

# AE333

## Mechanics of Materials

### Lecture 23 - Mohr's Circle

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1 Apr, 2019

## schedule

- 1 Apr - Mohr's Circle
- 3 Apr - Strain Transformation, HW7 Due
- 5 Apr - Deflection of Beams
- 8 Apr - Deflection of Beams, HW8 Due

# outline

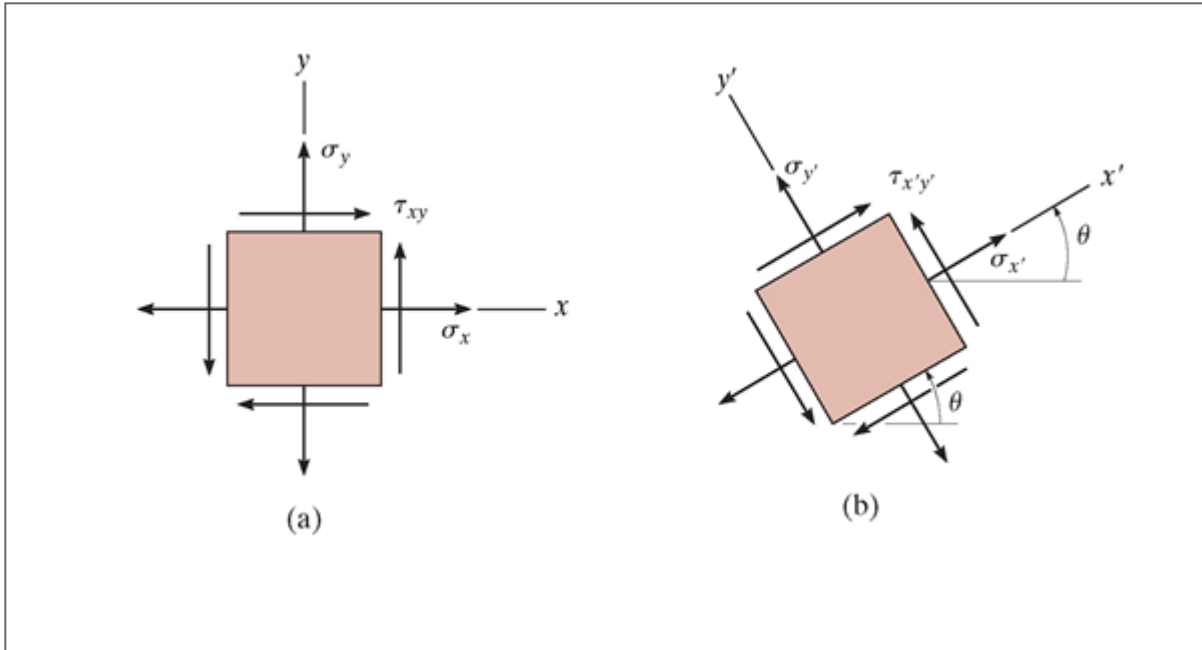
- plane stress  
transformation
- general equations
- principal stresses
- mohr's circle

# plane stress transformation

## plane stress

- In general, the state of stress at a point is characterized by six stress components
- In practice, this is rare, as most stresses and forces act in the same plane
- This case is referred to as plane stress

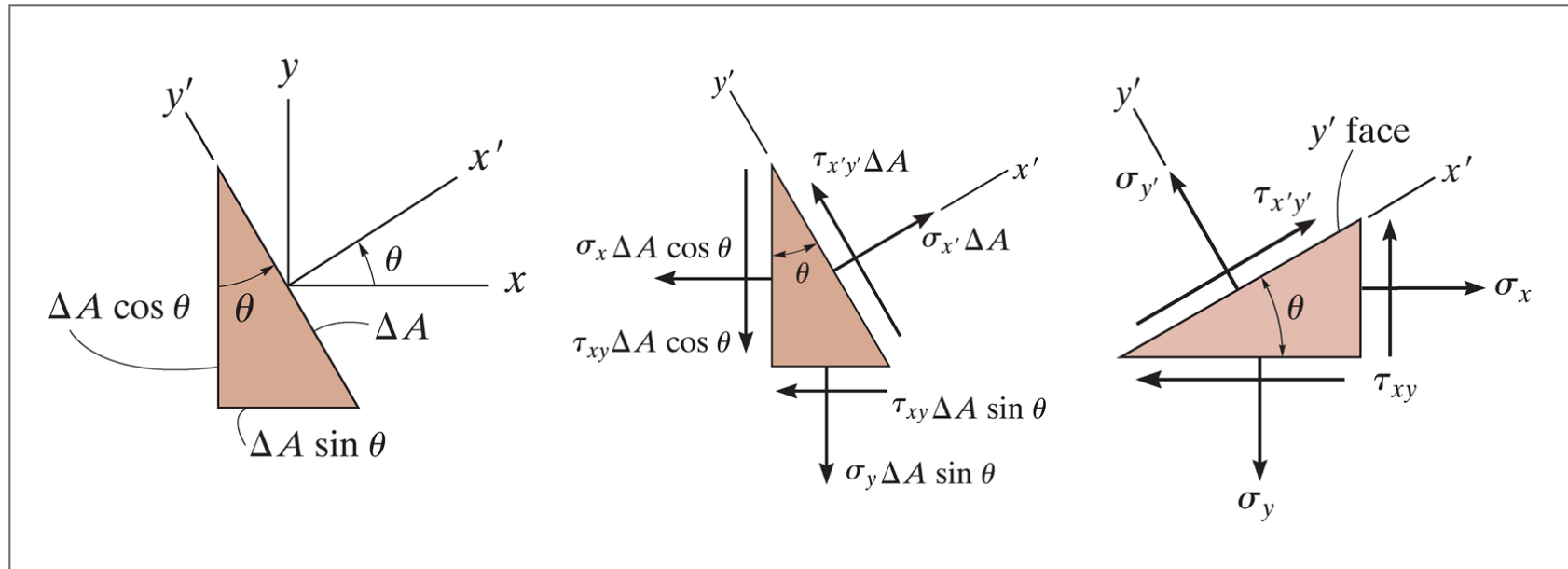
# transformation



## procedure

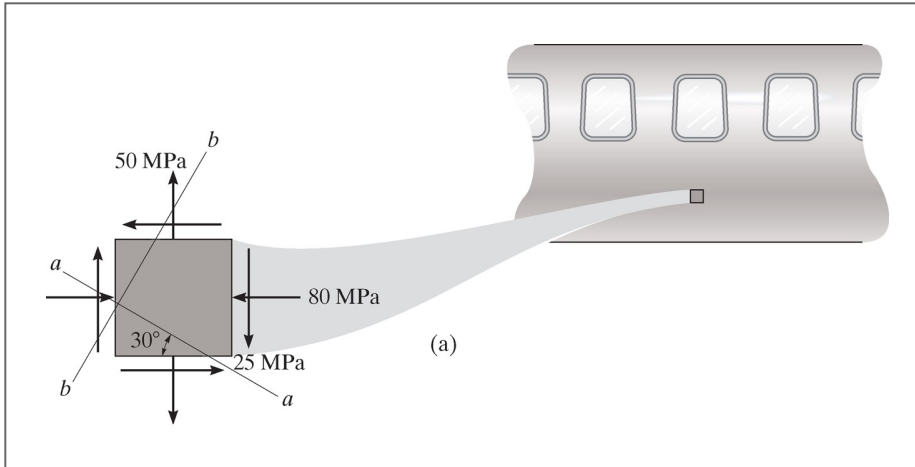
- If the state of stress ( $\sigma_x, \sigma_y, \tau_{xy}$ ) is known for a known axis system  $x$  and  $y$ , we can find the stress relative to some rotated coordinate system
- We do this by considering a section of the element perpendicular to the  $x'$  axis
- Sum of forces in  $x$  and  $y$  will give  $\sigma_{x'}$  and  $\tau_{x'y'}$
- A second section is needed to find  $\sigma_{y'}$ , perpendicular to the  $y'$  axis

# procedure





## example 9.1



Represent the state of stress shown on the fuselage section on an element rotated  $30^\circ$  clockwise from the position shown.

