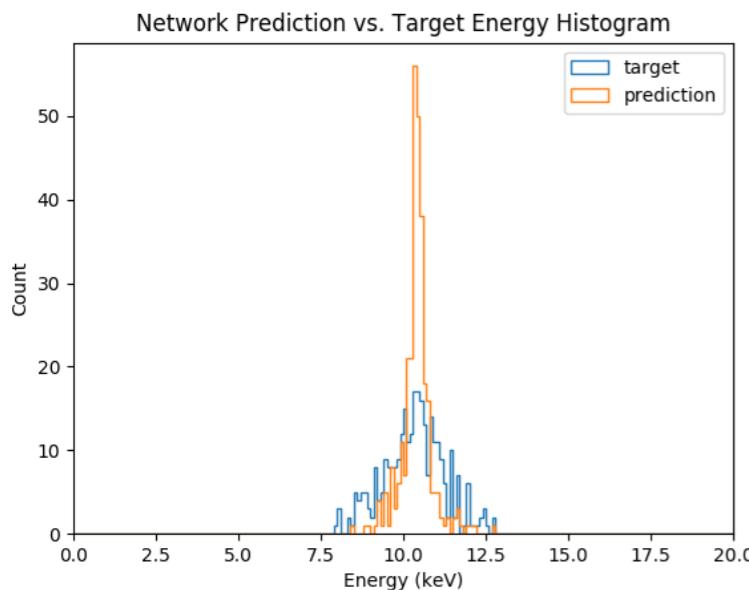
- Try to find the "right" channel list. Not easy!
- Determine where / why we beat the WF. Focus on 20-85 Hz to aid the stochastic search
- Do waveform injections followed by NN-cleaning and then perform parameter estimation. Compare to WF



THANK YOU!

--- Extra Slides ---

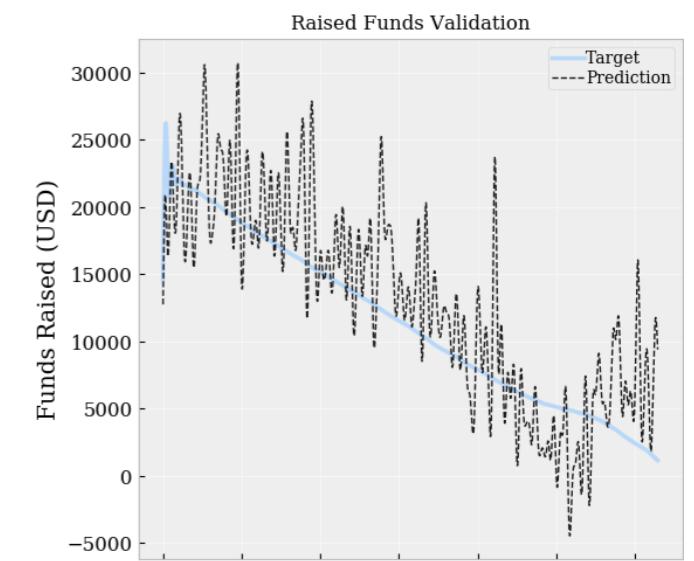
Nonlinear MLAs Outside LIGO



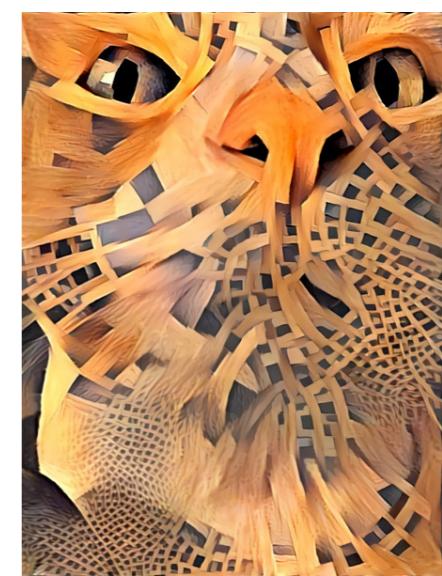
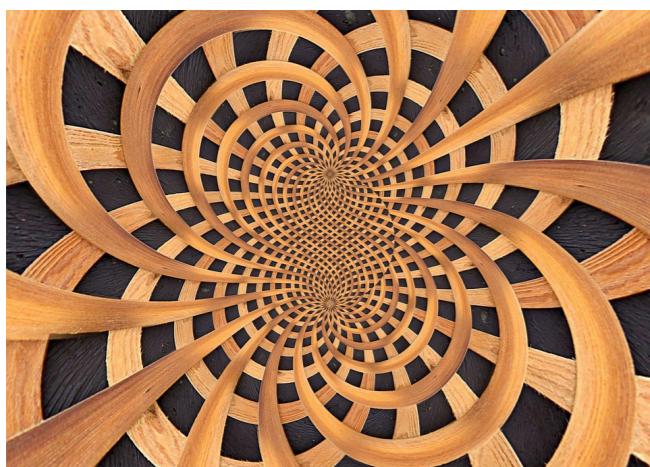
Reconstructing event energies
in CDMS



Enticing jellyfish



Determining how much money
a candidate needs to raise to
win an election (yes, really)



Transverse Traceless Gauge

Under the transformation $x \rightarrow x' = x + \xi$, we find

$$h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

Using the trace reversed gauge where

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

We find

$$\bar{h}_{\mu\nu} \rightarrow \bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} + \partial_\mu \xi_\nu - \partial_\nu \xi_\mu - \eta_{\mu\nu}\eta^{\alpha\beta}\partial_\alpha \xi_\beta$$

We can choose the Lorenz gauge where

$$0 = \partial_\mu \bar{h}_\nu^{\mu'} = \partial_\mu \bar{h}_\nu^\mu - \square \xi_\nu$$

Lastly, we can pick $\square \xi_\nu = 0$ which implies that

$$\partial_\mu h^{\mu\nu} = 0$$

From GR to GW

Solutions to the GW equation can be readily found in the near and far fields

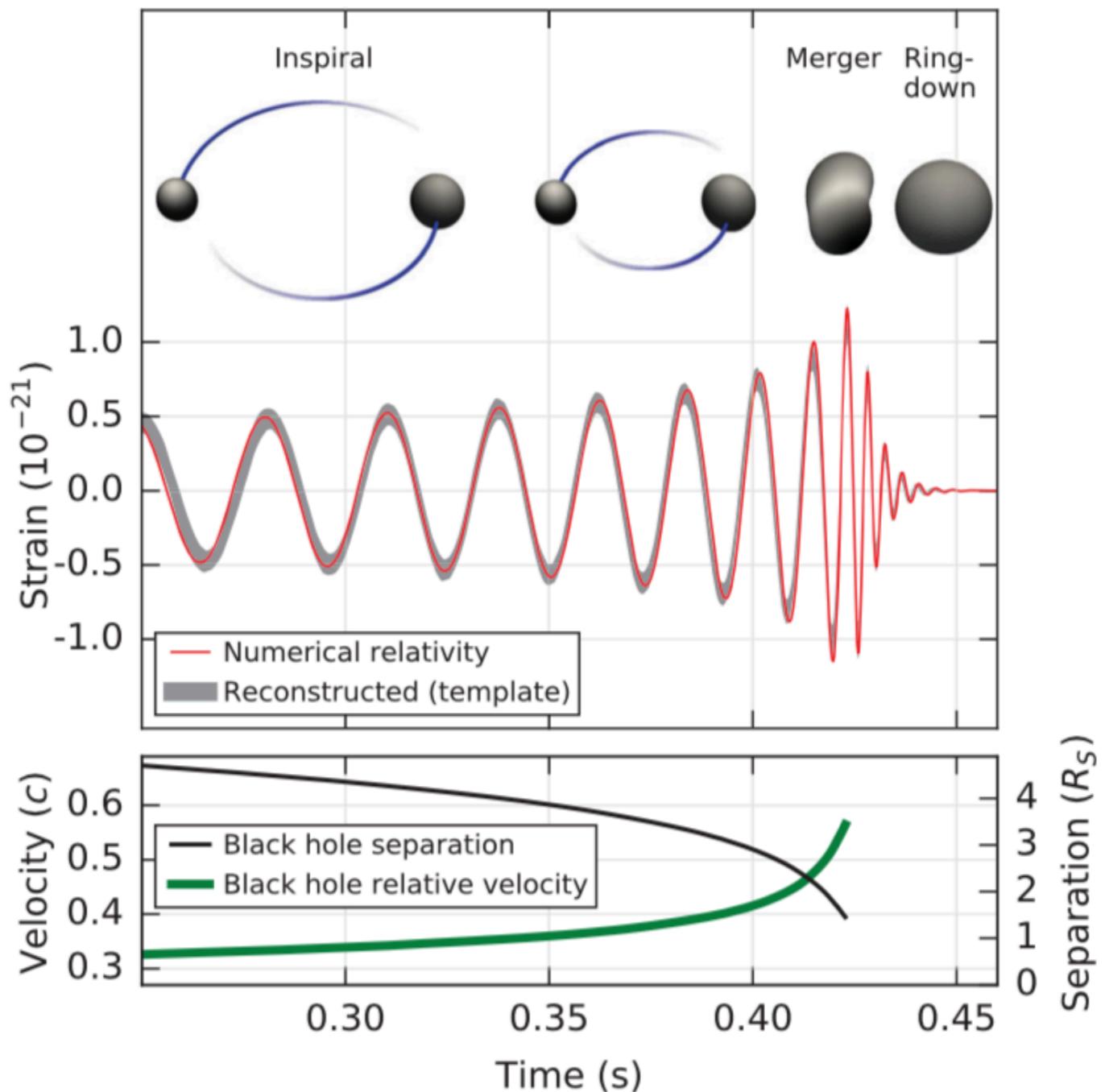
Far Field: $r \gg \lambda_{GW} \gg R_s$

$$\bar{h}_{ij}(t, \mathbf{x}) \simeq \frac{2G}{c^4 r} \ddot{I}_{ij}(t - r/c) \quad I_{ij}(t) = \int x_i x_j T_{00}(t - r/c, \mathbf{x}) d^3 \mathbf{x}$$

