ASSIGNMENT III

Problem 1

* From the table, $P(X=1, Y=1) = 0.1$

* $P(X=0) = \sum_i P(X=0, Y_i) = 0.06 + 0.03 + 0.02 + 0.01 = 0.12$

* $P(X/Y) = \frac{P(X,Y)}{P(Y)} = \frac{0.2}{0.1+0.03+0.04+0.2} = 0.54$

Problem 2

* For a $f(x,y)$ to be a pdf, the area under the curve should be equal to 1.

$$\int_0^1 \int_0^1 c(x+y^2)\,dy\,dx = c \int_0^1 \int_0^1 (x+y^2)\,dy\,dx = c\,\frac{5}{6} \Rightarrow 1$$

$$\therefore c = 6/5$$

* $$g(X \mid Y = 1/2) = \frac{f(X, Y=1/2)}{f_Y(Y=1/2)}$$

$$f_Y(y) = \int_0^1 f(x,y)\,dx = \int_0^1 \frac{6}{5}(x+y^2)\,dx = \left(y^2 + \frac{1}{2}\right)6/5 \Rightarrow f_Y(1/2) = \frac{3}{4}$$

$$g(x \mid Y = 1/2) = \frac{6/5\,(x+1/4)}{3/4} = \frac{24}{15}\,(x+1/4)$$

$$P(0<X<1/4 \mid y=1/2) = \int^{1/4} \frac{24}{} (x + 1/) dx = 0.1$$

$$P(0 \le X \le 1/4 \mid Y = 1/2) = \int_0^{1/4} \frac{24}{15}(x + 1/4)\,dx = 0.15$$

* $E(X \mid Y = 1/2) = \int_0^1 x\, g(x \mid y = 1/2)\,dx = \int_0^1 \frac{24}{15}(x^2 + x/4)\,dx = 0.73$

Problem III

* Yes, I have all necessary information. For continuous independent random variables, the joint probability is equivalent to the product of the probability density function.

* $f(x,y) = f_X(x) \cdot f_Y(y)$

$$f(x,y) = 2e^{-2x} \cdot 3e^{-3y} = 6e^{-(2x+3y)}$$

$$P(X+Y \ge 3) = P(X, Y \ge 3 - X) = \int_0^3 \int_0^{3-x} 6e^{-(2x+3y)}\,dy\,dx$$

$$P(X+Y \ge 3) = 0.99281$$

* No, it doesn't because X and Y are independent variables

Problem IV

* It's a curve with vertical asymptote at origin while $\lim_{x \to \infty} y = 0$ (similar to $y = 1/x^2$)

? * $f(x,y)$ is the same thing as a function $f(r)$ through combination of x and y by equation $x^2+y^2=r^2$ where r is the location where the dart hits

$$P(X^2+Y^2 \le R/36) = 4\int_0^{R/8} \int_0^{\sqrt{R/6 - x^2}} \frac{1}{\pi R^2}\,dy\,dx = \frac{1}{\pi R} \int_0^{R/8} \int_0^{\sqrt{R^2/64 - x^2}} dy\,dx$$

$$= \frac{4}{\pi R^2}\int_0^{R/8} \sqrt{R^2/64 - x^2}\, dx$$

$$= \frac{4}{\pi R^2}\left(\frac{\pi R^2}{256}\right)$$

$$= \frac{1}{64}$$

* $f_x(x) = 4\int_0^{\sqrt{R^2-x^2}} \frac{1}{\pi R^2}\, dy$, $f_y(y) = 4\int_0^{\sqrt{R^2-y^2}} \frac{1}{\pi R^2}\, dx$

* No, they are not independent

* $f_x(x) = 4\int_0^{\sqrt{R^2-(R^2/4)}} \frac{1}{\pi R^2}\, dy = \frac{4}{\pi R^2}\, y\Big|_0^{\frac{\sqrt{3}}{2}R} = \frac{4}{\pi R^2}\left(\frac{\sqrt{3}}{2}R\right) = \frac{2\sqrt{3}}{\pi R}$

$g(y|x) = \frac{f(x,y)}{f_x(x)} = \frac{1}{\pi R^2} \cdot \frac{\pi R}{2\sqrt{3}} = \frac{1}{2\sqrt{3}\, R}$

$P(y>0 \mid x = R/2) = \int_0^R g(y|x)dy = \int_0^R \frac{1}{2\sqrt{3}R}\, dy = \frac{1}{2\sqrt{3}R}\, y\Big|_0^R$

$\therefore\ P = \frac{1}{2\sqrt{3}}$

* $E(g(y|x)) = \int_{-\infty}^{\infty} g(y|x)\cdot f(x,y)\, dy$

$= \int^R \ldots$

$$= \int_{-R}^{R} \frac{1}{2\sqrt{3}R} \cdot \frac{1}{\pi R^2} \, dy$$

$$= 2 \cdot \frac{1}{2\sqrt{3}R^3\pi} \, y \Big|_0^R$$

$$= \frac{1}{\pi\sqrt{3}R^2}$$

Problem V

* Parameters are $N\left(10, \frac{10}{\sqrt{10}}\right)$ i.e $N\left(\bar{x}, \frac{\sigma}{\sqrt{n}}\right)$

* Yes, because the sample size is small so we don't have a good estimation of the pdf.

