

### Problem 2

Assume that the 6 numbers to win the lottery are a set of  $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$

We can combine 6 numbers randomly from a bin of 30 in  ${}^{30}C_6$  ways.

To get the exact 6 numbers i.e.,  $x_1, x_2, x_3, x_4, x_5, x_6$

$$p = \frac{{}^6C_6}{{}^{30}C_6} = 0.0001694$$

$\frac{-6!}{10!}$

If two numbers are drawn, then

$$p = \frac{{}^4C_4}{{}^{28}C_2} = 0.000476$$

It is easier, or we have more chances, when two numbers are picked.

### Problem 3

When an infinite amount of the sample is

## ASSIGNMENT 1.

### Problem 1

a) The possible number of ways is a combination of  $N$  molecules to be randomly arranged in  $N$  pores i.e.

$${}^N C_N = {}^8 C_3 = 56$$

b) The total number of ways  $M$  molecules can be arranged is equal  ${}^N C_M$  but  $M/4 < \frac{1}{2}$  i.e.  $M$  is 1, 2 or 3.

The total number of ways molecules can only be arranged in the left side is  ${}^4 C_M$  such that  $\frac{M}{4} < \frac{1}{2}$

$\therefore M \leq 2 \Rightarrow M$  is 1

The probability of having molecules on the left hemispherical side "P"

$$P = \frac{{}^4 C_1}{{}^8 C_1 + {}^8 C_2 + {}^8 C_3} = 0.043$$

c) 
$$\frac{\binom{N/2}{P/2}}{{}^N C_M}$$

### Problem 3

There exists infinite elements in the sample space

$$A = \{S^X, M^X M^Y S^X, M^X M^Y M^X M^Y S^X\}$$

$$P(X \text{ wins}) = \frac{1}{3}$$

$$P(X \text{ loses}) = \frac{2}{3}$$

$$P(Y \text{ wins}) = \frac{1}{4}$$

$$P(Y \text{ loses}) = \frac{3}{4}$$

Let  $n$  be the number of attempts made by X before making the shot.

$n$	Event	$P$
0	$S^X$	$\frac{1}{3}$
1	$M^X M^Y S^X$	$\frac{1}{3} \cdot \frac{3}{4} \cdot \frac{1}{3}$
2	$M^X M^Y M^X M^Y S^X$	$\frac{1}{3} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{3}$
$\vdots$	$\vdots$	$\vdots$
$\infty$	$M^X \dots S^X$	$\frac{1}{3} \left(\frac{2}{3}\right)^n \left(\frac{3}{4}\right)^n$

The probability that X wins the game is the summation of the terms as they go to infinity i.e.

$$\sum_{n=0}^{\infty} \frac{1}{3} \left(\frac{2}{3}\right)^n \left(\frac{3}{4}\right)^n = \sum_{n=0}^{\infty} \frac{1}{3} \left(\frac{2}{3} \cdot \frac{3}{4}\right)^n = \sum_{n=0}^{\infty} \frac{1}{3} \left(\frac{3}{4}\right)^n = \frac{2}{3}$$

$$\text{Generally, for } p \leq q, E(X) = p \sum_{n=0}^{\infty} ((1-p)(1-q))^n = p \frac{1}{1 - (1-p)(1-q)} = \frac{p}{-pq + q + p}$$



