Continuum Mechanics

Lecture 17 - Non-Newtonian Fluids

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schedule

- 17 Nov - Non-Newtonian Fluids
- 1 Dec - Final Review, Research Project, HW9 Due
- 3 Dec - Final Review

outline

- non-newtonian fluids
- maxwell fluid
- other viscoelastic models
- dynamic properties
- time temperature superposition
- boltzman superposition
- experimental characterization

non-newtonian fluids

- For a fluid to be Newtonian we assumed that T'_{ij} is linearly dependent on D_{ij} and nothing else
- In many fluids T'_{ij} is not linearly dependent on D_{ij}
- These fluids are most commonly referred to as shear-thinning or shear-thickening fluids
- Their viscosity is a function of the strain-rate applied

shear thinning fluids

- Shear-thinning fluids decrease in viscosity with an increase in strain-rate
- Some examples are ketchup, paint, whipped cream, quicksand and blood
- Shear-thinning is generally only an effect in complex fluids (polymers and solutions)
- At the molecular level, research is still being done to discover why a fluid would be shear-thinning

shear-thickening fluids

- Shear-thickening fluids increase viscosity with an increased strain-rate
- Generally occurs in suspensions (many polymer melts are suspensions)
- Cornstarch and water, silica and polyethylene glycol
- Uses include body armor and brake pads

generalized newtonian fluids

- Many non-newtonian fluids have a viscosity which does not depend on the history, only on the current shear-rate
- In this case, the fluid can be modeled as a generalized Newtonian fluid
$$T'_{ij} = \mu(D_{ij})D_{ij}$$
- Some common models for $\mu(D_{ij})$ are Power-Law, Cross, and Carreau

linear viscoelasticity

- We discussed fluids where T'_{ij} is not linearly dependent on D_{ij}, but if T'_{ij} depends on some other measure (such as strain), the fluid is also non-newtonian
- Linear viscoelastic materials have a linear dependence on D_{ij}, but also depend on E_{ij}
- Viscoelasticity can be used to model behavior in both liquids (die swell in polymer melts) and solids (creep and stress relaxation)

maxwell fluid

- In general, a Maxwell fluid is defined by the constitutive equation

$$T_{ij} = -p\delta_{ij} + S_{ij}$$

- Where S is the "extra stress"

$$S_{ij} + \lambda \frac{\partial S_{ij}}{\partial t} = 2\mu D_{ij}$$

- In 1D, a Maxwell fluid can be considered as a spring and dashpot connected in series

maxwell fluid

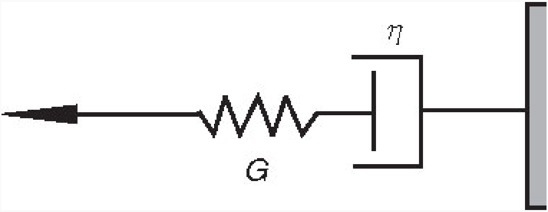


Figure 1: image

creep

- We can model the creep behavior of a 1D Maxwell fluid by considering a constant applied force, F_0
- For a 1D Maxwell fluid we have

$$F + \lambda \frac{dF}{dt} = \eta \frac{d\epsilon}{dt}$$

- If $F = F_0$, then $\frac{dF}{dt} = 0$ so we have

$$F_0 = \eta \frac{d\epsilon}{dt}$$

- So the strain will be given by

$$\epsilon = \int \frac{F_0}{\eta} dt = \frac{F_0}{\eta} t + \epsilon_0$$

creep

- Since ϵ_0 represents the initial strain, we can write $\epsilon_0 = F_0/G$ where G is the spring constant
- In shear the creep compliance is often called $J(t)$, and represents the strain divided by the applied force
- For this fluid the creep compliance function is

$$J(t) = \frac{\epsilon}{F_0} = \frac{1}{\eta} t + \frac{1}{G}$$

stress relaxation

- To model stress relaxation, we consider a constant applied strain, ϵ_0, and see what effect that has on the stress
- In this case $\frac{d\epsilon}{dt} = 0$ and we have

$$F + \lambda \frac{dF}{dt} = 0$$

- solving this differential equation gives

$$F = F_0 e^{-tG/\eta} = G\epsilon_0 e^{-tG/\eta}$$

- The stress relaxation function, $\phi(t)$ is F/ϵ_0 which gives

$$\phi(t) = Ge^{-tG/\eta}$$

notation

- Creep and stress relaxation work often uses a different notation than what we are accustomed to
- $J(t)$ is shear creep compliance, $D(t)$ is tensile creep compliance
- Compliance is most commonly used, but sometimes $G(t)$ for shear stiffness and $E(t)$ for tensile stiffness are used
- Due to the time history dependence, in general

