ASSIGNMENT 2

Sunday, October 2, 2022 4:37 PM

ASSIGNMENT II

(1)

$P(X) = 0.8$
$P(Y) = 0.18$
$P(Z) = 0.02$

Let's define a new variable H that stands for over heating

$P(X \cap H) = 0.02$

$P(Y \cap H) = 0.08$

$P(Z \cap H) = 0.35$

$* P(H) = P(X \cap H) + P(Y \cap H) + P(Z \cap H) = 0.02 + 0.08 + 0.35 = 0.45$

$* P(X | H) = \frac{P(X \cap H)}{P(H)} = \frac{0.02}{0.45} = 0.044$

$* P(Z | H) = \frac{P(Z \cap H)}{P(H)} = \frac{0.35}{0.45} = 0.778$

(2)

* Y is a continuous random variable ??

$* Y = \{ \{-x\} \{-x+1\} \{-x+2\} \{-x+3\} \{-x+10\} \{-x+5\} \}$

Y	$P(Y=y)$
$\{-x\}$	$9/13$
$\{-x+1\}$	$1/13$
$\{-x+2\}$	$1/13$
$\{-x+3\}$	$1/13$
$\{-x+10\}$	$1/52$
$\{-x+5\}$	$3/52$

$E(P(y)) = \sum_y y\,p(y) = 1/13 + 2/13 + 3/13 + 10/52 + 15/52 - x$
$= 49/52 - x$

$E(P(y))$ is the amount of money the player expects to win/loss

For you to gain \$0.25, then $0.25 = -\left(\frac{49}{52} - x\right)$

$x = 1.19$

(3) $f(x) = \dfrac{c}{(x+2)^4}$ for $x > 0$ and $f(x) = 0$ for rest

* If $f(x)$ is a pdf, then the integral over the whole range of x should be equal to one i.e.

$$\int_0^\infty f(x)\,dx \Rightarrow \int_0^\infty \frac{c}{(x+2)^4}\,dx$$

$$= c \int_0^\infty \frac{1}{(x+2)^4}\,dx$$ let ∞ be replaced by b

$$\therefore c\int_0^\infty \frac{1}{(x+2)^4}\,dx = c\int_0^b \frac{1}{(x+2)^4}\,dx = c\left[-\frac{1}{3(x+2)^3}\right]_0^b$$

$$c\left[\lim_{b\to\infty}\left(-\frac{1}{3(b+2)^3}\right) + \frac{1}{3(0+2)^3}\right]$$

$$c\,(0 + 1/24) = 1 \Rightarrow c/24 = 1$$

$$\therefore\ c = 24 \quad \text{and} \quad f(x) = \frac{24}{(x+2)^4}$$

* $P(X > 2) = \displaystyle\int_2^\infty \frac{24}{(x+2)^4}\,dx = \frac{1}{8}$

* $E(X) = \displaystyle\int_0^\infty x f(x)\,dx = \int_0^\infty x \cdot \left(\frac{24}{(x+2)^4}\right) dx = 24\int_0^\infty \frac{x}{(x+2)^4}\,dx = 1$

* $f(x) = \dfrac{d}{dx}\left(F(x)\right) \Rightarrow F(x) = \displaystyle\int_{-\infty}^{x} f(x)\,dx \Rightarrow F(x) = \int_0^x f(x)\,dx$

because pdf is made equal to zero $-\infty \le x \le 0$.

$$F(x) = \int_0^x \frac{24}{(t+2)^4}\,dt = \left[-\frac{8}{(t+2)^3}\right]_0^x = -\frac{8}{(x+2)^3} + 1$$

* The median is a random variable x ; $F(x) = 0.5$

$$-\frac{8}{(x+2)^3} + 1 = 0.5$$

$$-\frac{8}{(x+2)^3} = -0.5$$

$$16 = (x+2)^3$$

$$x+2 = \sqrt[3]{16}$$

$$x = \sqrt[3]{16} - 2$$

* Yes I would because the expectation is 1, therefore it is very likely for an amazon battery to live up to a year

(4)

Poisson Distribution.

* Parameters are number of occurence "k" and the expected value "λ"
* k and λ should be positive non-zero integers
* The expected value and variance are all equal to λ
* Calls per hour at a call centre
  Number of arrivals at a resturant

Exponent Distribution.

