# Continuum Mechanics

Lecture 15 - Experimental Rheology

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# schedule

- 5 Nov - Experimental Rheology
- 10 Nov - Energy, Rotation, Vorticity
- 12 Nov - Compressible Flow, HW8 Due
- 17 Nov - Non-Newtonian Fluids

## plane couette flow

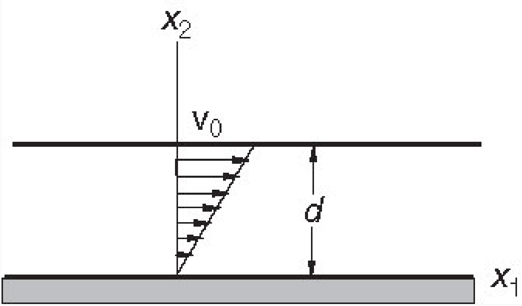



**Figure 1:** Plane Couette flow



## plane couette flow

- Steady unidirectional flow of an incompressible fluid between two horizontal plates with no pressure gradient in the flow direction is known as plane Couette flow
- one plate is fixed and the other moves with a constant velocity $v_0$
- From the continuity equation

$$v_i = \langle v(x_2), 0, 0 \rangle$$

- From Navier-Stokes we find

$$v(x_2) = \frac{v_0 x_2}{d}$$

## plane poiseuille flow

- Steady, unidirectional flow between two fixed plates
- Initial form for $v_i$ is same as plane Couette flow
- Navier-Stokes gives

$$\frac{\partial p}{\partial x_1} = \mu \frac{d^2 v}{dx_2^2}$$
$$\frac{\partial p}{\partial x_2} = 0$$
$$\frac{\partial p}{\partial x_3} = 0$$



## plane poisseuille flow

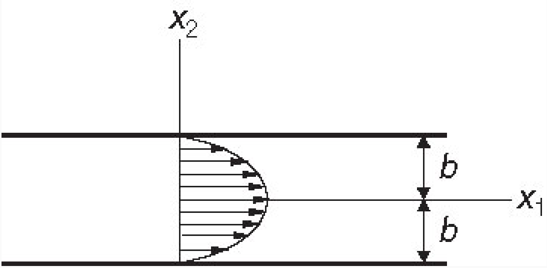



**Figure 2:** plane Poiseuille flow

## hagen-poiseuille flow

- Steady, unidirectional axisymmetric flow in a circular cylinder

$$v_r = v_\theta = 0, \qquad v_z = v(r)$$

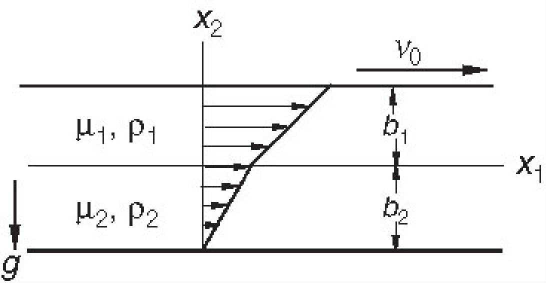
- From the Navier-Stokes equations, we find

$$\begin{aligned} \frac{\partial p}{\partial r} &= 0 \\ \frac{\partial p}{\partial \theta} &= 0 \\ -\frac{\partial p}{\partial z} + \mu \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{dv}{dr} \right) \right] &= 0 \end{aligned}$$



## example

- Find the veloc-
ity field if there are two fluids (in layers) in plane couette flow



## couette flow

- Laminar, steady flow of an incompressible fluid between two rotating coaxial cylinders is called Couette flow
- The velocity field has the form

