Name:

Exam 1

1. Any coordinate transformation involving any combination of rotations can be expressed in terms of three successive rotations. For some second-order tensor, T_{ij}, write the coordinate transformation equations for T'_{ij}, T''_{ij} and T'''_{ij} and use these to find one effective transformation, Q^e_{ij}, which captures all three rotations, Q^1_{ij}, Q^2_{ij}, and Q^3_{ij}. Express the resulting expression for Q^e_{ij} in both index notation and matrix notation.

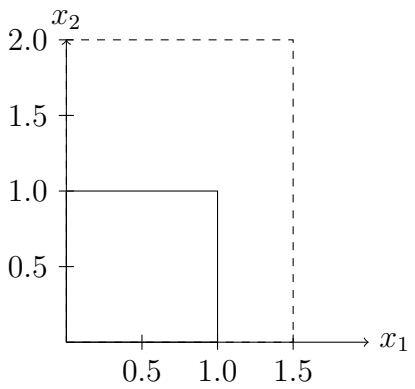

Figure 1: Biaxial tension for problem 2, deformation shown at $t = 1$ sec.

2. As part of a study on the crystallization of polymer films, you set up an experiment to study how a temperature field affects a thin film under biaxial tension.

   (a) Write equations for the motion shown in Figure 1.

   (b) The imposed temperature field is given as

   $$\Theta = x_1 x_2 \tag{1}$$

   find the time rate of change of temperature for the material point $X_i = (0.50, 0.50)$ after 0.5 seconds.

   (c) Find the time rate of change at the spatial point $x_i = (0.50, 0.50)$ after 0.5 seconds.

3. Given the velocity field

$$v_i = \langle \alpha x_1 t, \alpha x_2 t, 0 \rangle \tag{2}$$

determine how a fluid's density would vary in time if in a spatial description it is a function of time only.

4. For the following deformation gradient, use the polar decomposition theory to find the left stretch tensor, V_{ij} and show how the rotation tensor, R_{ij} is found.

$$F = \frac{1}{\sqrt{3}} \begin{bmatrix} 3 & 0 \\ \sqrt{2} & \sqrt{10} \end{bmatrix} \tag{3}$$