* Parameter of distribution is "λ"
* λ should be greater than zero
* The expected value $E(x) = 1/\lambda$ while $Var(x) = 1/\lambda^2$
* Rate of incoming phone calls
  Radioactive decay

Gamma Distribution.

* Parameters are shape parameter "α" and inverse scale "β"
* α and β should be greater than zero
* $E(x) = \alpha/\beta$ and $Var(x) = \alpha/\beta^2$
* The waiting time for cell division
  Amount of rainfall accumulated might be modeled by a gamma distribution

Weibull Distribution

* Parameters are shape parameter "k" and scale "λ".
* k and λ are greater than zero
* $E(x) = \lambda\Gamma(1+1/k)$ and $Var(x) = \lambda^2\left[\Gamma(1+2/k) - (\Gamma(1+1/k))^2\right]$
* Extreme value theory
  Overvoltage occurence in in an electrical system

5.

The experiment can be modeled using bernouille distribution. I chose the probability for success to be "not being sick" i.e., P=1-0.001=0.999

To get the probability that a group of a hundred people tests negative, I used 100C100*P^100

To get the probability that a group of a hundred people tests positive, I used 1-(100C100*P^100)

I set Y to be a random variable that represents the number of groups that test negative

P(y)=(10Cy)*(p^y)*((1-p)^(10-y)) where p is the probability for a group to test negative

Probability of being sick	0.001
Probability of group of 100 tests negative	0.904792
Probability of group of 100 tests positive	0.095208

Y	**P(y)**	**n**	**nP(y)**
0	6.11967E-11	1010	6.1809E-08
1	5.81572E-09	910	5.2923E-06
2	2.48709E-07	810	0.00020145
3	6.30285E-06	710	0.00447502
4	0.000104822	610	0.06394124
5	0.001195387	510	0.60964734
6	0.009466802	410	3.88138901
7	0.051409256	310	15.9368693
8	0.183209773	210	38.4740523
9	0.386911978	110	42.5603176
10	0.367695425	10	3.67695425
Expectation			**105.207853**

In [19]:
```
from scipy.stats import bernoulli
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import seaborn as sns

def member_health_status(n, p):
# Get random variable
    H = bernoulli(p)

# Return health status
    return H.rvs(size=n)

print(member_health_status(100,0.001))
print(type(member_health_status(100,0.001)))
```

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
<class 'numpy.ndarray'>
```

In [20]:
```
def number_test_kits (n,p):
    Y=member_health_status(n,p)
    X=0
    for i in Y:
        X=X+i

    return 10+(X*100)

print(number_test_kits(100,0.001))
```

```
10
```

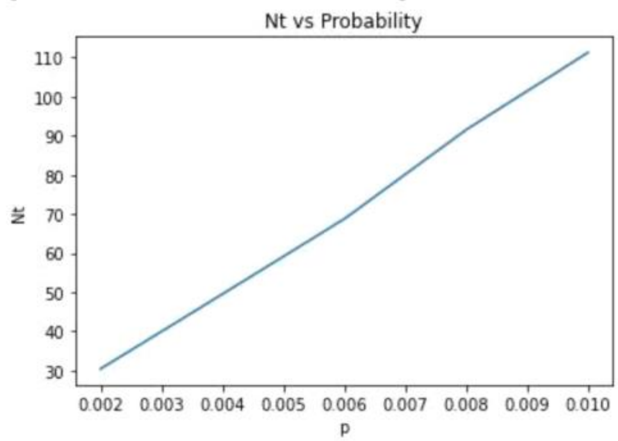
In [23]:
```
def expected_number_kits (n,p,Nu):
    x=0
    for i in range (0,Nu):

        a=number_test_kits(n,p)

        x=x+a
    return x/Nu

print(expected_number_kits(100,0.01,10000))
```

```
111.5
```

In [24]:
```
def make_a_plot(x):
    y=[]
    for i in x:
        #member_health_status(100, i)
        b=expected_number_kits(100,i,10000)
        y.append(b)
    print(x)
    print(y)


    plt.plot(x, y)
```

```
    plt.xlabel('p')

    plt.ylabel('Nt')


    plt.title('Nt vs Probability')


