

# AE333

## Mechanics of Materials

### Lecture 8 - Axial Load

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## schedule

- 13 Feb - Axial Load
- 15 Feb - Torsion
- 18 Feb - Torsion, HW3  
Due
- 20 Feb - Bending

# outline

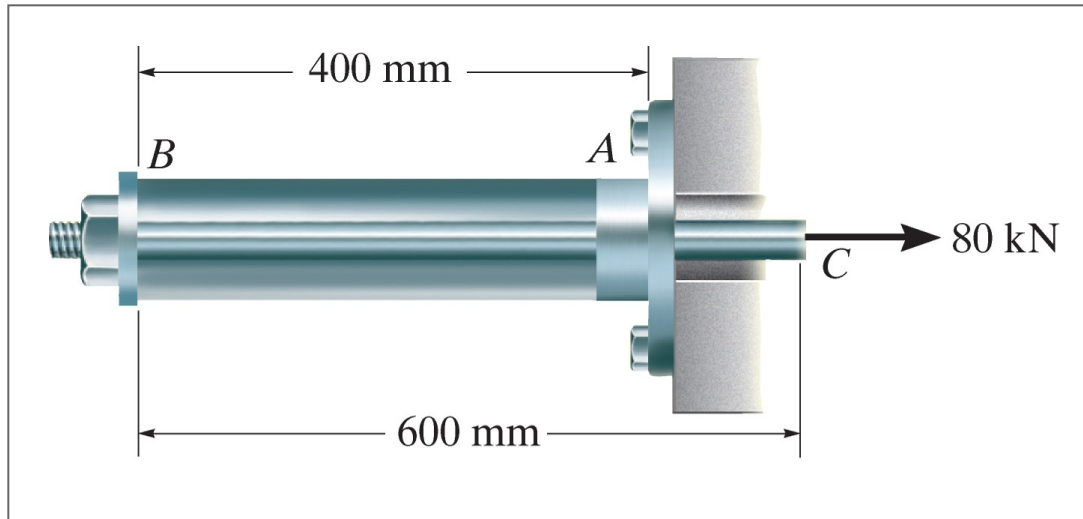
- review
- superposition
- statically indeterminate
- force method
- thermal stress

# review

## review

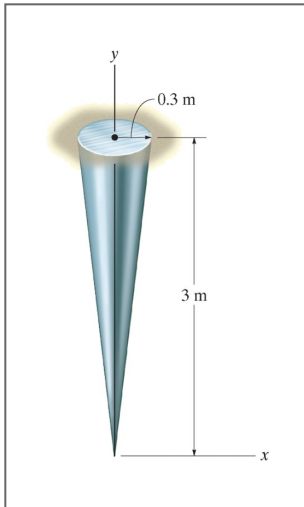
- What is Saint Venant's Principle?
- How do we find axial deformation, in general?

## example 4.2



A steel rod with a 10mm diameter is attached to a rigid collar passing through an aluminum tube with cross-sectional area of  $400 \text{ mm}^2$ . Find the displacement at C if  $E_{st} = 200 \text{ GPa}$  and  $E_{al} = 70 \text{ GPa}$ .

## example 4.4



The cone shown has a specific weight of  $\gamma = 6 \text{ kN/m}^3$  and  $E = 9 \text{ GPa}$ . Determine how far the end is displaced due to gravity.

# superposition



## superposition

- Some problems are too complicated to solve all at once
- Instead, we break them up into two simpler problems
- Each "sub-problem" must still satisfy equilibrium
- Problem must be linear and the deformation should be small enough that it does not cause moment-equilibrium issues