Near Field: $\lambda_{GW} \gg r \gg R_s$

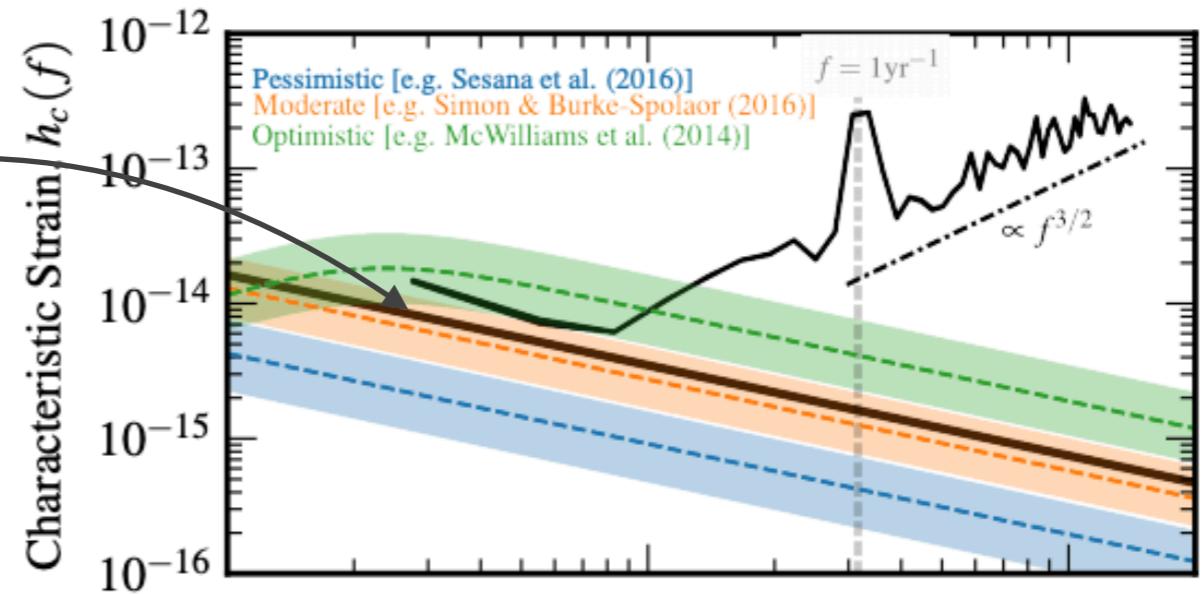
$$\bar{h}_{ij}(t, \mathbf{x}) \simeq \frac{2G}{c^4 r} \ddot{I}_{ij}(t - r/c) \quad I_{ij}(t) = \int (x_i x_j - \frac{1}{3} \delta_{ij} r^2) T_{00}(t - r/c, \mathbf{x}) d^3 \mathbf{x}$$

CBC GW Waveform

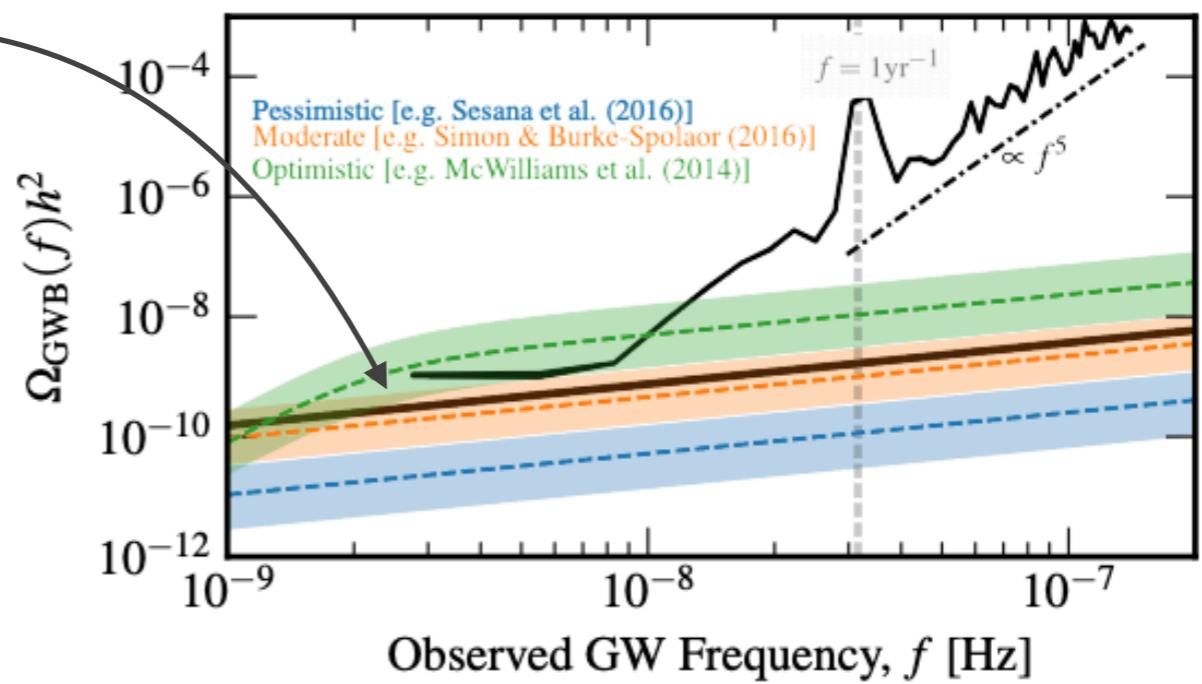


NanoGrav 11 Year Stochastic Paper

$$\Omega_{GW}(f) \propto f^{2/3} \propto f^3 S_h(f) \propto f^2 h_c^2(f)$$
$$\rightarrow h_c(f) \propto f^{-2/3}$$



$$\Omega_{GW}(f) \propto f^{2/3}$$

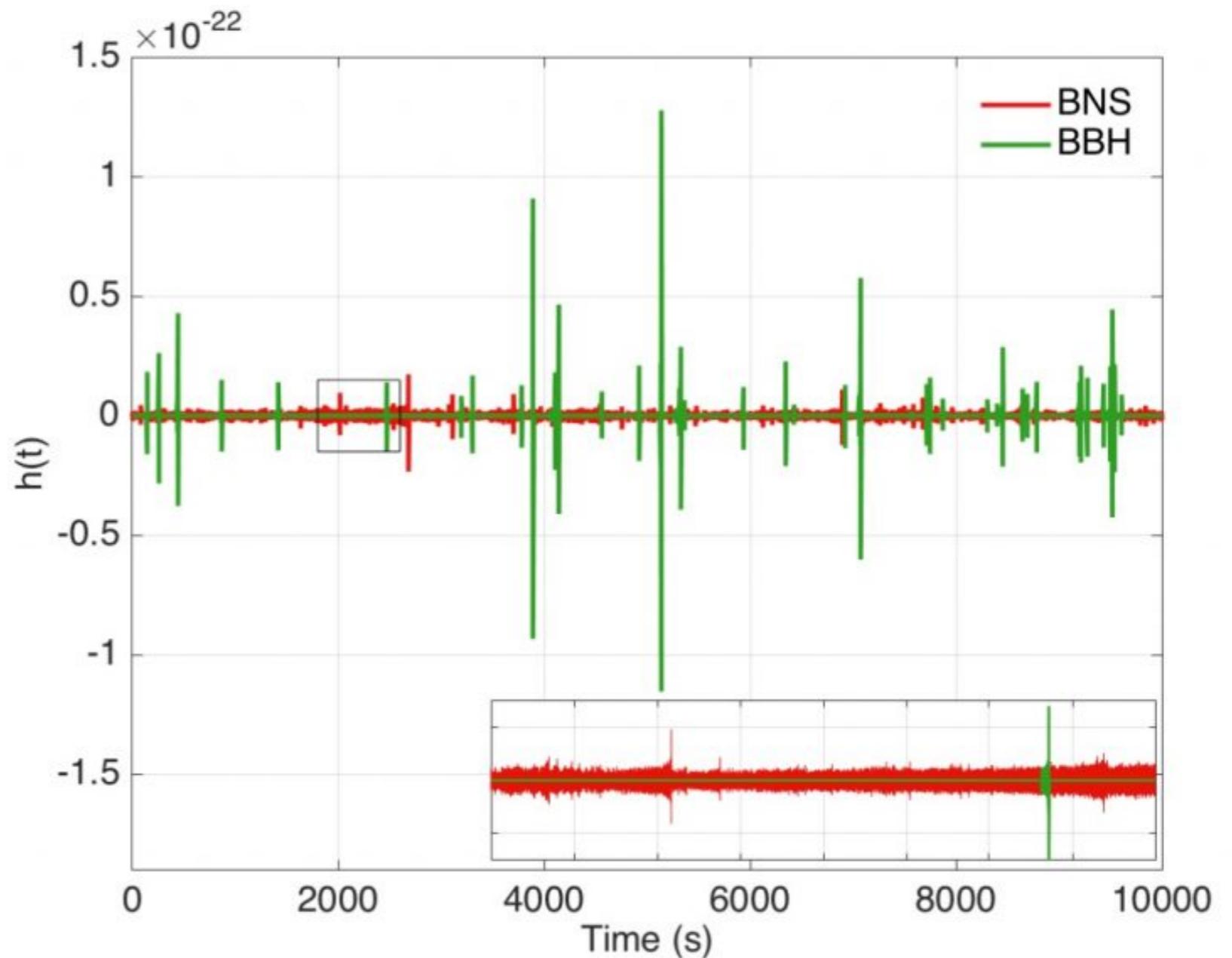


Ref: [arxiv 1801.02617](https://arxiv.org/abs/1801.02617)

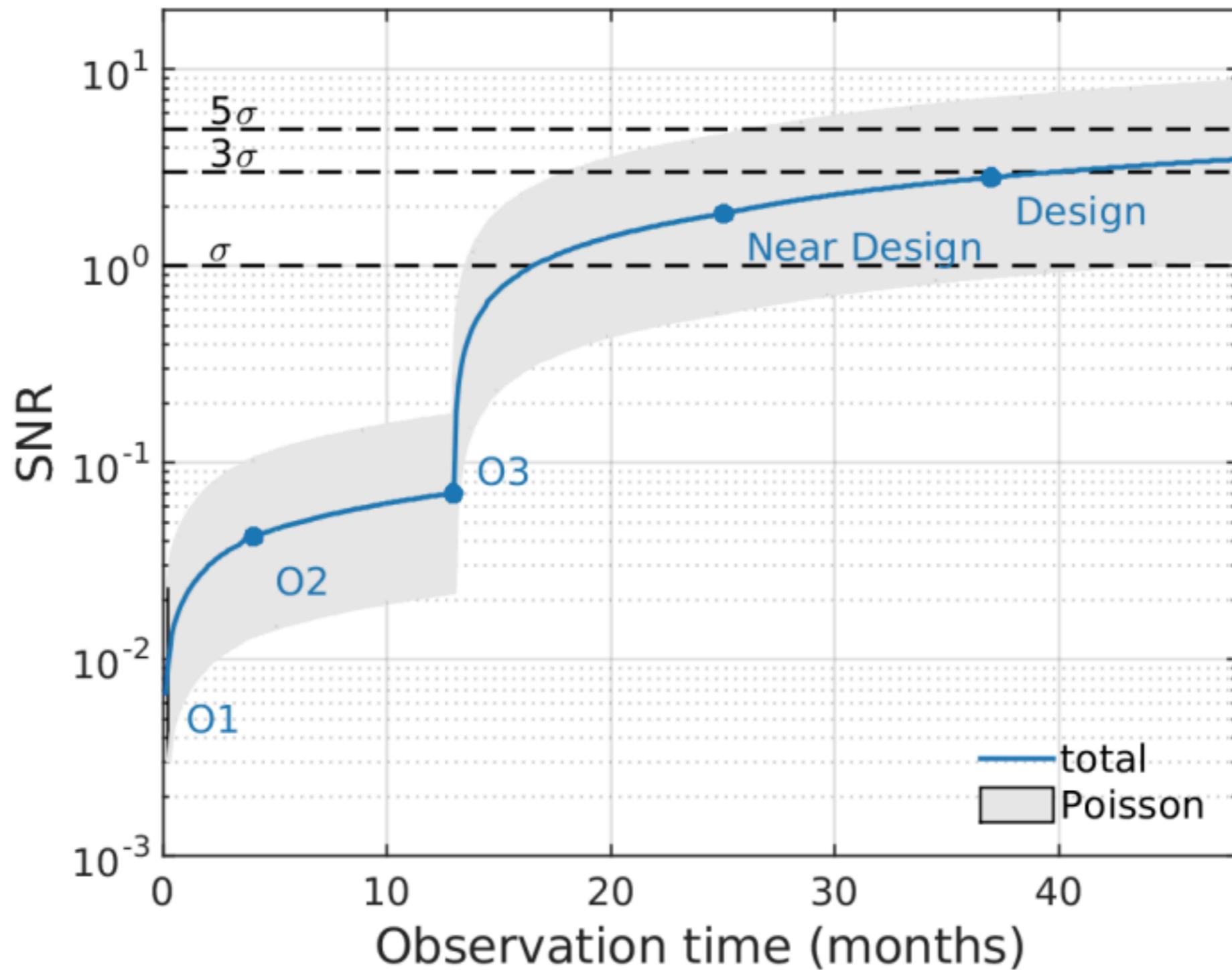
The Stochastic CBC Background

"Popcorn" BBH background
and persistent BNS
background.

[arxiv ref](#)



The Stochastic GW Background



Isotropic SGWB Bayesian Search

	Uniform prior		Log-uniform prior	
α	O1+O2	O1	O1+O2	O1
0	6.0×10^{-8}	1.7×10^{-7}	3.5×10^{-8}	6.4×10^{-8}
2/3	4.8×10^{-8}	1.3×10^{-7}	3.0×10^{-8}	5.1×10^{-8}
3	7.9×10^{-9}	1.7×10^{-8}	5.1×10^{-9}	6.7×10^{-9}
Marg.	1.1×10^{-7}	2.5×10^{-7}	3.4×10^{-8}	5.5×10^{-8}

TABLE II. 95% credible upper limits on Ω_{ref} for different power law models (fixed α), as well as marginalizing over α , for combined O1 and O2 data (current limits) and for O1 data (previous limits) [66]. We show results for two priors, one which is uniform in Ω_{ref} , and one which is uniform in the logarithm of Ω_{ref} .

90% Confidence Upper Limits

$$0.90 = \int_{-\infty}^{\Omega_\alpha^{90}} p(\hat{\Omega}_\alpha | \Omega_\alpha) p(\Omega_\alpha) d\Omega_\alpha = \int_0^{\Omega_\alpha^{90}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\hat{\Omega}_\alpha - \Omega_\alpha)^2}{2\sigma^2}} d\Omega_\alpha$$

Isotropic SGWB Models

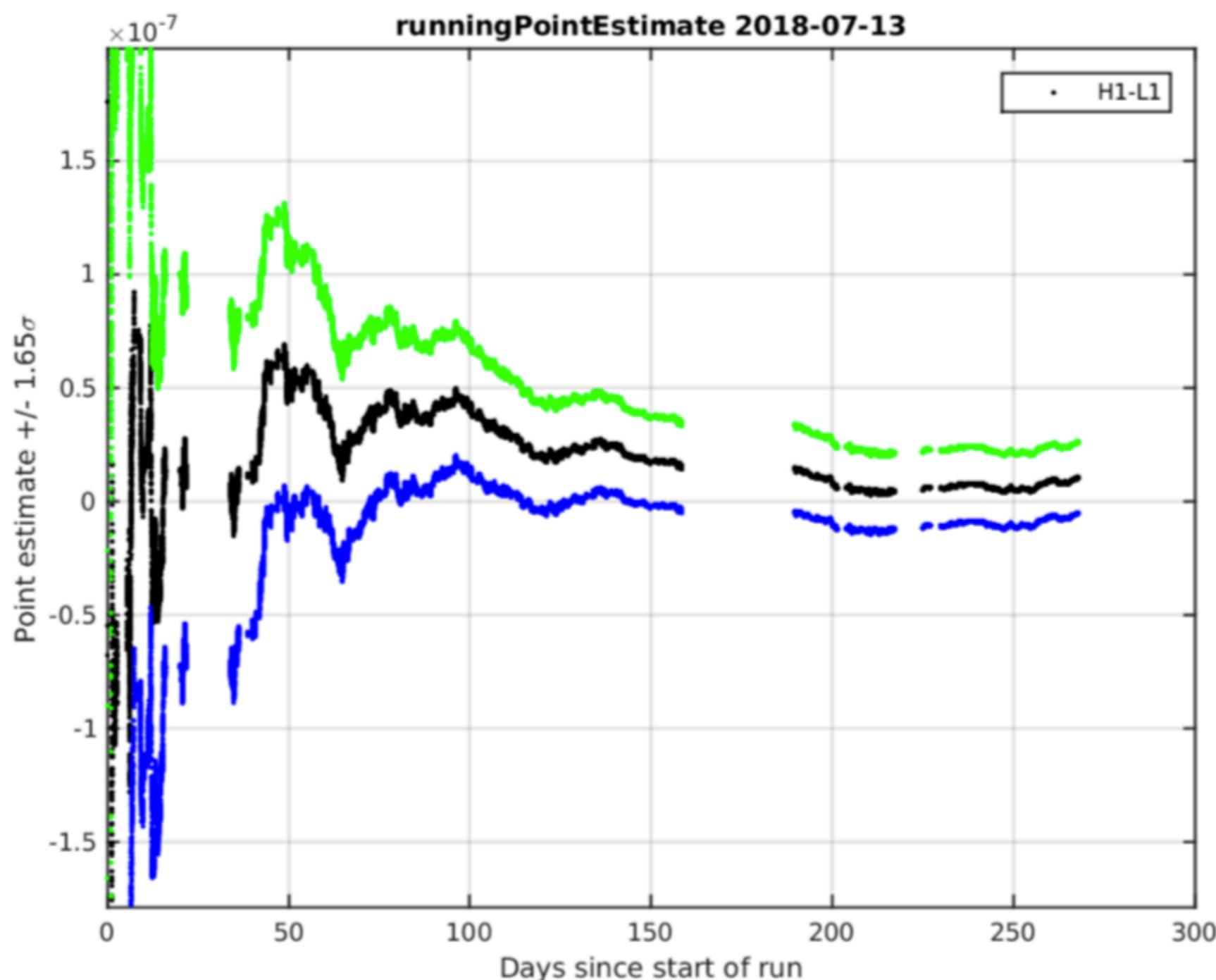
GW170817: Implications for the Stochastic Gravitational-Wave Background from Compact Binary Coalescences

The LIGO Scientific Collaboration and The Virgo Collaboration
(Dated: October 17, 2017)

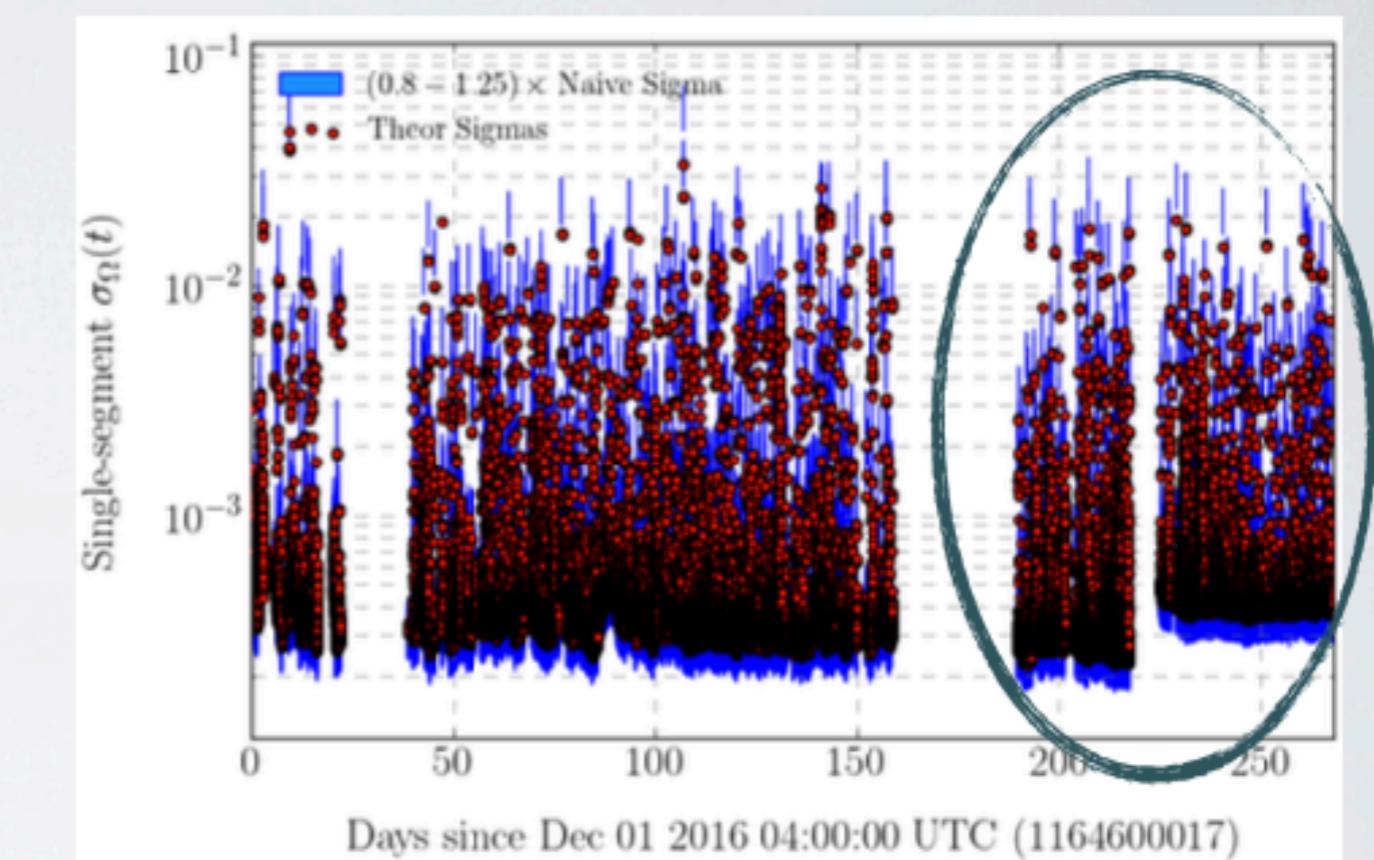
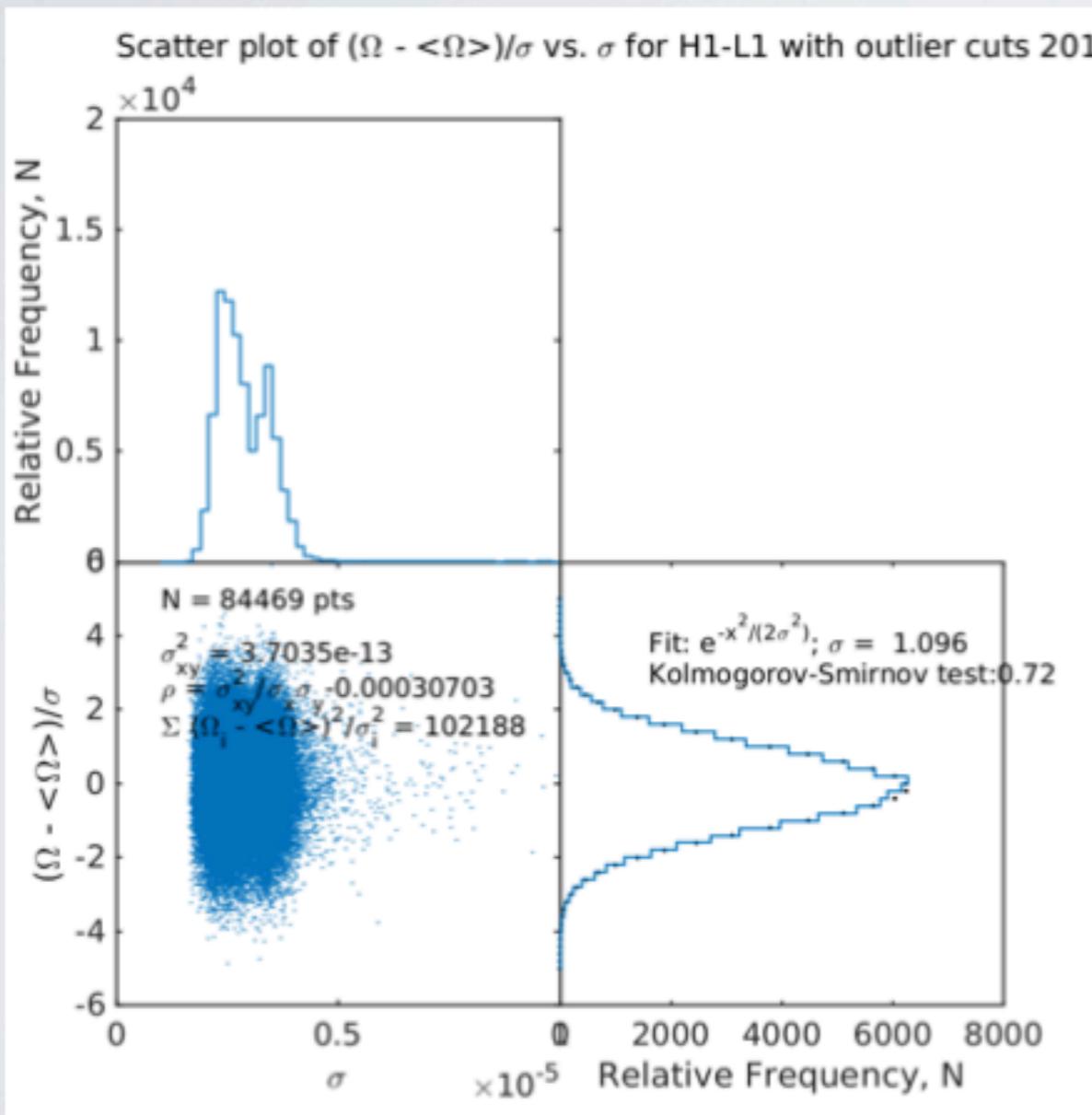
TABLE I. Estimates of the background energy density $\Omega_{\text{GW}}(f)$ at 25 Hz for each of the BNS, BBH and total background contributions, along with the 90% Poisson error bounds. We also show the average time τ between events as seen by a detector in the frequency band above 10 Hz, and the number of overlapping sources at a given time λ . We quote the number given the median rate and associated Poisson error bounds.

	$\Omega_{\text{GW}}(25 \text{ Hz})$	τ [s]	λ
BNS	$0.7^{+1.5}_{-0.6} \times 10^{-9}$	13^{+49}_{-9}	15^{+30}_{-12}
BBH	$1.1^{+1.2}_{-0.7} \times 10^{-9}$	223^{+352}_{-115}	$0.06^{+0.06}_{-0.04}$
Total	$1.8^{+2.7}_{-1.3} \times 10^{-9}$	12^{+44}_{-8}	15^{+31}_{-12}

Running Point Estimate



BIMODAL SIGMA DISTRIBUTION

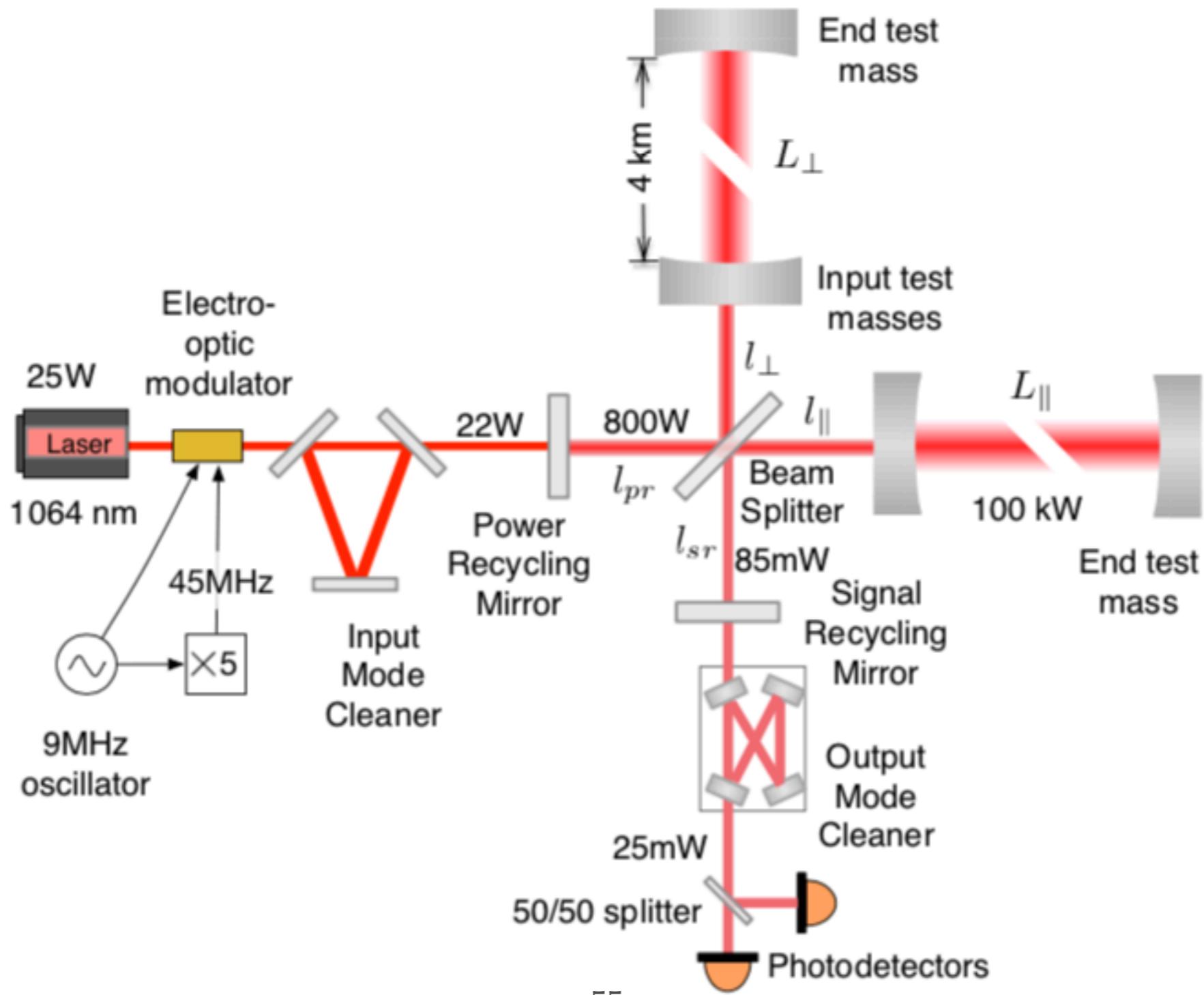


Systematic jump in sigma around July caused by earthquakes near Hanford, WA

SGWB Constraints

- COBE Multipole moments ($2 \leq \ell \leq 30$): $\Omega_{GW} h_{100}^2 < 7 \times 10^{-11} \left(\frac{H_0}{f}\right)^2$ for $H_0 < f < 30H_0$
- Pulsar Timing Arrays: $h_c \leq 10^{-14} \rightarrow \Omega_{GW}(f = 10^{-8}) < 10^{-8}$
- Nucleosynthesis constrains expansion rate which constrains energy density which constrains energy-density in a cosmological background of GWs: $\int \Omega_{GW} d(\ln f) \leq 0.07 \times (1 + z_{eq})^{-1} \approx 10^{-5}$ where $z_{eq} = 6000$

IFO Schematic



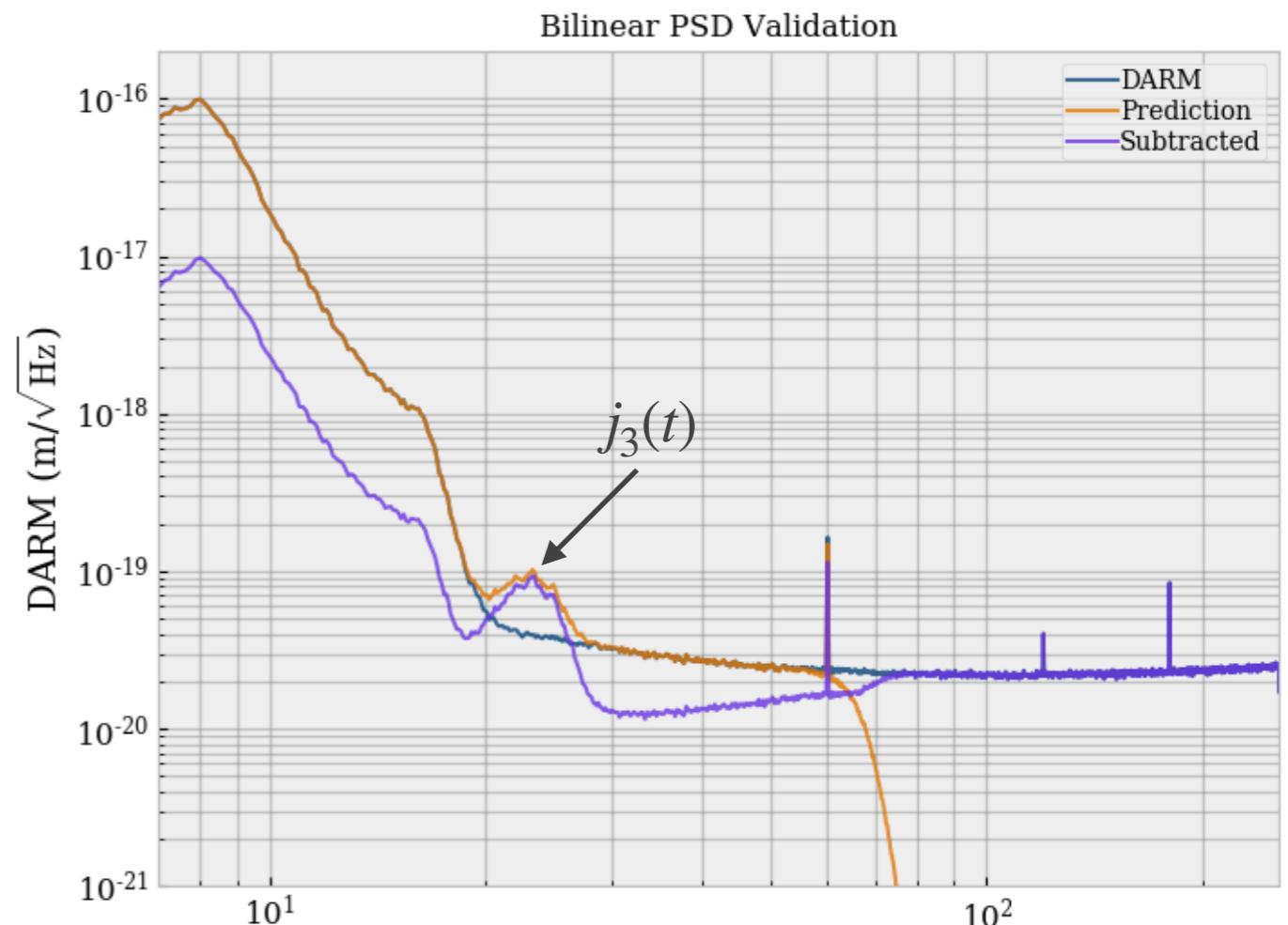
Mock Data Test

Bilinear noise added to
mock DARM spectrum

Jitter Channels = $[j_1(t), j_2(t)]$

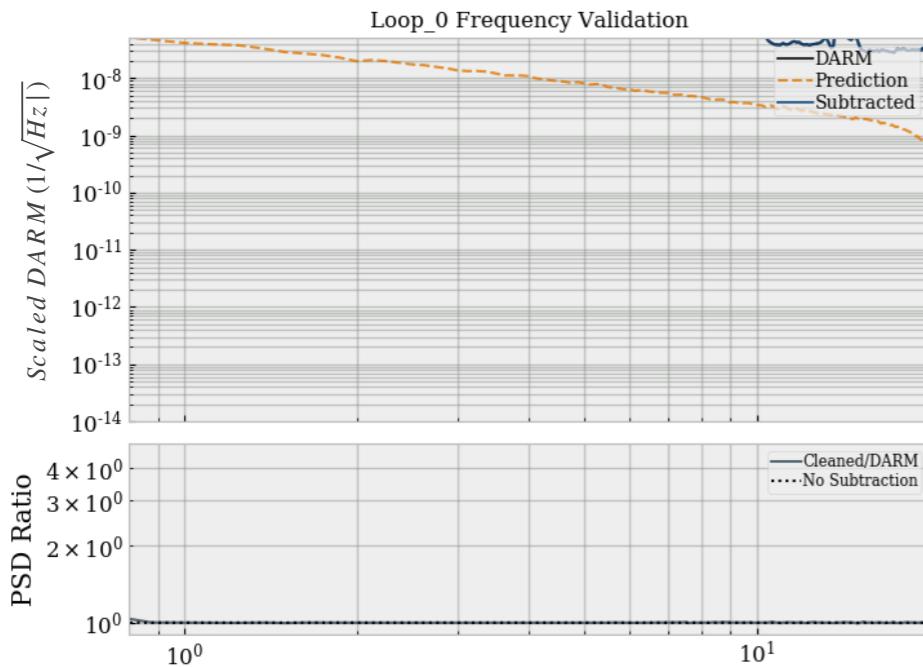
Witness Channels = $[j_1(t), j_2(t)]$

DARM = $h(t) + \alpha * j_1(t) * j_2(t)$
 $+ \beta * j_3(t)$

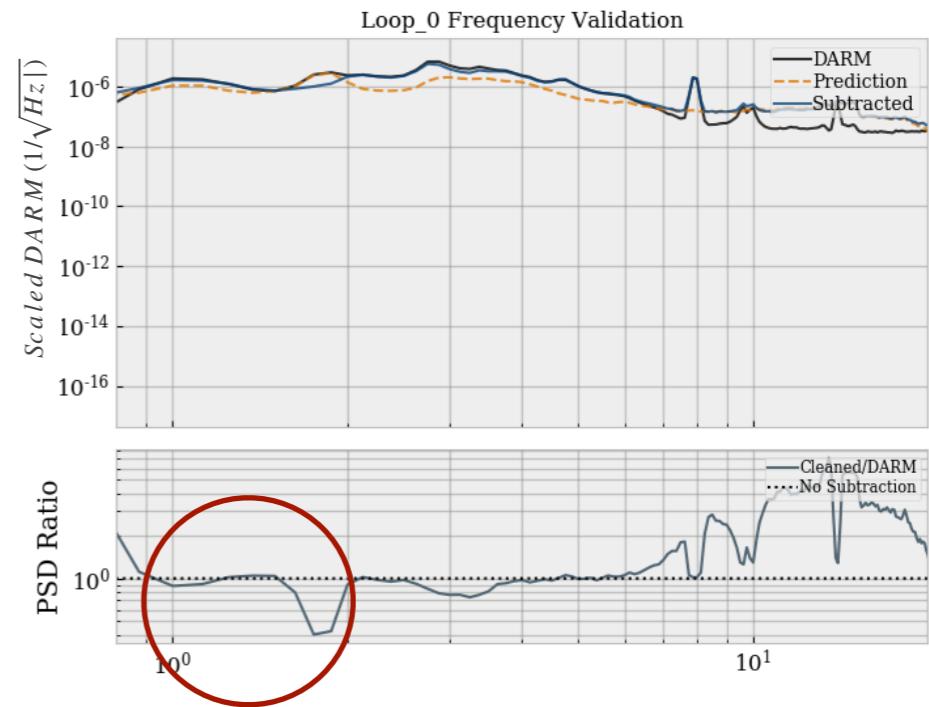


Removing low frequency bilinear
jitter added to mock data (generated
using the [MockData](#) repo on GitLab)