$$G(t) \neq 1/J(t)$$

example

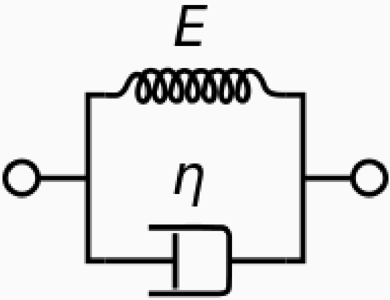
- A rod is initially 12" long and 2" in diameter
- Find the stretch after a 20 lb weight is hung for 24 hours
- Tensile compliance is given by

$$D = 0.5 - \exp(-0.03t)\ \text{Mpsi}^{-1}$$

solids

- A Maxwell fluid has an elastic portion, but behaves mostly like a fluid
- Many viscoelastic models are for solids, which have some viscous or damping behavior, but are mostly solid-like
- The Kelvin-Voigt model connects a spring and dashpot in parallel, and is the simplest form of viscoelastic model for solids

kelvin-voigt



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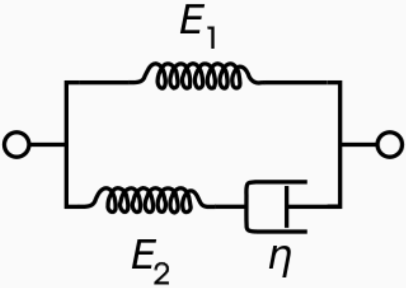
zener model

- Both the Maxwell and Kelvin-Voigt models are overly simple to describe many real viscoelastic materials
- The Zener model combines the two, it can be viewed either as a spring in series with a Kelvin-Voigt solid or as a spring in parallel with a Maxwell fluid
- Also called a standard linear solid
- Can be further extended for polymers with a distribution of relaxation times
- Multiple Kelvin-Voigt elements are connected in series, with a spring connected in series

zener model



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dynamic properties

- You have probably noticed that plastic cups do not ring as long as glasses or metal objects
- This is due to the damping properties of viscoelastic materials
- Polymers are often used to dampen vibrations for this reason
- Damping will vary with frequency, we can model the effects

dynamic properties

- If we assume some sinusoidal applied strain, $\epsilon(t) = \epsilon_0 \sin \omega t$
- In general, the viscoelastic stress response will be out of phase

$$\sigma = \sigma_0 \sin(\omega t + \delta)$$

- We can re-write this as

$$\sigma = (\sigma_0 \cos \delta) \sin \omega t + (\sigma_0 \sin \delta) \cos \omega t$$

dynamic properties

- We can further re-write the equation in terms of a so-called "storage modulus" and "loss modulus"

$$\sigma = \epsilon_0 [E' \sin \omega t + E'' \cos \omega t]$$

- The storage modulus, E' corresponds to the elastic response
- The loss modulus, E'' corresponds to the viscous response

complex representation

- The dynamic response is analogous to electric circuits, and can be expressed in a similar fashion using a complex representation

$$\epsilon^* = \epsilon_0 \exp i\omega t$$
$$\sigma^* = \sigma_0 \exp i(\omega t + \delta)$$

- This gives the complex modulus as

$$\begin{aligned} E^* &= \sigma^*/\epsilon^* \\ &= \frac{\sigma_0}{\epsilon_0}\exp i\delta \\ &= \frac{\sigma_0}{\epsilon_0}(\cos\delta + i\sin\delta) \\ &= E' + iE'' \end{aligned}$$

damping

- We can characterize the amount of damping in a viscoelastic material with δ

$$\tan\delta = \frac{D''}{D'} = \frac{E''}{E'}$$

- When $\delta = 0$ there is no viscous damping (most metals have $\delta \approx 0$)
- Polymers in certain temperature ranges can have δ as high as 30°
- Note: at the same temperature and frequency, $E^* = 1/D^*$ and $G^* = 1/J^*$

example

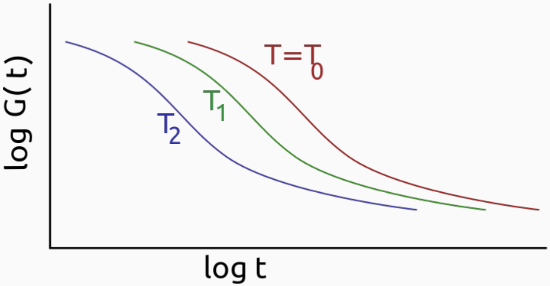
- Consider the same bar as before (12" long and 2" in diameter)
- The bar is subjected to a sinusoidal force with $F_0 = 50$ lbs and 80 Hz.

$$D'' = 40 \text{ ksi}$$

temperature dependency

- In both the time and frequency domain, many viscoelastic materials are temperature dependent
- In terms of our 1D models, the viscous portion is much more temperature dependent than the elastic portion
- The two most common models of temperature dependence are the Arrhenius model and the Williams-Landel-Ferry (WLF) model

time-temperature



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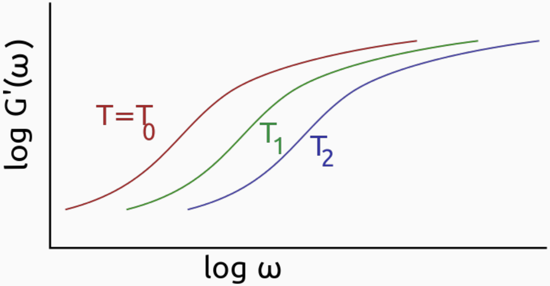
frequency-temperature



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temperature shift

- Both Arrhenius and WLF models us the parameter a_T as the temperature shift
- $a_T = a_T(T, T_0)$
- For compliance (J and D)

$$J^{T_0}(t) = J^T(a_T t)$$

- And for stiffness (G and E)

$$D^{T_0}(t) = D^T\left(\frac{t}{a_T}\right)$$

arrhenius

- The Arrhenius function only uses one curve-fit parameter

$$a_T = \exp\left(\frac{\Delta H}{R}\left[\frac{1}{T} - \frac{1}{T_R}\right]\right)$$

- R is the universal gas constant
- Theoretically, ΔH is the activation enthalpy of the relaxation
- T is the temperature and T_0 is the reference temperature
- Popular model because the constants have some physical meaning, but they are still curve fit

williams-landel-ferry

- The Williams-Landel-Ferry model is given by

$$\log(a_T) = \frac{-C_1(T - T_R)}{C_2 + (T - T_R)}$$

- C_1 and C_2 are the curve-fit parameters
- Most optimization algorithms perform poorly with exponential functions, so it is best to solve for $\log(a_T)$

characterization example

rendered version of example can be found at here

boltzman superposition

- What if we had one load applied at $t_0 = 0$, we would have

$$\epsilon_0 = \sigma_0 D(t)$$

- And some other load applied at t_1 would give

$$\epsilon_1 = \sigma_1 D(t - t_1)$$

- Thus the total strain would be:

$$\epsilon = \epsilon_0 + \epsilon_1 = \sigma_0 D(t) + \sigma_1 D(t - t_1)$$

boltzman superposition

- For N applied loads, we have

$$\epsilon(t) = \Delta\sigma_1 D(t - t_1) + \Delta\sigma_2 D(t - t_2) + ... + \Delta\sigma_N D(t - t_N)$$

- For some general, arbitrary loading function, this gives

$$\epsilon(t) = \int_0^t D(t - u)\frac{d\sigma}{du}du$$

example

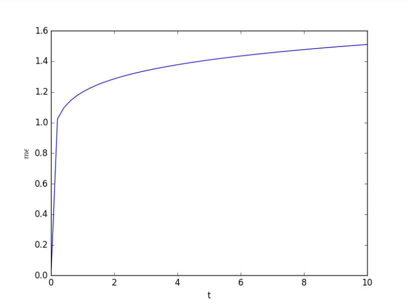
- For example, when we apply a load and then remove it, we have

$$\epsilon(t) = \sigma_0 D(t) - \sigma_0 D(t - t_1)$$

- If we consider $D = 1.2t^{0.1}$ GPa^{-1} and let $\sigma_0 = 1$ MPa and $t_1 = 1$ s, then we have

$$\epsilon(t) = \left(1.2t^{0.1} - 1.2(t-1)^{0.1}\right)$$

example



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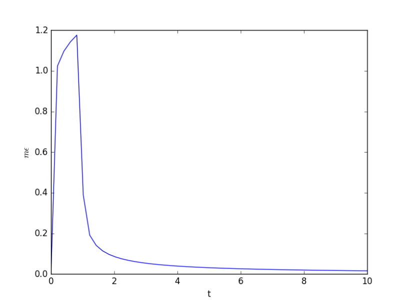
example



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experimental characterization

- In practice, both the dynamic properties and time-temperature superposition are often used to characterize viscoelastic materials
- Dynamic experiments are much faster than creep experiments, and can give the same information
- There are physical limitations to the frequencies that can be applied
- To go beyond those frequencies, a range of temperatures are tested
- This can typically be done using a Dyanmic Mechanical Analyzer (DMA)

dynamic mechanical analyzer

- To characterize a viscoelastic material in a DMA, tests are run over a range of frequencies
- At each frequency we measure both the stress and strain
- Plotting both, we can find the complex modulus (or compliance) at that frequency
- Over a range of frequencies this gives the complex modulus as a function of frequency
- Only a few cycles are needed to fit the curve

other considerations

- When creep and/or stress relaxation experiments must be done, time-temperature superposition can still be used
- Creep or relaxation experiments are run under various temperatures, as in our example