Continuum Mechanics

Lecture 6 - Polar Decomposition

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schedule

- 3 Sep - Polar Decomposition
- 8 Sep - Exam Review, HW3 Due
- 10 Sep - Exam 1
- 15 Sep - Stress
- 17 Sep - Stress

outline

lagrangian strain tensor

- Recall the Lagrangian strain tensor

$$E^*_{ij} = \frac{1}{2}(C_{ij} - \delta_{ij})$$

- Following the same development as done previously, we can find the physical meaning of the Lagrangian strain tensor

$$E^*_{11} = \frac{ds_1^2 - dS_1^2}{2dS_1^2}$$

$$2E^*_{12} = \frac{ds_1 ds_2}{dS_1 dS_2}\cos(n_i, m_i)$$

- We can compare this to the infinitesimal strain tensor

$$E_{11} = \frac{ds - dS}{dS}$$

$$2E_{12} = \gamma$$

lagrangian strain tensor

- We can also write the Lagrangian strain tensor in terms of displacement
- Recall that $C = F^T F$ and $F = I + \nabla u$

$$E^* = \frac{1}{2}(F^T F - I) = \frac{1}{2}\left[(I + \nabla u)^T (I + \nabla u) - I\right] =$$
$$\frac{1}{2}\left[(\nabla u)^T \nabla u + \nabla u + (\nabla u)^T\right]$$

eulerian strain tensor

- The Eulerian strain tensor is defined as

$$e^* = \frac{1}{2}(I - B^{-1})$$

- Following the same procedure for identifying physical meaning, but using the inverse of F_{ij}, we find

$$e^*_{11} = \frac{ds_1^2 - dS_1^2}{2ds_1^2}$$
$$e^*_{12} = -\frac{dS_1 dS_2}{ds_1 ds_2}\cos(n_i, m_i)$$

eulerian strain tensor

- If we express the eulerian strain tensor in terms of displacement, we find

$$e^* = \frac{1}{2}\left[-(\nabla_x u)^T \nabla_x u + \nabla_x u + (\nabla_x u)^T\right]$$

- Notice that for small deformations, $e^* \approx E^*$

change in volume

- If we consider three material elements, $dX_i^{(1)} = dS_1 e_1$, $dX_i^{(2)} = dS_2 e_2$ and $dX_i^{(3)} = dS_3 e_3$ the volume in the reference configuration is given by

$$dV_0 = dS_1 dS_2 dS_3$$

- After deformation, we find that

$$dV = dV_0 |\det F|$$

- For convenience, $J = |\det F|$ is often used

finite element mapping

- Finite elements are often used to solve continuum mechanics problems
- It is helpful to know how important quantities, such as the deformation gradient, are calculated
- Reference on this topic can be found from the open source continuum mechanics text

finite element mapping

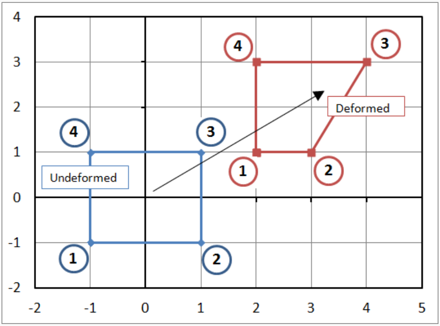

Figure 1: image

finite element mapping

- The equations to map this deformation are generally written as
  $u(X, Y) = \phi_1(X, Y)u_1 + \phi_2(X, Y)u_2 + \phi_3(X, Y)u_3 + \phi_4(X, Y)u_4$
  $v(X, Y) = \phi_1(X, Y)v_1 + \phi_2(X, Y)v_2 + \phi_3(X, Y)v_3 + \phi_4(X, Y)v_4$
- Where ϕ_i are the shape functions for each node in the element

$$\phi_1 = \frac{1}{4}(1 - X)(1 - Y)$$
$$\phi_2 = \frac{1}{4}(1 + X)(1 - Y)$$
$$\phi_3 = \frac{1}{4}(1 + X)(1 + Y)$$
$$\phi_4 = \frac{1}{4}(1 - X)(1 + Y)$$

deformation gradients

- Recall that the deformation gradient in terms of displacement is

$$F_{ij} = \delta_{ij} + u_{i,j}$$

- We can readily calculate this for individual terms of the deformation gradient