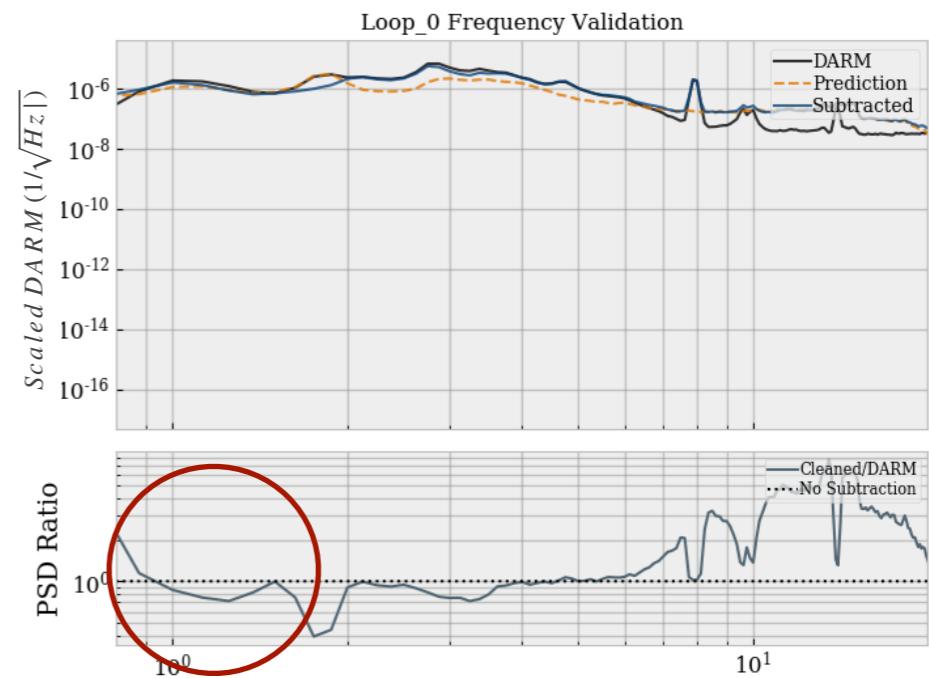
LHO 01 GW150914



Top Left: Length Sensing and Control
(LSC)channels only



Top Right: Alignment Sensing and Control
(ASC) channels only



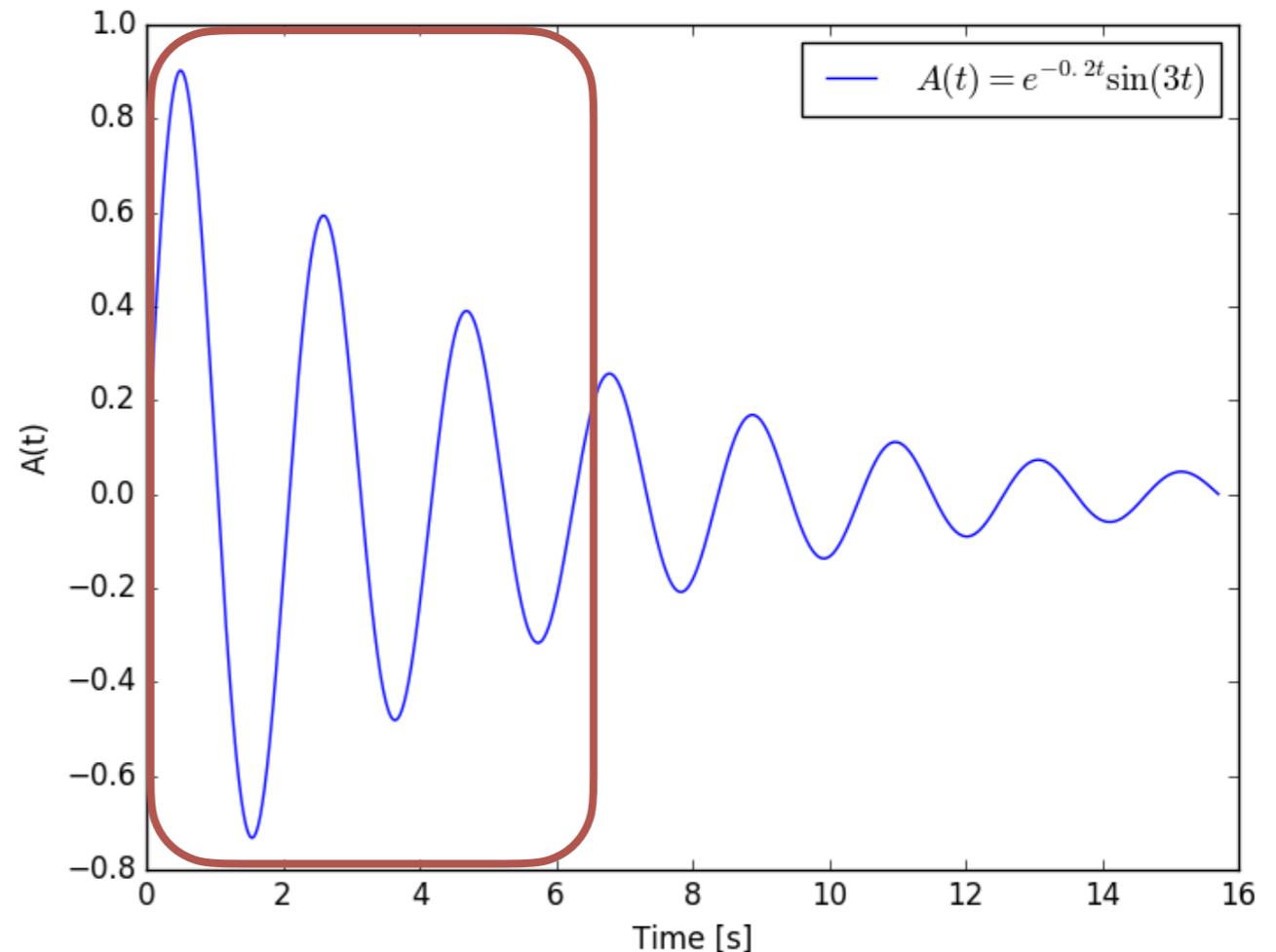
Bottom Right: LSC + ASC channels

Picking up low-frequency mode?

Recurrent Neural Networks - 1

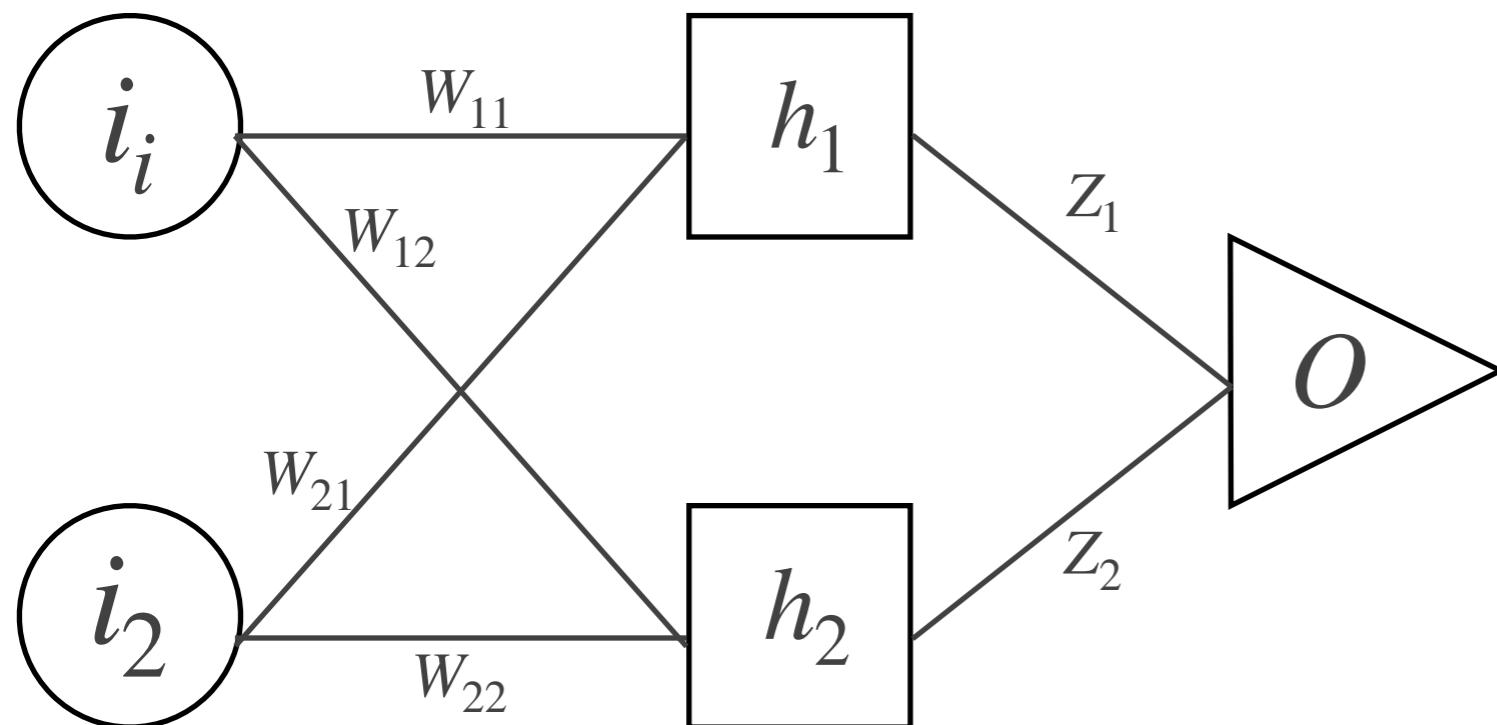
How can we capture long-term dependencies?

For the LIGO detectors, it can take seismic noise several seconds to reach the corner station and couple into the IFO, but our data is synced with GPS times. So, constantly shift data streams or try and *remember* them



Given the data in the red box, can the data that follows be predicted?

