AE333

Mechanics of Materials

Lecture 25 - Strain Transformation

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schedule

- 5 Apr - Deflection of Beams
- 8 Apr - Deflection of Beams, HW8 Due
- 10 Apr - Deflection of Beams
- 12 Apr - Deflection of Beams

outline

- plane strain
- principal strains and mohr's circle
- strain rosettes
- material property relationships
- deflection of beams and shafts

plane strain

plane strain

- Under plane stress we assume no out-of-plane stresses are present
- This is typically a good assumption for very thin materials
- Under plane strain we assume no out-of-plane strains are present
- Typically a good assumption for very thick materials

sign convention

- Normal strains, ϵ_x and ϵ_y, are considered positive when they cause elongation, and negative when they cause contraction
- Shear strains, γ_{xy} are positive if the interior angle becomes smaller than 90°, and negative if the angles becomes larger than 90°

general equations

- We derive the general strain transformation equations by comparing infinitesimal elements before and after deformation
- To find $\gamma_{x'y'}$ we compare the angle between dx and dy before and after deformation

general equations

- The equations are nearly exactly the same as the stress transformation equations
- Pay attention to the difference, strain transformation equations are NOT on the equation sheet

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

- As with $\sigma_{y'}$, we find $\epsilon_{y'}$ by letting $\theta_y = \theta_x + 90°$

engineering strain

- Side note: there is another definition of shear strain known as tensorial shear strain, where $\gamma_{xy} = 2\epsilon_{xy}$
- Under tensorial strain, the transformation equations are exactly the same (as in this case both stress and strain are tensors)
- γ_{xy} is known as engineering strain, you will need to pay attention to which strain convention is used when extracting data from finite elements or other sources

principal strains and mohr’s circle

principal strains

- As you might imagine, since the transformation equations are nearly identical so are the principal strain equations

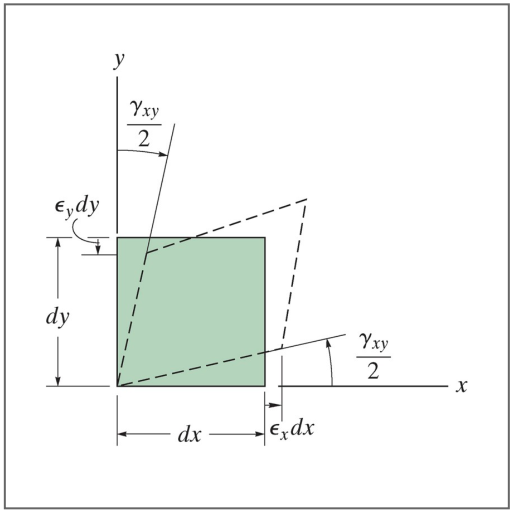
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

mohr's circle

- Mohr's circle can also be used in exactly the same way for strain as it is for stress
- The only difference is that the vertical axis is tensor strain, or $\gamma_{xy}/2$

example 10.4



The state of plane strain at a point has components of $\epsilon_x = 250\mu\epsilon$, $\epsilon_y = -150\mu\epsilon$, and $\gamma_{xy} = 120\mu\epsilon$. Determine the principal strains and the direction they act.