# general equations

## general equations

- We can follow the methodology from the previous section to develop equations for some arbitrary rotation and a completely general state of stress
- We use some trig identities to simplify the formulae

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

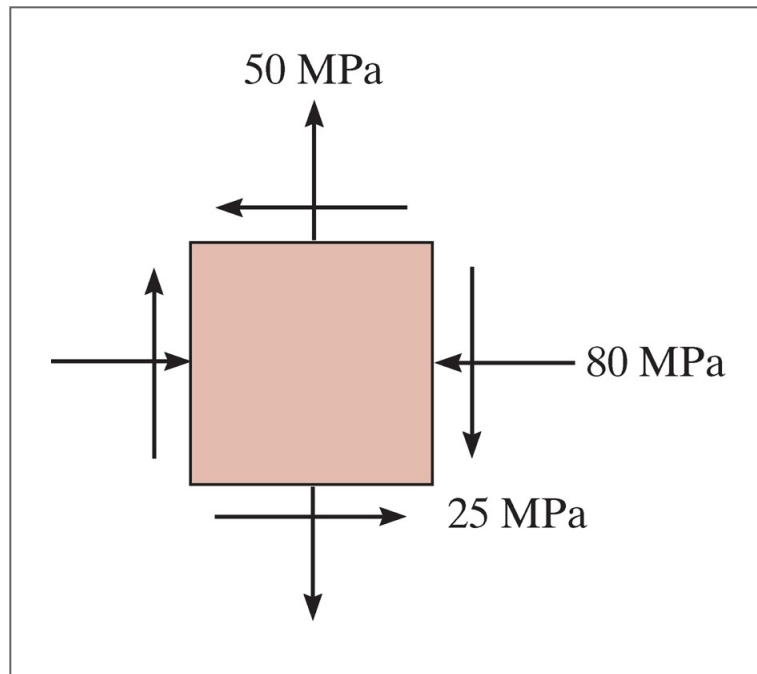
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

- To find  $\sigma_{y'}$  we can simply add  $90^\circ$  to  $\theta$

## procedure

- The procedure in general is mostly “plug and chug”
- The only thing we need to be cautious about is sign convention: stresses are positive in tension, shear is positive with arrows pointing to the 1st and 3rd quadrants,  $\theta$  is measured counter-clockwise from the  $x$ -axis

## example 9.2



Determine the stress at this point on an element rotated  $30^\circ$  clockwise from the position shown.

# principal stresses

## principal stresses

- Since the local stresses only change with the rotation angle, we might like to find the angle which gives the maximum stress
- This is known as the principal direction, and the stresses are known as principal stresses
- We can find this direction by differentiating the equation for  $\sigma_{x'}$

## principal stress

- We find the angle as

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- The principal stresses are then