Linear Example



$$h_1 = i_1 W_{11} + i_2 W_{21}$$

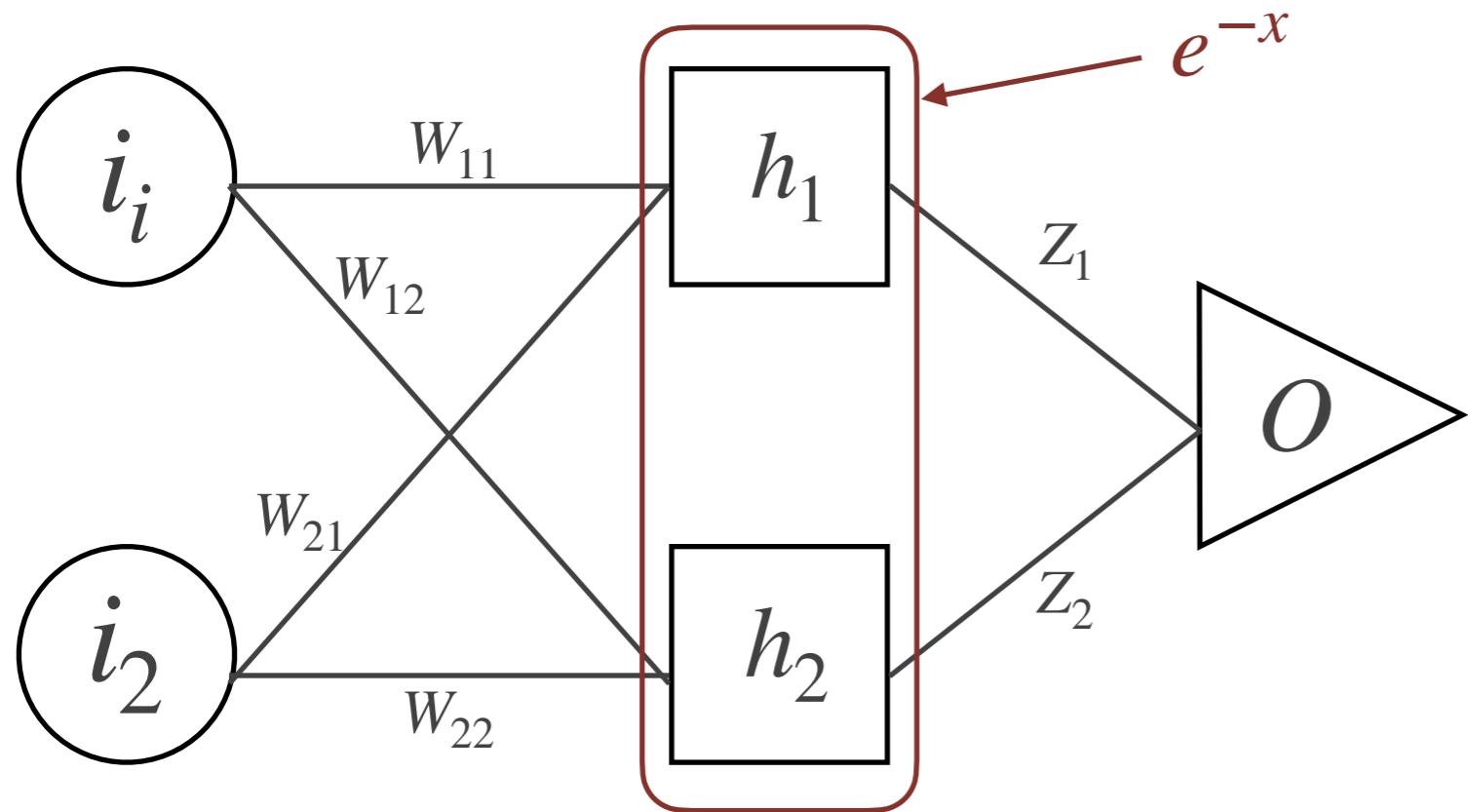
$$h_2 = i_1 W_{12} + i_2 W_{22}$$

$$O = h_1 Z_1 + h_2 Z_2$$

$$\rightarrow O = i_1(W_{11}Z_1 + W_{12}Z_2) + i_2(W_{21}Z_1 + W_{22}Z_2)$$

Output is a *linear* function of the input

Nonlinear Example



$$h_1 = e^{-(i_1 W_{11} + i_2 W_{21})}$$

$$h_2 = e^{-(i_1 W_{12} + i_2 W_{22})}$$

$$\rightarrow O = Z_1 e^{-(i_1 W_{11} + i_2 W_{21})} + Z_2 e^{-(i_1 W_{12} + i_2 W_{22})}$$

Output is a *nonlinear* function of the input

Layers != Complexity

Layer 1:

$$y = Wx + b$$

Layer 2:

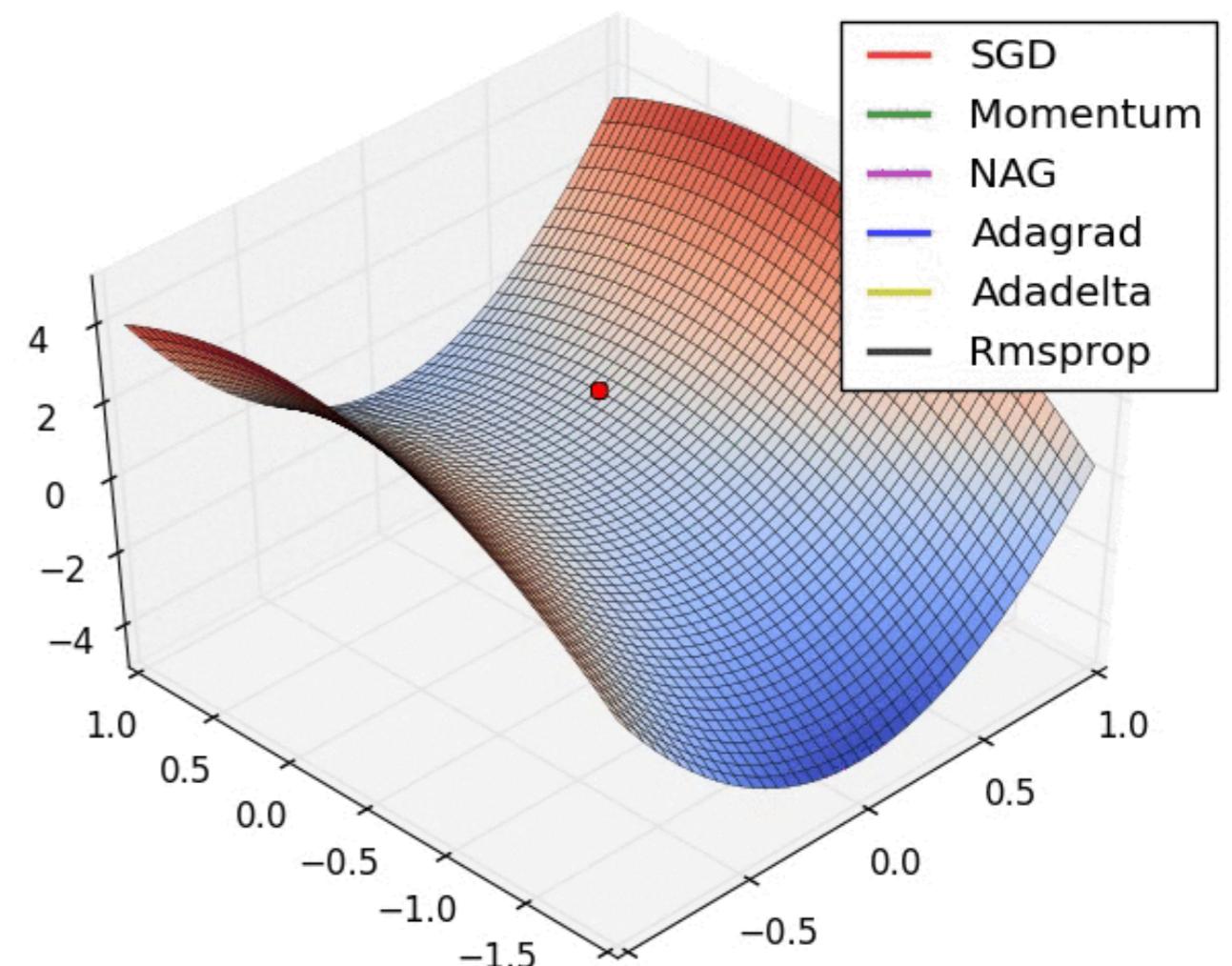
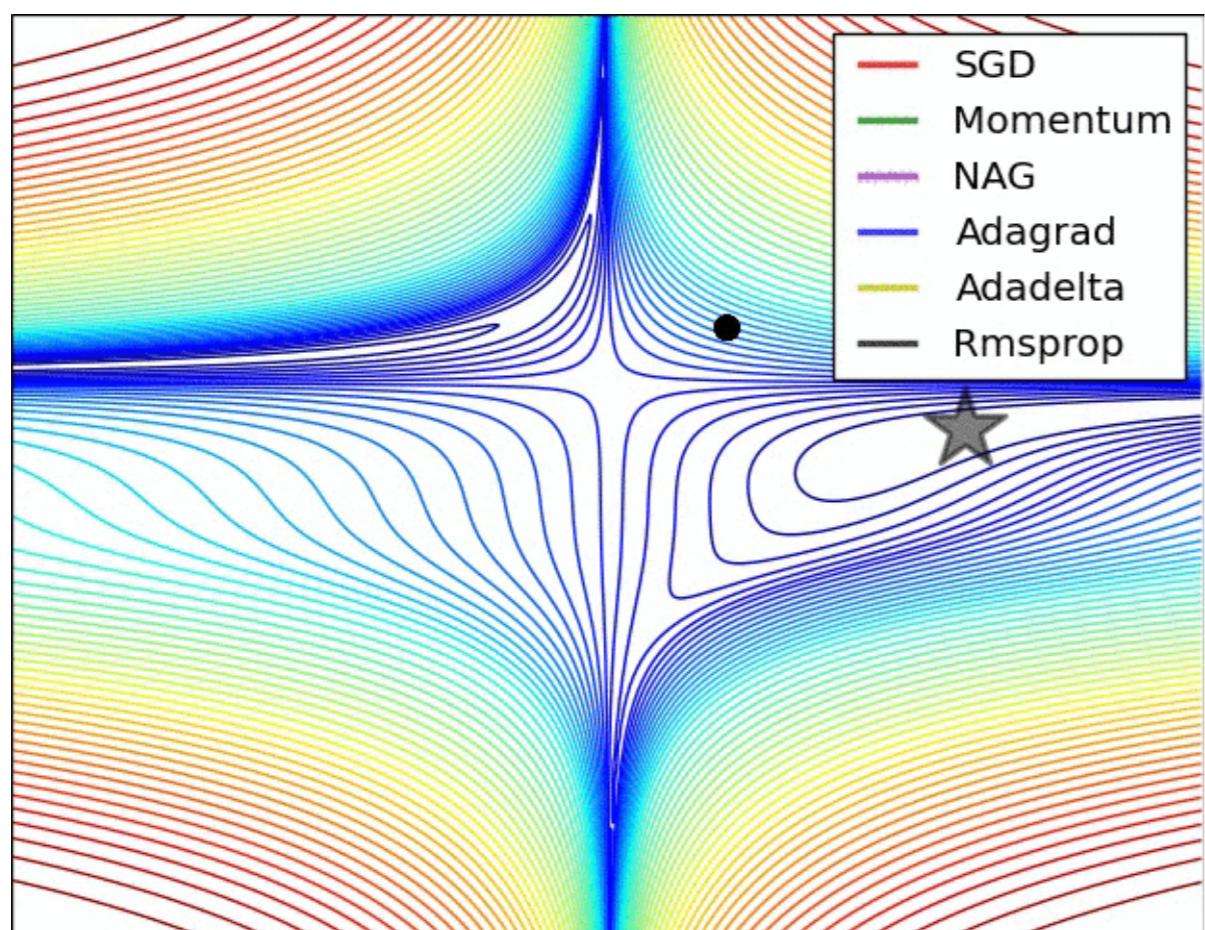
$$z = Vy + c$$

Result?

