AE333

Mechanics of Materials

Lecture 24 - Strain Transformation

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schedule

- 3 Apr - Strain Transformation, HW7 Due
- 5 Apr - Deflection of Beams
- 8 Apr - Deflection of Beams, HW8 Due
- 10 Apr - Deflection of Beams

outline

- mohr's circle
- absolute maximum shear
- plane strain
- principal strains and mohr's circle
- strain rosettes
- material property relationships

mohr’s circle

mohr's circle

- We can visualize plane stress transformation using a technique known as Mohr's circle
- If we re-write the stress transformation equations we find

$$\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right) = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

mohr's circle

- If we square each equation and add them together, we find

$$\left[\sigma_{x'} - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

mohr's circle

- Since σ_x, σ_y and τ_{xy} are known constants, we can write a more compact form by letting

$$(\sigma_{x'} - \sigma_{avg})^2 + \tau_{x'y'}^2 = R^2$$

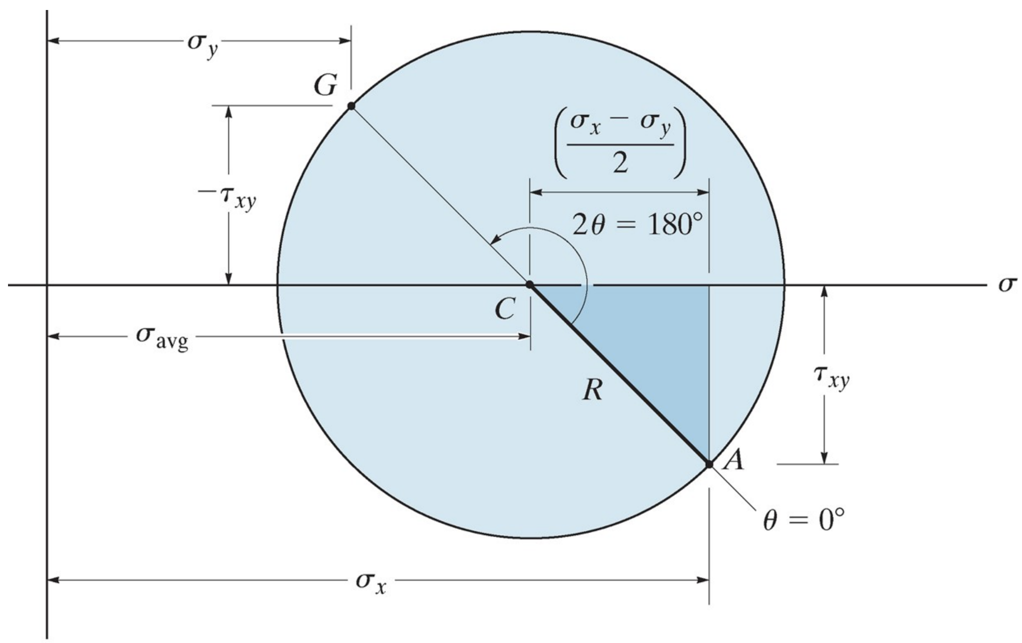
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

mohr's circle

- Re-written in this way, we can see that the previous equation is the equation of a circle on the σ, τ axis
- The center of the circle is at $\tau = 0$ and $\sigma = \sigma_{avg}$
- The radius of the circle is $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- Each point on the circle represents $\sigma_{x'}, \tau_{x'y'}$

mohr’s circle



(c)

visual construction of Mohr’s circle

- By convention, positive τ points down, use this convention to plot the center of the circle and a reference point at (σ_x, τ_{xy}) where the x' axis is coincident with the x axis
- Use these two points to sketch the circle

principal stress

- The principal stresses, σ_1 and σ_2 are the coordinates where Mohr's circle intersects the σ axis
- The angles θ_{p1} and θ_{p2} can be found by calculating the angle between the reference line and the σ axis (note that this angle is equal to $2\theta_p$)
- Note that the direction from the the reference point to the σ axis will be the same as the direction from the x axis to the principal axis

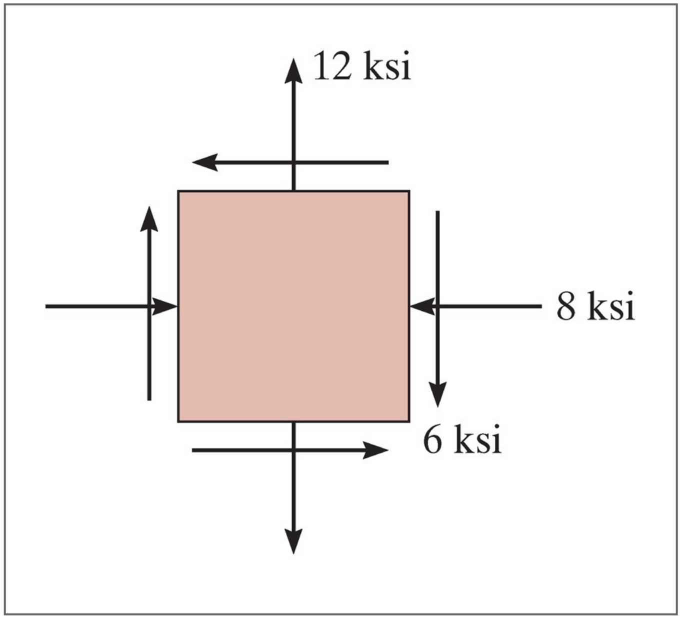
maximum shear stress

- The top and bottom of the circle represent the maximum shear stress
- The angles θ_{s1} and θ_{s2} can be found in a similar method to that described for the principal stress

stress on arbitrary plane

- To find the stress at some arbitrary plane some known angle θ away from our reference plane, we find the angle 2θ away from the reference line on Mohr's circle
- We can use trigonometry to find the value of the coordinates at that point
- We must draw our angle in the same direction as the desired rotation

example 9.9



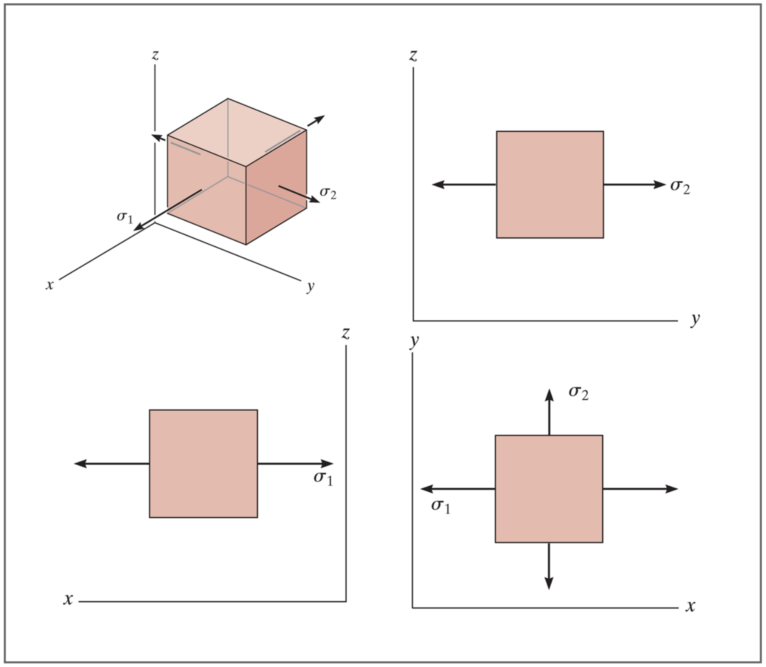
Represent the state of stress shown on an element rotated 30° counterclockwise from the position shown.

absolute maximum shear

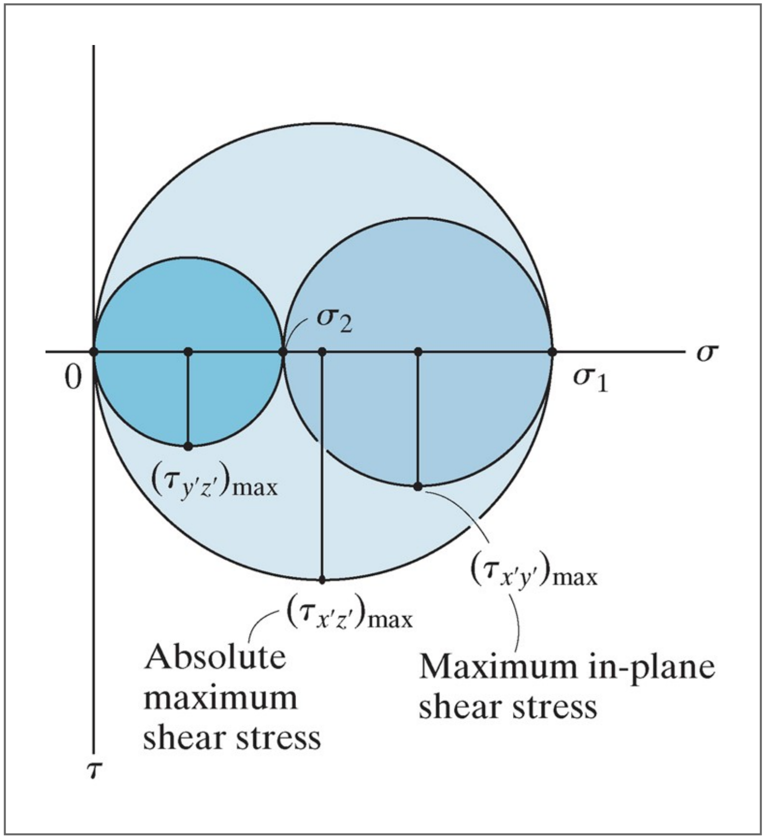
absolute maximum shear

- We already know how to find the maximum in-plane shear, but sometimes the maximum shear stress can occur in another plane
- We can do this (without treating it as a fully 3D problem) by treating each plane as its own plane stress problem

mohr’s circle



mohr's circle



absolute max shear

- The maximum absolute shear will depend on whether σ_1 and σ_2 are in the same or opposite directions

$$\tau_{abs,max} = \frac{\sigma_1}{2} \qquad \text{same direction}$$

$$\tau_{abs,max} = \frac{\sigma_1 - \sigma_2}{2} \qquad \text{opposite directions}$$

- Which of the three mohr's circles the maximum occurs in will determine which plane the shear acts in

plane strain

plane strain

- Under plane stress we assume no out-of-plane stresses are present
- This is typically a good assumption for very thin materials
- Under plane strain we assume no out-of-plane strains are present
- Typically a good assumption for very thick materials

sign convention

- Normal strains, ϵ_x and ϵ_y, are considered positive when they cause elongation, and negative when they cause contraction
- Shear strains, γ_{xy} are positive if the interior angle becomes smaller than $90°$, and negative if the angles becomes larger than $90°$

general equations

- We derive the general strain transformation equations by comparing infinitesimal elements before and after deformation
- To find $\gamma_{x'y'}$ we compare the angle between dx and dy before and after deformation

general equations

- The equations are nearly exactly the same as the stress transformation equations
- Pay attention to the difference, strain transformation equations are NOT on the equation sheet

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

- As with $\sigma_{y'}$, we find $\epsilon_{y'}$ by letting $\theta_y = \theta_x + 90°$

engineering strain

- Side note: there is another definition of shear strain known as tensorial shear strain, where $\gamma_{xy} = 2\epsilon_{xy}$
- Under tensorial strain, the transformation equations are exactly the same (as in this case both stress and strain are tensors)
- γ_{xy} is known as engineering strain, you will need to pay attention to which strain convention is used when extracting data from finite elements or other sources

principal strains and mohr’s circle

principal strains

- As you might imagine, since the transformation equations are nearly identical so are the principal strain equations

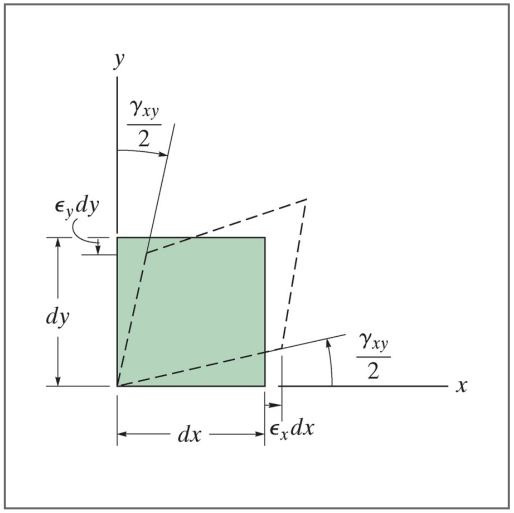
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

mohr's circle

- Mohr's circle can also be used in exactly the same way for strain as it is for stress
- The only difference is that the vertical axis is tensor strain, or $\gamma_{xy}/2$

example 10.4



The state of plane strain at a point has components of $\epsilon_x = 250\mu\epsilon$, $\epsilon_y = -150\mu\epsilon$, and $\gamma_{xy} = 120\mu\epsilon$. Determine the principal strains and the direction they act.

strain rosettes

rosettes

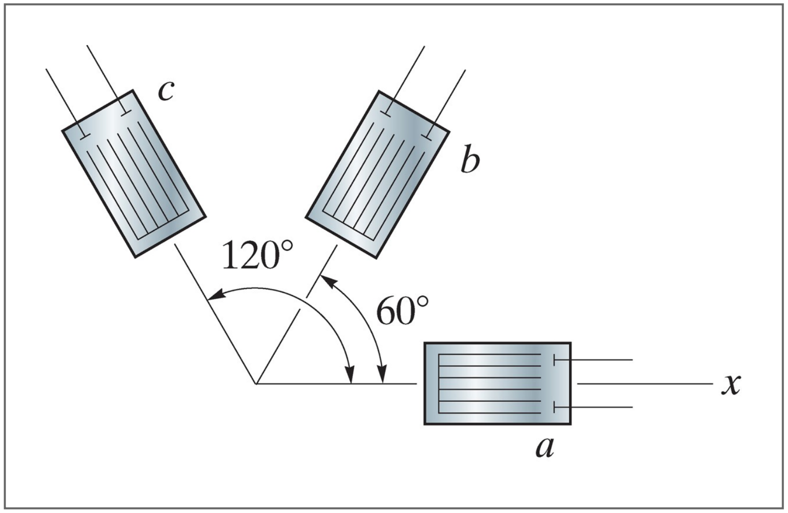
- Normal strain is fairly easy to measure using a strain gage
- Shear strain is more difficult to measure directly, so instead a "rossette" of normal strain gages is used
- We can use the strain transformation equations to determine τ_{xy}

rosettes

- Usually, we have $\theta_a = 0$, $\theta_b = 90$ and $\theta_c = 45$ OR $\theta_a = 0$, $\theta_b = 60$ and $\theta_c = 120$

$$\epsilon_a = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_a + \frac{\gamma_{xy}}{2}\sin 2\theta_a$$
$$\epsilon_b = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_b + \frac{\gamma_{xy}}{2}\sin 2\theta_b$$
$$\epsilon_c = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_c + \frac{\gamma_{xy}}{2}\sin 2\theta_c$$

example 10.8



The readings from the rosette shown are $\epsilon_a = 60\mu\epsilon$, $\epsilon_b = 135\mu\epsilon$ and $\epsilon_c = 264\mu\epsilon$. Find the in-plane principal strains and their directions.

material property relationships

generalized hooke's law

- We have previously used Hooke's Law in 2D, in 3D we have

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

generalized hooke's law

- And in shear

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$
$$\gamma_{yz} = \frac{1}{G}\tau_{yz}$$
$$\gamma_{xz} = \frac{1}{G}\tau_{xz}$$

dilatation

- When a material deforms it often changes volume
- The change in volume per unit volume is called “volumetric strain” or dilatation

$$e = \frac{\partial V}{dV} = \epsilon_x + \epsilon_y + \epsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

hydrostatic pressure

- One way of characterizing volumetric response is to apply hydrostatic pressure (equal compression on all sides with no shear)
- Under this case, we have

$$\frac{p}{e} = -\frac{E}{3(1-2\nu)}$$

- We call the term on the right (with no negative sign) the bulk modulus, k