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



## maximum shear stress

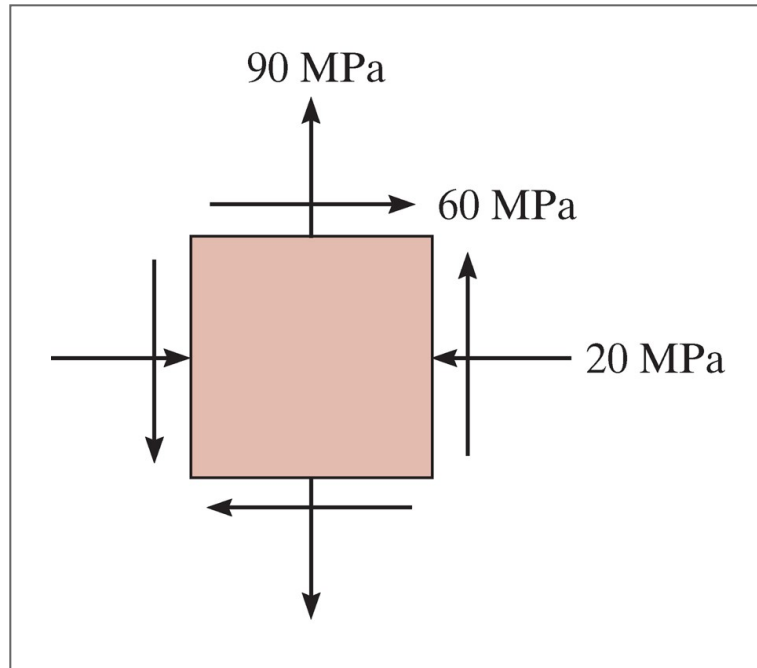
- Similarly, we might want to find the direction of maximum shear stress

$$\tan 2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$

- And the maximum shear stress is

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

## example 9.3



Find the principal stress for the stress state shown.

## example 9.5

- When torsional loading  $T$  is applied to a circular bar it produces a state of pure shear stress.
- Find the maximum in-plane shear stress and the associated average normal stress
- Find the principal stresses

# mohr's circle

## mohr's circle

- We can visualize plane stress transformation using a technique known as Mohr's circle
- If we re-write the stress transformation equations we find

$$\sigma_{x'} - \left( \frac{\sigma_x + \sigma_y}{2} \right) = \left( \frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$
$$\tau_{x'y'} = - \left( \frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

## mohr's circle

- If we square each equation and add them together, we find

$$\left[ \sigma_{x'} - \left( \frac{\sigma_x + \sigma_y}{2} \right) \right]^2 + \tau_{x'y'}^2 = \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2$$

## mohr's circle

- Since  $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$  are known constants, we can write a more compact form by letting

$$(\sigma_{x'} - \sigma_{avg})^2 + \tau_{x'y'}^2 = R^2$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

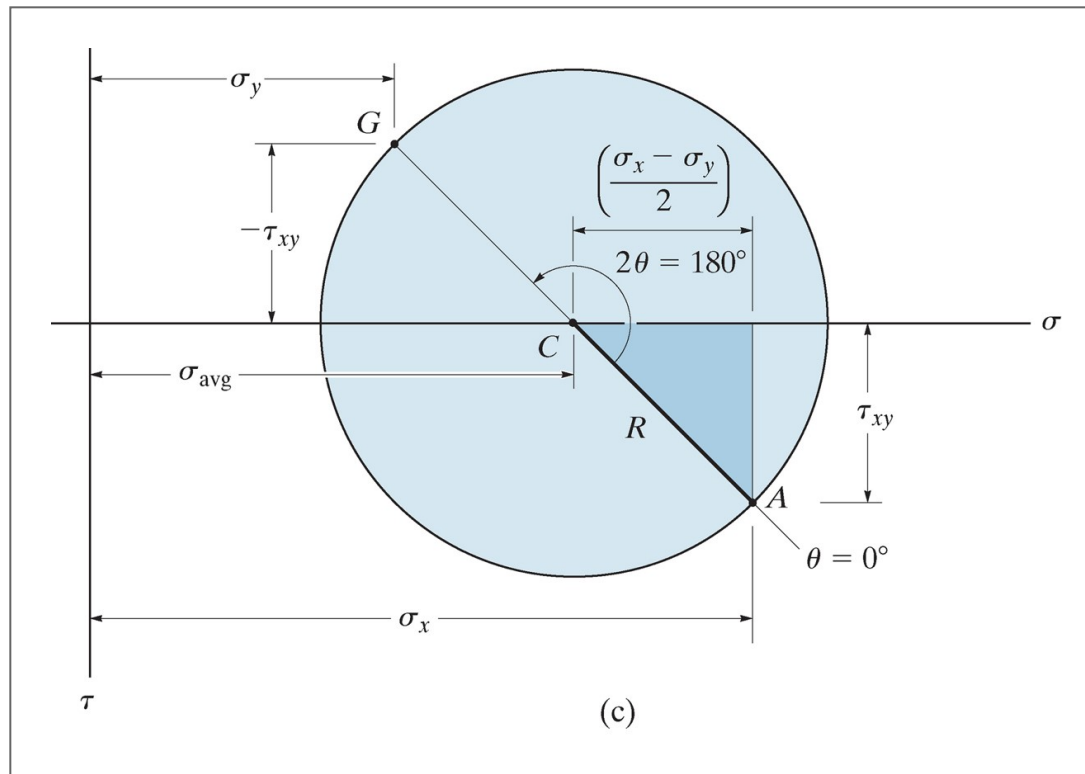
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

## mohr's circle

- Re-written in this way, we can see that the previous equation is the equation of a circle on the  $\sigma, \tau$  axis
- The center of the circle is at  $\tau = 0$  and  $\sigma = \sigma_{avg}$
- The radius of the circle is  $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- Each point on the circle represents  $\sigma_{x'}, \tau_{x'y'}$



# mohr's circle



## visual construction of Mohr's circle

- By convention, positive  $\tau$  points down, use this convention to plot the center of the circle and a reference point at  $(\sigma_x, \tau_{xy})$  where the  $x'$  axis is coincident with the  $x$  axis
- Use these two points to sketch the circle

## principal stress

- The principal stresses,  $\sigma_1$  and  $\sigma_2$  are the coordinates where Mohr's circle intersects the  $\sigma$  axis
- The angles  $\theta_{p1}$  and  $\theta_{p2}$  can be found by calculating the angle between the reference line and the  $\sigma$  axis (note that this angle is equal to  $2\theta_p$ )
- Note that the direction from the the reference point to the  $\sigma$  axis will be the same as the direction from the x axis to the principal axis

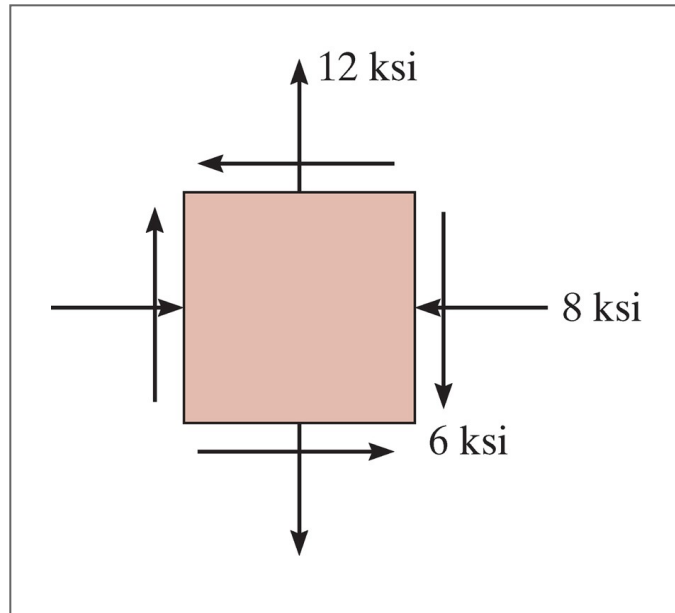
## maximum shear stress

- The top and bottom of the circle represent the maximum shear stress
- The angles  $\theta_{s1}$  and  $\theta_{s2}$  can be found in a similar method to that described for the principal stress

## stress on arbitrary plane

- To find the stress at some arbitrary plane some known angle  $\theta$  away from our reference plane, we find the angle  $2\theta$  away from the reference line on Mohr's circle
- We can use trigonometry to find the value of the coordinates at that point
- We must draw our angle in the same direction as the desired rotation

## example 9.9



Represent the state of stress shown on an element rotated  $30^\circ$  counterclockwise from the position shown.