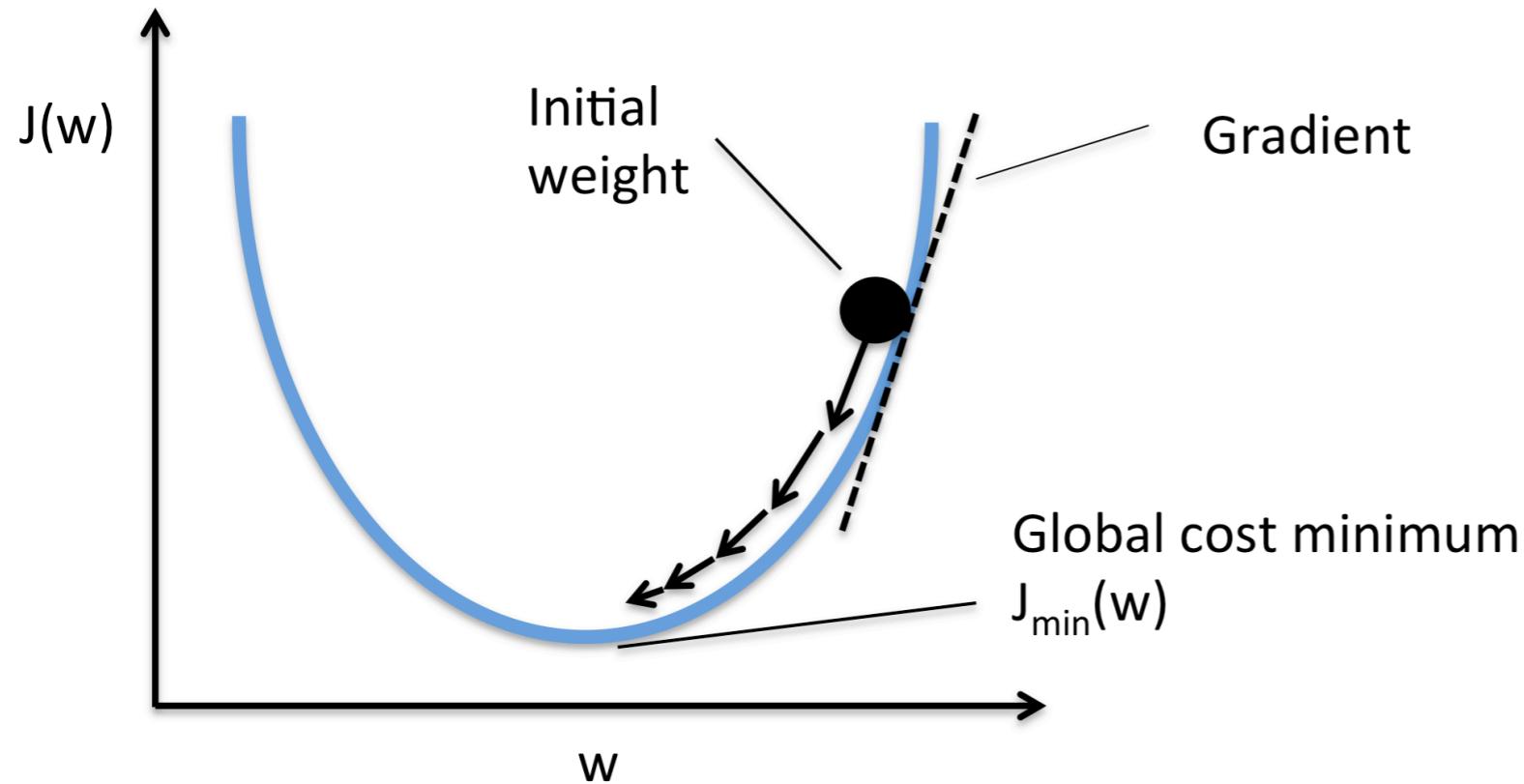
$$z = V(Wx + b) + c = W'x + (Vb + c) = W'x + b'$$

Increased training time without gaining functionality

Optimizers



Gradient Descent & Cost



$$\mathbf{w} \rightarrow \mathbf{w} - \eta \vec{\nabla} \sum_i J(w_i)$$

The learning rate, eta, sets the step size as represented by the arrows in the plot. Often, the learning rate is a decaying function of the epoch to improve convergence.

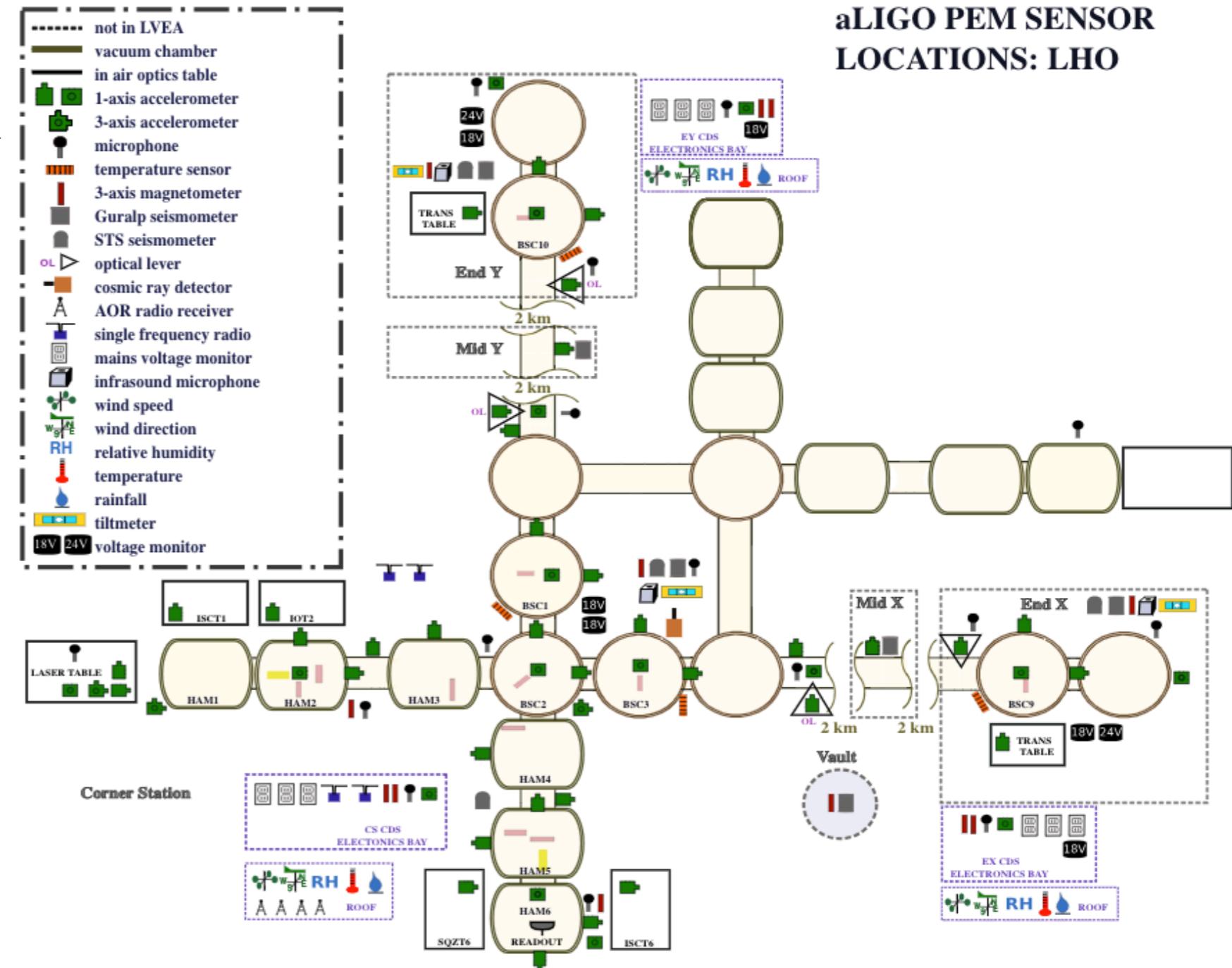
Easy to converge with simple parameter spaces. Extremely difficult and computationally costly with high dimensionality spaces.

Hanford PEM Locations

Approach - Put a large variety of physical environmental monitors everywhere

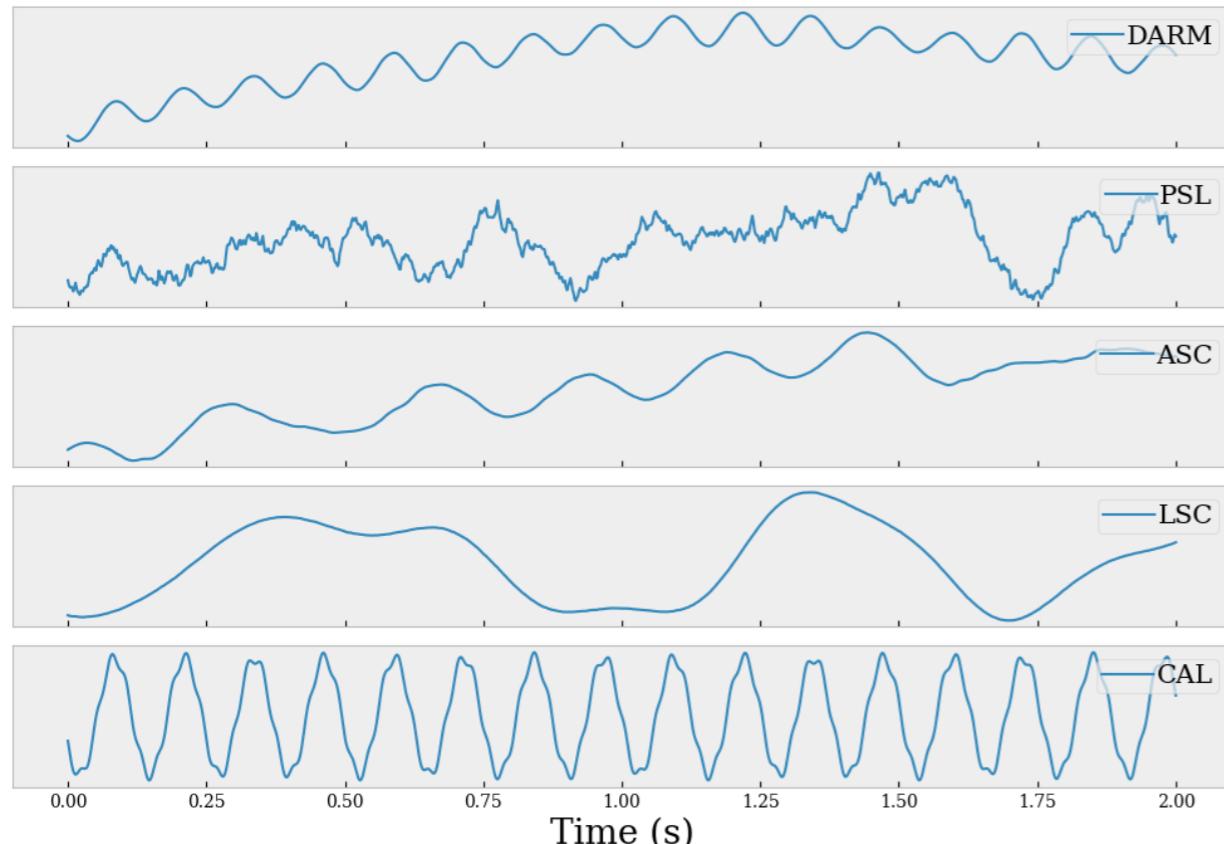
1055 channels at LHO
1000 channels at LLO

~500,000 2-channel combos
~2_E 3-channel combos



DeepClean Training

Input Data



Time Series Prediction