strain rosettes

rosettes

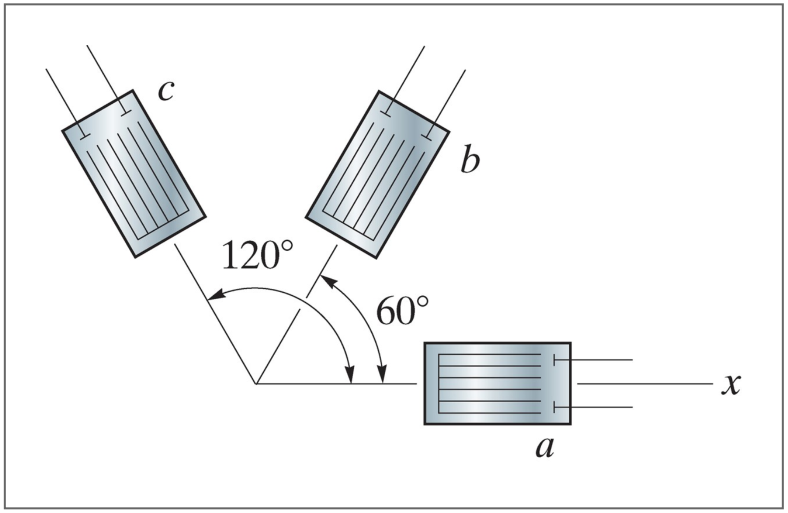
- Normal strain is fairly easy to measure using a strain gage
- Shear strain is more difficult to measure directly, so instead a “rossette” of normal strain gages is used
- We can use the strain transformation equations to determine τ_{xy}

rosettes

- Usually, we have $\theta_a = 0$, $\theta_b = 90$ and $\theta_c = 45$ OR $\theta_a = 0$, $\theta_b = 60$ and $\theta_c = 120$

$$\epsilon_a = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_a + \frac{\gamma_{xy}}{2}\sin 2\theta_a$$
$$\epsilon_b = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_b + \frac{\gamma_{xy}}{2}\sin 2\theta_b$$
$$\epsilon_c = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_c + \frac{\gamma_{xy}}{2}\sin 2\theta_c$$

example 10.8



The readings from the rosette shown are $\epsilon_a = 60\mu\epsilon$, $\epsilon_b = 135\mu\epsilon$ and $\epsilon_c = 264\mu\epsilon$. Find the in-plane principal strains and their directions.

material property relationships

generalized hooke's law

- We have previously used Hooke's Law in 2D, in 3D we have

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

generalized hooke's law

- And in shear

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$
$$\gamma_{yz} = \frac{1}{G}\tau_{yz}$$
$$\gamma_{xz} = \frac{1}{G}\tau_{xz}$$

dilatation

- When a material deforms it often changes volume
- The change in volume per unit volume is called “volumetric strain” or dilatation

$$e = \frac{\partial V}{dV} = \epsilon_x + \epsilon_y + \epsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

hydrostatic pressure

- One way of characterizing volumetric response is to apply hydrostatic pressure (equal compression on all sides with no shear)
- Under this case, we have