Problem VI

* The plot of p vs N has a horizontal assymptote at $p=0.5$ which means that as you increase the number of trials the probability of getting a head or tail goes close to the real probability (½)

of trials the probability of getting a head or tail goes close to the real probability ($\frac{1}{2}$)

The plot N_e vs N has a horizontal asymptote at $N_e = 0$ which suggest that the expectation approaches the real mean as you increase sample size

In [18]:
```
from scipy.stats import expon
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import seaborn as sns

def get_service_times_blackbox(n):
    # Return service times of n tellers
    rv = expon(scale=10)
    return rv.rvs(size=n)
```

In [19]:
```
print(get_service_times_blackbox(20))
```

```
[ 1.43762997  4.76287196  5.18890875 13.37856343  8.73393117  4.91481367
  1.90131632 22.79099897 40.71722654  4.72582637  0.53100067 19.75444886
  5.28066227  5.08101482  9.14303588 10.8628755  23.51425576  0.12702145
  8.8309475   0.34637471]
```

In [20]:
```
def average_get_service(n):
    x=sum(get_service_times_blackbox(n))

    return x/n
```

In [21]:
```
print(average_get_service(20))
```

```
8.17742954888861
```

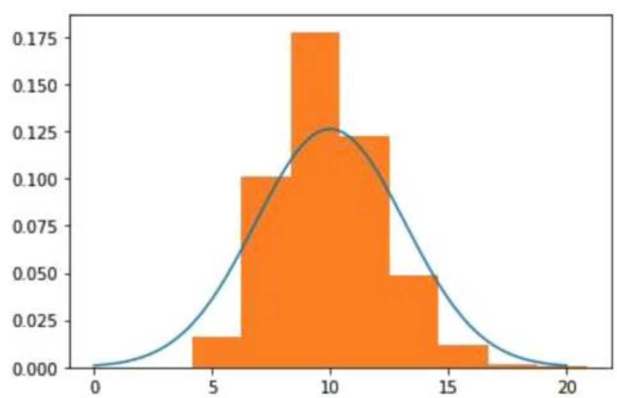
In [44]:
```
def histogram_for_average(m):
    x=[0]
    for i in range (0,m):
        y=average_get_service(20)
        x.append(y)
    return x   #plt.hist(x, density=True)
```

In [46]:
```
x = np.linspace(0,20,100)

#Creating a Function.
def normal_dist(x , mean , sd):
    prob_density = (1/(sd*(2*np.pi)**0.5)) * np.exp(-0.5*((x-mean)/sd)**2)
    return prob_density


#Apply function to the data.
y1 = histogram_for_average(10000)
y2 = normal_dist(x,10,3.162277)
#Plotting the Results
#plt.plot(x,y1)
plt.plot(x,y2)
plt.hist(y1, density=True)
```

Out[46]:
```
(array([4.79975256e-05, 1.91990102e-04, 1.58391834e-02, 1.00698809e-01,
        1.77638842e-01, 1.22681675e-01, 4.86694909e-02, 1.20953764e-02,
        1.77590845e-03, 3.83980205e-04]),
 array([ 0.        ,  2.08323241,  4.16646483,  6.24969724,  8.33292965,
        10.41616206, 12.49939448, 14.58262689, 16.6658593 , 18.74909172,
        20.83232413]),
 <BarContainer object of 10 artists>)
```



In [ ]:

In [ ]:

In [35]:
```
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import seaborn as sns


def get_n_coin_tosses ( n = 1 ):

    return np.random.randint (2, size = n)
```

In [36]:
```
print(get_n_coin_tosses(10))
```

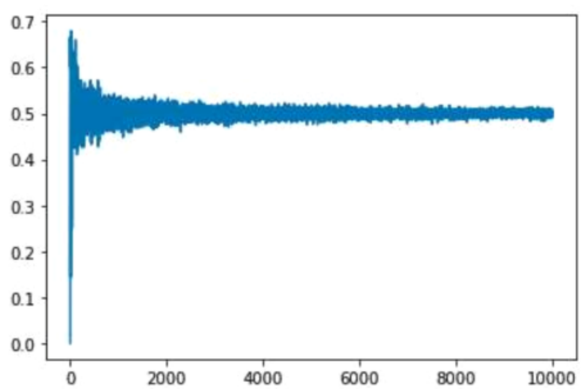
```
[1 0 0 1 1 0 0 1 1 1]
```

In [37]:
```
def fraction_heads (N):
    x=get_n_coin_tosses (N)
    a=sum(x)
    return a/N
```

In [38]:
```
def simulation (m):
    x=np.arange(1,m+1)
    y=[]
    for i in x:
        y.append(fraction_heads(i))
    return x,y
```

In [40]:
```
x,y=simulation(10000)
plt.plot(x,y)
```

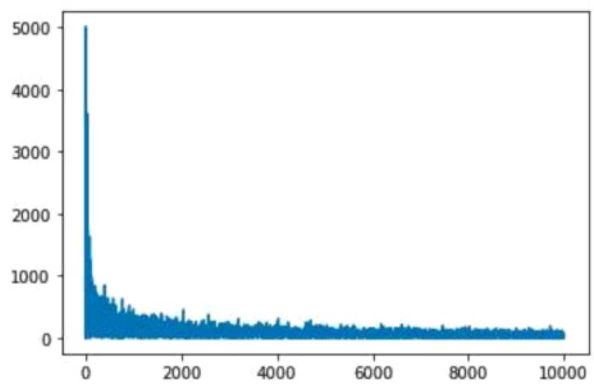
Out[40]: [<matplotlib.lines.Line2D at 0x2c1d64d0d30>]



In [48]:
```
def get_Ne (N):
    x,y= simulation(N)
    Ne=[]
    for i in y:
        c= N * abs(i-1/2)
        Ne.append(c)
    return plt.plot(x, Ne)
```

In [50]:
```
print(get_Ne(10000))
```

```
[<matplotlib.lines.Line2D object at 0x000002C1D6819730>]
```



In [ ]: