AE333

Mechanics of Materials

Lecture 18 - Transverse Shear
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schedule

- 20 Mar - Transverse Shear
- 22 Mar - Transverse Shear
- 25 Mar - Combined Loading, HW6 Due

outline

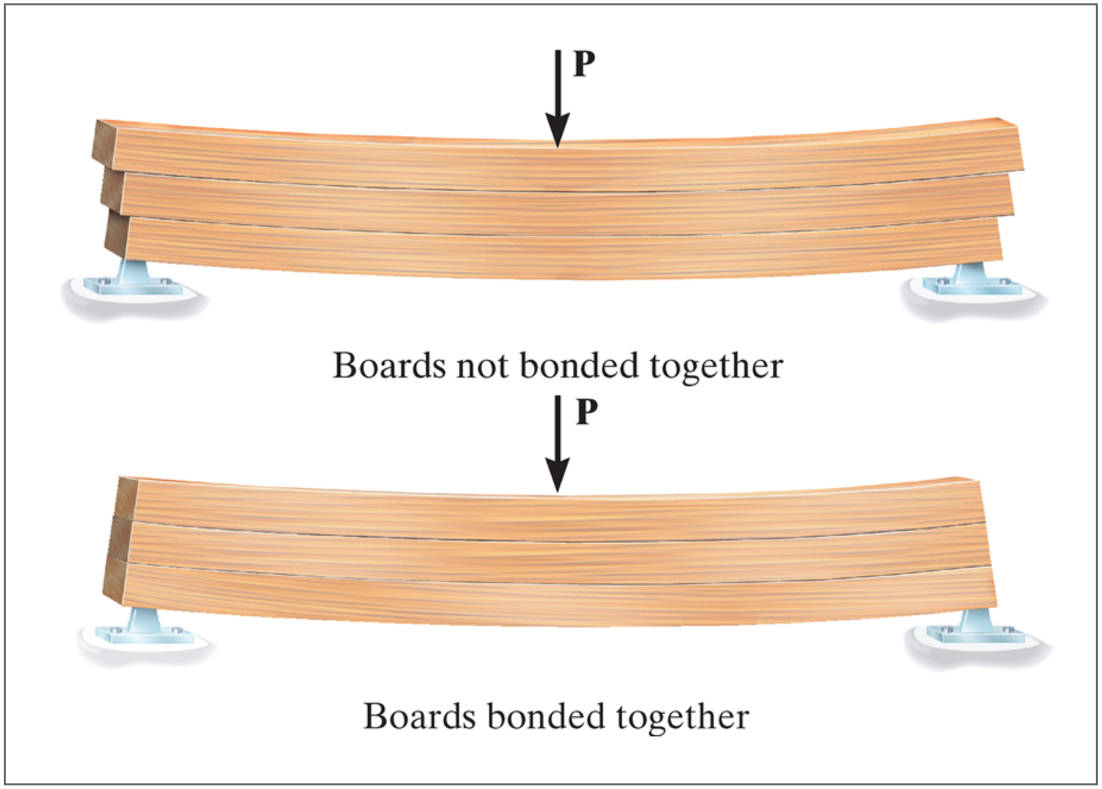
- shear in straight members
- the shear formula
- group problems
- shear flow in built-up members

shear in straight members

shear

- We have discussed the internal stresses caused by the internal moment M
- There are also internal shear stresses caused by the internal shear force V
- We can illustrate the effect of internal shear stress by considering three boards, either resting on top of on another or bonded

shear

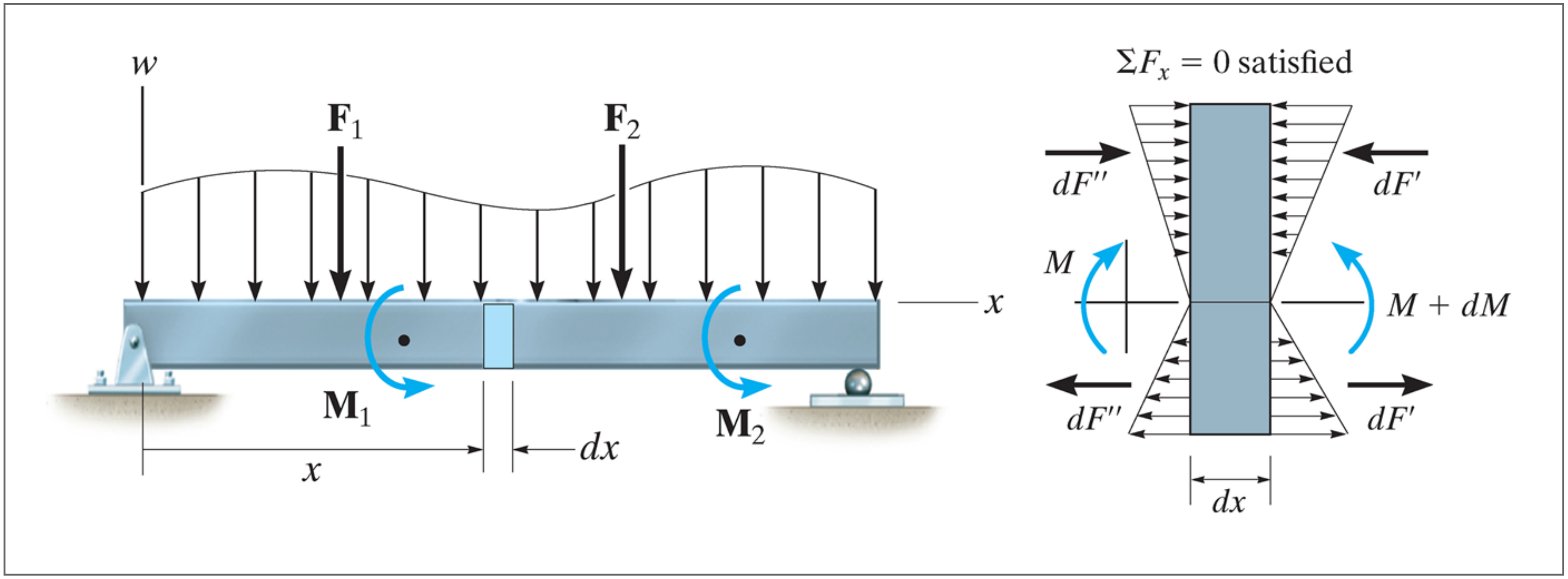


the shear formula

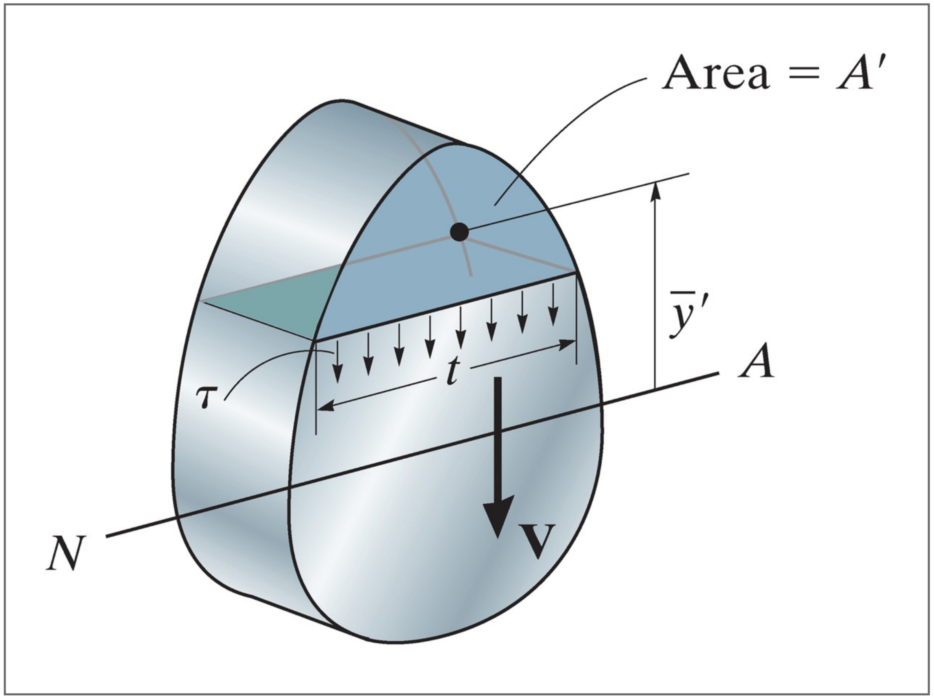
shear formula

- Internal shear causes a more complicated deformation state, so we will use an indirect method to find the shear stress-strain distribution
- We will consider equilibrium along a section of a beam, then we will consider another section

equilibrium



equilibrium



equilibrium

- There must be a shear force along the bottom to equilibrate the different stresses on either side of the section
- If we assume that this shear is constant through the thickness, we obtain the following from equilibrium

$$\sum F_x = 0 = \int_{A'} \sigma' dA' - \int_{A'} \sigma dA' - \tau(t dx)$$

equilibrium

$$0 = \int_{A'} \left(\frac{M + dM}{I}\right) y dA' - \int_{A'} \left(\frac{M}{I}\right) y dA' - \tau(t dx)$$

$$\left(\frac{M}{I}\right) \int_{A'} y dA' = \tau(t dx)$$

$$\tau = \frac{1}{It} \left(\frac{dM}{dx}\right) \int_{A'} y dA'$$

shear formula

- In this formula, recall that $V = \frac{dM}{dx}$
- We also call Q the moment of the area A' which is equal to $\int_{a'} y dA'$
- We can also write Q in terms of the centroid
  $Q = \bar{y}' A'$

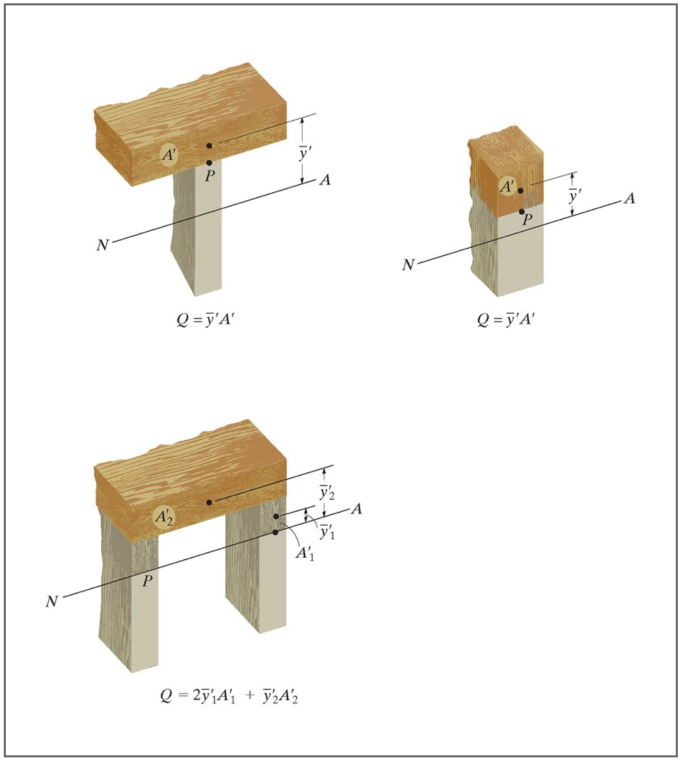
shear formula

- Simplified, the shear formula is
  $$\tau = \frac{VQ}{It}$$
- Q poses the greatest difficulty in calculations, so we will consider a few examples

Q

- Q represents the moment of the cross-sectional area above (or below) the point at which the shear stress is being calculated
- We apply the formula to that area

Q


A'
P
$\bar{y}'$
N
A
$Q = \bar{y}'A'$
A'
P
$\bar{y}'$
N
A
$Q = \bar{y}'A'$
A'_2
$\bar{y}'_2$
$\bar{y}'_1$
A'_1
P
N
A
$Q = 2\bar{y}'_1 A'_1 + \bar{y}'_2 A'_2$

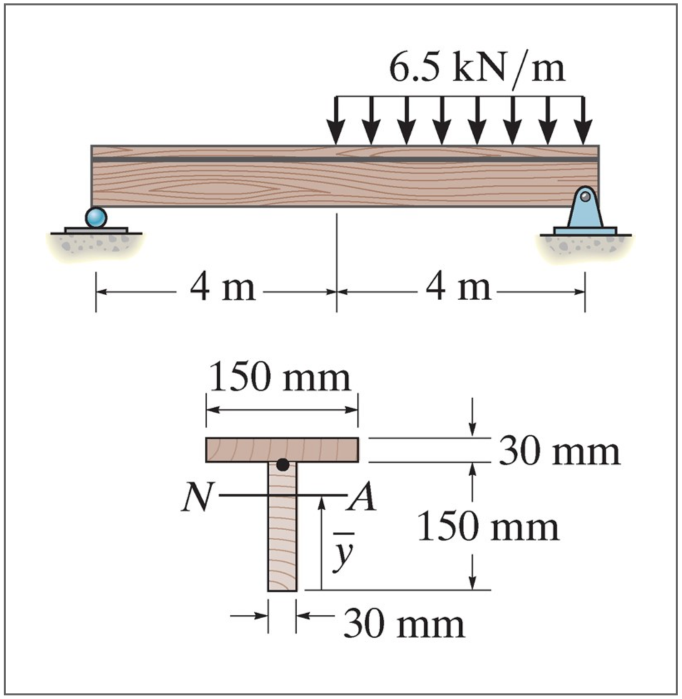
shear formula limitations

- A major assumption made is that the shear stress was constant through the thickness, essentially we have found the average shear
- This is more accurate the more slender a beam is (small b and large h)
- The formula is also not accurate for cross sections that change or have boundaries that are not right angles

procedure

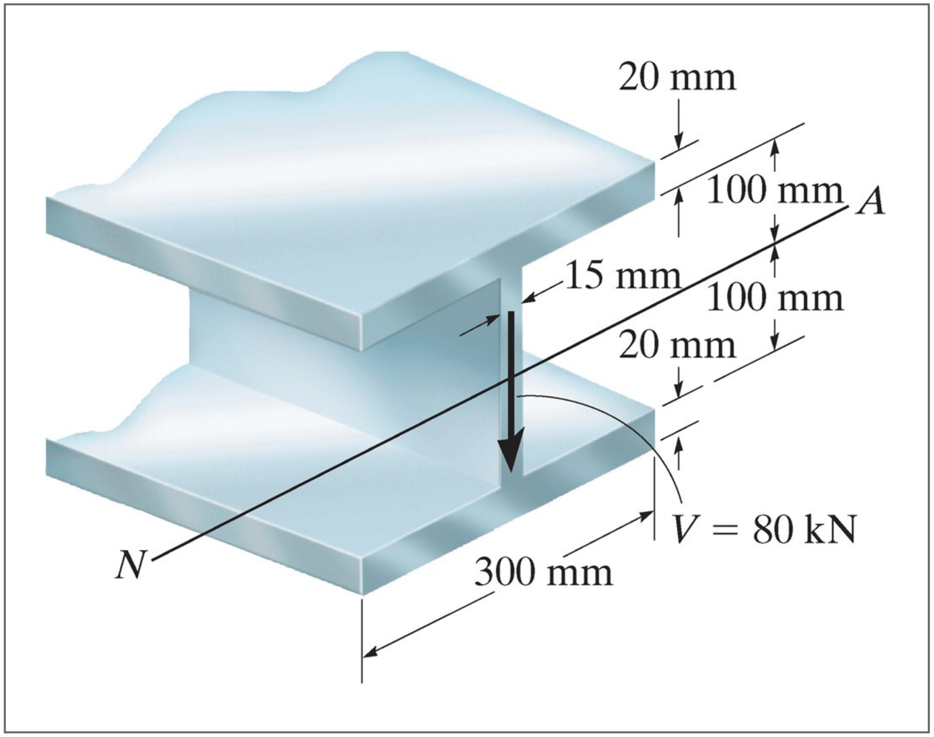
- First we find the internal shear, V
- Find I, the moment of inertia (of the entire section) about the neutral axis
- Find t from an imaginary cross-section at the point of interest
- Calculate Q from either the area above or below the point of interest
- τ acts in the same direction as V (and must be equilibrated on other faces)

example 7.1



Determine the maximum stress needed by a glue to hold the boards together.

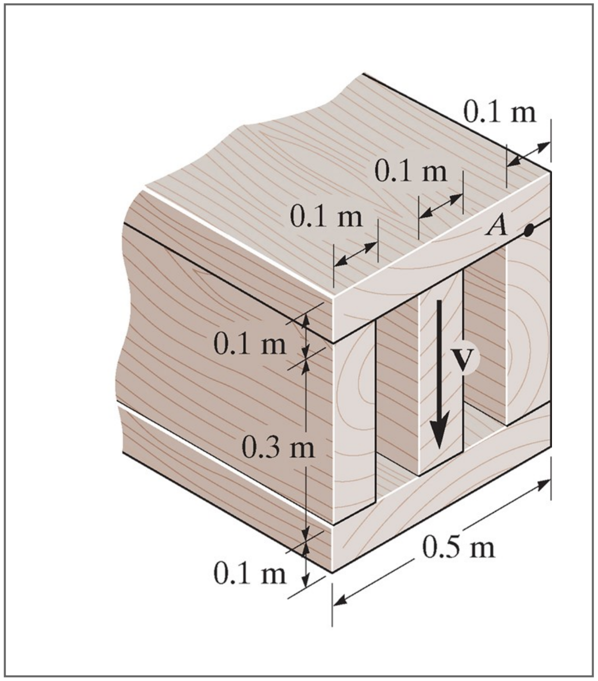
example 7.3



Plot the shear stress distribution through the beam.

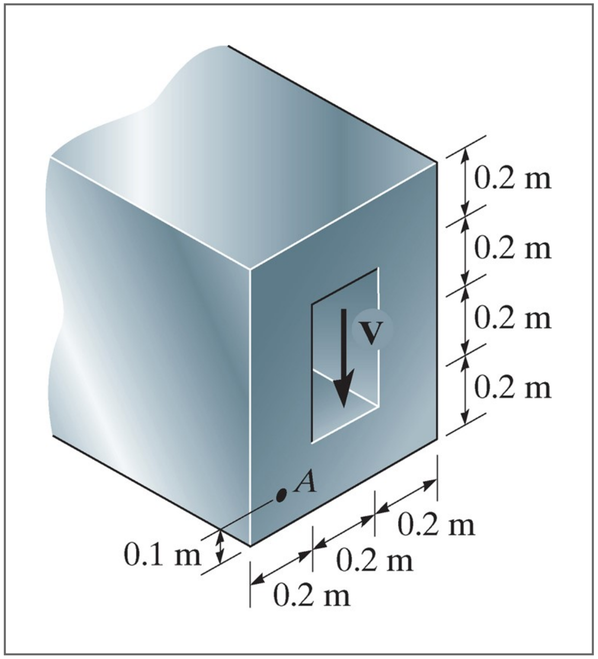
group problems

group one



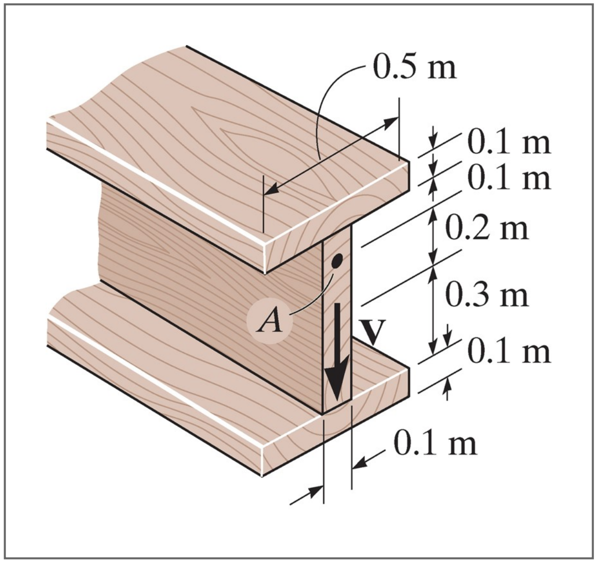
Find Q and t that would be used to find the shear stress at A.

group two



Find Q and t that would be used to find the shear stress at A.

group three



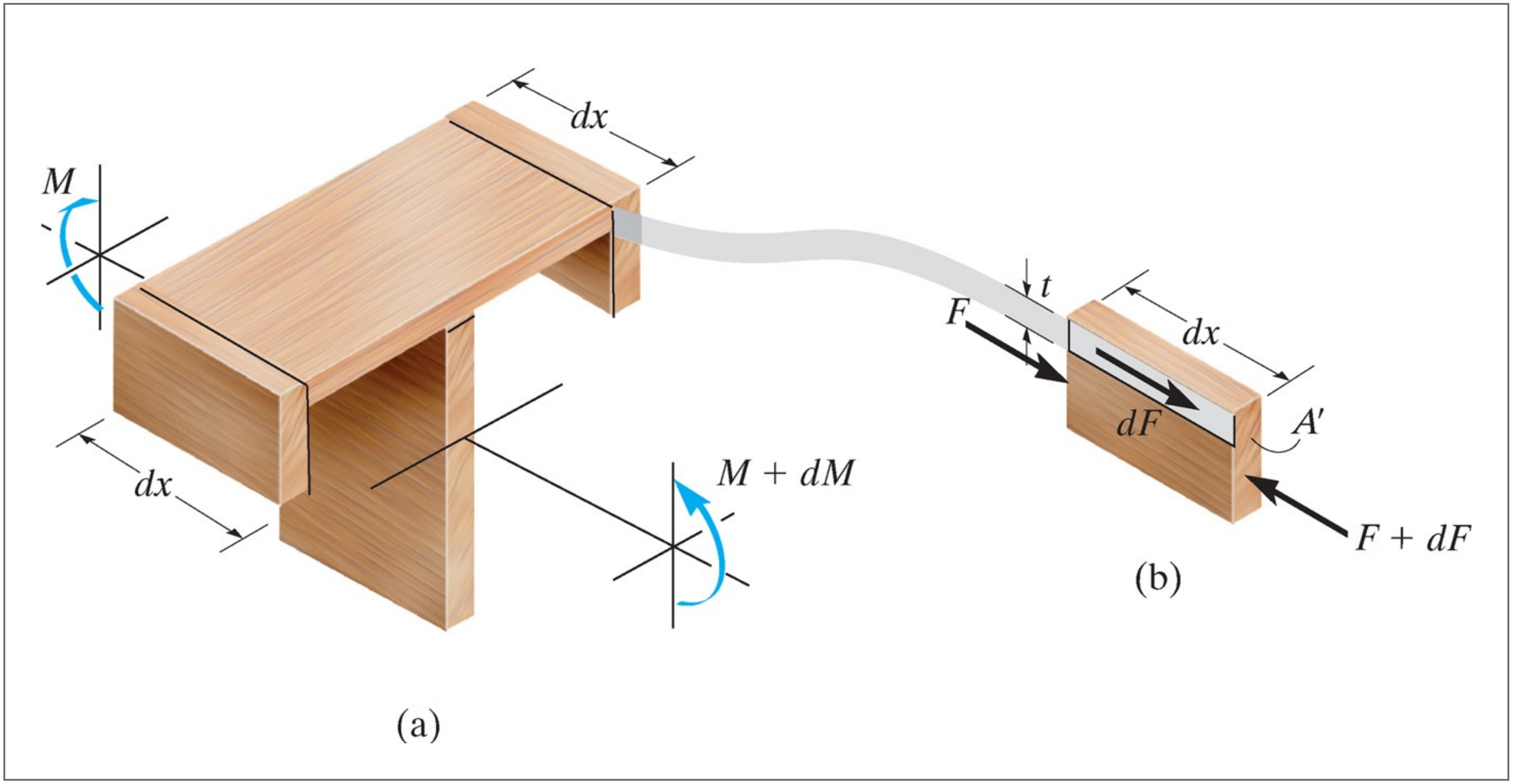
Find Q and t that would be used to find the shear stress at A.

shear flow in built-up members

built-up members

- Often in practice, structural members are "built-up"
- This refers to parts that are comprised of several other parts to have greater strength in certain areas
- We need to analyze the shear between these members to choose appropriate adhesives or fasteners

equilibrium



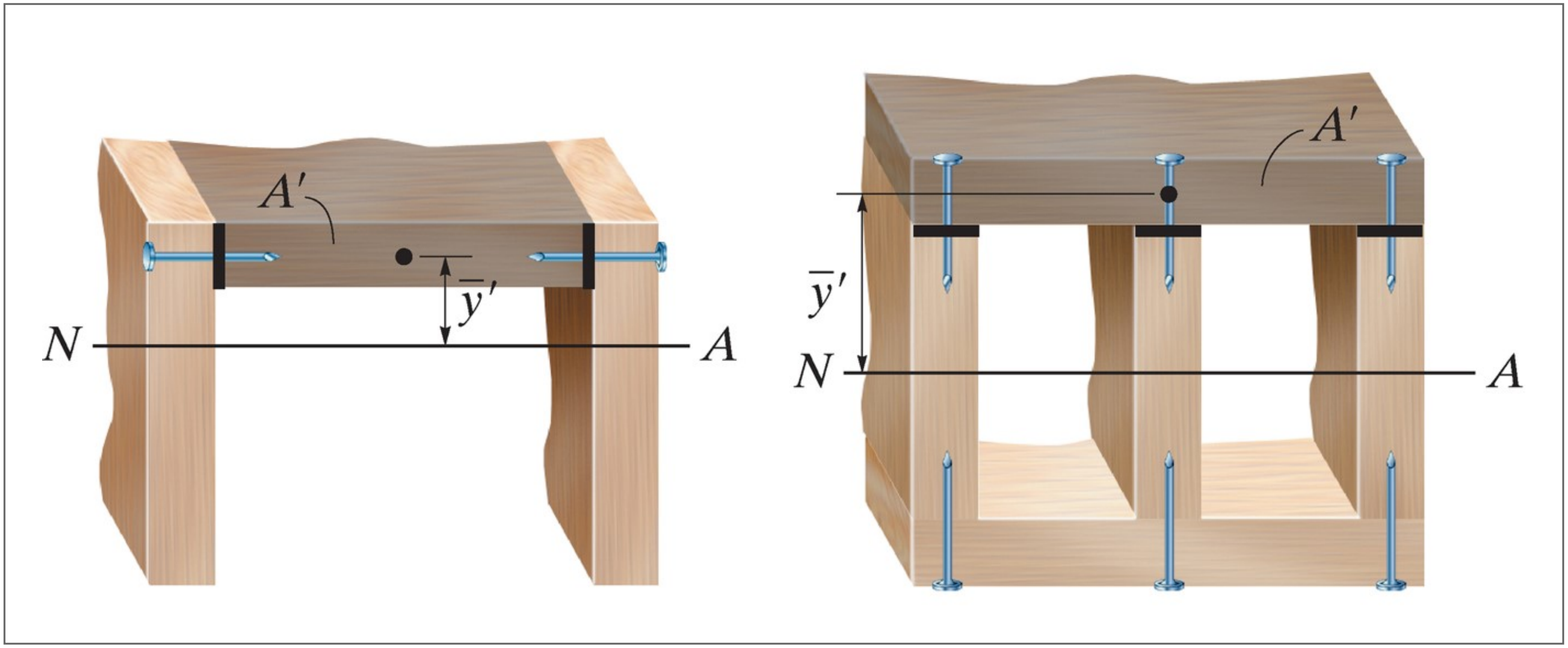
equilibrium

- From equilibrium we see that
  $$dF = \frac{dM}{I}\int_{A'} y dA'$$
- We recall that this integral represents Q, we can also define the shear flow as $q = dF/dx$ and recall that $dM/dx = V$ to find
  $$q = \frac{VQ}{I}$$

fastener spacing

- We can use shear flow to determine fastener spacing
- Say a fastener can support a shear force of F_0 before failure
- The shear flow (force/distance) times the spacing (distance) will give the shear force per fastener $F=qs$