# statically indeterminate

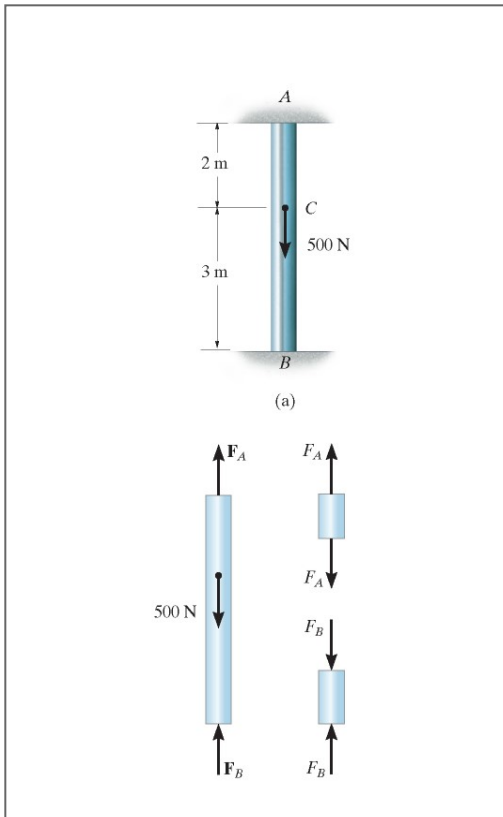
## statically indeterminate

- There are many problems that are at least slightly over-constrained
- While this is common engineering practice, it creates too many variables for statics analysis
- These problems are called "statically indeterminate"

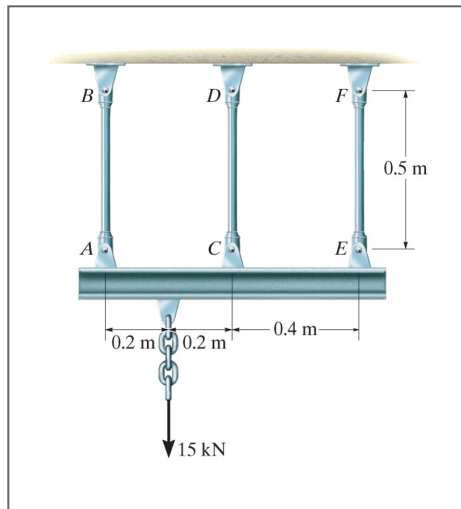
## statically indeterminate

- One extra equation we can use is called "compatibility" or the "kinematic condition"
- We know that at the displacement must be equal on both sides of any arbitrary section we make in a member
- We can separate a member into two parts, then use compatibility to relate the two unknown forces

# statically indeterminate



## example 4.7



Assuming the bottom bar is rigid, find the force developed in each bar. AB and EF have cross-sectional areas of  $50 \text{ mm}^2$  while CD has a cross-sectional area of  $30 \text{ mm}^2$ .