$$\frac{p}{e} = -\frac{E}{3(1-2\nu)}$$

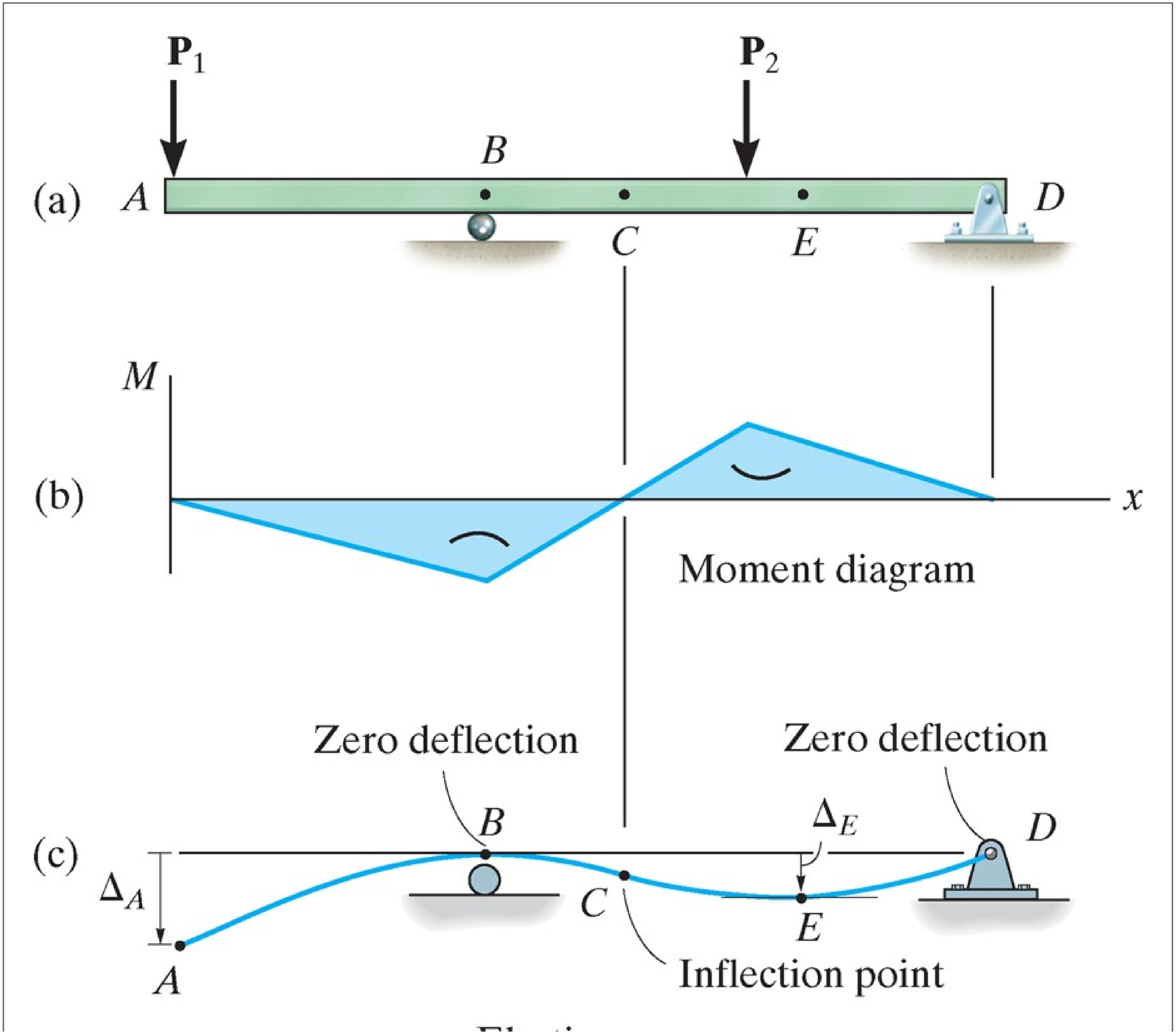
- We call the term on the right (with no negative sign) the bulk modulus, k

deflection of beams and shafts

elastic curve

- Before finding the exact displacement, it is useful to sketch the approximate deformed shape of a beam
- For difficult beams, it is useful to draw the moment curve first
- Positive internal moment tends to bend the beam concave up, while negative moment tends to bend the beam concave down