$$v_r = 0, \qquad v_\theta = v(r), \qquad v_z = 0$$

- Continuity is automatically satisfied, Navier-Stokes gives

$$\frac{d^2 v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{v}{r^2} = 0$$



## couette flow

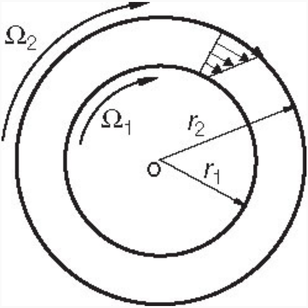



**Figure 3:** couette flow

## oscillating plane

- Flow near an oscillating plane will have the form

$$v_i = \langle v(x_2, t), 0, 0 \rangle$$

- The only non-trivial Navier-Stokes equation gives

$$\rho \frac{\partial v}{\partial t} = \mu \frac{\partial^2 v}{\partial x_2^2}$$

- Which has the solution

$$v = a e^{-\beta x_2} \cos(\omega t - \beta x_2 + \epsilon)$$



## rotational cylinder

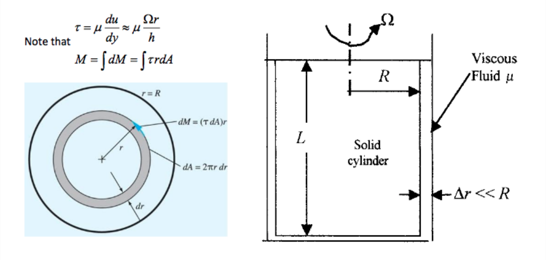



**Figure 4:** rotating cylinder viscometer

# rotational cylinder

- Couette flow with one of the cylinders fixed
- Shear rate is applied through a constant angular velocity of one of the cylinders
- Torque on the other cylinder is measured
- Used for fluids with very low viscosity



# cone and plate

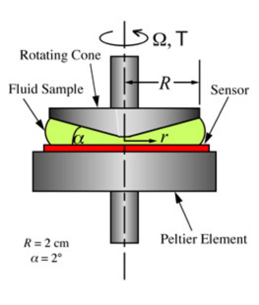



**Figure 5:** Cone and plate rheometer

## cone and plate

- One of the two plates is held fixed and torque is measured
- The other rotates at an applied angular velocity
- Cones are used to provide a constant shear rate, also use less fluid volume
- Parallel plates are more flexible in the spacing, also used for very temperature-sensitive tests



## capillary rheometer

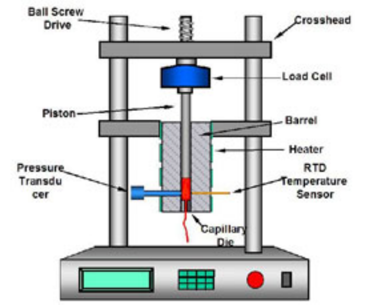



**Figure 6:** capillary rheometer

## capillary rheometer

- Used for testing higher shear-rates than rotational rheometers
- Commonly used for polymers (which are non-newtonian and have rate-dependent viscosity)
- Usually flow-rate is controlled and pressure drop is measured (but either can be controlled)
- Flow-rate can be converted to find the shear-rate, pressure drop can be converted to shear stress



## dynamic

- For materials which are viscoelastic, dynamic tests are used
- Either an oscillating rotational rheometer (for more liquid)
- Or an oscillating tensile test (for more elastic materials)

## extensional rheometry

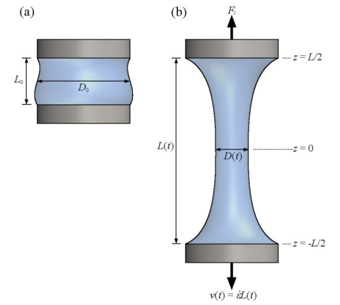



**Figure 7:** extensional rheometry



## extensional rheometry

- Another common test for polymer melts and viscoelastic materials
- Extensional viscosity is a function of both the applied stretch rate and the total deformation of the material
- Extrusion-based processes depend on extensional viscosity

## group 1

- Find velocity field of water pumped up a hill through a narrow channel
- Slope of channel is $45^\circ$
- Assume the only body forces present are due to gravity
- Note: solution will be in terms of pressure



## group 2

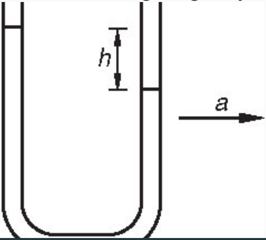
- Given the following polar coordinate velocity field

$$v_r = \frac{Q}{2\pi r} \qquad v_\theta = 0$$

- Find the streamline passing through some arbitrary point $(r_0, \theta_0)$ and the pathline originating from $(R, \Theta)$ at $t = 0$

## group 3

- A slender U-tube moves to the right with some acceleration, $a$.
- Find the relation between $a$, the width of the tube, $l$ and the difference in height levels of water at different ends of the tube, $h$
- Note: do not neglect gravity






## reading

- pp 375-402