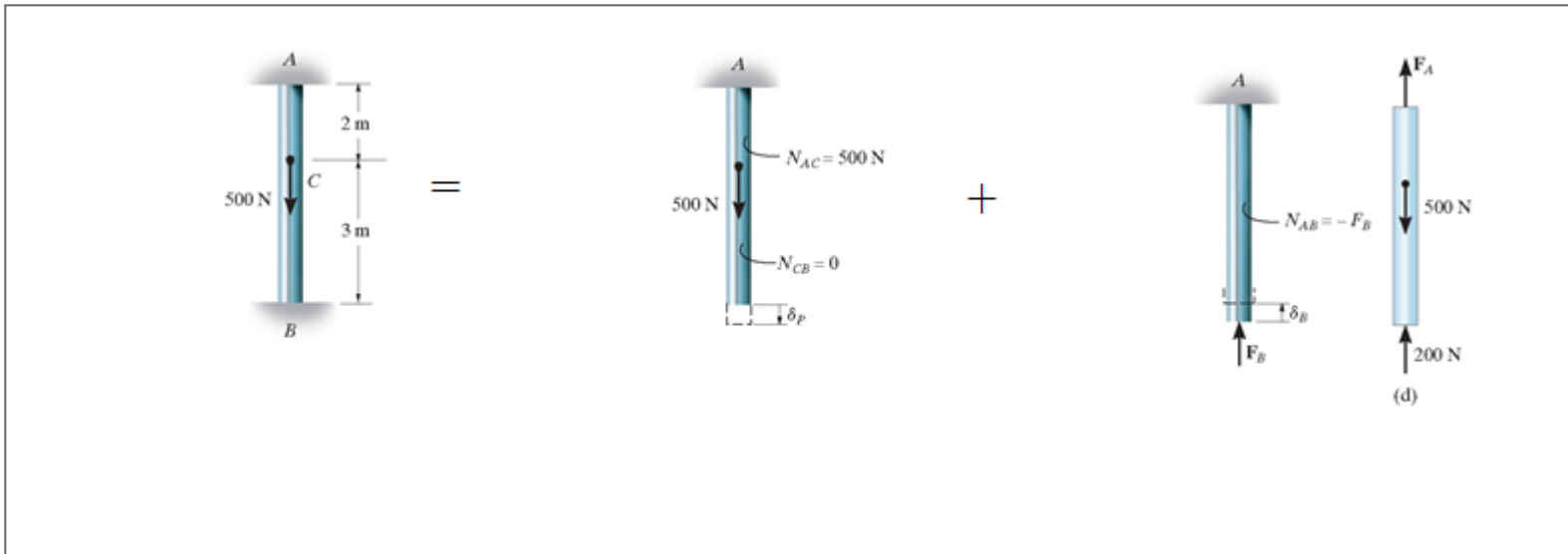
# force method

## force method

- One way to solve statically indeterminate problems is using the principle of superposition
- We choose one redundant support and remove it
- We then add it back as a force separately (without the other forces in the problem)



# force method



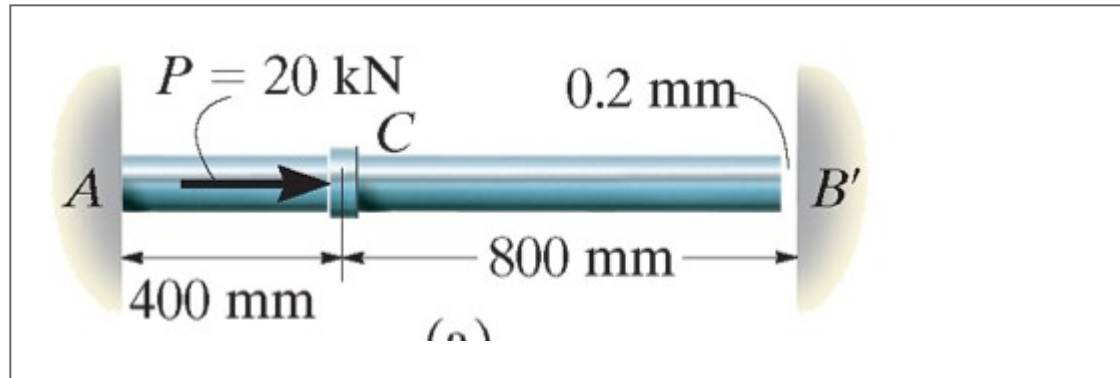
## force method

- We connect the two problems by requiring that the displacement in both frames adds to 0 to meet the support requirements
- This is referred to as the equation of compatibility

## procedure

- Choose one support as redundant, write the equation of compatibility
- Express the external load and redundant displacements in terms of load-displacement relationship
- Draw free body diagrams and use the equations of equilibrium to solve

## example 4.9



The steel rod shown has a diameter of 10 mm. Determine the reactions at A and B'.

# thermal stress

## thermal stress

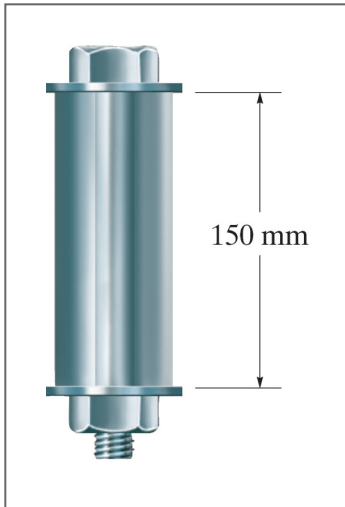
- A change in temperature causes a material to either expand or contract
- For most materials this is linear and can be described using the coefficient of linear expansion

$$\delta_T = \alpha \Delta T L$$

## thermal stress

- When a body is free to expand, the deformation can be readily calculated using
- If it is not free to expand, however, thermal stresses develop
- We can use the force method described previously to find the thermal stresses developed

## example 4.12



An aluminum tube with cross-section of  $600 \text{ mm}^2$  is used as a sleeve for a steel bolt with cross-sectional area of  $400 \text{ mm}^2$ . When  $T=15$  degrees Celsius there is negligible force and a snug fit, find the force in the bolt and sleeve when  $T=80$  degrees Celsius.