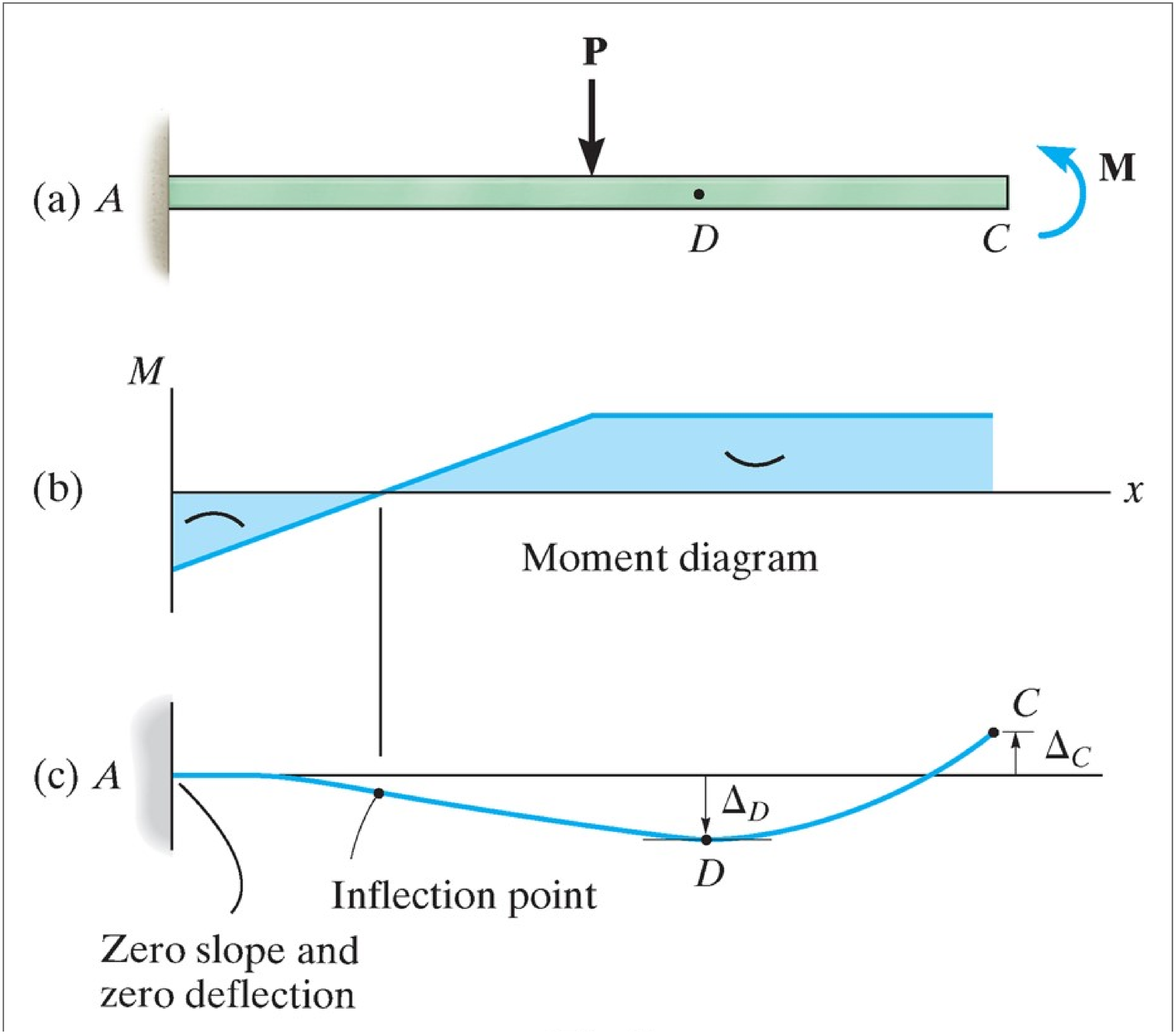
elastic curve



P_1
P_2
(a)
A
B
C
E
D
M
(b)
x
Moment diagram
Zero deflection
Zero deflection
(c)
Δ_A
B
Δ_E
D
C
E
A
Inflection point


Elastic curve

elastic curve


P
M
(a) A
D
C
M
(b)
x
Moment diagram
C
Δ_C
(c) A
Δ_D
D
Inflection point
Zero slope and
zero deflection

Elastic curve

moment-curvature

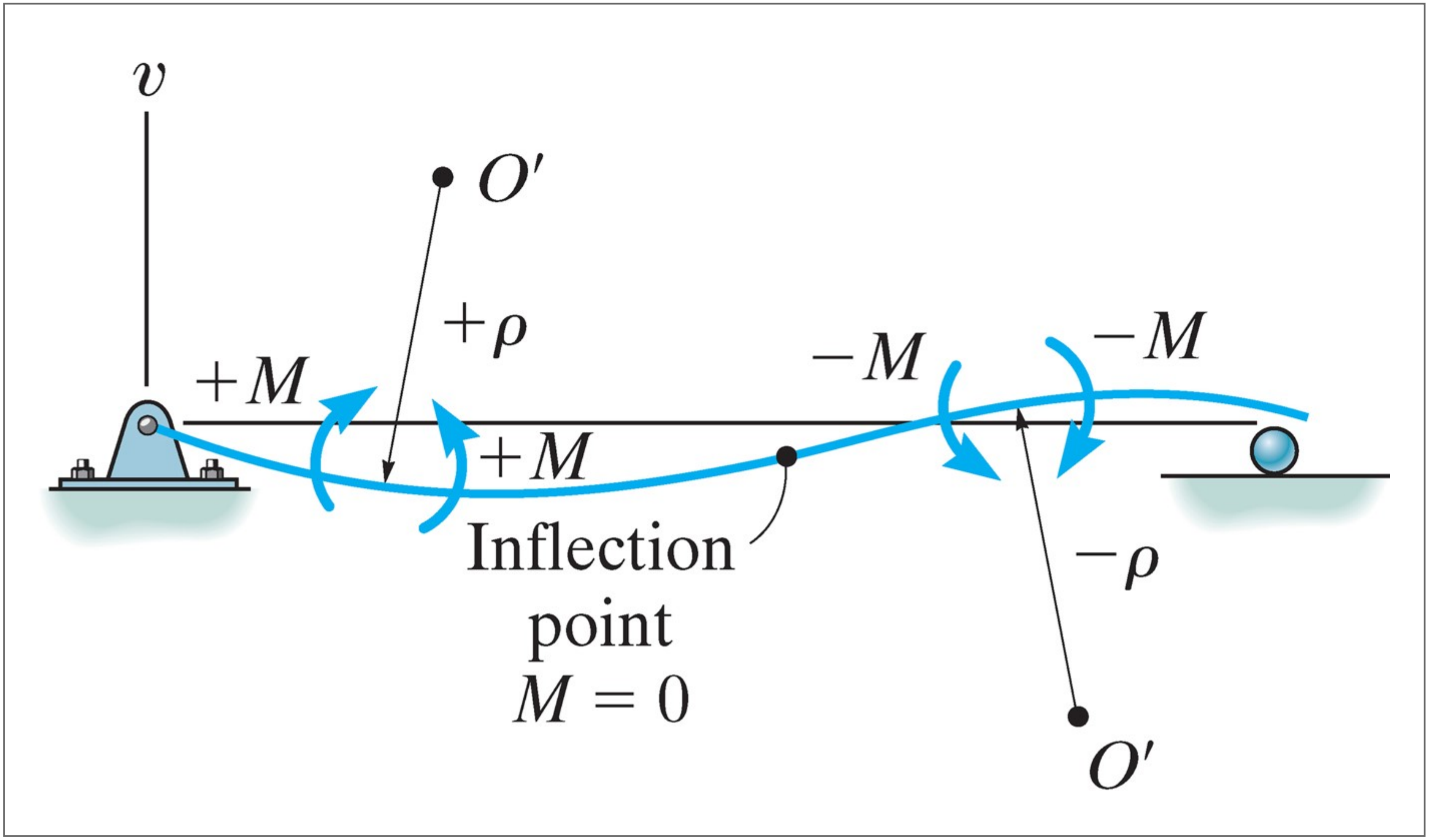
- In Chapter 6 we compared the strain in a segement of a beam to the radius of curvature and found

$$\frac{1}{\rho} = -\frac{\epsilon}{y}$$

- Since Hooke's Law applies, $\epsilon = \sigma/E = -My/EI$, substituting gives

$$\frac{1}{\rho} = \frac{M}{EI}$$

sign convention



ρ is positive when the center of the arc is above the beam, negative when it is below.