    plt.show()

print(make_a_plot([0.002, 0.004, 0.006, 0.008, 0.01]))
```

```
[0.002, 0.004, 0.006, 0.008, 0.01]
[30.4, 49.52, 68.72, 91.52, 111.27]
```



None

In [ ]:

(7) $0 \leq x \leq 1$ where x is the fraction of maximum possible time

$Y = 8X^3$

For $x=0$, $y=0$

For $x=1$, $y = 8(1)^3 = 8$

$0 \leq y \leq 8$

let $y_0 \in [0,8]$ be a random variable for Y such that

$8x^3 \leq y \Rightarrow x \leq \frac{1}{2}\sqrt[3]{y_0}$

$F(y_0) = P(y < y_0) = \int_0^{\frac{1}{2}\sqrt[3]{y_0}} f(x)\,dx = \int_0^{\frac{1}{2}\sqrt[3]{y_0}} 3x^2\,dx = x^3 \Big|_0^{\frac{1}{2}\sqrt[3]{y_0}}$

$F(y_0) = \frac{1}{8}y_0$

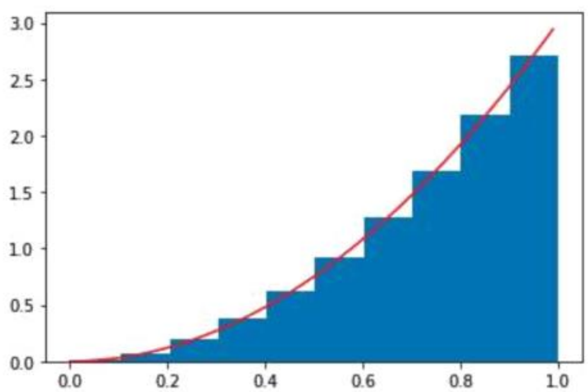
$f(y_0) = \frac{d}{dy}F(y_0) = \frac{d}{dy}\left(\frac{1}{8}y_0\right) = \frac{1}{8}$

$f(y_0) = \begin{cases} 1/8, & 0 \leq y \leq 8 \\ 0 & \text{elsewhere} \end{cases}$

In [1]:
```
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
import seaborn as sns


def get_driver_work_fraction():
    return (np.random.random())**(1/3)
```

In [2]:
```
def collect_work_function():
    x=[]
    for i in range (0,1000000):
        x.append(get_driver_work_fraction())
    x=np.array(x)
    return x
```

In [22]:
```
x=np.arange(0,1,0.01)

y=3*x**2

plt.plot(x, y, color='r')
a=collect_work_function()

plt.hist(a, density=True)

plt.show()
```

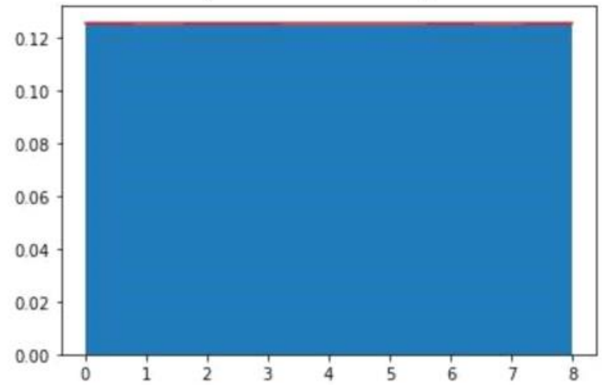


In [4]:
```
def get_hourly_wage():
    x=collect_work_function()
    y=8*x**3
    return y
```

In [25]:
```
x=np.arange(0,8,0.01)
y_star=np.ones(800)
y= (1/8)*y_star
plt.plot(x,y, color='r')
b=get_hourly_wage()
plt.hist(b,range=[0,8],density=True)
```

Out[25]:
```
(array([0.1250075 , 0.124795  , 0.1254525 , 0.125195  , 0.1249275 ,
        0.1248425 , 0.12477375, 0.12531375, 0.124615  , 0.1250775 ]),
```

```
array([0. , 0.8, 1.6, 2.4, 3.2, 4. , 4.8, 5.6, 6.4, 7.2, 8. ]),
<BarContainer object of 10 artists>)
```



In [ ]: