AE333

Mechanics of Materials

Lecture 9 - Axial Load, Torsion

Dr. Nicholas Smith
Wichita State University, Department of Aerospace Engineering

February 13, 2019

schedule

- 15 Feb - Axial Load, Torsion
- 18 Feb - Torsion, HW3 Due
- 20 Feb - Torsion
- 22 Feb - Bending

outline

- superposition
- statically indeterminate
- force method
- thermal stress
- torsion

superposition

superposition

- Some problems are too complicated to solve all at once
- Instead, we break them up into two simpler problems
- Each "sub-problem" must still satisfy equilibrium
- Problem must be linear and the deformation should be small enough that it does not cause moment-equilibrium isssues

statically indeterminate

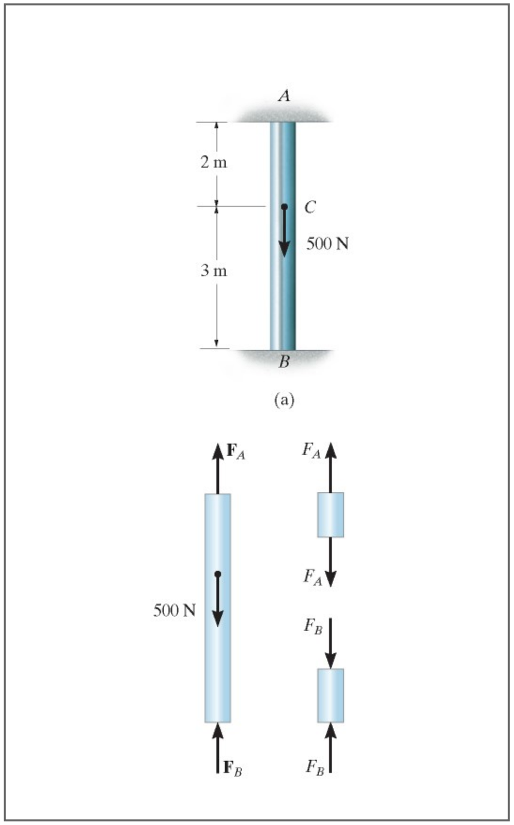
statically indeterminate

- There are many problems that are at least slightly over-constrained
- While this is common engineering practice, it creates too many variables for statics analysis
- These problems are called "statically indeterminate"

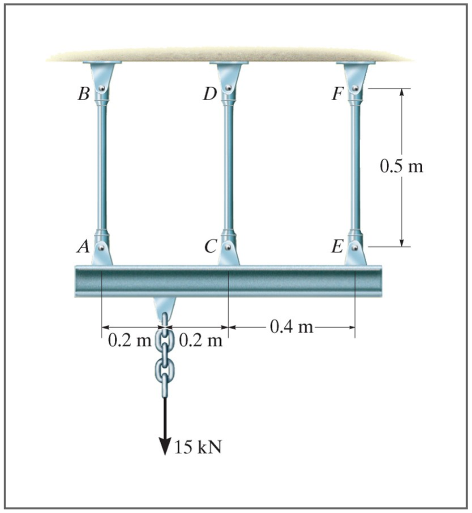
statically indeterminate

- One extra equation we can use is called "compatibility" or the "kinematic condition"
- We know that at the displacement must be equal on both sides of any arbitrary section we make in a member
- We can separate a member into two parts, then use compatibility to relate the two unknown forces

statically indeterminate



example 4.7



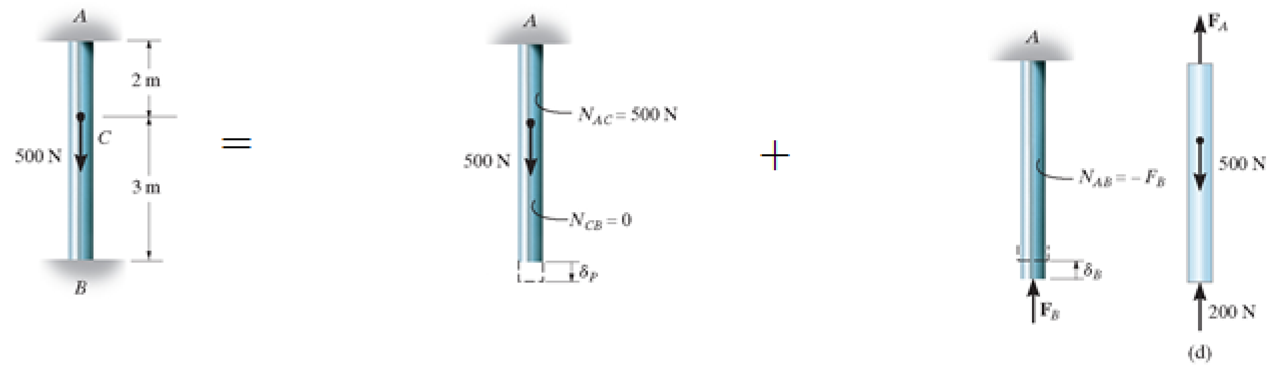
Assuming the bottom bar is rigid, find the force developed in each bar. AB and EF have cross-sectional areas of 50 mm^2 while CD has a cross-sectional area of 30 mm^2.

force method

force method

- One way to solve statically indeterminate problems is using the principle of superposition
- We choose one redundant support and remove it
- We then add it back as a force separately (without the other forces in the problem)

force method



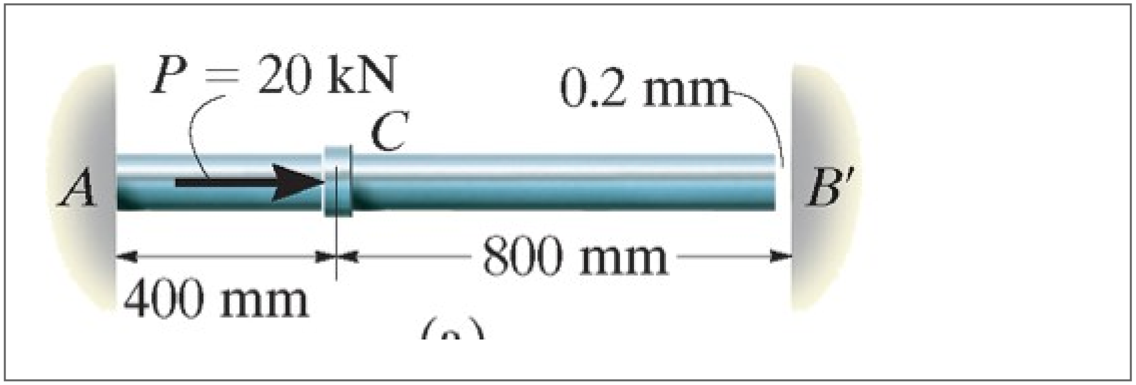
force method

- We connect the two problems by requiring that the displacement in both frames adds to 0 to meet the support requirements
- This is referred to as the equation of compatibility

procedure

- Choose one support as redundant, write the equation of compatibility
- Express the external load and redundant displacements in terms of load-displacement relationship
- Draw free body diagrams and use the equations of equilibrium to solve

example 4.9



The steel rod shown has a diamater of 10 mm. Determine the reactions at A and B'.

thermal stress

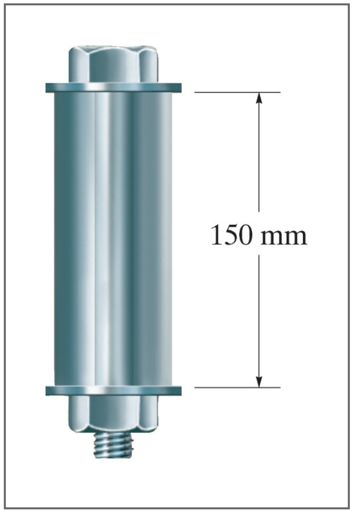
thermal stress

- A change in temperature cases a material to either expand or contract
- For most materials this is linear and can be described using the coefficient of linear expansion
  $\delta_T = \alpha \Delta T L$

thermal stress

- When a body is free to expand, the deformation can be readily calculated using
- If it is not free to expand, however, thermal stresses develop
- We can use the force method described previously to find the thermal stresses developed

example 4.12



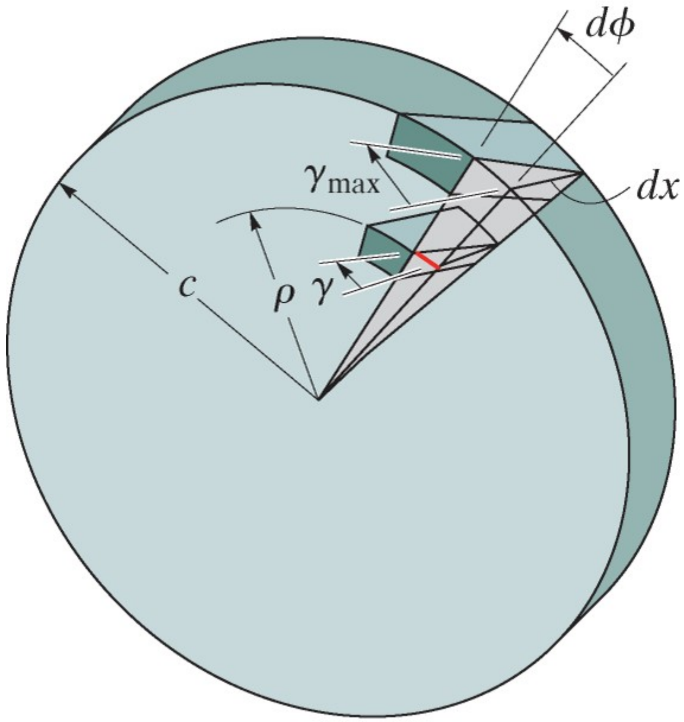
An aluminum tube with cross-section of 600 mm^2 is used as a sleeve for a steel bolt with cross-sectional area of 400 mm^2. When T=15 degrees Celsius there is negligible force and a snug fit, find the force in the bolt and sleeve when T=80 degrees Celsius.

torsion

torsion

- Torque is a moment that tends to twist a member about its axis
- For small deformation problems, we assume that the length and radius do not change signicantly under torsion
- The primary deformation we are concerned with in torsion is the angle of twist, denoted with ϕ

shear



The shear strain at points on the cross section increases linearly with ρ, i.e., $\gamma = (\rho/c)\gamma_{max}$.

torsion formula

- For a linearly elastic material, Hookeâ™s Law in shear will hold ($\tau = G\gamma$)
- This means that, like shear strain, shear stress will vary linearly along the radius

torsion formula

- We can find the total force on an element, dA by multiplying the shear stress by the area
  $dF = \tau dA$
- The torque ($dT = \rho dF$) produced by this force is then
  $dT = \rho(\tau dA)$

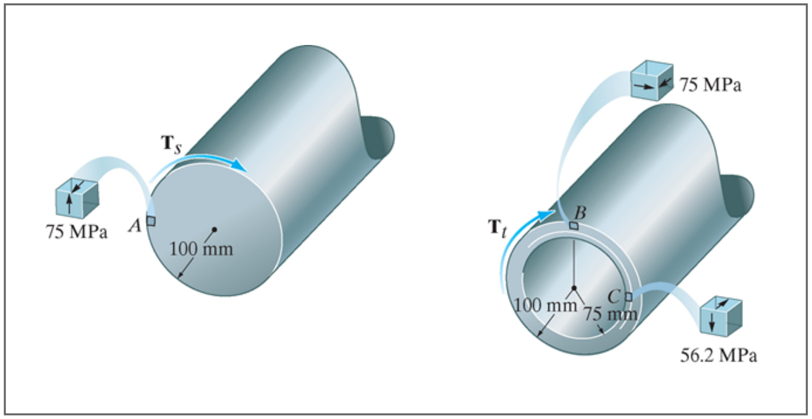
torsion formula

- Integrating over the whole cross-section gives
  $$T = \int_A \rho(\tau dA) = \frac{\tau_{max}}{c} \int_A \rho^2 dA$$
- The integral $\int_A \rho^2 dA$ is also called the Polar Moment of Inertia, J, so we can write
  $$\tau_{max} = \frac{Tc}{J}$$

polar moment of inertia

- We know that $J = \int_A \rho^2 dA$, so we can compute this for some common shapes
- For a solid circular cross-section, we have
  $$J = \int_0^c \rho^2 (2\pi\rho d\rho) = \frac{\pi}{2} c^4$$
- For a circular tube we have
  $$J = \int_{c_1}^{c_2} \rho^2 (2\pi\rho d\rho) = \frac{\pi}{2} (c_2^4 - c_1^4)$$

example 5.1



The allowable shear stress is 75 MPa. Determine the maximum torque that can be applied to each of the cross-sections shown and find the stress acting on a small element at A, B and C.