Uber Problem - Hamster Huey and Algebra Alex

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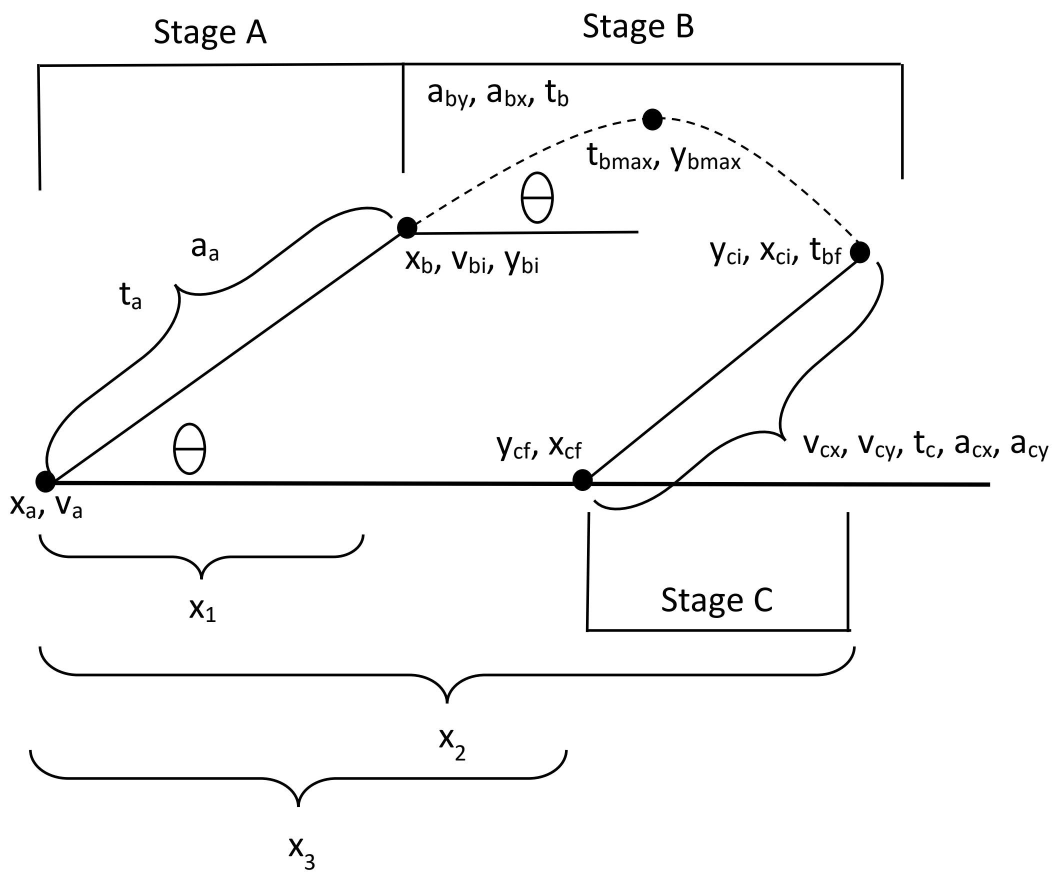
Description

One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at a specified angle above the East horizontal. The rocket engine is designed to burn for specified time while producing a constant net acceleration for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls a specified distance from its maximum height. When the parachute opens the rocket instantly changes speed and descends at a constant vertical speed. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind. Assume the wind affects the rocket only during the parachute stage.

Diagram



Givens

$t_a = 6.2$ s
$a_a = 4.5$ m/s^2
$y_{ci} = y_{bmax} - 42$m
$v_{cx} = -10$ m/s
$v_{cy} = -15$ m/s
$x_a = 0$ m
$v_a = 0$ m/s
$y_{bi} = x_b$
$a_{by} = -9.8$ m/s^2
a_{bx}, a_{cx}, $a_{cy} = 0$ m/s^2
$y_{cf} = 0$ m
$\Theta = 38°$

Strategy

1. For stage A, use kinematic equations in 1-dimensions to calculate the distance traveled diagonally. Afterwards, use a distance triangle to calculate the x-displacement (x_1) using the Pythagorean theorem.
2. For stage B, use kinematic equations in 2-dimesions, using both x and y vectors to calculate x-displacement (x_2) for this stage.
3. For stage C, use kinematic equations in 1-dimension to find the distance traveled diagonally. Afterwards, use a distance triangle to calculate the final x-displacement (x_3), which is the final solution.

Work

Stage A

$x_b = ½\, a_a\, t_a^2 + v_a\, t_a + x_a$

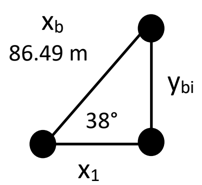
$x_b = ½\, (4.5)\, (6.2)^2$

$x_b = 86.49$ m

$v_{bi}^2 = v_a^2 + 2\, a_a\, (x_b - x_a)$

$v_{bi}^2 = 2\, (4.5)\, (86.49)$

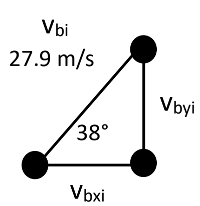
$v_{bi} = 27.9$ m/s



$x_1 = 86.49 * \cos(38°) = 68.155$ m

$y_{bi} = 86.49 * \sin(38°) = 53.25$ m

Stage B



$v_{byi} = 27.9 * \sin(38°) = 17.18$ m/s

$v_{bxi} = 27.9 * \cos(38°) = 21.99$ m/s

y-direction:

$y_b[t] = ½\, (a_{by})\, t_b^2 + v_{byi}\, t_b + y_{bi}$

$y_b[t] = ½\, (-9.8)\, t_b^2 + (17.18)\, t_b + 53.25$

$t_{bmax} = -b/2a = -17.18 / -9.8 = 1.753$ sec

$y_b[t_{bmax}] = 68.302$ m

$y_{ci} = y_b[t_{bmax}] - 42$

$y_{ci} = 26.302$ m

$y_b[t_{bf}] = 26.302 = ½\, (-9.8)\, t_{bf}^2 + (17.18)\, t_{bf} + 53.25$ -> Solve for t_{bf}

$t_{bf} =$ ~~-1.175 sec~~ or 4.68 sec

$t_{bf} = 4.68$ sec

x-direction:

$x_2 = ½\, (a_{bx})\, t_b^2 + v_{bxi}\, t_b + x_1$

$x_2 = (21.99)\, t_b^2 + 68.155$ -> Substitute t_{bf} to get final x_2

$x_2 = 171.05$ m

Stage C

y-direction:

$y_{cf} = ½\, (a_{cy})\, t_c^2 + v_{cy}\, t_c + y_{ci}$

$0 = (-10)\, t_c + 26.302$

$t_c = 2.6302$ sec

x-direction:

$x_3 = ½\, (a_{cx})\, t_c^2 + v_{cx}\, t_c + x_2$

$x_3 = (-15)\, (2.6302) + 171.05$

$\boxed{x_3 = 131.6 \text{ m}}$

* Note: Some exact values used during actual computation and only estimates are shown (ex: 27.9 * cos (38°) instead of 21.99)