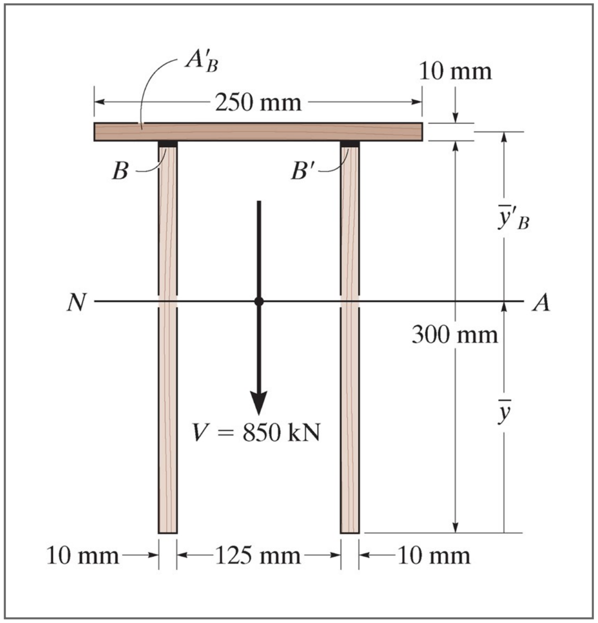
multiple fasteners



multiple fasteners

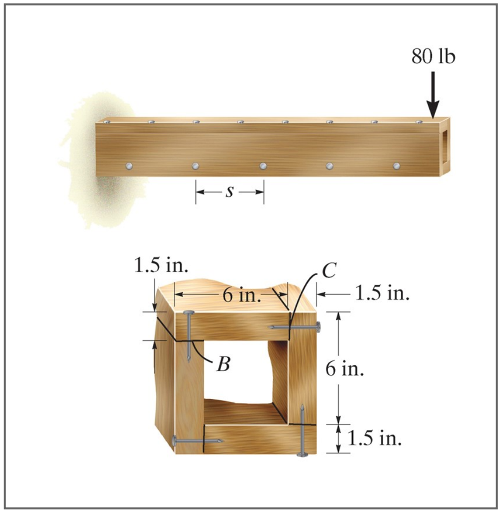
- When multiple arms are connecting the same area (as shown in the previous slide)
- The shear flow "seen" by each fastener is q/n where n is the number of fastners per area

example 7.4



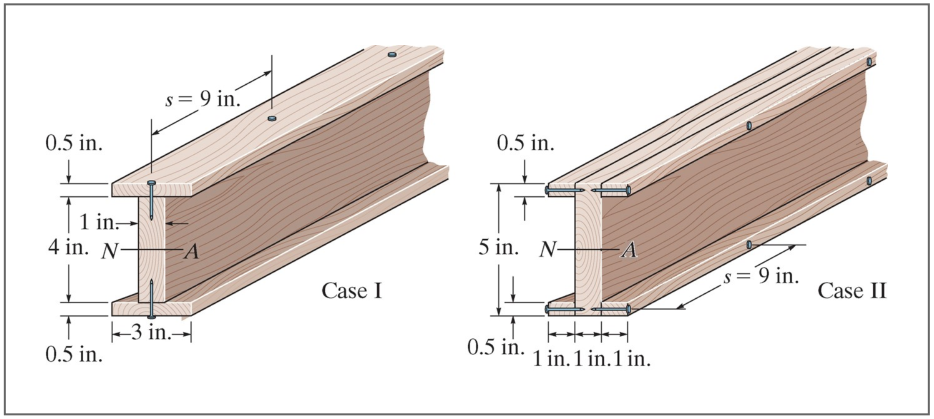
Determine the shear flow at B and B' that must be resisted by glue to bond the boards together.

example 7.5



If each nail can support a maximum shear force of 30 lb, determine the maximum spacing of the nails at B and at C so that the beam can support the force of 80 lb.

example 7.6



Nails with a shear strength of 40 lb are used in a beam that can be constructed as shown in Case I or Case II. If the nails are spaced at 9 in determine the largest vertical shear that can be supported.